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Regulating Credit Booms from Micro and Macro Perspectives

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(Preliminary and Incomplete)

Abstract

This study examines how micro- and macro-prudential policies work and interact with each other over the credit cycles using a dynamic general equilibrium model of financial intermediaries. Micro-prudential policies restrict the excess risk-taking of individual institutions, while taking real interest rates (prices) as given. By contrast, macro-prudential policies control the aggregate credit supplied (equilibrium outcome) by internalizing prices or the general equilibrium effect. The proposed model indicates that: (i) micro-prudential policy alone cannot completely remove inefficient credit cycles; (ii) when macro-prudential policy is conducted jointly with the micro-prudential one, policymakers can improve banks’ credit quality and remove inefficient credit cycles completely without sacrificing the total credit supply; and (iii) the contributions of micro- and macro-prudential policies to the improvement in social welfare are roughly comparable.

Keywords: Micro-prudential policy; Macro-prudential policy; Moral hazard problem; General equilibrium; Inefficient credit cycles

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1 Introduction

We have learned, from the global financial crisis, that the traditional micro-prudential regulation (or supervision) of banks, whose primary goal is to prevent the excessive risk taking of individual financial institutions, is insufficient to fully achieve system-wide financial stability. Meanwhile, macro-prudential bank regulation, developed to contain systemic risk, has been thought to be a powerful policy tool after the crisis (e.g., Hanson et al. (2011); Crockett (2000); Saporta (2009); Bakker et al. (2012)).

One of the policy concerns is how we can harmonize the macro-prudential policy with the traditional micro-prudential policy? How to design and arrange those two types of policies? While micro- and macro-prudential policies have different focuses (i.e., one focuses on the individual institutions and the other on the system as a whole), functions, and toolkits, they have some interaction, complementarity, and synergies (e.g., Osinski et al. (2013); Boissay et al. (2014); Brunnermeier et al. (2009)). Finally, to what extent can micro-prudential policy improve the efficiency, and to what extent and how can macro-prudential policy add to that? Even though there is a global regulatory shift toward macro-prudential policy, these questions remain unanswered sufficiently with a theoretical foundation.

This paper aims to fill this gap. It helps us to understand how micro- and macro-prudential policies work and interact with each other using a general equilibrium framework, and answers the above questions. To the best of my knowledge, this paper is the first to investigate the relationship and the interaction of micro- and macro-prudential policies through the lens of a general equilibrium model.

My model is built upon Martinez-Miera and Repullo (2017), but also embeds the roles of micro- and macro-prudential policies into them. Both micro- and macro-prudential policies have their own strengths and weaknesses. A micro-prudential policy can be contingent on individual bank states and choices, while taking real interest rates (prices) as given. By contrast, a macro-prudential policy can internalize the real interest rates or the general equilibrium effect, while it cannot be contingent on individual bank choices or states (see Figure 1). All these assumptions are consistent with Hanson et al. (2011).1 2

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1 Hanson et al. (2011) summarize: “A microprudential approach is one in which regulation is partial equilibrium in its conception and aimed at preventing the costly failure of individual financial institutions. By contrast, a “macroprudential” approach recognizes the importance of general equilibrium effects, and seeks to safeguard the financial system as a whole. In the aftermath of the crisis, there seems to be agreement among both academics and policymakers that financial regulation needs to move in a macroprudential direction.”

2 The difference between micro- and macro-prudential policies can come from the “technology” each authority owns. For example, at the Bank of Japan, the micro-prudential department has
In the model, individual banks intermediate a unit of goods from investors to entrepreneurs. Each penniless entrepreneur has a project that requires a unit of investment and yields a stochastic return that follows a Poisson process. The success probability of the project depends on the individual banks’ monitoring efforts (banks’ choices), which are costly and not observable by investors. Alternatively, monitoring effort can be interpreted as credit risk or credit quality for banks. Then, by imposing some macroeconomic conditions (e.g., credit market clearing and contestability conditions) on the system, macroeconomic variables (lending rate and aggregate credit supplied) are determined in the general equilibrium.

Under the above environment, I formulate and solve different equilibria with and without micro- or macro-prudential policies. Because the monitoring effort is not contractible and banks are protected by limited liability (i.e., banks can default on deposit repayments), there is a moral hazard problem: their monitoring intensities are generally smaller than the socially optimal ones and credit rationing may occur (e.g., Holmström and Tirole (1998)). A micro-prudential policy mitigates the moral hazard problem and associated credit rationing by making individual banks properly monitor loans, which boosts the total credit supplied to the economy. Micro-prudential policy (or bank supervision), taking real interest rates as given, aims to restore the efficiency at the partial equilibrium level.

many practical and field-oriented staff (supervisors), while the macro-prudential department has many more academic and research-oriented staff including macroeconomists.
However, the micro-prudential policy is not sufficient to achieve efficiency as a system, since it does not internalize real interest rates (prices), or the general equilibrium effect, by assumption. Since the social benefits of banks’ monitoring are not fully reflected in the return of loans, private banks take excessive risk (more than socially optimal) even under the optimal micro-prudential policy in the general equilibrium.

Macro-prudential policy targets the macroeconomic variables or equilibrium outcomes. In my model, macro-prudential policy controls the total credit supplied (and wealth accumulation in the dynamic case) by, for instance, imposing bank capital requirements, or taxes on the borrowing costs, and thereby controls the lending rates (and risk-free rates in the dynamic case), aiming to fix the excess risk taking as a whole system (which micro-prudential policy cannot fix).

Finally, we incorporate these mechanisms into a model of endogenous credit boom and bust cycles by endogenizing investors’ wealth accumulation. Historically, credit booms sometimes ended up in financial crises (busts), which left long-lasting and devastating damages on the real economy (Schularick and Taylor (2012); Reinhart and Rogoff (2009)). Prolonged credit booms raise the buds of crises and have real costs.

In my model, investors’ wealth accumulates over time, which pushes down the risk-free rates and credit spreads gradually (stage of the credit boom). The decrease in real interest rates discourages banks from monitoring loans and encourages them to take larger risks, which is known as the search for yield. Because the risk of entrepreneurs is correlated, some credit booms unfortunately end up in devastating busts, while others persist. In the case of credit busts, investors lose a large fraction of their wealth and then start to accumulate their wealth again. I characterize these credit cycles using ergodic distributions, and investigate how each policy changes the profile of the cycles.

My model answers the following questions.

- Can we remove inefficient credit boom and bust cycles perfectly if micro-prudential policy is conducted appropriately? The answer is negative. My model shows micro-prudential policy alone cannot completely remove inefficient credit cycles. While micro-prudential policy mitigates the moral hazard problem and, thereby, increases the total credit supplied to the economy, it does not take account of how wealth accumulation decreases real interest rates and discourages banks’ monitoring incentives. As a result, banks’ monitoring intensities become insufficient, and there still are inefficient boom and bust cycles even if the micro-prudential policy is optimally implemented.
Can we do away with a micro-prudential policy after the complete enforcement of a macro-prudential policy? The model also provides a negative answer to this question. In other words, micro-prudential policy continues being important and cannot be substituted by macro-prudential policy. Using only a macro-prudential policy (without a micro-prudential policy) results in the reduction of the aggregate credit supply, which seriously damages the macroeconomy. This is because macro-prudential policy improves banks’ monitoring incentives basically by curbing the total credit supply and keeping real interest rates from decreasing, which distorts the credit market. When macro-prudential policy is conducted jointly with micro-prudential policy, policymakers can recover the credit reduction induced by the macro-prudential policy. Namely, policymakers can improve banks’ credit quality (monitoring intensities) and remove inefficient credit cycles completely, without sacrificing the total credit supply.

How much do micro- and macro-prudential policies contribute to improving social welfare, respectively? Both static and dynamic models indicate that the contributions of micro-and macro-prudential policies to social welfare improvement are roughly comparable. Moreover, conducting macro-prudential policy jointly with micro-prudential policy improves social welfare to a level close to the first-best.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces the static model and illustrates the mechanisms by which micro- and macro-prudential policies work individually and interact with each other. Section 4 incorporates the mechanisms into the dynamic model by embedding investors’ consumption and saving decisions. Section 5 concludes the paper.

2 Literature Review

The model is built upon the works of Martinez-Miera and Repullo (2017); Dell’Ariccia et al. (2014); Boyd and De Nicolo (2005), all of which embed endogenous credit risk-taking and moral hazard problems into the banking sector. Martinez-Miera and Repullo (2017), as well as Boissay et al. (2016), model endogenous credit boom and bust cycles using a general equilibrium framework. I extend these studies by incorporating two types of policies, namely micro- and macro-prudential policies, to understand how these two types of policies interact, contribute to financial stability, and mitigate inefficient credit cycles.
This study is also related to some literature strands on the role of macro-prudential policies. First, Bianchi (2011, 2010); Lorenzoni (2008); Jeanne and Korinek (2019); Bianchi and Mendoza (2018) investigate the role of the policies in mitigating the negative externalities caused by general equilibrium effects, or pecuniary externalities. In these studies, the general equilibrium effect interacts with borrowing constraints, and generates a distortion between the solution for the social planner’s problem and that for the private agent’s one. In connection with this, Korinek and Simsek (2016); Farhi and Werning (2016) propose models in which the macro-prudential policy is effective in limiting the excessive leverage or borrowing caused by aggregate demand externalities. In these studies, the general equilibrium effects interact with nominal frictions. In my model, by contrast, the general equilibrium effect interacts with the moral hazard problem of banks associated with their monitoring intensity (Holmstrom and Tirole (1997); Holmström and Tirole (1998)). Since the social benefits of banks’ monitoring are not fully reflected in the return of loans in the general equilibrium, private banks take excessive risk even under the optimal micro-prudential policy. Macro-prudential policy limits the total credit supplied to the economy and tries to maintain banks’ monitoring incentives appropriately as a whole system.

Secondly, Farhi and Tirole (2012); Bianchi (2016); Jeanne and Korinek (2020); Diamond and Rajan (2012) develop models to study the role of macro-prudential policies in view of collective moral hazard. In short, this literature stream studies how ex-post macro bailout policies make strategic banks invest collectively ex-ante and lead to inefficient systemic risk accumulation. Macro-prudential policies, such as capital requirements, are effective in mitigating this ex-ante risk accumulation. In my dynamic model, the credit risks of banks are exogenously correlated, which could result in devastating credit busts even under the optimal micro-prudential regulations of the partial equilibrium level. I show that macro-prudential policy, combined with micro-prudential policy completely removes the harmful credit cycles caused by the risk correlations.

As another rationale for macro-prudential policies, Gertler et al. (2020) construct a model in which banks fail to internalize the impact of their leverage decisions on the likelihood of financial panic, which they call systemic run externality.

This study is also related to the literature that quantitatively investigates the impact of capital regulations on the macro economy using general equilibrium frameworks (e.g., Elenev et al. (2020); Corbae et al. (2021); Van den Heuvel (2008); Malherbe (2020); Begnau (2020)). In these studies, regulations are generally macro-prudential. I add to this literature stream by incorporating the important role of micro-prudential policy and investigating their interactions.
3 The Model

The model is built upon Martinez-Miera and Repullo (2017). Specifically, I embed the roles of micro- and macro-prudential policies in their framework.

I consider an economy with two dates \((t = 0, 1)\). The economy consists of a continuum of penniless entrepreneurs, risk-neutral investors, and banks. Each entrepreneur has a project that requires a unit of investment at \(t = 0\) and yields a stochastic return \(R^L\) when successful, and zero otherwise (following a Poisson process) at \(t = 1\). Individual banks can only fund a limited set of projects, taken to be just one for simplicity; thereby, individual banks intermediate a unit of investment goods from investors to entrepreneurs. The success probability of the project corresponds to the bank’s monitoring intensity, \(m \in [0, 1]\). While monitoring increases the success probability, it is costly to the bank. Since monitoring is unobservable for the investors, the deposit rates cannot be contingent on the monitoring intensity \(m\): monitoring intensity is not contractible.

The monitoring cost function, \(c(m)\), satisfies \(c(0) = c'(0) = 0\), \(c''(m) > 0\), and \(c'''(m) \geq 0\). Specifically, for the numerical exercises, I formulate it as:

\[
c(m) = \frac{\gamma}{2} m^2,
\]

where \(\gamma > 0\).

Individual banks choose the monitoring intensity, \(m\). Micro-prudential policy is targeted to impact individual choices of \(m\) taking interest rates as given in the model.

In the general equilibrium, by imposing some macroeconomic conditions (i.e., credit market clearing and contestability conditions) on the system, macroeconomic variables (i.e., lending rate \(R^L\) and total credit supplied \(X\)) are determined. Macro-prudential policy is aimed to impact those macroeconomic variables or equilibrium

\(^3\)In Martinez-Miera and Repullo (2017), the success probability is assumed to be \((1 - p) + m\), given an exogenous parameter \(p \in [0, 1]\). This formulation means that the project can be successful without monitoring efforts \((m = 0)\). They associate the case \((m = 0)\) with institutions that originate-to-distribute (through securitization) and the case \((m > 0)\) with traditional banks that originate-to-hold. Here, we assume the success probability as exactly equal to bank’s monitoring effort \(m\) for the simplicity of the analyses (focusing solely on the traditional banks without securitization), as in Dell’Ariccia et al. (2014). The model implications do not change qualitatively by this.
3.1 Partial Equilibrium

3.1.1 Individual Bank (Partial Equilibrium)

Given lending rate, $R^L$, an optimal contract between a bank and an investor is the deposit rate and monitoring intensity ($R^{D^*}, m^*$) that solves:

$$\max_{(R^D, m \in [0,1])} \left[ m (R^L - R^D) - c(m) \right],$$

subject to the bank’s incentive compatibility constraint:

$$m^* = \arg \max_{m \in [0,1]} \left[ m (R^L - R^{D^*}) - c(m) \right],$$

the bank’s participation constraint:

$$m^* (R^L - R^{D^*}) - c(m^*) \geq 0,$$

and the investor’s participation constraint:

$$m^* R^{D^*} = R^0,$$

where $R^0$ is the risk-free rate, which is assumed to be exogenous here but is endogenized in the later part of the paper.

An interior solution to (3) is characterized by the following first-order condition:

$$R^L - R^{D^*} - c'(m^*) = 0.$$
Solving for \( R^D \) in participation constraint (5), substituting it into first-order condition (6), and rearranging gives the following equation:

\[
c'(m^*) + \frac{R^0}{m^*} = R^L.
\]  

(7)

Since we have assumed \( c'''(m) \geq 0 \), the function in the left-hand side of (7) is convex in \( m \).

**Proposition 1.** Bank finance is feasible if equation (7) has any solutions in the feasible range \([0,1]\), in which case the optimal contract between the bank and the investor is given by:

\[
m^* = \max \left\{ m \in [0,1] \mid c'(m) + \frac{R^0}{m} \leq R^L \right\}
\]

(8)

and

\[
R^D = \frac{R^0}{m^*}.
\]

(9)

The profit of the bank is thus larger than zero:

\[
Prof = m^* R^L - R^0 - c(m^*) = m^* (R^L - c'(m^*) \frac{R^0}{m^*}) + m^* c'(m^*) - c(m^*) \geq 0
\]

(10)

where we use the convexity of the cost function \( c'(m^*) > \frac{c(m^*)}{m^*} \).

By using the functional form of monitoring cost function (1), the interior solution for \( m^* \) can be expressed as:

\[
m^* = \frac{R^L + \sqrt{(R^L)^2 - 4\gamma R^0}}{2\gamma},
\]

(11)

where we assume that \( R^L \geq 2\sqrt{\gamma R^0} \). From (11), holding lending rate \( R^L \) as constant, an increase in the risk-free rate \( R^0 \) would lead to a reduction in the monitoring
intensity $m^\ast$. This is the \textit{risk-shifting effect} induced by an increase in the funding cost for the bank; that is, an increase in funding cost exacerbates the moral hazard problem associated with limited liability. Notice that when we endogenize lending rate $R^L$ in the next subsection, the implication becomes the opposite (Dell’Ariccia et al. (2014) explain this issue in detail).

### 3.1.2 Micro-Prudential Authority (Partial Equilibrium)

Taking prices $(R^L, R^0)$ as given, the micro-prudential authority fixes the excessive risk-taking of individual banks, which is caused by limited liability. By assumption, micro-prudential authority cannot internalize the general equilibrium effects and other price-determination mechanisms.

By supervision, the micro-prudential authority makes the individual banks monitor at the level of $m^{mic\ast}$ to maximize the joint surplus, such that:

$$\max_{m \in [0,1]} \left[ mR^L - R^0 - c(m) \right].$$

(12)

An interior solution to (12) is characterized by the first-order condition:

$$c'(m^\ast) = R^L.$$  

(13)

More generally, since $R^L > c'(0)$, the solution to (12) is given by:

$$m^{mic\ast} = \max \left\{ m \in [0,1] \mid c'(m) \leq R^L \right\}.$$  

(14)

The gap between equations (13) and (7) comes from the moral hazard problem caused by the limited liability of banks. This gap is larger when the success rate $(m^\ast)$ is smaller, which means that decreasing the success probability exacerbates the moral hazard problem. Micro-prudential policy removes this gap by inspecting the risk profile of individual banks and penalizing them depending on the risks they are taking. Roughly speaking, if a bank is penalized by $(1 - m)R^D$ depending on the risk it is taking, its problem becomes $\max_m \left[ m \left( R^L - R^D \right) - c(m) - (1 - m)R^D \right]$ (instead of (3)) and the solutions of these two problems match each other (i.e., efficiency is restored). Notice that this outcome cannot be implemented by a macro-prudential policy whose targets are macroeconomic or aggregate variables. Any macro-prudential policy, by assumption, cannot be contingent on individual banks’ choices or states.
(individual risks).\(^6\)

### 3.2 General Equilibrium

This subsection incorporates the partial equilibrium model into a general equilibrium one with the exogenous risk-free rates of investors, \(R^0\). Other variables, including deposit rates \(R^D\), lending rates \(R^L\), and aggregate credit supply \(X\), are determined endogenously.\(^7\)

#### 3.2.1 Competitive Equilibrium without Policies

Since banks have positive profits (equation (10)), a usual zero-profit condition cannot be imposed to pin down the lending rate. In general equilibrium, the market solution for lending rate \(R^L\) is determined by the *contestability condition* (i.e., banks set the lowest feasible rate), as follows:

\[
R^L = \min_{m \in [0, 1]} \left( \frac{R^0}{m} + c'(m) \right). 
\]

(15)

Since we have \(\lim_{m \to 0} c''(m) = \frac{R^0}{m^2} < 0\), the solution is given by:

\[
m^* = \max \left\{ m \in [0, 1] \mid c''(m) \leq \frac{R^0}{m^2} \right\},
\]

(16)

\[
R^{L^*} = \frac{R^0}{m^*} + c'(m^*). 
\]

(17)

When the cost function is quadratic (1), the equilibrium value of monitoring \(m^*\) can be expressed as:

\[
m^* = \min \left( \sqrt{\frac{R^0}{\gamma}}, 1 \right).
\]

(18)

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\(^6\)Macro-prudential policy could penalize banks depending on the equilibrium value of monitoring \((m^*)\), instead of \(m\). However, since individual banks take \(m^*\) as given, the policy cannot directly impact individual banks’ choices.

\(^7\)In their static general equilibrium, Martinez-Miera and Repullo (2017) endogenize the risk-free rates \((R^0)\) by considering the aggregate credit supply \((w\) in their notation) as an exogenous variable. Here, I take aggregate credit supply, \(X\), as an endogenous variable to illustrate the effects of different policies. We (both papers) finally endogenize all these variables by embedding investors’ consumption and saving decisions in the dynamic parts.
Notice that (18) has the exact opposite implication of (11) (when the lending rate is exogenous) regarding the movement of monitoring intensity in reaction to the change in the risk-free rate: a reduction in risk-free rate $R^0$ leads to a reduction in monitoring intensity $m^*$. When there exists an interior solution, $m^* = \sqrt{\frac{R^0}{\gamma}}$, the equilibrium lending rate can be computed, using (17), as:

$$R^L = 2\sqrt{\gamma R^0}. \quad (19)$$

Equation (19) indicates that the equilibrium lending rate also decreases when the risk-free rate decreases \textit{(path-through effect)}. Since we have a relation $m^* = \frac{R^L - R^D}{\gamma}$ in the equilibrium, the fact that $m^*$ increases in $R^0$ means that a reduction in $R^0$ reduces $R^L = 2\sqrt{\gamma R^0}$ more than a reduction of $R^0$ reduces $\frac{R^0}{m^*}$, that is, the \textit{path-through effect} is larger than the \textit{risk-shifting effect}. This is the mechanism of the \textit{search for yield} (Dell’Ariccia et al. (2014) explain it in detail).

The credit market clears:

$$R^L = R(X), \quad (20)$$

where $R(.)$ is the aggregate demand function of the credit. Appendix shows the microfoundation of how the demand function of credit $R(X)$ is related to an economy’s production and social surplus.

**Definition 1.** Given the risk-free rate of investors, $R^0$, the competitive equilibrium \textit{without any policies} is defined as an allocation of $(m^*, X^*, R^L, R^D)$ that satisfies equations (9), (16), (17), and (20).

### 3.2.2 Competitive Equilibrium with Micro-Prudential Policy

**Definition 2.** Given the risk-free rate of investors, $R^0$, competitive equilibrium \textit{with micro-prudential policy} is defined as an allocation of $(m^{mic}, X^{mic}, R^{Lmic}, R^{Dmic})$ that satisfies equations (9), (14), (20), and the zero-profit condition of banks:

$$mR^L - R^0 - c(m) = 0. \quad (21)$$

When the cost function is quadratic as in (1), the equilibrium value of the
monitoring can be expressed as:

\[ m^{\text{mic}} = \min \left( \sqrt{\frac{2R^0}{\gamma}}, 1 \right). \] (22)

Notice that a reduction in the risk-free rate \( R^0 \) reduces the monitoring effort, \( m^{\text{mic}} \), which means we have the search for yield effect in the case with micro-prudential policy (similar to the case without any policies). Since we have relationship \( m^* = \frac{R^L_*}{\gamma} \) for the interior solution (by (13)), in the case with micro-prudential policy, only the path-through effect is present (there is no risk-shifting effect).

Figure 2 compares the competitive equilibrium without any policies \((m^*, R^L^*)\) and that with micro-prudential policy \((m^{\text{mic}}, R^{L^{\text{mic}}})\) when both solutions are interior. The horizontal axis represents the monitoring intensity (i.e., individual banks’ choice), and the vertical axis the lending rate (equilibrium outcome). In the figure, MM, ZPC, and MIC show the market solution for monitoring (7), the zero-profit condition (21), and the solution for micro authority (13), respectively.

In the equilibrium without any policies, bank credit provisions are insufficient because of the moral hazard problem. An increase in the aggregate credit supply reduces the lending rate and discourages banks from monitoring loans. When lending rate \( R^L \) decreases further to \( R^L < R^L^* \) in Figure 2, banks are unable to maintain their monitoring incentives, rendering the contract impossible: credit rationing occurs (e.g., Holmström and Tirole (1998) analyze the credit rationing caused by moral hazard problems and Stiglitz and Weiss (1981) that caused by adverse selection).

Micro-prudential policy (bank supervision) makes banks appropriately monitor loans. As a result, it mitigates the credit rationing caused by the moral hazard problem and boosts the aggregate credit supply (lending rates decrease).
Figure 2: Determinations of equilibria (without any policies and with micro-prudential policy)

Note: The horizontal axis shows the monitoring intensity (i.e., individual banks’ choice) and the vertical axis the lending rate (equilibrium outcome). MM, ZPC, and MIC show the market solution for monitoring (7), the zero-profit condition (21), and the solution for micro authority (13), respectively.

3.3 The First-Best Allocation

Here, we define the first-best allocation in the model.

A social planner chooses the total credit supplied, $X$, and the monitoring intensity of individual banks, $m$, to maximize social surplus without any constraints as:

$$\max_{X,m \in [0,1]} S(X)m - c(m)X - R^0 X,$$

where concave function $S(X)$ is the social surplus from the production associated with the credit supplied, $X$. Appendix shows the microfoundation for the social surplus using the demand of final good producers that use entrepreneurs’ output as intermediate input.
The optimal condition for $X$ can be expressed as:

$$S'(X)m - c(m) - R^0 = 0,$$  \hspace{1cm} (24)

$$\iff R^Lm - c(m) - R^0 = 0,$$  \hspace{1cm} (25)

where we use condition (52) ($S'(X) = R(X) = R^L$). Notice that this is the same as the zero-profit condition in (21).

The optimal condition for $m$ can be expressed as:

$$m^{FB} = \max\left\{ m \in [0, 1] \mid c'(m) \leq \frac{S(X)}{X}\right\}.$$  \hspace{1cm} (26)

Specifically, an interior solution is given by:

$$S(X) = c'(m).$$  \hspace{1cm} (27)

I compare conditions (13) for the micro-prudential authority and (27) for the social planner. Since $R^L = R(X) = \frac{ds(X)}{dX} < \frac{s(X)}{X}$ (from the concavity of $S(X)$), the social planner chooses larger monitoring than the micro-prudential authority does. This is because the micro-prudential authority, taking the price $R^L$ as given, cannot internalize the macroeconomic, or general equilibrium, effects. In other words, the social benefits of their monitoring are not fully reflected in the returns of loans, which makes banks take excessive risk and lend too much (more than socially optimal), even under the optimal micro-prudential policy in the general equilibrium (see Figure 4).

Following Martinez-Miera and Repullo (2017), I formulate:

$$S(X) = \frac{a^\sigma}{\sigma - 1} \left( \frac{X}{a} \right)^{-\frac{\sigma - 1}{\sigma}},$$  \hspace{1cm} (28)

which implies:

$$R(X) = \frac{dS(X)}{dX} = \left( \frac{X}{a} \right)^{-\frac{1}{\sigma}}.$$  \hspace{1cm} (29)

Appendix shows the microfoundation for the relation between credit demand function $R(X)$ and the economy’s production and social surplus $S(X)$.  

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Solving conditions (25) and (26) yields the first-best allocation:

\[
m^{FB} = \begin{cases} 
\min \left(\sqrt{\frac{R^0}{\gamma \left(\frac{2^2}{\sigma} - \frac{1}{2}\right)}}, 1\right) & \text{when } \sigma > 2 \\
1 & \text{when } \sigma \leq 2,
\end{cases}
\]  

(30)

\[
R_L^{FB} \equiv \left(\frac{x^{FB}}{a}\right)^{-\frac{1}{\sigma}} = \frac{\gamma}{2} m^{FB} + \frac{R^0}{m^{FB}}.
\]  

(31)

### 3.4 Macro-Prudential Policy

This subsection demonstrates how the macro-prudential authority improves social welfare by internalizing general equilibrium effects, which the micro-prudential authority cannot.

We saw that the micro-prudential policy can influence the choices of individual banks, while it takes prices (real interest rates) as given. By contrast, the macro-prudential policy targets macroeconomic variables, or equilibrium outcomes. In my model, it controls the total credit supplied (and associated lending rates). In practice, it can be implemented by capital requirements or by imposing taxes on borrowing costs.\(^8\) Here, we focus on the allocation rather than its implementation. In subsection 4.5, I will show how to implement the allocation by imposing taxes on borrowing costs for banks.

#### 3.4.1 Macro-Prudential Policy without Micro-Prudential Policy

When the economy has no micro-prudential authority, the equilibrium value of monitoring is determined by equation (7). Taking this condition as given, the macro-prudential authority chooses the total (aggregate) credit supplied to maximize social welfare as:

\[
\max_{X, m} S(X)m - c(m)X - R^0 X
\]

subject to (7).

The solution to this problem can be interpreted as a constrained-efficient allocation, in which the social planner faces the same constraints as the private economy in

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\(^{8}\text{Martinez-Miera and Repullo (2019) incorporate macro-prudential policy (capital requirements) and monetary policy into the framework of Martinez-Miera and Repullo (2017).}\)
(7), but she considers the relationship between the amount of investment and the equilibrium returns of loans.

The first-order condition with respect to \( X \) can be expressed as:

\[
R^L m - c(m) - R^0 + X \frac{dR(X)}{dX} \frac{dm}{dR^L} \left\{ \frac{S(X)}{X} - c'(m) \right\} = 0
\]

\[
\iff R^L m = \frac{R^L}{\sigma} \left\{ \frac{\sigma}{\sigma - 1} R^L - \gamma m \right\} \frac{m^2}{\gamma m^2 - R^0}
\]

benefit of marginal credit reduction through the increase in monitoring

\[
+ c(m) + R^0.
\]

Equation (32) indicates that macro-prudential authority balances the cost of the marginal credit reduction (LHS) and its benefit (RHS), taking the general equilibrium effects and private agents’ actions into account.

**Definition 3.** Given the risk-free rate of investors, \( R^0 \), a competitive equilibrium with macro-prudential policy but without micro-prudential policy is defined as an allocation of \((m^\text{mac}, X^\text{mac}, R^L^\text{mac})\) that satisfies conditions (7), (32), and (20).

Figure 3 compares the competitive equilibrium without any policies \((m^*, R^L^*)\) with that with only macro-prudential policy \((m^\text{mac}, R^L^\text{mac})\) when both are interior solutions. In the figure, MM and MAC show the market solution for monitoring (7) and the solution for macro-prudential authority (32), respectively.

As shown in Figure 3, to maintain banks’ monitoring incentives, macro-prudential policy sacrifices credit provision and raises lending rate \( R^L \). In this regard, the macro-prudential policy is inferior to the micro-prudential policy, which increases not only banks’ monitoring intensities, but also total credit supplied, as discussed in subsection 3.2.1. Meanwhile, the macro-prudential policy can internalize the general equilibrium effects, as can be seen from the above formulation, while the micro-prudential policy cannot internalize these effects. To summarize, both micro- and macro-prudential policies have their own strengths and weaknesses.
Figure 3: Determinations of equilibria (without any policies and with only macro-prudential policy)

Note: The horizontal axis represents the monitoring intensity (i.e., individual banks’ choice) and the vertical axis the lending rate (equilibrium outcome). MM and MAC show the market solution for monitoring (7) and the solution for the macro authority (32), respectively.

3.4.2 Macro-Prudential Policy with Micro-Prudential Policy

When the economy has micro-prudential authority, the equilibrium value of monitoring is determined by equation (13). Taking this condition as given, the macro-prudential authority chooses the total credit supplied to maximize social welfare as:

$$\max_{X,m} S(X)m - c(m)X - R^0X$$

subject to (13).

The first-order condition with respect to $X$ can be expressed as:

$$R^L m - c(m) - R^0 + X \frac{dR(X)}{dX} \frac{dm}{dR^L} \left\{ \frac{S(X)}{X} - c'(m) \right\} = 0$$

$$\iff R^L m = \frac{R^L}{\sigma} \left\{ \frac{\sigma}{\sigma - 1} R^L - \gamma m \right\} \left\{ \frac{1}{\gamma} \right\}$$

benefit of marginal credit reduction through the increase in monitoring

$$+ c(m) + R^0.$$ (33)
Equation (33) means that macro-prudential policy balances the cost of the marginal credit reduction (LHS) and its benefit (RHS), taking the general equilibrium effects and micro-prudential policy into account.

**Definition 4.** Given the risk-free rate of investors, $R^0$, competitive equilibrium with both macro- and micro-prudential policies is defined as an allocation of $(m^{both}, X^{both}, R^{L^{both}})$ that satisfies conditions (13), (20), and (33).

Figure 4 compares the competitive equilibria with only the micro-prudential policy $(m^{mic}, R^{L^{mic}})$, both macro- and micro-policies $(m^{both}, R^{L^{both}})$, and the first-best solution $(m^{FB}, R^{L^{FB}})$ when they are interior solutions. In the figure, SPM, SPX, MIC, and MAC show the social planner’s solutions for monitoring (27) and total credit amount (25) (the same as the zero-profit condition), the solution for micro-prudential authority (13), and the solution for macro-prudential authority (33), respectively.

This figure explains how much the micro-prudential policy can accomplish alone and how much the macro-prudential policy can supplement it. The difference between the first-best allocation $(m^{FB}, R^{L^{FB}})$ and the equilibrium with micro-prudential policy $(m^{mic}, R^{L^{mic}})$ comes from the difference between SPM and MIC (i.e., the fact that the micro-prudential policy cannot internalize the general equilibrium effect). Then, the macro-prudential authority attempts to increase individual banks’ monitoring intensity from $m^{mic}$ to $m^{FB}$, but the authority cannot do that directly because the authority cannot impact individual banks’ choices. Instead, the macro-prudential authority increases banks’ monitoring incentives by decreasing the aggregate credit supplied and increasing the associated real interest rates. However, an excessive credit reduction is harmful to the economy. At point $(m^{both}, R^{L^{both}})$, the merit (increasing banks’ monitoring intensity) and demerit (decreasing total credit supplied) of the macro-prudential policy balance each other, and social welfare is maximized under constraint (13).
Figure 4: Determinations of the first-best allocation and the equilibria (with only micro-prudential policy and both policies)

Note: The horizontal axis shows the monitoring intensity (i.e., individual banks’ choice) and the vertical axis the lending rate (equilibrium outcome). SPM, SPX, MIC, and MAC show the social planner’s solutions for monitoring (27) and total credit amount (25) (the same as the zero-profit condition), the solution for micro authority (13), and the solution for macro authority (33), respectively.

3.5 Implications for Static Analyses

Here, I compute and compare outcomes for each equilibrium. The model parameters are shown in Table 1. An important caveat is that the purpose of the numerical analyses (conducted in this subsection and the next section) is not to obtain some rigid quantitative evaluations of the model equilibria, but rather obtain qualitative evaluations or rough figures of the equilibria, as in Martinez-Miera and Repullo (2017). Parameter values are chosen for this purpose and are not calibrated to yield realistic values of the endogenous variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target (Source)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>4</td>
<td>Martinez-Miera and Repullo (2017)</td>
</tr>
<tr>
<td>$a$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Martinez-Miera and Repullo (2017)</td>
</tr>
<tr>
<td>$R^0$</td>
<td>1</td>
<td>Risk-free rate = 0%</td>
</tr>
</tbody>
</table>
Table 2 shows the equilibrium outcomes for each equilibrium. In the table, social welfare is defined as $S(X)m - c(m)X - R^0X$. As demonstrated in subsection 3.4, the macro-prudential policy sacrifices credit provisions to keep banks’ monitoring incentives. However, when the policy is jointly used with the micro-prudential one, policymakers can make up for the decrease in the credit supplied without deteriorating the credit quality (i.e., keeping banks monitoring appropriately).

Using the macro-prudential policy jointly with the micro-prudential policy improves social welfare to a level (0.313) close to the first-best (0.333).

Contributions of micro- and macro-prudential policies to improving social welfare are roughly comparable. Specifically, the micro-prudential policy boosts social welfare by 60 percent, from 0.156 to 0.25. The macro-prudential policy boosts welfare by an additional 40 percent, from 0.25 to 0.313.

<table>
<thead>
<tr>
<th></th>
<th>No policies</th>
<th>Micro</th>
<th>Macro</th>
<th>Both</th>
<th>FB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitoring intensity ($m$)</td>
<td>0.5</td>
<td>0.707</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Lending rate ($R^L$)</td>
<td>4</td>
<td>2.83</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Total credit supplied ($X$)</td>
<td>0.0625</td>
<td>0.125</td>
<td>0.04</td>
<td>0.0625</td>
<td>0.111</td>
</tr>
<tr>
<td>Social welfare</td>
<td>0.156</td>
<td>0.25</td>
<td>0.28</td>
<td>0.313</td>
<td>0.333</td>
</tr>
</tbody>
</table>

4 Dynamic Analyses: Incorporation into Credit Cycles

So far, we reviewed the static version of the model and its implications. In this section, I extend the model to a dynamic one (with an infinite time horizon) by endogenizing investors’ wealth accumulation and associated risk-free rates, following Martinez-Miera and Repullo (2017). The wealth accumulation at any period is the outcome of investors’ consumption and saving decisions in the previous period.

At each date $t$, there is a continuum of one measure of infinitely-lived investors with $W_t$ wealth ($W_t$ is the only state variable in the economy). Investors have a
risk-neutral utility, $u(c_t) = c_t$, and a discount future by $0 < \beta < 1$. These investors fund banks, which in turn fund one-period-lived penniless entrepreneurs’ projects.

The returns of entrepreneurs’ projects are imperfectly correlated. The project of entrepreneur $i$ is driven by the realization of a latent random variable:

$$y_{it} = \Phi^{-1}(1 - m_{it}) + \sqrt{\rho}z_t + \sqrt{1 - \rho}\varepsilon_{it}, \quad (34)$$

where $z_t$ is a systemic risk factor that affects all entrepreneurs, and $\varepsilon_{it}$ is an idiosyncratic risk factor that only affects entrepreneur $i$’s project. We assume that $z_t$ and $\varepsilon_{it}$ are standard normal random variables, independently distributed from each other and over time, as well as, in the case of $\varepsilon_{it}$, across entrepreneurs. Parameter $0 < \rho < 1$ determines the extent of the correlation between the returns of entrepreneurs’ projects, $\Phi(.)$ denotes the cumulative distribution function (CDF) of a standard normal variable, and $\Phi^{-1}(.)$ denotes its inverse.\(^9\)

The project of entrepreneur $i$ fails at date $t$ when $y_{it} < 0$, whose probability is:

$$Pr(y_{it} < 0) = Pr\left[\sqrt{\rho}z_t + \sqrt{1 - \rho}\varepsilon_{it} < \Phi^{-1}(1 - m_{it})\right] = 1 - m_{it}. \quad (35)$$

We focus on the symmetric equilibrium, in which all entrepreneurs are monitored by banks equally, as $m_{it} = m_t$, since entrepreneurs are ex-ante identical.

By assumption, the dynamic behavior of aggregate wealth can be expressed as:

$$W_{t+1} = R_t^D p(m_t; z_t) X_t, \quad (36)$$

where $X_t$ is the aggregate credit supplied ($X_t \leq W_t$), $R_t^D$ is the deposit rate determined in equilibrium, and $p(m_t; z_t)$ is a function defined as:

$$p(m_t; z_t) \equiv Pr(y_{it} \geq 0 \mid z_t) = \Phi\left(\frac{\sqrt{\rho}z_t - \Phi^{-1}(1 - m_t)}{\sqrt{1 - \rho}}\right).$$

In the following, we analyze the dynamic equilibrium of the three scenarios: (i) without any policies, (ii) only with micro-prudential policy, and (iii) with both macro- and micro-prudential policies. Similar to the static cases, macro-prudential policy impacts the macroeconomic variables by internalizing the general equilibrium effects; it thus controls the total credit supplied, $X_t$, by considering how it affects both

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\(^9\)The analyses in the previous section are based on the assumption that there is no idiosyncratic risk in the economy, $\rho = 1$, that is risk is perfectly correlated over entrepreneurs.
wealth accumulation (36) and banks’ monitoring intensity, \( m_t \), through changes in real interest rates.

The values of the parameters are listed in Table 1. \( R^0 \) is determined endogenously in the dynamic model. The parameters unique to the dynamic model are \( \rho = 0.15 \) and \( \beta = 0.96 \), which are the same as those used by Martinez-Miera and Repullo (2017).

### 4.1 Dynamic Equilibrium without Any Policies

This case is the same as that analyzed by Martinez-Miera and Repullo (2017). Since the problem of measureless investors is constant-returns-to-scale to their wealth holdings, we formulate the problem of an investor with one unit of wealth as follows.

Let the value of an investor with one unit of wealth, when the aggregate wealth of the economy is \( W \), be denoted as \( v(W) \). The Bellman equation of the investor is expressed as:

\[
v(W_t) = \max_{s_t} 1 - s_t + \beta s_t R^D_t E_z t [p(m_t; z_t) v(W_{t+1})],
\]

where \( s_t \) is the investment rate of the investor \( (c_{nt} \equiv 1 - s_t \) is the consumption rate), \( R^D_t \) is the deposit rate, and \( E_z t \) is the expectation over the realization of \( z_t \).

Notice that, in equilibrium, \( v(W_t) \geq 1 \) since the investor can always obtain one unit of utility by consuming that amount. Furthermore, \( v(W_t) > 1 \) must be accompanied by \( c_{nt} = 0 \), since the marginal increase in the investment provides the investor with more than one unit of utility in this case, that is, he prefers investment to consumption.

Let us define:

\[
\hat{W} = \inf \{ W \mid v(W) = 1 \}.
\]

Then, we have:

\[
v(W) = 1 \quad \text{for all } W \geq \hat{W}.
\]

Moreover, aggregate consumption \( CON(W) \) of investors is given by:

\[
W - X = CON(W) = \begin{cases} 
W - \hat{W} & \text{when } W > \hat{W} \\
0 & \text{when } W \leq \hat{W}.
\end{cases}
\]
When $W_t \leq \hat{W}$, we have $\text{const}_t = 0$, and equation (37) becomes:

$$v(W_t) = \beta R^P_t E_{zt} [p(m_t; z_t)v(W_{t+1})] = \beta R^0_t E_{zt} [v(W_{t+1})],$$

(41)

where $R^0_t$ is the risk-free rate of investors.

Following the same analyses as in the static version of the model (subsection 3.2.1), bank monitoring intensity $m_t$ is the solution of:

$$R^L_t = R(W_t) = \min_{m_t} R^P_t (m_t) + c'(m_t)$$

$$= \min_{m_t} v(W_t) \beta E_{zt} [p(m_t; z_t)v(W_{t+1})] + c'(m_t),$$

(42)

where we use equation (41).

**Definition 5.** A dynamic equilibrium without any policies is composed of functions \{v(W_t), m(W_t), R^L_t(W_t), R^P_t(W_t), R^0_t(W_t), CON(W_t)\}, and a value $\hat{W}$ those satisfy conditions (36), (38), (39), (40), (41), and (42).

Figure 5 shows the numerical results for the equilibrium without any policies. As the economy accumulates wealth, the real interest rates decrease, which encourages banks to take excessive risks. This phenomenon is known as the *search for yield* (e.g., Rajan (2006)). Its detailed mechanism is the same as that described in the static case in subsection 3.2.1.
Figure 5: Equilibrium variables as functions of aggregate wealth (without any policies)

Note: When $W > \hat{W}$, the system immediately goes to point $W = \hat{W}$ by consuming $W - \hat{W}$.

4.2 Social Value

Let the social value of an economy (i.e., the value for the social planner) with aggregate wealth $W_t$ be denoted as $V^S(W_t)$. Similar to subsection 4.1, let $\hat{W}$ denotes the threshold of investors’ wealth above which they start to consume.

When $W_t \leq \hat{W}$, given the law of motion for aggregate wealth (36) $(W_{t+1} = R_t^D p(m_t; z_t) X_t = R_t^D p(m_t; z_t) W_t)$, $V^S(W_t)$ can be expressed recursively as:

$$V^S(W_t) = \beta E_{z_t} \left[ p(m_t; z_t) S(W_t) - c(m_t) W_t - W_{t+1} + V^S(W_{t+1}) \right], \quad (43)$$

where $S(X)$ is the static social surplus from the production associated with credit.
supplied $X$, which was introduced in section 3.3. Notice that $V^S(W_t)$ is, by definition, a dynamic counterpart of the social surplus in static version (23). When we put the solution of subsection 4.1 into $(\hat{W}, m_t, R_t^D)$, the solution of the above problem, $V^S(W_t)$, corresponds to the social value at equilibrium without any policies.

Figure 6 shows the social value defined in (43) and an ergodic distribution when the economy is at equilibrium without any policies. Notice that the social value in Figure 6 has a plateau with a slope smaller than one. In this region, accumulating wealth decreases interest rates and discourages intermediaries’ monitoring incentives (see Figure 5). In the same region, as the ergodic distribution shows, some credit booms end up in painful credit busts ($W_t$ drops significantly) because of the risk correlation, while other booms maintain (staying at $W_t = \hat{W}$). Overall, those credit cycles are socially inefficient, as the slope of social value is smaller than one. In the dividend distributing region, $W_t > \hat{W}$, the slope of social value is exactly one.

Figure 6: Social value and ergodic distribution of credit cycles (without any policies)

Note: When $W > \hat{W}$, the system immediately goes to point $W = \hat{W}$ by consuming $W - \hat{W}$.

---

This recursive formulation implies that an economic surplus that does not go to investors directly through the return on their investment (i.e., profits of production and banking sectors) is transferred to investors and consumed immediately by them, without being added to their wealth.
4.3 Macro-Prudential Policy

As demonstrated in the previous subsection, in the plateau region of the social value in Figure 6, inefficiency occurs, that is, distributing wealth to investors rather than accumulating it improves social welfare. In this case, restricting wealth accumulation such as $W_t \leq \hat{W}_{mac} < \hat{W}$ pushes up interest rates and keeps banks’ monitoring intensity from decreasing, which is welfare improving. We interpret this restriction to wealth accumulation, $W_t \leq \hat{W}_{mac}$, as a macro-prudential policy. This is the dynamic counterpart of the static version analyzed in subsection 3.4. Notice that, as in the static case, the macro-prudential policy works at the expense of the total credit supplied.

4.4 Micro-Prudential Policy

In the case with micro-prudential policy, the equilibrium is pinned down by the zero-profit condition (instead of the contestability condition), as in the static version (see subsection 3.2.2). The zero-profit condition of banks can be expressed as:

$$m_t(R^L_t - R^D_t) - c(m_t) = 0$$

$$\iff R(W_t) = R^L_t = R^D_t + \frac{c(m_t)}{m_t}$$

$$= \frac{v(W_t)}{\beta E_{z_t} [p(m_t; z_t)v(W_{t+1})]} + \frac{c(m_t)}{m_t}. \quad (44)$$

**Definition 6.** A dynamic equilibrium with micro-prudential policy is composed of functions

$$\{v_{mic}(W_t), m_{mic}(W_t), R^L_{mic}(W_t), R^D_{mic}(W_t), CON_{mic}(W_t)\}$$

and a value $\hat{W}$ those satisfy conditions (36), (38), (39), (40), (41), (44), and the micro-prudential policy:

$$R(W_t) = R^L_t = \hat{c}(m). \quad (45)$$

Figure 7 shows the numerical results for the equilibrium with micro-prudential policy. Figure 8 shows the social value defined in (43) and the ergodic distribution of wealth. The results look similar to those of the case without any policies. When aggregate wealth accumulates, the risk-free rate decreases, which discourages banks from monitoring, and we have the effect of search for yield in the case with micro-prudential policy, similar to the case without any policies (see subsection 3.2.2 (static...
case) for more details).

However, the major differences between them are as follows. Banks’ monitoring intensity is larger when the aggregate wealth increases in the case with micro-prudential policy than it is in the case without any policies. Because of this, value of a unit of wealth decreases more slowly when aggregate wealth increases in the case with a micro-prudential policy than it does in the case without any policies. As a result, the economy accumulates larger wealth in the case with micro-prudential policy than it does in the case without any policies. Roughly speaking, micro-prudential policy mitigates the moral hazard problem (and the associated credit rationing) by making banks appropriately monitor, which boosts the total credit supplied to the economy. This implication is the same as the static case discussed in subsection 3.2.2.

Similar to the economy without any policies, the social value in Figure 8 has a plateau whose slope is smaller than one. In this region, accumulating wealth decreases interest rates and discourages intermediaries’ monitoring incentives (see Figure 7). In the region, as the ergodic distribution shows, some credit booms end up as credit busts ($W_t$ drops significantly) because of the risk correlation, while other booms are maintained (stay at $W_t = \hat{W}$). Overall, these credit cycles are socially inefficient, as the slope of the social value is smaller than one. As we will see in the next subsection, a macro-prudential policy combined with a micro-prudential policy can completely remove these inefficient cycles by limiting aggregate credit supplied and keeping banks’ monitoring incentives.
Figure 7: Equilibrium variables as functions of aggregate wealth (with micro-prudential policy)

Note: When $W > \hat{W}$, the system immediately goes to point $W = \hat{W}$ by consuming $W - \hat{W}$. 
Note: When $W > \hat{W}$, the system immediately goes to point $W = \hat{W}$ by consuming $W - \hat{W}$.

4.5 Both with Macro- and Micro-Prudential Policies

A macro-prudential policy can improve the efficiency of the macroeconomy by controlling the total credit supplied, $X_t$. Here, I assume that a macro-prudential policy is implemented by taxing on borrowing for banks. Letting $\tau$ be the tax charged on the borrowing, then the investors’ Euler equation (36) can be rewritten as:

$$v(W_t) = \beta \left( R^D_t - \tau \right) E_{z_t} [p(m_t; z_t)v(W_{t+1})] = \beta R^D_t E_{z_t} [v(W_{t+1})].$$  \hspace{1cm} (46)

As usual in the literature, tax revenue is rebated as a lump sum transfer to investors; thus the dynamics of aggregate wealth is unchanged from (36). This means that the tax can only impact the wealth accumulation by changing investors’ intertemporal saving decisions through $\hat{W}$ (i.e., the amount of wealth beyond which investors start to consume).

The macro-prudential authority chooses $\tau$ to maximize social value, internalizing the dynamics of wealth accumulation (36), investors’ Euler equation (46), and micro-prudential policy (45).
Let the value of the macro-prudential authority (i.e., social value) when the wealth of the economy is $W_t$ be denoted as $V^S(W_t)$. Then, the authority solves the Ramsey problem:

$$V^S(W_t) = \max_{\tau, X_t} \beta E_{z_t} \left[ p(m_t; z_t)S(W_t) - c(m_t)W_t - W_{t+1} + V^S(W_{t+1}) \right], \quad (47)$$

subject to aggregate wealth dynamics (36), investors’ equilibrium conditions ((38), (39), (40), and (46)), and banks’ equilibrium conditions ((44) and (45)). This problem can be divided into two parts: the problem of allocation and the one of policy implementation.

The problem of allocation is expressed as:

$$V^S(W_t) = \max_{X_t} \beta E_{z_t} \left[ p(m_t; z_t)S(W_t) - c(m_t)W_t - W_{t+1} + V^S(W_{t+1}) \right],$$

subject to aggregate wealth dynamics (36) and banks’ conditions ((44) and (45)).

When the cost function is quadratic as in (1), this problem can be solved directly by inputting (45) $\iff$ $R(W_t) = c'(m) \iff m(W_t) = \frac{R(W_t)}{\beta}$, (44) $\iff$ $R^D(W_t) = R(W_t) - \frac{c(m)}{m} = R(W_t) - \frac{1}{2} m(W_t)$, and (36) $\iff$ $W_{t+1} = R^D(W_t)p(m(W_t); z_t)X_t$ into the above equation.

Let us define:

$$\hat{W}_S = \inf \left\{ W \mid V'^S(W) = 1 \right\}. \quad (48)$$

Then, we have $V'^S(W) = 1$ for all $W \geq \hat{W}_S$. The consumed part of aggregate wealth (without being invested in the next period) $\Pi(W)$ is given by:

$$W - X = \Pi(W) = \begin{cases} W - \hat{W}_S & \text{when } W > \hat{W}_S \\ 0 & \text{when } W \leq \hat{W}_S. \end{cases} \quad (49)$$

The problem for the implementation, $\tau$, is chosen so that investors’ conditions ((38), (39), (40) and (46)) imply $\hat{W}$, which is equal to the $\hat{W}_S$ derived above.

By construction, the solution to the Ramsey problem can be implemented with macro-prudential policy (taxes on borrowing $\tau$) and micro-prudential policy.

**Definition 7.** A dynamic equilibrium with both macro- and micro-prudential policies is composed of functions

$$\{V^S_{both}(W_t), v_{both}(W_t), m_{both}(W_t), \Pi_{both}(W_t), R^L_{both}(W_t), R^D_{both}(W_t), R^0_{both}(W_t)\}, \text{ tax } \tau, \text{ and}$$
a value $\hat{W}$ those solve Ramsey problem (47).

Figure 9 shows the numerical results for the equilibrium with both micro- and macro-prudential policies. Figure 10 shows the social value and ergodic distribution of wealth. Compared to the case with only micro-prudential policy (in the previous subsection), micro-prudential policy combined with macro-prudential policy can completely remove the inefficient credit cycles by limiting the aggregate credit supplied and maintaining banks’ monitoring incentives.

The ergodic distribution in Figure 10 shows that, under the optimal policies, there are no credit cycles since wealth stays at $W_t = \hat{W}$. Notice that, in reality, the economy faces other different types of shocks (shocks to productivity, foreign exchange rates, monetary policy, etc.), all of which can cause credit (and long-wealth $\hat{W}$) to fluctuate over time. In the model, I abstract the credit movements caused by these forces.

Figure 9: Equilibrium variables as functions of aggregate wealth (with both policies)

Note: When $W > \hat{W}$, the system immediately goes to point $W = \hat{W}$ by consuming $W - \hat{W}$.
4.6 Implications for the Dynamic Analyses

Table 3 shows the outcomes for each equilibrium. In the table, social value is defined by equation (43). Expectation is taken over the ergodic distribution (therefore, time average). In general, the results are consistent both qualitatively and quantitatively with those of the static model.

The contributions of micro-and macro-prudential policies to improving social welfare are roughly comparable. More concretely, the micro-prudential policy boosts the expected social value by 57 percent, from 3.88 to 6.08. The macro-prudential policy boosts the value by an additional 32 percent, from 6.08 to 7.31.

When it is combined with the micro-prudential policy, the macro-prudential policy improves banks’ credit quality and removes inefficient credit boom and bust cycles completely without sacrificing the total credit supply (see Figure 10).
Table 3: Comparison of dynamic equilibrium outcomes

<table>
<thead>
<tr>
<th></th>
<th>No policies</th>
<th>Micro policy</th>
<th>Both policies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected monitoring intensity ($m$)</td>
<td>0.546</td>
<td>0.729</td>
<td>1.00</td>
</tr>
<tr>
<td>Expected lending rate ($R^L$)</td>
<td>4.22</td>
<td>2.92</td>
<td>4.00</td>
</tr>
<tr>
<td>Expected total credit supplied ($X$)</td>
<td>0.0594</td>
<td>0.120</td>
<td>0.0624</td>
</tr>
<tr>
<td>Long-run wealth ($\hat{W}$)</td>
<td>0.0732</td>
<td>0.133</td>
<td>0.0624</td>
</tr>
<tr>
<td>Expected social value ($V^S$)</td>
<td>3.88</td>
<td>6.08</td>
<td>7.31</td>
</tr>
</tbody>
</table>

5 Concluding Remarks

This study examines how micro- and macro-prudential policies work and interact with each other over the credit cycles using a dynamic general equilibrium model of financial intermediaries. Micro-prudential policies restrict the excess risk-taking of individual institutions, while taking real interest rates (prices) as given. By contrast, macro-prudential policies control the aggregate credit supplied (equilibrium outcome) by internalizing prices or the general equilibrium effect. The proposed model indicates that: (i) micro-prudential policy alone cannot completely remove inefficient credit cycles; (ii) when macro-prudential policy is conducted jointly with the micro-prudential one, policymakers can improve banks’ credit quality and remove inefficient credit cycles completely without sacrificing the total credit supply; and (iii) the contributions of micro- and macro-prudential policies to the improvement in social welfare are roughly comparable.
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Appendix A

Proof. Proof of Proposition 1

If equation (7) has no solutions for \( m^* \) in feasible range \([0,1]\), \( c'(m) + \frac{R_0}{m} > R_L \) holds for all \( m \in [0,1] \), since \( \lim_{m \to 0} c'(m) + \frac{R_0}{m} = \infty \). In this case, banks have an incentive to reduce \( m \) to 0. However, for \( m^* = 0 \), \( m^*(R_L - R_D^*) - c(m^*) = -R_0 - c(m^*) < 0 \), which violates the bank’s participation constraint (4).

If equation (7) has any solutions, by the convexity of the LHS of (7), there exists an interval \([m^-, m^*] \in [0,1]\) so that:

\[
R_L - \frac{R_0}{m} - c'(m) \geq 0 \text{ if and only if } m \in [m^-, m^*].
\]

In ranges \( m \leq m^- \text{ or } m^* \leq m \), the bank has an incentive to reduce \( m \). In range \( m \in (m^-, m^*) \), the bank has an incentive to increase \( m \). These results indicate that banks prefer \( m = 0 \) or \( m^* \). Notice that \( m = 0 \) breaks the bank’s participation constraint (4) as shown above, while \( m = m^* \) satisfies \( m^*(R_L - R_D^*) - c(m^*) = m^*(R_L - \frac{R_0}{m^*}) - c(m^*) \geq m^* c'(m^*) - c(m^*) > 0 \), where we use the convexity of the cost function \( c'(m^*) > \frac{c(m^*)}{m^*} \). \( \square \)

Appendix B

Following Martinez-Miera and Repullo (2017), I provide a microfoundation for the relationship between credit demand, production, and social surplus in terms of the demand for a set of final goods producers that use entrepreneurs’ output as an intermediate input.

Each entrepreneur produces (if successful) a unit of intermediate input sold at a price \( R_L \) to a set of final good producers. There is a continuum of final goods producers with heterogeneous productivity \( \theta \); each producer can transform a unit of intermediate input into \( \theta \) units of the final good. \( \theta \) is distributed according to: density \( g(\theta) = a\sigma(\theta)^{-(\sigma+1)} \), where \( a > 0 \) and \( \sigma > 1 \). For any price of intermediate input \( R_L \), only producers with productivity larger than \( R_L (\theta \geq R_L) \) will operate.
Hence, given supply of the intermediate input $X$, we must have:

$$
\int_{R^L}^{\infty} g(\theta) d\theta = a(R^L)^{-\sigma} = X,
$$

which implies:

$$
R^L = R(X) = \left( \frac{X}{a} \right)^{\frac{1}{\sigma}}. \tag{50}
$$

$\sigma$ is the elasticity of the demand for the intermediate input, while $a$ is a (proportional) demand shifter related to the productivity of the final good producers.

The social surplus $S(X)$ from the final good production associated with the intermediate input $X$ is expressed as follows:

$$
S(X) = \int_{R^L}^{\infty} \theta g(\theta) d\theta = \frac{a \sigma}{\sigma - 1} (R^L)^{-\sigma+1} = \frac{a \sigma}{\sigma - 1} \left( \frac{X}{a} \right)^{\frac{\sigma-1}{\sigma}}. \tag{51}
$$

By conditions (50) and (51), we have:

$$
R(X) = \frac{dS(X)}{dX}. \tag{52}
$$