Effects of patent policy on growth and inequality: A perspective of exogenous and endogenous quality improvements

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Effects of patent policy on growth and inequality: A perspective of exogenous and endogenous quality improvements

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Abstract

In this study, we explore the effects of patent protection on growth and inequality under exogenous versus endogenous quality improvements. With an exogenous step size of quality improvement, strengthening patent protection promotes economic growth; the strengthening in patent protection has an ambiguous effect on income inequality but a negative effect on consumption inequality. However, with an endogenous step size, the growth effect of patent protection becomes ambiguous; the strengthening in patent protection still has an ambiguous effect on income inequality but a negative effect on consumption inequality. Under our calibrated parameter values, we find that strengthening patent protection raises the degree of income inequality under exogenous quality improvements. In the case of endogenous quality improvements, our results show that the strengthening in patent protection has an inverted-U effect on economic growth; both income inequality and consumption inequality decrease with the strength of patent protection.

JEL classification: D30, O30, O40

Keywords: innovation, patent protection, economic growth, inequality

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1. Introduction

It is a common belief that R&D is characterized by positive externality, resulting in the investment in R&D below its socially optimal level. Patent policy is an important tool for government to intervene in R&D activities.[1] As a result, a large number of studies have explored the effects of patent protection on innovation and economic growth.[2] Recently, the relationship between innovation and income inequality is receiving more attention (Grossman and Helpman, 2018; Jones and Kim, 2018; Aghion et al., 2019). Moreover, Saez and Zucman (2016) argue that many countries have experienced higher income inequality over the past few decades. Therefore, exploring how patent protection affects income inequality is also important for the assessment of patent policy. Several recent studies are devoted to this important issue (Chu and Cozzi, 2018; Chu et al., 2021; Kiedaisch, 2021).

The Schumpeterian quality-ladder models typically assume that the step size of innovation is constant (Grossman and Helpman, 1991; Aghion and Howitt, 1992; Chu and Cozzi, 2018; Yang, 2018). However, the assumption of exogenous quality improvements contradicts the empirical evidence that the step size and the value of innovations are not identical. For example, Silverberg and Verspagen (2007) and Minniti et al. (2013) suggest that the distribution of the innovation sizes is close to the Pareto or logarithmic distribution. Acemoglu and Cao (2015) and Akcigit and Kerr (2018) argue that large firms invest in incremental innovation, while small firms engage in more radical innovation. Given the above facts, it would be interesting to explore how the endogenous step size of innovation affect the effects of patent protection on economic growth and inequality.

Our study contributes to the literature by exploring the implications of patent protection on economic growth, income inequality, and consumption inequality under exogenous versus endogenous quality improvements. A Schumpeterian model featuring sequential innovations and heterogeneous households is established. To introduce household heterogeneity, we assume that households possess different levels of assets as in Chu and Cozzi (2018) and Chu et al. (2021). Therefore, in the economy, different levels of asset income are the source of income and consumption inequality.[3] Each intermediate industry is temporarily dominated by a monopolistic industry leader until the next innovation arrives. The current industry leader holds a patent on the latest

[1] See Sampat (2018) for a survey of empirical evidence on patent protection and innovation.
[2] See Becker (2015) for a survey of this strand of literature.
[3] See Atkinson (2000, 2003) and Piketty (2014) for empirical evidence that unequal asset income has a substantial impact on the degree of income inequality.

innovation but infringes the patent of the previous industry leader.[4] As a result of this patent infringement, the current industry leader must pay a licensing fee to the previous industry leader. In line with O'Donoghue and Zweimuller (2004) and Chu and Pan (2013), we assume that the current industry leader transfers a share of its profits to the previous industry leader and the profit-division ratio decreases with the step size of innovation.[5] Obviously, with an endogenous step size, the profit-division ratio between the most and second most recent innovators is also endogenously determined.

Within this theoretical framework, we arrive at some new findings. In an environment with sequential innovations and exogenous quality improvements, strengthening patent protection raises the arrival rate of innovation and promotes economic growth. Given the growth effect is positive, the strengthening in patent protection increases the real interest rate and thus leads to a higher asset income, which is the source of income inequality. Accordingly, strengthening patent protection has a positive *interest-rate* effect on income inequality through the real interest rate. However, strengthening patent protection will increase the value of monopolistic producers and thus raise the real wage rate. Therefore, the strengthening in patent protection also carries a negative *asset-value* effect on income inequality by decreasing the asset-to-wage ratio. The above two opposing forces give rise to an ambiguous effect of patent protection on income inequality when the quality step size is exogenous. In contrast, the strengthening in patent protection has only a negative asset-value but no interest-rate effect on consumption inequality. As a result, in this case, the degree of consumption inequality decreases with the strength of patent protection.

However, in the case of endogenous quality improvements, strengthening patent protection increases the arrival rate of innovation but decreases the quality step size. The reason is that a higher innovation rate increases the expected return of an R&D firm, which makes it willing to invest in innovation with a smaller step size. Therefore, the overall effect of patent protection on economic growth becomes ambiguous. Consequently, the strengthening in patent protection also generates an ambiguous interest-rate effect on income inequality. Furthermore, with an endogenous step size, the asset-value effect remains negative. As a result, in this case, the microeconomic effect of patent protection on income inequality is generally ambiguous. Moreover, as in the case of exogenous quality improvements, strengthening patent protection has

[4] Due to Arrow's replacement effect, the most and second most recent innovations are owned by different innovators; for a discussion of the Arrow effect, see Cozzi (2007).

[5] This setup captures the fact that investing in more radical innovation reduces the chance of infringement. As a result, the current quality leader is less likely to be required to pay a licensing fee.

only a negative asset-value effect on consumption inequality. Thus, with an endogenous step size of innovation, consumption inequality is decreasing in the strength of patent protection.

We also calibrate the model to quantify the growth and inequality effects of patent protection. Under our calibrated parameter values, we find that strengthening patent protection stimulates economic growth when the step size of innovation is exogenous. As for the microeconomic implications of patent protection on inequality, we find that strengthening patent protection raises income inequality but reduces consumption inequality. However, in the case of endogenous step size of innovation, our results show that strengthening patent protection generates an inverted-U effect on economic growth. Moreover, in this case, both income and consumption inequality decrease with the strength of patent protection.

Literature review

This study is associated with the literature on quality improvements and economic growth; see Grossman and Helpman (1991) and Aghion and Howitt (1992) for pioneering works and Aghion et al. (2014) for a survey of this literature.[6] Several subsequent studies, such as Bessen and Maskin (2009), Cozzi and Galli (2014), and Yang (2018), explore the relationship between quality improvements and economic growth in the Schumpeterian economy with sequential innovations. However, all the studies mentioned above assume an exogenous step size of quality improvements. One important exception is Chu and Pan (2013), who extend the Grossman and Helpman (1991) model by allowing for endogenous step size to analyze the impact of different patent instruments on growth. A recent study by Hu et al. (2021) explores the macroeconomic effect of inflation on economic growth in a Schumpeterian economy with an endogenous step size of innovation. This study contributes to this literature by developing a Schumpeterian growth model with sequential innovations and heterogeneous households. More importantly, this model is flexible enough to allow us to consider both exogenous and endogenous quality improvements.

This study also relates to the literature on the effects of patent policy on R&D and economic growth. The pioneering study by Judd (1985) analyzes the impact of patent length on innovation and economic growth and argues that an infinite patent length is optimal. Subsequent studies, such as Iwaisako and Futagami (2003), Futagami and

[6] For other seminal studies on R&D-based endogenous economic growth, see also Romer (1990), Segerstrom et al. (1990), Jones (1995), and Peretto (1998).

Iwaisako (2007), and Acemoglu and Akcigit (2012), also explore the relationship between patent length and R&D. Moreover, an earlier study by O'Donoghue and Zweimuller (2004) discusses the effects of an alternative patent instrument, patentability requirement, on innovation and economic growth.[7] Instead of patent length and patentability requirement, we consider patent breadth as Li (2001), who finds that increasing patent breadth stimulates R&D and promotes economic growth. Following Li (2001), a large number of studies, such as Goh and Olivier (2002), Kwan and Lai (2003), Furukawa (2007), Chu and Furukawa (2011), Cysne and Turchick (2012), Iwaisako and Futagami (2013), Saito (2017), Chu and Cozzi (2018), Iwaisako (2020), and Yang (2021), also explore the effects of patent breadth within variants of the Schumpeterian growth model.[8] The present paper complements this strand of literature by investigating the growth effect of patent protection in a quality-ladder model with sequential innovations and providing a comparison of the effects of patent protection under exogenous and endogenous quality improvements. Given that few studies examine the effects of patent protection with an endogenous step size, a novel contribution of this paper is to find that strengthening patent protection may generate an inverted-U effect on economic growth under endogenous quality improvements.

This study also relates to the strand of literature on innovation and inequality. Some studies explore wage inequality in R&D-based models; see, for example, Acemoglu (1998, 2002), Spinesi (2011), and Cozzi and Galli (2014). Instead of wage inequality, studies by Chou and Talmain (1996), Zweimuller (2000), Foellmi and Zweimuller (2006), Grossman and Helpman (2018), Jones and Kim (2018), and Aghion et al. (2019) focus on the relationship between innovation and income inequality. Moreover, several recent studies explore how government policies affect economic growth and income inequality. For instance, Chu et al. (2019), Zheng (2020), and Zheng et al. (2020) incorporate heterogeneous households and money demand in R&D-based growth models to analyze the impact of monetary policy on innovation and income inequality. The present paper contributes to this literature by exploring the effects of patent protection rather than monetary policy on income inequality as well as consumption inequality.

Studies by Chu and Cozzi (2018), Chu et al. (2021), and Kiedaisch (2021) also explore the effects of patent protection on growth and inequality. Kiedaisch (2021)

[7] See also Kiedaisch (2015), who argues that raising patentability requirement causes a negative effect on innovation.
[8] Some studies examine the effects of blocking patents, see, for instance, Chu (2009), Chu and Pan (2013), Cozzi and Galli (2014), Yang (2018). To focus on the effect of patent breadth under exogenous and endogenous quality improvements, we do not consider blocking patents in this paper.

explores the implications of patent protection in a variety-expanding model with hierarchical preferences. Unlike Kiedaisch (2021), Chu and Cozzi (2018) model household heterogeneity by assuming that they own different levels of wealth to analyze the effects of patent protection on growth and equality. Chu et al. (2021) explore the dynamic effects of patent protection on inequality in a Schumpeterian model featuring both horizontal and vertical R&D. We complement their studies by investigating the implications of patent protection in an environment with sequential innovations. More importantly, Chu and Cozzi (2018) and Chu et al. (2021) assume that the step size of innovation is exogenous whereas our analysis considers both exogenous and endogenous quality improvements.

The rest of this paper proceeds as follows. Section 2 presents the model. Section 3 analyzes the effects of patent protection under exogenous quality improvements. In Section 4, we consider the case of endogenous quality improvements. The final section concludes.

2. The model

To investigate the effects of patent protection on growth and inequality, we extend the seminal growth model in Grossman and Helpman (1991) by (i) introducing heterogeneous households owning different levels of wealth as in Chu and Cozzi (2018) and Chu et al. (2021), (ii) incorporating patent protection which determines the market power of monopolistic intermediate-goods producers as in Goh and Olivier (2002) and Iwaisako and Futagami (2013), and (iii) considering a profit-division rule between sequential innovators as in O'Donoghue and Zweimuller (2004) and Chu and Pan (2013). Throughout this study, we choose the final good as the numeraire. To conserve space, we describe the standard features of the model briefly.

2.1. Households

There is a unit continuum of heterogeneous households indexed by $h \in [0,1]$. These households own different levels of wealth but have identical preferences over consumption $c_t(h)$. Household h's lifetime utility function is given by

$$U(h) = \int_0^\infty e^{-\rho t} \ln c_t(h)\, dt, \tag{1}$$

where $\rho > 0$ denotes the subjective discount rate. Household h maximizes utility subject to an asset-accumulation equation given by

$$\dot{a}_t(h) = r_t a_t(h) + w_t - c_t(h). \tag{2}$$

$a_t(h)$ represents the amount of wealth owned by household h, and r_t is the real interest rate. Household h inelastically supplies one unit of labor to earn a real wage w_t. Standard dynamic optimization yields the familiar Euler equation given by

$$\frac{\dot{c}_t(h)}{c_t(h)} = r_t - \rho . \tag{3}$$

2.2. Final good

The unique final good (numeraire) is produced by competitive firms using a Cobb-Douglas aggregator given by

$$y_t = \exp\left\{\int_0^1 \ln x_t(i)\, di\right\}, \tag{4}$$

where $x_t(i)$ is the quantity of intermediate good $i \in [0,1]$. From Profit maximization, the conditional demand function for intermediate good i is

$$x_t(i) = \frac{y_t}{p_t(i)}, \tag{5}$$

where $p_t(i)$ denotes the price of intermediate good i.

2.3. Intermediate goods

There is a continuum of intermediate industries indexed by $i \in [0,1]$, which produce differentiated intermediate products. In each industry, there is a monopolistic industry leader who holds a patent on the most recent innovation and temporarily dominates the market until the next innovation arrives. The production function of the leader in industry i is

$$x_t(i) = z^{q_t(i)} l_{x,t}(i), \tag{6}$$

where $z > 1$ represents the step size of quality improvements and $q_t(i)$ denotes the number of quality improvements that have occurred in industry i. $l_{x,t}(i)$ represents the labor employed to produce intermediate good i. Given $z^{q_t(i)}$, the marginal production cost in industry i is given by

$$MC_t(i) = \frac{w_t}{z^{q_t(i)}} . \tag{7}$$

To maximize profit, the industry leader charges a constant markup over this marginal cost. In the quality-ladder models, the Bertrand competition between current and previous industry leaders leads to an unconstrained profit-maximizing markup ratio that is determined by the step size z. To analyze the impact of patent policy, we assume that the markup ratio is equal to the level of patent protection $\mu < z$, which is set by the government, as in prior studies such as Goh and Olivier (2002), Iwaisako and Futagami (2013), Chu and Cozzi (2018), and Yang (2018). As a result, the profit-maximizing price is given by $p_t(i) = \mu MC_t(i)$. Then, in industry i, the monopolistic producer's profit and production cost are respectively given by

$$\pi_t(i) = p_t(i)x_t(i) - \frac{1}{\mu}p_t(i)x_t(i) = \frac{\mu-1}{\mu}y_t, \tag{8}$$

$$w_t l_{x,t}(i) = \frac{1}{\mu}p_t(i)x_t(i) = \frac{1}{\mu}y_t. \tag{9}$$

(8) and (9) imply that $\pi_t(i) = \pi_t$ and $l_{x,t}(i) = l_{x,t}$, respectively. Therefore, industry leaders employ the same amount of labor and obtain the same amount of profit.

2.4. R&D

In each industry, the most recent innovator (i.e., the current industry leader) infringes the patent of the second most recent innovator (i.e., the previous industry leader). As a result of this patent infringement, the most recent innovator needs to transfer a share $s \in (0,1)$ of the monopolistic profit to the previous innovator as a licensing fee. In line with Chu and Pan (2013), the profit-division rule is given by $s = \beta/z$, where $\beta \in (0, z)$ determines the previous innovator's bargaining power. As is obvious, a larger step size z results in the most recent innovator paying a smaller licensing fee to the previous innovator. This setup captures the fact that an innovation that is more different from previous innovations is less likely to result in patent infringement. We focus on the symmetric equilibrium[9] and denote by $V_{2,t}(i)$ the value of the second most recent innovation in industry i. Since industry leaders obtain the same amount of profit, we have $V_{2,t}(i) = V_{2,t}$. The no-arbitrage condition for $V_{2,t}$ is then given by

$$r_t V_{2,t} = s\pi_t + \dot{V}_{2,t} - \lambda_t V_{2,t}, \tag{10}$$

where λ_t denotes the Poisson arrival rate of quality improvements. The right-hand side of (10) is the sum of three terms. $s\pi_t$ is the licensing fee received by the previous innovator, and $\dot{V}_{2,t}$ represents the potential capital gain. The last term, $-\lambda_t V_{2,t}$, denotes the expected value loss due to creative destruction (at the rate λ_t, the next innovation arrives and thus the previous industry leaser loses its claim to the profit).

[9] In a symmetric equilibrium, innovation arrival rates are equal across industries. See Cozzi et al. (2007) for a detailed discussion.

Similarly, the no-arbitrage condition for the value of the most recent innovation $V_{1,t}$ is

$$r_t V_{1,t} = (1-s)\pi_t + \dot{V}_{1,t} - \lambda_t (V_{1,t} - V_{2,t}). \tag{11}$$

$(1-s)\pi_t$ and $\dot{V}_{1,t}$ denote the profit share of the current industry leader and the capital gain, respectively. The third term, $-\lambda_t (V_{1,t} - V_{2,t})$, represents the expected value loss resulting from the most recent innovator becoming the second most recent innovator.

At any time, there is a unit continuum of potential entrants (i.e., R&D firms). They invest in R&D to improve the quality of existing intermediate goods that they do not currently own. When an R&D firm's innovation is successful, the firm will enter the market and become the new industry leader. The innovation arrival rate of an R&D firm is given by

$$\lambda_t = \frac{\varphi l_{r,t}}{z}, \tag{12}$$

where $l_{r,t}$ is the labor used for R&D and $\varphi > 0$ determines the R&D productivity. (12) indicates that the arrival rate λ_t is decreasing in the step size z, which captures the effect that more radical innovations are less likely to succeed. Then, an R&D firms' expected return is given by $\pi_{r,t} = \lambda_t V_{1,t} - w_t l_{r,t}$. Combining this expression and (12), we obtain the zero-expected profit condition given by[10]

$$\frac{\varphi V_{1,t}}{z} = w_t. \tag{13}$$

(13) determines the allocation of labor inputs between intermediate goods production and R&D investment.

2.5. Equilibrium

The decentralized equilibrium consists of a time path of allocations $\{c_t(h), a_t(h), y_t, x_t(i), l_{x,t}, l_{r,t}\}_{t=0}^{\infty}$ and a time path of prices $\{p_t(i), w_t, r_t, V_{1,t}, V_{2,t}\}_{t=0}^{\infty}$. Also, at each instance of time,

- households maximize lifetime utility taking $\{w_t, r_t\}$ as given;
- competitive final-good firms produce y_t and choose $x_t(i)$ to maximize profits taking $p_t(i)$ as given;
- the monopolistic industry leader in industry i produce intermediate good $x_t(i)$ and choose $\{p_t(i), l_{x,t}\}$ to maximize profit taking w_t as given;
- each R&D firm employs an amount $l_{r,t}$ of labor to maximize expected

[10] The free entry of potential entrants implies that the expected profit $\pi_{r,t}$ must be equal to zero.

revenue taking $\{w_t, V_{1,t}\}$ as given;

- the market for final good clears such that $y_t = c_t$;
- the market for labor clears such that $l_{x,t} + l_{r,t} = 1$;
- the market for assets clears such that the value of assets owned by households is equal to the value of all monopolistic firms: $\int_0^1 a_t(h)\,dh = V_{1,t} + V_{2,t}$.

2.6. Aggregation

Substituting (6) into (4) to obtain the aggregate production function for the final good given by

$$y_t = \exp\left\{\int_0^1 q_t(i)\ln z di\right\} l_x. \tag{14}$$

Following Chu and Cozzi (2018) and Chu et al. (2019), we define the level of aggregate technology as

$$Z_t = \exp\left\{\int_0^1 q_t(i)\ln z di\right\}. \tag{15}$$

Taking the logarithm of Z_t yields

$$\ln Z_t = \left(\int_0^1 q_t(i)\,di\right)\ln z = \left(\int_0^t \lambda_\tau d\tau\right)\ln z, \tag{16}$$

where the second equality follows from the law of large numbers. Differentiating (16) with respect to time and using $y_t = c_t$ and (14), we obtain

$$g = \frac{\dot{Z}_t}{Z_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = \lambda \ln z. \tag{17}$$

As a result, the long-run economic growth rate g is determined by the innovation arrival rate λ and the step size z. Hereafter, we focus on the balanced growth path (BGP). From (8)-(10) and (13), we can show that along the BGP, $\dot{V}_1/V_1 = \dot{V}_2/V_2 = g$. Therefore, from (10), we can derive the value V_2 as

$$V_2 = \frac{s\pi}{r-g+\lambda} = \frac{s\pi}{\rho+\lambda}, \tag{18}$$

where the second equality uses the Euler equation (3). Similarly, from (11), we can derive the value V_1 as

$$V_1 = \frac{(1-s)\pi}{r-g+\lambda} + \frac{\lambda V_2}{r-g+\lambda} = \frac{(1-s)\pi}{\rho+\lambda} + \frac{\lambda V_2}{\rho+\lambda}. \tag{19}$$

In (19), both λ and V_2 are determined by the next innovator rather than the current industry leader.

3. Patent, growth, and inequality: Exogenous quality improvements

In this section, we discuss how does patent protection affects economic growth and inequality under an exogenous step size of quality improvements. Subsections 3.1 and 3.2 explore the macroeconomic impact of patent policy on economic growth and the microeconomic impact on inequality, respectively. In Subsection 3.3, we provide a quantitative analysis for this case.

3.1. Effects of patent protection on growth

From (9) and (13) we immediately obtain $\varphi V_1/z = y/\mu l_x$. Substituting (8), (18) and (19) into this equation yields

$$\frac{\varphi(\mu-1)}{z(\rho+\lambda)}\left(1-s+\frac{\lambda s}{\rho+\lambda}\right)=\frac{1}{l_x}, \tag{20}$$

which determines the labor for the production of intermediate goods. Then, we substitute $l_x = 1-l_r$ and $l_r = \lambda z/\varphi$ into (20) to obtain

$$(\mu-1)\left[\left(1-\frac{\beta}{z}\right)\rho+\lambda\right]=\frac{(\rho+\lambda)^2}{\varphi/z-\lambda}. \tag{21}$$

Obviously, the left-hand side (LHS) of (21) is a linear and increasing function of the arrival rate λ while the right-hand side (RHS) of (21) is a convex and increasing function of the arrival rate λ. To ensure that there is a unique $\lambda > 0$ that satisfies (21), we impose the following parameter restriction.

$$\text{Condition } \alpha: \ \varphi > \frac{z^2\rho}{(\mu-1)(z-\beta)}.$$

Under Condition α, the inequality $LHS\,|_{\lambda=0} = (\mu-1)(1-\beta/z)\rho > z\rho^2/\varphi = RHS\,|_{\lambda=0}$ holds. Then, the unique intersection of the left-hand and right-hand sides of (21) determines the equilibrium innovation arrival rate λ^*. Moreover, an increase in μ shifts up the LHS of (21), resulting in a higher arrival rate λ^*; see Figure 1 for an illustration. With an exogenous step size, the equilibrium economic growth rate $g^* = \lambda^* \ln z$ also increases with the level of patent protection μ. This is the traditional positive effect of strengthening patent protection; see, for example, Li (2001), Chu (2010), Chu and Cozzi (2018), and Yang (2018). We summarize this result below.

Proposition 1. *In an environment characterized by sequential innovations and exogenous quality improvements, strengthening patent protection increases the equilibrium arrival rate of innovation* λ^* *and the equilibrium growth rate* g^*.

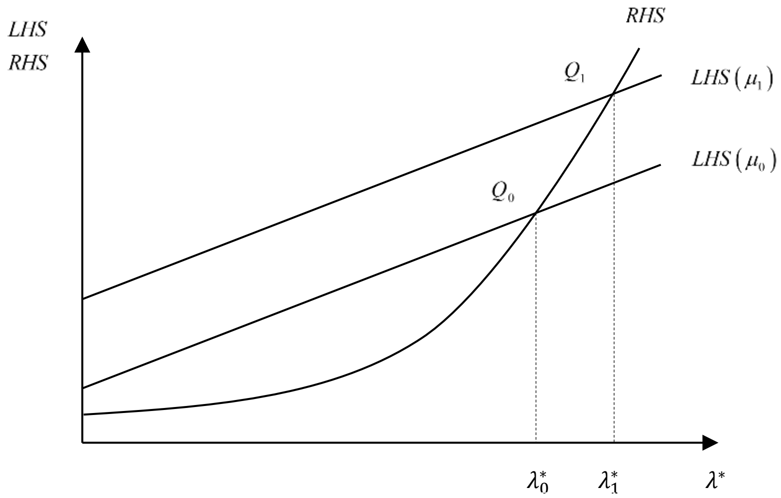

Figure 1. Effect of patent protection on the arrival rate: Exogenous step size.

3.2. Effects of patent protection on inequality

We are now ready to explore the impact of patent policy on the degree of inequality. We first demonstrate that, as in Chu and Cozzi (2018) and Chu et al. (2021), wealth inequality is exogenously determined by its initial level. Then, we show how patent policy affects income and consumption inequality.

3.2.1. Wealth distribution

Aggregating (2) for all households, we have

$$\dot{a}_t = r_t a_t + w_t - c_t, \tag{22}$$

where a_t represents the total value of financial assets (i.e., the total wealth) owned by households. In line with Chu and Cozzi (2018) and Chu et al. (2021), we denote $\theta_{a,t}(h) \equiv a_t(h)/a_t$ as the share of household h's wealth and assume that the initial share $\theta_{a,0}(h) \equiv a_0(h)/a_0$ has a distribution with a mean of one and an exogenous standard deviation of $\sigma_a > 0$. Combining (2) and (22), we obtain

$$\frac{\dot{\theta}_{a,t}(h)}{\theta_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{c_t - w_t}{a_t} - \frac{c_t(h) - w_t}{a_t(h)}. \tag{23}$$

Similarly, we define $\theta_{c,t}(h) \equiv c_t(h)/c_t$ as the share of household h's consumption. Then, we rearrange the terms and (23) can be re-expressed as

$$\dot{\theta}_{a,t}(h) = \frac{c_t - w_t}{a_t}\theta_{a,t}(h) - \frac{\theta_{c,t}(h)c_t - w_t}{a_t}. \tag{24}$$

From (3), we immediately have $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$ and thus $\dot{\theta}_{c,t}(h)/\theta_{c,t}(h) = 0$. Therefore, household h's consumption share $\theta_{c,t}(h)$ is time-invariant and

$\theta_{c,t}(h)=\theta_{c,0}(h)$ for all time t. Furthermore, from (3) and (22), we can derive that $(c_t - w_t)/a_t = r_t - \dot{a}_t/\dot{a} = \rho > 0$.[11] As a result, (24) is a one-dimensional differential equation and has a positive coefficient $(c_t - w_t)/a_t$ on $\theta_{a,t}(h)$. Given that the consumption share $\theta_{a,t}(h)$ is a state variable, along the BGP, $\dot{\theta}_{a,t}(h)$ must be equal to 0 such that $\theta_{a,t}(h)=\theta_{a,0}(h)$ for all time t. Therefore, the wealth inequality measured by the standard deviation $\theta_{a,t}(h)$ is not affected by patent protection and equal to its initial level σ_a.

3.2.2. Income distribution

Household h's real income is $I_t(h)=r_t a_t(h)+w_t$, which consists of asset income $r_t a_t(h)$ and wage income w_t. Aggregating $I_t(h)$ for all h yields the aggregate level of real income given by $I_t = r_t a_t + w_t$. Then, household h's income share is

$$\theta_{I,t}(h) \equiv \frac{I_t(h)}{I_t} = \frac{r_t a_t \theta_{a,0}(h) + w_t}{r_t a_t + w_t}, \tag{25}$$

where the second equality applies $\theta_{a,t}(h)=\theta_{a,0}(h)$. In line with Chu and Cozzi (2018) and Chu et al. (2021), we use the standard deviation of the distribution of $\theta_{I,t}(h)$ to measure the degree of income inequality. (25) implies that the mean of the distribution of $\theta_{I,t}(h)$ is equal to one. Therefore, the distribution of $\theta_{I,t}(h)$ has a standard deviation given by

$$\sigma_{I,t} = \sigma_I \equiv \sqrt{\int_0^1 \left[\theta_{I,t}(h) - 1\right]^2 dh} = \frac{r_t a_t}{r_t a_t + w_t}\sigma_a = \frac{r_t a_t / w_t}{r_t a_t / w_t + 1}\sigma_a. \tag{26}$$

(26) clearly shows that the degree of income inequality σ_I is increasing in the real interest rate r_t and the ratio of asset to wage a_t/w_t. Recall that households own different levels of assets and the asset income $r_t a_t$ is the source of income inequality in the economy. Thus, the increase in r_t or a_t/w_t raises the ratio of asset income to wage income $r_t a_t/w_t$, which in turn will lead to a higher degree of income inequality. Hereafter, we refer to the effect of patent protection on inequality via the real interest rate r_t and the ratio of asset to wage a_t/w_t as the interest-rate effect and the asset-value effect of patent protection, respectively.

With an exogenous step size of quality improvements z, from (13), (18) and (19), we obtain

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[11] Given that $a_t = V_1 + V_2$ and $\dot{V}_1/V_1 = \dot{V}_2/V_2 = g$, we have that along the BGP, $\dot{a}_t/\dot{a}_t = g$.

$$\frac{a_t}{w_t} = \frac{z}{\varphi} \frac{\rho + \lambda^* + \lambda^* s}{(1-s)\left(\rho + \lambda^*\right) + \lambda^* s}. \tag{27}$$

From (27), we immediately have $\partial\left(a_t/w_t\right)/\partial\lambda^* < 0$. Together with the fact that $\partial\lambda^*/\partial\mu > 0$, we have $\partial\left(a_t/w_t\right)/\partial\mu < 0$. As a result, strengthening patent protection generates a negative asset-value effect on income inequality. The intuition can be explained as follows. On the one hand, by (8), an increase in μ will raise the monopolistic profits of intermediate-goods producers, leading to a higher value of all monopolistic firms a_t (i.e., the total wealth of households). On the other hand, by (13), the increase in V_1 will lead to a higher real wage rate w_t, which in turn raises the wage income of households. As shown in (27), the latter effect is greater than the former one, thus causing the ratio of asset to wage a_t/w_t decreases with μ.

Moreover, Proposition 1 shows that with an exogenous step size, an increase in μ stimulates economic growth. Given that $r^* = g^* + \rho$, the real interest rate is increasing in μ. Thus, strengthening patent protection has a positive interest-rate effect on income inequality. Together with the negative asset-value effect, in this case, the overall effect of strengthening patent protection on income inequality is ambiguous. To see this,

$$\frac{\partial\sigma_I}{\partial\mu} = \underbrace{\frac{\partial\sigma_I}{\partial\left(r_t a_t/w_t\right)}}_{+} \frac{\partial g^*}{\partial\mu} \frac{z}{\varphi} \frac{\rho + \lambda^* + \lambda^* s}{\rho - s\rho + \lambda^*} - \frac{s\left(\rho + g^*\right)\left(\rho + \lambda^*\right)}{\left(\rho - s\rho + \lambda^*\right)^2} \underbrace{\frac{\partial\lambda^*}{\partial\mu}}_{+}. \tag{28}$$

Chu and Cozzi (2018) also explore the impact of patent protection on income inequality in a Schumpeterian economy, which has an exogenous step size but does not feature sequential innovations. In their model, both the interest-rate effect and the asset-value effect are positive, so that strengthening patent protection increases income inequality. Moreover, a recent study by Chu et al. (2021) investigates the effect of patent protection on income inequality in a Schumpeterian economy with endogenous market structure. They find that in the long run, both the interest-rate effect and the asset-value effect are negative, such that strengthening patent protection decreases income inequality. This paper complements their studies by showing that in an environment with sequential innovations, strengthening patent protection has a positive interest-rate effect as well as a negative asset-value effect on income inequality, thereby generating an overall ambiguous effect on income inequality.

Proposition 2. *In an environment characterized by sequential innovations and* *exogenous quality improvements, strengthening patent protection causes a positive interest-rate effect as well as a negative asset-value effect on income inequality. Therefore, the overall effect of patent protection on income inequality is ambiguous.*

3.2.3. Consumption distribution

From the asset-accumulation equation (2), household h's consumption is given by $c_t(h) = \rho a_t(h) + w_t$. Aggregating $c_t(h)$ for all h yields the aggregate level of consumption given by $c_t = \rho_t a_t + w_t$. Then, household h's consumption share is

$$\theta_{c,t}(h) \equiv \frac{c_t(h)}{c_t} = \frac{\rho a_t \theta_{a,0}(h) + w_t}{\rho a_t + w_t}, \tag{29}$$

where $\theta_{a,t}(h) = \theta_{a,0}(h)$ is used again. Similarly, the degree of consumption inequality is measured by the standard deviation of the distribution of $\theta_{c,t}(h)$. (29) implies that $\theta_{c,t}(h)$ has a mean of one and a standard deviation given by

$$\sigma_{c,t} = \sigma_c \equiv \sqrt{\int_0^1 \left[\theta_{c,t}(h) - 1\right]^2 dh} = \frac{\rho a_t}{\rho a_t + w_t}\sigma_a = \frac{\rho\, a_t/w_t}{\rho\, a_t/w_t + 1}\sigma_a, \tag{30}$$

which is increasing in the ratio of asset to wage a_t/w_t. Based on the discussion in 3.2.2, (30) shows that strengthening patent protection has only an asset-value effect on consumption inequality via a_t/w_t but no interest-rate effect. Given that $\partial(a_t/w_t)/\partial\mu < 0$, the asset-value effect of strengthening patent protection on consumption inequality is negative. This result also differs from Chu and Cozzi (2018) and Chu et al. (2021). Chu and Cozzi (2018) find a positive relationship between patent protection and consumption inequality, while Chu et al. (2021) find that strengthening patent protection leads to a one-time permanent decline in the degree of consumption inequality. Therefore, the present paper complements these two studies by showing that in an environment with sequential innovations, the degree of consumption inequality is decreasing in the strength of patent protection.

Proposition 3. *In an environment characterized by sequential innovations and exogenous quality improvements, strengthening patent protection causes only a negative asset-value effect on consumption inequality. Therefore, the effect of patent protection on consumption inequality is negative.*

3.3. Quantitative analysis

In this subsection, we quantify the effects of patent protection on economic growth and inequality under exogenous quality improvements. Following Acemoglu and

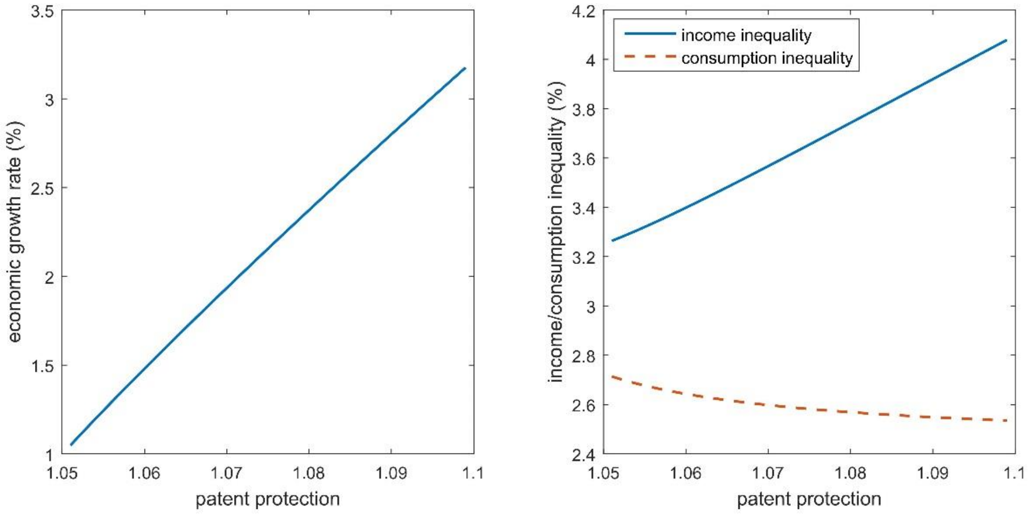
Akcigit (2012) and Chu and Cozzi (2018), we set the subjective discount rate $\rho = 0.05$. For the exogenous step size z, we consider a conventional value of 1.08 as in Akcigit and Kerr (2018) and Akcigit et al. (2021). For the arrival rate of innovation, we choose a value of 0.2 as our benchmark, which implies a long-run economic growth rate of 1.5% as in Chu and Furukawa (2011) and Chu and Pan (2013). We consider the value of the R&D share of GDP (i.e., R&D intensity) of 3% for the United States. Then, we set the profit-division ratio $s = 0.6$, thus implying a markup (i.e., the level of patent protection) of 1.05, which is within the reasonable range estimated by the empirical literature.[12] From the above values, we can set the structural parameters β=0.65 and $\varphi = 6.35$. In addition, given that the estimates of the innovation arrival rate range widely in the literature, we also consider the cases of λ^*=0.1 and λ^*=0.3, respectively.[13] Under these calibrated parameter values, we can verify that Condition α always holds.[14]

Table 1. Calibration: Exogenous step size

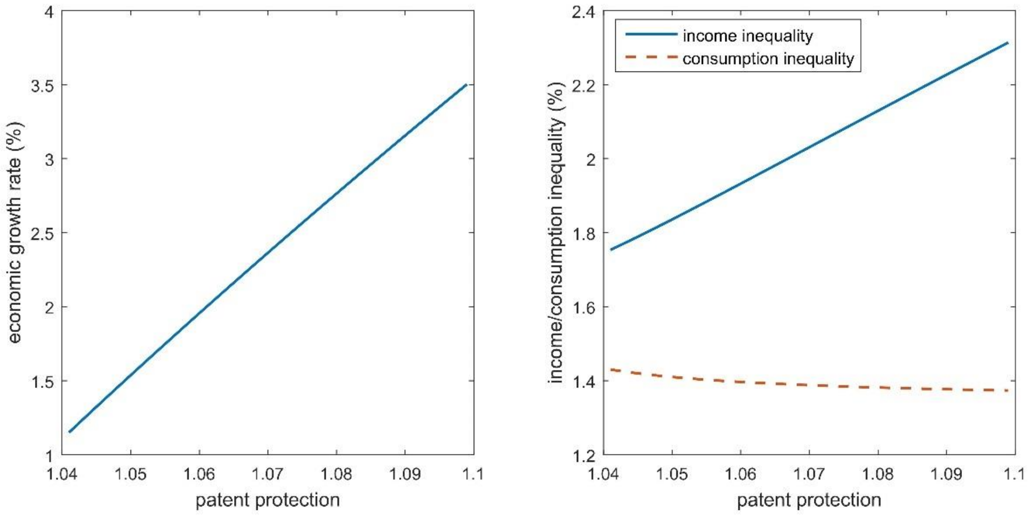
Parameters	λ^*	ρ	s	z	β	μ	φ
	0.1	0.05	0.6	1.16	0.70	1.06	3.74
values	0.2	0.05	0.6	1.08	0.65	1.05	6.35
	0.3	0.05	0.6	1.05	0.63	1.04	10.36

Figure 2 depicts the effects of patent policy in this case. Under exogenous quality improvements, strengthening patent protection stimulates economic growth, as shown in Proposition 1 and illustrated in the left panels of Figure 2. The results in Figures 2(a)-2(b) clearly show that the degree of income inequality is greater than that of consumption inequality, a finding consistent with Chu (2010) and Chu and Cozzi (2018); see, for example, Krueger and Perri (2006) and Blundell et al. (2008) for empirical evidence. More importantly, the right panels of Figure 2 show that in all the three cases, λ^*=0.1, λ^*=0.2, and λ^*=0.3, strengthening patent protection raises the degree of income inequality. In other words, the positive interest-rate effect of strengthening

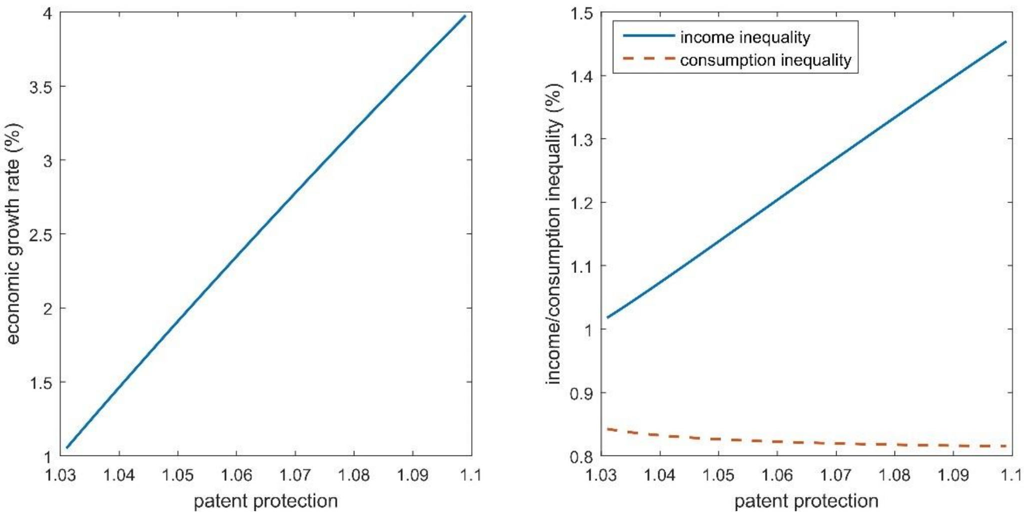
[12] See, for example, Jones and Williams (1998) estimate the markup ranging from 1.05 to 1.40.
[13] For example, Caballero and Jaffe (2002) estimate the arrival rate of innovation to be 0.04, Acemoglu and Akcigit (2012) calibrate the arrival rate to be 0.33, and Chu and Cozzi (2018) consider a value of 0.125.
[14] One can see that in Table 1, the markup ratio μ is smaller than the step size z (i.e., the unconstrained profit-maximizing markup ratio).



(a) $\lambda = 0.1$



(b) $\lambda = 0.2$



(c) $\lambda = 0.3$

Figure 2. Effects of patent protection on economic growth and inequality: Exogenous step size.

patent protection on income inequality dominates the associated negative asset-value effect. Interestingly, while strengthening patent protection worsens income inequality with an exogenous step size, the strengthening in patent protection suppresses consumption inequality, as shown in Proposition 3 and illustrated in Figure 2.

4. Patent, growth, and inequality: Endogenous quality improvements

In contrast to Section 3, in this section, we consider the case of endogenous quality improvements. More importantly, we compare growth and inequality effects of patent protection under exogenous versus endogenous quality improvements.

4.1. Effects of patent protection on growth

Substituting (19) and $s = \beta/z$ into $\varphi V_1/z$ yields

$$\frac{\varphi V_1}{z} = \frac{\varphi}{z}\left[\frac{\left(1-(\beta/z)\right)\pi}{\rho+\lambda} + \frac{\lambda V_2}{\rho+\lambda}\right]. \tag{31}$$

As mentioned above, λ and V_2 in (31) are not chosen by the innovator itself. Therefore, under endogenous quality improvements, an R&D firm takes λ and V_2 as given and chooses the step size z to maximize $\varphi V_1/z$ such that the zero-expected profit condition in (13) holds.[15] Taking the derivative of (31) with respect to z and using (18), we have

$$\frac{\partial\left(\varphi V_1/z\right)}{\partial z} = -\frac{\varphi}{z^2}\left[\frac{\left(1-(\beta/z)\right)\pi}{\rho+\lambda} + \frac{(\beta/z)\lambda\pi}{(\rho+\lambda)^2}\right] + \frac{\varphi}{z}\left(\frac{\beta\pi}{\rho+\lambda}\frac{1}{z^2}\right). \tag{32}$$

Then, the optimal step size is

$$z = \frac{\beta(2\rho+\lambda)}{\rho+\lambda}. \tag{33}$$

Given that $s = \beta/z$, the optimal profit-division ratio is

$$s = \frac{\rho+\lambda}{2\rho+\lambda}. \tag{34}$$

We now derive the equilibrium innovation arrival rate in this case. Combining (9) and (13) yields

$$\frac{\varphi V_1}{z} = \frac{y}{\mu l_x}. \tag{35}$$

Substituting (8), (18), and (19) into (35), we have

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[15] Note that potential entrants also take w_t and π as given.

$$\frac{\varphi(\mu-1)}{z(\rho+\lambda)}\left(1-s+\frac{\lambda s}{\rho+\lambda}\right)=\frac{1}{l_x}. \tag{36}$$

Then, we substitute (33), (34), $l_x = 1-l_r$, and $l_r = \lambda z/\varphi$ into (36) to obtain

$$\varphi(\rho+\lambda)=\beta\left[\frac{(2\rho+\lambda)^2}{\mu-1}+\lambda(2\rho+\lambda)\right]. \tag{37}$$

As in (21), the LHS of (37) is a linear and increasing function of λ while the RHS of (37) is a convex and increasing function of λ. Similarly, to ensure that there is a unique $\lambda > 0$ that satisfies (37), we impose the following condition.

$$\text{Condition } \gamma\text{: } \varphi > \frac{4\beta\rho}{\mu-1}.$$

Under Condition γ, the equilibrium arrival rate λ^* is determined by the unique intersection of the left-hand and right-hand sides of (37). Then, the equilibrium economic growth rate g^*, the equilibrium step size of quality improvements z^*, and the equilibrium profit-division rule s^* are given by (17), (33), and (34), respectively.

In this case, an increase in μ shifts down the RHS of (37), leading to a higher arrival rate λ^*.[16] Thus, strengthening patent protection increases the innovation arrival rate, which is consistent with the case of exogenous quality improvements. However, given that $\partial\lambda^*/\partial\mu > 0$, from (33), we immediately have $\partial z^*/\partial\mu < 0$. Therefore, with an endogenous step size of innovation, strengthening patent protection generates an additional negative effect on economic growth by decreasing the step size z^*. Intuitively, the increase in μ allows a monopolistic producer to charge a higher markup, thereby increasing the producer's expected profit. As a result, R&D firms have an incentive to set a higher arrival rate of innovation. This effect is the same as in the case of exogenous step size of innovation. However, in response to the increase in expected profits, R&D firms are willing to invest in innovation with a relatively small step size and pay a higher licensing fee once they successfully enter the market. Thus, in this case, strengthening patent protection reduces the equilibrium step size z^*.

The above two opposing forces imply that with an endogenous step size, the effect of strengthening patent protection on economic growth becomes ambiguous. To see this,

$$\frac{\partial g^*}{\partial\lambda^*}=\ln z^*+\lambda^*\frac{\partial \ln z^*}{\partial\lambda^*}\approx \ln\beta+\frac{2\rho^2}{(2\rho+\lambda^*)(\rho+\lambda^*)}, \tag{38}$$

[16] Figure 1 also applies to this case. We can get this result immediately when the *RHS* curve in Figure 1 shifts downward.

where the approximate equation in (38) applies the log approximation $\ln\left[\left(2\rho+\lambda^*\right)/\left(\rho+\lambda^*\right)\right] \approx \rho/\left(\rho+\lambda^*\right)$. Given that $\partial\lambda^*/\partial\mu > 0$, we can show that $\partial g^*/\partial\mu = \left(\partial g^*/\partial\lambda^*\right)\left(\partial\lambda^*/\partial\mu\right) > 0(<0)$ if $\ln\beta + 2\rho^2/\left[\left(2\rho+\lambda^*\right)\left(\rho+\lambda^*\right)\right] > 0(<0)$. Specifically, if the parameter β is sufficiently large, then strengthening patent protection stimulates economic growth; if the parameter β is sufficiently small, then strengthening patent protection deters economic growth; if the parameter β is neither sufficiently large nor sufficiently small and $\ln\beta + 1 > 0$, strengthening patent protection may have an interesting inverted-U effect on economic growth.[17]

Proposition 4. *In an environment characterized by sequential innovations and* endogenous *quality improvements, strengthening patent protection increases the equilibrium arrival rate of innovation* λ^* *but decreases the equilibrium step size* z^*. *Therefore, strengthening patent protection has an overall ambiguous effect on economic growth.*

4.2. Effects of patent protection on inequality

In this subsection, we explore the relationship between patent protection and inequality under endogenous quality improvements. Our results show that, in this case, the overall effects of patent protection on income and consumption inequality remain the same as under exogenous quality improvements.

4.2.1. Income distribution

From (13), (18)-(19), and (33)-(34), the ratio of asset to wage becomes

$$\frac{a_t}{w_t} = \frac{2\beta}{\varphi}\frac{2\rho+\lambda^*}{\rho+\lambda^*}. \tag{39}$$

Given that $\partial\lambda^*/\partial\mu > 0$, we immediately have $\partial\left(a_t/w_t\right)/\partial\mu < 0$. Thus, with an endogenous step size, strengthening patent protection also has a negative asset-value effect on income inequality. Moreover, substituting (33) into (39) yields $a_t/w_t = 2z^*/\varphi$. Therefore, given that the step size z^* is endogenous, an increase in μ leads to a smaller ratio of asset to wage a_t/w_t by decreasing z^*. However, Proposition 4 shows that in this case, the interest-rate effect of strengthening patent protection on income inequality becomes ambiguous. As a result, with an endogenous step size of quality improvements, the overall effect of strengthening patent protection on income equality

[17] From (38), we have $\partial\left(\partial g^*/\partial\lambda^*\right)/\partial\lambda^* < 0$. Together with $\partial\lambda^*/\partial\mu > 0$, we have $\partial\left(\partial g^*/\partial\lambda^*\right)/\partial\mu < 0$. This implies that if the relationship between patent protection and economic growth rate is positive at the relatively low levels of μ, there may be a threshold value beyond which the relationship will become negative.

remains ambiguous. To see this,

$$\sigma_I = \frac{2\left(\rho+g^*\right)\left(\beta/\varphi\right)\left(2\rho+\lambda^*\right)/\left(\rho+\lambda^*\right)}{2\left(\rho+g^*\right)\left(\beta/\varphi\right)\left(2\rho+\lambda^*\right)/\left(\rho+\lambda^*\right)+1}\sigma_a, \tag{40}$$

which is increasing in $\Omega=\left(\rho+g^*\right)\left(\beta/\varphi\right)\left(2\rho+\lambda^*\right)/\left(\rho+\lambda^*\right)$. Differentiating Ω with respect to μ yields

$$\frac{\partial\Omega}{\partial\mu} = \underset{+,-}{\frac{\partial g^*}{\partial\mu}}\frac{\beta}{\varphi}\frac{2\rho+\lambda^*}{\rho+\lambda^*} - \frac{\beta\rho\left(\rho+g^*\right)}{\varphi\left(\rho+\lambda^*\right)^2}\underset{+}{\frac{\partial\lambda^*}{\partial\mu}}. \tag{41}$$

Therefore, the degree of income inequality σ_I can be increasing or decreasing in the level of patent protection μ. We summarize these results below.

Proposition 5. *In an environment characterized by sequential innovations and endogenous quality improvements, strengthening patent protection causes an ambiguous interest-rate effect as well as a negative asset-value effect on income inequality. Therefore, the overall effect of patent protection on income inequality is ambiguous.*

4.2.2. Consumption distribution

As discussed in Subsection 3.2.2, strengthening patent protection affects consumption inequality only through the ratio of asset to wage a_t/w_t. (39) shows that a_t/w_t decreases with the level of patent protection μ. Therefore, with an endogenous step size of innovation, strengthening patent protection causes only a negative asset-value effect on consumption inequality, thereby decreasing the degree of consumption inequality. This result is the same as in the case of exogenous quality improvements. Proposition 6 summarizes the effect of patent protection on consumption inequality in this case.

Proposition 6. *In an environment characterized by sequential innovations and endogenous quality improvements, strengthening patent protection causes only a negative asset-value effect on consumption inequality. Therefore, the effect of patent protection on consumption inequality is negative.*

4.3. Quantitative analysis

In this subsection, we recalibrate the parameters to quantify the effects of patent protection on economic growth and inequality under endogenous quality improvements. As in the case of exogenous quality improvements, we consider the arrival rate of innovation $\lambda^*=0.2$ as our benchmark and set the discount rate $\rho=0.05$, the long-run economic growth rate $g^*=1.5\%$, and the R&D intensity $\text{R\&D/GDP}=3\%$. Then, in this case, we calibrate the endogenous step size of innovation $z^*=1.08$, the bargaining power $\beta=0.90$, the level of patent protection $\mu=1.05$, the endogenous profit-division ratio $s^*=0.83$, and the R&D productivity $\varphi=7.91$, respectively. As before, we also consider the cases of $\lambda^*=0.1$ and $\lambda^*=0.3$, respectively. Under these calibrated parameter values, we can verify that Condition γ always holds.[18]

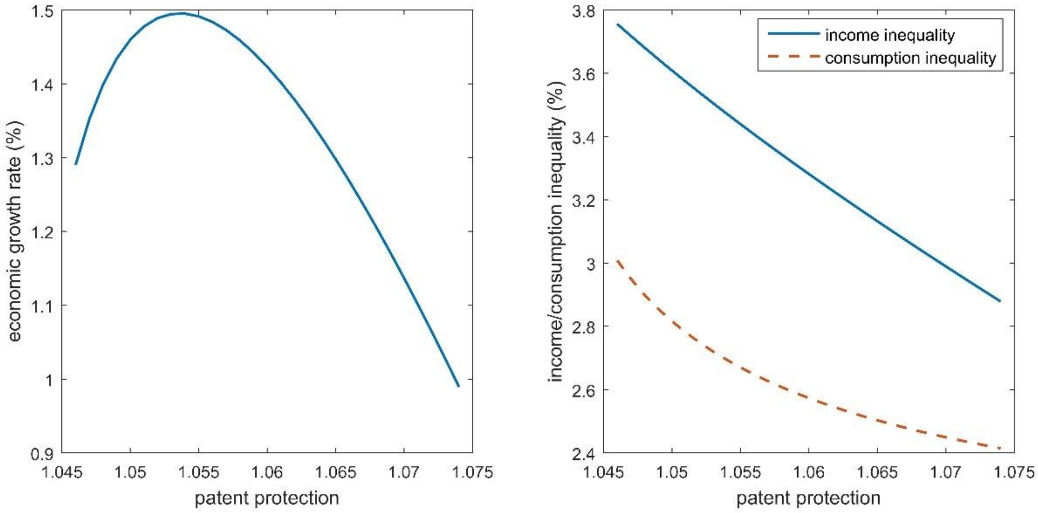
Table 2. Calibration: Endogenous step size

Parameters	λ^*	ρ	s^*	z^*	β	μ	φ
	0.1	0.05	0.75	1.16	0.87	1.06	4.26
values	0.2	0.05	0.83	1.08	0.90	1.05	7.91
	0.3	0.05	0.87	1.05	0.92	1.04	11.40

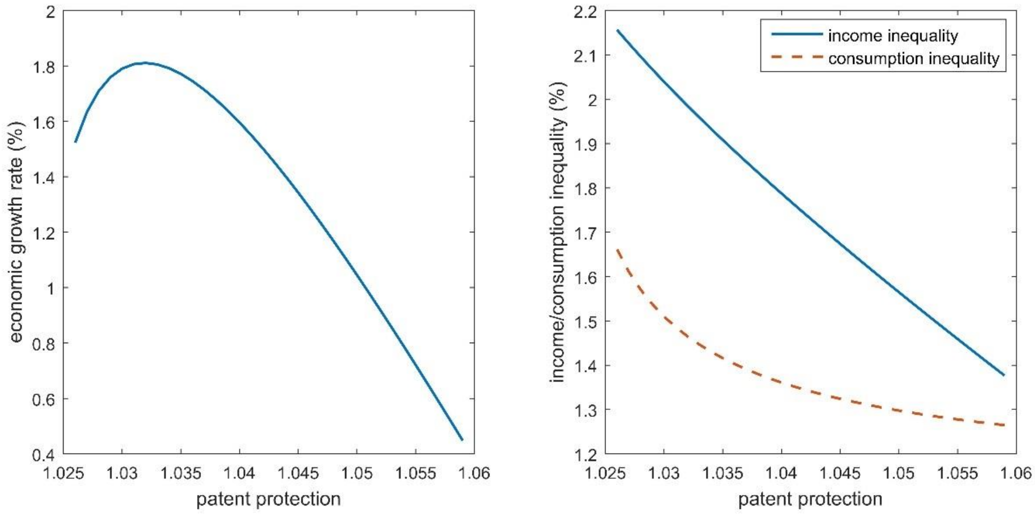
Figure 3 depicts the effects of patent protection in this case. The left panels of Figure 3 show that when we consider endogenous instead of exogenous quality improvements, the macroeconomic effect of strengthening patent protection on economic growth becomes an inverted-U function in all the three cases, $\lambda^*=0.1$, $\lambda^*=0.2$, and $\lambda^*=0.3$.[19] Moreover, as in Figure 2, income inequality is greater than consumption inequality, and the degree of consumption inequality decreases with the strength of patent protection. However, in this case, strengthening patent protection suppresses income inequality, as illustrated in the right panels of Figure 3. The intuition behind this result is straightforward. On the upward-sloping side of the inverted U, while strengthening patent protection has a positive interest-rate effect on income inequality, this effect is dominated by the associated negative asset-value effect. On the downward-sloping side of the inverted U, both the interest-rate effect and the asset-value effect of strengthening patent protection on income inequality are negative. As a result, with an endogenous step size of innovation, the degree of income inequality decreases with the level of patent protection.

[18] Again, one can see that in Table 2, the markup ratio μ is smaller than the equilibrium step size z^*.

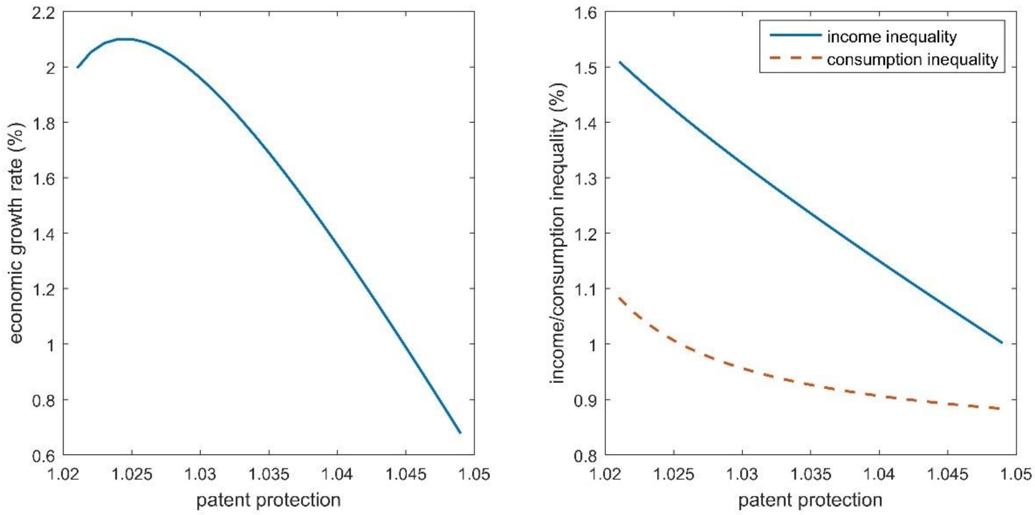
[19] Iwaisako and Futagami (2013) consider an endogenous growth model featuring both innovation and capital accumulation and also find that strengthening patent protection has an inverted-U effect on economic growth.



(a) $\lambda = 0.1$



(b) $\lambda = 0.2$



(c) $\lambda = 0.3$

Figure 3. Effects of patent protection on economic growth and inequality: Endogenous step size.

5. Conclusion

In this paper, we revisit the impact of patent policy on economic growth and inequality in a Schumpeterian economy with sequential innovations and heterogeneous households. We find that the effects of patent protection on growth and inequality in an environment with sequential innovations are different from Chu and Cozzi (2018) and Chu et al. (2021). More importantly, we provide a comparison of the effects of patent policy under exogenous and endogenous quality improvements. Our results show that with an endogenous step size of innovation, the growth and inequality effects of patent protection can be quite different from those with an exogenous step size. Under exogenous quality improvements, strengthening patent protection stimulates economic growth. Whereas income inequality increases with the strength of patent protection, consumption inequality decreases with the strength of patent protection. However, under endogenous quality improvements, strengthening patent protection generates an inverted-U effect on economic growth, and both income inequality and consumption inequality are decreasing in the strength of patent protection.

Our analysis models household heterogeneity by assuming that households own different levels of wealth, and focuses on income and consumption inequality. Alternatively, one can explore with other types of heterogeneity, such as heterogeneous preferences and wage heterogeneity, how endogenous quality improvements affects the relationship between patent protection and inequality. Furthermore, for tractability, in our analysis we assume that the most recent innovation only infringes on the patent of the second most recent innovation. We leave the interesting extension of how endogenous quality improvements affect the effect of patent policy in a more general case to future research.

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