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Abstract

We analyse a two-period model in which a monopolistic seller may adopt behavior-based price discrimination (BBPD) and charge consumers different prices based on their purchasing histories. We show that if there is quality uncertainty and prices convey valuable information about product quality, BBPD can be profitable for the seller both when the seller can and can not commit to future prices, contrasting the traditional view that the seller would like to avoid BBPD due to strategic delay of consumption on the consumers' side. BBPD increases consumers' sensitivity to a price change in the first period and enables the high type seller to signal product quality with relatively low prices, effectively reducing signaling costs in comparison to uniform pricing. In the separating equilibria that survive the intuitive criterion, first-time purchasers pay lower prices than repeat purchasers.

Keywords: Behavior-based Price Discrimination (BBPD), Quality Uncertainty, Signaling

JEL classifications: D82, L11, L15

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1 Introduction

We analyse a two-period model in which a monopolistic seller may adopt behavior-based price discrimination (BBPD) and charge consumers different prices in the second period based on their purchasing record in the first period. When the consumers have uncertainty regarding product quality, the prices the seller posts convey valuable information about product quality. We investigate how the option of BBPD interacts with the signaling role of prices and how BBPD affects seller profit, consumer surplus and total welfare.

The development of information technology enables the seller to keep an easy track of consumers' purchasing history and then exploit such information in subsequent trade by charging consumers different prices on the basis of their purchasing records. Such pricing strategy of BBPD is wildly observed in many markets, including retail, data plan for cell phones, plane tickets, hotels, etc. When the seller has the option to use BBPD, consumers rationally adjust their initial purchase decisions when they anticipate that their purchase history will affect the prices they face in the future. An important insight from the literature is that when consumers' valuations of the product being sold are constant across periods, a monopolistic seller does not want to condition price offers on the consumers' purchasing history because the consumers strategically delay their consumptions or hide their identities when they anticipate BBPD. (See, e.g. Taylor, 2004; Acquisti and Varian, 2005.)

In the existing literature on BBPD, consumers usually have perfect knowledge about the quality of the product they are going to purchase. However, in many real life situations, this may not be true. When a seller launches a new product or a consumer considers buying a product that she is not familiar with, a widely-recognized problem is that consumers do not know perfectly whether the product has high or low quality. With the existence of quality uncertainty on the consumers' side, prices posted by the seller naturally convey information about the product quality. Classic wisdoms include "high prices signal high quality", or "high and declining prices signal product quality". (See, e.g., Wolinsky, 1983; Bagwell and Riordan, 1991.) The marketing literature has also produced various evidence on the price-quality relationships. When a seller is able to condition prices on consumers' purchasing history, whether the seller uses BBPD and the prices he posts naturally serve a signaling role about product quality.

In this paper we set up a model in which a monopoly seller and the consumers interact in two periods. The consumers have a unit demand for the seller's product in each period. The consumers have the same valuation (v_L) for a low quality product (type L) but their valuations

(v_i) for high quality product (type H) are heterogeneous and private. At the beginning of the first period product quality is the seller's private information. The seller can also keep track whether a consumer makes a purchase or not. In the second period, information about product quality is perfectly revealed to all consumers and the seller is able to post different prices for consumers on the basis of their purchasing history in the first period. We consider two regimes: 1) the no-commitment regime in which the seller can not commit to future prices and thus posts short-term prices at the beginning of each period; and 2) the commitment regime in which the seller is able to commit to future prices and post long-term prices at the beginning of the first period.

In the no-commitment regime, the seller posts one-period price at the beginning of each period. Absent asymmetric information about product quality, type L seller charges a flat price to all consumers that equals v_L, the consumers' valuation for low quality product in each period. If the seller is not allowed to use BBPD, type H seller will post the static per-period monopoly price ($\tilde{s}$), and consumers with valuations $v_i \geq \tilde{s}$ purchase in both periods. However, when BBPD is allowed, standing at the second period, it is optimal for the type H seller to charge consumers different prices conditional on whether they have made a purchase in the first period. Anticipating BBPD, consumers strategically adjust their first-period demand. In equilibrium, given type H seller's first-period price p_{1H}, there exists a marginal consumer $\hat{v} > p_{1H}$ above which consumers prefer to purchase in both periods than wait and purchase only in the second period. Thus, consumers with valuations $v_i \in [p_{1H}, \hat{v}]$ will not consume a high quality product in the first period although their valuations are above the posted price. This strategic delay of consumptions by the consumers force the type H seller to lower first-period price below $\tilde{s}$. Overall, the type H seller's expected profit is lower when BBPD is allowed than when the seller is restricted to uniform pricing and charges the same price to all consumers independent of their purchasing history. Under complete information, although it is optimal to use conditional prices in the second period, type H seller would be better off if he could have refrained from BBPD.

What happens when there is asymmetric information about product quality in the first period and consumers have to infer product quality from the posted prices? Type H seller's first-period price then serves as a direct signal of product quality. However, the consumers' first-period demand is affected not only by the first-period price but also by the expected second-period prices, including whether BBPD will be used or not. When BBPD is not allowed and the seller has to adopt uniform pricing in the second period, type H seller needs to post a sufficiently high first-period price to prevent the mimicking of type L seller due to the production cost of high-quality product. When BBPD is allowed, type H seller will indeed find it optimal to adopt such pricing strategy in the second period. Anticipating this, consumers become more sensitive to a price change in the first period in comparison to uniform pricing. Without BBPD, consumers reduce their demands by one unit if there is a one-unit price increase in the first period. With BBPD, this demand reduction exceeds one unit because consumers' purchasing decision is not only affected by the first period price but also by the second-period prices through $\hat{v}$. As a result BBPD makes consumers more sensitive to a price change in the first period and this reduces type L's potential gain from mimicking type H seller's choice. As a consequence, the option of BBPD lowers the first-period price that type H seller needs to set to prevent type L from imitation, and reduces the signaling cost of type H seller.

We show that with the option of BBPD there always exists a unique separating equilibrium that survives intuitive criterion. The equilibrium pricing pattern exhibits first-time purchaser discounts for high quality product and flat prices for low quality product,[1] and thus consumers pay lower prices for their first units than their second units if they purchase in both periods. In comparison to uniform pricing, BBPD increases type H seller's profit when the product cost is sufficiently low and decreases his profit when production cost is high, while type L seller's profit is unaffected by the option of BBPD because low quality product can only be sold at price v_L in equilibrium. However, the possibility of BBPD is always beneficial to the consumers because more consumers consume the high quality product at a lower average price and this benefit dominates the decrease in type H's profit when production cost is high, and as a result the option of BBPD always increases total welfare.

When the seller can commit to future prices and post prices for both periods at the beginning of the first period, a key difference from the no-commitment regime is that the prices of both periods serve as a signaling instrument. In a separating equilibrium in which type H seller posts some prices above v_L for the second period, imitating type H's choice implies zero profit from the second period, further reducing type L's imitation incentives in comparison to the no-commitment regime. As a result, the first-period price that is needed to prevent imitation is further lowered and the option of BBPD allows type H to signal product quality with a first-period price that is even below the static monopoly price ($\tilde{s}$). Consequently, when the seller can commit to future prices, the option of BBPD always increases type H seller's profit in comparison

[1] First-time purchaser discounts are widely observed in retailing. E-commerce platforms, including Taobao, Meituan, Jingdong, etc., all use first-order discounts to attract new consumers.

to uniform pricing. In the separating equilibria surviving intuitive criterion, the price pattern has similar features as that under no-commitment regime, and consumers pay lower price for first purchases and larger price for repeat purchases. The option of BBPD increases type H seller's profit, consumer surplus and total welfare.

The remainder of this paper is organized as follows. We relate our study to the literature in the rest of this section. In Section 2 we present the model setup and analyse the case that the seller has no commitment power and thus posts short-term prices at the beginning of each period. We first derive the equilibrium of complete information benchmark and then proceed to analyse the game under quality uncertainty with and without BBPD. In Section 3 we analyse the case when the seller can commit to future prices. Concluding remarks can be found in Section 4. The proof of Lemma 1 is substantiated in the main text and all the remaining proofs are relegated to the Appendix.

Related Literature

The literature on BBPD has been growing very fast over the past decade. Early works, see, for example, Hart and Tirole (1988), Fudenberg and Tirole (2000), Villas-Boas and Miguel (2004), Acquisti and Varian (2005), etc., highlight the key observation that consumers strategically delay their initial consumptions or defend themselves by hiding their identities when they anticipate higher prices to be charged to repeat purchasers, and consequently if the consumers' valuations for the product being sold remain constant across periods, sellers do not want to condition current price offers on consumers' past behavior.[2] The literature has also explored under what conditions BBPD could be potentially profitable to a monopoly seller. Hart and Tirole (1988) argues that BBPD can increase the seller's profit if the consumer's valuations are revealed in the initial purchase so that the seller can extract all the surplus in the future. In Acquisti and Varian (2005), if the consumers are myopic such that they do not anticipate future price changes based on their current behavior, or if the seller is able to provide repeat purchasers value-added services, BBPD could generates larger profits than uniform pricing. Jing (2011) shows that for an experience good, BBPD can generate larger profit than time-consistent pricing when the mean consumer valuation is sufficiently high.[3]

[2]For comprehensive reviews of the BBPD literature, see, for example, Fudenberg and Villas-Boas (2006, 2012) and Acquisti, Taylor and Wagman (2016).

[3]Conitzer, Taylor and Wagman (2012) and Lagerlöf (2018) analyse the consumers' incentives to hide purchase history when the seller can adopt BBPD and explores the welfare effects of anonymous shopping.

In a competitive environment, BBPD often decreases firms' profits due to intensified competition and harms total welfare due to inefficient consumer switching (See, for example, Chen 1997 and Fudenberg and Tirole 2000). Chen and Zhang (2009) considers a model in which firms actively pursue consumer recognition, and find that BBPD can benefit firms in a competitive environment rather than monopoly because of milder competition between firms in order to gain consumers' private information. Li and Jain (2016) shows that firms' profits and total welfare increase compared with no consumer recognition if consumers have fairness concerns. Jing (2016) finds that BBPD often increases firm's profits when the product has an experience attribute. However, consumer surplus and total welfare decrease whenever firm profits increase. Jing (2017) shows that BBPD could potentially increase firms' profits when product qualities are endogenously chosen, but social welfare generally decreases due to inefficient mismatch between consumers and products and excessive product differentiation. In a spatial competition modek, Choe, King and Matsushima (2018) shows that personalized prices based on customer information lowers firms' profits relative to simpler pricing scheme regardless of whether product differentiation is exogenously given or chosen by firms endogenously. Garella, Laussel and Resende (2021) shows that behavior based price personalization lowers firm profits compared to uniform pricing in a duopoly market with vertical product differentiation. Esteves, Liu and Shuai (2021) shows that BBPD can boost competitive firms' profits at the expense of consumer surplus when consumers' preferences are non-uniformly distributed.

Our work differs from the existing literature on BBPD in several important aspects. First, our model captures the novel feature that consumers have uncertainty regarding product quality and prices convey valuable information about product quality. Second, our analysis identifies a new channel through which BBPD affects the seller's profits and consumer welfare. BBPD lowers the high quality seller's signaling cost by increasing the consumer's sensitivity to a price change in the first period. In equilibrium, first time purchasers pay lower price than repeat purchasers, not for the purpose of poaching rivals' consumers, but rather to convey quality information to the consumers. Third, we show that BBPD can benefit type H seller by lowering the signaling costs of high quality product in comparison to uniform prices, thus rendering BBPD a potentially profitable pricing strategy for a monopoly seller. Fourth, due to the existence of quality uncertainty, BBPD increases consumer surplus by lowering the average price consumers pay, both for consumers purchasing two units and consumers purchasing one unit. Moreover, BBPD always increases total welfare because a larger quantity of high quality product is consumed under BBPD relative to uniform pricing.

Our paper also relates to the literature that studies the signaling role of prices when consumers hold incomplete information about product quality. Wolinsky (1983) introduces a signaling model where product quality is endogenously chosen by the firms and shows that there exists a separating equilibrium in which each price signals a unique quality level. Bagwell and Riordan (1991) shows high introductory price signals product quality and price tends to decline over time as information about the product quality diffuses among the consumers. While positive correlation between quality and production cost results in high price serving as a signal of high cost, Judd and Riordan (1994) relaxes this correlation and proves that high price can still serve as a signal of high quality when consumers also have some private information about the product quality. Different from these works, our focus is to analyse how the option of BBPD affects the signaling cost and we show that BBPD increases the consumers' sensitivity to price changes in the first period, lowers type L seller's imitation incentives and thus lowers the signaling cost of type H seller. The option of BBPD allows the type H seller to signal product quality using a first-period price that is even lower than the static monopoly price under the price-commitment regime.

2 The Model

A monopolist seller introduces a new product to the market. The quality of the product is either high or low, $q \in \{H, L\}$. The production cost of type H product is constant and equals $c > 0$, while the production cost of type L product is normalized to 0. There is a continuum of consumers with total mass normalized to 1. Consumers have common valuation $v_L > 0$ for type L product, but have heterogeneous valuations for type H product. In particular, a consumer i's valuation for type H product is v_i, which is a random draw following a uniform distribution on the support $[v_L, v_L + 1]$, and the realization of v_i is the consumer's private information.

The seller and the consumers interact in two periods. Each consumer has a unit demand for the product in each period. A consumer's valuation, v_i, remains unchanged across the two periods. At the beginning of the first period, nature draws the quality of the product and reveals the information privately to the seller; the consumers do not observe the realized quality of the product and they believe that the product is type H with probability ρ and type L with probability $1 - \rho$. However, the consumers can infer some information about the product quality from the prices posted by the seller. At the beginning of the second period, product quality becomes public information and consumers learn whether the product is type H or type L before making their second-period purchasing decisions. The seller can not observe the realization of v_i, however, he can observe whether a consumer has made a purchase at the first period or not and update his information about v_i from the consumer's first-period purchasing record. Both the seller and the consumers are risk neutral and there is no discounting.

The seller has no commitment power and thus can only post short-term price in each period. We use $p = \{p_{1q}, p_{2q}\}$ and $s = \{s_{1q}, s_{2q}\}$, $q \in \{H, L\}$, to denote respectively the seller's pricing strategies when *behavior-based price discrimination (BBPD)* is allowed and when BBPD is not allowed and *uniform pricing (UP)* (UP) has to be adopted. The time structure of the game is as follows:

1. At $t = 1$, the seller privately learns product quality $q \in \{H, L\}$ and posts the first-period price p_{1q} when BBPD is allowed (resp. s_{1q} under UP). A consumer learns privately her valuation v_i about type H product. The consumer observes the posted price and updates her belief about the product quality, $\mu(p_{1q}) = \Pr\{H \mid p_{1q}\}$, the probability that the product is type H given price p_{1q}, and decides whether to buy the product at $t = 1$.

2. At $t = 2$, information about product quality becomes common knowledge. The seller posts the second-period price p_{2q} when BBPD is allowed (resp. s_{2q} under UP). Consumers make their second-period purchasing decisions.

At $t = 2$, since there is no asymmetric information about product quality, a consumer will never buy a low quality product at price over v_L, thus it is optimal for type L seller to charge v_L to all consumers under BBPD or UP. However, the option of BBPD makes a difference to type H seller because consumers have heterogenous valuations for high quality product. Under uniform pricing, type H seller has to charge the same price, s_{2H}, to all consumers independent of their purchasing histories. Under BBPD, the seller has the option to charge the consumers different prices on the basis of their purchasing history, and thus $p_{2H} = (p_2^R, p_2^N)$ and the price charged to a repeat purchaser, p_2^R, may differ from the price, p_2^N, charged to a first-time purchaser in the second period.

At $t = 1$, price p_{1q} (resp. s_{1q}) conveys valuable information about product quality. Although the second period price p_{2q} (resp. s_{2q}) does not serve as a signal of product quality directly, it may affect the consumers' first period demands because whether the consumers make a first-period purchase or not affects the price they will face in the second period.[4]

Note that type H seller never sells her product to anyone if $c \geq v_L + 1$. On the other hand, if $v_L \geq 1$, type H seller would post a monopoly price above v_L which type L seller would never imitate.To focus on the interesting cases, we make the following assumptions in subsequent analysis:

$$v_L < 1; \qquad v_L < c < v_L + 1. \tag{1}$$

The solution concept of the game is perfect Bayesian Nash equilibrium (PBE) that satisfies: 1) the seller's choice of p_{1q} (resp. s_{1q}) is optimal given his anticipation of the consumers' beliefs and purchasing strategy; 2) the consumers' purchasing decision is rational given their updated beliefs about the product quality; 3) the consumer's beliefs about the product quality are consistent with the seller's pricing strategies on the equilibrium path. Since signaling games have the disconcerting feature of multiple equilibria, we focus on equilibria that survive Intuitive Criterion (Cho and Kreps, 1987).

2.1 Complete Information Benchmark

Suppose consumers can also observe the product quality at $t = 1$. Under BBPD and uniform pricing, type L product will be sold at a constant price $\tilde{p}_{1L} = \tilde{p}_{2L} = v_L$ in both periods. All consumers purchase in both periods and the resulting total profits of the seller are $\tilde{\Pi}^b_L = 2v_L$.

For type H seller, first consider uniform pricing. The first period price s_{1H} can not affect the demand the seller faces at $t = 2$ and thus the optimal price for the two periods must be the same, $s_{1H} = s_{2H}$. The seller's per-period profit is:

$$\pi_H(s_{1H}) = (v_L + 1 - s_{1H})(s_{1H} - c). \tag{2}$$

It follows that the optimal price $\tilde{s}_H = \{\tilde{s}_{1H}, \tilde{s}_{2H}\}$ and type H seller's total profits are respectively

$$\tilde{s}_{1H} = \tilde{s}_{2H} = \frac{v_L + 1 + c}{2} \equiv \tilde{s}, \qquad \tilde{\Pi}^u_H = 2\frac{(v_L + 1 - c)^2}{4} \equiv 2\tilde{\pi}, \tag{3}$$

where $\tilde{s}$ represents type H's static monopoly price and $\tilde{\pi}$ is his static per-period monopoly profit

[4] The option of BBPD differentiates our model from Bagwell and Riordan (1991) which assumes that the seller can only charge uniform prices to all consumers in each period. On the other hand, if $\rho = 1$ and the product is always high quality, our model is a continuous version of Acquisti and Varian (2005). This setup allows us to have a clean comparison with the key insights in Bagwell and Riordan (1991) and Acquisti and Varian (2005) to isolate the effects BBPD potentially has on the signaling cost under quality uncertainty.

under complete information when BBPD is not allowed.

Remark 1. *When there is no asymmetric information about product quality, under uniform pricing, type H seller's optimal price is* $\tilde{s}_H = \{\tilde{s}, \tilde{s}\}$ *which brings total profit* $\tilde{\Pi}_H^u = 2\tilde{\pi}$*, and type L seller's optimal price is* $\tilde{s}_L = \{v_L, v_L\}$ *which brings total profit* $\tilde{\Pi}_L^u = 2v_L$*.*

Now consider type H seller's choice under BBPD. A first-period price p_{1H} is followed by a continuation game in which consumers update their beliefs about the product quality and make their first-period purchasing decision, and at $t = 2$ the seller posts $p_{2H} = (p_2^R, p_2^N)$ and the consumer makes their second period purchase decisions after learning about the product quality. At $t = 2$, repeat purchasers buy a second-unit of type H product if and only if $v_i \geq p_2^R$ and first-time purchasers buy their first unit of type H product if and only if $v_i \geq p_2^N$. Solving the game backward, we will be able to pin down the optimal pricing strategy of type H seller.

Observing p_{1H} and anticipating $p_{2H} = (p_2^N, p_2^R)$, a consumer with value v_i purchases her first unit at $t = 1$ only if

$$(v_i - p_{1H}) + \max\{v_i - p_2^R, 0\} \geq \max\{v_i - p_2^N, 0\}, \tag{4}$$

where the two terms on the LHS are respectively the consumer's utility from purchasing her initial unit at price p_{1H} in the first period and then make a repeat purchase at price p_2^R in the second period, and the RHS of (4) is her utility from making a first-time purchase at price p_2^N in the second period. For given p_{1H}, all consumers purchase at $t = 1$ if (4) holds for all $v_i \in [v_L, v_L + 1]$, and no consumer purchases at $t = 1$ if (4) is violated even for $v_i = v_L + 1$.

Define marginal consumer as one with valuation $\hat{v} \in [v_L, v_L + 1)$ such that a consumer purchases in the first period if and only if $v_i \geq \hat{v}$. When a marginal consumer indeed exists, $\hat{v}$ divides the consumers into high valuation segment where consumers with $v_i \geq \hat{v}$ face price p_2^R and low valuation segment where consumers with $v_i < \hat{v}$ face price p_2^N in the second period. Given p_{1H}, by setting $p_2^R \in [v_L, v_L + 1]$ at $t = 2$, type H seller's demand in the high valuation segment is

$$Q_2^R = \begin{cases} v_L + 1 - p_2^R & \text{if} \quad p_2^R \in [\hat{v},\ v_L + 1] \\ v_L + 1 - \hat{v} & \text{if} \quad p_2^R \in [v_L,\ \hat{v}] \end{cases}.$$

Type H seller's profit in the high valuation segment is $\pi_{2H}^R = Q_2^R(p_2^R - c)$ and is maximized at $p_2^R = \max\{\frac{v_L+1+c}{2},\ \hat{v}\} = \max\{\tilde{s},\ \hat{v}\}$.

By setting $p_2^N \in [v_L, v_L+1]$, type H seller's demand in the low valuation segment is:

$$Q_2^N = \begin{cases} 0 & \text{if} \quad p_2^N \in [\hat{v},\ v_L+1] \\ \hat{v} - p_2^N & \text{if} \quad p_2^N \in [v_L,\ \hat{v}] \end{cases}.$$

Type H seller's profit in the low valuation segment is $\pi_{2H}^N = Q_2^N(p_2^N - c)$ and is maximized at $p_2^N = \max\{\frac{\hat{v}+c}{2}, c\}$. Anticipating the second period prices, the valuation of marginal consumer $\hat{v}$ can thus be uniquely determined by:

$$(\hat{v} - p_{1H}) + \underbrace{\max\{\hat{v} - \max\{\tilde{s},\ \hat{v}\}, 0\}}_{=0} = \max\{\hat{v} - \max\{\frac{\hat{v}+c}{2}, c\}, 0\}. \tag{5}$$

Since $\hat{v} - p_2^R = \hat{v} - \max\{\tilde{s}, \hat{v}\} \leq 0$, we have $\max\{\hat{v} - p_2^R, 0\} = 0$ and the marginal consumer anticipates zero utility from making a repeat purchase at $t = 2$, and thus is indifferent between purchasing her first unit at price p_{1H} in the first period and at price p_2^N in the second period. Moreover, if $\hat{v} \geq c$, $\max\{\hat{v} - p_2^N, 0\} = \frac{\hat{v}-c}{2}$, and if $\hat{v} < c$, $\max\{\hat{v} - p_2^N, 0\} = 0$. The analysis leads to the following relationship between $\hat{v}$ and type H seller's first period price p_{1H}:

Remark 2. *If $p_{1H} \in [c, \tilde{s}]$, then $\hat{v} = 2p_{1H} - c$, and if $p_{1H} \in [v_L, c)$, then $\hat{v} = p_{1H}$.*

At $t = 1$, type H seller chooses p_{1H} to maximize his total expected profits from the two periods. First note that a price with $p_{1H} \in (\tilde{s}, v_L + 1]$ where $\tilde{s} = \frac{v_L+1+c}{2}$ as given in (3) is strictly dominated for the type H seller. Given such a price no consumer makes a purchase in the first period, and following this it is optimal for the seller to choose $p_2^R = p_2^N = \tilde{s}$ at $t = 2$ and the seller's total profit is $\Pi_H(p_{1H}) = \tilde{\pi}$. However, by setting some price $p_{1H} \in [c, \tilde{s}]$, type H seller earns a strictly positive profit from $t = 1$ and then $\tilde{\pi}$ from $t = 2$.

Then consider $p_{1H} \in [v_L, \tilde{s}]$ and the marginal consumer indeed exists. Type H seller's expected profit is:

$$\Pi_H(p_{1H}) = (v_L + 1 - \hat{v})(p_{1H} - c) + (\hat{v} - p_2^N)(p_2^N - c) + (v_L + 1 - p_2^R)(p_2^R - c), \tag{6}$$

where the first term is the seller's profit from $t = 1$, and the last two terms are his profit from the low valuation and high valuation segment at $t = 2$. Making use of Remark 2, we have $p_2^N = p_{1H}$

and $p_2^R = \max\{\tilde{s}, \hat{v}\}$ if $p_{1H} \in [c, \tilde{s}]$, and $p_2^N = c$ and $p_2^R = \tilde{s}$ if $p_{1H} \in [v_L, c)$. Thus

$$\Pi_H(p_{1H}) = \begin{cases} (v_L + 1 - \max\{\tilde{s}, 2p_{1H} - c\})(\max\{\tilde{s}, 2p_{1H} - c\} - c) & \\ \qquad\qquad + (v_L + 1 - p_{1H})(p_{1H} - c) \quad \text{if } p_{1H} \in [c, \tilde{s}], & \\ (v_L + 1 - p_{1H})(p_{1H} - c) + (v_L + 1 - \tilde{s})(\tilde{s} - c) \quad \text{if } p_{1H} \in [v_L, c). & \end{cases} \tag{7}$$

Solving for p_{1H} that maximizes $\Pi_H(p_{1H})$ leads to the equilibrium outcome under BBPD in the complete information benchmark which we summarise in the next lemma.

Lemma 1. *Suppose there is no asymmetric information about product quality. Type L seller charges a flat price v_L in both periods, $\tilde{p}_L = \{v_L, v_L\}$. Type H seller charges discriminatory prices $\tilde{p}_H = \{\tilde{p}_{1H}, (\tilde{p}_2^R, \tilde{p}_2^N)\}$ with*

$$\tilde{p}_{1H} = \tilde{p}_2^N = \frac{3v_L + 7c + 3}{10}, \quad \tilde{p}_2^R = \frac{3v_L + 2c + 3}{5}.$$

The two types' expected profits are respectively

$$\tilde{\Pi}_L^b = 2v_L; \qquad \tilde{\Pi}_H^b = \frac{9}{20}(v_L + 1 - c)^2.$$

The type H seller is worse off with the option of behavior-based price discrimination than when such an option is not available.

If BBPD is not allowed or the seller can commit not to condition future prices on the consumers' purchasing histories, type H seller would post the static monopoly price $\tilde{s}$, pocketing the static monopoly profits in both periods. With the option of BBPD, posting $\tilde{s}$ in both periods is no longer an equilibrium. Given $p_{1H} = \tilde{s}$, at the second period it is optimal for the seller to make use of the available information about the consumers' purchasing history, charging a price higher than $\tilde{s}$ to repeat purchasers to maximize profit. However, consumers anticipate that they are going to face higher price in the second period if they make a purchase in the first period, and adjust their initial purchasing decisions accordingly. When there is no future price discrimination, consumers would purchase in the first period as long as $v_i \geq p_{1H}$. However, when they anticipate BBPD, consumers with $v_i \in [p_{1H}, \hat{v}]$ will make no purchase in the first period and choose to wait until the second period to purchase their first unit. This strategic delay of consumption on the consumers' side forces type H seller to lower the first-period price below $\tilde{s}$ and overall the seller's profit is also decreased in comparison to the case of uniform pricing. Thus the possibility of conditioning prices on purchasing histories makes type H seller worse off when there is no uncertainty regarding product quality. Despite the fact that BBPD can potentially benefit the consumers because prices are lower under BBPD, the seller is reluctant to adopt such pricing strategies.

2.2 Signaling Equilibrium with BBPD

We now analyse the game when the seller holds private information about product quality at $t = 1$. Since product quality becomes common knowledge at $t = 2$, type L seller can only charge price v_L in the second period independent of the consumers' purchasing histories. Thus type L's benefits from mimicking the price choice of type H seller comes solely from his first-period profit. When type H seller chooses p_{1H} from different intervals, given the consumers' beliefs, the valuation of a marginal consumer changes, and this in turn affects the demands of the high valuation section and low valuation section at $t = 2$.

In a separating equilibrium with $p^*_{1H} \neq p^*_{1L}$, the equilibrium path beliefs are given by $\mu(p^*_{1H}) = 1$ and $\mu(p^*_{1L}) = 0$. Thus $p^*_{1L} = v_L$ must hold. Different from the complete information benchmark, type H seller may also choose $p_{1H} < c$ or $p_{1H} > \tilde{s}$ since price has the role of signaling. If $p_{1H} > \tilde{s}$, no consumers purchase in the first period and the seller can achieve the highest profit $\tilde{\pi}$ in the second period by setting $p^R_2 = p^N_2 = \tilde{s}$.

Let $\Pi_q(p_1, \mu)$ denotes the total profits of type q, $q \in \{H, L\}$, by posting a first-period price p_1 and being believed to be type H with probability μ. Suppose the off-equilibrium path belief is such that $\mu(p_1) = 0$ for $p_1 \neq p^*_{1H}$. In a separating equilibrium, if a type H deviates to some $p_1 \neq p^*_{1H}$, he can only sell a positive quantity of the product at price equal to v_L at $t = 1$ and all consumers buy at this price but this leads to a loss $v_L - c$, or set $p_1 > v_L$ and no consumer purchases at $t = 1$ which brings a first period profit of 0; at $t = 2$ since all consumers have the same purchasing history, the seller can not use conditional prices, and it is optimal for him to set the static monopoly price $\tilde{s}$ and receive $\tilde{\pi}$ from the second period. Thus, if type H seller deviates from the equilibrium choice p^*_{1H} at $t = 1$, the highest deviation payoff is given by $\max_{p_1 \neq p^*_{1H}} \Pi_H(p_1, 0) = \tilde{\pi}$.

Which p^*_{1H} can be supported in a separating equilibrium? Suppose $p^*_{1H} \in [v_L,\ c)$, then $\hat{v} = p^*_{1H}$ and all consumers with $v_i \geq p^*_{1H}$ purchase the product at p^*_{1H} at $t = 1$, and it is optimal for the seller to choose $p_2^R = \tilde{s}$ and $p_2^N = c$ at $t = 2$. Type H is willing to post p_{1H}^* if and only if

$$\Pi_H(p_{1H}^*, 1) = (v_L + 1 - p_{1H}^*)(p_{1H}^* - c) + \tilde{\pi} \geq \max_{p_1 \neq p_{1H}^*} \Pi_H(p_1, 0) = \tilde{\pi} \tag{8}$$

which can not be satisfied since $p_{1H}^* < c$. Thus $p_{1H}^* \in [v_L,\ c)$ can not be supported in a separating equilibrium.

Now consider $p_{1H}^* \in [c, \tilde{s}]$, it follows $\hat{v} = 2p_{1H}^* - c$. Consumers with $v_i \geq \hat{v}$ purchase the product at price p_{1H}^* at $t = 1$ given their belief $\mu(p_{1H}^*) = 1$. Using (7), the corresponding IC constraints of the two types can be written as:

$$\Pi_L(p_{1H}^*, 1) = (v_L + 1 - 2p_{1H}^* + c)p_{1H}^* + v_L \leq \Pi_L(v_L, 0) = 2v_L, \tag{9}$$

$$\begin{aligned} \Pi_H(p_{1H}^*, 1) &= (v_L + 1 - p_{1H}^*)(p_{1H}^* - c) + (v_L + 1 - \max\{\tilde{s}, 2p_{1H}^* - c\})(\max\{\tilde{s}, 2p_{1H}^* - c\} - c) \\ &\geq \max_{p_1 \neq p_{1H}^*} \Pi_H(p_1,\ 0) = \tilde{\pi}. \end{aligned} \tag{10}$$

For $p_{1H}^* \in [c, \tilde{s}]$, (10) is always satisfied. Thus a separating equilibrium with $p_{1H}^* \in [c, \tilde{s}]$ exists if and only if (9) holds.

Lastly, suppose $p_{1H}^* \in (\tilde{s}, v_L + 1]$, no consumers make a purchase at $t = 1$ given their belief $\mu(p_{1H}^*) = 1$ and the IC constraints are:

$$\Pi_L(p_{1H}^*, 1) = 0 + v_L \leq \Pi_L(v_L, 0) = 2v_L, \tag{11}$$

$$\Pi_H(p_{1H}^*, 1) = \tilde{\pi} \geq \max_{p_1 \neq p_{1H}^*} \Pi_H(p_1, 0) = \tilde{\pi}. \tag{12}$$

Note that both (11) and (12) are always satisfied, any price with $p_{1H}^* \in (\tilde{s}, v_L + 1]$ can be supported in a separating equilibrium, but such equilibria can be ruled out by intuitive criterion, and a separating equilibrium that survives intuitive criterion must satisfy $p_{1H}^* \in [c, \tilde{s}]$. We state this result in the next lemma.

Lemma 2. *In a separating equilibrium that survives the intuitive criterion, $p_{1L}^* = v_L$ and $p_{1H}^* \in [c, \tilde{s}]$ and satisfies (9).*

When constraint (9) is binding, we get two thresholds: (Once we write $\bar{p}$ in the following form, we are assuming $(v_L + 1 + c)^2 - 8v_L \geq 0$.)

$$\underline{p} = \frac{v_L + 1 + c - \sqrt{(v_L + 1 + c)^2 - 8v_L}}{4}, \quad \bar{p} = \frac{v_L + 1 + c + \sqrt{(v_L + 1 + c)^2 - 8v_L}}{4}. \tag{13}$$

Since $\Pi_L(p_{1H}, 1)$ is a parabola in p_{1H}, type L has no incentive to imitate the price choice of type H seller if and only if $p_{1H} \geq \bar{p}$ or $p_{1H} \leq \underline{p}$. Moreover, $\Pi_H(p_{1H}, 1)$ increases in p_{1H} for $p_{1H} \leq \tilde{p}_{1H} = \frac{3v_L+7c+3}{10}$, and as a result, $p^*_{1H} \leq \underline{p}$ can be ruled out using the argument of intuitive criterion.[5] As a result, $p^*_{1H} \geq \bar{p}$ must hold for p^*_{1H} to survive the intuitive criterion.

In the next proposition, we show that $p^*_{1H} = \max\{\tilde{p}_{1H}, \bar{p}\}$ forms the unique separating equilibrium that survives the intuitive criterion. When $\tilde{p}_{1H} > \bar{p}$, the optimal first-period price under complete information is supported in equilibrium; when $\tilde{p}_{1H} < \bar{p}$, type H seller needs to choose a first-period price that is higher than $\tilde{p}_{1H}$ to convince consumers of his product quality.

Proposition 1. *Under BBPD, there is a unique separating equilibrium that survives the intuitive criterion:*

1. *If $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 > 0$, the equilibrium outcome under complete information is supported. Type L seller chooses $p^*_L = \{v_L, v_L\}$ and type H seller charges $p^*_H = \{p^*_{1H}, (p^{*R}_2, p^{*N}_2)\}$ with*

$$p^*_{1H} = p^{*N}_2 = \frac{3v_L + 7c + 3}{10}, \quad p^{*R}_2 = \frac{3v_L + 2c + 3}{5}.$$

The two types' expected profits are respectively

$$\Pi^b_L = 2v_L; \qquad \Pi^b_H = \frac{9}{20}(v_L + 1 - c)^2. \tag{14}$$

2. *If $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 \leq 0$, type L seller chooses $p^*_L = \{v_L, v_L\}$ and type H seller charges $p^*_H = \{p^*_{1H}, (p^{*R}_2, p^{*N}_2)\} = \{\bar{p}, (2\bar{p} - c, \bar{p})\}$, where*

$$\bar{p} = \frac{v_L + 1 + c + \Delta}{4}, \quad \textit{with} \quad \Delta \equiv \sqrt{(v_L + 1 + c)^2 - 8v_L}. \tag{15}$$

The two types' expected profits are respectively

$$\Pi^b_L = 2v_L; \qquad \Pi^b_H = \frac{(1 - 3c + v_L + \Delta)(7 + 3c + 7v_L - 5\Delta)}{16}. \tag{16}$$

The signaling equilibrium in Proposition 1 exhibits flat price for low quality product and BBPD for high-quality product, with first-time purchasers paying a lower price than repeat

[5] Since $\Pi_H(p_{1H}, 1)$ is a parabola with the maximum point at $\tilde{p}_{1H} \geq \frac{\underline{p}+\bar{p}}{2}$, $\Pi_H(\underline{p}, 1) < \Pi_H(\bar{p} + \epsilon, 1)$ for small positive ϵ. Moreover $\Pi_L(\bar{p} + \epsilon, 1) < \Pi_L(v_L, 0) = 2v_L$. Intuitive criterion requires $\mu(\bar{p} + \epsilon) = 1$ and thus $\underline{p}$ can not be optimal for type H seller. Similar arguments can be used to rule out $p^*_{1H} < \underline{p}$.

purchasers. Interestingly, for given v_L production cost c of high quality product has an important impact on the equilibrium price.

For $1 - 5v_L + v_L^2 \geq 0$, $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 \gtreqless 0$ is equivalent to $c \gtreqless \frac{1}{7}\left(2 + 2v_L + 5\sqrt{1 - 5v_L + v_L^2}\right)$. For given v_L, when c is relatively large, the complete information outcome can be supported as a unique separating equilibrium. However, when this cost is low, it is no longer the case and type H seller needs to set a first-period price higher than the optimal price under complete information, $\tilde{p}_{1H}$, to prevent mimicking by type L and to convince consumers of his product quality. From type L's incentive compatibility constraint (9), when c increases marginally, the first-period demand drops faster than the equilibrium price p_{1H}^* increases because $\hat{v} = 2p_{1H}^* - c$, not p_{1H}^*, determines the first-period demand under BBPD. The equilibrium $\hat{v}$ is closer to p_{1H}^* when c is large than when c is small.

2.3 Welfare of BBPD

To analyse the welfare effect of BBPD, we need to compare the equilibrium outcome in Proposition 1 with the equilibrium outcome when BBPD is not allowed. Without BBPD, the seller has to charge uniform price to all consumers at $t = 2$, and our model is a two-period extension of the setup in Bagwell and Riordan (1991). Since there is no asymmetric information about product quality at $t = 2$, it is optimal for type H seller to set $s_{2H}^* = \tilde{s}$ in the second period. At $t = 1$, consumers make their purchasing decisions on the basis of s_{1H} only because the seller can not charge conditional prices in the future. Therefore, the first period demand will be $v_L + 1 - s_{1H}$, different from $v_L + 1 - \hat{v}$ under the case of BBPD.

In a separating equilibrium with $s_{1H}^* \neq s_{1L}^*$, $s_{1L}^* = v_L$ always holds and s_{1H}^* satisfies type L's incentive compatibility constraint:

$$\Pi_L(s_{1H}^*, 1) = (v_L + 1 - s_{1H}^*)s_{1H}^* + v_L \leq \Pi_L(v_L, 0) = 2v_L. \tag{17}$$

Binding constraint (17) leads to $\bar{s} = 1$ and $\underline{s} = v_L$. Type H seller needs to post a sufficiently high first-period price, $s_{1H}^* \geq \bar{s} = 1$, to convince consumers of his product quality. As a result, the price chosen by type H seller in a separating equilibrium is given by $s_{1H}^* = \max\{\tilde{s}, 1\}$ where $\tilde{s}$ is the static monopoly price given in Lemma 1. We characterize the equilibrium in the next proposition.

Proposition 2. *If BBPD is not allowed, there is a unique separating equilibrium that survives* *the intuitive criterion:*

1. *If $c \geq 1 - v_L$, the equilibrium outcome under complete information is supported. Type L chooses $s_{1L}^* = s_{2L}^* = v_L$ and type H chooses $s_{1H}^* = s_{2H}^* = \tilde{s}$. The two types' expected profits are*

$$\Pi_L^u = 2v_L; \qquad \Pi_H^u = \frac{(v_L + 1 - c)^2}{2}. \tag{18}$$

2. *If $c < 1 - v_L$, type L seller chooses $s_{1L}^* = s_{2L}^* = v_L$ and type H seller sets $s_{1H}^* = \bar{s} = 1$ and $s_{2H}^* = \tilde{s}$. The two types' expected profits are*

$$\Pi_L^u = 2v_L, \qquad \Pi_H^u = v_L(1-c) + \frac{(v_L + 1 - c)^2}{4}. \tag{19}$$

Comparing the equilibrium outcomes in Propositions 1 and 2, we are able to get a clear picture on the benefits and costs of conditional pricing. At $t = 2$, it is optimal for type H seller to adopt BBPD when conditional pricing is allowed. However, at $t = 1$, the potential of conditioning prices on consumers' purchasing history leads to strategic adjustment of purchasing decisions by the consumers, which forces type H seller to lower the price for first-time purchasers. In equilibrium, second period price discrimination increases type H seller's profit under BBPD, while first-period purchasers delay consumption under BBPD and this affects type H seller's profit negatively. Absent signaling considerations, the negative effect dominates the positive effect and the option of BBPD leads to a decrease in type H seller's profits relative to uniform pricing. This is the result we obtained in Lemma 1 under complete information, and part 1 in Propositions 1 and 2 when there is quality uncertainty and complete information outcome is supported in separating equilibria under BBPD and uniform pricing.

However, when the complete information outcome can not be supported in a separating equilibrium, type H seller needs to post a sufficiently high first-period price, a price larger than the respective equilibrium price under complete information, to convince consumers of his product quality. Under BBPD $p_{1H}^* \geq \bar{p}$ and uniform pricing $s_{1H}^* \geq \bar{s} = 1$. Note that $\bar{p} < \bar{s}$. Under uniform pricing one unit of increase in s_{1H} leads to one unit of decrease in demand $(v_L + 1 - s_{1H})$ because consumers with $v_i \geq s_{1H}$ purchase in the first period, while under BBPD, the decrease in demand is given by $\hat{v} = 2p_{1H} - c > p_{1H}$ because consumers with $v_i > \hat{v}$ purchase in the first period while consumers with $v_i \in [p_{1H}, \hat{v}]$ strategically delay their consumptions even though their valuations are above price. As a result, the impact on the first-period demand by a marginal price change is amplified under BBPD relative to uniform pricing and this in turn lowers the threshold $(\bar{p} < \bar{s})$

type H seller needs to set to prevent type L's mimicry and thus effectively lowers the signaling cost of type H seller.

In summary, BBPD affects type H seller's profits in two opposite directions. Without signaling considerations the negative effect on profits dominates. When there is quality uncertainty, BBPD also has the benefit of lowering signaling cost of type H seller, and this positive effect reinforces the positive effect from second period purchasers. When the production cost of high quality product is sufficiently low, the divergence between $\hat{v}$ and p_{1H} is big and the signaling effect is prominent, and then the overall positive effects dominate the negative effect and BBPD increases type H seller's profits relative to uniform pricing. When the production cost of type H product is high, $\hat{v}$ is close to p_{1H} and the signaling effect is small and as a result BBPD lowers type H seller's profit relative to uniform pricing. In the next corollary, we show that the production cost of high quality product indeed affects the profitability of BBPD relative to uniform pricing.

Corollary 1. *For given v_L with $v_L \in [0, \frac{5-\sqrt{21}}{2})$, there exist thresholds c_1 and c_2 with $c_1, c_2 \in [v_L, v_L+1]$ and $c_1 \leq c_2$ such that BBPD increases type H seller's profit in comparison to uniform pricing if $c \leq c_1$, and lowers type H seller's profit in comparison to uniform pricing if $c \geq c_2$.*

Although BBPD may decrease type H seller's profit in comparison to uniform pricing, it always increases consumer surplus and the incremental benefits to the consumers dominate the loss in type H seller's profits, and overall total surplus increases when the price regime moves from uniform pricing to BBPD. We summarize this result in the next corollary.

Corollary 2. *BBPD increases consumer surplus and total welfare in comparison to uniform pricing.*

When $c \geq 1 - v_L$, complete information outcome is supported as the unique equilibrium under both BBPD and uniform pricing. Under uniform pricing, products are sold at price $\tilde{s}$ with demand $v_L + 1 - \tilde{s}$ in both periods. Under BBPD, $p_{1H}^* < \tilde{s}$ and $p_2^{*R} > \tilde{s}$, repeat purchasers pay a larger price while first-time purchasers (consumers that make their first-time purchases at $t = 1$ and $t = 2$) pay a lower price under BBPD. Note that $\tilde{s} - p_{1H}^* = 2(p_2^{*R} - \tilde{s})$, the price change for first-time purchasers is twice as large as that for repeat purchasers. This implies that when the price regime moves from uniform pricing to BBPD, consumers with $v_i \geq \tilde{s}$ gain more from purchasing their first unit than they lose from purchasing the second unit, and consumers with $v_i \in [p_{1H}^*, \tilde{s}]$ purchases under BBPD while they do not consume under uniform pricing.[6] Moreover, the benefits BBPD brings to the consumers are always larger than the harm it does to type H seller's profits, total welfare increases as a result.

When $c < 1 - v_L$, consider the case that type H seller needs to set a first-period price higher than that under complete information to signal product quality with or without BBPD. Under uniform pricing, type H products are sold at price $s_{1H}^* = 1$ with demand $v_L + 1 - s_{1H}^*$ in the first period and sold at price $s_{2H}^* = \tilde{s}$ with demand $v_L + 1 - \tilde{s}$ in the second period. Under BBPD, repeat purchasers pay the price $p_2^{*R} = 2\bar{p} - c$ with demand $v_L + 1 - (2\bar{p} - c)$, and consumers pay the price $p_{1H}^* = p_2^{*N} = \bar{p}$ for their first unit and total demand is $v_L + 1 - \bar{p}$. Note that $\bar{p} < \tilde{s}$ and $2\bar{p} - c < \bar{s} = 1$, all consumers who purchase type H product under uniform pricing pay less with BBPD, leading to an increase in consumer surplus. Moreover, total demand under BBPD is also larger than that under uniform pricing, thus BBPD increases consumer surplus relative to uniform pricing. Furthermore, the incremental consumer surplus dominates the potential loss to type H seller's profit and as a result total welfare is always higher under BBPD.

2.4 Numerical Example

We close this section with a numerical example illustrating the welfare of BBPD relative to uniform pricing. Suppose $v_L = 0.1$ and $c \in (0.1, 1.1)$. Under BBPD, by Proposition 1, when $c \geq 0.824$, $p_{1H}^* = p_2^{*N} = \frac{3.3+7c}{10}$, and $p_2^{*R} = \frac{3.3+2c}{5}$; when $c < 0.824$, $p_{1H}^* = p_2^{*N} = \frac{1.1+c+\sqrt{c^2+2.2c+0.41}}{4}$, and $p_2^{*R} = \frac{1.1-c+\sqrt{c^2+2.2c+0.41}}{2}$. Type H seller's profit and consumer surplus associated with high quality product under BBPD are given by

$$\Pi_H^b = \begin{cases} \frac{(1.1-3c+\sqrt{c^2+2.2c+0.41})(7.7+3c-5\sqrt{c^2+2.2c+0.41})}{16} & \text{if } c < 0.824 \\ \frac{9(1.1-c)^2}{20} & \text{if } c \geq 0.824 \end{cases}.$$

$$CS_H^b = \begin{cases} \frac{(3.3-c-\sqrt{c^2+2.2c+0.41})^2+4(1.1+c-\sqrt{c^2+2.2c+0.41})^2}{32} & \text{if } c < 0.824 \\ \frac{(7.7-7c)^2+4(2.2-2c)^2}{200} & \text{if } c \geq 0.824 \end{cases}.$$

Under uniform pricing, applying the results from Proposition 2 we have $s_{1H}^* = s_{2H}^* = \tilde{s} = \frac{1.1+c}{2}$ if $c \geq 0.9$, and $s_{1H}^* = 1$ and $s_{2H}^* = \frac{1.1+c}{2}$ if $c < 0.9$. Type H seller's profit and consumer

[6] The necessary and sufficient condition for total surplus to increase is that $p_{1H}^* - c > \frac{p_2^{*R} - \tilde{s}}{2}$, which is always true given $c < v_L + 1$.

surplus associated with high quality product are respectively

$$\Pi_H^u = \begin{cases} 0.25c^2 - 0.65c + 0.403 & \text{if } c < 0.9 \\ 0.5c^2 - 1.1c + 0.605 & \text{if } c \geq 0.9 \end{cases}, \quad CS_H^u = \begin{cases} 0.005 + \frac{(1.1-c)^2}{8} & \text{if } c < 0.9 \\ \frac{(1.1-c)^2}{4} & \text{if } c \geq 0.9 \end{cases}.$$

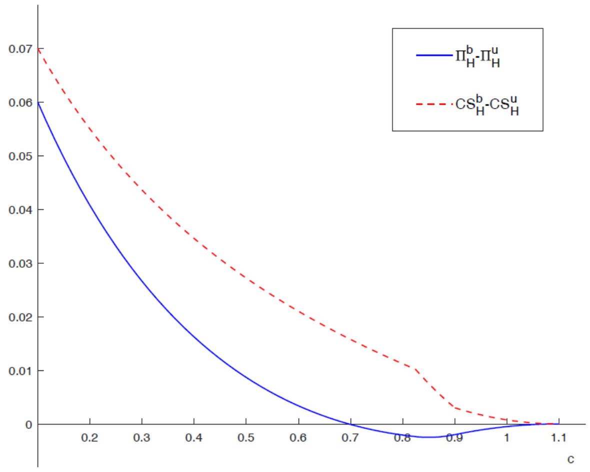

Figure 1: BBPD versus uniform pricing for $v_L = 0.1$ and $c \in (0.1, 1.1)$.

In this example, $\Pi_H^b > \Pi_H^u$ if and only if $c < 0.696$.[7] When $c = 0.2$, $\Pi_H^u = 0.323$ and $\Pi_H^b = 0.283$, BBPD increases type H's profit by 14.42%. When $c = 0.8$, $\Pi_H^u = 0.040$ and $\Pi_H^b = 0.043$, BBPD decreases type H's profit by 4.90%. We illustrate the incremental change of the seller's profits and consumer surplus associated with type H product in Figure 1. The magnitude of increment consumer surplus is always larger than the marginal change in type H seller's profit, thus total surplus is always higher under BBPD than uniform pricing.

3 Price Commitment

So far we have assumed that the seller can not commit to future prices and has to post short-term prices at the beginning of each period. In many cases, the seller may have the ability to post prices for multiple periods at $t = 1$ and commit to such prices out of reputation concerns in later period. In such cases the seller of type $q \in \{H, L\}$ posts price scheme $p_q = \{p_{1q}, p_{2q}\}$ under BBPD (resp. $s_q = \{s_{1q}, s_{2q}\}$ under uniform pricing) at $t = 1$. Different from the no commitment

[7]Note that this is not in contradictory to the statement in Corollary 1 because $c \leq c_1$ and $c \geq c_2$ are sufficient but not necessary conditions for the profitability of BBPD.

case, since the second period price p_{2q} (resp. s_{2q}) is posted together with first-period price p_{1q} (resp. s_{1q}), the choice of second-period prices (including whether BBPD is used) forms a part of the quality signal for $t = 1$.

Before proceeding to the formal analysis, we establish in the next Lemma that when BBPD is allowed and the seller can commit to future prices, for any price scheme $p_H = \{p_{1H}, (p_2^R, p_2^N)\}$ with $p_{1H} \neq p_2^N$ there always exists an alternative scheme $\hat{p}_H$ with $\hat{p}_{1H} = \hat{p}_2^N$ that brings type H seller (weakly and sometimes strictly) larger profit in the complete information benchmark or in a separating equilibrium while keeping type L's incentive constraint satisfied under asymmetric information.

Lemma 3. *Suppose BBPD is allowed and the seller can commit to future prices. When there is complete information about product quality or in a separating equilibrium when there is asymmetric information about product quality, type L seller's optimal price scheme is $p_L = \{v_L, v_L\}$; for type H seller, price scheme $p_H = \{p_{1H}, (p_2^R, p_2^N)\}$ with $p_{1H} \neq p_2^N$ is weakly (sometimes strictly) dominated by a price scheme $\hat{p}_H = \{\hat{p}_{1H}, (\hat{p}_2^R, \hat{p}_2^N)\}$ with $\hat{p}_{1H} = \hat{p}_2^N$.*

The observation in Lemma 3 allows us to focus on price scheme in the form of $p_H = \{p_{1H}, (p_2^R, p_2^N)\} = \{\tau, (\beta\tau, \tau)\}$ with $\beta \in (0, +\infty)$ under BBPD, and this greatly simplifies the subsequent analysis. If $\beta > 1$ (first-time purchaser discount), type H seller charges a lower price to first-time purchasers and a higher price to repeat purchasers. If $\beta < 1$ the price scheme exhibits the feature of repeat purchaser discount. If $\beta = 1$ (no price conditioning), the seller charges the same price across periods and across consumers. In the following, we first analyse complete information benchmark and then proceed to the separating equilibrium under asymmetric information about product quality and compare the welfare under BBPD with that under uniform pricing.

3.1 Complete Information Benchmark

Suppose there is no asymmetric information about product quality $t = 1$. Recall from Remark 1 that if type H seller can not condition the second-period prices upon consumers' purchasing history, the static monopoly price is $\tilde{s} = \frac{v_L+1+c}{2}$. We show in the next Lemma that when the seller can commit to future prices, it is optimal for type H seller to post uniform price $\tilde{s}$ for both periods even though BBPD is allowed.

Lemma 4. *Under complete information about product quality, if the seller can commit to future prices, type H seller posts the same price under BBPD and uniform pricing, $\tilde{p}_H^c = \tilde{s}_H^c = \{\tilde{s}, \tilde{s}\}$*

and obtains profit $\tilde{\Pi}_H^c = 2\tilde{\pi}$*; type L seller posts the same price* $\tilde{p}_L^c = \tilde{s}_L^c = \{v_L, v_L\}$ *and obtains* $\tilde{\Pi}_L^c = 2v_L$*. It is not optimal for type H seller to condition second-period price on the consumers' first period purchasing history.*

Although the option of BBPD enlarges the seller's feasible choice set, it does not increase his profit when there is no asymmetric information about product quality. Recall from Lemma 1 that if the seller can not commit to future prices, type H seller adopts BBPD in the second period and this makes him worse off. Lemma 4 confirms this result by showing that if the seller can commit to future prices, he will choose uniform pricing although BBPD is an option.

3.2 Signaling Equilibrium with BBPD

In the price-commitment regime a seller of type q posts price scheme $p_q = \{p_{1q}, p_{2q}\}$ at $t = 1$ while in the no-commitment regime the seller posts p_{tq} in period t. This distinction leads to two important differences between the two regimes. First, since p_{2q} is posted together with p_{1q}, the second period price p_{2q} (including whether price conditioning is used or not) conveys information about product quality directly, while in the no-commitment regime p_{2q} is posted at $t = 2$ and does not convey quality signal. Second, since quality becomes public information in the second period, in the price-commitment regime type L seller will get zero profit from $t = 2$ by imitating type H's price choice if the second period price is higher than v_L, while in the no-commitment regime type L seller can always ensure himself a positive profit v_L by setting a price equal to v_L at $t = 2$. The first feature shows that type H seller has more instruments to signal product quality in the first period and the second feature lowers type L's second period profit from mimicry in a separating equilibrium.

In a separating equilibrium with $p_H^c \neq p_L^c$, consumers' equilibrium path beliefs satisfy $\mu(p_H^c) = 1$ and $\mu(p_L^c) = 0$. Suppose the off-equilibrium path belief is $\mu(p) = 0$ for $p \notin \{p_H^c, p_L^c\}$. Then $p_L^c = \{v_L, v_L\}$ must hold and type L seller's equilibrium profit is $\Pi_L^{c,b} = 2v_L$.

For p_H^c to be an equilibrium, type H seller needs to be better off choosing p_H^c rather than deviating to an alternative price which brings him the maximal deviation payoff $\tilde{\pi}$. In price-commitment regime, this always holds true. In the next Lemma we show that in a separating equilibrium surviving the intuitive criterion, p_H^c maximizes type H's equilibrium-path profit subject to the incentive compatibility constraint of type L seller.

Lemma 5. *In a separating equilibrium that survives the intuitive criterion,* $p_L^c = \{v_L, v_L\}$ *and*

p_H^c *solves the following maximization program* Γ

$$p_H^c = \arg\max_{\hat{p}_H} \Pi_H(\hat{p}_H, 1) \tag{20}$$

$$\textit{such that } \Pi_L(p_H^c, 1) \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{21}$$

Recall from Lemma 3 that in the analysis of a separating equilibrium under BBPD it is without loss of generality to focus on prices in the form of $p_H = \{p_{1H}, (p_2^R, p_2^N)\} = \{\tau, (\beta\tau, \tau)\}$. Thus finding equilibrium price p_H^c is equivalent to finding a combination of β and τ that solves the maximization program Γ. Due to the assumption $c > v_L$, it is never optimal for type H seller to choose $\tau < v_L$ or $\beta\tau < v_L$. Thus in equilibrium type L receives zero profit from the second period by imitating type H's price choice. For different values of β, the first-period demand varies and type L's profits from imitating the price choice of type H is also different.

1. First-time purchaser discount ($\beta > 1$). Given $\mu(p_H^c) = 1$, consumers purchase at $t = 1$ if and only if $v_i \geq \beta\tau$. Type L's IC constraint (21) becomes

$$\Pi_L(p_H^c, 1) = (v_L + 1 - \beta\tau)\tau + 0 \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{22}$$

2. Repeat purchaser discount ($\beta < 1$). Given $\mu(p_H^c) = 1$, consumers purchase at $t = 1$ if and only if $v_i \geq \frac{1+\beta}{2}\tau$. Type L's IC constraint (21) becomes

$$\Pi_L(p_H^c, 1) = (v_L + 1 - \frac{1+\beta}{2}\tau)\tau + 0 \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{23}$$

3. No price conditioning ($\beta = 1$). Given $\mu(p_H^c) = 1$, consumers purchase at $t = 1$ if and only if $v_i \geq \tau$. Thus type L's IC constraint (21) becomes

$$\Pi_L(p_H^c, 1) = (v_L + 1 - \tau)\tau + 0 \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{24}$$

A natural question is whether the complete information outcome in Lemma 4 can be supported when there is asymmetric information about product quality. For $\tilde{p}_H^c = \{\tilde{s}, \tilde{s}\}$ to be sustained in a separating equilibrium, type L's incentive compatibility constraint (24) requires

$$\Pi_L(\tilde{p}_H^c, 1) = (v_L + 1 - \tilde{s})\tilde{s} + 0 = \frac{(v_L + 1)^2 - c^2}{4} \leq \Pi_L(p_L, 0) = 2v_L.$$

which holds if and only if $(v_L + 1)^2 - 8v_L \leq c^2$. When this condition holds, $\tilde{p}_H^c$ obviously satisfies (20) and forms the unique equilibrium that survives the intuitive criterion. If $(v_L + 1)^2 - 8v_L \leq c^2$ does not hold, the complete information outcome can not be supported in a separating equilibrium, and we show in the next proposition that type H seller uses BBPD and the equilibrium price scheme exhibits first-time purchaser discounts ($\beta > 1$).

Proposition 3. *When consumers have asymmetric information about product quality and the seller can commit to long term prices, there exists a separating equilibrium that satisfies the intuitive criterion.*

1. *If $(v_L + 1)^2 - 8v_L \leq c^2$, the complete information outcome is supported as the unique separating equilibrium. Type L seller chooses $p_L^c = \{v_L, v_L\}$ and type H seller chooses $p_H^c = \{\tilde{s}, \tilde{s}\}$.*

2. *If $(v_L + 1)^2 - 8v_L > c^2$, type H seller uses BBPD and offers a price scheme $p_H^c = \{\tau^c, (\beta^c \tau^c, \tau^c)\}$ in which $\tau^c \in [\frac{v_L+1-\sqrt{(v_L+1)^2-8v_L}}{2}, \tilde{s})$ and $\beta^c = \frac{1}{\tau^c}(v + 1 - \frac{2v}{\tau^c}) > 1$, type L seller chooses $p_L^c = \{v_L, v_L\}$.*

Similar to the no-commitment regime, for given v_L, when c is relatively small, the complete information outcome can not be supported in a separating equilibrium and type H seller needs to post a price that is sufficiently high to signal product quality. Usually one would expect type H seller to post a price higher than the monopoly price under complete information to signal high quality. This, however, is not necessarily true when the seller can commit to future prices and BBPD can be used in the second period, as shown in part 2 of Proposition 3. In equilibrium, type H can signal his quality with a first-period price strictly smaller than $\tilde{s}$ in combination with second-period prices that are conditional upon the consumers' purchasing history. The reason behind this outcome is that BBPD increases consumers' sensitivity to a price change in the first period and at the same time commitment to future prices drives type L's future imitation profit down to zero. These two effects reinforce each other and work in the same direction of lowering type L's imitation incentive.

3.3 Welfare Analysis

When the seller can commit to future prices, by revealed preference principle, type H seller's profit must be (weakly) higher with the option of BBPD than when the seller is restricted to uniform pricing. However, to evaluate how BBPD affects consumer welfare, we still need to know how the equilibrium outcome looks like when the seller adopts uniform pricing.

When the seller is not allowed to use BBPD, price scheme $s_H^c = (s_{1H}^c, s_{2H}^c)$ and $s_L^c = (v_L, v_L)$ form a separating equilibrium if type L has no incentive to mimic type H's choice of s_H^c and type H has no incentive to deviate from s_H^c either. Since the seller can commit to future prices, if $s_{2H}^c > v_L$, type L receives zero profit from the second period by mimicking the price choice of type H. Thus type L's incentive compatibility constraint can be rewritten as

$$\Pi_L(s_H^c, 1) = (v_L + 1 - s_{1H}^c)s_{1H}^c + 0 \leq \Pi_L(s_L^c, 0) = 2v_L. \tag{25}$$

In the next proposition, we summarize the equilibrium outcome when BBPD is not allowed and the seller has to charge the same price to all consumers in the second period.

Proposition 4. *If BBPD is not allowed, there is a unique separating equilibrium that survives the intuitive criterion under price commitment:*

1. *If $(v_L + 1)^2 - 8v_L \leq c^2$, both types choose their optimal prices under complete information, that is, $s_L^c = \{v_L, v_L\}$ and $s_H^c = \{\tilde{s}, \tilde{s}\}$. The equilibrium profits of the two types are*

$$\Pi_L^{c,u} = 2v_L, \qquad \Pi_H^{c,u} = \frac{(v_L + 1 - c)^2}{2}. \tag{26}$$

2. *If $(v_L + 1)^2 - 8v_L > c^2$, type L seller chooses $s_L^c = \{v_L, v_L\}$ and type H seller sets $s_H^c = \{s_{1H}^c, s_{2H}^c\}$ with $s_{1H}^c = \frac{v_L+1+\sqrt{(v_L+1)^2-8v_L}}{2}$ and $s_{2H}^c = \tilde{s}$. The equilibrium profits of the two types are*

$$\Pi_L^{c,u} = 2v_L, \qquad \Pi_H^{c,u} = (v_L + 1 - s_{1H}^c)(s_{1H}^c - c) + \frac{(v_L + 1 - c)^2}{4}. \tag{27}$$

From Proposition 4, when complete information outcome can not be supported in a separating equilibrium, type H seller has to charge a high price $s_{1H}^c > s_{2H}^c = \tilde{s}$ as a convincing signal of product quality. Without price discrimination, first-period demand is determined solely by the first period price. With BBPD, the second period price affects the first-period demand and in turn affects the signaling cost. Under uniform pricing, one unit of price increases transforms to one unit of demand reduction in the first period, while with BBPD one unit of price increases transmits to $\beta^c > 1$ units of demand reduction, and this increase in the price sensitivity of the first-period demand lowers the price threshold type H seller needs to set to prevent imitation of type L seller.

By comparing the equilibrium prices under BBPD and uniform pricing, we can see that $s^c_{1H} > \tilde{s} > p^c_{1H}$ when $(v_L + 1)^2 - 8v_L > c^2$ holds and type H seller needs to set a relatively high price to signal product quality under uniform pricing. Moreover, both first-time purchasers and repeat purchasers pay lower prices under BBPD, and demand is also higher under BBPD than that under uniform pricing. Thus, when the seller can commit to future prices, the option of BBPD always increases consumer surplus.

Corollary 3. *In comparison to uniform prices, when the seller can commit to future prices and there is asymmetric information about product quality, behavior-based price discrimination (i) increases type H seller's profit; (ii) increases consumer surplus; and (iii) increases social welfare.*

Numerical Example. We close this section with a revisit to the example in Section 2.4 in which $v_L = 0.1$. Let $c = 0.5$, thus $(v_L + 1)^2 - 8v_L > c^2$ holds and $\tilde{s} = 0.8$. Under uniform pricing, applying the results from Proposition 4, the equilibrium prices for type H seller is $\{s^c_{1H}, s^c_{2H}\} = \{0.870, 0.8\}$. Seller profit, consumer surplus and social surplus associated with type H product are respectively $\Pi^{c,u}_H = 0.175$, $CS^{c,u} \approx 0.071$ and $TS^{c,u} \approx 0.247$.

Under BBPD, by Proposition 3, numerical calculation shows that the equilibrium prices for type H seller is $\{p^c_{1H}, p^c_{2H}\} = \{p^c, (\beta^c\tau^c, \tau^c)\} = \{0.785, (0.845, 0.785)\}$ and $\beta^c = 1.07$. Both first-time purchasers and repeat purchasers pay lower prices than those under uniform pricing. Seller profit, consumer surplus and social surplus associated with type H product are respectively $\Pi^{c,b}_H \approx 0.178$, $CS^{c,b} \approx 0.082$ and $TS^{c,b} \approx 0.260$. Relative to uniform pricing, BBPD increases type H seller's profit by 1.48%, increases consumer surplus by 14.7%, and total welfare by 5.31%.

4 Conclusions

We analyse a two-period model in which a monopolistic seller may adopt behavior-based price discrimination (BBPD) and charge consumers different prices in the second period based on their purchasing history in the first period. When the consumers have uncertainty about product quality, the prices the seller posts convey valuable information about product quality. We investigate how the option of BBPD affects the signaling role of prices and how BBPD affects seller profit, consumer surplus and total welfare. Contrary to the existing insight in the literature that BBPD harms the seller if consumers' valuations remain constant over time, our analysis shows that BBPD can potentially benefit the seller of high quality product by lowering signaling cost if there is asymmetric information about product quality. Under quality uncertainty, type H seller needs to set sufficiently high first-period price to signal product quality. BBPD lowers the signaling cost by increasing the price sensitivity of first period demand and lowers type L seller's imitation incentives, thus lowering the first-period price that is needed to prevent mimicry of type L seller. We also show that BBPD always increases consumer surplus and total welfare because the average price is lower and demand is higher under BBPD. The equilibrium price patterns under both no-commitment regime and price-commitment regime have the feature of first-time purchaser discount that is widely observed in practice.

In our analysis we have assumed a simple information structure that in the first period all the consumers only know the prior distribution and need to make inference about product quality from the price posted by the seller, and in the second period information about product quality is fully revealed. An alternative structure is that information about product quality is diffused among consumers gradually, with coexistence of informed and uninformed consumers in each period. Informed consumers have perfect knowledge about product quality while uninformed consumers only know the prior distribution of product quality and the fraction of informed consumers increases as time elapses. Our main insight that BBPD can increase type H seller's profit by lowering the signaling cost qualitatively holds in this more general information setup. One issue that arises with tracking of consumers' purchasing history is that some consumers are averse to the idea of revealing personal identity and may take costly measures to maintain anonymity and avoid being identified as repeat purchasers in their interaction with the seller. The incentive of remaining anonymity on the consumers' side may reduce the significance how BBPD affects type H seller's first-period price. Our main insights will still hold if the cost of remaining anonymity is sufficiently large or the portion of consumers that wish to remain anonymous is sufficiently small.

The present set-up considers a monopoly seller. For oligopolistic market with multiple firms, the sellers may need to engage in competitive signaling to convince consumers of their product quality. Prices for first-time purchasers and repeat purchasers may be driven down by competition and the threat of consumer poaching. In this case, BBPD may be beneficial to the sellers due to lower signaling costs but at the same time harmful to the sellers due to endogenous segmentation of market and intensified competition in the second period. BBPD's overall impact upon the sellers' profits becomes more subtle and we leave this issue for future research.

Appendix

This Appendix contains the proofs of Lemmas 2–5, Propositions 1–4, Remark 3 and Corollaries 1–3. The proof of Lemma 1 is substantiated in the main text.

Proof of Lemma 2. Given the off-equilibrium path belief $\mu(p_1) = 0$ for $p_1 \neq p^*_{1H}$, constraint (9) ensures that type L seller will not mimic the price choice of type H seller and p^*_{1H} can indeed be supported in a separating equilibrium.

Next, if p^*_{1H} is a separating equilibrium that survives the intuitive criterion, then $p^*_{1H} \in [c, \tilde{s}]$. Suppose $p^*_{1H} > \tilde{s}$ and $p^*_{1L} = v_L$ form a separating equilibrium. Then $\Pi_H(p^*_{1H}, 1) = \tilde{\pi}$ and $\Pi_L(v_L, 0) = 2v_L$. Consider deviation $p^d = \tilde{s} - \epsilon$ where ϵ is an infinitely small positive number. With this price, $\hat{v} = 2p^d - c$ and $\tilde{s} < 2p^d - c$. Then we have

$$\begin{aligned}\Pi_H(p^d, 1) &= (v_L + 1 - p^d)(p^d - c) + (v_L + 1 - \max\{\tilde{s}, 2p^d - c\})(\max\{\tilde{s}, 2p^d - c\} - c) \\ &= (p^d - c)(3v_L - 5p^d + 2c + 3) = 4\epsilon(\tilde{s} - c) - 5\epsilon^2 + \tilde{\pi} > \Pi_H(p^*_{1H}, 1) = \tilde{\pi} \qquad (28)\end{aligned}$$

$$\Pi_L(p^d, 1) = (v_L + 1 - 2p^d + c)p^d + v_L = 2\epsilon(\tilde{p} - \epsilon) + v_L < \Pi_L(v_L, 0) = 2v_L. \qquad (29)$$

Thus, intuitive criterion requires $\mu(p^d) = 1$. Given such belief, it is better for the type H seller to choose $p^d = \tilde{s} - \epsilon$ instead of p^*_{1H}, in contradiction to the assumption that $p^*_{1H} > \tilde{s}$ forms a separating equilibrium. Our discussion in the text also rules out $p_{1H} \in [v_L,\ c)$ to be a separating equilibrium. Therefore, $p^*_{1H} \in [c,\ \tilde{s}]$ must hold in a separating equilibrium surviving the intuitive criterion. □

Proof of Proposition 1. We show that $p^*_{1H} = \max\{\bar{p}, \tilde{p}_{1H}\}$ forms the unique separating equilibrium that survives the intuitive criterion, where $\tilde{p}_{1H} = \frac{3v_L+7c+3}{10}$ as given in Lemma 1. Since $p^*_{1H} \geq \bar{p}$, constraint (9) is satisfied and type L seller will not mimic the price choice of type H seller.

When $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 > 0$, $p^*_{1H} = \tilde{p}_{1H}$. Since $\tilde{p}_{1H}$ is a global maximizer of $\Pi_H(p_{1H}, 1)$, any $p^d \neq \tilde{p}_{1H}$ can not survive the intuitive criterion. Suppose $p^d \neq \tilde{p}_{1H}$ is indeed a separating equilibrium, since $\Pi_H(p^d, 1) < \Pi_H(\tilde{p}_{1H}, 1)$ and

$$\Pi_L(\tilde{p}_{1H}, 1) = (v_L + 1 - 2\tilde{p}_{1H} + c)\tilde{p}_{1H} + v_L < \Pi_L(v_L, 0) = 2v_L, \qquad (30)$$

where the inequality holds because $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 > 0$. Thus, intuitive criterion requires $\mu(\tilde{p}_{1H}) = 1$ and type H seller prefers $\tilde{p}_{1H}$ over p^d which is a contradiction.

When $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 \leq 0$, $\bar{p} \geq \tilde{p}_{1H}$ and $p^*_{1H} = \bar{p}$. Note that $\bar{p}$ is the local maximizer of $\Pi_H(p_{1H}, 1)$ for $p_{1H} \in [\bar{p}, \tilde{s}]$. Any $p_{1H} \neq \bar{p}$ that satisfies (9) can not survive the intuitive criterion. Suppose $p^d > \bar{p}$ is indeed a separating equilibrium, there exists a small positive ϵ such that $p^d - \epsilon > \bar{p}$, $\Pi_H(p^d, 1) < \Pi_H(p^d - \epsilon, 1)$, and $\Pi_L(p^d - \epsilon, 1) < \Pi_L(v_L, 0)$. Intuitive criterion requires $\mu(p^d - \epsilon) = 1$ and type H seller prefers $p^d - \epsilon$ to p^d, which is a contradiction. Using similar logic we can also rule out $p^d \leq \underline{p}$ to be a separating equilibrium. Thus $p^*_{1H} = \bar{p}$ forms the unique separating equilibrium that survives the intuitive criterion.

Plugging in the equilibrium p^*_{1H} into $\Pi_H(p^*_{1H}, 1)$ in (10) gives us the claimed equilibrium profits of type H seller in (14) and (16). □

Proof of Proposition 2. At $t = 2$, since the seller has to charge uniform prices to all consumers independent of their purchasing history, the unique optimal price for type H seller is the static monopoly price, $s^*_{2H} = \tilde{s}$. At $t = 1$, in a separating equilibrium with $s^*_{1L} = v_L$ and $s^*_{1H} \neq s^*_{1L}$, the equilibrium path belief must be $\mu(s^*_{1L}) = 0$ and $\mu(s^*_{1H}) = 1$. Suppose the off-equilibrium path belief is $\mu(s_1) = 0$ for $s_1 \neq s^*_{1H}$. For s^*_{1H} and s^*_{1L} to be supported in a separating equilibrium, the following constraints must be satisfied:

$$\Pi_L(s^*_{1H}, 1) = (v_L + 1 - s^*_{1H})s^*_{1H} + v_L \leq \Pi_L(v_L, 0) = 2v_L, \tag{31}$$

$$\Pi_H(s^*_{1H}, 1) = (v_L + 1 - s^*_{1H})(s^*_{1H} - c) + (v_L + 1 - \tilde{s})(\tilde{s} - c) \geq \max_{s_1 \neq s^*_{1H}} \Pi_H(s_1, 0) = \tilde{\pi}. \tag{32}$$

Note that (32) is always satisfied for $s^*_{1H} \geq c$ and it is never optimal for type H seller to set $s^*_{1H} < c$. So we can focus on $s^*_{1H} \geq c$ that satisfies (31). Solving

$$(v_L + 1 - s^*_{1H})s^*_{1H} + v_L = 2v_L$$

for s^*_{1H} gives to boundaries: $\bar{s} = 1$ and $\underline{s} = v_L$. Recall that $v_L < 1$ by assumption (1). To satisfy (31) we need $s^*_{1H} \geq \bar{s} = 1$ or $s^*_{1H} \leq \underline{s} = v_L$. Obviously $s^*_{1H} \leq v_L$ can not be optimal for type H seller. Thus in a separating equilibrium $s^*_{1H} \geq 1$ must hold.

When $c \geq 1 - v_L$, $\tilde{s} = \frac{v_L + 1 + c}{2} \geq \bar{s} = 1$ which means type L has no incentive to mimic type H when type H chooses the optimal price $\tilde{s}$ under complete information. So $s^*_{1H} = \tilde{s}$ is supported in a separating equilibrium. Furthermore, any price with $s^*_{1H} > \tilde{s}$ can not survive the intuitive criterion because there exists $\epsilon > 0$ such that $s^d = s_{1H}^* - \epsilon > \tilde{s}$ and

$$\Pi_H(s_{1H}^*, 1) = (v_L + 1 - s_{1H}^*)(s_{1H}^* - c) + \tilde{\pi} < \Pi_H(s^d, 1) = (v_L + 1 - s^d)(s^d - c) + \tilde{\pi},$$
$$\Pi_L(s_{1L}^*, 0) = 2v_L > \Pi_L(s^d, 1) = (v_L + 1 - s^d)s^d + v_L.$$

Therefore, intuitive criterion requires $\mu(s^d) = 1$ and given such belief, type H seller is better off by choosing s^d instead of s_{1H}^* which is a contradiction. Thus, $s_{1H}^* > \tilde{s}$ can not survive the intuitive criterion. Using similar arguments, we can also rule out $s_{1H}^* < \tilde{s}$ to be a separating equilibrium.

When $c < 1 - v_L$, $s_{1H}^* = \bar{s}$, the minimum price that prevents type L's mimicry. Type H's IC constraints (32) is also satisfied. Thus, $s_{1H}^* = 1$ is supported in a separating equilibrium. Using similar arguments as above, we can show that no $s_{1H}^* > \bar{s}$ can survive the intuitive criterion.

The equilibrium profits in (18) and (19) are obtained by plugging the equilibrium prices into type H seller's profits:

$$\Pi_H^u = (v_L + 1 - s_{1H}^*)(1 - c) + \tilde{\pi} = \begin{cases} \frac{(v_L+1-c)^2}{2} & \text{if } s_{1H}^* = \tilde{s} \\ v_L(1-c) + \frac{(v_L+1-c)^2}{4} & \text{if } s_{1H}^* = \bar{s} = 1 \end{cases}.$$

□

Proof of Corollary 1.

1. Consider $c \geq 1 - v_L$ which implies $7c^2 - 4(v_L+1)c + 25v_L - 3(v_L+1)^2 > 0$. The complete information outcome forms the unique separating equilibrium under both BBPD and uniform pricing. We have

$$\Pi_H^u = \frac{(v_L + 1 - c)^2}{2} > \Pi_H^b = \frac{9(v_L + 1 - c)^2}{20}.$$

Let $c_2 \equiv 1 - v_L$. Thus when $c \geq c_2$, BBPD lowers type H seller's profits.

2. Consider $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 \leq 0$ which is equivalent to

$$c \leq \frac{1}{7}\left(2 + 2v_L + 5\sqrt{1 - 5v_L + v_L^2}\right)$$

for $v_L \leq \frac{5-\sqrt{21}}{2}$, it follows that $c < 1 - v_L$. From (16) and (19), type H seller's equilibrium profits under uniform pricing and BBPD are respectively

$$\Pi_H^u = (v_L + 1 - 1)(1-c) + \tilde{\pi}_H = v_L(1-c) + \frac{(v_L+1-c)^2}{4},$$
$$\Pi_H^b = (\bar{p} - c)(3v_L - 5\bar{p} + 2c + 3).$$

Note that at $c = v_L$, $\bar{p} = \frac{1}{2}$ and

$$\Pi_H^b(c = v_L) = (\frac{1}{2} - v_L)(3v_L - \frac{5}{2} + 2v_L + 3) = \frac{1}{4} + 2v_L - 5v_L^2 > \Pi_H^u(c = v_L).$$

It is always true that $\Pi_H^b \leq \frac{9(v_L+1-c)^2}{20}$, with strict inequality when $p_{1H}^* = \bar{p}$. It follows that

$$\Pi_H^b(c = 1 - \frac{3+\sqrt{5}}{2}v_L) \leq \Pi_H^u(c = 1 - \frac{3+\sqrt{5}}{2}v_L).$$

By continuity there must exist a threshold value $\hat{c} \in [v_L, 1 - \frac{3+\sqrt{5}}{2}v_L]$ such that $\Pi_H^b \geq \Pi_H^u$ if $c \leq \hat{c}$. Let $c_1 \equiv \min\{\hat{c}, \frac{1}{7}\left(2 + 2v_L + 5\sqrt{1 - 5v_L + v_L^2}\right)\}$. BBPD increases type H seller's profits in comparison to uniform pricing when $c \leq c_1$.

□

Proof of Corollary 2. 1. Consider $c \geq 1 - v_L$. Under uniform pricing, from Proposition 3, the complete information outcome is supported in equilibrium. Consumer surplus and social welfare when the product is type H are given by

$$CS^u = 2\int_{\tilde{s}}^{v_L+1}(x - \tilde{s})dx = \frac{(v_L+1-c)^2}{4},$$
$$TS^u = \Pi_H^u + CS^u = \frac{3(v_L+1-c)^2}{4}.$$

Under BBPD, the equilibrium price is given by the equilibrium prices under complete information, that is, $p_L^* = (v_L, v_L)$ and

$$p_{1H}^* = p_2^{*N} = \frac{3v_L + 7c + 3}{10}, \quad p_2^{*R} = \frac{3v_L + 2c + 3}{5}.$$

Consumers with $v_i \geq \hat{v} = 2p_{1H}^* - c = \frac{3v_L+2c+3}{5}$ purchase at $t = 1$; consumers with $v_i \geq p_2^{*R}$ purchase a second unit at $t = 2$, and consumers with $v_i \in [p_2^{*N}, \hat{v}]$ purchase their first unit at $t = 2$. Thus, the consumer surplus and total welfare when the product is type H are

$$\begin{aligned}
CS^b &= \int_{\hat{v}}^{v_L+1} (x - p_{1H}^*)dx + \int_{p_2^{*N}}^{\hat{v}} (x - p_2^{*N})dx + \int_{p_2^{*R}}^{v_L+1} (x - p_2^{*R})dx \\
&= \frac{(v_L + 1 - p_{1H}^*)^2}{2} + \frac{(v_L + 1 - p_2^{*R})^2}{2} = \frac{13(v_L + 1 - c)^2}{40} > CS^u, \\
TS^b &= \Pi_H^b + CS_H^b = \frac{31(v_L + 1 - c)^2}{40} > TS^u.
\end{aligned}$$

2. Consider $c < 1 - v_L$. The equilibrium prices under uniform pricing are given by $s_{1L}^* = s_{2L}^* = v_L$ and $s_{1H}^* = 1$ and $s_{2H}^* = \tilde{s}$. Following this, the consumer surplus and total surplus when the product quality is high are respectively:

$$\begin{aligned}
CS^u &= \int_{1}^{v_L+1} (x - 1)dx + \int_{\tilde{s}}^{v_L+1} (x - \tilde{s})dx = \frac{v_L^2}{2} + \frac{(v_L + 1 - c)^2}{8}, \\
TS^u &= \Pi_H^u + CS^u = v_L(1 - c) + \frac{v_L^2}{2} + \frac{3(v_L + 1 - c)^2}{8}.
\end{aligned}$$

Under BBPD, we differentiate two cases following Proposition 1:

(a) If $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 > 0$, we have $p_{1H}^* = p_2^{*N} = \frac{3v_L+7c+3}{10}$, $p_2^{*R} = \frac{3v_L+2c+3}{5}$, and subsequent consumer surplus and total surplus related to type H product are

$$CS^b = \frac{13(v_L + 1 - c)^2}{40} > CS^u, \quad TS^b = \frac{31(v_L + 1 - c)^2}{40} > TS^u.$$

(b) If $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 \leq 0$, $p_{1H}^* = p_2^{*N} = \bar{p}$ and $p_2^{*R} = 2\bar{p} - c$. The consumer surplus and total surplus under BBPD are:

$$\begin{aligned}
CS^b &= \int_{\bar{p}}^{v_L+1} (x - \bar{p})dx + \int_{2\bar{p}-c}^{v_L+1} (x - 2\bar{p} + c)dx = \frac{(v_L + 1 - \bar{p})^2}{2} + \frac{(v_L + 1 - 2\bar{p} + c)^2}{2}, \\
TS^b &= (\bar{p} - c)(3v_L - 5\bar{p} + 2c + 3) + \frac{(v_L + 1 - \bar{p})^2}{2} + \frac{(v_L + 1 - 2\bar{p} + c)^2}{2}.
\end{aligned}$$

To show $CS^b > CS^u$, it is sufficient to show the equilibrium prices are lower in both periods under BBPD than those under uniform pricing, that is $\bar{p} \leq \tilde{s}$ and $2\bar{p} - c \leq 1$. $\bar{p} \leq \tilde{s}$ is obvious and $2\bar{p} - c \leq \bar{s} = 1$ holds because

$$\begin{aligned}
\bar{p}(v_L = 0) &= \frac{1 + c + \sqrt{(1 + c)^2}}{4} = \frac{1 + c}{2}, \\
\frac{\partial \bar{p}}{\partial v_L} &= \frac{1}{4}[1 + \frac{(v_L + 1 + c) - 4}{\sqrt{(v_L + 1 + c)^2 - 8v_L}}] < 0
\end{aligned}$$

in which the inequality in the second line holds because $c < 1 - v_L$. Since the equilibrium prices under BBPD and uniform pricing are above the production cost c, lower prices for all purchasers also lead to larger social welfare.

$\square$

Proof of Lemma 3. Given price scheme $p_H = \{p_{1H}, (p_2^R, p_2^N)\}$, consumers have four options, purchasing in both periods at price p_{1H} and p_2^R, purchasing only in the first period at price p_{1H}, purchasing only in the second period at price p_2^N, or not purchasing any unit.

1. First consider $p_{1H} < p_2^N$. No consumers purchase only in the second period. By lowering p_2^N to p_{1H}, type H seller's profit does not change. Therefore price scheme p_H is at least weakly dominated by price scheme $\hat{p}_{1H} = \{p_{1H}, (p_2^R, p_{1H})\}$ for type H seller under complete information. When there is incomplete information about product quality, since the first period demand does not increase, type L seller has no incentive to mimic type H under the same first period price.

2. Next consider $p_{1H} > p_2^N$. We differentiate two cases:

   (a) $p_{1H} + p_2^R \leq 2p_2^N$. Consumers with $v_i \geq \frac{p_{1H}+p_2^R}{2}$ purchase twice at prices p_{1H} and p_2^R, and those with $v_i < \frac{p_{1H}+p_2^R}{2}$ do not make a purchase. By instead setting $\hat{p}_2^N = p_{1H}$ does not change demand, therefore it does not change the profits of type H seller, nor does it change type L seller's imitation incentive under asymmetric information.

   (b) $p_{1H} + p_2^R > 2p_2^N$. Consumers with $v_i \geq p_{1H} + p_2^R - p_2^N$ purchase in both periods at prices p_{1H} and p_2^R, and consumers with $v_i \in [p_2^N, p_{1H} + p_2^R - p_2^N]$ purchase only at $t = 2$ at price p_2^N.[8] Type H seller's profits from the two periods are

$$\Pi_H(p_H) = \left[v_L + 1 - (p_{1H} + p_2^R - p_2^N)\right](p_{1H} - c + p_2^R - c) + (p_{1H} + p_2^R - p_2^N - p_2^N)(p_2^N - c) \quad (33)$$

   where the first term is type H's total profits from the high valuation segment and the second term is his total profits from the low valuation segment.

   By instead posting price scheme $\hat{p}_H = \{\hat{p}_{1H}, (\hat{p}_2^R, \hat{p}_2^N)\}$ with $\hat{p}_{1H} = \hat{p}_2^N = p_2^N$ and $\hat{p}_2^R = p_{1H} + p_2^R - p_2^N > p_2^R$, type H seller earns exactly the same profits both from the high valuation

[8] If $p_{1H} + p_2^R - p_2^N > v_L + 1$, consumers with $v_i \geq p_2^N$ purchase only at $t = 2$. Then posting $\hat{p}_H = \{\hat{p}_{1H}, (\hat{p}_2^R, \hat{p}_2^N)\}$ instead with $\hat{p}_{1H} = \hat{p}_2^N = p_2^N$ and $\hat{p}_2^R > v_L + 1$ neither changes type H's profits nor type L's imitation incentive.

segment and the low valuation segment:

$$\begin{aligned}\Pi_H(\hat{p}_H) =&[v_L + 1 - (\hat{p}_{1H} + \hat{p}_2^R - \hat{p}_2^N)](\hat{p}_{1H} - c + \hat{p}_2^R - c) + (\hat{p}_{1H} + \hat{p}_2^R - \hat{p}_2^N - \hat{p}_2^N)(\hat{p}_2^N - c)\\ =&[v_L + 1 - (p_{1H} + p_2^R - p_2^N)](p_{1H} - c + p_2^R - c) + (p_{1H} + p_2^R - p_2^N - p_2^N)(p_2^N - c)\\ =&\Pi_H(p_H).\end{aligned}$$

Furthermore, since $\hat{p}_{1H} < p_{1H}$, under $\hat{p}_H$, type L seller has strictly less incentive to mimic the type H seller when there is asymmetric information about product quality and p_H forms a separating equilibrium

$$\begin{aligned}\Pi_L(\hat{p}_H, 1) = \left(v_L + 1 - (\hat{p}_{1H} + \hat{p}_2^R - \hat{p}_2^N)\right)(\hat{p}_{1H} - c) = (v_L + 1 - [p_{1H} + p_2^R - p_2^N])(\hat{p}_{1H} - c)\\ < (v_L + 1 - [p_{1H} + p_2^R - p_2^N])(p_{1H} - c) = \Pi_L(p_H, 1) \le \Pi_L(p_L^*, 0) = 2v_L.\end{aligned}$$

Thus, by adjusting $\hat{p}_2^R$ optimally while keeping $\hat{p}_{1H} = \hat{p}_2^N = p_2^N$ may bring type H seller a profit strictly larger than $\Pi_H(p_H)$, while keeping type L's imitation incentive satisfied. In particular, note that $\hat{p}_{1H} = \hat{p}_2^N$ and $\hat{p}_2^R$ cannot be both equal to $\tilde{s}$. Suppose $p_2^R \lesseqgtr \tilde{s}$ there must exist $\hat{p}'_H = \{\hat{p}_{1H}, (\hat{p}'^R_2, \hat{p}_2^N)\}$, where $\hat{p}'^R_2 = \hat{p}_2^R \pm \epsilon$ ($\epsilon > 0$) which is closer to $\tilde{s}$ than $\hat{p}_2^R$, such that type H seller earns strictly higher profits:

$$\begin{aligned}\Pi_H(\hat{p}'_H) =&(v_L + 1 - \hat{p}'^R_2)(\hat{p}'^R_2 - c) + (v_L + 1 - \hat{p}_{1H})(\hat{p}_{1H} - c)\\ >&(v_L + 1 - \hat{p}_2^R)(\hat{p}_2^R - c) + (v_L + 1 - \hat{p}_{1H})(\hat{p}_{1H} - c) = \Pi_H(\hat{p}_H).\end{aligned}$$

And type L has no incentive to mimic type H given that ϵ is sufficiently small and $\Pi_L(\hat{p}_H, 1) < 2v_L$:

$$\Pi_L(\hat{p}'_H, 1) = [v_L + 1 - (\hat{p}_2^R \pm \epsilon)](\hat{p}_{1H} - c) = \Pi_L(\hat{p}_H, 1) \pm \epsilon(\hat{p}_{1H} - c) < 2v_L.$$

Suppose $p_2^R = \tilde{s}$, it follows that $\hat{p}_{1H} = \hat{p}_2^N < \tilde{s}$. Then there exists $\hat{p}'_H = \{\hat{p}'_{1H}, (\hat{p}_2^R, \hat{p}'^N_2)\}$, where $\hat{p}'_{1H} = \hat{p}'^N_2 = \hat{p}_{1H} + \epsilon$ ($\epsilon > 0$), such that type H seller earns strictly higher profits:

$$\begin{aligned}\Pi_H(\hat{p}'_H) =&(v_L + 1 - \hat{p}_2^R)(\hat{p}_2^R - c) + (v_L + 1 - \hat{p}'_{1H})(\hat{p}'_{1H} - c)\\ >&(v_L + 1 - \hat{p}_2^R)(\hat{p}_2^R - c) + (v_L + 1 - \hat{p}_{1H})(\hat{p}_{1H} - c) = \Pi_H(\hat{p}_H),\end{aligned}$$

and type L has no incentive to mimic type H given that ϵ is sufficiently small:

$$\Pi_L(\hat{p}'_H, 1) = (v_L + 1 - \hat{p}_2^R)(\hat{p}_{1H} + \epsilon - c) = \Pi_L(\hat{p}_H, 1) + \epsilon(v_L + 1 - \hat{p}_2^R) < 2v_L.$$

$\square$

Proof of Lemma 4. Under complete information about product quality, it is optimal for type L seller to choose price equal to v_L for each period under BBPD and uniform pricing. Thus $\tilde{p}_L^c = \tilde{s}_L^c = \{v_L, v_L\}$, and type L seller's profit is $\tilde{\Pi}_L^c = 2v_L$. Type L seller can not benefit from the option of BBPD.

Consider type H's choice under complete information in the price-commitment regime. Suppose BBPD is not allowed and the seller has to adopt uniform pricing, $s_H = \{s_{1H}, s_{2H}\}$, type H seller's profit is

$$\Pi_H(s_H) = (v_L + 1 - s_{1H})(s_{1H} - c) + (v_L + 1 - s_{2H})(s_{2H} - c), \tag{34}$$

where the two terms are respectively the seller's profits from the first and second period. Since the seller's profits from the two periods are independent, it is optimal for type H to choose the static monopoly price for both periods and $s_{1H}^c = s_{2H}^c = \tilde{s}$.

Now let's turn to the case with BBPD, $p_H = \{\tau, (\beta\tau, \tau)\}$. When $\beta = 1$, consumers purchase in each period if and only if $v_i \geq \tau$. Thus

$$\Pi_H(p_H) = (v_L + 1 - \tau)(\tau - c) + (v_L + 1 - \tau)(\tau - c). \tag{35}$$

The optimal choice is $\tau = \tilde{s}$ and the associated profit is $\Pi_H(p_H) = 2\tilde{\pi}$.

When $\beta > 1$, consumers with $v_i \geq \beta\tau$ purchase in both periods and consumers with $v_i \in [\tau, \beta\tau)$ purchase one unit either in the first or in the second period. Type H seller's profit is

$$\Pi_H(p_H) = (v_L + 1 - \beta\tau)(\tau - c) + (\beta\tau - \tau)(\tau - c) + (v_L + 1 - \beta\tau)(\beta\tau - c), \tag{36}$$

where the first term is the seller's profit from $t = 1$ and the second and third term are the seller's profit from $t = 2$. The derivatives of $\Pi_H(p_H)$ with respect to τ and β are respectively

$$\frac{\partial \Pi_H(p_H)}{\partial \tau} = (1 + \beta)(v_L + 1 + c) - 2(1 + \beta^2)\tau \tag{37}$$

$$\frac{\partial \Pi_H(p_H)}{\partial \beta} = \tau\,(v_L + 1 + c - 2\beta\tau) \tag{38}$$

Setting (37) to 0, we have

$$\tau = \frac{(1 + \beta)(v_L + 1 + c)}{2(1 + \beta^2)},$$

and (38) leads to $\frac{\partial \Pi_H(p_H)}{\partial \beta} < 0$ for all $\beta > 1$. Thus $\Pi_H(p_H) < 2\tilde{\pi}$ for $\beta > 1$. Therefore, p_H with $\beta > 1$ is dominated by p_H with $\beta = 1$ for type H seller.

When $\beta < 1$, consumers with $v_i \geq \frac{1+\beta}{2}\tau$ purchase twice and the other consumers do not buy the product. Thus, type H seller's profit is

$$\Pi_H(p_H) = (v_L + 1 - \frac{1+\beta}{2}\tau)(\tau - c) + (v_L + 1 - \frac{1+\beta}{2}\tau)(\beta\tau - c). \tag{39}$$

The first order conditions are

$$\frac{\partial \Pi_H(p_H)}{\partial \tau} = (1+\beta)\left(v_L + 1 + c - (1+\beta)\tau\right) = 0 \tag{40}$$

$$\frac{\partial \Pi_H(p_H)}{\partial \beta} = \tau\left(v_L + 1 + c - (1+\beta)\tau\right) = 0 \tag{41}$$

From (40) and (41), we have

$$\tau(\beta + 1) = v_L + 1 + c. \tag{42}$$

Thus p_H with $\beta < 1$ is (weakly) dominated by p_H with $\beta = 1$ and $\tau = \tilde{s}$ for type H seller, and type H seller's profit is $\Pi_H(p_H) = 2\tilde{\pi}$.

Therefore, when BBPD is allowed, type H seller maximizes his profit by setting a flat price with $\tau = \tilde{s}$ and $\beta = 1$ instead of using price conditioning. Consumers with $v_i \geq \tilde{s}$ purchase in both periods while others purchase in neither period. Type H seller's profit equals $\Pi_H^c = 2\tilde{\pi} = \frac{(v_L+1-c)^2}{2}$. □

Proof of Lemma 5. Constraint (21) ensures that type L seller will not mimic the price choice of type H and p_H^c is indeed supported in a separating equilibrium. We now show that for $p_H^c = \{\tau^c, (\beta^c\tau^c, \tau^c)\}$ and $p_L^c = \{v_L, v_L\}$ to survive the intuitive criterion, (20) must be satisfied. Suppose there exists some p^d that is also supported in a separating equilibrium and $\Pi_H(p^d, 1) < \Pi_H(p_H^c, 1)$. Condition (21) implies $\Pi_L(p_H^c, 1) \leq \Pi_L(p_L^c, 0)$. We differentiate two cases:

1. $\Pi_L(p_H^c, 1) < \Pi_L(p_L^c, 0)$. Intuitive criterion requires $\mu(p_H^c) = 1$ and thus type H seller prefers p_H^c over p^d, contradicting p^d as the optimal choice of type H.

2. $\Pi_L(p_H^c, 1) = \Pi_L(p_L^c, 0)$. If $\Pi_L(p_H^c, 1)$ decreases in τ at $\tau = \tau^c$, then let $\hat{p}_H = \{\hat{\tau}, (\beta\hat{\tau}, \hat{\tau})\}$ where $\hat{\tau} = \tau^c + \epsilon$ and ϵ is a small positive number. Then we have $\Pi_L(\hat{p}_H, 1) < \Pi_L(p_L^c, 0)$ and $\Pi_H(p^d, 1) < \Pi_H(\hat{p}_H, 1)$ violating the intuitive criterion. Similarly there exists $\hat{\tau} = \tau^c - \epsilon$ so that p^d can not survive the intuitive criterion if $\Pi_L(p_H^c, 1)$ increases in τ at $\tau = \tau^c$.

$\square$

We prove 3 below to prepare for the proof of Proposition 3.

Remark 3. *There exists no separating equilibrium with $\beta \leq 1$ that survives the intuitive criterion when $(v_L+1)^2 - 8v_L > c^2$.*

Proof of Remark 3. Suppose $(v_L+1)^2 - 8v_L > c^2$ holds and we show in sequence that an equilibrium candidate p_H with $\beta < 1$ or $\beta = 1$ violates the intuitive criterion.

Suppose there exists a separating equilibrium with $p_H^c = \{\tau^c, (\beta^c\tau^c, \tau^c)\}$ and $\beta^c < 1$ and $p_L^c = \{v_L, v_L\}$. The optimal β^c for type H seller must satisfy

$$\Pi_L(p_H^c, 1) = (v_L + 1 - \frac{1+\beta^c}{2}\tau^c)\tau^c + 0 \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{43}$$

Then consider price scheme $\hat{p}_H = (\hat{\tau}, (\hat{\beta}\hat{\tau}, \hat{\tau}))$ with $\hat{\tau} = \frac{1+\beta^c}{2}\tau^c$ and $\hat{\beta} = 1$. We have

$$\begin{aligned}\Pi_L(\hat{p}_H, 1) &= (v_L + 1 - \hat{\tau})\hat{\tau} + 0 = (v_L + 1 - \frac{1+\beta^c}{2}\tau^c)\frac{1+\beta^c}{2}\tau^c \\ &< \Pi_L(p_H^c, 1) \leq \Pi_L(p_L^c, 0).\end{aligned}$$

Thus under $\hat{p}_H$ type L seller has no incentive to mimic the price choice of type H seller. Furthermore, note that type H seller's profits under p_H^c and $\hat{p}_H$ are the same:

$$\begin{aligned}\Pi_H(p_H^c, 1) &= (v_L + 1 - \frac{1+\beta^c}{2}\tau)(\tau - c) + (v_L + 1 - \frac{1+\beta^c}{2}\tau^c)(\beta^c\tau^c - c) \\ &= 2(v_L + 1 - \frac{1+\beta^c}{2}\tau^c)(\frac{1+\beta^c}{2}\tau^c - c), \\ \Pi_H(\hat{p}_H, 1) &= (v_L + 1 - \hat{\tau})(\hat{\tau} - c) + (v_L + 1 - \hat{\tau})(\hat{\tau} - c) \\ &= 2(v_L + 1 - \frac{1+\beta^c}{2}\tau^c)(\frac{1+\beta^c}{2}\tau^c - c).\end{aligned}$$

Moreover, since $(v_L+1)^2 - 8v_L > c^2$ holds, complete information outcome $\tilde{p}_H^c = \{\tilde{s}, \tilde{s}\}$ can not be supported in a separating equilibrium and we have $\Pi_L(\tilde{p}_H^c, 1) > \Pi_L(p_L^c, 0) = 2v_L$, thus $\hat{\tau} \neq \tilde{s}$ must hold. Then for $\hat{\tau} \lesseqgtr \tilde{s}$ there exists $\check{\tau}_H$ with $\check{\tau} = \hat{\tau} \pm \epsilon$ and $\check{\beta} = 1$ such that for sufficiently small positive ϵ we have

$$\begin{aligned}&\Pi_L(\check{p}_H, 1) = (v_L + 1 - \check{\tau})\check{\tau} + 0 = (v_L + 1 - (\hat{\tau} \pm \epsilon))(\hat{\tau} \pm \epsilon) < \Pi_L(p_L^c, 0) \\ &\Pi_H(\check{p}_H, 1) = 2(v_L + 1 - \check{\tau})(\check{\tau} - c) > \Pi_H(\hat{p}_H, 1) = \Pi_H(p_H^c, 1).\end{aligned}$$

Thus the intuitive criterion requires $\mu(\check{p}_H) = 1$ and type H seller prefers price scheme $\check{p}_H$ over p_H^c, which is a contradiction. Therefore, a separating equilibrium p_H^c with $\beta^c < 1$ can not survive the intuitive criterion.

Next we show that p_H^c with $\beta^c = 1$ can not survive the intuitive criterion either. Suppose there exists a separating equilibrium with $p_H^c = \{\tau^c, (\tau^c, \tau^c)\}$. The optimal $\tau^c > \tilde{s}$ for a type H seller must satisfy

$$\Pi_L(p_H^c, 1) = (v_L + 1 - \tau^c)\tau^c + 0 \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{44}$$

Then consider price scheme $\hat{p}_H = \{\tilde{s}, (\hat{\beta}\tilde{s}, \tilde{s})\}$ with $\hat{\beta}\tilde{s} = \tau^c$ where $\hat{\beta} > 1$. We have

$$\Pi_L(\hat{p}_H, 1) = (v_L + 1 - \hat{\beta}\tilde{s})\tilde{s} + 0 = (v_L + 1 - \tau^c)\tilde{s} < \Pi_L(p_H^c, 1) \leq \Pi_L(p_L^c, 0). \tag{45}$$

Thus under $\hat{p}_H$ type L seller will also not mimic the price choice of type H seller. Moreover, type H seller's profits under $\hat{p}_H$ are higher than that under p_H^c:

$$\begin{aligned}\Pi_H(\hat{p}_H, 1) &= (v_L + 1 - \tau^c)(\tau^c - c) + (v_L + 1 - \tilde{s})(\tilde{s} - c) \\ &> (v_L + 1 - \tau^c)(\tau^c - c) + (v_L + 1 - \tau^c)(\tau^c - c) = \Pi_H(p_H^c, 1).\end{aligned}$$

Thus both $\Pi_H(\hat{p}_H, 1) > \Pi_H(p_H^c, 1)$ and $\Pi_L(\hat{p}_H, 1) < \Pi_L(p_L^c, 0)$ hold, intuitive criterion requires $\mu(\hat{p}_H) = 1$ and as a result type H seller prefers $\hat{p}_H$ over p_H^c which is a contradiction. Therefore, p_H^c with $\beta^c = 1$ can not be supported in a separating equilibrium that survives the intuitive criterion. $\square$

Proof of Proposition 3. Part 1 of the statement follows directly from the discussion in the text before Proposition 3. We already show in Remark 3 that price scheme $p_H = \{\tau, (\beta\tau, \tau)\}$ with $\beta \leq 1$ and $p_L = \{v_L, v_L\}$ can not be supported in a separating equilibrium that survives the intuitive criterion. In the following we prove that if $(v_L + 1)^2 - 8v_L > c^2$,[9] there exists a separating equilibrium with $\beta > 1$ that survives the intuitive criterion. That is, there exists a

[9]This implies $v_L < \frac{1}{6}$ and $c < \sqrt{v_L^2 - 6v_L + 1}$.

combination of $\beta^c > 1$ and τ^c that maximizes

$$\Pi_H(p_H, 1) = (v_L + 1 - \beta\tau)(\tau - c) + (v_L + 1 - \beta\tau)(\beta\tau - c) + (\beta\tau - \tau)(\tau - c) \tag{46}$$

$$\text{such that} \qquad \Pi_L(p_H, 1) = (v_L + 1 - \beta\tau)\tau \leq \Pi_L(p_L) = 2v_L. \tag{47}$$

The lagrangian function is written as follow:

$$\mathcal{L}(\beta, \tau) = (v_L + 1 - \beta\tau)[(1 + \beta)\tau - 2c)] + (\beta - 1)\tau(\tau - c) + \lambda[2v_L - (v_L + 1 - \beta\tau)\tau]$$

in which λ is the lagrangian multiplier. The first order conditions with respect to β and τ are respectively

$$(1 + \beta)(v_L + 1 - \beta\tau) - \beta[(1 + \beta)\tau - 2c)] + (\beta - 1)(2\tau - c) - \lambda(v_L + 1 - 2\beta\tau) = 0, \tag{48}$$

$$-\tau[(1 + \beta)\tau - 2c)] + \tau(v_L + 1 - \beta\tau) + \tau(\tau - c) + \lambda\tau^2 = 0. \tag{49}$$

Suppose $\lambda = 0$, equation (49) implies that $\beta\tau = \tilde{s}$. Plugging this back into (48), we get $\beta = 1$, which leads to $\tau = \beta\tau = \tilde{s}$ as the optimal price scheme of type H seller. This is a contradiction because under $(v_L + 1)^2 - 8v_L > c^2$, the complete information outcome $\tilde{p}_H^c = \{\tilde{s}, \tilde{s}\}$ can not be supported in a separating equilibrium. Therefore, in a separating equilibrium, $\lambda > 0$ must hold and type L's IC constraint (47) must be binding, which implies

$$(v_L + 1 - \beta\tau)\tau = 2v_L \Leftrightarrow \beta(\tau) = \frac{v_L + 1}{\tau} - \frac{2v_L}{\tau^2}. \tag{50}$$

Thus, type H seller's profit maximization program simplifies to:

$$\max_{\tau} \ \Pi_H(p_H, 1) = \frac{2v_L}{\tau}(\tau + v_L + 1 - \frac{2v_L}{\tau} - 2c) + (v_L + 1 - \frac{2v_L}{\tau} - \tau)(\tau - c). \tag{51}$$

Then the derivative with respect to τ is

$$\frac{\partial \Pi_H(p_H, 1)}{\partial \tau} = -\frac{2v_L(v_L + 1 - c)}{\tau^2} + \frac{8v_L^2}{\tau^3} + v_L + 1 - 2\tau + c \tag{52}$$

Note that for $\tau \leq \frac{(v_L+1)-\sqrt{(v_L+1)^2-8v_L}}{2} \in [v_L, \tilde{s}]$ in which $\tilde{s} = \frac{v_L+1+c}{2}$, we have

$$\begin{aligned}\frac{\partial \Pi_H(p_H, 1)}{\partial \tau} &= \frac{v_L}{\tau^3}[8v_L - 2\tau(v_L + 1 - c)] + v_L + 1 - 2\tau + c \\ &\geq \frac{v_L}{\tau^3}[8v_L - (v_L + 1 - \sqrt{(v_L+1)^2 - 8v_L})(v_L + 1 - c)] + c + \sqrt{(v_L+1)^2 - 8v_L} \\ &= \frac{v_L}{\tau^3}(v_L + 1 - \sqrt{(v_L+1)^2 - 8v_L})(\sqrt{(v_L+1)^2 - 8v_L} + c) + c + \sqrt{(v_L+1)^2 - 8v_L} \\ &> 2c > 0.\end{aligned}$$

For $\tau \geq \tilde{s}$, we have

$$\begin{aligned}\frac{\partial \Pi_H(p_H, 1)}{\partial \tau} &= -\frac{2v_L(v_L + 1 - c)\tau - 8v^2}{\tau^3} + v_L + 1 + c - 2\tau \\ &\leq -\frac{v_L[(v_L + 1 - c)(v_L + 1 + c) - 8v_L]}{\tau^3} + v_L + 1 + c - (v_L + 1 + c) \\ &< 0.\end{aligned}$$

Since the objective function (51) is differentiable for $\tau \in [v_L, v_L + 1]$, there exists a $\tau^c \in (\frac{(v_L+1)-\sqrt{(v_L+1)^2-8v_L}}{2}, \tilde{s})$ that maximizes the objective function (51). Furthermore, since $(v_L + 1 - \tau)\tau > 2v_L$ for all $\tau \in (\frac{(v_L+1)-\sqrt{(v_L+1)^2-8v_L}}{2}, \tilde{s})$, $\beta^c = \frac{1}{\tau^c}(v_L + 1 - \frac{2v_L}{\tau^c}) > 1$.

Finally, we confirm that $p_H^c = \{\tau^c, (\beta^c\tau^c, \tau^c)\}$ indeed constitutes a separating equilibrium by showing that type H seller has no incentive to deviate from this price scheme. Consider $\hat{p}_H = (\tilde{s}, (\hat{\beta}\tilde{s}, \tilde{s}))$ with $\hat{\beta} = \frac{1}{\tilde{s}}(v_L + 1 - \frac{2v_L}{\tilde{s}})$. Then making use of (46), we have

$$\begin{aligned}\Pi_H(\hat{p}_H, 1) &= (v_L + 1 - \tilde{s})(\tilde{s} - c) + (v_L + 1 - \hat{\beta}\tilde{s})(\hat{\beta}\tilde{s} - c) \\ &= \tilde{\pi} + \frac{2v_L}{\tilde{s}}\left(v_L + 1 - \frac{2v_L}{\tilde{s}} - c\right) > \tilde{\pi}\end{aligned}$$

because $(v_L + c)^2 - 8v_L > c^2$ implies $v_L + 1 - \frac{2v_L}{\tilde{s}} - c > 0$. Since p_H^c is a maximizer of type H seller's program Γ, let the consumer's off-equilibrium belief be $\mu(p^d) = 0$ for $p^d \neq p_H^c$, we have

$$\Pi_H(p_H^c, 1) \geq \Pi_H(\hat{p}_H, 1) > \max_{p^d \neq p_H^c} \Pi_H(p^d, 0) = \tilde{\pi}.$$

Thus a type H seller indeed has no incentive to deviate from p_H^c. Since p_H^c is a maximizer of type H seller's profit (20), it also survives the intuitive criterion. $\square$

Proof of Proposition 4. In a separating equilibrium with $s_H^c = \{s_{1H}^c, s_{2H}^c\}$ and $s_L^c = \{v_L, v_L\}$, a type L seller does not mimic s^c_H if

$$\Pi_L(s^c_H, 1) = (v_L + 1 - s^c_{1H})s^c_{1H} + 0 \leq \Pi_L(s^c_L, 0) = 2v_L. \tag{53}$$

Under uniforming pricing, in a separating equilibrium surviving the intuitive criterion, $s^c_H = \{s^c_{1H}, s^c_{2H}\}$ maximize

$$\Pi_H(s_H, 1) = (v_L + 1 - s_{1H})(s_{1H} - c) + (v_L + 1 - s_{2H})(s_{2H} - c) \tag{54}$$

subject to constraint (53). It follows that $s^c_{2H} = \tilde{s}$, and when $(v_L + 1 - \tilde{s})\tilde{s} \leq 2v_L$, that is, $(v_L + 1)^2 - 8v_L \leq c^2$, $s^c_{1H} = \tilde{c}$; when $(v_L + 1)^2 - 8v_L > c^2$, $s^c_{1H} = \frac{v_L+1+\sqrt{(v_L+1)^2-8v_L}}{2}$, which is the price generating the highest profits for type H seller subject to binding constraint (53). The equilibrium profit of type H seller in (27) is obtained by plugging the equilibrium prices into (54). □

Proof of Corollary 3. From Proposition 3 and 4, if $(v_L + 1)^2 - 8v_L \leq c^2$ holds, the complete information outcome is supported as the unique separating equilibrium under both BBPD and uniform pricing, and type H seller's profits under the two pricing regimes are the same: $\Pi^c_H = 2\tilde{\pi}$. If $(v_L + 1)^2 - 8v_L > c^2$ holds, Lemma 3 implies that under BBPD price scheme $p_H = \{s^c_{1H}, (s^c_{2H}, s^c_{2H})\}$ is (weakly) dominated by some price scheme $\hat{p}_H = \{s^c_{1H}, (s^c_{2H}, s^c_{1H})\}$. The analysis in Proposition 3 suggests that when the price scheme takes the form of $p_H = \{\tau, (\beta\tau, \tau)\}$ the equilibrium prices are $p^c_H = \{\tau^c, (\beta^c\tau^c, \tau^c)\}$ as given in Proposition 3. Thus type H seller's profit with BBPD must be larger than that under uniform pricing in the price-commitment regime.

To prove part (ii) of the claim, note that under BBPD consumers with $v_i \geq \beta^c\tau^c$ purchase at price τ^c in the first period, consumers with $v_i \in [\tau^c, \beta^c\tau^c)$ purchase their first unit in the second period at price τ^c, and consumers with $v_i \in [\beta^c\tau^c, v_L + 1]$ purchase their second unit in the second period at price $\beta^c\tau^c$, while under uniform pricing consumers with $v_i \geq s^c_{1H}$ purchase in the first period at price s^c_{1H} and consumers with $v_i \geq \tilde{s}$ purchase in the second period at price $\tilde{s}$. Thus the consumer surplus associated with type H product under BBPD and uniform price are respectively

$$
\begin{aligned}
CS^{c,b} &= \int_{\beta^c\tau^c}^{v_L+1}(t-\tau^c)dt + \int_{\tau^c}^{\beta^c\tau^c}(t-\tau^c)dt + \int_{\beta^c\tau^c}^{v_L+1}(t-\beta^c\tau^c)dt \\
&= \int_{\tau^c}^{v_L+1}(t-\tau^c)dt + \int_{\beta^c\tau^c}^{v_L+1}(t-\beta^c\tau^c)dt. \\
CS^{c,u} &= \int_{s_{1H}^c}^{v_L+1}(t-s_{1H}^c)dt + \int_{\tilde{s}}^{v_L+1}(t-\tilde{s})dt.
\end{aligned}
$$

In Proposition 3 we have shown that $\tau^c < \tilde{s}$. Moreover,

$$
\beta^c\tau^c = v_L + 1 - \frac{2v_L}{\tau^c} < v_L + 1 - \frac{2v_L}{s_{1H}^c} = s_{1H}^c.
$$

Thus $CS^{c,u} < CS^{c,b}$ holds. Since both consumer surplus and type H seller's profits are higher, total surplus is also higher under BBPD than uniform pricing. $\square$

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