An Essay on Labor Supply Decisions and Reference Dependent Preferences

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This article presents a brief analysis of labor supply decision models using the concept of reference dependent preference. This reference dependent preference leads the model towards a behavioral aspects where "Gain-Loss utility" can be derived form standard "consumption utility" and the reference point is determined endogenously by the economic environment. At the first two sections of the article labor supply decision and reference dependent preference has been discussed in a brief. Then three model has been discussed where the economists used the concept of reference - dependent preference to make labor supply model decisions. *I would like to thank Dr Sandip Kumar Agarwal (Assistant Professor Department of Economic Science, Indian Institute of Science Education and Research, Bhopal) for suggesting me this topic and for the valuable guidance while writing this article.*

I. Introduction - Labor Supply Decision

Each of us must decide whether to work and, once employed, how many hours to work. At any point in time, the from the economic point of view the labor supply is given by adding the work choices made by each person in the population. Total labor supply also depends on the total population which is directly connected by the fertility decisions made by earlier generations. So, the main concept of labor supply decision is a frame work used by economists to study that how an individual maximize their well-being by consuming goods and leisure. In this mentioned framework of labor supply decision, goods have to be purchased from a market place and as because most of the population are not independently wealthy so they have to work in order to earn money which is required to get goods and services from the market but in the same place they are sacrificing many valuable leisure times.

So it can be said that if we do not do work then we can get a lot of leisure but if we do work then we will be able to make money which can buy required goods and services. This is the main intuition behind the theory of labor-leisure choice. The model of labor-leisure choice isolates wage rate and income of an individual as the key economic variables that guide the time allocation between

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the labor market and leisure activities. The labor supply decision is also determined by the opportunity wage (which is the wage rate of individual could earn in the labor market), non-labor income (the amount of income available to the individuals from other sources), and the individual's preferences for leisure versus works.

This economic framework of labor supply decision helps us to determine many important socio-economic questions related to economic policy and social consequences like why women's work propensities rose and hours of work declined? do welfare programs reduce incentives to work? Does a cut in the income tax rate increase hours of work? And what factors explain the rapid growth in the number of women who choose to participate in the labor market? Many of these questions are related to many behavioral concepts like bounded rationality, cumulative prospect theory and so on. Here in this article we will see that how the concept of reference-dependent preference have been applied to the labor supply decision models which helps us to connect labour-leisure choice and gain-loss utility by various endogenous reference point.

II. Reference - Dependent Preference

A. A Brief Overview

In this section we will discuss about the basic framework behind the concept of reference - dependent preference following the model made by Kőszegi and Rabin.[1]. According to their model of reference - dependent preference and loss aversion, the gain -loss utility can be derived from standard consumption utility with endogenous reference points. This model is based on the basic intuition of prospect theory[2] and also other models of reference dependence. By applying this model it can be shown that the willingness to pay for any good got increased in the expected probability of purchase and in the expected price conditional on purchase. In the article by Kőszegi and Rabin it also has been proposed that a person's reference point is the probabilistic belief he held in the recent past from the outcomes and in deterministic environment preferred personal equilibrium can direct the decision makers to get their maximum consumption utility by the replications of the predictions of classical reference independent utility theory.[1]

B. Model for Reference - Dependent Utility

Let assume a person's utility for riskless outcome as $u(c \mid r)$, where $c = (c_1, c_2, ..., c_K) \in \mathbb{R}^K$ is the consumption bundle and $r = (r_1, r_2, ..., r_K) \in \mathbb{R}^K$ is the set for "reference level" of individual consumption. Now, if is drawn from $\mathbb{F}$ (Probability measure), then the utility of the person can be defined as:

$$U(F \mid r) = \int u(c \mid r) dF(c) \tag{1}$$

Here in this model, as it has been assumed that the reference points are the pre-beliefs about the outcomes, so they can be stochastic themselves. So, if reference point of a person is G (probability measure) over $\mathbb{R}^K$ then the utility will be:

$$U(F \mid G) = \iint u(c \mid r) dG(r) dF(c) \tag{2}$$

From this formulation, it can be said that from a given consumption, the sense of loss or gain can be derived by comparing it with all other possible outcomes under the reference theory. So gain and losses are not dependent on references though the preferences are reference dependent.

In this model they also assume that the overall utility has two parts: $u(c \mid r) \equiv m(c)+n(c \mid r)$, where $m(c)$ is the consumption utility and $n(c \mid r)$ is the gain - loss utility.[3]. Here it has been assumed that the "consumption utility" is additively separable across all dimensions, i.e. $m(c) \equiv \sum_{k=1}^{K} m_k(c_k)$, where every $m_k(.)$ is strictly increasing and differentiable. The "gain - loss utility" can be represented as: $n(c \mid r) \equiv \sum_{k=1}^{K} n_k(c_k \mid r_k)$. This assumptions tells us that anyone while taking any decisions, he or she can assesses the gain - loss utility in every dimension separately.[4] In combination with loss aversion, this separability of gain - loss utility is at the crux of many effects of reference - dependent utility such as endowment effect. How a person feels about gaining or losing in a dimension depends on the changes in consumption utility as:[5]

$$n_k(c_k \mid r_k) \equiv \mu(m_k(c_k) - m_k(r_k) \tag{3}$$

Where $\mu(.)$ is a "universal gain-loss function" which satisfies the following assumptions:[6]
A1: $\mu(x)$ is strictly increasing.
A2: If $y > x > 0$, then $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.
A3: $\mu''(x) \leq 0$ for $x > 0$, and $\mu''(x) \geq 0$ for $x < 0$.
A4: $\mu'_-(0)/\mu'_+(0) \equiv \lambda > 1$, where $\mu'_+(0) \equiv \lim_{x\to 0} \mu'(|x|)$ and $\mu'_-(0) \equiv \lim_{x\to 0} \mu'(-|x|)$
Loss aversion is captured by A2 for large stakes and A4 for small stakes and A3 is the "diminishing sensitivity" of gain - loss utility. To connect the implications of reference - dependence with loss aversion without diminishing sensitivity as a force on behavior, we need to take an alternative A3 as:

$$A3' : \forall x \neq 0, \mu\prime\prime(x) = 0$$

Now the following two propositions[1] will be about utility function which replicates properties which are generally associated with reference - dependent preference.

Proposition 1: If μ satisfies assumptions A0 - A4 then for riskless consumption bundles, the followings are true:

1) For all F, G, G' such that the marginals of G' firstorder stochastically dominate the marginals of G in each dimension, $U(F \mid G) \geq U(F \mid G')$.

2) For any $c, c' \in \mathbb{R}^K, c \neq c', u(c \mid c') \geq u(c' \mid c') \Rightarrow u(c \mid c) > u(c' \mid c)$

3) Suppose that μ satisfies A3′. Then, for any F, F' that do not generate the same distribution of outcomes in all dimensions, $U(F \mid F') \geq U(F' \mid F') \Rightarrow U(F \mid F) > U(F' \mid F)$

The first point of this proposition establishes that lower reference point makes a person happier. The next two points tells us about the exhibition of the status quo bias i.e. if a person wants to leave the reference point for an alternative then that person strictly prefers that alternative if that is the reference point.

Proposition 2: If m is linear and μ satisfies assumptions A0 - A4, then there exists $\{v_k\}_{k=1}^{K}$ satisfying Assumptions A0 -A4 such that, $\forall c \& r$,

$$u(c \mid r) - u(r \mid r) = \sum_{k=1}^{K} v_k (c_k - r_k) \tag{4}$$

This second proposition tells us that in case of linear $m(.)$, the utility function $u(c \mid r)$ shows same properties as $\mu(.)$. So for small changes in the utility function shares qualitative properties of standard prospect theory.

C. The Reference Point as Endogenous Expectations

In this model by Kőszegi and Rabin (2006)[1] it has been assumed that the reference points are fully determined from a person's expectations in the recent past. So, the reference point of a person is the probabilistic belief about that particular consumption outcomes in the time between the person took the first decision and just before the latest consumption occurs. This assumption helps this model to reconcile and unify the existing interpretations.[7] The reference points are also has been assumed as status quo which make relevant the reference point being expectations, where expectations are fully rational.
So, if the decision maker has some beliefs with probabilistic distribution of $\mathbb{Q}$ over $\mathbb{R}$, which draws a distribution over all possible choice sets for the decision maker $\{D_l\}_{l \in \mathbb{R}}$, where every $D_l \in \Delta(\mathbb{R}^K)$. Now on the basis of previous mentioned propositions we will see two definitions state a new concept of personal equilibrium which makes this model by Kőszegi and Rabin very unique. This concept of 'personal equilibrium' is very similar to the concept of 'loss aversion equilibrium' by Shalev [2000] which is Nash equilibrium for multiplayer games, where the reference points are equal to player's reference - dependent expected utility.[8]

Definition 1: A selection $\{F_l \in D_l\}_{l \in \mathbb{R}}$ is a personal equilibrium (PE) if for all $l \in \mathbb{R}$ and $F_l' \in D_l, U\left(F_l \mid \int F_l dQ(l)\right) \geq U\left(F_l' \mid \int F_l dQ(l)\right)$

Definition 2: A selection $\{F_l \in D_l\}_{l \in \mathbb{R}}$ is a preferred personal equilibrium (PPE) if it is a PE, and $U\left(\int F_l dQ(l) \mid \int F_l dQ(l)\right) \geq U\left(\int F_l' dQ(l) \mid \int F_l' dQ(l)\right)$ for all PE selections $\{F_l' \in D_l\}_{l \in \mathbb{R}}$

So, from these two definition it can be said that from a set of choices $\mathbb{D}_l$ if a decision maker expect to choose $\mathbb{F}_l$, then the decision maker expect an outcome of $\int \mathbb{F}_l dQ(l)$ and this expectation is the reference point of that decision maker. Now if we apply the concept of loss aversion equilibrium using utility function to individual decision making then personal equilibrium (PE) generates. There may be multiple PE and a decision maker can rank the PEs according to the previous expected utility.

III. Models for Labor Supply Decision with Reference - Dependent Preferences

In this section, we will see how the concept of reference - dependent preference can be helpful while making a model for labor supply decision. For this we will discuss following three models and will also observe their empirical analysis with the real world data. The three models are by: Camerar et al.[9], Farber[10] and Crawford et al.[11]. In the model by Camerar et al. [9] they has tried to figure out and analyse the positive relationship between changes in wages and amount of hours spent. They also tried to analyze and understand the elasticities of cab driers. In the next two models by Farber[10] and Crawford and Meng[11] we will see the support of Camerer's empirical finding through the concept of reference - dependence preference which we have discussed earlier.
For every model, there are three section in this article. At first we discussed about the theoretical framework of the model and then at the second part we have discussed the methodology of the analysis and the brief results of the analysis. At the third section, we have told about the conclusion we can say from those models.

A. General description of the Data

Here in the three discussed models of labor supply, the empirical studies has been done on the data of New York cab drivers. The ideal test of labor supply responses to transitory wage increases would use a context where the wages are comparatively constant within a day but uncorrelated across days. In this type of situation, all dynamic optimization models predict positive relationships between hours worked and wages because of the negligible impact on daily life wealth of a one-day increase in wage.
Such data are available for at least one group of workers: New York City cab-drivers. The labor supply of cab drivers is consistent with the existence of inter-temporal substitution. Drivers face wages which fluctuate on a daily basis due to various kinds of demand shocks caused by weather, subway breakdowns, day-of-the-week effects, holidays, conventions, etc. But the rates per mile are set by

law. So, on busy days drivers spend less time searching for customers and thus earn a higher hourly wage. These wages tend to be correlated within days and uncorrelated across days (i.e., transitory).

B. Model by Camerar et al

Camerar et al. (1997)[9] have done extensive research to find the relationship between daily working hours and wages of New York cab drivers. In his model of labor supply it has been assumed that the cab drivers take the cabs on lease for a particular amount of time or day with a fixed fee. The drivers themselves are responsible for the fuel and maintenance expenses. After paying the fixed amount to the owner, he can keep 100 percent of the fare, the driver gonna earn. The divers themselves also can choose that for how much time they want to work during that specific lease period. The agreement for lease is very close to the first best solution of the incentive theorist's to the firm-worker principal agent problem of selling the farm to the worker.
The main direction of their model is computing the daily wage rate as a ratio of daily income to daily hours. Then the model do a regression between logarithms of daily hours and logarithms of daily wage rate which generate a significant and substantial negative elasticity for labor supply and this is consistent with a specific target earning model where cab drivers generally stops working when they reach that target value of daily income.
There is another similar type of study by Chou (2000)[12], where he analyzed the labor supply of cab drivers on Singapore market and got the same negative relationship between logarithm of daily wages and logarithm of daily working hours. Chou came to a conclusion like cab drivers appear to set targets over short horizons.

Analysis of the model with Data

To analyze the model they have used the trip sheets of New York cabdrivers. The trip sheets are sequential lists of trips that a driver took on a specific day. In that trip sheet, the driver lists every fare (tip excluded) and their time of picking up and dropping off. Camerar et al. analyzed three trip sheets named TLC1, TLC2, TRIP. The summary of the satistical analysis of the three data sheet are in the table 3.

There is one feature, which need to be noticed properly, i.e. the variation in trip count and the working hours. There is a large difference can be seen for these two features between TRIP dataset and TLC datasets. The TRIP sample is almost half of TLC1 and TLC2. The reason behind this is because TRIP dataset consists of the data from fleet drivers, while TLC2 consists of lease and owner drivers and TLC1 consist of fleet, lease and owner, all three type of drivers. After rigorous analysis of the above mentioned three data sets, Colin Camerar has found following points:

	Mean	Median	Std. dev
TRIP ($n = 70$)			
Hours worked	9.16	9.38	1.39
Average wage	16.91	16.20	3.21
Total revenue	152.70	154.00	24.99
# Trips listed on sheet	30.17	30.00	5.48
# Trips counted by meter	30.70	30.00	5.72
High temperature for day	75.90	76.00	8.21

Correlation log wage and log hours = −.503. The standard deviation of log hours is .159, log wage is .183, and log total revenue is .172. The within-driver standard deviation of log revenue is .155 and across drivers standard deviation is .017.

TLC1 ($n = 1044$)			
Hours worked	9.62	9.67	2.88
Average wage	16.64	16.31	4.36
Total revenue	154.58	154.00	45.83
# Trips counted by meter	27.88	29.00	9.15
High temperature for day	65.16	64.00	8.59

Correlation log wage and log hours = −.391. The standard deviation of log hours & is .263, log wage is .351, and log total revenue is .347. The within-driver standard deviation of log revenue is .189 and across drivers standard deviation is .158.

TLC2 ($n = 712$)			
Hours worked	9.38	9.25	2.96
Average wage	14.70	14.71	3.20
Total revenue	133.38	137.23	40.74
# Trips counted by meter	28.62	29.00	9.41
High temperature for day	49.29	49.00	2.01

Correlation log wage and log hours = −.269. The standard deviation of log hours is .382, log wage is .259, and log total revenue is .400.

Table 1—Summary of Statistical Examination by Camerar[9]

Variation of wages between days and within days:
Here it has been tried to estimate the function for labor supply. For this it has been assumed that the drivers take the decision of when to stop on the basis of average wage rate rather than the wage's fluctuation on that particular day. Though it is important to consider the fluctuation of wages within day, as because within day negatively auto correlated wage rate of every hours can make the driver

to drive for the time-duration predicted from the standard theory to behave as they were violating it.

So from the statistical analysis of the data sets and the negative correlations, it can be said that if the auto-correlation is negative on a day started with high wage, then any driver will stop driving early because high wage hours likely to be followed by low wage hours. Similarly on a day started with low wage hour, driver will drive for longer hours expecting the high wage hour later on that day.

Wage elasticities:

From the above table (3) the simple correlation between logarithm of wages and logarithm of hours can be seen for the three sets of data ($-0.503, -0.391, -0.269$). The scatter plots between log of hours and log of wages for three data sets are in the below figure (5). Camerar also did OLS regression between log of these two

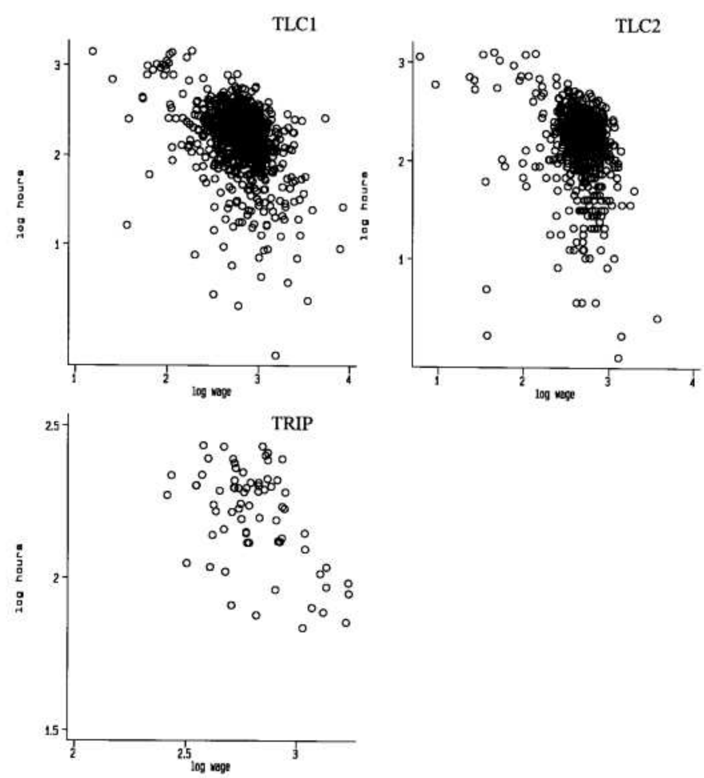

Figure 1. Scatter Plot of log(Hours) vs log(Wage) by Camerar [9]

variables with taking two factors relating the weather condition and one dummy variable for the difference in demand between weekdays and weekend. From the regression, they came up to a conclusion that for TRIP data, the wage elasticity depends on whether we include the driver fixed effects or not. For no driver fixed effects its -0.411 and with driver fixed effects the elasticity is less negative -0.186. For the TLC1 and TLC2 data sets, the elasticities are strongly negative (ranges from -0.355 to -0.618). They are estimated more precisely and are quiet robust to include the fixed effects.{All the mentioned values are taken from the article by C. Camerar[9]}. Another possible conclusion we can say is that elasticities are quiet less negative in case of fleet drivers than the owner drivers and lease drivers.

Wage Elasticity vs Experience of the driver: By time drivers get experienced and learn that its rational to drive more on high wage days and less on low wage days. This can give them better income and more leisure time also. This experience can results to more positive wage elasticity for experienced drivers than the inexperienced drivers.

Wage Elasticity vs Payment Structure of the Cab-drivers: The payment of the drivers also can impact the wage-elasticities. The payment structure also can impact on the degree to which they can vary their driving hours across the day. Here it can be said that the fleet drivers have lower elasticity (lesser negative) as they can drive for only a period of twelve hours.

So why the elasticities are negative?

From the above discussion it is very clear that the wage elasticities are negative, though are significantly higher for experienced drivers than the inexperienced ones. So, according to Camerar the explanation for negative elasticities are as follows:

Tagetting daily income: The negative elaticity is because the cab-drivers generally take an one day horizon target to achieve for their wage and quit driving when that target got achieved. Like many other mental accounts, daily income targets can help to mitigate self control problems.[13] Driving is a very tiring job and drivers can quit anytime they want, so keeping a daily income target establishes an output based guideline of when to quit. The utility function for daily income target can also lead us to some more points like as a decision maker, its easy for a diver to make decision about when to quit as this is computationally easier than tracking the ongoing balance of marginal income utility and forgone leisure utility which depend on the expected future wages required for optimal inter-temporal substitution. So, a daily target rule is a much simpler rule for the drivers to keep track on balance. The daily income target also supports the impact of experience for daily wages. Experienced drivers have larger elaticities because either they learn over time to take a longer horizon target or to adopt the simple rule of driving for a fixed number of hours every day.

Liquidity constraints: Another possible reason for the negative elaticities are that the drivers face strongly binding liquidity constraints. Liquidity constrained cab - drivers who must earn a certain amount of money each day must drive long hours in a low wage day. From figure 5 the variation of hours worked can be seen for three different type of drivers. So, the cab drivers who are not liquidity constrained such as those with substantial wealth, they shouldn't show negative

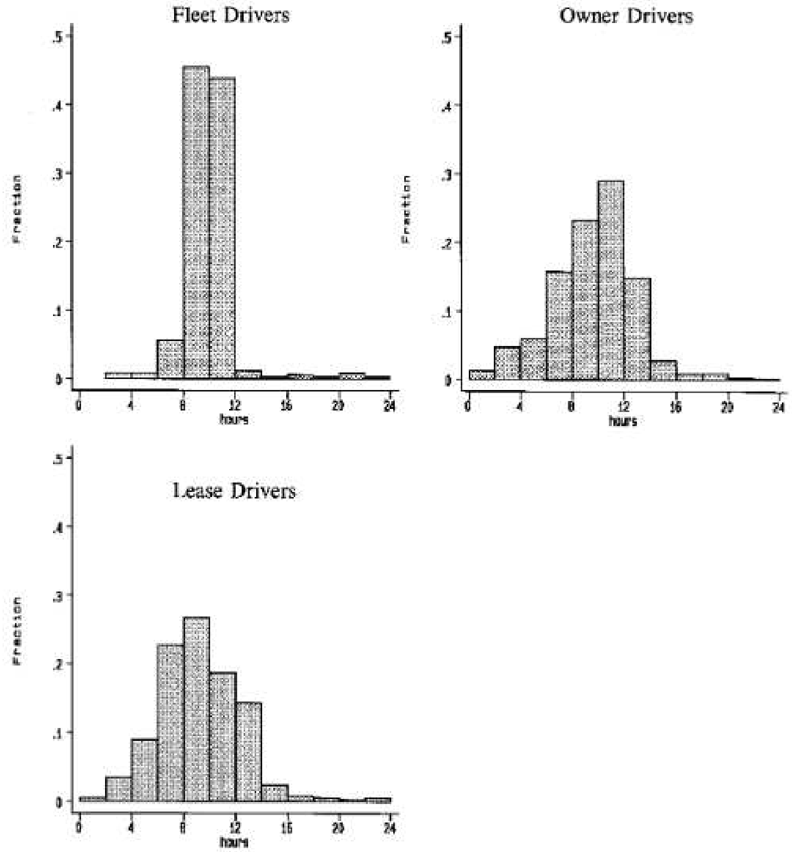

Figure 2. Histograms of Hours Worked by Driver Ownership Class(for TLC1 data)[9]

elasticity
Beside these two points, there may be other factor also for the reason of negative elasticities, such as breaks, increasing the dis-utility of effort and participation.

Conclusion from this model

According to dynamic theory of labor supply, there should be a positive loabor supply response for the fluctuations in wages. But from the above mentioned statistical analysis by C.Camerar it is very evident that the sign for elasticity is negative. Cab drivers take decisions on their own about thier working hours and earn wages which is highly correlated within days, but only weekly correlated between days. From this it can be said that the fluctuation in wages are transitory. The negative sign of wage elasticities tells us that the elasticities of inter - temporal substitution around zero may lead us to a real behavioral regularity[9]. It tells us that this negative elasticity is not just an artifact of computation or any special

condition for the cab drivers.
In the next two following model of labor supply we will see the what happens to these results if we analyze the labor supply using the concept of reference - dependent preference.

C. *Model by Henry S. Farber*

Prof H Farber has done a lot of research works on labor supply decision and deviations from the standard neoclassical consumer behavior model. With the same dataset of new york city cab drivers he also has analyzed the predictions with the help of *reference - dependent preference*. He has published three articles about this particular topic[14] [10] [15]. Here we will discuss the model he made on the article named *"Reference - Dependent Preference and Labor Supply: The Case of New York City Taxi Drivers"* published on 2008.[10] In this article he applied the model to the data on daily labor supply of new york cab drivers where the reference point of the drivers are the daily income. His findings are directly opposite to those of Camerar et al[9] which we discussed in the earlier section.
In the article Farber (2005) [14] he sketched a model for daily labor supply for taxi drivers in New York. The fully optimizing model is based on the solution of a dynamic programming problem in which a driver at a given point in his shifts compares his utility between the expected utility of if he stops and continue to work. The final Lagrangian expression for constrained maximization of the utility function of this model is as follows:

$$\mathbb{V} = \sum_{t=0}^{T} (1+\rho)^{-t} \left[a(x_t) + b(l_t)\right] + \lambda \left\{ Y_0 + \sum_{t=0}^{T} (1+r)^{-t} [y_t(1-l_t) - x_t] \right\} \tag{5}$$

where,

λ = marginal utility of lifetime wealth,

x_t = daily goods consumption,

l_t = daily leisure consumption,

Y_0 = the initial wealth, and

$y_t(.)$ = daily earning which has been generated as a function of work time $(1-l_t)$.

In his article Farber (2008)[10] he used an alternative approach considering the end of each passenger trip as a decision point for continue or stop driving. Assuming the future earnings are stochastic, he derive the implication of reference - dependent preference for a driver's labor supply decision. Here he formulated the driver's utility after trip t during a shift which incorporate a reference point is as follows:

$$U\left(Y_t, h_t\right) = \left(1 + \alpha I\left[Y_t < \mathrm{T}\right]\right)\left(\mathrm{Y} - \mathrm{T}\right) - \frac{\psi}{1+\nu} h_t^{1+\nu} \tag{6}$$

where $I[\cdot]$ is the indicator function and

h_t = hours worked on shift at end of trip t,

Y_t = income earned on shift at end of trip t,

T = reference income

α = parameter determining sharpness of "kink" in utility function ($\alpha > 0$),

ψ = parameter determining dis-utility of work, and

ν = elasticity parameter (wage elasticity of labor supply = $1/\nu$, defining $Y = Wh$).

The above utility function is continuous, but kinked at income level T, with the marginal utility of income discontinuously higher below the kink ($a > 0$) than above the kink. So, now if we express this utility function at $(t+1)$ level, then the utility will be like as follows:
(7)

$$U\left(Y_{t+1}, h_{t+1}\right) = \left(Y_t + f_{t+1} - T\right)\left(1 + \alpha I\left[Y_t + f_{t+1} < T\right]\right) - \frac{\psi}{1+\nu}\left(h_t + \tau_{t+1}\right)^{1+\nu}$$

The marginal utility for the next trip will be depend upon the fare and needed time to complete the ride. So the expression for the marginal utility for the next trip at $(t+1)$ time period will be:

$$MU_{t+1} = f_{t+1}\left(1 + \alpha I\left[Y_t < \mathrm{T}\right]\right) - \tau_{t+1}\psi h_t^{\nu} \tag{8}$$

We can see here that the marginal value of continuing to drive is a linear function of the uncertain wage on the $t+1$ trip. Let $\mathrm{E}\left(\omega_t\right) = \mu_t^{\omega}$ so that expected marginal value is:

$$E\left[MV_{t+1}\right] = \mu_{t+1}^{\omega}\left(1 + \alpha I\left[Y_t < \mathrm{T}\right]\right) - \psi h_t^{\nu} \tag{9}$$

Now in the next section we will see the estimation of labor supply model based on these formulations referring Farber (2008) article.[10]

Analysis of the model with Data

A large value of δ is not sufficient for reference - dependent preferences to be useful to predict the labor supply. An individual's reference income level also need to be roughly constant over a reasonable period of time. The standard deviation (σ_μ) of the daily random component determine the reference income which is a measure of how much reference income level deviates every day. If the standard deviation is relatively small to the mean (θ_i), then only it can be said that the reference income level is quiet stable. The θ_i also can tell us that when driver i is going to stop.
From the intuitive point of view, it can be said that most of the shifts will end

Parameters	(1)	(2)	(3)	(4)
$\hat{\beta}$(contprob)(constant)	-0.691	-	-	-
	(0.243)			
$\hat{\theta}$(mean ref inc)	159.02	206.71	250.86	-
	(4.99)	(7.98)	(16.47)	
$\hat{\delta}$(cont increment)	3.40	5.35	4.85	5.38
	(0.279)	(0.573)	(0.711)	(0.545)
$\hat{\sigma}$(ref inc var)	3,199.4	10,440.0	15,944.3	8,236.2
	(294.0)	(1,660.7)	(3,652.1)	(1,222.2)

Table 2—Maximum Likelihood Estimates of Reference-Dependent Labor Supply Model from Farber(2008)[10]

with drivers reaching their reference income level and if that doesn't happen, then the reference dependence is not likely to have any significant effect on behavior. So, the model has to be examined for prediction of probability that the reference income level has been reached at the end of the of shift. Total income observed on each shift lead us to a the computation of estimated parameters of the model. Here Farber presented the maximum likelihood estimates of the parameters ($\boldsymbol{\beta}, \theta_i, \sigma_\mu^2$ and δ) of the reference - dependent labor supply model on the NY cab drivers data. The estimation values are in the table 3. These estimates are for the restricted version of the model that constraints all the drivers to have the same mean reference income level and the X factor to contain only a constant. From the table 3 it can be observed that the mean reference income level is \$159.02 and the δ estimation inversely index the change in probability of continuing to drive once the reference level for the day is reached. The value for this is 3.40 which is statistically very different from zero. Thus tells us that strongly about the possible impact of reference - dependence for the labor supply decision. So, all the factors are showing that reference - dependent preference has an important role in determining the daily labor supply for the taxi drivers. We are getting following evidences in support of this statement by the analysis of the data by Farber[10].

- There is a very clear decline in the continuation value when the drivers reach the reference income level.

- The estimated value of σ_μ is very small relative to the estimated mean reference income level. (θ_i)

- The θ_i can predict the daily income very precisely.

Conclusion from this model

So from the analysis as a summary we can say the results is showing that the reference - dependent preference is impacting the labor supply in four following points Farber[10]:

- From table 3 it can be seen that the continuation probability got decreased so much when the income level has been reached by cab drivers. The value of δ is 5.38 implies a decrease of five standard deviation in the standerd normal distribution of the continuation value at the time when driver reach their reference income level. This makes us almost certain that the driver will stop after they reach their reference income level.
- The estimation of within- driver inter - day variation is very large at the reference income level which says us that there is little consistency in a driver's reference income level in each day.
- There is a strong positive correlation can be seen between the average hours of driving and the average income of them and θ_i. These correlations are 0.86 and 0.76 respectively.[10]. To see whether the reference income level can really predict the daily income, Farber did a regression of:

  $$Y_{ij} = \alpha_0 + \alpha_1 \hat{\theta}_i + \gamma_{ij} \tag{10}$$

  where the Y_{ij} is the income of driver i on shift j. The R^2 score he got from the regression was 0.07.
- The probability that income Y_{ij} exceeds the reference level after the j-th shift is the CDF of a normal distribution where θ_i is the mean and the variance is 8236.2. The formulation for this is as follows:

  $$P\left(T_{ij} \leq Y_{ij}\right) = \Phi\left[\left(Y_{ij} - \theta_i\right)/\sigma_\mu\right] \tag{11}$$

  The following figure 5 shows the plot for fraction of shifts where the probability at reference income level was reached during the shift falls in ten intervals of zero to one.

From the above points we end up getting a puzzle. The concept of reference - dependent preference utility make a non - linear model for the relationship between the large estimated value of δ and the income with the continuation probability which tells us that the cab drivers generally stop if they reach at their respective reference income level. But the reference income level of the drivers also varies everyday, however from the analysis it is very evident that most of the shifts end before reaching the reference income level. The data also shows us smoothness between reference level income and the probability for stop or to continue driving where this reference - dependent preference plays an important role. The within - driver variation of income across different shifts also direct us

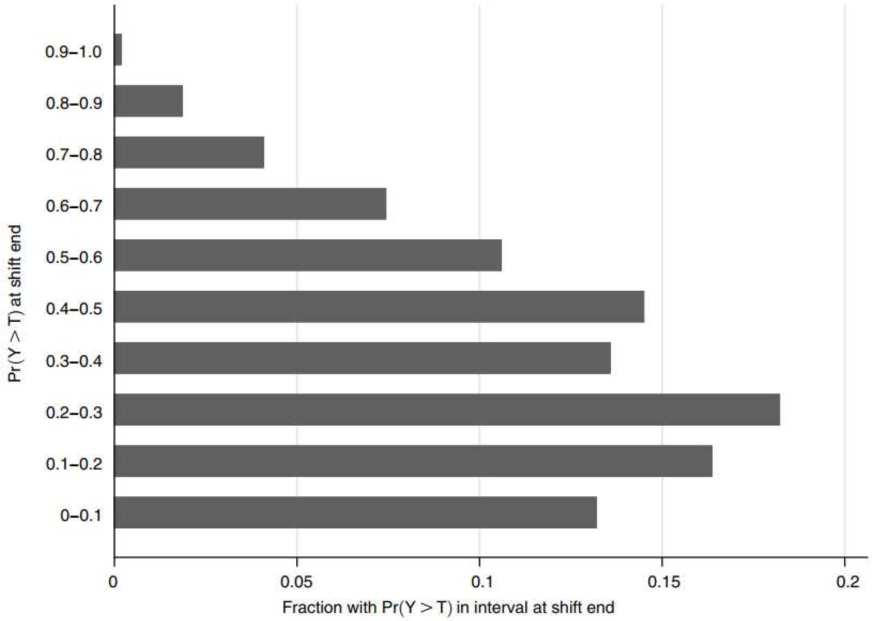

Figure 3. Distribution of Predicted Probability of Income Exceeding the Reference Level at Shift End (*Based on parameters in column 4 of Table 2*)

to the importance of the concept of reference - dependent preferences though the clear reason behind this variation has not been explained precisely.

D. Model by Vincent P. Crawford and Juanjuan Meng

This model is based on a paper named "New York City Cab Drivers' Labor Supply Revisited: Reference-Dependent Preferences with Rational Expectations Targets for Hours and Income" by Crawford and Meng[11]. This model targets for hours as well as income, determined by proxied rational expectations. The model by Camerar et. al[9] can be explained by the neo classical economic theories where the strongly negative elasticity can be obtained via an implausibly large negative serial correlation of realized earnings per hour. Farber (2008)[10] suggests that the labor supply is reference dependent and that can have strong implications on policies. Farber suggested about a rich parametrization which

> *Evaluation of much government policy regarding tax and transfer programs depends on having reliable estimates of the sensitivity of labor supply to wage rates and income levels. To the extent that individuals' levels of labor supply are the result of optimization with reference-dependent preferences, the usual estimates of wage and income elasticities are likely to be misleading.*

we have seen in the previous model analysis section, that is better than the neo classical economic theories to explain this problem. In this article Crawford and Meng tried to frame a better model following Farber and also allowing the gain - loss utility with some other factors such as how the targets for drivers can be determined and expected earnings are the rational expectations.
So, this model also treats each day separately as the other two models discussed before. Let I and H denote his income earned and hours worked that day, and let I^r and H^r denote his income and hours targets for the day. We write the driver's total utility, $V\left(I, H \mid I^r, H^r\right)$, as a weighted average of consumption utility $U_1(I)+U_2(H)$ and gain-loss utility $R\left(I, H \mid I^r, H^r\right)$, with weights $1-\eta$ and $\eta($ where $0 \leq \eta \leq 1$), as follows:

$$V\left(I, H \mid I^r, H^r\right)=(1-\eta)\left(U_1(I)+U_2(H)\right)+\eta R\left(I, H \mid I^r, H^r\right) \tag{12}$$

where gain-loss utility:

$$\begin{aligned} R\left(I, H \mid I^r, H^r\right) &= 1_{\left(I-I^r \leq 0\right)} \lambda\left(U_1(I)-U_1\left(I^r\right)\right) \\ &+1_{\left(I-I^r>0\right)}\left(U_1(I)-U_1\left(I^r\right)\right)+1_{\left(H-H^r \geq 0\right)} \lambda\left(U_2(H)-U_2\left(H^r\right)\right) \\ &+1_{\left(H-H^r<0\right)}\left(U_2(H)-U_2\left(H^r\right)\right) \end{aligned} \tag{13}$$

Here in this model a simple characterization has been allowed that the driver's optimal stopping decision with a target for hours as well as income which make this model parallel to the model by Farber's model (2005, 2008)[14] [10] characterization of optimal stopping at the reference income level. This suggests that the marginal utility of income is approximately constant and, treating $u_1(.)$ as a *von Neumann-Morgenstern utility function*, that consumption utility is approximately risk-neutral in daily income, a restriction Farber (2008) and which has been imposed here in the structural analyses.
The optimal stopping decision then maximizes $V\left(I, H \mid I^r, H^r\right)$ as in previous two equations, subject to the linear menu of expected income-hours combinations $I=w^e H$. When $U_1(\cdot)$ and $U_2(\cdot)$ are concave, $V\left(I, H \mid I^r, H^r\right)$ is concave in I and H for any targets I^r and H^r. Thus the driver's decision is characterized by a first-order condition, generalized to allow kinks at the reference points: he continues if expected earnings per hour exceeds the relevant marginal rate of substitution and stops otherwise. So, if the marginal rates of substitution with reference - dependenct preference in all possible domains are will be: From the table 3 it can be seen that the marginal rates of substitution in the interiors of the four possible domains of gain - loss have been expressed as hours disutility cost per income. Here this gain - loss utility is additively separable in all dimensions following the model of reference - dependent preference and determined component by component by the difference between realized and target consumption utilities, when hours and income are both in the interior of the gains or loss domain,

	Hours gain ($H < H^r$)	Hours loss ($H > H^r$)
Income gain ($I > I^r$)	$-U_2'(H)/U_1'(I)$	$-[U_2'(H)/U_1'(I)][1-\eta+\eta\lambda]$
Income loss ($I < I^r$)	$-[U_2'(H)/U_1'(I)]/[1-\eta+\eta\lambda]$	$-U_2'(H)/U_1'(I)$

Table 3—Marginal Rates of Substitution with Reference-Dependent Preferences by Domain

the marginal rate of substitution is the same as for consumption utilities alone and the stopping decision follows the neoclassical first-order condition. But when hours and income are in the interiors of opposite domains, the marginal rate of substitution equals the consumption-utility trade-off times a factor that reflects the weight of gain-loss utility and the coefficient of loss aversion, $(1-\eta+\eta\lambda)$ or $1/(1-\eta+\eta\lambda)$. On boundaries between regions, where $I = I^r$ and/or $H = H^r$, the marginal rates of substitution are replaced by generalized derivatives whose left-hand and right-hand values equal the interior values. The following is the plot (figure 5) to show the linearity of the utility of consumption.

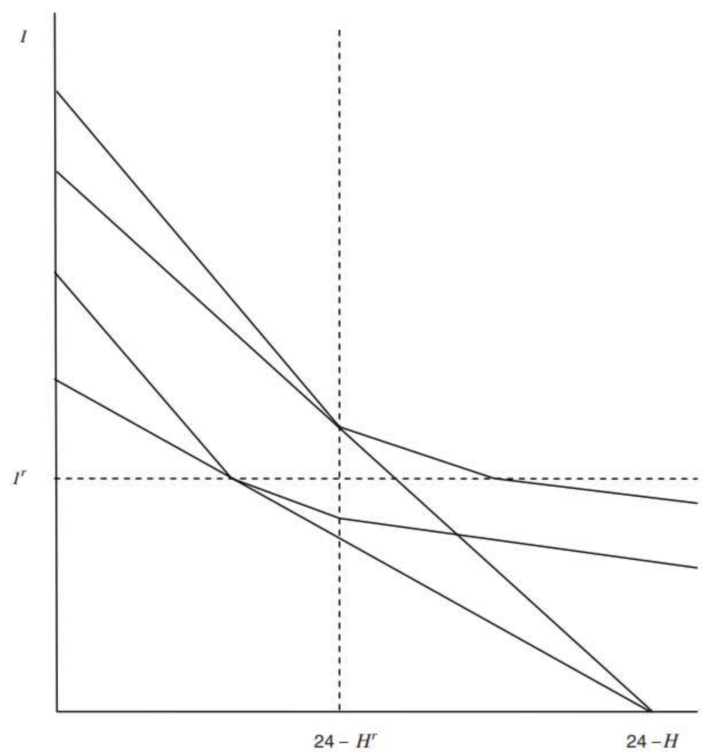

Figure 4. A Reference-Dependent Driver Stops at the Second Target He Reaches: Income on a Bad Day (lower budget line), Hours on a Good Day (upper budget line)

Analysis of the model with Data

Here for the data analysis, they have used the exact same data used by Farber. For the analysis, Farber choose only hours not the efforts, so here in this

[10] Farber generously shared his data with us. His 2005 paper gives a detailed description of the data cleaning and relevant statistics. The data are converted from trip sheets recorded by the drivers. These contain information about starting/ending time/location and fare (excluding tips) for each trip. There are in total 21 drivers and 584 trip sheets, from June 2000 to May 2001. Drivers in the sample all lease their cabs weekly so they are free to choose working hours on a daily basis. Because each driver's starting and ending hours vary widely, and 11 of 21 work some night and some day shifts, subleasing seems unlikely. Farber also collected data about weather conditions for control purposes.

model this is a new inclusion to the analysis factors. Early earnings, unlike total earnings, should be appraximately correlated with errors in stopping decision to avoid the problems related to the sample selection via endogenous variable which is a special requirement for reference - dependent preference. The summary of

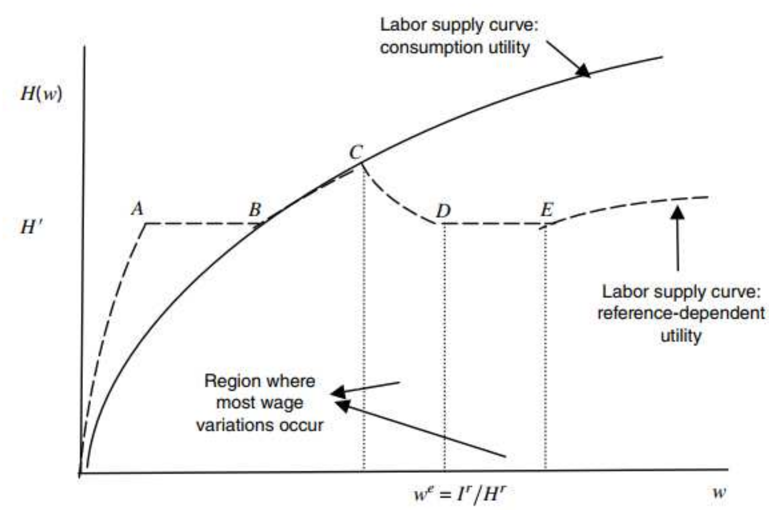

Figure 5. A Reference-Dependent Driver's Labor Supply Curve

the analysis can be represented by the figure 5. This diagram here compares labor-supply curves for a neoclassical and a reference-dependent driver with the same consumption utility functions. The solid curve is the neoclassical supply curve, and the dashed curve is the reference-dependent one. The shape of the reference-dependent curve depends on which target has a larger influence on the stopping decision, which depends on the relation between the neoclassical optimal stopping point (that is, for consumption utility alone) and the targets.

Conclusion from this model

In the conclusion, we can refer to the the figure 5 which suggests that This model, estimated with Farber's data, suggests that reference dependence is an important part of the labor-supply story in his dataset, and that using Koszegi Rabin model to take it into account does yield a useful model of cab drivers' labor supply. Overall, the results for this model are more comprehensive investigation of the behavior of cab drivers and other workers with similar choice sets, with larger datasets and more careful modeling of targets, will yield a reference-dependent model of labor supply that significantly improves upon the neoclassical model.

IV. Conclusion

In the conclusion it can be said that the neo classical economic theories are not sufficient in many cases where we need to look the problem from the behavioral aspect and this labor supply decision problem is a very strong example of this type. Here Camerar suggested the negative elasticities though satisfy the neo classical theories but were not satisfying the empirical analysis without some strong assumptions. But in the real world those assumptions are not valid, so for a better model we need to consider every possible angle of view and reference - dependence preference theory is an important tool for this. But there are still a lot of unexplained questions floating and for that we need to formulate models with more consideration to all possible factors in all possible domains and dimensions, which we expect to see in near future.

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