Presumptive taxation and firms' efficiency: an integrated approach for tax compliance analysis

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Presumptive taxation and firms' efficiency: an integrated approach for tax compliance analysis

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Abstract

Presumptive taxation methods are policy tools widespread adopted by fiscal authorities with the aim to improve voluntary tax compliance and to fight tax evasion. Such methods allow authorities to uncover firms' under-reporting, but face several limits. In particular, presumptive taxation methods do not allow to disentangle when the presence of under-reporting is ascribable to tax evasion behaviour or to the lack of managerial skills and inefficiency. To overcome the main presumptive taxation weakness, we propose combining presumptive frameworks with a measure of technical efficiency, thus developing an integrated approach for tax evasion analysis able to support the audit activities of fiscal authorities. Further, we provide some considerations in terms of tax compliance and support our approach with evidence obtained from an empirical application based on Italian firms.
Keywords: Tax Compliance, Presumptive Taxation, Efficiency, Stochastic Frontier, Business Sector Studies
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1. Introduction

Increasing voluntary tax compliance in order to fight tax evasion represents a long-standing issue in the policy and academic debate. The economics-

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of-crime literature, stemming from the study by Allinhgam and Sadmo (1972), suggests that the probability of detection and penalties are key elements to fight tax evasion and to increase voluntary tax compliance. However, over the past years, audit rates and punishment probabilities worldwide have scarcely succeeded in deterring cheaters, increasing the interest in the adoption of alternative tax enforcement methods, such as presumptive taxation.[1]

Presumptive taxation is an alternative methodology for the assessment of tax liability different from the regular method used to compute actual taxable income based on taxpayers' accounts (as defined by law). Presumptive taxation methods reconstruct taxpayers' income through administrative practices using information supplementary to accounting and fiscal data. Usually, this information refer to variables that are not included in the standard computation of taxable income but are linked to income generation and easily achievable by tax authorities. This approach is used whenever taxpayers are small and medium enterprises and self-employed, the so-called hard-to-tax, for which a significant degree of tax evasion and noncompliance is observed (Logue and Vettori, 2011, Bruhn and Loeprick, 2016).[2]

One of the main consequences of presumptive taxation methods is the possibility to uncover situations of under-reporting of revenues and costs by comparing reported and presumed values. When there is a disagreement with the presumed value, firms could either decide to align with the presumed result by adjusting their reported data, or they could appeal to the fiscal authority and provide evidence that their actual performance is lower than the value reconstructed through the presumptive method (De Jantscher and Tanzi, 1987).

Several advantages could originate from the adoption of presumptive tax regimes. First, when evasion is due to complex tax systems, in terms of the lack of knowledge/understanding of fiscal obligations and/or high compliance costs, a presumptive tax system, having a low compliance burden and simple rules, would increase voluntary tax compliance and decrease fiscal evasion. Second, from the fiscal authority's point of view, presumptive taxa-

[1]For a through review of recent studies using various tax enforcement instruments see Slemrod (2019).

[2]Strong incentives for SMEs and self-employed taxpayers to fail voluntary tax compliance and to evade taxes are provided by complex legal provisions and administrative practices, often combined with high administrative and compliance costs, making it arduous for fiscal authorities to verify and to monitor numerous small entities (Pope, 2008).

tion methods could significantly reduce the compliance costs by overcoming administrative weaknesses to ascertain the actual tax base in the case of poor record-keeping, avoiding the costs linked to the conventional verification and assessment procedures.[3] Third, presumptive taxation can lead to higher levels of horizontal equity, such as between salaried employees in the formal sector who are generally unable to avoid taxes and self-employed taxpayers who potentially could have the opportunity to evade taxes (De Jantscher and Tanzi, 1987).

However, presumptive taxation methods face also several limits. The presumptive rules are known by the taxpayers and, consequently, firms have a strong incentive to manipulate reported revenues and costs in order to reach the presumed value. Another weakness of presumptive approach is that it usually adopts a regression framework to model the behaviour of the average firm. This average setting could induce firms above the presumed values to reduce their reported numbers, hiding their extra performance to the tax authority. Finally, presumptive taxation methods allow authorities to uncover situations of firms' under-reporting (i.e., presumptive values are higher than the data reported by taxpayers) without discerning the motivations for such discrepancies.

During the last decades, literature has long investigated the role of presumptive taxation methods in increasing voluntary tax compliance. While the theoretical studies succeeded in providing an assessment of the risks and benefits of presumptive methods (Sadka and Tanzi, 1993), empirical studies failed in reaching a shared consensus about the efficacy of these policy tools. In a comprehensive survey of empirical literature, Bucci (2020) concludes that presumptive taxation methods usually are weakly effective in increasing tax revenues collection and voluntary tax compliance, both in developed and developing context.

With the aim to exploiting the potential advantages of presumptive taxation methods, solving their main weakness, we propose an integrated approach for tax evasion analysis. The integrated methodology combines presumptive taxation systems with a measure of technical efficiency. In particular, we propose to estimate a production function using the stochastic

[3] Presumptive taxation methods may be very useful when the books and records are difficult to verify for the tax authorities or when such values do not correctly reflect the taxable capacity (Slemrod and Yitzhaki, 1994, Martins and Sa, 2018).

frontier analysis based on generalized additive model specification (Vidoli and Ferrara, 2015), setting the threshold at the level of the most efficient firm (Kumbhakar and Lovell, 2000). We provide a relative measure of technical efficiency, notified to taxpayers only *ex-post*, in order to reduce the potential for further manipulation of reported data. We solve a critical issue of presumptive taxation methods, represented by the presence of under-reporting (accidental or not) of inputs, by adopting a two-step methodology to estimate the production inputs potentially affected by under-reporting. Our integrated approach reduces the unfair behaviours generated by both the consideration of the average firm and the advantage of revenues/costs manipulation - i.e., taxpayers' learning-by-doing attitude. The main contribution of such integrated methodology relies on the capacity to disentangle if situations of firms' under-reporting are ascribable to tax evasion behaviour or to the lack of managerial skills, implying firms' inefficiency.

Further, we develop a tax-compliance analysis, aimed at evaluating taxpayers' inclination to evasive behaviours, and provide an empirical application based on Italian firms supporting the potentiality of our approach.

This paper contributes to existing literature in several ways. First, we contribute to the literature on presumptive taxation methods pointing out that, if integrated with efficiency measures, such methods could be effective in identifying taxpayers adopting potentially anomalous behaviours ascribable to tax evasion. Second, we show that the concept of firms' efficiency is suitable for application in tax evasion analysis. The existing literature has long investigated the determinants of firms' efficiency (Bottaso and Sembenelli, 2004, Heshmati, 2003, Alvarez and Crespi, 2003), but the role of efficiency in firms' choice has rather been neglected. It is possible to find scant evidence on the role of firm's efficiency on corporate capital structure(Margaritis and Psillaki, 2007, Berger and Bonaccorsi, 2006), but, to the best of our knowledge, this is the first paper that takes into consideration the effect of taxpayers' efficiency in tax evasion analysis. Third, the tax-compliance analysis provides a significant contribution to the tax evasion literature, which has long studied if taxpayers characteristics could affect evasive behaviors (Slemrod, 2007). For example, analysing taxpayers belonging to different economic sectors would allow to identify different compliance patterns for firms within different industries (Gokalp, et al., 2017, Tedds, 2010).

The integrated approach combined with the tax compliance analysis could provide an useful policy tool able to support controls by tax authorities, aimed at improving voluntary tax compliance and fighting tax evasion by

SMEs and self-employed taxpayers.

The remainder of this paper is organized as follows. Section 2 provides some background information and briefly describes several presumptive taxation methods. In Section 3, we develop and discuss the empirical model. In Section 4, we describe the empirical application and present the main findings. The final section offers some concluding remarks and discusses the main issues for future research.

2. Institutional background

Tax evasion represents an urgent issue for many governments worldwide. Several reasons could explain the urgency. The most common reason is that, due to tax evasion, significant revenue losses occur, implying decreased public expenditures and cuts in the public services that citizens receive. The misallocation of resources is another consequence of tax evasion, which could cause economic agents to alter their behaviour, by modifying, for example, the labour supply or investment schedules (Alm, 2019). Further, tax evasion could prevent the socially-optimal redistribution of income and resources by altering the distribution of income in unfair ways. Finally, feelings of unfair behaviour and non-observance of the law may arise in a fiscal system affected by tax evasion, undermining citizens' trust in their governments.

The necessity to improve the efficiency of the tax system by minimizing the costs of taxation, in the spirit of optimal taxation (Slemrod, 1990), and decreasing evasion has led to the adoption of presumptive taxation methods.

The concept of presumptive taxation entails different alternatives for reconstructing taxable income. Usually, these methods are based on administrative practices and use the information reported by the taxpayers but that are not all included in the standard computation of taxable income. These information concern characteristics presumably related to income generation (i.e., sales, the number of employees, company size, company location, and the nature of the business).

It is possible to find elements of presumptive taxation in several countries.[4] The most known examples in this field are provided by the Israeli

[4]Weichenrieder (2007) provides a description of several presumptive taxation methods adopted by OECD countries, including Japan with the *Taxation by Estimation Method* and Spain with the *Estimacion Objetiva*.

Tachshiv, the French *Forfait*, and the Italian *Business Sector Studies*.[5]

The *Tachshiv* of Israel is widely referred to as the most complex presumptive taxation method. It was designed in order to compute the value of net profit upon which the tax is imposed. The computation of net profit is based on a two-step procedure. In the first step, a negotiation process between the tax authority and industries' representatives results in the definition of several indicators that are different for specific industries (these indicators concern the average sales per worker, the average ratio between inventory and turnover, and the correlation between water consumption and sales). These average values are then used to estimate the value of the firm's turnover. In the second step, the fiscal authority defines the value of the taxable profits, subtracting a presumed amount of business expenses from the estimated turnover.

The French *Forfait* is a contractual method involving a negotiation between the taxpayer and the tax authority on the amount of taxes due. It is applicable only to SMEs with an annual turnover below a specified threshold. To comply with the presumptive tax regime, the taxpayer has to provide the fiscal authority with various types of information that can be compared to the previous year (i.e., purchases and sales, the value of the closing inventory, the number of employees, the amount of wages paid). Then, the tax authority estimates the value of taxable income by combining the information provided by the taxpayers with general business expenses. The proposal of tax liabilities formulated by the tax authority is then subject to agreement by the taxpayer.

The Italian *Business Sector Studies (BSS)* estimates the value of the presumptive turnover of SMEs and self-employed using the information declared by these taxpayers.[6] The *BSS* is based on the hypothesis that two firms produce the same turnover and should have to pay the same amount of taxes if they belong to the same economic sector and face the same combination of input costs (i.e., the two firms share the same production function). Therefore, given that the "true" level of turnover is unknown, an econometric regression model is used to estimate the presumptive value, conditional on taxpayers' characteristics, obtaining a vector of coefficients showing the presumptive

[5]For detailed descriptions of these presumptive taxation methods see Arachi and Santoro (2007), Thuronyi (2004), and Yitzhaki (2007).

[6]The *BSS* concerns SMEs and the self-employed reporting annual revenues up to 5.165 million euros. For a detailed description of the *BSS* structure see Fiorio et al. (2013).

productivity of each input. The reported turnover is then compared with the presumed value. Whenever firms report a value of turnover below the presumptive threshold, they are defined "non-congruous" and could be audited by the Italian tax authority.[7] Therefore, the *BSS* could be considered an audit selection mechanism. However, the *BSS* could also be viewed as a method of presumptive taxation, given that the rules used to estimate the value of the turnover are known by the firms. As a consequence, for the dishonest firms, the reported values converge to the ones presumed by the fiscal authority, generating tax evasion (Arachi and Santoro, 2007, Santoro, 2008, Santoro and Fiorio, 2011).

Regardless of the technicalities of the methodology adopted, presumptive taxation methods could allow authorities to uncover situations of under-reporting of revenues and costs, by comparing reported and presumed values. In the case of discrepancies between reported and presumed values, firms have the possibility to adapt to the presumed results or to appeal to the fiscal authority by providing evidence that their actual performance is lower than the presumed one.

The risk of abuse by firms represents one of the main limits of presumptive taxation systems. When the presumptive regime is too simple and too favourable, firms could choose to alter their behaviour by manipulating reporting data in order to rely on the presumptive regime because of the opportunity to reduce their tax burden (Memon, 2013, Dube, 2018).

An extremely important element of the presumptive framework concerns the timing. Usually, when firms report revenues and costs, the rules used by the tax authority to estimate presumptive values are known. Consequently, firms have the possibility of manipulating reported numbers, in order to reach the presumed values.[8]

Another weakness of presumptive systems is represented by the average setting: the presumptive values are often estimated using an OLS regression

[7]For details on the effectiveness of the *BSS* methodology in forcing tax compliance, by highlighting the gaps between declared and presumed values see Senato (2019) and Vellutini et al. (2019).

[8]With respect to the Italian context, Convenevole et al. (2006) find that the share of firms reporting turnover lower than the presumed values strongly decreased over the years. According to Pisani (2004), this decreasing trend has been caused by a learning-by-doing process of manipulation of the reported variables, which lowered the presumed level of turnover.

to catch the average firm's behaviour. This average setting could be exploited to under-report revenues for firms above the presumed values, inducing the firms to hide their extra performance to the tax authority.[9] Further, this process could have caused the lowering of the presumed values over time.

Finally, an important limit of the presumptive taxation methods is the inability to identify the motivations for the presence of under-reporting (i.e., presumptive values higher than reported numbers). These methods do not allow one to disentangle when the under-reporting is ascribable to tax evasion behaviour or to the lack of managerial skills, implying firms' inefficiency.

3. The empirical model: the integrated approach

With the aim to exploit the potential advantages of presumptive taxation, solving their main weakness, we propose an integrated approach for tax evasion analysis. The integrated approach consists of combining efficiency analysis with the standard presumptive taxation framework.

The proposed methodology has been developed on presumptive taxation methods that estimate the value of turnover, such as the Israeli *Tachshiv* or the Italian *Business Sector Studies*. However, it can be applied to different presumptive performance measures.

3.1. The efficiency score estimation

The starting point of our integrated approach is the computation of a measure of technical efficiency for each firm, obtained by implementing the stochastic frontier model.

A critical issue of presumptive taxation methods is represented by the under-reporting (accidental or not) of inputs by firms. Therefore, with the aim of solving this issue, we propose a two-step procedure. In particular, in the first step we estimate a linear regression model for each input potentially affected by under-reporting x_i, using an input-specific vector of explanatory variables $\boldsymbol{Z_i}$:

$$x_i = \alpha + \boldsymbol{\gamma} \boldsymbol{Z_i} + \epsilon_i \qquad i = 1, \dots, n \tag{1}$$

[9]The evidence in this filed has been provided by Pisani (2004), who shows that over the years a growing number of taxpayers declared revenues around the mean value of the distribution, highlighting a process of learning-by-doing in terms of *BSS* manipulation by Italian firms, thus generating tax evasion.

When the input reported by firm x_i is lower than the fitted value ($\hat{x}_i$), then the fitted value is used in the production function estimation, according to the following formula:

$$x_i^* = \max(x_i, \hat{x}_i) \qquad i = 1, \ldots, n \tag{2}$$

The second step proceeds with the estimation of the production function parameters. Our starting point is the parametric stochastic frontier model introduced by Aigner et al. (1977) and Meeusen and van den Broeck (1977).[10] In particular, we define the production function as a combination of a response function and a composite error term:

$$y_i = f(\boldsymbol{x_i}, \beta) + v_i - u_i \quad i = 1, \ldots, n \tag{3}$$

where $f(\cdot)$ represents the production relationship between the vector of inputs ($\boldsymbol{x}_i$), potentially resulting from the combination of Eq. (1) and Eq. (2), and the maximum level of output achievable by firm i (y_i), and β is the vector of parameters to be estimated through the stochastic frontier model. The composite error term is given by the term v_i, which represents symmetric disturbance and is assumed to be *i.i.d.* ($N(0, \sigma_v^2)$), and the term u_i, which represents the error component reflecting technical inefficiency and is assumed to be distributed independently of v_i as a half-normal truncated distribution above zero ($N^+(0, \sigma_u^2)$).

In spite of its simple computation and interpretation, this model does not allow for too much flexibility. Indeed, forcing $f(\cdot)$ to belong to a fully parametric family of functions (*i.e.*, translog or Cobb-Douglas) might lead to a non negligible bias in the model specification and to misleading conclusions about the link between inputs and output.

To overcome drawbacks due to the specification of a particular production function, Fan et al. (1996) introduce a two-step pseudo-likelihood procedure for the estimation of stochastic frontier model where the functional form of the frontier is unknown and estimated via kernel regression.[11] In this work, we consider the Vidoli and Ferrara (2015) model that extends Fan et

[10]In the literature different stochastic frontier models have been proposed. Overviews of developments in this area have been provided by Sickles and Zelenyuk (2019) and Kumbhakar and Lovell (2000).

[11]More recently, Kumbhakar et al. (2007) propose a new approach based on the local maximum likelihood principle.

al. (1996) approach by specifying a Generalized Additive Model framework (GAM). In a regression context with Normal response, the model can be expressed as:

$$E(Y|\boldsymbol{X} = \boldsymbol{x}) = \psi_0 + \sum_{j=1}^{p} \psi_j(X_j), \tag{4}$$

where the $\psi_j(\cdot)'s$ are smooth functions standardized so that $E[\psi_j(X_j)] = 0$ (Hastie and Tibshirani, 1990). This model takes into account the variability of the response through an additive function of the inputs, as in the corresponding stochastic frontier model.

The main advantages of using the GAM specification for the stochastic frontier analysis (GAM-SFA) over the standard approaches are: a) the considerations that the transformations ψ_j's are determined simultaneously and the non-linear fits can potentially make more accurate prediction of the response; b) the non-parametric estimators of the unknown functions ψ_j are able to avoid the *curse of dimensionality* since each additive terms is estimated using a univariate smoother; c) the partial response function ψ_j shows how the prediction changes with respect to X_j, as in an additive liner model. Furthermore, the gradients of the non-parametric model can be interpreted as partial output elasticities and their sum (i.e. elasticity of scale: eos) highlights useful information about specific return to scale.

In a cross-sectional setting, model (3) becomes:

$$y_i = \Psi(\boldsymbol{x}_i) + v_i - u_i, \qquad i = 1, \ldots, n, \tag{5}$$

where the unknown function $\Psi(\cdot)$ is modelled *via* GAM (4). When the response is measured in logs, in frontier models the relative estimates of the technical efficiency for each unit is obtained by:[12]

$$TE_i = \exp\{-\hat{u}_i\}. \tag{6}$$

[12]The model estimation has been carried out through the `R` Environment (R Core Team, 2020) using the `semsfa` and the `mgcv` packages. In particular, the $f_j's$ smooth functions are represented using thin plate regression splines avoiding the knot placement problems of conventional regression spline modelling (Wood, 2003). Please see Wood (2006) for the test statistics related to smooth terms and the graphical representations for the analysis and interpretation of the ψ_j's. For a recent review of the stochastic frontier analysis using R see (Ferrara, 2020).

3.2. Integrated analysis

Once we have estimated the efficiency scores for each firm, we combine these measures with the presumptive results. In particular, we consider the efficiency score and the relative distance between the reported and presumed turnover simultaneously for each firm.[13] We assume that in the presence of a negative gap between the recorded and presumed values associated with a low efficiency score, it is more likely that the under-reporting is due to firm's inefficiency rather than to tax evasion. Otherwise, tax evasion behaviour is more likely in presence of highly efficient firms exhibiting negative differences between their reported and presumed values.

The integrated approach is suitable to be represented through a graph analysis, like Figure 1, where we represent each taxpayer in terms of its efficiency score and its presumptive result (i.e., the difference between reported and presumed turnover). Therefore, by merging the efficiency-presumptive analysis, we categorize the taxpayers into four different quadrants: the firms in quadrants I and II report a turnover higher than the value presumed by the fiscal authority. Therefore, it is less likely that such firms are adopting tax evasion behaviours.[14]

In contrast, firms in quadrants III and IV have negative gaps between their declared and presumed turnover. The discriminant between quadrants III and IV relies on a threshold value established for the efficiency score, which allows one to distinguish the group of high efficiency firms (quadrant III) from the low efficiency ones (quadrant IV). In this graph analysis, for illustrative purposes, we set the threshold value equal to 0.5, but in the empirical analysis, it is possible to define the threshold value using different measures (such as the first quartile of the corresponding distribution, or the value of a different percentile).[15]

Therefore, while for quadrant IV the negative gap between reported and

[13]In what follows, the relative distance is computed as the difference between recorded and presumed turnover as a share of the presumed value.

[14]In order to simplify our integrated approach we excluded from the group of potential tax evaders firms reporting a turnover higher than the presumed value. However, tax evaders that reach the presumed turnover due to the manipulation of reported variables are also included among those firms exceeding the presumed values. Allowing this possibility implies the necessity to include taxpayers just above the presumed values in the group of potential tax evaders.

[15]We consider all the potential values that the efficiency score could assume, ranging from 0 for the least efficient firm to 1 for the most efficient firm.

presumed turnover is linked to a low efficiency score, the firms in quadrant III do not reach the presumed values, despite a high level of efficiency. Therefore, the combined presumptive-efficiency analysis allows us to identify quadrant III as the area where there is more likely to be a mass of dishonest firms that adopt potentially anomalous behaviours ascribable to tax evasion. Therefore, the firms in quadrant III could require additional investigations by the fiscal authority.

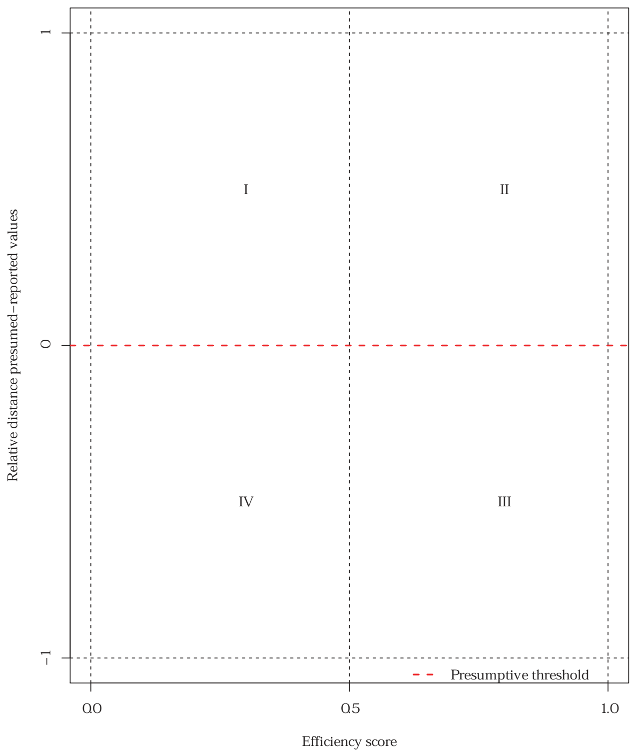

Figure 1: Integrated analysis

3.3. Tax compliance analysis

Further, we develop a tax-compliance analysis, which could provide a robustness check for the validity of our integrated methodology and could allow to obtain an outline of the potential tax evasion.

The empirical analysis of tax compliance is notoriously difficult due to the lack of reliable information on taxpayers' compliance, which gets complicated to define and to measure such phenomenon (Alm, 2019, Slemrod and Weber, 2012, Torgler, 2016).

We solve this issue by assuming that a difference between the value of the presumed and reported turnover is due to taxpayers' non-compliance, implying a difference between the potential taxes collected and the amount of taxes actually paid.[16]

Therefore, we compute a simple non-compliance measure (*SSR*) based on the sum of squared residuals of the deviation of a firm's turnover from the presumed threshold.

Then, to take into account the efficiency level, we weight the *SSR* measure for firms' efficiency scores (*W-SSR*). The higher the difference between the *W-SSR* and *SSR* is, the lower the efficiency level.

Finally, taken into account that exclusively such firms below the presumed value need further investigation by the tax authority, we define a more stringent measure for non-compliance: *NW-SSR*. It is based on the sum of squared residuals of the deviation of the turnover from the presumed threshold of firms reporting a turnover lower than the presumed value, which is weighted based on the efficiency scores. A higher *NW-SSR* implies lower tax compliance and, consequently, higher fiscal evasion.

The tax-compliance analysis would allow us to compare and to rank different groups of taxpayers in terms of their tax compliance and exposure to tax evasion behaviours.[17]

4. Empirical application

In order to test the potentiality of our integrated approach we implement an empirical application based on the Italian method of presumptive taxation: *Business Sector Studies (BSS)*. In particular, we take advantage of the

[16]The idea behind this is that the higher the deviation of each taxpayer from the presumed value is, the higher the value of the under-reporting of turnover and, consequently, the lower the tax compliance.

[17]It is important to highlight that a simple comparison between *NW-SSR* measures could result in conclusions distorted by different numerousness of taxpayers' groups. Therefore, we suggest weighting such measures according to the number of firms below the presumed turnover value before comparing these measures.

availability of a cross-sectional data-set concerning the *BSS* regime provided by Ferrara (2011).

The empirical results show that adding an efficiency dimension into the presumptive framework can disentangle when under-reporting is ascribable to tax evasion or to firms' inefficiency.

In what follows, we run the empirical analysis for two subsamples of firms belonging to two different economic sectors: the retail and the services.

4.1. Data description

The data set includes firms active in the retail and in the services economic sectors and that were subject to the *Business Sector Studies* in the 2006 fiscal year. In particular, we used a sample composed of 222 companies belonging to two different sectors: the retail (102 firms) and the services (120 firms).

Table 1 provides the descriptive statistics of the main variables used in the empirical analysis: the number of employees; capital stock; input costs, which were divided into the costs for goods and costs for services. Additionally, information concerning two structural variables, the size in square metres of the company's office/shop and the amount of gas used in cubic metres, is provided only for firms belonging to the services sample (Panel (b) of Table 1).

By comparing each sample with the corresponding *BSS* values, we observe that although the two samples have not been randomly selected, their average values are rather close to the ones of the corresponding *BSS*.

By comparing the values by economic sectors, it emerges that the retail sample reports on average higher turnover and higher costs in terms of both goods and services purchased. Further, the retail sample employs fewer employees and records lower capital stock. Finally, taxpayers belonging to the services sample have an average headquarters size of 41 square metres and use more than one thousand cubic metres of gas.

4.2. The retail sector

The empirical application for the estimation of the efficiency scores starts with the regression frontier analysis.

As illustrated in 3.1, in order to avoid the problem of under-reporting (accidental or not) of inputs by taxpayers, we adopt a two-step procedure. In the first step, we estimate a linear regression model for the labour variable,

Table 1: Summary statistics

(a) Retail Sector

Variables	Sample (n=102)		BSS (n = 10,235)	
	Mean	St. Dev.	Mean	St. Dev.
Turnover	230	143	203	310
Capital stock	93	79	83	125
Number of employees	2	1	2	5
Costs of materials	116	78	108	183
Costs of services	23	23	19	39

(b) Services Sector

Variables	Sample (n=120)		BSS (n=88,110)	
	Mean	St. Dev.	Mean	St. Dev.
Turnover	177	169	189	259
Capital stock	104	154	77	129
Number of employees	3	1	3	3
Costs of materials	65	61	78	104
Costs of services	19	21	22	41
Headquarter size	41	100	28	28
Gas consumption	1,124	2,201	3,500	7,700

Note: All variables are expressed in thousands of euros, except for headquarters size, expressed in square metres, and gas consumption, expressed in cubic metres.

the input potentially mainly affected by under-reporting. We specify the labour equation as follows:

$$L_i = \gamma_1 K_i + \gamma_2 M_i + \gamma_3 S_i + \nu_i \qquad i = 1, \ldots, n \tag{7}$$

where L_i is the number of employees employed by company i, K_i is the capital stock, S_i is the costs for services, and M_i is the costs for materials (all the variables are expressed in logs).

The second step proceeds with the estimation of the production function by implementing the stochastic frontier analysis based on the GAM-SFA model:

$$y_i = \psi(\beta_0 + \beta_1 K_i + \beta_2 L_i^* + \beta_3 M_i) + v_i - u_i, \qquad i = 1, \ldots, n, \tag{8}$$

where y_i is the log value of the turnover reported by company i and L_i^* is the higher number between the reported and the fitted value for the labour variable obtained following Eq. (1) and Eq. (2).

In Figure 2 we show the estimated partial effects of the GAM frontier, providing evidence about the non-linearity (in contrast with the corresponding Cobb-Douglas specification of the production function - linear or logs). [18]

Figure 3 shows that the efficiency scores estimated for retail range from 0.66 to 0.96 and that a high share of firms in this sample are quite efficient, since most of the firms report an efficiency score higher than 0.9 (approximately 54%). Given that the nonparametric specification allows for subject specific partial elasticities, we report also the distribution of the elasticity of scale, given by the sum of the partial elasticities, highlighting increasing return to scale for all firms.

The efficiency scores estimated through the GAM-SFA model for each firm in the sample are then combined with the relative distances between the reported turnover and the presumptive values estimated by the Italian tax authority under the *BSS* regime: the *BSS* congruity threshold.[19] The congruity analysis shows that taxpayers lie both above and below the *BSS* threshold, with some predominance above. However, when we jointly consider both dimensions (efficiency and congruity), interesting evidence emerges. The low

[18]The estimation results are reported in Appendix.

[19]As explained in Section 2, firms reporting a value of turnover below the presumed threshold are defined to be non-congruous.

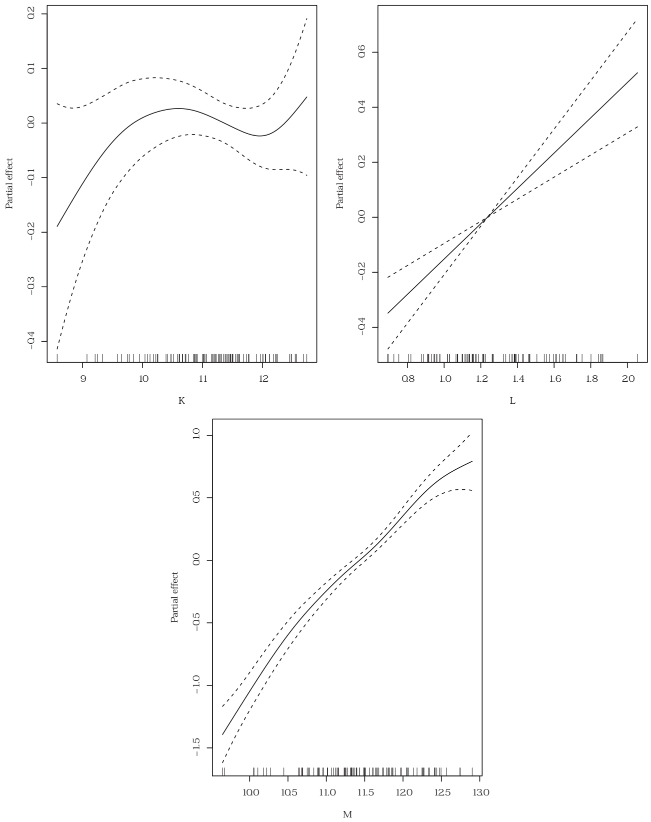

Figure 2: Partial effects with confidence intervals (dotted lines) of the estimated GAM frontier: the retail sector

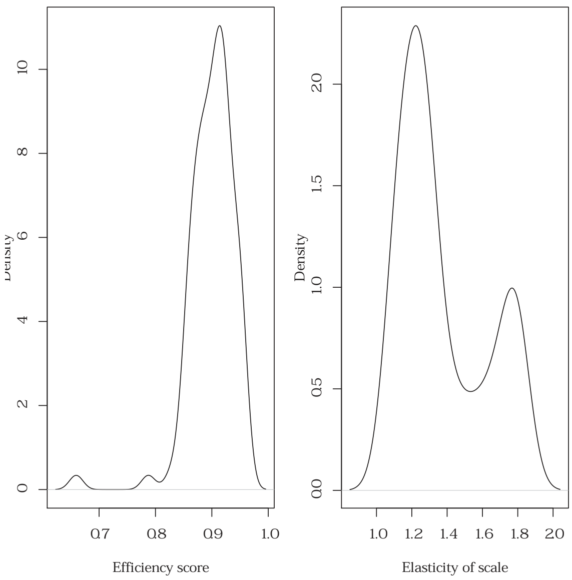

Figure 3: Efficiency and elasticity of scale for the retail sector

efficiency firms (i.e., efficiency score below 0.9) are quite uniformly spread out along the estimated threshold. For the high efficiency scores (i.e., higher than 0.9), the number of firms recording turnover above the estimated threshold is higher than the share of firms with reported turnover below the *BSS* threshold (54% *vs.* 46%, respectively). This evidence highlights that most of the efficient firms in the retail sample honestly declare their turnover.

This analysis provides an useful tool able to support the audit activities of fiscal authorities, since it could help to distinguish non congruous cheating firms from firms with economic problems. For example, although firm **A** in Figure 4 is highly efficient, the reported turnover value is significantly below the *BSS* congruity thresholds. This evidence could inform authorities that this firm is engaging in anomalous behaviour that is probably ascribable to tax evasion activity. In contrast, firm **B** has a non-congruity status, that could be in part explained by the presence of a low efficiency score rather than a tax evasion behaviour.[20]

[20] In this case we fix the threshold efficiency value to the first quartile of the corresponding distribution.

Therefore, the combined analysis allows authorities to identify which are the non-congruous taxpayers needing additional controls by the fiscal authority because of their higher likelihood of tax evasion behaviour.

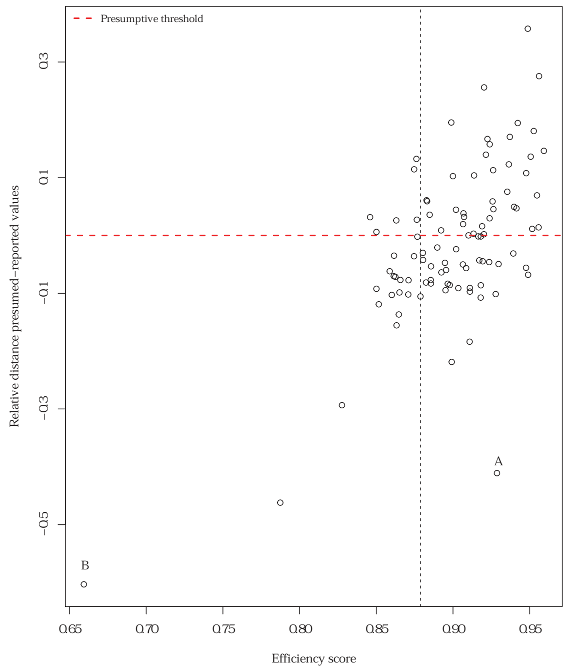

Figure 4: Efficiency-congruity: the retail sector

To test the robustness of our findings, we replicated the combined analysis conditioned by quartiles using a single-index.[21] Figure 5 shows that smaller firms (i.e., the firms in quartiles I and II) have higher efficiency scores and are more likely to record turnovers higher than the estimated threshold. On

[21]We divided the firms in the sample into quartiles based on the value of the sum of the standardized values of K and M rather than turnover (which was already used to estimate both the efficiency score and to compute the relative distance from the *BSS* congruity threshold). This choice originates from considering that the single-index measures the level of inputs, and, therefore, it is strongly correlated with revenues and could be used as a proxy for firm size.

the contrary, larger firms (quartiles III and IV) have higher spreads in terms of their efficiency scores and are more likely to declare turnover lower than the *BSS* estimated threshold. In particular, in quartile IV, several cases with under-reported turnover are not connected with low efficiency scores.

Finally, we provide some insights in terms of tax compliance by measuring the non-compliance level for the retail sample, both overall and by quartiles. The findings are summarized in Table 2. The overall gap between the declared and presumed turnover estimated for the retail is equal to 1.836. If we take into account firms' size, we find that the *SSR* is much higher in quartile III and IV, while in first two quartile the value of the gap is lower.

As expected, the non-compliance measure is reduced quite proportionally when we weight the measure by taking into account the efficiency scores (*W-SSR*). If we compute the non-compliance measure by restricting the firms to the subset of taxpayers reporting a turnover lower than the presumed value, we find that the *NW-SSR* is almost half the *SSR*. However, the most important result emerges when analysing the non-compliance measures by quartiles. The values of the *NW-SSR* for smaller firms (quartiles I and II) are much lower than those of larger ones (III and IV). This evidence, in line with the findings in Figure 5, allows us to conclude that in the retail sample smaller firms are more compliant, while the high non-compliance measure for larger firms signals more frequent anomalous behaviours.

Table 2: Tax compliance in the retail sector

	Turnover (€ x 1000)	*SSR*	*W-SSR*	*NW-SSR*
Full sample	230	1.836	1.557	0.952
I quartile	109	0.398	0.367	0.020
II quartile	181	0.274	0.254	0.088
III quartile	218	0.626	0.468	0.447
IV quartile	433	0.537	0.466	0.397

4.3. The services sector

As for the retail sample, the empirical application starts with the estimation of the labour variable, which is the input potentially affected by

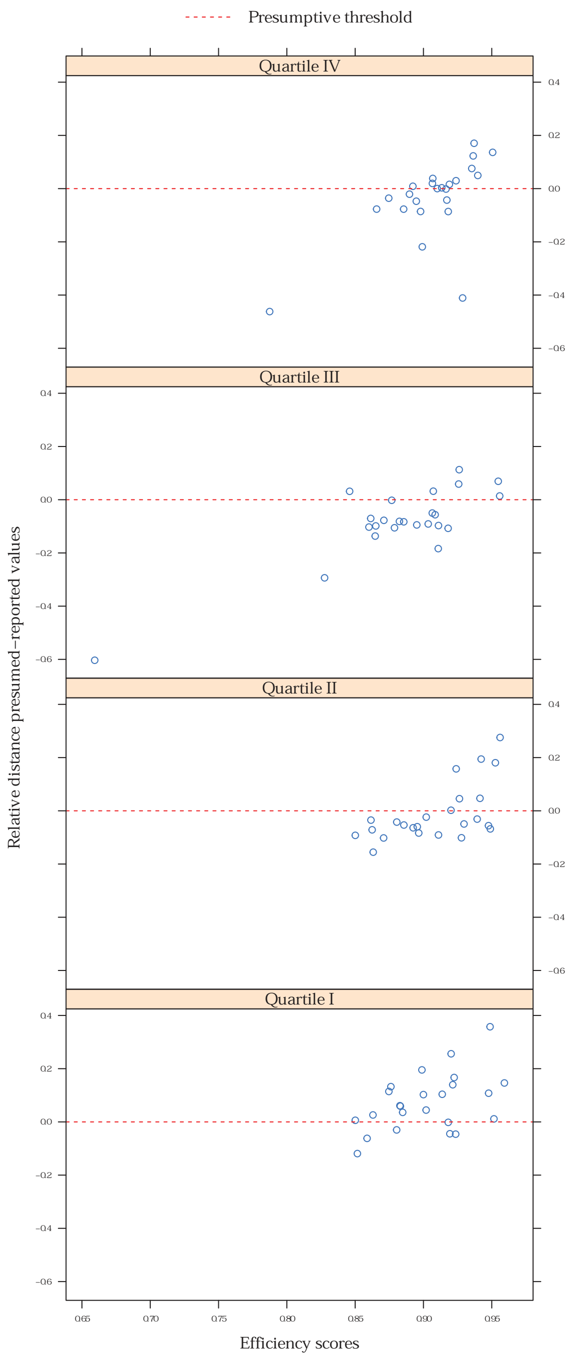

Figure 5: Efficiency-congruity by size: the retail sector

under-reporting. In particular, we specify the labour equation as it follows:

$$L_i = \gamma_1 K_i + \gamma_2 M_i + \gamma_3 S_i + \gamma_4 Lab_i + \gamma_5 Gas_i + \nu_i, \qquad i = 1, \dots, n, \qquad (9)$$

Therefore, we define the number of employees (L_i) as a function of the capital stock (K_i), the costs for services and for materials (S_i and M_i, respectively), the office/shop size (Lab_i) and the amount of gas used (Gas_i).[22]

In Figure 6 we report the estimated frontier in terms of partial effects providing, as for the retail sector, an evidence about the non linearity of the production function estimated via $\psi(\cdot)$.

Figure 7 shows that the efficiency scores estimated range from 0.85 to 0.95 and that a high share of firms in this sample are quite efficient, since most of the firms report an efficiency score higher than 0.9 (approximately 86%). As for the retail sector, we report the distribution of the elasticity of scale, highlighting increasing return to scale for all firms.

Figure 8 combines the efficiency scores estimated with the outcome of the *BSS* analysis in terms of the congruity for each firm.[23]

When we compare the reported turnover with the *BSS* threshold, both positive and negative differences emerge. However, if we focus on the tails of the estimated efficiency scores, interesting results emerge. On the one hand, less efficient firms (i.e., those having efficiency scores lower than first quartile) declare turnover lower than the presumed *BSS* values and, as a consequence, they are non-congruous. On the other hand, most of the more efficient firms declare a turnover higher than the *BSS* congruity threshold (86% of such firms are congruous). This evidence suggests that in the services sample the *BSS* and the SIF approaches lead to similar results: most of the non-congruous firms exhibit low efficiency scores, while, most of the high efficiency firms are congruous based on the *BSS* presumptions.

However, as in the retail, in the services sample, it is possible to find some examples of firms with diverging results. For example, despite firm **A** has an high efficiency score, it results non-congruous, thus highlighting the potential need for further investigations by the fiscal authority. In contrast, firms **B**, with lower efficiency score and high positive distance from the *BSS* presumed threshold, indicates a potential redundancy of further investigations for potential under-reporting data.

[22]As for the retail sample, all these variables are expressed in logs.

[23]The results are available in the Appendix.

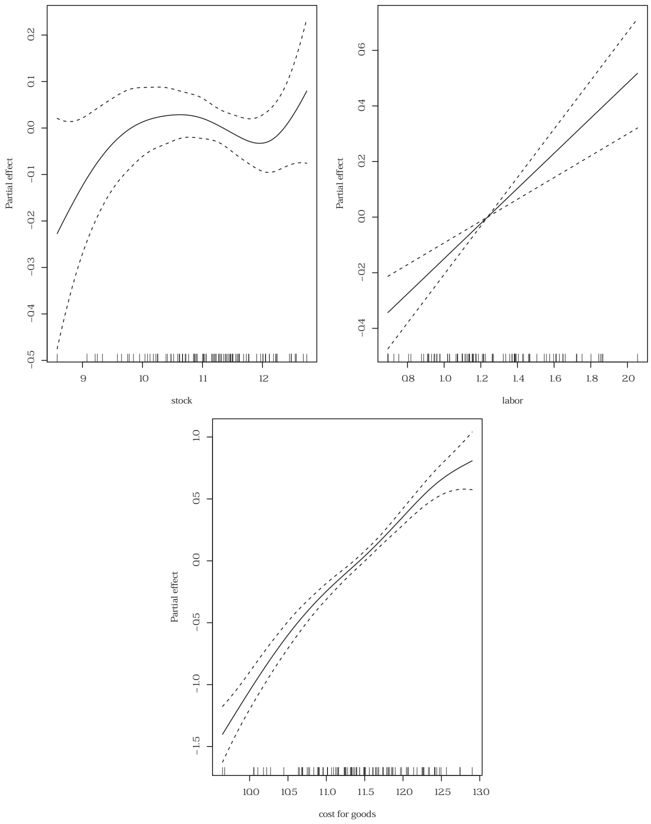

Figure 6: Partial effects with confidence intervals (dotted lines) of the estimated GAM frontier: the services sector.

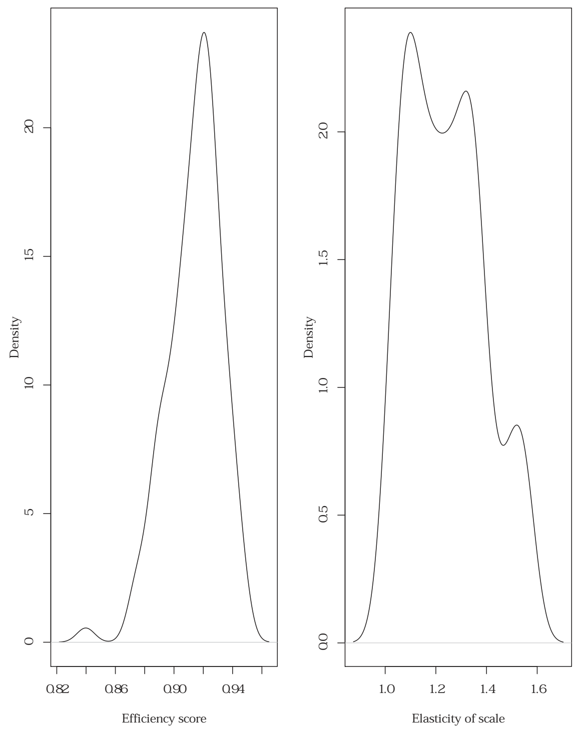

Figure 7: Efficiency and elasticity of scale for the services sector

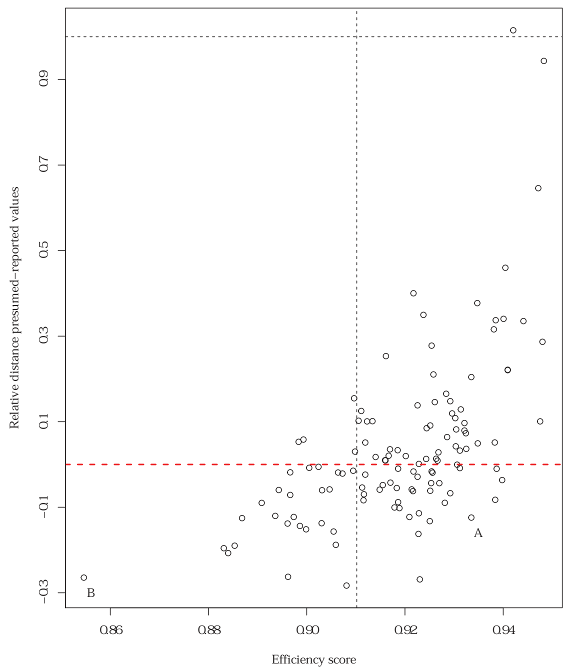

Figure 8: Efficiency-congruity: the services sector

In line with the retail sector, we have replicated the analysis by quartiles using a single-index, which is used as a proxy for the firm size. The results are summarized in Figure 9, which shows that the smallest firms (quartile I) exhibit high heterogeneity in terms of both measures: being above/below the presumed *BSS* threshold is not necessarily linked with being ranked as having low/high efficiency. Larger firms belonging to quartiles II, III and IV are rather condensed in terms of both their efficiency performance and congruity results.

Finally, in Table 3, the analysis in terms of tax compliance shows that the *SSR* estimated for the whole services sample is 2.569.[24] The higher value of non-compliance measure is concentrated in the first quartile, while lower values are recorded for the larger firms. Weighting the *SSR* for the efficiency scores reduces the non-compliance measure both for the whole sample and for the four quartiles without affecting the ranking of the four quartiles in terms of compliance. However, if we focus on the *NW-SSR* measure, a split in non-compliance values related to firms' dimension emerges: the smallest firms (quartiles I and II) record the highest values, while, the larger firms (quartiles III and IV) record the lower ones. These findings, in line with Figure 9, highlight that in the services sector, smaller firms are less compliant than the larger ones.

Table 3: Tax compliance in services sector

	Turnover (€ x 1000)	*SSR*	*W-SSR*	*NW-SSR*
Full sample	181	2.569	2.371	0.721
I quartile	58	0.938	0.859	0.356
II quartile	108	0.617	0.574	0.136
III quartile	166	0.555	0.516	0.125
IV quartile	381	0.300	0.276	0.104

[24]We exclude the three largest positive distances potentially outliers in the *BSS* regression model.

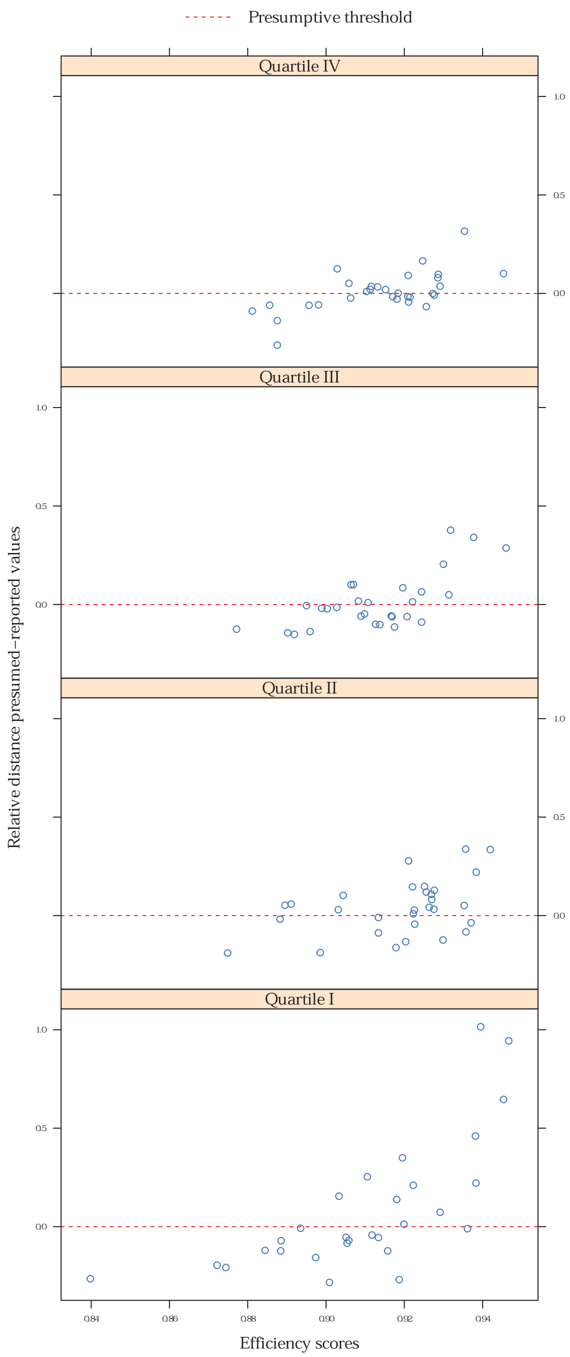

Figure 9: Efficiency-congruity by size: the services sector

4.4. Retail-services sector comparison

The empirical application of our integrated methodology to two different sectors leads to quite different conclusions. By comparing Figure 4 with Figure 8 in terms of efficiency, we find that firms in the services sector are more efficient than those in the retail sector.

The combined efficiency-congruity analysis shows that in both sectors negative differences between the reported and presumed turnover are quite frequent. However, the retail and the services sectors differ in terms of their tax evasion behaviour, with anomalous firms' reporting being less ascribable to low efficiency in the retail than in the services sector.

Further, the tax compliance analysis shows that the firms in the retail sample exhibit lower tax compliance. The *NW-SSR* computed for the services sector is lower than the one computed for the retail, both in the absolute and weighted values for the number of firms reporting a turnover lower than the presumed value.[25]

Therefore, the combined efficiency-congruity analysis and the tax compliance considerations highlight the presence of more anomalous behaviours in the retail than in the services sample, requiring more investigation by the fiscal authorities.

5. Concluding remarks

This paper develops an integrated approach for tax evasion analysis, combining presumptive taxation systems with a measure of technical efficiency estimated through the stochastic frontier analysis. This integrated methodology allows us to distinguish when firms' under-reporting is linked to the taxpayer's inefficiency from the situation where discrepancies between reported and presumed values are more presumably ascribable to the taxpayer's evasive behaviour. Further, we provide some considerations in terms of tax compliance. In particular, we compute a non-compliance measure based on the difference between reported and presumed revenues, supposing that the higher the value of under-reporting of revenues is, the lower the tax compliance.

Our integrated methodology allows to disentangle tax evasion from inefficiency in tax declaration, providing an useful policy tool able to support

[25]The value of the *NW-SSR* weighted for the number of non-congruous firms in the retail sample is equal to 0.952 while for the services, the value decreases to 0.721.

fiscal authorities' audit activities. Further, the empirical application of our integrated framework to different groups of taxpayers would allow a ranking in terms of higher/lower tax compliance, investigating if the level of evasion changes in relation to the taxpayers' characteristics, such as, for example, the different economic sectors.

We support our integrated approach with evidence obtained from an empirical application based on data from Italian firms subject to the *Business Sector Studies*. A limit of the empirical application is the very scant number of taxpayers analysed. Therefore, it would be very interesting to test the robustness of this integrated methodology using a larger sample and moving from cross-section to panel analysis. The possibility to take into account individual-specific heterogeneity would allow us to study the dynamics and to investigate taxpayers' behaviours more completely.

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Labour regression estimation results

(a) Retail Sector

Variables	Estimate	SE
Capital stock	0.064***	(0.028)
Costs for materials	0.322*	(0.043)
Costs for services	0.019	(0.016)
F Statistic		44.570***

(b) Services Sector

Variables	Estimate	SE
Capital stock	0.021*	(0.011)
Costs for materials	0.051*	(0.019)
Costs for services	0.025	(0.019)
Headquarter size	0.048	(0.041)
Gas consumption	0.021*	(0.010)
F Statistic		258.700***

Note: Significance codes: 0 “***”, 0.001 “**”, 0.01 “*”.

Appendix A

Gam frontier estimation results

(a) Retail Sector

Variables	F
Capital stock	1.126
Labor	28.571***
Costs for materials	40.740***

(b) Services Sector

Variables	F
Capital stock	10.169***
Labor	8.584**
Costs for materials	111.998***

Note: Significance codes: 0 "***", 0.001 "**", 0.01 "*".