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Estimating the Social Welfare Function of Amartya Sen for Latin America*

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Abstract

The present article establishes a set of empirical approximations related to the theoretical formulation of the social welfare function of Sen applied to the Latin-American context between 1995 & 2018, this in order to provide estimates of the welfare trends derived from its original version and the generalized version proposed by Mukhopadhaya (2003a; 2003b). The article equally contributes exploring the long-run relationships among welfare, inequality and income with the panel data methodology. The results indicated that long-run relationships exists among these variables, a higher elasticity on the welfare comes from the income distribution rather than the economic growth, in the short-run, only the economic growth was significant to explain the welfare. Finally, for the Latin American countries the ranking of welfare was done considering the predicted values of welfare at country level with the linear predictions of the econometrical model of the long-run, which matches in terms of behavior and rank in the welfare with the original approach of Sen applied to Latin America.

Keywords: welfare; inequality; growth; income; Latin America.

JEL: D63; E25; O54; O57.

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Introduction

One of the existing problematics in all societies from the world since the introduction of capitalism as an economic system, has been the intrinsic relationship between welfare, inequality and income. Important debates from the vision of the welfare have been given from different perspectives associated to the argumentation process of the decisions involving economic policies, in which we can mention the theory of social election of Arrow (1963), the —*welfarism*— view (Roberts, 1980) or even the classic utilitarian considerations of Bentham (1781/2000) or John Stuart Mill (1863/2001) applied to the neoclassical reality in authors like Blackorby et al. (2002).

The necessity to discuss, analyze and study this problematic maintains its validity in the present, since the patterns of income concentration have tended upwards (increasing) all over the world in the XXI century (Berman et al., 2016; Chancel et al., 2017; Piketty, 2013), affecting the levels of social welfare (McGillivray & Markova, 2010). This matter, by the level of its complexity has to considerate separate investigations for homogeneous economies in terms of economic development. Examples of this kind of investigations can be found in the studies of Milanovic (2013) and Cobham et al. (2016) among others.

Within this article, a set of empirical estimations are considered from the theoretical instrument of the *Social Welfare Function*—SWF— formulated by Sen (1974), which will allow to establish -on a first matter- the causal order of the levels of welfare of an economy, explained through the existing conditions of the income, relating the existing inequality levels, and also -on a second matter- permits the classification and comparative positioning between the levels of social welfare by country in the Latin-American context from 1995 to 2018. The contribution is augmented with an econometrical estimation of the SWF differentiating the relationships among the short-run and the long-run terms for Latin-America, where it is also provided the comparative analysis of the ranking of welfare among the countries for the year 2018.

The contribution of this article -and specially the estimation- takes place within the scarcity of empirical analysis in the economic literature of the topic with Sen's Instrument. In fact, only eight empirical works have been done until the date the specific proposal of Sen (1974). The first, presented in the article of Sen (1976) which uses the social welfare function to establish intra-state comparatives between the real income of India during the years 1961 and 1962 for the purpose to provide an approximation of the welfare expressed by the income and the inequality. The second is allocated in Berrebi and Silber (1987) in which the authors provide an analysis of the changes of welfare using the social welfare function of Sen during the years 1960, 1970, and 1980 for the United States. The third, -based on the Latin-American context- was developed by Gasparini and Sosa Escudero (2001), estimating empirically the series of welfare with different approximations for

Argentina between the years 1980 and 1998¹. The fourth, developed by Mukhopadhaya (2003a) where it also proposed by the first time, a generalized version of the social welfare function of Sen of 1974, disaggregating the analysis by components of the national income applying the Gini coefficient, leading to establish the welfare trends for Australia from 1984 to 1994. The fifth, developed by the same author (Mukhopadhaya, 2003b) analyze the estimation of the social welfare by using Sen's (1976) approximation and uses a generalized version that he proposed before to establish the changes and positioning of the levels of social welfare in Singapore. The sixth study in Baluch and Razi (2007), estimate the trend of welfare during 1970 to 2002 in Pakistan, with the traditional approach of the function of Sen (1974). The seventh, done by Bishop et al. (2009) presents the statistical procedure of an asymptotic theory in which the SWF of Sen works, the authors ranked the 50 states of United States for the year 1980 with the respective levels of welfare. Finally, the eighth empirical study, belongs to Mukhopadhaya (2014) following the same line of his previous studies, but deepened in Singapore for the years 1984 to 2011, with a special focus of the elements associated to the trade-off between efficiency and equity for the country.

In this article, the approximations of Sen and Mukhopadhaya are used to estimate in the Latin-American context, the trends of social welfare by country. In this sense, it also presents the special inclusion of an econometrical approximation which seeks to calculate the measure of the elasticity of welfare by changes on the per capita income within the income concentration for Latin America for the years 1995 to 2018. This econometrical approximation is done using the Engle & Granger (1987) cointegration analysis extended to the application of panel data framework.

The generalized relationship between inequality, welfare and income.

From its origins in Bergson (1938), the SWF has evolved significantly inside the economic theory, in which some interesting points of discussion about the debate of equity and efficiency has risen over the time. Just as it was mentioned by Mukhopadhaya (2003a) in the praxis, the function it's mainly used in the consideration of distributive impact analysis for the society in terms of the dilemma of efficiency and equity in the social welfare. The shortest expression of the SWF with the simplest approximation possible can be defined as:

$$W = W(S, \theta) , \quad [1]$$

Where W is the level of social welfare, and S y θ are functions of the income profile x of the society. The characterization of S in equation (1) is that it is a representation of the total income of the society, which seeks to capture the aspect of efficiency of

¹ Similarly, Gasparini y Weinschelbaum (1991) presented a study in which they analyze the measures of welfare for Latin-America and Argentina in the period of 1963 to 1980 – Study which is not strictly ligated to the Sen's Theory used in the article.

the economy. On the other hand, θ represents the level of inequality marked in the society, and seeks to capture the aspect of equity in the analysis.

The properties of function (1) must satisfy:

$$\frac{\partial W}{\partial S} > 0 \quad [2]$$

$$\frac{\partial W}{\partial \theta} < 0 \quad [3]$$

Which means that the SWF of equation [1] must be increasing respect to the income profile of the society, and decreasing respective to the levels of inequality. We will impose that [1] must also be concave to reflect the criteria of a preference for the equity necessary in the analysis. Mukhopadhaya (2003a) at this point argues that the preference for the equity is given when there exists a transfer of the income of a person with higher level to a person which has a lowest level of income; in the sense, that such transfer increases the level of social welfare greatly in comparison by just changes of income with high level positionings.

The function in equation [1] also has to be ruled by the Paretian Principle, in which, if there's an increase in the income of a person, holding all the other variables as constants, the level of welfare will increase. Symbolically this can be represented as:

$$\frac{\partial W}{\partial x_i} > 0, i = 1, 2, \dots, N. \quad [4]$$

Where x_i is the income level of individual i considering N individuals in the population. The relation expressed in [4] indicates that a positive change in the income level of the individual i without considering his positioning in the income distribution, also increases the level of aggregated social welfare, regardless his positioning in such distribution.

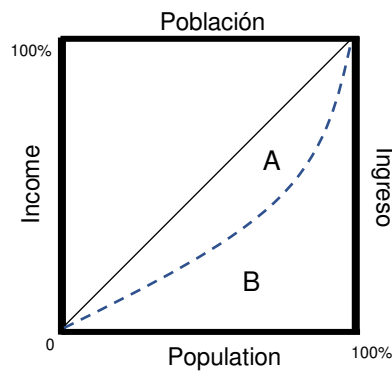
Sen (1974) assumes that the marginal utility of the society, is inversely related to its positioning in the income distribution. This implies that, when the transfers of income between the population group with highest levels of income towards the population group with the lowest levels, there's a general increase in the levels of social welfare of the society. In this perspective, the axiomatic introduction of the Sen's vision of the SWF is defined as:

$$W = y(1 - G) \quad [5]$$

Where y is the mean income of the society, and G is the Gini Coefficient in the income distribution. The SWF in [5] is the so-called social welfare function of Sen, and satisfies conditions [2] and [3]. The Gini index in this sense considers the ordering of population percentiles related to the general income distribution of the economy, which satisfies with the conditions that increases in marginal utility of the society is associated by transfers of the income considering their special location in the population ordering.

The Gini coefficient is defined as the double of the area between the perfect equality distribution (45°-degree line) and the Lorenz curve. The mentioned area is highlighted in the Figure 1 as point A. For this reason, the complement of the Gini coefficient ($1 - G$) is represented as the double of the area of B, defined below the Lorenz curve.

Figure 1. Lorenz Curve



— Line of Perfect Equality Distribution - - - Lorenz Curve

Source: Own Elaboration

The inference from before is that; from higher levels of income concentration, there's a decrease in levels of general welfare of the society -- from the approximation of the SWF of Sen--, given since the double of the area in B --that is $1 - G$ denominated complement of Gini-- is reduced when the area marked as A increases. The rate of substitution of inequality and efficiency in the context of the SWF of Sen is given as:

$$\frac{dG}{dy} = \frac{1 - G}{y} \quad [6]$$

By [6] it is stated that substitution rate is highly sensitive to changes in the income y , in comparison to the changes in the level of inequality G , considering that these two variables are determinate by the income profile x of the society.

An important discussion emerges in this point. Mukhopadhaya (2003a) argues that both G and y are jointly determinate, and therefore the decision maker inside the economy cannot change or alter these variables for different levels of economic growth or the ordering of the income distribution. However, this aspect can be debated in more than one sense. To start, the State and the public administration play an important role in process of redistribution of income, affecting directly the levels of economic inequality (Tomaszewicz & Trębska, 2015) and economic development (Gründler & Köllner, 2017). In this sense, Zhang (2007) states that not only the channels of public expenditure or governmental transfers influence the levels of inequality, also they influence the process of public investment --in special those who are related to education-- which affects the levels of income concentration and in general the evolution of economic development.

By this, it is argued that the value of G can also be time variant from the set of decisions related of redistributive policies in the economies which the governments enforce and do effective. In this sense the process of redistribution of income plays an important role in the aggregated improvements of welfare, having a close a relationship with the decisions of public policies, but with a lesser impact in economic growth given the rate of substitution in [6]

The Sen's formulation based on expression [5] indicates that it can be interpret as an extreme Paretian function of Social Welfare (Mukhopadhaya, 2003b) which seeks to cover the deficiencies of the Lorenz curve, since it allows to estimate the general trends of welfare from the components of efficiency and equity (Ndamsa et al., 2020).

The generalization process of the Sen's SWF emerges as a critic for the compliance of the Pareto principle, this generalization proposed by Mukhopadhaya² (2003a) and assumes that the statement can be re-written as:

$$W = y^\beta (1 - G). \quad [7]$$

Note that if $\beta < 1$, the SWF stops having the behavior of Pareto optimal defined in [4] and this derived from the theorem number 1 of Mukhopadhaya (2003a), which can be found in the Appendix A of this article. The justification for this approach is that not all positive changes in the income of all the populational groups increase in the same manner the aggregated social welfare, the critic is that the Pareto principle in this set up may not apply to the highest population percentiles with the largest income levels. In this sense, positive variations of income in this higher percentile may have a null or even negative effect in the aggregated social welfare, and by this, is not applicable the mentioned principle; this perspective is shared also with Bluch & Razi (2007), making emphasis that positive variations in the population groups with highest income levels do not bring always positive variations in the levels of aggregated social welfare. Here is important to note that when the generalized SWF of [7] has a value of $\beta = 1$, the result would Sen's original formulation from 1974, expressed in equation [5].

[T1]Methodological Aspects

[T2]Estimation of Traditional SWF of Sen

From the previous, there exists two functions which were birth from the perspective of Sen: The first, from the traditional function stated in equation [5], and the second, from the generalized form of Mukhopadhaya (2003a), represented in equation [7]. In this part it is discussed the forms of the estimation from both perspectives.

² Before this formal publication, workpapers detailing the generalization process can be found, see Mukhopadhaya (2001a) y Mukhopadhaya (2001b).

As a general approach, considering an annual periodicity between 1995 and 2018 for the countries of Latin America³, it is proposed the estimation of the original social welfare function stated by Sen (1974) of the form:

$$W_{it} = y_{it} (1 - G_{it}), \quad [8]$$

Where i represents the subindex associated to a certain country, t represents the time —measured in years—. Within this perspective W is simply the result of the calculus between the product of the mean income of the society y , assumed by the Gross Domestic Production per Capita in real terms in US of 2011 for all the countries of the sample, and the complement of the Gini coefficient $(1 - G)$. This approximation is used with comparative purposes (Baluch & Razi, 2007; Mukhopadhaya, 2003a; Sen, 1976) in order to determinate quantitatively the trends of welfare by country.

On a second place, with the generalized function of Sen proposed by Mukhopadhaya (2003a) of the form:

$$W_{it} = y_{it}^{\beta} (1 - G_{it}). \quad [9]$$

It is proposed to execute the calculus assuming constant values of β , in order to produce the quantitative differences of the levels of social welfare by their trends implying the optic which rejects the Pareto Principle. According to the study of Mukhopadhaya (2003b), using a constant value of $\beta = 0.5$, allows to identify an important difference with the original function of Sen, difference that emerges from the comparisons and positionings between the levels of welfare⁴. At this point, it is only estimated the traditional approach of the SWF of Sen, where we assume that the level of social welfare W is unknown and only measurable through the expression of the mean income of the economy along with the Gini coefficient G .

In this article it is proposed the possibility to use a proxy variable to capture the essence of W in order to determinate econometrically the value of β , which is a more realistic approximation than assuming that the elasticity of welfare-income is constant in all periods of time for all the countries of the analysis. The choice of the proxy variable for welfare has been extensively debated in the literature and the theory (Anand & Harris, 1994). However, here it is considered the synthetic indicator proposed by the United Nations for the Development (PNUD) denominated Human Development Index (HDI). The variable measures important features of welfare given by the dimensions of health, education, and worthy life style which directly affect the levels of welfare of an economy (PNUD, 2019). In fact, as it was proposed by Sen (1974) and Mukhopadhaya (2003a) from the SWF, the analysis of welfare tends to be mainly explained by the levels of income in the determination of the welfare W , which is a principal component in the construction of the IDH, but not

³ La muestra abarca los países de Argentina, Bolivia, Brasil, Chile, Colombia, Costa Rica, República Dominicana, Ecuador, El Salvador, Honduras, México, Panamá, Paraguay, Perú y Uruguay.

⁴ Es de aclarar que el valor de comparación se encuentra consignado en dicho estudio, el cual se considera como un valor arbitrario pero pertinente por parte de Mukhopadhaya (2003b).

uniquely determinate alone by this variable, since it is jointly related to the levels of health, and knowledge of a society and the levels of welfare of a population for each country.

[T2] Econometrical Estimation

From the generalized function of social welfare in equation [9], it is proposed that β represents the elasticity of welfare-income in the theoretical formulation of Sen, hence, the complement of the Gini Coefficient ($1 - G$) can possess as exponent, a parameter α to measure the elasticity related to the levels of welfare W , along this it is considered that exists an autonomous parameter of income and the inequality, also the influence of individual specific effects to capture the individual heterogeneity from the countries of analysis. In this form, the model is specified as:

$$W_{it} = y_{it}^{\beta_1} (1 - G_{it})^\alpha e^{\beta_0 + \mu_i + u_{it}}, \quad [10]$$

Where W_{it} represents the level of social welfare of the country i for the year t measured by the HDI. y_{it} is the mean income of the economy measured by the GDP per capita in real terms in USD of 2011. β_1 is the welfare-income elasticity, G is the Gini coefficient, α is the elasticity associated to the welfare from the complement of the Gini Coefficient ($1 - G$), β_0 is the autonomous coefficient (parameter) independent of the income and the inequality, μ is the fixed effects component which captures individual heterogeneity (Wooldridge, 2001) existing between the countries of Latin America and u_{it} is the residual of the panel regression model.

The linearization of the above function using natural logarithms is proposed as the econometrical model to execute the estimation of the form of:

$$\ln W_{it} = \beta_0 + \beta_1 \ln y_{it} + \alpha \ln(1 - G) + \mu_i + u_{it}. \quad [11]$$

The panel data structure is composed with $T > N$ dimensions, and considering the characteristics of the set of variables in the econometrical model stated in [11], it is possible to segment the analysis of the short-run and the long-run dynamics by the Engle y Granger (1987) methodologies extending the analysis of panel cointegration (McCoskey & Kao, 1998). In fact, equation [11] —since it only contains contemporaneous values over time— can be understand as a long-run relationship between the variables of analysis, these have the characteristics to possess unit-roots in levels but are stationary at first differences, defining a set of $I(1)$ (first-order integrated) variables according to the unit-root tests of first and second generation for panel data (see Appendix B).

For the analysis of cointegration between the variables —extended to the application in panel data— an error correction model is required, where it is first verified the existence of a cointegration relationship in the long-run expressed in equation [11] by specific tests to confirm the cointegrated process (Kao, 1999; Pedroni, 1999), after testing it, an error correction model is proposed using the variations from the

variables in the long-run —variations understood as first-differences— and using the information of the long-run in an error correction term, deriving in the short-run model of the form of:

$$\Delta \ln W_{it} = \beta_0 + \beta_1 \Delta \ln y_{it} + \alpha \Delta \ln(1 - G_{it}) + \varphi ECT_{it-1} + u_{it}, \quad [12]$$

Where Δ is the first-difference operator. The error correction term is defined as $ECT_{it-1} = \hat{u}_{t-1}$. Which means is a variable that contains lagged values from the residuals estimated of the long-run equation [11]. The parameter associated to the long-run information φ associated to the error correction term must be negative, between 0 and 1 and statistically significant to indicate a stable adjustment process towards the long-run.

Considering that the panel structure is $T > N$, the relationship of the long-run in [11] and the short-run in [12] are susceptible to contain problems of heteroskedasticity, serial correlation and cross-sectional dependence. To avoid a wrongful inference in terms of statistical significance, the proposed estimator for the regressions is the one formulated by Driscoll y Kraay (1998) in which it is presented the use of robust standard errors to account the problems of serial correlation, heteroscedasticity and cross-sectional dependence in the residuals on the multivariate regression framework, ideal for panels with larger or equal time periods of time in comparison to the number of individuals of the sample (Hoechle, 2007).

With this, the relationships from the short-run and the long-run between welfare, income and inequality can be estimated for the countries of Latin America in the period between 1995 and 2018. The sources of information of the variables -GDP per capita in real terms in USD of 2010, and the Gini coefficient-- for the different countries in Latin America were taking from the World Bank [Banco Mundial (s. f.)], the information of the HDI was taken from PNUD (s. f.).

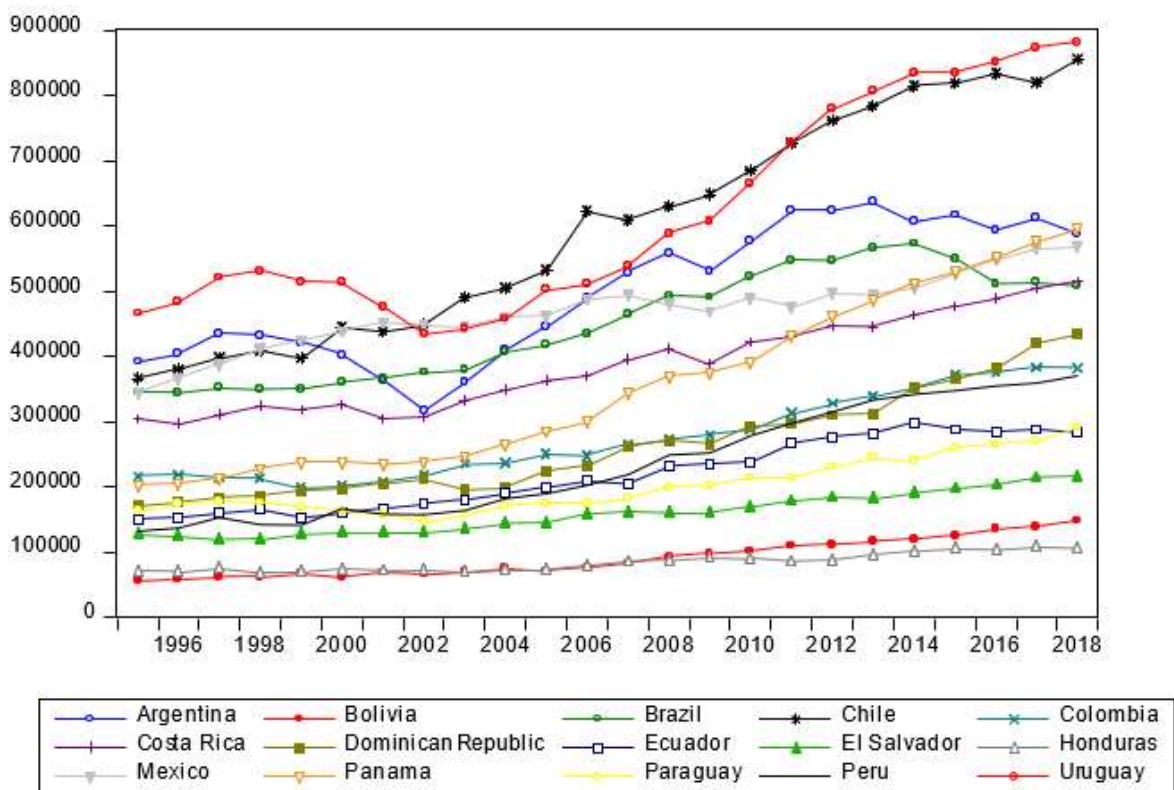
[T1] Results

In this section, the results are presented divided in two parts. The first part contains the traditional estimation of the SWF of Sen which considers $\beta = 1$ and the generalized proposition of the function with an elasticity of welfare-income of $\beta = 0,5$. In the second section, the results of the econometrical estimations are presented with the general results.

[T2] *Results of the traditional estimation*

The empirical results considering the SWF original of Amartya Sen (1974) for Latin America are presented in Figure 2, where it is remarked the elasticity parameter used for such calculations as $\beta = 1$ as the value of the welfare-income elasticity.

Figure 1. Estimation of the SWF of Sen (1974) by Country ($\beta=1$)



Source: Own Elaboration using World Bank (Banco Mundial (s. f.))

It is noted that for the generality of the countries, the trend is positive in the levels of welfare by the original SWF of Sen. Punctually, Uruguay and Chile are the countries that possess the best levels of welfare in the region, meanwhile Bolivia and Paraguay are the ones with the lowest levels of welfare. For the year 2018, the ranking of countries with the highest levels from this approach are: 1) Uruguay, 2) Chile, 3) Panamá, 4) Argentina, 5) México, 6) Costa Rica, 7) Brazil, 8) Dominican Republic, 9) Colombia, 10) Perú, 11) Paraguay, 12) Ecuador, 13) El Salvador, 14) Bolivia y 15) Honduras. The descriptive statistics of the calculated social welfare are presented in the Table 1.

Table 1. Descriptive statistics of the aggregated welfare using the original SWF of Sen (1974)

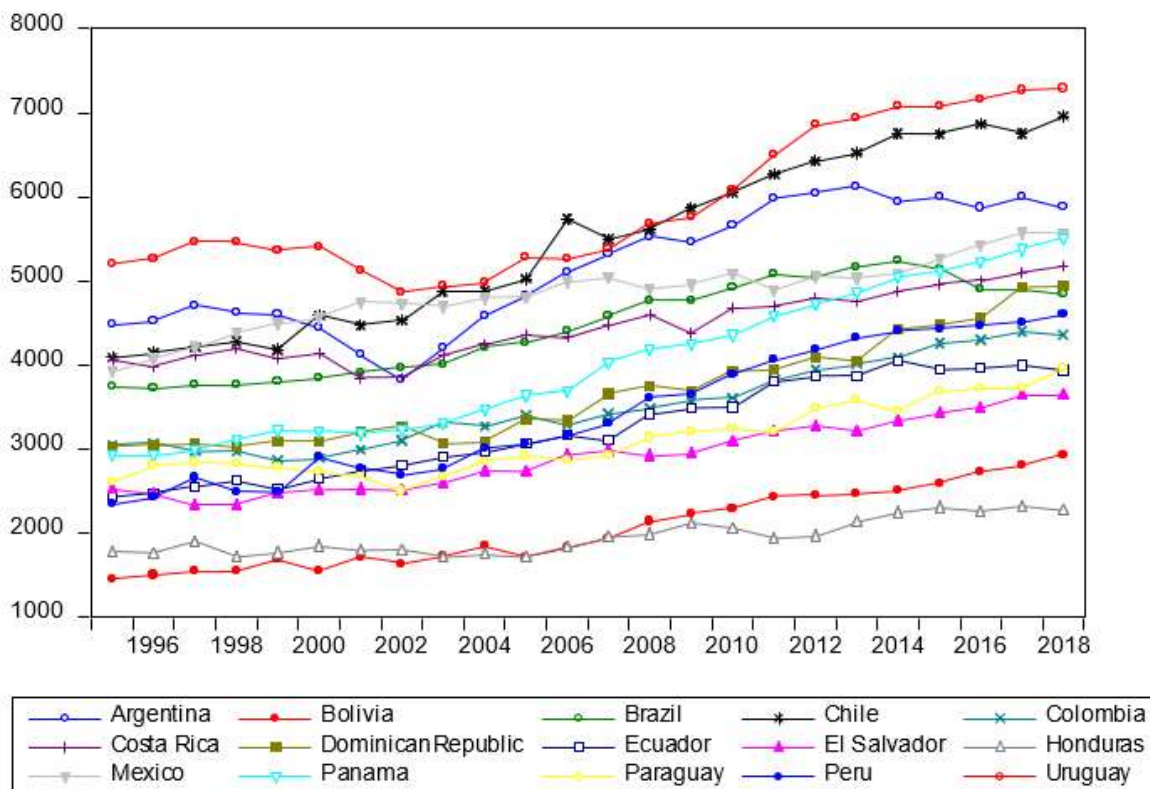
ID	N	Min	Max	Mean	Mediana	Desv. Est.	Varianza
Argentina	24	316668,4	636293,2	498558,7	508870,9	102799	1,06e+10
Bolivia	24	56086,77	147939,8	90172,32	78843,48	28849,59	8,32e+08
Brasil	24	344499,7	572463,1	448563,3	449806,2	82926,23	6,88e+09
Chile	24	365797,2	855618	600645,0	615406	170638,8	2,91e+10
Colombia	24	198077,2	382970,4	274804,8	257135,2	64227,86	4,13e+09
Costa Rica	24	295592,7	514266,7	386557,6	378451,9	71364,58	5,09e+09
Dom. Rep.	24	170892,1	433381,8	263743,9	246197	80055,19	6,41e+09
Ecuador	24	150168,3	297667,3	217987,3	207086,5	53379,8	2,85e+09
Salvador	24	118217	215603,3	158098,4	158391,6	31116,63	9,68e+08
Honduras	24	67966,66	107670,2	83384,88	81829,48	13593,03	1,85e+08
México	24	343881,3	568033,3	468415,3	472102,1	56471,33	3,19e+09
Panamá	24	201984,2	595575,7	354527,7	320769,1	132467,3	1,75e+10
Paraguay	24	145446,1	290330,7	200384,4	179365,7	41887,42	1,75e+09

Perú	24	130436,7	369143,7	233790,2	209505	85186,59	7,26e+09
Uruguay	24	434436,6	881433,1	618525,6	534362,4	159109,4	2,53e+10
Total	360	56086,77	881433,1	326544	296232,8	188707,1	3,56e+10

Source: Own Elaboration.

Going on with the estimation of the SWF with the generalized versión proposed by Mukhopadhaya with $\beta = 0,5$ —the case when the welfare do not satisfy Pareto's principle— the trends of welfare are presented in Figure 3.

Figure 2. Generalized SWF with $\beta=0,5$



Source: Own Elaboration.

It is noted that some slight differences emerge in the calculus of the social welfare by country in comparison to the original SWF of Sen (1974). Likewise, a small reduction of the gaps within this calculus of the SWF generalized can be seen. The trend component of welfare still remains as positive for Latin America like in the original estimation of Sen, but a change in the *raking* can be noticed for the year 2018 when we compare the country-welfare by this approach. The ranking has the following order: 1) Uruguay, 2) Chile, 3) Argentina, 4) México, 5) Panamá, 6) Costa Rica, 7) Dominican Republic, 8) Brazil, 9) Perú, 10) Colombia, 11) Paraguay, 12) Ecuador, 13) Salvador, 14) Bolivia, 15) Honduras.

In the same manner from the empirical analysis of Mukhopadhaya (2003a), the position of each country varies for certain countries in the ranking when it is compared to the original SWF of Sen. In Latin America --from the generalized version-- Panama descend and occupies the fifth place, Argentina ascends to the third place, and Mexico to the fourth place. Brazil -on the other hand- descends to

the eighth place with Colombia, which also descends to the tenth place. This situation indicates the existence of small variations in the measures of welfare by changes in the welfare-income elasticity assumed by the parameter β . The descriptive statistics of welfare calculated by the generalized versión assuming $\beta = 0,5$ are presented in Table 2:

Table 2. Descriptive Statistics of Social Welfare using the Generalized SWF with $\beta=0,5$

ID	N	Min	Max	Media	Mediana	Desv. Est.	Varianza
Argentina	24	3824,929	6127,096	5156,831	5209,453	734,1332	538951,6
Bolivia	24	1451,342	2924,195	2043,848	1881,097	469,4054	220341,5
Brasil	24	3716,778	5236,505	4443,777	4486,186	550,8025	303383,4
Chile	24	4088,635	6955,979	5468,679	5556,704	1027,159	1055055
Colombia	24	2860,173	4389,011	3513,412	3408,176	507,9851	258048,8
Costa Rica	24	3833,792	5171,254	4445,988	4368,486	405,2136	164198,1
Dom. Rep.	24	3014,039	4939,575	3665,88	3498,289	633,9395	401879,4
Ecuador	24	2422,511	4046,196	3236,696	3128,07	576,7686	332662,1
El Salvador	24	2329,145	3638,412	2903,879	2919,846	420,3152	176664,9
Honduras	24	1701,582	2308,609	1942,906	1905,889	208,3668	43416,71
México	24	3914,389	5572,109	4843,178	4896,404	426,189	181637,1
Panamá	24	2919,543	5500,477	4001,132	3854,772	871,2411	759061
Paraguay	24	2492,097	3952,188	3093,171	2913,155	417,6352	174419,2
Perú	24	2340,028	4595,108	3419,605	3231,145	787,8224	620664,1
Uruguay	24	4865,881	7290,433	5900,728	5462,23	858,2002	736507,7
Total	360	1451,342	7290,433	3871,981	3849,065	1293,752	1673793

Source: Own Elaboration.

[T2] Results of the econometrical estimation.

The unit-root results (see Appendix B) through the cointegration test of first and second generation of Levin et al. (2002), and Im et al. (2003) indicated that the set of variables of the model in the long-run do have unit-roots in levels, meanwhile they are stationary in first differences, confirming the integration order as I(1), condition required to execute the cointegration analysis.

The formal tests of cointegration of Pedroni (1999) and Kao (1999) (see Appendix C) reflect the existence of cointegration within the variables for the econometrical estimation of the long-run equation. The general results are presented with the Driscoll-Kraay estimator in table 3.

Table 3. Regression of the Long-run Equation.

VARIABLES	ln HDI
ln y	0,174*** (0,0150)
ln(1 - G)	0,184*** (0,0227)
β_0	2,036*** (0,163)
Country Fixed Effects	Yes
F- Test p-value	0,0000
Within R-squared	0,8354
Observations	360
Number of groups	15

Note: Natural logarithms of the HDI, the complement of the Gini coefficient and the constant are defined as $\ln HDI$, $\ln(1 - G)$, and β_0 respectively. Robust Standard errors are in parenthesis, significance levels of *** $p < 0,01$, ** $p < 0,05$, * $p < 0,1$. Estimator Driscoll-Kraay. Calculus done using Stata 16.

Source: Own Elaboration.

About the revision of the assumptions of the long-run equation (see Appendix D), the regression satisfies the diagnostics of no-multicollinearity, normality and correct specification. However, it presents the existence of heteroscedasticity and serial correlation⁵, problems that are captured by the Driscoll-Kraay estimator to provide robust statistical inferences.

The results of the regression are considered consistent given the existing long-run relationship established by the panel data cointegration tests (Madsen, 2005), and they indicate that GDP per capita and the complement of the Gini Coefficient are statistically significant at a 1% level, to explain the welfare measured by the HDI. The impacts of both variables over the welfare are positive. It is noted that the welfare-income elasticity ($\beta = 0,174$) is lower than the elasticity of the complement of the Gini Coefficient ($\alpha = 0,184$), this situation puts in evidence that improvements in the welfare for Latin America are influenced on a first manner by the reduction of the levels of income concentration, and in a second way, by the increments in the income per capita over the long-run. The above considering that the parameters reflect that for each increase of 1% on the income per capita, there exists an increase of 0.17% in average across countries in the HDI —*ceteris paribus*— statistically significant at a 1%, consolidating an inelastic impact over the welfare. On the other hand, by each 1% increase of the complement of the Gini coefficient, there exists an increase of 0.18% in average across countries on the HDI, *ceteris paribus*, statistically significant at 1%, consolidating too an inelastic impact on the welfare.

The short-run estimation is presented in table 4 and it is noted that it posses a coefficient of adjustment towards the long-run statistically significant at 1%, negative, between zero and one expressing a stable dynamic towards the long-run relationship previously estimated. The results indicate that the speed of adjustment towards the long-run from this short-run error correction model is about 9.64% of annual correction between the variables of analysis.

Table 4. Regression from the Short-run equation

VARIABLES	$\Delta \ln HDI$
$\Delta \ln y$	0,0649*** (0,0120)
$\Delta \ln(1 - G)$	-0,00399 (0,00864)
ECT_{t-1}	-0,0964***

⁵ Particularly, the serial-correlation is expected and do not constitute a problem in the regression of the long-run equation given that the set of variables are $I(1)$ and cointegrated. The serial correlation is not an impediment to establish unbiased estimators, and in the case of cointegration, consistent, given that when a long-run relationship is proved, the property of super-consistency emerges in the parameters. See Karakitsos y Varnavides (2014) for more information.

	(0,0125)
β_0	0,00532***
	(0,000602)
Country Fixed Effects	Yes
F- Test p-value	0,0000
Within R-squared	0,1639
Observations	345
Number of groups	15

Note: Δ is the first-difference operator, ECT_t is the error correction term. Levels of statistical significant are given by *** $p < 0,01$, ** $p < 0,05$, * $p < 0,1$. Estimator: Driscoll-Kraay.

Source: Own Elaboration.

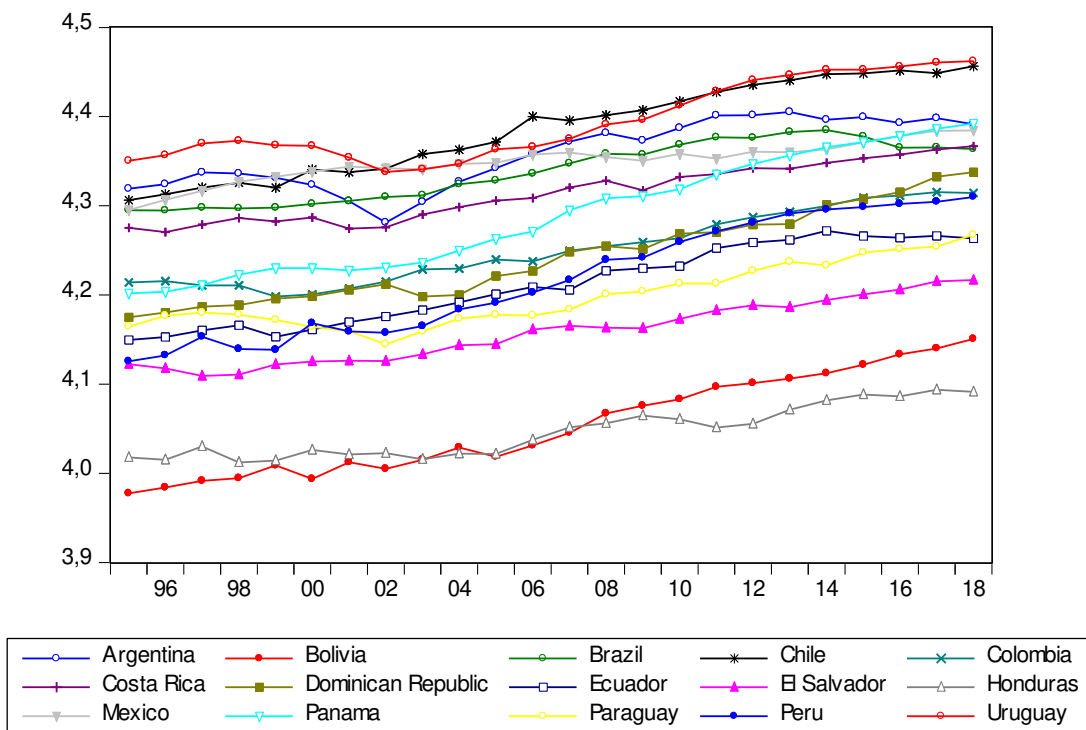
For the case of the relationship in the short-run, it is noted that only the growth of the GDP per capita is statistically significant at 1% to explain the growth in the HDI, meanwhile, the growth rate of the complement of the Gini coefficient is not statistically significant at a 10% level to explain the welfare. In the case of the GDP per capita, the elasticity of the short-run implies that for each 1% increase in the growth rate of this variable, there exists an impact of 0.06% increase in the growth of the HDI—*ceteris paribus*— in average across the countries of analysis.

Given that the complement of the Gini coefficient is not statistically significant to explain the changes in the HDI for the short-run, it is concluded that only the modifications of the structure of the income concentration do not have immediate short-run impacts, meanwhile on the long-run they tend to be determinant --and even larger-- than the income to improve the social welfare for the countries of Latin America.

The assumption checking of this short-run relationship (see Appendix D) is satisfactory, the model even when there's in presence of heteroscedasticity -- this corrected by the Driscoll-Kraay estimator -- it satisfies the assumptions of normality, no-multicollinearity, no serial-correlation and correct specification, providing a robust statistical inference of the relations in the short-run and the long-run.

As a final empirical exercise, given the regression from the long-run, the linear prediction in the sample period is done -- method known as in-sample forecast- to corroborate the behavior of the welfare trends and their existing differences in the ranking for the welfare in Latin America. The general results are presented in Figure 4 and the comparative positioning for the year 2018 in Latin America with the diverse estimation are presented in Table 5.

Figure 3. In-Sample forecasts of the long-run equation



Note: The model used for the linear prediction is the long-run equation estimated in table 3.

Fuente: elaboración propia.

It is noted that the evolution given the prediction of the HDI from the long-run reequation related to the trend and the positioning of welfare of the different countries of Latin America is similar with the results of the traditional SWF of Sen and the generalized version proposed by Mukhopadhaya.

Table 5. Ranking of countries by the measure of welfare for 2018

SWF of Sen $\beta = 1$	Generalized SWF $\beta = 0,5$	Linear Prediction $\beta = 0,174$ y $\alpha = 0,184$
1) Uruguay	1) Uruguay	1) Uruguay
2) Chile	2) Chile	2) Chile
3) Panamá	3) Argentina	3) Panamá
4) Argentina	4) México	4) Argentina
5) México	5) Panamá	5) México
6) Costa Rica	6) Costa Rica	6) Costa Rica
7) Brasil	7) República Dominicana	7) Brasil
8) República Dominicana	8) Brasil	8) Republica Dominicana
9) Colombia	9) Perú	9) Colombia
10) Perú	10) Colombia	10) Perú
11) Paraguay	11) Paraguay	11) Paraguay
12) Ecuador	12) Ecuador	12) Ecuador
13) El Salvador	13) El Salvador	13) El Salvador

14) Bolivia

14) Bolivia

14) Bolivia

15) Honduras

15) Honduras

15) Honduras

Fuente: elaboración propia.

The positioning of the countries by the level of welfare given the linear prediction estimated econometrically coincides exactly with the positioning of the countries by the traditional calculus of the original SWF of Amartya Sen for the year 2018. In this sense, it is noted that the theoretical construction of Sen (1974; 1976) is a robust instrument for the approximation and ranking of the social welfare even when it is represented by an expression of the mean income of the economies. However, it has to be noted that the SWF of Sen and the generalized version do work to establish comparisons by the levels of welfare, but not to indicate an absolute variable for the integral measure of welfare. The econometric results provide evidence that HDI can be used as a variable of analysis for the social welfare, which -in terms of trend and positioning behavior- coincides with the empirical application of the theoretical formulation of the SWF proposed by Sen (1974) in order to establish a measure of the levels of welfare.

[T1]Conclusiones

In this article it was developed a series of empirical approximations for Latin America between the period of 1995 and 2018 of the social welfare function developed by Sen (1974), and its generalized version, proposed by Mukhopadhaya (2003a). With this, the estimation of the trends of welfare by country and the ranking (by the relative positioning) respect the highest level of social welfare for the year 2018. The research -from the theoretical formulation of the SWF of Sen- explored the existence of long-run relationships between the variables of welfare, income and inequality, variables that were measured by the human development index (HDI), the real GDP per capita and the Gini Coefficient respectively. The results of the cointegration analysis confirmed the existence of stable long-run relationships between the variables and an econometrical estimation for the SWF for Latin American was done.

The econometrical methodology used an extension of the generalized SWF of Sen assuming a fixed effects component to capture the heterogeneity at a country level for Latin America, with this, a long-run relationship was estimated followed by the formulation of an error correction model extending the methodology of Engle and Granger (1987) applied to the scheme of panel data in order to establish the short-run dynamics and the stability of adjustment parameter towards the long-run equilibria between the welfare, the income and the concentration of income. The regressions used the Driscoll-Kraay estimator to provide robust statistical inferences in the presence of heterocedasticity, serial-correlation and cross-sectional dependency in the estimation of the models of the short-run and the long-run.

The econometrical results indicate that, in the long-run, the mean income of the economy and the levels of equality in the income of the Latin American countries tend both to increase the levels of social welfare. The elasticity analysis from this long-run relationship establishes a higher impact over the welfare associated on a to the changes in the income distribution, and in a lesser impact, by changes in the mean income of the economy. In the short-run, the dynamics do change substantially given that only the growth of the income is significant to explain changes in the social welfare. Respect to these general results, there exists empirical evidence that suggest that for the Latin American societies, improvements of the welfare are slightly higher when the situation of the income distribution improves compared when only the mean income of the economy increases over the long-run. For the short-run dynamics, the variable associated to the income distribution (as the complement of the Gini Coefficient) is not statistically significant, and indicates that the redistribution process of the income must be continued over time to exists some positive impact for the social welfare according to the estimations.

From this econometrical estimation of the long-run, the linear prediction was established to estimate the general trends of welfare by country. The results do coincide with the increasing evolution of the welfare trends from the estimations of the traditional SWF of Sen and the generalized version. Likewise, respect to the comparative ranking across countries by the levels of social welfare also coincide with the original ranking using the SWF formulation of Sen. These results indicate that the theoretical formulation of the traditional SWF of Sen is a useful and robust approximation to establish the behavior of the trends of welfare of an economy. It is remarked that the human development index, as a proxy variable, coincide with the behavior of the measure of the welfare from Amartya Sen's SWF.

The contribution of the theoretical formulation of the SWF of Sen is a topic barely applied in the empirical literature. For this reason, in this article --also-- it is presented the first econometrical approximation for this formulation of the social welfare function for Latin America using panel data. The general recommendation is that governments from the different countries in Latin America, attempt to implement for more re-distributional policies related to the income in order to reduce the inequality levels, given that -- and according to estimations-- the distribution of the income plays an important role over the welfare, which is slightly larger than the economic growth for the long-run.

The introduction and permanent continuation of the re-distributional policies of the income are likely to improve the social welfare over the long-run, in comparison to the actual bet focused mainly on economic growth. For this reason, it is suggested as final recommendation that the priority must be targeted -on a first place- towards the distributional policies in order to achieve equality levels of the income, and --in a second place-- towards the economic growth in order to elevate the levels of social welfare for Latin America.

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[T1]Appendix A

Aquí Mukhopadhaya (2003a) modifica la función original planteada por Sen (1974) y pone en duda que la SWF siga un comportamiento basado en el principio de Pareto. Para demostrarlo, el autor sigue el siguiente proceso planteado como teorema 1 basado en:

$$\begin{aligned} \frac{\partial W}{\partial x_i} &= \frac{\partial}{\partial x_i} [y^\beta (1 - G)] \\ &= \frac{\partial}{\partial x_i} \left[\left(\frac{\sum x_i}{n} \right)^\beta \left(1 - \frac{\sum (s_i - n - 1)x_i}{n \sum x_i} \right) \right] \\ &= \beta y^{\beta-1} \frac{1}{n} [1 - G] + y^\beta \left[\frac{0 - (2i - n - 1)n^2 y + n \sum (2i - n - 1)x_i}{(n \sum x_i)^2} \right] \\ &= \frac{\frac{1}{n} \beta y^{\beta-1} n^4 y^2 (1 - G) - n^2 y^{\beta+1} (2i - n - 1) + y^\beta n \sum (2i - n - 1)x_i}{(n^2 y)^2} \end{aligned} \quad [13]$$

La expresión anterior para regirse por el principio de Pareto tiene que ser mayor que 0. Lo que significa que

$$\begin{aligned} \frac{1}{n} \beta y^{\beta-1} n^4 y^2 (1 - G) + y^\beta n \sum (2i - n - 1)x_i &> n^2 y^{\beta+1} (2i - n - 1) \\ \rightarrow n^3 \beta y^{\beta+1} (1 - G) + y^\beta n \sum (2i - n - 1)x_i &> n^2 y^{\beta+1} (2i - n - 1) \\ \rightarrow n\beta(1 - G) + \frac{1}{ny} \sum (2i - n - 1)x_i &> (2i - n - 1) \\ \rightarrow \beta(1 - G) + \frac{\sum (2i - n - 1)x_i}{n^2 y} &> \frac{2i - n - 1}{n} \\ \rightarrow \beta - \beta G + G &> \frac{2i - n - 1}{n}, \end{aligned} \quad [14]$$

para todo $i = 1, 2, \dots, n$

Por ende, la función $W = y^\beta (1 - G)$ es paretiana si:

$$\rightarrow \beta - \beta G + G > \frac{n-1}{n}, \quad [15]$$

En este caso sí n es muy grande. La expresión se reduce a:

$$\beta - \beta G + G \geq 1, \quad [16]$$

Aquí entonces para $W = y^\beta (1 - G)$ donde $\beta < 1$ es una SWF no paretiana e indica que el bienestar social disminuirá o no aumentará si los beneficios del crecimiento económico caen solo en las manos de las personas más ricas en la sociedad.

[T1]Appendix B

Pruebas de raíces unitarias Levin-Lin-Chu

Tabla 6. Pruebas de raíces unitarias Levin-Lin-Chu

Variable	Type	Statistic	p-value	Decision
ln_HDI	Unadjusted t	-8,1038	0,0122	Unit-Root
	Adjusted t*	-2,2496		
d. ln_HDI	Unadjusted t	-14,6527	0,000	Stationary
	Adjusted t*	-6,0254		
ln_CG	Unadjusted t	-3,6502	0,0814	Unit-Root
	Adjusted t*	-1,396		
d.ln_CG	Unadjusted t	-19,2283	0,000	Stationary
	Adjusted t*	-10,2943		
ln_gdp_p	Unadjusted t	-0,7685	0,8239	Unit-Root
	Adjusted t*	0,9305		
d.ln_gdp_p	Unadjusted t	-12,1868	0,000	Stationary
	Adjusted t*	-5,278		

Nota: el símbolo d. significa primeras diferencias, GDP_P hace referencia a los valores del PIB per cápita en dólares a precios constantes de 2010. Los resultados se analizaron con un nivel de significancia del 5%.

Fuente: elaboración propia.

Pruebas de raíces unitarias Im-Pesaran-Shin

Tabla 7. Pruebas de raíces unitarias Im-Pesaran-Shin

Variable	N.Statistic	Type	Statistic	p-value	Decision
ln_HDI	t	bar	-1,6019	0,4143	Unit-Root
	t	tilde-bar	-1,4609		
	Z	t-tilde-bar	-0,2165		
d. ln_HDI	t	bar	-4,6611	0,000	Stationary
	t	tilde-bar	-3,2539		
	Z	t-tilde-bar	-9,1937		
ln_CG	t	bar	-1,1244	0,9524	Unit-Root
	t	tilde-bar	-1,0821		
	Z	t-tilde-bar	1,6685		
d.ln_CG	t	bar	-5,6567	0,0000	Stationary
	t	tilde-bar	-3,5328		
	Z	t-tilde-bar	-10,5856		
ln_gdp_p	t	bar	0,0384	1,0000	Unit-Root
	t	tilde-bar	0,0457		

	Z	t-tilde-bar	7,2814		
	t	bar	-3,4215		
d.ln_gdp_p	t	tilde-bar	-2,7396	0,0000	Stationary
	Z	t-tilde-bar	-6,6266		

Nota: el símbolo d. significa primeras diferencias, GDP_P hace referencia a los valores del PIB per cápita en dólares a precios constantes de 2010. Los resultados se analizaron con un nivel de significancia del 1%.

Fuente: elaboración propia.

[T1] Appendix C

Pruebas de cointegración

Prueba de cointegración de Pedroni en panel de datos

Tabla 8. Prueba de Cointegración de Pedroni

Tests	Statistic	p-value
Modified Phillips-Perron t	-0.0908	0,4638
Phillips-Perron t	-3.6094	0,0002
Augmented Dickey-Fuller t	-2.9901	0,0014

Nota: Ho: no cointegración, Ha: todos los paneles están cointegrados, número de paneles: 15, número de periodos= 23. Lags: 2,00 (Newey-West). Medias de panel incluidas, medias de corte transversal removidas, parámetro panel AR. Por mayoría de pruebas se acepta que existe evidencia de cointegración. Panel específico.

Fuente: elaboración propia.

Prueba de cointegración de Kao en panel de datos

Tabla 9. Prueba Kao de Cointegración

Name of the Statistics	Statistic	p-value
Modified Dickey-Fuller t	-0,9493	0,1712
Dickey-Fuller t	-2,5251	0,0058
Augmented Dickey-Fuller t	-2,4139	0,0079
Unadjusted modified Dickey Fuller t	-1,5155	0,0648
Unadjusted Dickey-Fuller t	-2,8324	0,0023

Nota: Ho: no cointegración, Ha: todos los paneles están cointegrados. número de paneles: 15, número de periodos= 23. Lags: 1,67 (Newey-West), medias de panel incluidas, vector común de cointegración entre paneles. Por mayoría de pruebas se acepta la hipótesis de cointegración.

Fuente: elaboración propia.

[T1] Appendix D

[T2] Supuestos de la regresión de largo plazo

- Normalidad

Tabla 10. Oblicuidad/Curtosis. Prueba de normalidad

Variable	Obs	Pr(Skewness)	Pr(Kurtosis) adj	chi2(2)	Prob>chi2
u	360	0,0999	0,7922	2,79	0,2481

Nota: la hipótesis nula es la distribución normal de los residuales de la ecuación de largo plazo denominados u.

Fuente: elaboración propia.

- Multicolinealidad

Tabla 11 Prueba VIF de multicolinealidad

Variable	VIF	1/VIF
In_CG	1,15	0,868147
In_gdp_p	1,15	0,868147
Mean VIF	1,15	

Nota: se acepta la hipótesis de que no hay problemas de multicolinealidad fuerte en el modelo dado que el valor medio del factor inflación varianza es menor que la "rule of thumb" del valor de 5.

Fuente: elaboración propia.

- Heterocedasticidad

Tabla 12. Prueba Wald Modificada de heterocedasticidad para la regresión del modelo de efectos fijos

chi2 (15) =	474.33
Prob>chi2 =	0,0000

Nota: la hipótesis nula es la distribución homocedástica de los errores, en este caso se rechaza esta hipótesis y se acepta la hipótesis alterna de heterocedasticidad en la regresión de efectos fijos.

Fuente: elaboración propia.

- Autocorrelación

Tabla 13. Prueba Wooldridge de autocorrelación de primer orden

F(1, 14)=	84,431
Prob> F =	0,0000

Nota: la prueba involucró los efectos fijos, H0: es la ausencia de correlación serial de primer orden, aquí se rechaza esta hipótesis y se acepta que los residuales de la ecuación de largo plazo tienen problemas de correlación serial.

Fuente: elaboración propia.

- Especificación correcta / variables omitidas

Tabla 14. Test de Ramsey de variables omitidas

F(1, 14) =	0,76
Prob> F =	0,3974

Nota: la prueba se realiza al segundo poder de los residuales, la hipótesis nula es que el modelo no tiene variables omitidas. Aquí se acepta la hipótesis nula con un nivel de significancia del 1%.

Fuente: elaboración propia.

[T2]Supuestos de la relación de corto plazo

- Normalidad

Tabla 15. Oblicuidad/Curtosis. Prueba de normalidad

Variable	Obs	Pr(Skewness)	Pr(Kurtosis) adj	chi2(2)	Prob>chi2
u_short	345	0,0431	0,8060	4,17	0,1245

Nota: la hipótesis nula es la distribución normal de los residuales de la ecuación de corto plazo representados en u_short.

Fuente: elaboración propia.

- Multicolinealidad

Tabla 16. Prueba VIF de multicolinealidad

Variable	VIF	1/VIF
D. ln_CG	1,00	0,998015
D. ln_gdp_p	1,00	0,998015
Mean VIF	1,00	

Nota: D. es el operador de primeras diferencias. Se acepta la hipótesis de que no hay problemas de multicolinealidad fuerte en el modelo dado que el valor medio del factor inflación varianza es menor que la "rule of thumb" del valor de 5.

Fuente: elaboración propia.

- Heterocedasticidad

Tabla 17. Prueba Wald modificada de heterocedasticidad para la regresión del modelo de efectos Fijos

chi2 (15) =	160,96
Prob>chi2 =	0,0000

Nota: la hipótesis nula es la distribución homocedástica de los errores, en este caso se rechaza esta hipótesis con una significancia del 1% y se acepta la hipótesis alterna de heterocedasticidad en la regresión de efectos fijos.

Fuente: elaboración propia.

- Autocorrelación

Tabla 18. Prueba Box-Pierce de autocorrelación de primer orden

Type	Statistic
Panel Rho Value	-0,0706
Box-Pierce LM Test	1,7220
P-Value > Chi2(1)	0,1894

Nota: H0: es la ausencia de correlación serial de primer orden, aquí no se rechaza esta hipótesis con un nivel de significancia del 10% y se acepta que los residuales de la ecuación de corto plazo no tienen problemas de correlación serial.

Fuente: elaboración propia.

- Especificación correcta / variables omitidas

Tabla 19. Test de Ramsey de variables omitidas

F(1, 14) =	0,27
Prob> F =	0,6098

Nota: la prueba se realiza al segundo poder de los residuales, la hipótesis nula es que el modelo no tiene variables omitidas. Aquí se acepta la hipótesis nula con un nivel de significancia del 1%.

Fuente: elaboración propia.