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When government expenditure meets bank regulation:
The impact of government expenditure on credit supply*,**

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Abstract

I develop a banking model to examine the effects of government expenditures on the credit and money supply under Basel III regulations. Purchases of goods and services from real firms or transfer payments to households as conventional government expenditures (CGEs) inject reserves into banks. Purchases of equity from banks as unconventional government expenditures (UGEs) inject equity into banks. Three Basel III regulations are examined: the capital adequacy ratio, liquidity coverage ratio, and net stable funding ratio. My results demonstrate that the CGE or UGE causes multiplier effects on the credit supply. The multiplier greater (less) than one means that banks amplify (contract) the government expenditure. Multiplier effects on the money supply in response to the CGE or UGE are also presented. My paper sheds considerable light on how government expenditure and bank regulation simultaneously affect the credit and money supply.

Keywords: Bank credit supply, Government expenditure, Basel III, Multiplier effect, Balance sheet

JEL classification: E51, E61, E62, G21, G28

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1. Introduction

Government expenditure becomes much more important when the government responds to economic recessions, such as the COVID-19 pandemic crisis and 2008 financial crisis. In fact, government expenditure also increases the liquidity or capital position of banks; such increases can lead to expansion of their balance sheets. At the same time, the expansion will be limited by regulations. In particular, as a key regulation reform in response to the 2008 financial crisis, today’s banks have to comply with the strengthened regulations introduced under the Basel III accord. This gives rise to the main question in this paper. How does government expenditure affect the supply of bank credit under Basel III regulations?

To address this issue, I develop a banking model in which the government expenditure changes the liquidity or capital position of banks and thus affects the credit and money supply under Basel III regulations. Bank balance sheets are viewed as fundamental to modeling the capital and liquidity positions and credit and money supply (Adrian and Shin, 2010a,b, 2011; Bezemer, 2010; McLeay et al., 2014). Banks expanding their balance sheets describes their simultaneously creating credit and money (Bezemer, 2010; Li and Wang, 2020; Jakab and Kumhof, 2015; McLeay et al., 2014; Werner, 2014a,b, 2016). Such creation is constrained by bank regulations (Li et al., 2017; Xing et al., 2020; Xiong et al., 2020). I adopt the modeling approach based on the balance sheet of banks to describe both the government expenditure and the regulatory rules.

I consider two types of government expenditure: the conventional government expenditure (CGE) and unconventional government expenditure (UGE). CGEs include purchases of goods and services from real firms and transfer pay-
ments to households. UGEs are purchases of equity from banks.\textsuperscript{1} Using bank balance sheets, I describe the two types of government expenditure as two types of injection shocks to the balance sheet quantities. The CGE injects reserves into banks. On the other hand, the UGE injects equity into banks.

Banks are subject to one of the three Basel III regulations: the capital adequacy ratio (CAR) (Basel Committee on Banking Supervision, 2011), the liquidity coverage ratio (LCR) (Basel Committee on Banking Supervision, 2013), and the net stable funding ratio (NSFR) (Basel Committee on Banking Supervision, 2014b).\textsuperscript{2} By using the balance sheet, I describe the regulations as the relationships between the balance sheet quantities.

The CGE as the reserve injection affects the credit and money supply by increasing reserves and deposits.\textsuperscript{3} The UGE as the equity injection affects the credit and money supply by increasing equity and reserves.\textsuperscript{4} In order to show the effects, I develop the model with three dates. At date 0, banks determine the credit supply by maximizing their profits under the regulation. At date 1, the expenditure takes place; then the balance sheets are changed by the CGE or UGE. At date 2, in response to the CGE or UGE, banks have to adjust

\textsuperscript{1}The government buying equity from banks can be viewed as an unconventional measure aimed to stabilize banking sectors. For example, during the 2008 financial crisis, the U.S. Treasury conducted the Capital Purchase Program (CPP) of the Troubled Asset Relief Program (TARP) to recapitalize banks. For more details on the CPP of the TARP, see Bayazitova and Shivdasani (2012); Calomiris and Khan (2015).

\textsuperscript{2}Basel III has two capital regulations: the CAR, a risk-based capital regulation, and the leverage ratio (Basel Committee on Banking Supervision, 2014a), a non-risk-based capital regulation. I call the risk-based capital regulation as the capital adequacy ratio (CAR). Although I only discuss the CAR, my results include those associated with the leverage ratio regulation. In fact, when the risk weights for loans and securities take the value of one in the results of the CAR, I obtain the effect of the leverage ratio.

\textsuperscript{3}As government expenditure, the CGE leads to an increase in bank reserves. As a result, the banks increase the deposits by the same amount. Thus the CGE increases reserves and deposits by the same amount as the CGE.

\textsuperscript{4}As government expenditure, the UGE injects equity by increasing reserves. So the UGE increases equity and reserves by the same amount as the UGE.
their credit supply under the regulation to again maximize their profits. This adjustment or response must follow the regulation through the corresponding regulatory relationship by which the balance sheets that were changed by the CGE or UGE at date 1 then determine the credit supply at date 2. The changes in the credit supply between date 2 and date 0 indicate the effects of the CGE or UGE.

First, under the CAR, I find the CGE does not change the credit supply. By contrast, the UGE causes a multiplier effect on the credit supply. The multiplier is greater than or equal to one. So banks amplify the UGE; the amplification is given by the multiplier. Such a multiplier is decreasing in the risk weight for loans or stringency of the CAR.

Second, as the LCR rules require, there are two regulatory regimes: cash inflows greater than or equal to three-quarters of cash outflows, denoted State H, and cash inflows less than three-quarters of cash outflows, denoted State L.

In State H, both the CGE and the UGE lead to multiplier effects on the credit supply. In general, the multipliers are greater than one. This means that the CGE and UGE will be amplified by banks; the amplifications are determined by the multipliers. The multiplier on the CGE is decreasing in the run-off rate for the deposits injected by the CGE. This implies when the risk of losses of the deposits increases, banks reduce the multiplier on the CGE. In this multiplier, substituting the sum of the deposit rate and run-off rate for the deposits injected by the CGE with the rate of return on equity, I get the multiplier on the UGE. It is decreasing in the rate of return on equity. That is, when paying a higher rate of return on equity, banks decrease the multiplier on the UGE. Also, the multiplier will be decreased by the increase in the stringency of the LCR.

The multipliers can also be explained by introducing the cash outflows per deposit and cash outflows per equity. The multiplier on the CGE is decreasing in the cash outflows per deposit associated with the CGE. By contrast, the multiplier on the UGE is decreasing in the cash outflows per equity.

Even though the LCR constraints and solutions for credit and money supply in State L differ from those in State H, the above results also hold in State L.
Third, under the NSFR, either the CGE or the UGE has a multiplier effect on the credit supply. The multiplier on the CGE can be both larger and smaller than one. As a result, the CGE can be amplified or contracted. The multiplier is increasing in the available stable funding (ASF) factor for the deposits injected by the CGE. The UGE causes the multiplier greater than one: banks amplify the UGE. One can get the multiplier on the UGE by substituting the ASF factor for deposits injected by the CGE with that for equity in the multiplier on the CGE. Because the ASF factor for equity takes the value of one, the multiplier on the UGE scaled down by the ASF factor for the deposits injected by the CGE equals the multiplier on the CGE. Both multipliers fall when the stringency of the NSFR increases.

So far, I present the changes in the credit supply in reaction to the CGE and UGE. From the balance sheets, I can get the changes in the deposits; these changes determine the responses of the money supply. On the one hand, the CGE increases deposits by the same amount. Adding the size of the CGE to the changes in the credit supply yields the changes in the money supply. On the other hand, the UGE does not change deposits; the changes in the money supply are the same as the changes in the credit supply.

My results have significant policy implications. They suggest policymakers need to take account of banks’ responses in assessing the influence of government expenditure. First, I reveal the impacts of the CGE or UGE on the credit supply under the bank regulations. These findings are helpful for better coordination of the bank regulations and fiscal policies. Second, my discussion sheds light on the government expenditure multiplier. As my paper argues, banks play a role in transmitting government expenditure. This implies that the analysis of the government expenditure multiplier can be decomposed into two parts. The

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5In fact, there is a growing consensus that the position and behavior of banks significantly affect macroeconomic performance, especially the financial and macroeconomic stability. For example, bank credit supply has an isolated channel to influence the macroeconomy (Mian and Sufi, 2018; Mian et al., 2020).
first is the ratio of the change in aggregate demand to the change in investment. The second is the ratio of the change in the investment to the government expenditure. For the first ratio, there have been a series of significant discussions, such as the Keynesian multiplier theory. For the second, my study provides a way to understand it. If investment is financed by borrowing from banks, the increase in the investment equals the increase in the deposits resulting from the government expenditure.\(^6\) So, in the second ratio, the increase in the investment can be substituted by the increase in the money supply. Such a ratio becomes the increase in the money supply divided by the government expenditure. The values of the ratio under the different Basel III regulations are presented in this study.

**Related literature.** My paper belongs to the theoretical banking literature that examines the supply of bank credit under regulations.\(^7\) First, one strand in this literature explores the relationship between the credit supply and the regulatory stringency. Many of these papers develop the models based on the bank’s maximization problems subject to the CAR and show the solutions for assets and liabilities, credit and deposits in particular. The basic result is that the increase in the stringency of the CAR causes a significant fall in the credit supply (Francis and Osborne, 2009; Furfine, 2001; Stiglitz and Greenwald, 2003). More recently, De Nicolo et al. (2014) point out an inverted U-shaped relationship between bank lending and the stringency of the CAR. Also, they find banks subject to the CAR further reduce lending when the LCR is imposed on them. Balasubramanyan and VanHoose (2013) find under the LCR, banks increase loans and deposits when the spread between security and deposit rates

\(^6\)More specifically, if the CGE takes place, the increase in the investment equals the deposits increased by the injection at date 1 and created at date 2. If the UGE takes place, the increase in the investment equals the deposits created at date 2.

\(^7\)For a broad survey of the literature on effects of bank regulations, see Martynova (2015); VanHoose (2007). The basics of the banking models can be found in Freixas and Rochet (2008).
or between loan and security rates rises. Second, a few papers focus on the relationship between the quantity of capital and the supply of credit under the CAR. Van den Heuvel (2007) shows that under the CAR the capital position of banks affects their credit supply; the decrease in equity by increasing deposit rates lowers the credit supply. Similarly, (Kopecky and VanHoose, 2004) find that the credit supply increases in response to a rise in loan rates or a fall in deposit rates when the CAR binds. Third, several studies look at adjustments of equity ratios of banks complying with the CAR. Hyun and Rhee (2011) find that to raise the equity ratios under the CAR, banks prefer to reduce loans rather than issue new equity. Zhu (2008) compares the equity ratio and the probability of bank failure under risk-based capital regulations to those under non-risk-based capital regulations. Schmaltz et al. (2014) study the reactions of banks to meet four joint Basel III regulations, the CAR, leverage ratio, LCR, and NSFR, by presenting the numerical solutions to the bank’s profit maximization problem subject to the four regulations. They find, in order to meet the regulations, banks manage their capital and liquidity positions mainly by adjusting their liabilities and equity.

This paper adds to the literature in following ways. As the main contribution, I develop a theoretical banking model to present the analytical links between the government expenditure and the credit supply by banks under Basel III regulations. The links indicate how banks complying with the regulations react to the CGE (reserve injection) or the UGE (equity injection). Moreover, such a model allows me to provide a more detailed analysis on the regulations, especially the liquidity regulations. It also presents how the money supply responds to the CGE or UGE.

An extensive literature employs macroeconomic models to explore effects of

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8In this strand of the literature, the procyclical effect on the credit supply resulting from the CAR is also a hotly debated topic (e.g., Estrella (2004); Heid (2007)).
government expenditure. The efforts most closely related to my paper are those focusing on the money-financed government expenditure, which injects money into the economy. The money-financed government expenditure is similar to the CGE in my study. The 2008 financial crisis and the COVID-19 pandemic crisis have reignited the interest in understanding the money-financed government expenditure. The current discussions hark back to the Milton Friedman’s “monetary and fiscal framework” (Friedman, 1948) and “helicopter” drop of money (Friedman, 1969). How such government injections of money affect macroeconomic performance is the main concern in this strand of the literature. Galí (2020) shows the money-financed expenditure has much larger output multipliers than the debt-financed expenditure. Similarly, Buiter (2014) also argues that a money-financed fiscal stimulus is powerful than a debt-financed one in increasing nominal aggregate demand. In Auerbach and Obstfeld (2005), money injections into the economy by open market operations (seen as a fiscal policy tool) also lead to a rise in output. The above studies show the effectiveness of money injections in stimulating the economy. These studies use macroeconomic models and examine the effects of money injections on macroeconomic performance. But they abstract from banking sectors and describe money injected into non-financial sectors. My paper complements the macroeconomic studies by modeling the CGE as a reserve injection into banks. Then I focus on how the bank credit and money supply react to the injection under Basel III regulations.

More recently, Goodhart et al. (2019) develop a dynamic stochastic general equilibrium (DSGE) model incorporating banks with the ability of creating money. They examine the money-financed government expenditure. Such expenditure is financed by the deposits that banks create when purchasing government bonds. Unlike my paper, that paper concerns the injection of government bonds into banks and does not introduce liquidity regulations.

My paper also relates to the literature investigating the impact of the UGE,
i.e., equity injections into banks. Their impacts have received much attention since the U.S. Treasury purchased bank equity under the Capital Purchase Program (CPP) of the Troubled Asset Relief Program (TARP).\(^\text{10}\) He and Krishnamurthy (2013) find that equity injections into financial intermediaries have a very strong effect on reducing the high risk premiums during crises. Gertler and Kiyotaki (2010); Hirakata et al. (2013); Kollmann and Roeger (2012); Kollmann et al. (2013) use DSGE models incorporating banks with the function of intermediating funds. They find government support for increasing bank equity generates the positive effect on output. Furthermore, Faria e Castro (2020) shows bank equity injections cause larger multiplier effects than those resulting from conventional fiscal policy tools. A growing body of empirical work examining effects of the equity injections made under the CPP. Berger and Roman (2015); Li (2013); Puddu and Waelchli (2015) show that the equity injections improve the ability of the banks to supply credit. Chang et al. (2014) find banks receiving equity injections have lower cash-to-assets ratios, which implies increases in their lending, purchasing securities, or both. Acharya et al. (2021) argue that the equity injections are crucial for stabilizing banks during the crisis. As they present, if fiscally constrained governments cannot recapitalize banks, the undercapitalization of banks leads to the decrease in their credit supply and the increase in their portfolio risk. My results are consistent with these theoretical and empirical findings in that equity injections increase the bank credit supply. Relative to these papers, I emphasize the role of the capital or liquidity regulations in determining the credit and money supply. In fact, my research highlights banks respond to the CGE or UGE by their adjustments of balance sheets, or their creation of credit and money, under the regulations. It is, however, quite common to ignore this role of banks in macroeconomic models (with few notable exceptions, particularly Jakab and Kumhof (2015)).

\(^{10}\)Calomiris and Khan (2015) provide a broad survey of the literature assessing the CPP.
This paper also contributes to the literature on the credit and money creation of banks (Bezemer, 2010; Li and Wang, 2020; Jakab and Kumhof, 2015; McLeay et al., 2014; Werner, 2014a,b, 2016). All of the papers are motivated by the need to rethink the macroeconomic role of banks in the aftermath of the 2008 financial crisis. As the literature argues, banks lend or purchase securities by creating the same amount of money. The main role of banks is to create money rather than transfer money. In line with the rethinking, balance sheets prove powerful in modeling the credit and money creation mechanism (Adrian and Shin, 2010a,b, 2011; Bezemer, 2010; McLeay et al., 2014). Such a mechanism indicates the amount of money (deposits) banks can borrow is not a direct limit on the amount of loans banks can make. Bank regulations become one of the main constraints on the size of the balance sheet and thus the supply of credit and money. By developing the models founded on balance sheets of banks, Li et al. (2017); Xing et al. (2020); Xiong et al. (2020) examine effects of the Basel III regulations on the credit and money supply. My study advances these papers by extending their models to describe the changes in bank balance sheets or the credit and money supply in reaction to the CGE or UGE, while their models offer the static relationships between the regulations and the size of bank balance sheets. In addition, my model considers the rates of return on loans, securities, deposits, and equity in the regulatory constraints of banks, whereas the interest rates are not considered in the models of Li et al. (2017); Xing et al. (2020); Xiong et al. (2020).

The rest of the paper is organized as follows. In Section 2, I present the model. I discuss the impacts of the CGE and UGE on the credit and money supply under the CAR in Section 3, under the LCR in Section 4, and under the NSFR in Section 5. Section 6 concludes.
2. The model

2.1. Timelines for bank balance sheets

There are three dates: 0, 1, and 2. Balance sheets of banks and notations at date $t$ are presented in Table 1.\(^\text{11}\) The balance sheet quantities satisfy the balance sheet identity:

$$L_t + S_t + R_t = D_t + E_t. \quad (1)$$

Banks choose loans to maximize their profits. I assume that the amount of securities are constant at $S$. Banks receive interest on loans and securities; they pay interest on deposits and dividends on equity. Taking all the revenues and expenses into account, I get the expression of the profits as

$$\Pi_t = i_L L_t + i_S S - i_D D_t - i_E E_t, \quad (2)$$

where $i_L$ is the loan rate, $i_S$ is the security rate, $i_D$ is the deposit rate, and $i_E$ is the rate of return on equity. In Eq. (2), deposits $D_t$ are not independent of loans $L_t$: banks create deposits when making loans. Substitute for $D_t$ by using the balance sheet identity in Eq. (1) to obtain

$$\Pi_t = (i_L - i_D) L_t + (i_S - i_D) S - i_D R_t - (i_E - i_D) E_t, \quad (3)$$

\(^{11}\)The balance sheet quantities are stock variables. The quantity of a stock variable at date $t$ represents that of the variable at the end of the date $t$. By contrast, interest payment, dividend payment, and government expenditure are flow variables. The amount of a flow variable at date $t$ represents that of the variable during the date $t$. 

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which indicates, as I have said, banks maximize their profits by choosing loans.

Next, I show the evolution of their balance sheets. At date 0, the equity $E_0$ equals $E$, and the reserves $R_0$ equal $R$. Banks choose $L_0$ units of loans to maximize their profits. To get the deposits at date 0, $D_0$, I rearrange the balance sheet identity in Eq. (1) as

$$D_t = L_t + S + R - E_t.$$  \hspace{2cm} (4)

Substituting the solution for $L_0$ into Eq. (4) yields $D_0$.

At date 1, the conventional government expenditure (CGE) or unconventional government expenditure (UGE) takes place. The CGE or UGE changes the balance sheet of banks: (i) the CGE injects reserves into banks; (ii) the UGE injects equity into banks.

On the one hand, the CGE as the reserve injection $RI$ increases bank reserves by $RI$; simultaneously, there is an equal increase in deposits. Table 2 shows the timeline of changes in the balance sheet of banks if the CGE takes place.

<table>
<thead>
<tr>
<th>Table 2</th>
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<tbody>
<tr>
<td>Timeline when the CGE takes place</td>
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<tr>
<td>Banks</td>
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</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tr>
<td>$L_0$</td>
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<td>$D_0 + RI$</td>
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<td>$S$</td>
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<td>$R$</td>
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<td>$R + RI$</td>
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</tbody>
</table>

On the other hand, the UGE as the equity injection $EI$ increases equity by $EI$; at the same time, reserves also increase by $EI$. Table 3 presents the timeline of changes in the balance sheet of banks if the UGE occurs.

At date 2, in response to either the CGE or the UGE, banks adjust loans to again maximize their profits. As a result, banks hold $L_2$ units of loans. Substituting the solution for $L_2$ into Eq. (4), I get the deposits at date 2, $D_2$.  \hspace{2cm} (4)
### Table 3
Timeline when the UGE takes place

<table>
<thead>
<tr>
<th>Date</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Assets</th>
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<th>Assets</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Date 0</td>
<td>$L_0$</td>
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<td>$D_0$</td>
<td>$L_2$</td>
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<tr>
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<td>$S$</td>
<td>$E$</td>
<td>$S$</td>
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<td>$S$</td>
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<td></td>
<td>$R$</td>
<td>$E$</td>
<td>$R + EI$</td>
<td>$E + EI$</td>
<td>$R + EI$</td>
<td>$E + EI$</td>
</tr>
</tbody>
</table>

I focus on the effects of the CGE or UGE on the credit and money supply. Specifically, I show the differences between loans at date 2 and loans at date 0, $L_2 - L_0$, to indicate the effects on the credit supply. The differences between deposits at date 2 and deposits at date 0, $D_2 - D_0$, show the effects on the money supply.

### 3. Government expenditure under the capital adequacy ratio

In this section, I discuss the impacts of the CGE and UGE on the credit and money supply under the capital adequacy ratio (CAR).

The CAR requires banks to hold sufficient capital to absorb adverse shocks to their capital; then it helps to avoid their insolvency. According to Basel Committee on Banking Supervision (2011), I briefly describe the rules of the CAR as follows. To comply with the CAR, banks need to maintain a minimum ratio of capital to total risk-weighted assets. The CAR is given by

$$\frac{\text{Capital}}{\text{Total risk-weighted assets}} \geq \text{car},$$

where $\text{car}$ is the required ratio, the capital equals the equity, $E_t$, the total risk-weighted assets are given by the sum of the assets multiplied by their risk weights. Denote by $\gamma_L$ the risk weight for loans and by $\gamma_S$ that for securities.
Then, the CAR constraint in Eq. (5) can be expressed as

$$\frac{E_t}{\gamma_L L_t + \gamma_S S} \geq \text{car.} \quad (6)$$

At \( t = 0 \), \( L_0^C \) are loans supplied by banks. From the objective function in Eq. (3) and the CAR constraint in Eq. (6), the bank’s maximization problem is

$$\max \Pi_0^C = (i_L - i_D)L_0^C + (i_S - i_D)S - i_D R - (i_E - i_D)E$$

over \( L_0^C \) subject to

$$\text{car}(\gamma_L L_0^C + \gamma_S S) \leq E,$$

and the nonnegativity constraint \( L_0^C \geq 0 \). In the following, at date 1, banks are hit by the CGE or UGE.

3.1. Conventional government expenditure

At \( t = 1 \), the CGE takes place: reserves \( RI \) are injected into banks. At the same time, the deposits \( D_0^C \) are increased by the same amount as the CGE. As Table 2 shows, the CGE leads to the following changes:

$$R_1 = R + RI, \quad (7)$$
$$D_1^C = D_0^C + RI. \quad (8)$$

At \( t = 2 \), in response to the CGE, banks adjust their loans to maximize their profits. Let \( L_2^{CR} \) be the loans made at date 2. Although the CGE takes place, the form of the CAR constraint in Eq. (6) does not change:

$$\text{car}(\gamma_L L_2^{CR} + \gamma_S S) \leq E. \quad (9)$$

Then the bank solves

$$\max \Pi_2 = (i_L - i_D)L_2^{CR} + (i_S - i_D)S - i_D (R + RI) - (i_E - i_D)E$$

over \( L_2^{CR} \) subject to

$$\text{car}(\gamma_L L_2^{CR} + \gamma_S S) \leq E,$$

and the nonnegativity constraint \( L_2^{CR} \geq 0 \).
3.2. Unconventional government expenditure

On the other hand, if the UGE occurs at date 1, it increases equity from $E$ to $E + EI$ and reserves from $R$ to $R + EI$. As Table 3 illustrates, the UGE causes the following changes:

$$E_1 = E + EI,$$
$$R_1 = R + EI.$$  

(10)  
(11)

In response to the UGE, banks have to adjust their loans at date 2. Denote by $L_2^{CE}$ the loans made at $t = 2$. In response to the UGE, the CAR constraint in Eq. (6) becomes

$$\text{car}(\gamma_L L_2^{CE} + \gamma_S S) \leq E + EI,$$  

(12)

where, on the right-hand side, $EI$ represents the increase in the equity arising from the UGE. Their problem is to maximize

$$\Pi_2 = (i_L - i_D)L_2^{CE} + (i_S - i_D)S - i_D(R + EI) - (i_E - i_D)E$$

over $L_2^{CE}$ subject to

$$\text{car}(\gamma_L L_2^{CE} + \gamma_S S) \leq E + EI,$$

and the nonnegativity constraint $L_2^{CE} \geq 0$.

3.3. Solution

The Lagrangians of the maximization problems before and after the CGE or UGE and their first-order conditions for the problems are given in Appendix A.

First, I discuss the impacts of the CGE. From the first-order conditions, I get the equations to determine $L_0$ and $L_2^{CR}$ as follows:

$$0 = E - \text{car}(\gamma_L L_0 + \gamma_S S),$$
$$0 = E - \text{car}(\gamma_L L_2^{CR} + \gamma_S S).$$

The solutions for loans and deposits are also given in Appendix A. Then I have the following proposition.
Proposition 1. Under the CAR, the CGE injecting reserves into banks does not change the credit supply:

\[ L_2^{CR} - L_0^C = 0. \] (13)

Proposition 1 presents that the CGE does not affect the bank credit supply. When the CGE that is worth \( RI \) units of money takes place, the CGE increases the reserves by \( RI \). But it does not change the credit supply at date 2.

By contrast, the CGE increases deposits by \( RI \) at date 1. The CGE increases the money supply by \( RI \):

\[ D_2^{CR} - D_0^C = RI. \] (14)

Second, I show the impacts of the UGE. The first-order conditions yield the equations to determine \( L_0 \) and \( L_2^{CE} \):

\[
\begin{align*}
0 &= E - \text{car}(\gamma_L L_0 + \gamma_S S), \\
0 &= E + EI - \text{car}(\gamma_L L_2^{CE} + \gamma_S S).
\end{align*}
\]

The solutions are also given in Appendix A. Proposition 2 focuses on the effect of the UGE on the credit supply.

Proposition 2. Under the CAR, the changes in the credit supply \( L_2^{CE} - L_0^C \) in response to the UGE are

\[ L_2^{CE} - L_0^C = \frac{1}{\text{car} \cdot \gamma_L} \cdot EI. \] (15)

Proposition 2 clarifies the effect of the UGE. The UGE causes that banks amplify the credit supply: a one-unit UGE increases the credit supply by \( 1/(\text{car} \cdot \gamma_L) \) units. The UGE has a multiplier effect on the credit supply; the multiplier is equal to \( 1/(\text{car} \cdot \gamma_L) \geq 1 \). The increase in \( \text{car} \) or \( \gamma_L \) increases the stringency of the CAR. Thus, a more stringent CAR means a smaller multiplier effect.

Having the changes in the credit supply given by Proposition 2, I next show the changes in the money supply. The UGE does not increase deposits at date 1. So the UGE causes the same effect on the money supply as that on the credit supply, or

\[ D_2^{CE} - D_0^C = \frac{1}{\text{car} \cdot \gamma_L} \cdot EI. \] (16)
4. Government expenditure under the liquidity coverage ratio

In this section, I turn to examine the effects of the CGE and UGE on the credit and money supply when banks are subject to the liquidity coverage ratio (LCR). The definition and calculation of the LCR are presented in the following. See Basel Committee on Banking Supervision (2013) for details on the LCR rules. The LCR is defined as

\[
\frac{\text{Unencumbered high-quality liquid assets}}{\text{Net cash outflows for the subsequent 30 calendar days}} \geq lcr, \quad (17)
\]

where \(lcr\) is the required LCR ratio. For the numerator in Eq. (17), according to the balance sheet in Table 1, reserves \(R_t\) and securities \(S\) compose the high-quality liquid assets \(HQLA_t\). Securities are subject to the haircut \(\chi\). Then,

\[
HQLA_t = R_t + (1 - \chi)S. \quad (18)
\]

For the denominator in Eq. (17), the net cash outflows for the subsequent 30 calendar days are defined as

\[
\text{Net cash outflows for the subsequent 30 calendar days} = \text{Cash outflows} - \min(\text{Cash inflows}, 0.75 \times \text{Cash outflows}). \quad (19)
\]

Next, I calculate cash inflows \(IF_t\) and cash outflows \(OF_t\). Let \(\kappa\) be the inflow percentage, and let \(\mu\) be the fraction of loans repaid. Thus, I have the cash inflows as

\[
IF_t = \kappa(\mu + i_L)\text{\(L_t\).} \quad (20)
\]

The cash outflows result from deposit run-off and dividend payment. Denote by \(\alpha\) the deposit run-off rate. The cash outflows can be written as

\[
OF_t = (\alpha + i_D)D_t + i_EE_t. \quad (21)
\]

The LCR has two separate regimes corresponding to the two forms of the net cash outflows in Eq. (19). On the one hand, if \(IF_t \geq 0.75OF_t\), i.e., \(\kappa(\mu + i_L)L_t \geq 0.75((\alpha + i_D)D_t + i_EE_t)\), the net cash outflows are given by

\[
NCOF_t = 0.25((\alpha + i_D)D_t + i_EE_t). \quad (22)
\]
Therefore the expression for the LCR in Eq. (17) becomes

\[
R + (1 - \chi)S_t \geq \frac{I_c}{0.25((\alpha + i_D)D_t + i_E E_t)}; \quad (23)
\]

On the other hand, if \( IF_t < 0.75OF_t \), i.e., \( \kappa(\mu + i_L)L_t < 0.75((\alpha + i_D)D_t + i_E E_t) \), the net cash outflows become

\[
NCOF_t = (\alpha + i_D)D_t + i_E E_t - \kappa(\mu + i_L)L_t. \quad (24)
\]

The formula for the LCR becomes

\[
R + (1 - \chi)S_t \geq \frac{I_c}{(\alpha + i_D)D_t + i_E E_t - \kappa(\mu + i_L)L_t}; \quad (25)
\]

In date 0 (date 2), the cash flow position of banks satisfies either \( IF_t \geq 0.75OF_t \) or \( IF_t < 0.75OF_t \) for \( t = 1 \) (\( t = 2 \)). I assume that the impact of the CGE or UGE on the cash flow position is small, so that neither of them leads to the switch between the two LCR regimes. The cash flow positions of banks satisfying \( IF_t \geq 0.75OF_t \) for \( t \in \{0, 2\} \) are labeled Case H. The cash flow positions meeting \( IF_t < 0.75OF_t \) for \( t \in \{0, 2\} \) are labeled Case L. Cases H and L are presented in Table 4. Case H means that the constraints both before and after the CGE or UGE are in Eq. (23). Case L indicates the constraints both before and after the CGE or UGE are in Eq. (25).

**Table 4**

<table>
<thead>
<tr>
<th>Case</th>
<th>Date 0</th>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>( IF_0 \geq 0.75OF_0 )</td>
<td>( IF_2 \geq 0.75OF_2 )</td>
</tr>
<tr>
<td>L</td>
<td>( IF_0 &lt; 0.75OF_0 )</td>
<td>( IF_2 &lt; 0.75OF_2 )</td>
</tr>
</tbody>
</table>

Table 5 presents the conditions for Cases H and L, which I derive from \( IF_0 \geq 0.75OF_0 \) and \( IF_0 < 0.75OF_0 \), respectively. Detailed derivations of the above conditions can be found in Appendix B.2 for Case H and in Appendix B.4 for Case L. In the following sections, I examine the effects of the CGE and UGE in Cases H and L.
Table 5
Conditions for Cases H and L

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>$\kappa(\mu + i_L) \geq 0.75(\alpha + i_D)$</td>
</tr>
<tr>
<td>L</td>
<td>$\kappa(\mu + i_L) &lt; 0.75(\alpha + i_D)$</td>
</tr>
</tbody>
</table>

4.1. Case H

The conditions for Case H are $IF_t \geq 0.75OF_t$ for $t \in \{0, 2\}$; the LCR constraint is given by Eq. (23). At $t = 0$, the loans are denoted by $L_H^0$. From Eq. (3) and the LCR constraint in Eq. (23), the bank’s problem is written as

$$\max \Pi_0 = (i_L - i_D)L_H^0 + (i_S - i_D)S - i_D R - (i_E - i_D)E$$

over $L_H^0$ subject to

$$0.25 lcr((\alpha + i_D)(L_H^0 + S + R - E) + i_E E) \leq R + (1 - \chi)S,$$

and the nonnegativity constraint $L_H^0 \geq 0$.

4.1.1. Conventional government expenditure

At $t = 1$, the CGE injects reserves $RI$ into banks. Reserves $R$ and deposits $D_H^0$ are simultaneously increased by $RI$. As in Section 3.1, I have

$$R_1 = R + RI,$$

$$D_H^1 = D_H^0 + RI.$$ (27)

Denote by $L_H^{2R}$ the loans made at date 2. Let $\omega$ be the run-off rate for the deposits injected by the CGE. From Eq. (22), together with Eq. (4), the net cash outflows are given by

$$NCOF_2 = 0.25((\alpha + i_D)(L_H^{2R} + S + R - E) + (\omega + i_D)RI + i_E E).$$ (28)

On the right-hand side, $(\omega + i_D)RI$ indicate the cash outflows caused by run-offs of the deposits injected by the CGE. From Eq. (23), the LCR constraint
becomes

\[ 0.25 \text{lcr}((\alpha + i_D)(L_2^{HR} + S + R - E) + (\omega + i_D)RI + i_E E) \leq R + RI + (1 - \chi)S. \]  

At \( t = 2 \), the bank solves

\[ \max \Pi_2 = (i_L - i_D)L_2^{HR} + (i_S - i_D)S - i_D(R + RI) - (i_E - i_D)E \]

over \( L_2^{HR} \) subject to

\[ 0.25 \text{lcr}((\alpha + i_D)(L_2^{HR} + S + R - E) + (\omega + i_D)RI + i_E E) \leq R + RI + (1 - \chi)S, \]

and the nonnegativity constraint \( L_2^{HR} \geq 0 \).

4.1.2. Unconventional government expenditure

On the other hand, if the UGE occurs at date 1, the government buys \( EI \) units of equity. The UGE increases equity and reserves by \( EI \). As in Section 3.2, I have

\[ E_1 = E + EI, \]
\[ R_1 = R + EI. \]

Denote by \( L_2^{HE} \) the loans issued at date 2. From Eq. (22) and Eq. (4), the net cash outflows are

\[ NCOF_2 = 0.25((\alpha + i_D)(L_2^{HE} + S + R - E) + i_E(E + EI)). \]  

On the right-hand side, \( i_E \cdot EI \) indicate the cash outflows resulting from dividend payments on the equity injected by the UGE. From Eq. (23), the LCR constraint becomes

\[ 0.25 \text{lcr}((\alpha + i_D)(L_2^{HE} + S + R - E) + i_E(E + EI)) \leq R + EI + (1 - \chi)S. \]

At \( t = 2 \), the bank solves

\[ \max \Pi_2 = (i_L - i_D)L_2^{HE} + (i_S - i_D)S - i_D(R + EI) - (i_E - i_D)(E + EI) \]

over \( L_2^{HE} \) subject to

\[ 0.25 \text{lcr}((\alpha + i_D)(L_2^{HE} + S + R - E) + i_E(E + EI)) \leq R + EI + (1 - \chi)S, \]

and the nonnegativity constraint \( L_2^{HE} \geq 0 \).
4.1.3. Solution

The Lagrangians of the maximization problems before and after the CGE or UGE and their first-order conditions are given in Appendix B.1.

First, I solve the maximization problems before and after the CGE. By the first-order conditions, I have the formulas to determine $L^H_0$ and $L^{HR}_2$ as

$$0 = R + (1 - \chi)S - 0.25lcr((\alpha + i_D)(L^H_0 + S + R - E) + i_E E),$$

$$0 = R + RI + (1 - \chi)S - 0.25lcr((\alpha + i_D)(L^{HR}_2 + S + R - E) + (\omega + i_D)RI + i_E E).$$

The solutions for loans and deposits are shown in Appendix B.1. The impact of the CGE on the credit supply is given in Proposition 3.

**Proposition 3.** When banks are subject to the LCR with $IF_0 \geq 0.75OF_0$ and $IF_2 \geq 0.75OF_2$, the changes in the credit supply in response to the CGE are given by

$$L^{HR}_2 - L^H_0 = \frac{4 - lcr(\omega + i_D)}{lcr(\alpha + i_D)} \cdot RI.$$

Proposition 3 presents three main findings about the effect of the CGE. First, the effect of the CGE on the credit supply is expressed as the size of the CGE multiplied by a multiplier. Therefore, the CGE leads to a multiplier effect on the credit supply. I have $(4 - lcr(\omega + i_D))/(lcr(\alpha + i_D)) > 1$; the CGE is amplified by banks. Second, the multiplier is decreasing in the deposit run-off rates $\alpha$ or $\omega$. A higher run-off rate for the deposits injected by the CGE $\omega$ decreases the multiplier effect. Third, the multiplier is also decreasing in the required LCR ratio, $lcr$. The increase in $lcr$ increases the stringency of the LCR. So the more stringent the LCR, the smaller the multiplier.

Since the CGE increases the deposits at date 1, I get the changes in the money supply by adding the size of the CGE to the changes in the credit supply. The changes in the money supply can be expressed as

$$D^{HR}_2 - D^H_0 = \frac{4 - lcr(\omega + i_D)}{lcr(\alpha + i_D)} \cdot RI + RI.$$
Second, I solve the maximization problems before and after the UGE. Their first-order conditions give the equations to determine $L_0^H$ and $L_2^{HE}$ as

$$0 = R + (1 - \chi)S - 0.25 \text{lcr}((\omega + i_D)(L_0^H + S + R - E) + i_E E),$$

$$0 = R + E I + (1 - \chi)S - 0.25 \text{lcr}((\omega + i_D)(L_2^{HE} + S + R - E) + i_E (E + E I)).$$

The solutions for loans and deposits are shown in Appendix B.1. The effect of the UGE on the credit supply is given in Proposition 4.

**Proposition 4.** When banks are subject to the LCR with $IF_0 \geq 0.75 OF_0$ and $IF_2 \geq 0.75 OF_2$, the changes in the credit supply $L_2^{HE} - L_0^H$ in response to the UGE are given by

$$L_2^{HE} - L_0^H = \frac{4 - \text{lcr} \cdot i_E}{\text{lcr}(\omega + i_D)} \cdot E I. \quad (36)$$

The UGE causes a multiplier effect on the credit supply. Comparing Proposition 4 to Proposition 3, I find that the multiplier resulting from the UGE can be obtained by substituting $i_E$ for $\omega + i_D$ in that caused by the CGE. I have $(4 - \text{lcr} \cdot i_E)/(\text{lcr}(\omega + i_D)) > 1$, so the multiplier on $E I$ is greater than one. This leads to the amplification of the UGE. If banks pay a higher rate of return on equity, the multiplier on the UGE will fall. A smaller multiplier also arises from a more stringent LCR by increasing $\text{lcr}$.

Because of the UGE without injecting deposits into banks at date 1, the changes in the money supply are the same as the changes in the credit supply:

$$D_2^{HE} - D_0^H = \frac{4 - \text{lcr} \cdot i_E}{\text{lcr}(\omega + i_D)} \cdot E I. \quad (37)$$

Here, I further present the difference between the multipliers arising from the CGE and the UGE. To do so, I define two variables: the cash outflows per deposit and cash outflows per equity. Eq. (28) implies the cash outflows per deposit associated with the CGE equal $\omega + i_D$. Eq. (32) suggests the cash outflows per equity equal $i_E$. As Proposition 3 shows, the multiplier caused by the CGE is decreasing in the cash outflows per deposit associated with the CGE, $\omega + i_D$. As Proposition 4 presents, the multiplier arising from the UGE is decreasing in the cash outflows per equity, $i_E$. The difference between the two

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multipliers results from the difference between the CGE and UGE: the CGE injects reserves and deposits while the UGE injects reserves and equity.

4.2. Case L

I now move to examine the effects of the CGE and UGE in Case L. I also aim to get the loan and deposit changes in response to them. The conditions for Case L are $IF_t < 0.75OF_t$ for $t \in \{0, 2\}$; the LCR constraint is given by Eq. (25). Denote by $L^L_0$ the loans issued at date 0. From Eq. (3) and the LCR constraint in Eq. (25), the bank’s problem at date 0 is

$$\max \Pi_0 = (i_L - i_D)L^L_0 + (i_S - i_D)S - i_D R - (i_E - i_D)E$$

over $L^L_0$ subject to

$$lcr((\alpha + i_D)(L^L_0 + S + R - E) + i_E E - \kappa (\mu + i_L)L^L_0) \leq R + (1 - \chi)S,$$

and the nonnegativity constraint $L^L_0 \geq 0$.

4.2.1. Conventional government expenditure

At $t = 1$, a CGE $RI$ takes place. It increases reserves and deposits by the same amount:

$$R_1 = R + RI,$$

$$D^L_1 = D^L_0 + RI.$$ (38) (39)

At $t = 2$, banks make loans $L^{LR}_2$. As in Case H, the run-off rate for the deposits injected by the CGE is denoted by $\omega$. Using Eq. (24) and Eq. (4), I can express the net cash outflows as

$$NCOF_2 = (\alpha + i_D)(L^{LR}_2 + S + R - E) + (\omega + i_D)RI + i_E E - \kappa (\mu + i_L)L^{LR}_2.$$ (40)

On the right-hand side, $(\omega + i_D)RI$ are cash outflows caused by run-offs of the deposits injected by the CGE. From Eq. (25), the LCR constraint is given by

$$lcr((\alpha + i_D)(L^{LR}_2 + S + R - E) + (\omega + i_D)RI + i_E E - \kappa (\mu + i_L)L^{LR}_2) \leq R + RI + (1 - \chi)S.$$ (41)
The bank solves

$$\max \Pi_2 = (i_L - i_D) L_2^{LR} + (i_S - i_D) S_2^{LR} - i_D (R + RI) - (i_E - i_D) E$$

over $L_2^{LR}$ subject to

$$lcr((\alpha + i_D)(L_2^{LR} + S + R - E) + (\omega + i_D) RI + i_E E - \kappa(\mu + i_L)L_2^{LR})$$

$$\leq R + RI + (1 - \chi) S,$$

and the nonnegativity constraint on loans $L_2^{LR} \geq 0$.

### 4.2.2. Unconventional government expenditure

Alternatively, at $t = 1$, an UGE $EI$ occurs. It increases equity and reserves by the same amount:

$$E_1 = E + EI,$$

$$R_1 = R + EI.$$ (42) (43)

At $t = 2$, the loans are denoted by $L_2^{LE}$. From Eq. (24) and Eq. (4), the net cash outflows are

$$NCOF_2 = (\alpha + i_D)(L_2^{LE} + S + R - E) + i_E (E + EI) - \kappa(\mu + i_L)L_2^{LE}.$$ (44)

On the right-hand side, $i_E \cdot EI$ are cash outflows caused by dividend payments on the equity injected by the UGE. From Eq. (25), the LCR constraint is given by

$$lcr((\alpha + i_D)(L_2^{LE} + S + R - E) + i_E (E + EI) - \kappa(\mu + i_L)L_2^{LE})$$

$$\leq R + EI + (1 - \chi) S.$$ (45)

Then the bank’s maximization problem becomes

$$\max \Pi_2 = (i_L - i_D) L_2^{LE} + (i_S - i_D) S - i_D (R + EI) - (i_E - i_D)(E + EI)$$

over $L_2^{LE}$ subject to

$$lcr((\alpha + i_D)(L_2^{LE} + S + R - E) + i_E (E + EI) - \kappa(\mu + i_L)L_2^{LE})$$

$$\leq R + EI + (1 - \chi) S,$$

and the nonnegativity constraint on loans $L_2^{LE} \geq 0$. 

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4.2.3. Solution

The Lagrangians for the above maximization problems and their first-order conditions are given in Appendix B.3.

First, I concern the maximization problems before and after the CGE. By their first-order conditions, I obtain the following equations to solve for $L^L_0$ and $L^L_2$:

\[
0 = R + (1 - \chi)S - lcr((\alpha + i_D)(L^L_0 + S + R - E) + i_E E - \kappa(\mu + i_L)L^L_0),
\]
\[
0 = R + RI + (1 - \chi)S - lcr((\alpha + i_D)(L^L_2 + S + R - E) + (\omega + i_D)RI + i_E E - \kappa(\mu + i_L)L^L_2).
\]

The solutions for loans and deposits are given in Appendix B.3. From the solutions, Proposition 5 follows.

**Proposition 5.** If banks are subject to the LCR with $IF_0 < 0.75OF_0$ and $IF_2 < 0.75OF_2$, the changes in the credit supply in response to the CGE are

\[
L^L_2 - L^L_0 = \frac{1 - lcr(\omega + i_D)}{lcr(\alpha + i_D - \kappa(\mu + i_L))} \cdot RI. \tag{46}
\]

First, as Proposition 5 presents, the CGE has a multiplier effect on the credit supply. In general, $(1 - lcr(\omega + i_D))/(lcr(\alpha + i_D - \kappa(\mu + i_L))) > 1$. As a result, banks amplify the CGE; the amplification is given by the multiplier. Second, the multiplier is decreasing in the deposit run-off rates $\alpha$ or $\omega$. The multiplier becomes smaller when the deposits the government pays have a larger run-off rate. Third, the multiplier is also decreasing in $lcr$, or the stringency of the LCR.

As in Case H, adding the size of the CGE to the changes in the credit supply yields the changes in the money supply:

\[
D^L_2 - D^L_0 = \frac{1 - lcr(\omega + i_D)}{lcr(\alpha + i_D - \kappa(\mu + i_L))} \cdot RI + RI. \tag{47}
\]

Second, I turn to discuss the problems before and after the UGE. Using the
first-order conditions, I have the equations to determine $L_0^L$ and $L_2^{LE}$:

$$0 = R + (1 - \chi)S - lcr((\alpha + i_D)(L_0^L + S + R - E) + i_E E - \kappa(\mu + i_L)L_0^L),$$

$$0 = R + E I + (1 - \chi)S - lcr((\alpha + i_D)(L_2^{LE} + S + R - E)
+i_E(E + EI) - \kappa(\mu + i_L)L_2^{LE}) = 0.$$  

The solutions are shown in Appendix B.3. I have Proposition 6 to show the effect of the UGE on the credit supply.

**Proposition 6.** If banks are subject to the LCR with $IF_0 < 0.75OF_0$ and $IF_2 < 0.75OF_2$, the changes in the credit supply $L_2^{LE} - L_0^L$ in response to UGE are

$$L_2^{LE} - L_0^L = \frac{1 - lcr \cdot i_E}{lcr(\alpha + i_D - \kappa(\mu + i_L))} \cdot EI. \quad (48)$$

The UGE causes a multiplier effect. In comparison to Proposition 5, Proposition 6 tells us that the multiplier caused by the UGE can be obtained by substituting $i_E$ for $\omega + i_D$ in that resulting from the CGE. Because, in general, $(1 - lcr \cdot i_E)/(lcr(\alpha + i_D - \kappa(\mu + i_L))) > 1$, the multiplier on $EI$ is larger than one. So banks amplify the UGE. The multiplier effect falls when banks pay a higher rate of return on equity $i_E$. The decrease in the multiplier can also be caused by increasing $lcr$, or the stringency of the LCR.

As in Case H, in response to the UGE, the changes in the money supply are the same as those in the credit supply, or

$$D_2^{LE} - D_0^L = \frac{1 - lcr \cdot i_E}{lcr(\alpha + i_D - \kappa(\mu + i_L))} \cdot EI. \quad (49)$$

Here, I further compare the multiplier on the CGE and that on the UGE.

As in Case H, I discuss the multipliers by using the cash outflows per deposit and cash outflows per equity. As Proposition 5 says, the multiplier on the CGE is decreasing in the cash outflows per deposit associated with the CGE, $\omega + i_D$.

As Proposition 6 presents, the multiplier on the UGE is decreasing in the cash outflows per equity, $i_E$. This difference arises from the fact that the CGE injects reserves and deposits while the UGE injects reserves and equity.
5. Government expenditure under the net stable funding ratio

The net stable funding ratio (NSFR) is defined as

\[
\frac{\text{Total available stable funding}}{\text{Total required stable funding}} \geq \text{nsfr},
\]

where \( \text{nsfr} \) denotes the required ratio, the amount of total available stable funding (ASF) is given by the sum of the liabilities multiplied by their ASF factors, and the amount of total required stable funding (RSF) equals the sum of the assets multiplied by their RSF factors. Details on the NSFR rules can be found in Basel Committee on Banking Supervision (2014b).

Following the NSFR rules, the ASF factor for equity is 100%. The ASF factor for deposits is denoted by \( \beta \). Denote the RSF factor for loans by \( \phi_L \) and that for securities by \( \phi_S \). Based on Eq. (50) and the bank balance sheet in Table 1, I rewrite the formula for the NSFR in Eq. (50) as

\[
\frac{\beta D_t + E_t}{\phi_L L_t + \phi_S S} \geq \text{nsfr}.
\]

Having the NSFR constraint in Eq. (51), I can show the bank’s problem at \( t = 0 \). Banks choose loans \( L_0^N \) to maximize their profits:

\[
\Pi_0^N = (i_L - i_D)L_0^N + (i_S - i_D)S - i_D R - (i_E - i_D)E
\]

subject to

\[
\text{nsfr}(\phi_L L_0^N + \phi_S S) \leq \beta(L_0^N + S + R) + (1 - \beta)E,
\]

and the nonnegativity constraint on loans \( L_0^N \geq 0 \).

5.1. Conventional government expenditure

At \( t = 1 \), if a CGE \( RI \) takes place, the immediate effects on the reserves \( R \) and deposits \( D_0^N \) are given by

\[
R_1 = R + RI,
\]

\[
D_1^N = D_0^N + RI.
\]
At date 2, banks make $L_{2}^{NR}$ units of loans. Denote by $\sigma$ the ASF factor for the deposits injected by the CGE. From Eq. (51) and Eq. (4), the NSFR constraint becomes

$$nsfr(\phi_L L_{2}^{NR} + \phi_S S) \leq \beta(L_{2}^{NR} + S + R) + (1 - \beta)E + \sigma \cdot RI.$$  \hspace{1cm} (54)

On the right-hand side, $\sigma \cdot RI$ represents the increase in the ASF resulting from the CGE. Then the bank's problem at date 2 becomes

$$\max \Pi_{2}^{NR} = (i_L - i_D) L_{2}^{NR} + (i_S - i_D) S - i_D (R + RI) - (i_E - i_D) E$$

over $L_{2}^{NR}$ subject to

$$nsfr(\phi_L L_{2}^{NR} + \phi_S S) \leq \beta(L_{2}^{NR} + S + R) + (1 - \beta)E + \sigma \cdot RI,$$

and the nonnegativity constraint on loans $L_{2}^{NR} \geq 0$.

5.2. Unconventional government expenditure

At $t = 1$, if an UGE $EI$ occurs, I have the increases in the equity and reserves as

$$E_1 = E + EI,$$  \hspace{1cm} (55)

$$R_1 = R + EI.$$  \hspace{1cm} (56)

Next, at $t = 2$, banks adjust loans to $L_{2}^{NE}$. Using Eq. (51) and Eq. (4), I express the NSFR constraint as

$$nsfr(\phi_L L_{2}^{NE} + \phi_S S) \leq \beta(L_{2}^{NE} + S + R) + (1 - \beta)E + EI.$$  \hspace{1cm} (57)

On the right-hand side, $EI$ represents the increase in the ASF resulting from the UGE. The maximization problem is

$$\max \Pi_{2}^{NE} = (i_L - i_D) L_{2}^{NE} + (i_S - i_D) S - i_D (R + EI) - (i_E - i_D) (E + EI)$$

over $L_{2}^{NE}$ subject to

$$nsfr(\phi_L L_{2}^{NE} + \phi_S S) \leq \beta(L_{2}^{NE} + S + R) + (1 - \beta)E + EI,$$

and the nonnegativity constraint on loans $L_{2}^{NE} \geq 0$.  

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5.3. Solution

The Lagrangians of the maximization problems before and after the CGE or UGE and their first-order conditions are shown in Appendix C.

First, I focus on the maximization problems before and after the CGE. By the first-order conditions, I have the equations determining $L^N_0$ and $L^{NR}_2$:

$$0 = \beta (L^N_0 + S + R) + (1 - \beta)E - nsfr(\phi_L L^N_0 + \phi_S S),$$

$$0 = \beta (L^{NR}_2 + S + R) + (1 - \beta)E + \sigma \cdot RI - nsfr(\phi_L L^{NR}_2 + \phi_S S).$$

The solutions for loans and deposits are given in Appendix C. The effect of the CGE on the credit supply is shown in Proposition 7.

**Proposition 7.** Under the NSFR, banks respond to the CGE by changing the credit supply as

$$L^{NR}_2 - L^N_0 = \frac{\sigma}{nsfr \cdot \phi_L - \beta} \cdot RI. \quad (58)$$

Proposition 7 shows three main findings about the effect of the CGE on the credit supply. First, the CGE causes a multiplier effect. The multiplier ranges from less than to more than one. Therefore the CGE will be amplified or contracted. Second, the multiplier is increasing in the ASF factors for deposits $\sigma$ and $\beta$. If the deposits injected by the CGE have a higher ASF factor, the CGE generates a larger multiplier. That is, the CGE leads to a larger amplification or smaller contraction of the injection. Third, the multiplier is decreasing in $nsfr$. The increase in the stringency of the NSFR decreases the multiplier.

For the money supply, the CGE increases the deposits by its size at date 1. So I get the changes in the money supply by adding the size of the CGE to the changes in the credit supply:

$$D^{NR}_2 - D^N_0 = \frac{\sigma}{nsfr \cdot \phi_L - \beta} \cdot RI + RI. \quad (59)$$

Second, I show the effects of the UGE. I obtain such effects by solving the maximization problems before and after the UGE. From the first-order condi-
tions, I have the equations determining $L_0^N$ and $L_2^{NE}$ as

$$0 = \beta(L_0^N + S + R) + (1 - \beta)E - \text{nsfr}(\phi_L L_0^N + \phi_S S),$$

$$0 = \beta(L_2^{NE} + S + R) + (1 - \beta)E + EI - \text{nsfr}(\phi_L L_2^{NE} + \phi_S S).$$

The solutions can also be found in Appendix C. The effect on the credit supply arising from the UGE is presented in Proposition 8

**Proposition 8.** Under the NSFR, in response to the UGE, banks adjust the credit supply as follows:

$$L_2^{NE} - L_0^N = \frac{1}{\text{nsfr} \cdot (\phi_L - \beta)} \cdot EI. \quad (60)$$

The UGE leads to a multiplier effect on the credit supply. By comparing Proposition 8 to Proposition 7, one can get the multiplier caused by the UGE by substituting the ASF factor for the deposits, $\sigma$, with the ASF factor for equity, 1, in the multiplier resulting from the CGE. Put differently, the multiplier on the UGE equals the multiplier on the CGE scaled up by the reciprocal of the ASF factor. Because of $1/(\text{nsfr} \cdot (\phi_L - \beta)) > 1$, banks amplify the UGE. The multiplier is decreasing in nsfr: a more stringent NSFR can reduce the multiplier effect.

In addition, because the UGE does not change deposits at date 1, the changes in the money supply are the same as those in the credit supply, i.e.,

$$D_2^{NE} - D_0^N = \frac{1}{\text{nsfr} \cdot (\phi_L - \beta)} \cdot EI. \quad (61)$$

6. Conclusion

There have been a vast literature examining the effects of government expenditure on macroeconomic performance. I pay attention to a particular characteristics of it: government expenditure changes the liquidity or capital position of banks. The conventional government expenditure (CGE) injects reserves into banks. The unconventional government expenditure (UGE) injects equity into banks. CGEs include purchases of goods and services from real firms and transfer payments to households. UGEs are purchases of equity from banks. Both
the CGE and the UGE stimulate banks to expand their balance sheets: the credit and money supply increase simultaneously. However, such expansion of the balance sheets are restricted by regulations. I consider three Basel III regulations: the capital adequacy ratio (CAR), liquidity coverage ratio (LCR), net stable funding ratio (NSFR). Considering the regulations, I aim to clarify what effects the CGE or UGE causes on the supply of bank credit and money.

I develop a model based on the balance sheet of banks. My model describes the CGE and UGE as injection shocks to banks. Each of the Basel III regulations becomes a regulatory relationship between the balance sheet quantities. I obtain the changes in the balance sheet in response to the CGE or UGE under each of the regulations. The changes in the credit supply are used to measure the effects of the CGE or UGE. At the same time, using the balance sheet identity, I obtain the changes in the money supply.

Under the CAR, the CGE does not change the credit supply. By contrast, the UGE has a multiplier effect on the credit supply. The multiplier greater than or equal to one; banks amplify the UGE. The amplification decreases when the stringency of the CAR increases.

Under the LCR, the increases in the credit supply responding to the CGE are given by multiplying the size of the CGE by multipliers. Because of the multipliers greater than one, there exist the amplifications of the CGE. The multipliers are decreasing in the run-off rate for the deposits injected by the CGE. That is, the higher the risk of the deposit outflow, the smaller the multipliers. The UGE also leads to multiple increases in the credit supply and thus multiplier effects on the credit supply. Because of the multipliers greater than one, the UGE is amplified. The multipliers can be obtained by substituting the sum of the deposit rate and the run-off rate for the deposits injected by the CGE with the rate of return on equity in the multipliers on the CGE. When banks pay a higher rate of return on equity, the multipliers will fall. In addition, the multipliers caused by both the CGE and the UGE are decreasing in the stringency of the LCR.

Under the NSFR, the CGE has a multiplier effect on the credit supply. Be-
cause the multiplier ranges from less than to more than one, banks can amplify or contract the CGE. The multiplier is increasing in the available stable funding (ASF) factor for the deposits injected by the CGE. When the deposits are more stable, the multiplier becomes larger. The UGE also causes a multiplier effect on the credit supply. The multiplier is greater than one; banks amplify the UGE. I can get the multiplier on the UGE by substituting the ASF factor for the deposits injected by the CGE with the rate of return on equity in the multiplier on the CGE. Either of the multipliers falls when the stringency of the NSFR increases.

Based on the above findings about the credit supply, I obtain how the money supply reacts to the CGE or UGE. Because of the CGE increasing deposits by the same amount, the changes in the money supply equal the size of the CGE plus the changes in the credit supply. By contrast, because of the UGE not changing deposits, the changes in the credit and money supply are the same.

My results have important policy implications. The reason is that they shed light on how government expenditure and bank regulation, two significant policy interventions, simultaneously affect banks. As I have presented, government expenditure and bank regulation are opposite: one to expand their balance sheets and the other to limit such expansion. I present the “equilibrium” positions of banks determined by the two opposite interventions. Moreover, my analysis points out a transmission mechanism of government expenditure that operates through their effects on bank credit supply.

The model I have developed is simple. The findings and insights I have provided should be seen as the foundations for future research. First, this framework can be used to show the relationships between the credit supply and the costs incurred by banks to adjust loans, deposits, or equity. To do so, I could extend my analysis by adding the adjustment costs to the bank’s objective function. Second, this framework can be a useful starting point to discuss how financial intermediations and markets react to fiscal stimuli, and how such reactions influence real sectors and macroeconomy. Such future studies help policymakers to better understand the interactions between financial regulations.
and fiscal stimulus policies.

CRedit authorship contribution statement

Boyao Li: Conceptualization; Formal analysis; Funding acquisition; Investigation; Methodology; Resources; Software; Validation; Writing - original draft; Writing - review & editing.

Declaration of Competing Interest

None.

Appendix A. The capital adequacy ratio

Lagrangians and first-order conditions. Denote by $\lambda^C_0$ the Lagrangian multiplier for the CAR constraint at date 0. Then the Lagrangian is given by

$$L^C_0 = (i_L - i_D)L^C_0 + (i_S - i_D)S^C_0 - i_D R - (i_E - i_D)E + \lambda^C_0 (E - car(\gamma_L L^C_0 + \gamma_S S^C_0)).$$

I show the first-order conditions for the problem as

$$0 = i_L - i_D - car \cdot \lambda^C_0 \gamma_L,$$
$$0 = E - car(\gamma_L L^C_0 + \gamma_S S^C_0) = 0.$$

Denote by $\lambda^CR_2$ the Lagrangian multiplier for the CAR constraint at date 2 after the CGE. Then the Lagrangian can be expressed as

$$L^CR_2 = (i_L - i_D)L^CR_2 + (i_S - i_D)S - i_D (R + RI) - (i_E - i_D)E + \lambda^CR_2 (E - car(\gamma_L L^CR_2 + \gamma_S S)).$$

I derive the first-order conditions for the problem as follows:

$$0 = i_L - i_D - car \cdot \lambda^CR_2 \gamma_L = 0,$$
$$0 = E - car(\gamma_L L^CR_2 + \gamma_S S).$$
After the UGE, the Lagrangian multiplier at date 2 is denoted by $\lambda_2^{CE}$. The Lagrangian is given by

$$L_2^{CE} = (i_L - i_D)L_2 + (i_S - i_D)S_2 - i_D(R + EI) - (i_E - i_D)E + \lambda_2^{CE}(E + EI - car(\gamma_LL_2^{CE} + \gamma_SS)).$$

The first-order conditions for the problem are

$$0 = i_L - i_D - car \cdot \lambda_2^{CE}\gamma_L,$$
$$0 = E + EI - car(\gamma_LL_2^{CE} + \gamma_SS).$$

Solutions for loans and deposits. At $t = 0$, the solutions for loans and deposits are

$$L_0^C = \frac{E - car \cdot \gamma_SS}{car \cdot \gamma_L},$$
$$D_0^C = \frac{E - car \cdot \gamma_SS}{car \cdot \gamma_L} + R + S - E.$$  

After the CGE, the date-2 loans and deposits can be expressed as

$$L_2^{CR} = L_0^C,$$
$$D_2^{CR} = D_0^C + RI.$$  

After the UGE, the date-2 loans and deposits are given by

$$L_2^{CE} = L_0^C + \frac{1}{car \cdot \gamma_L} \cdot EI,$$
$$D_2^{CE} = D_0^C + \frac{1}{car \cdot \gamma_L} \cdot EI.$$  

Appendix B. The liquidity coverage ratio

In this section, I show the Lagrangians and solutions in Cases H and L and the conditions for the cases.

Appendix B.1. Case H: Lagrangians and solutions

Lagrangians and first-order conditions. Denote by $\lambda_0^H$ the Lagrangian multiplier for the LCR constraint at date 0. Next, I show the Lagrangian at date 0 as

$$L_0^H = (i_L - i_D)L_0^H + (i_S - i_D)S_0^H - i_DR - (i_E - i_D)E + \lambda_0^H(R + (1 - \chi)S_0^H - 0.25lcr((\alpha + i_D)(L_0^H + S_0^H + R - E) + i_EE)).$$
I get the first-order conditions as follows:

\[ 0 = i_L - i_D - 0.25 \text{lcr} \cdot \lambda_H^0 (\alpha + i_D), \]
\[ 0 = R + (1 - \chi) S - 0.25 \text{lcr} ((\alpha + i_D)(L_H^0 + S + R - E) + i_E E). \]

Denote by \( \lambda_{HR}^2 \) the Lagrangian multiplier for the LCR constraint at date 2 after the CGE. Then the Lagrangian is

\[ L_{HR}^2 = (i_L - i_D)L_{HR}^2 + (i_S - i_D)S - i_D(R + RI) - (i_E - i_D)E + \lambda_{HR}^2 (R + RI + (1 - \chi)S - 0.25 \text{lcr} ((\alpha + i_D)(L_{HR}^2 + S + R - E) + i_E (E + EI)). \]

I get the first-order conditions as follows:

\[ 0 = i_L - i_D - 0.25 \text{lcr} \cdot \lambda_{HR}^2 (\alpha + i_D), \]
\[ 0 = R + RI + (1 - \chi) S - 0.25 \text{lcr} ((\alpha + i_D)(L_{HR}^2 + S + R - E) + (\omega + i_D)RI + i_E E). \]

Denote by \( \lambda_{HE}^2 \) the Lagrangian multiplier at date 2 after the UGE. Then the Lagrangian is given by

\[ L_{HE}^2 = (i_L - i_D)L_{HE}^2 + (i_S - i_D)S - i_D(R + EI) - (i_E - i_D)(E + EI) + \lambda_{HE}^2 (R + EI + (1 - \chi)S - 0.25 \text{lcr} ((\alpha + i_D)(L_{HE}^2 + S + R - E) + i_E (E + EI)). \]

The problem yields the first-order conditions as

\[ 0 = i_L - i_D - 0.25 \text{lcr} \cdot \lambda_{HE}^2 (\alpha + i_D), \]
\[ 0 = R + EI + (1 - \chi) S - 0.25 \text{lcr} ((\alpha + i_D)(L_{HE}^2 + S + R - E) + i_E (E + EI)). \]

Solutions for loans and deposits. The loans at date 0 are given by

\[ L_H^0 = \frac{(4 - lcr(\alpha + i_D))R + (4(1 - \chi) - lcr(\alpha + i_D))S + lcr(\alpha + i_D - i_E)E}{lcr(\alpha + i_D)}. \]  
\[ (B.1) \]

The deposits at date 0 are given by

\[ D_H^0 = \frac{(4 - lcr(\alpha + i_D))R + (4(1 - \chi) - lcr(\alpha + i_D))S + lcr(\alpha + i_D - i_E)E}{lcr(\alpha + i_D)} + R + S - E. \]  
\[ (B.2) \]
After the CGE, the date-2 loans and deposits are as follows:

\[ L_2^{HR} = L_0^H + \frac{4 - lcr(\omega + i_D)}{lcr(\alpha + i_D)} \cdot RI, \quad (B.3) \]
\[ D_2^{HR} = D_0^H + \left( \frac{4 - lcr(\omega + i_D)}{lcr(\alpha + i_D)} + 1 \right) \cdot RI. \quad (B.4) \]

After the UGE, the date-2 loans and deposits are given by

\[ L_2^{HE} = L_0^H + \frac{4 - lcr \cdot i_E}{lcr(\alpha + i_D)} \cdot EI, \quad (B.5) \]
\[ D_2^{HE} = D_0^H + \frac{4 - lcr \cdot i_E}{lcr(\alpha + i_D)} \cdot EI. \quad (B.6) \]

Appendix B.2. Case H: Conditions

The conditions for Case H are

\[ IF_t \geq 0.75OF_t \text{ for } t \in \{0, 2\}. \]

Before the CGE or UGE, from Eqs. (20) and (21), \( IF_0 \geq 0.75OF_0 \) becomes

\[ \kappa(\mu + i_L)L_0 \geq 0.75((\alpha + i_D)D_0 + i_E E). \quad (B.7) \]

Plug Eqs. (B.1) and (B.2) into Eq. (B.7) to get

\[ 0 \geq -(\kappa(\mu + i_L) - 0.75(\alpha + i_D) - 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))R \]
\[ -((1 - \chi)(\kappa(\mu + i_L) - 0.75(\alpha + i_D)) - 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))S \]
\[ +0.25lcr \cdot \kappa(i_E - (\alpha + i_D)))(\mu + i_L)E. \quad (B.8) \]

After the CGE, from Eqs. (20) and (21), \( IF_2 \geq 0.75OF_2 \) can be written as

\[ \kappa(\mu + i_L)L_2^{HR} \geq 0.75((\alpha + i_D)D_2^{HR} + i_E E). \quad (B.9) \]

Using Eqs. (B.3) and (B.4), I rewrite Eq. (B.9) as

\[ (\kappa(\mu + i_L) - 0.75(\alpha + i_D) - 0.25lcr \cdot \kappa(\omega + i_D)(\mu + i_L))RI \]
\[ \geq -(\kappa(\mu + i_L) - 0.75(\alpha + i_D) - 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))R \]
\[ -((1 - \chi)(\kappa(\mu + i_L) - 0.75(\alpha + i_D)) - 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))S \]
\[ +0.25lcr \cdot \kappa(i_E - (\alpha + i_D)))(\mu + i_L)E. \quad (B.10) \]
After the UGE, from Eqs. (20) and (21), the condition for $IF_2 \geq 0.75OF_2$ can be written as

$$\kappa(\mu + i_L)L_2^{HR} \geq 0.75((\alpha + i_D)D_2^{HR} + i_E(E + EI)).$$

(B.11)

Substitute Eqs. (B.5) and (B.6) into Eq. (B.11) to obtain

$$\kappa(\mu + i_L) - 0.75(\alpha + i_D) - 0.25lcr \cdot \kappa \cdot i_E(\mu + i_L))EI$$

$$\geq -(\kappa(\mu + i_L) - 0.75(\alpha + i_D) - 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))R$$

$$-((1 - \chi)(\kappa(\mu + i_L) - 0.75(\alpha + i_D)) - 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))S$$

$$+0.25lcr \cdot \kappa(i_E - (\alpha + i_D))(\mu + i_L)E.$$

(B.12)

The above inequalities can be further simplified. To do so, I present the orders of magnitude of the variables. Equity, $E$, and reserves, $R$, are large and of the order of magnitude of $10^Q$. On the other hand, the loan rate, $i_L$, security rate, $i_S$, deposit rate, $i_D$, and rate of return on equity $i_E$ are small and of the order of magnitude of $10^{-j}$. In practice, $Q$ and $j$ are greater than zero; $Q$ is much greater than $j$. As $i_L$, $i_S$, $i_D$, and $i_E$, the deposit run-off rates $\alpha$ and $\omega$ and the fraction of loans repaid $\mu$ are also small. For simplicity, I assume that they also have the order of magnitude of $10^{-j}$, the same as that of the rates of return on loans, securities, deposits, and equity. In addition, $lcr \approx 1$ and $0 < \kappa \leq 1$ are of the order of 1.

Retaining only the highest-order terms, I simplify the condition at date 0 in Eq. (B.8) to

$$\kappa(\mu + i_L) \geq 0.75(\alpha + i_D).$$

(B.13)

This is the condition for Case H, as shown in Table 5. Also retaining only the highest-order terms, I simplify the conditions at date 2 in Eqs. (B.10) and (B.12) to

$$R + RI + (1 - \chi)S \geq 0$$

(B.14)

and

$$R + EI + (1 - \chi)S \geq 0,$$

(B.15)
respectively. These conditions must hold.

**Appendix B.3. Case L: Lagrangians and solutions**

**Lagrangians and first-order conditions.** Let \( \lambda_0^L \) be the Lagrangian multiplier for the LCR constraint at date 0. Then the Lagrangian at date 0 can be written as

\[
L_0^L = (i_L - i_D)L_0^L + (i_S - i_D)S - i_D R - (i_E - i_D)E \\
+ \lambda_0^L(R + (1 - \chi)S - lcr((\alpha + i_D)(L_0^L + S + R - E) + i_E E - \kappa(\mu + i_L)L_0^L)).
\]

I obtain the first-order conditions as follows:

\[
0 = i_L - i_D - lcr \cdot \lambda_0^L(i_D + \alpha - \kappa(\mu + i_L)), \\
0 = R + (1 - \chi)S - lcr((\alpha + i_D)(L_0^L + S + R - E) + i_E E - \kappa(\mu + i_L)L_0^L).
\]

Let \( \lambda_2^{LR} \) be the Lagrangian multiplier for the LCR constraint at date 2 after the UGE. I get the Lagrangian as

\[
L_2^{LR} = (i_L - i_D)L_2^{LR} + (i_S - i_D)S - i_D(R + RI) \\
- (i_E - i_D)E + \lambda_2^{LR}(R + RI + (1 - \chi)S) \\
- lcr((\alpha + i_D)(L_2^{LR} + S + R - E) + (\omega + i_D)RI + i_E E - \kappa(\mu + i_L)L_2^{LR})).
\]

The first-order conditions are obtained as

\[
0 = i_L - i_D - lcr \cdot \lambda_2^{LR}(i_D + \alpha - \kappa(\mu + i_L)), \\
0 = R + RI + (1 - \chi)S - lcr((\alpha + i_D)(L_2^{LR} + S + R - E) \\
+ (\omega + i_D)RI + i_E E - \kappa(\mu + i_L)L_2^{LR}).
\]

After the UGE, the Lagrangian multiplier at date 2 is denoted by \( \lambda_2^{LE} \). Then the Lagrangian is

\[
L_2^{LE} = (i_L - i_D)L_2^{LE} + (i_S - i_D)S - i_D(R + EI) - (i_E - i_D)(E + EI) + \lambda_2^{LE} (R + EI) + (1 - \chi)S - lcr((\alpha + i_D)(L_2^{LE} + S + R - E) + i_E(E + EI) - \kappa(\mu + i_L)L_2^{LE})).
\]

The problem yields the first-order conditions:

\[
0 = i_L - i_D - lcr \cdot \lambda_2^{LE}(i_D + \alpha - \kappa(\mu + i_L)), \\
0 = R + EI + (1 - \chi)S - lcr((\alpha + i_D)(L_2^{LE} + S + R - E) \\
+ i_E(E + EI) - \kappa(\mu + i_L)L_2^{LE}) = 0.
\]

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Solutions for loans and deposits. The loans at date 0 are given by
\[ L_0^L = \frac{(1 - lcr(\alpha + i_D))R + (1 - \chi - lcr(\alpha + i_D))S - lcr(i_E - \alpha - i_D)E}{lcr(\alpha + i_D - \kappa(\mu + i_L))}. \]  
(B.16)

The deposits at date 0 are given by
\[ D_0^L = \frac{(1 - lcr(\alpha + i_D))R + (1 - \chi - lcr(\alpha + i_D))S - lcr(i_E - \alpha - i_D)E}{lcr(\alpha + i_D - \kappa(\mu + i_L))} + R + S - E. \]  
(B.17)

After the CGE, I solve for the loans and deposits at date 2 as
\[ L_2^{LR} = L_0^L + \frac{1 - lcr(\omega + i_D)}{lcr(\alpha + i_D - \kappa(\mu + i_L))} \cdot RI, \]  
(B.18)
\[ D_2^{LR} = D_0^L + \left( \frac{1 - lcr(\omega + i_D)}{lcr(\alpha + i_D - \kappa(\mu + i_L))} + 1 \right) \cdot RI. \]  
(B.19)

After the UGE, the solutions for loans and deposits at date 2 are
\[ L_2^{LE} = L_0^L + \frac{1 - lcr \cdot i_E}{lcr(\alpha + i_D - \kappa(\mu + i_L))} \cdot EI, \]  
(B.20)
\[ D_2^{LE} = D_0^L + \frac{1 - lcr \cdot i_E}{lcr(\alpha + i_D - \kappa(\mu + i_L))} \cdot EI. \]  
(B.21)

Appendix B.4. Case L: Conditions

The conditions for Case L are \( IF_i < 0.75OF_i \) for \( t \in \{0, 2\} \). Before the CGE or UGE, from Eqs. (20) and (21), I rewrite \( IF_0 \geq 0.75OF_0 \) as
\[ \kappa(\mu + i_L)L_0 < 0.75((\alpha + i_D)D_0 + i_E E). \]  
(B.22)

Plugging Eqs. (B.16) and (B.17) into Eq. (B.22), I have
\[ 0 \geq - (0.75(\alpha + i_D) - \kappa(\mu + i_L)) + 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))R \]
\[ - ((1 - \chi)(0.75(\alpha + i_D) - \kappa(\mu + i_L)) + 0.25lcr \cdot \kappa(\alpha + i_D)(\mu + i_L))S \]
\[ + 0.25lcr \cdot \kappa(\alpha + i_D - i_E)(\mu + i_L)E. \]  
(B.23)

After the CGE, from Eqs. (20) and (21), \( IF_2 \geq 0.75OF_2 \) can be given by
\[ \kappa(\mu + i_L)L_2^{LR} < 0.75((\alpha + i_D)D_2^{LR} + i_E E). \]  
(B.24)
Substituting Eqs. (B.18) and (B.19) into Eq. (B.24) yields

\[(0.75(\alpha+i_D) - \kappa(\mu+i_L) + 0.25lcr \cdot \kappa(\omega+i_D)(\mu+i_L))RI \]
\[\geq -(0.75(\alpha+i_D) - \kappa(\mu+i_L) + 0.25lcr \cdot \kappa(\alpha+i_D)(\mu+i_L))R \]
\[-((1-\chi)(0.75(\alpha+i_D) - \kappa(\mu+i_L)) + 0.25lcr \cdot \kappa(\alpha+i_D)(\mu+i_L))S \]
\[+0.25lcr \cdot \kappa(\alpha+i_D-i_E)(\mu+i_L)E. \]
(B.25)

After the UGE, using Eqs. (20) and (21), I have the condition for \(IF_2 \geq 0.75OF_2\) as

\[\kappa(\mu+i_L)L_{2E}^L < 0.75((\alpha+i_D)D_{2E}^L+i_E(E+EI)). \quad (B.26)\]

Substituting Eqs. (B.20) and (B.21) into Eq. (B.26), I have

\[(0.75(\alpha+i_D) - \kappa(\mu+i_L) + 0.25lcr \cdot \kappa \cdot i_E(\mu+i_L))EI \]
\[\geq -(0.75(\alpha+i_D) - \kappa(\mu+i_L) + 0.25lcr \cdot \kappa(\alpha+i_D)(\mu+i_L))R \]
\[-((1-\chi)(0.75(\alpha+i_D) - \kappa(\mu+i_L)) + 0.25lcr \cdot \kappa(\alpha+i_D)(\mu+i_L))S \]
\[+0.25lcr \cdot \kappa(\alpha+i_D-i_E)(\mu+i_L)E. \]
(B.27)

In the following, I use the same approximation as in Appendix B.2 to Eqs. (B.23), (B.25) and (B.27). Retaining only the highest-order terms, I simplify Eq. (B.8) to

\[\kappa(\mu+i_L) < 0.75(\alpha+i_D). \quad (B.28)\]

I obtain the condition for Case L, as shown in Table 5. Similarly, Eqs. (B.25) and (B.27) can be reduced to

\[R + RI + (1-\chi)S > 0 \quad (B.29)\]
and
\[R + EI + (1-\chi)S > 0, \quad (B.30)\]
respectively. The two conditions must hold.
Appendix C. The net stable funding ratio

Lagrangians and first-order conditions. Let $\lambda_0^N$ be the Lagrangian multiplier for the NSFR constraint at date 0. Then the Lagrangian at date 0 can be written as

$$\mathcal{L}_0^N = \left( i_L - i_D \right) L_0^N + (i_S - i_D)S - i_D R - (i_E - i_D)E$$
$$+ \lambda_0^N \left( \beta(L_0^N + S + R) + (1 - \beta)E - \text{nsfr}(\phi_L L_0^N + \phi_S S) \right).$$

I have the first-order conditions:

$$0 = i_L - i_D - \lambda_0^{NR} (\text{nsfr} \cdot \phi_L - \beta),$$
$$0 = \beta(L_0^{NR} + S + R) + (1 - \beta)E + \sigma \cdot RI - \text{nsfr}(\phi_L L_0^{NR} + \phi_S S).$$

Let $\lambda_2^{NR}$ be the Lagrangian multiplier for the NSFR constraint at date 2 after the CGE. Then the Lagrangian is as follows:

$$\mathcal{L}_2^{NR} = \left( i_L - i_D \right) L_2^{NR} + (i_S - i_D)S - i_D (R + RI) - (i_E - i_D)E$$
$$+ \lambda_2^{NR} \left( \beta(L_2^{NR} + S + R) + (1 - \beta)E + \sigma \cdot RI - \text{nsfr}(\phi_L L_2^{NR} + \phi_S S) \right).$$

The first-order conditions are

$$0 = i_L - i_D - \lambda_2^{NR} (\text{nsfr} \cdot \phi_L - \beta),$$
$$0 = \beta(L_2^{NR} + S + R) + (1 - \beta)E + \sigma \cdot RI - \text{nsfr}(\phi_L L_2^{NR} + \phi_S S).$$

After the UGE, the Lagrangian multiplier for the NSFR constraint at date 2 is denoted by $\lambda_2^{NE}$. I have the Lagrangian as

$$\mathcal{L}_2^{NE} = \left( i_L - i_D \right) L_2^{NE} + (i_S - i_D)S - i_D (R + EI) - (i_E - i_D)(E + EI)$$
$$+ \lambda_2^{NE} \left( \beta(L_2^{NE} + S + R) + (1 - \beta)E + EI - \text{nsfr}(\phi_L L_2^{NE} + \phi_S S) \right).$$

The first-order conditions are given by

$$0 = i_L - i_D - \lambda_2^{NE} (\text{nsfr} \cdot \phi_L - \beta),$$
$$0 = \beta(L_2^{NE} + S + R) + (1 - \beta)E + EI - \text{nsfr}(\phi_L L_2^{NE} + \phi_S S).$$

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Solutions for loans and deposits. The solution for date-0 loans is

\[ L_0^N = \frac{(1 - \beta)E + \beta R - (\text{nsfr} \cdot \phi_S - \beta)S}{\text{nsfr} \cdot \phi_L - \beta}. \] (C.1)

The solution for date-0 deposits is

\[ D_0^N = \frac{(1 - \beta)E + \beta R - (\text{nsfr} \cdot \phi_S - \beta)S}{\text{nsfr} \cdot \phi_L - \beta} + R + S - E. \] (C.2)

After the CGE, I solve for the loans and deposits at date 2 as

\[ L_2^{NR} = L_0^N + \sigma \frac{\text{nsfr} \cdot \phi_L - \beta}{\text{nsfr} \cdot \phi_L - \beta} \cdot RI, \] (C.3)

\[ D_2^{NR} = D_0^N + \left(\sigma \frac{\text{nsfr} \cdot \phi_L - \beta}{\text{nsfr} \cdot \phi_L - \beta} + 1\right) \cdot RI. \] (C.4)

After the UGE, the loans and deposits at date 2 are

\[ L_2^{NE} = L_0^N + \frac{1}{\text{nsfr} \cdot \phi_L - \beta} \cdot EI, \] (C.5)

\[ D_2^{NE} = D_0^N + \frac{1}{\text{nsfr} \cdot \phi_L - \beta} \cdot EI. \] (C.6)

Appendix D. Table of notations

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<th>Description</th>
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<tr>
<td>(S)</td>
<td>Securities</td>
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<td>(R)</td>
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<td>(E)</td>
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<td><strong>Panel B: Government expenditure</strong></td>
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<tr>
<td>$EI$</td>
<td>Unconventional government expenditure (UGE)</td>
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Panel C: Return rates

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<tr>
<td>$i_S$</td>
<td>Rate of return on Securities</td>
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<td>$i_D$</td>
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<tr>
<td>$i_E$</td>
<td>Rate of return on equity</td>
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Panel D: Regulations

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<td>$\gamma_S$</td>
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<td>High-quality liquid assets</td>
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<td>$OF$</td>
<td>Cash outflows</td>
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References


