Maybe I Should Just Stay Home

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Abstract

We model decisions to apply for a job or admission, attend an audition, or run for office as costly entry in contests where the outcomes solely depend on entrants' talent or ability. Player ability is privately known and players generically differ in commonly-known characteristics such as ability distributions, entry costs, or payoffs from winning. Any infinitesimal difference in these characteristics leads to wildly different equilibrium entry probabilities. Minute differences in characteristics imply large dispersions in representation even in a pure meritocracy. We can improve social welfare by handicapping advantaged players or by surcharging advantaged players to subsidize disadvantaged players.

JEL Classifications: D44, L86, D82, C73

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1 Introduction

Consider two violinists deciding whether to attend a blind audition for a position at the New York Philharmonic. If both attend the audition, the judges choose the better violinist. If only one attends, she is chosen by default as the selection committee have already determined that the two violinists possess the minimum required ability for the position based on their credentials, prior recordings, and demo tapes. Attending the audition is costly. While a violinist knows her ability, she does not know the other player's ability. Hence, she will enter only if the probability of landing the position and the return from the position are high enough to justify the cost of attending. Otherwise, she will just stay home. In this paper, we characterize how the violinists would make their entry decisions. We show that, when their publicly-known characteristics are different, even if slightly, their entry probabilities will differ greatly. If one views the composition of the orchestra to have resulted from a series of such auditions, a systematic difference in entry behavior of potential applicants of different backgrounds may lead to a lack of diversity. Nonetheless, this can be somewhat mitigated via affirmative action policies.

In our contest model, contestants may vary in terms of their ability distributions, sizes of the reward, or entry costs, which are all publicly known. Once a contestant enters, her ability is observed by the contest designer, who awards the prize to the entrant with the highest ability. One should think of ability as the accumulation of lifetime effort in becoming proficient at a job, broadly speaking. It is a combination of inherent facility at the task as well as thousands of hours of preparation. Relative to accumulation of ability, the entry cost is more short-run. For music auditions, this may represent practicing an assigned piece at the cost of other performance opportunities, work, and leisure. For admissions to college or specialized high schools, the entry cost represents application fees and preparation costs, mental stress, etc. The admission office takes into account applicants' cumulative record over years and chooses the person who fits the opportunity the best. Reward from gaining admission can be thought of as an increase in lifetime income net of tuition fees and other costs that are incurred only by those who join the program. In a business setting, ability represents the accumulated competency and human capital of the organization. When an acquisition opportunity appears, a business must decide whether to devote scarce resources—lawyers, investment bankers, the CFO—toward making an offer, or not. It is this local "gearing up" effort that represents the short-run entry cost.

The main finding from this model is the easiest to see when there are two players. Even though the players can differ in a myriad of ways, there is always a unique equilibrium, which provides a very sharp prediction about who enters the contest. In the generic case, where players differ in some characteristics, the entry probabilities across players are strictly different no matter how small the differences are. Specifically, one of the players will be *advantaged* and will enter the contest even when her merit or ability is very low. The other player will enter only when her ability is high enough. In the non-generic case of ex ante identical players, however, both players will stay out of the contest with the same, positive, probability. Equilibrium discontinuity between these two cases implies that in this game, assuming that player characteristics are identical and focusing on the symmetric equilibrium is not an innocuous simplification of the case when player characteristics are *almost* identical. Suppose a workplace is composed of winners of a series of contests, each with one player from the advantaged population group and one from the disadvantaged population group. Because of the lopsided entry probabilities, the number of contest winners from the advantaged group will likely be much greater. Even when the highest-ability entrant is always chosen, the advantage group will be disproportionately highly represented.

Difference across the players may arise from the publicly known player characteristics discussed above. Such difference may also arise from sociocultural misperception about how the ability of a player is distributed or psychological factors such as competition aversion. Even minor misperception or stereotyping or lack of awareness of opportunities lead to obstacles to equitable representation. Thus, barriers to equity are not only about financial incentives and constraints— cost or reward. Social, cultural, or perceptual factors can lead to the same problem.

We can increase representation of the *disadvantaged* player and increase the contest designer's better off by handicapping the advantaged player. When abilities of both players are on the low side, a handicapping rule increases the likelihood that the higher ability player is chosen by reducing the entry threshold for the disadvantaged player. While this rule increases the probability of choosing the lower ability player when the advantaged player has a slightly higher ability, that effect is of second order. Hence, small handicaps are optimal. Representation can also be improved by providing a subsidy to the disadvantaged player that can be supported by a surcharge on the advantaged player. This reduces the entry threshold for the disadvantaged player without reducing the advantaged player's entry probability. This increases the likelihood of an efficient outcome and the societal benefits from the contest. Even if one does not care about diversity per se, increasing representativeness by increasing entry probabilities of the disadvantaged players makes economic sense. Heterogeneity across participants have a major impact in entry into meritocracy when entry is costly. Even if no mistake is made in choosing the best entrant, relative to the actual population, we may make mistakes as people with the same ability may enter the playing field with different likelihoods. Considering endogeneity in entry, hence, is of supreme importance in meritocratic contests.

Our model of meritocratic contest with endogenous entry is closely related to a number of different strands of literature. The large literature on contest theory focuses on players' effort choice under exogenous entry.[1] Our focus is more on entry decisions in highly competitive and high-stake contests where a contestant cannot win unless she exerts a high level of effort. Any lackluster performance would be detected and can be considered a non-entry. We believe that it represents a wide variety of settings that have been overlooked in the contest literature. Our game may be more relevant to the large experimental literature on entry games.[2] In most of these papers, ex ante heterogeneity across players is not thoroughly explored. The entry game experiments in Camerer and Lovallo (1999) are isomorphic to our model. They found excess entry relative to a theoretical benchmark and attributed this mainly to subjects' overconfidence regarding their winning probability. Our model provides conditions when such incorrect perceptions can lead to a large amount of excess entry.

Our work contributes to the significant literature on affirmative action (see Holzer and Neumark, 2000, for a survey of the literature) by showing that such policies can be welfare improving when it leads to greater participation by the disadvantaged population. There is also a large literature on affirmative action in asymmetric contests with exogenous entry. Chowdhury, Esteve-Gonzalez, and Mukherjee (2020) provides a comprehensive survey of the literature. However, that literature focuses on effort provision while our focus is on increasing the representation of disadvantaged players, which is somewhat closer to the common usage of the term. There is a recent, but growing, literature that theoretically consider representation and affirmative action in a similar way as we do— see Fershtman and Pavan (2021) and Siniscalchi and Veronesi (2021), for example. However, entry is exogenous in those models.

The remainder of the paper proceeds as follows: Section 2 introduces the model in

[1] See Corchón (2007), Konrad (2009), and Corchón and Serena (2018) for detailed discussions of the literature.

[2] See Kahneman (1988), Rapoport (1995), Rapoport et al. (1998), and Duffy and Ochs (2012), for example.

a general format and presents the equilibrium. In Section 3, we focus on applications of the model. We illustrate the model predictions with 2-player contests to keep the expositions simple and derive insights regarding how this model fits into different applications. In Section 4, we derive welfare implications of interventions such as affirmative action for a disadvantaged player. Section 5 concludes the paper.

2 Model Setup

We study meritocratic contests with endogenous entry. A player's sole strategic decision is whether to enter the contest, which requires paying an entry cost. Potential entrants may differ in ability, distribution of ability, entry cost, or prize valuation. Ability is private information to each player and the other characteristics are commonly known. Among those who choose to enter, ability alone determines success. While we refer to a player's type as ability, it should be understood to be the combination of natural aptitude and cumulative effort undertaken prior to the start of the contest. Importantly, we assume that contemporaneous effort during the contest itself does not influence performance.

Formally, there are N risk-neutral players, indexed by $i \in \{1, 2, ..., N\}$, who simultaneously and independently decide whether to enter the contest. Player i privately learns her ability α_i, drawn independently from an atomless distribution F_i with continuous density function f_i on the common support $[\underline{\alpha}, \bar{\alpha}]$. Player i's entry cost is $c_i \in [\underline{c}, \overline{c}]$ where $\underline{c} > 0$. If there is only one entrant, she wins by default. If there are multiple entrants, the entrant with the highest ability wins.[3] The benefit of winning the contest is a reward $V_i \in \left[\underline{V}, \overline{V}\right]$, where $\underline{V} > \overline{c}$. An entrant who loses the contest, without loss of generality, receives a reward of zero. The payoff from not entering the contest is 0. Save for the privately-observed realized ability α_i, all other aspects of the game are common knowledge. The players are *generically* asymmetric or ex ante non-identical. We analyze the Bayesian Nash equilibria of this game. The equilibrium will describe each player's entry strategy. As a tie-breaking rule, we assume that when a player is indifferent between entering and not entering, she does not enter.[4]

[3] If multiple entrants have the highest ability, which happens with zero probability, one of them is randomly chosen.

[4] This tie-breaking rule is inspired by unprofitable games argument in Harsanyi (1966). Uniqueness of equilibria in Theorem 1 depends on this rule only in the non-generic case.

2.1 Equilibrium Entry Decisions

In the game described above, generically, the players will be non-identical; i.e., they will systematically differ in terms of their reward values, distributions of ability, or entry costs. This game has a unique Bayesian Nash Equilibrium, which can be described by a set of cutoff values α_i^* such that player i enters if and only if $\alpha_i > \alpha_i^*$. Deriving the relevant cutoffs require notations reflecting an iterative process.

Equilibrium Algorithm: We first find the player who has the highest cutoff value of ability to render her indifferent between entering and not entering when all other players enter for sure. We denote this player as player N and the cutoff ability level as α_N^*. Then we find player $k \in \{2, 3, ..., N-1\}$ iteratively by finding the player who has the highest cutoff value to be indifferent between entering and not entering when player $j > k$ enter using cutoff α_j^* and the other players enter for sure. We denote this player as player k and the associated cutoff value as α_k^*. That is, this algorithm's $N-k+1$st iteration yields $a_k^{(i)}$, where $a_k^{(i)}$ solves for $i \leq k$

$$\prod\nolimits_{l=k+1}^{N} F_l\left(\alpha_l^*\right) \prod\nolimits_{j \in \{1,2,...,k-1,k\} \backslash i} F_j\left(a_k^{(i)}\right) = \frac{c_i}{V_i}. \tag{1}$$

By construction, $\underline{\alpha} < \alpha_2^* \leq \alpha_3^* \leq ... \leq \alpha_{N-1}^* \leq \alpha_N^*$ and $a_2^{(1)} \leq a_2^{(2)}$.

While we allow the players to vary in many different ways, the nature of the equilibria depends on only a simple set of statistics. Specifically, the equilibrium looks very different depending on whether the indifference ability levels of the two players in the last (i.e., $(N-1)$st) iteration of the algorithm, $a_2^{(1)}$ and $a_2^{(2)}$, are equal. Generically, the cutoffs $a_2^{(1)}$ and $a_2^{(2)}$ generated by the "equilibrium algorithm" and characterized by equation (1) are distinct. In that case, player 1 enters the contest for any value of α_1, including $\underline{\alpha}$. Other players enter if and only if their ability is above a cutoff strictly greater than $\underline{\alpha}$. On the other hand, in the special case of $a_2^{(1)} = a_2^{(2)}$, *all players* enter using a cutoffs that are strictly greater than $\underline{\alpha}$. Theorem 1 characterizes this equilibrium.

Theorem 1 *Characterize α_i^* by equation (1) for $i \geq 2$. There is a unique Bayesian Nash equilibrium of this game. Player $i \geq 2$ enters if and only if $\alpha_i > \alpha_2^*$. If $a_2^{(1)} < a_2^{(2)}$, then player 1 enters for all $\alpha_1 \in [\underline{\alpha}, \overline{\alpha}]$ and if $a_2^{(1)} = a_2^{(2)}$ then player 1 enters if and only if $\alpha_1 > \alpha_2^*$.*

Proof. We derive each player's entry cutoff using an iterative process. No matter what other players' entry strategies are, player i's expected payoff from entry is at least $V_i \prod_{j \neq i} F_j(\alpha_i) - c_i$. This is strictly positive if $\alpha_i > a_N^{(i)}$ where $a_N^{(i)}$ can be characterized by $\prod_{j \neq i} F_j\left(a_N^{(i)}\right) = \frac{c_i}{V_i}$. Denote the player with the maximal value of $a_N^{(i)}$ as player N, with the corresponding cutoff $\alpha_N^* = a_N^{(N)}$. If multiple players have the maximal value of $a_N^{(i)}$, we denote one of them as player N randomly. As any other player j will not enter with probability of $F_j(\alpha_N^*)$ or lower, player N gets strictly positive expected payoff from entry only if $\alpha_N > \alpha_N^*$. In any equilibrium, player N enters if and only if $\alpha_N > \alpha_N^*$. Following this general method, we propose an algorithm whose $N - k + 1$st iteration yields $a_k^{(i)}$, where $a_k^{(i)}$ solves for $i \leq k$

$$\prod_{l=k+1}^{N} F_l(\alpha_l^*) \prod_{j \in \{1,2,\ldots,k-1,k\} \setminus i} F_j\left(a_k^{(i)}\right) = \frac{c_i}{V_i}.$$

We run this algorithm iteratively starting at $k = N$ and move in a decreasing order for all $k \in \{2, 3, ..., N-1, N\}$ to find $a_k^{(k)}$. Denote the player who maximizes $a_k^{(i)}$ as player k and also let $\alpha_k^* = a_k^{(k)}$. Our construction implies that $\alpha_2^* \leq \alpha_3^* \leq ... \leq \alpha_{N-1}^* \leq \alpha_N^*$. Given that, player k receives positive payoff from entering if $\alpha_k > \alpha_k^*$ and receives weakly negative utility from entering if $\alpha_k \leq \alpha_k^*$. Thus, in any equilibrium, player $k \geq 2$ enters the contest if and only if $\alpha_k > \alpha_k^*$. Repeated elimination of strictly dominated strategies leads to the remaining player, player 1, getting an expected payoff of $V_1 \prod_{j=2}^{k} F_j\left(\max\left\{\alpha_1, \alpha_j^*\right\}\right) - c_1$ in any equilibrium.

When $a_2^{(1)} < a_2^{(2)}$, the above expected payoff is strictly positive for any $\alpha_1 \in [\underline{\alpha}, \overline{\alpha}]$. In the unique Bayesian Nash equilibrium, player 1 will enter for any α_1 and player $i > 1$ will enter if and only if $\alpha_i > \alpha_i^*$, as defined above.

When $a_2^{(1)} = a_2^{(2)}$, player 1's expected payoff is strictly positive for any $\alpha_1 > \alpha_2^*$, zero for $\alpha_1 = \alpha_2^*$, and strictly negative for $\alpha_1 < \alpha_2^*$. In the unique Bayesian Nash equilibrium, player $i \in \{1, 2\}$ will enter if and only if $\alpha_i > \alpha_2^*$ and player $i > 2$ will enter if and only if $\alpha_i > \alpha_i^*$. ■

This theorem suggests that while we allow player characteristics to vary quite generally, a simple set of statistics that incorporates all those characteristics is sufficient to characterize player behavior. More importantly, the nature of the equilibria depends only on the relation of the statistics for the two "strongest" players. The case where these two players are equally strong, which happens when $F_2^{-1}\left(\frac{c_1}{V_1 \prod_{l=3}^{N} F_l(\alpha_l^*)}\right) = F_1^{-1}\left(\frac{c_2}{V_2 \prod_{l=3}^{N} F_l(\alpha_l^*)}\right)$, is not the limiting case of when the two players are "slightly different." Thus, assuming players are ex ante identical and focusing on a symmetric equilibrium is not an innocuous simplification even when differences across players are small. To illustrate the stark change in equilibrium characteristic between the generic and non-generic cases, we present the equilibrium with two players.

Corollary 1 *Suppose $N = 2$ and $\alpha_i^* = a_2^{(i)} = F_j^{-1}\left(\frac{c_i}{V_i}\right)$. There is a unique Bayesian Nash equilibrium. If $\alpha_1^* < \alpha_2^*$, then player 1 enters for all $\alpha_1 \in [\underline{\alpha}, \overline{\alpha}]$ and player 2 enters using the cutoff α_2^*. If $\alpha_1^* = \alpha_2^*$, then both players enter using the same cutoff α_2^*.*

Given costly entry, a player only enters when she believes that she has a sufficiently large chance of winning the prize. As the probability of winning depends on their ability, contestants will have a *threshold* ability above which they will enter in equilibrium. When contestant characteristics are identical, so too are the equilibrium thresholds. A contestant of marginal ability is just indifferent between entering or not, given the entry propensity of her rival. This marginal type only wins by default—when her rival enters, she loses with probability one. Additionally, a player with ability below the marginal type will also be indifferent between entering and not entering when the other player does not enter for abilities below the marginal type.

When contestants differ in terms of a publicly known characteristic, however, these weak best responses become strict for the more advantaged player. To see this, suppose the two players have the same ability distribution F and reward value V. However, their entry costs are different, with $c_1 = c_2 - \varepsilon$ for some $\varepsilon > 0$. For any entry strategy of the other player, player i's expected payoff from entry is at least $VF(\alpha_i) - c_i$. Thus, in any equilibrium, player i enters if $\alpha_i > F^{-1}\left(\frac{c_i}{V}\right)$. Hence, player 2's expected payoff from entry is at most $VF\left(\max\left\{\alpha_2, F^{-1}\left(\frac{c_1}{V}\right)\right\}\right) - c_2$. Given that $\frac{c_1}{V} < \frac{c_2}{V}$, this is strictly negative if $\alpha_2 < F^{-1}\left(\frac{c_2}{V}\right)$. Player 2 will not enter at such ability levels. As a consequence, player 1's expected payoff from entry is at least $VF\left(F^{-1}\left(\frac{c_2}{V}\right)\right) - c_1$ for any α_1. This is strictly positive no matter how small ε is. In the unique equilibrium, player 1 enters *regardless of her ability* and player 2 enters if and only if $\alpha_2 > F^{-1}\left(\frac{c_2}{V}\right)$.[5]

[5] The equilibrium with non-identical players is robust to other tie-breaking rules and different refinements. Additional equilibria arise without the tie-breaking assumption if players are ex ante identical. Nonetheless, only the symmetric equilibrium survives when we restrict attention to cautiously rationalizable strategies proposed by Pearce (1984).

3 Applications

The economic intuitions of the model are most easily seen when two players compete for the reward, as discussed in Corollary 1. In the remainder of the paper, we focus solely on contests with two players and illustrate how differences in player characteristics along different dimensions capture different real-life scenarios. We also discuss some welfare implications of endogenous entry in meritocratic contests. The results can be extended to contests with more than two players.

3.1 Non-identical Payoffs

Suppose that players have identical ability distributions, i.e. $F_1 = F_2 = F$, but possibly different prize valuations and entry costs. The player who has the lower entry cost relative to her reward value will be advantaged. When $\frac{c_1}{V_1} < \frac{c_2}{V_2}$, player 1 will be advantaged and enter for sure no matter how close the two ratios are. However, if the two ratios are equal, player 1's entry probability will drop to $1 - \frac{c_1}{V_1}$, illustrating the discontinuity that is central to all our findings. Although we have assumed that all players are risk-neutral, similar discontinuity would also occur if one player is more risk-averse or more loss-averse than the other.

This model can help explain the persistence of gender and racial disparities in upper management.[6] If women must sacrifice more to compete for promotion to managerial positions, derive less benefit from a promotion (perhaps due to male-female wage gaps), or are more risk-averse than their male counterparts, then they will compete for promotions at wildly lower rates than men. Consequently, women will be vastly underrepresented among the population of managers. Similarly, if it is more costly for racial minorities to apply or prepare or train for a position, they will be under represented. In a political campaign, if a candidate believes that her past is more likely to be scrutinized than her equally talented competitor's, she may be a lot more likely to not run and stay home. Even when the direct effect of smaller rewards or larger costs is mild, our model highlights an indirect effect that is not mild and can disproportionately harm disadvantaged people. This also harms firm performance: asymmetric entry implies that a less competent male may be promoted over a more competent female.

[6] Women held 14.2% of the top-5 leadership positions in the S&P 500 companies in 2015 (Egan, 2015) and 4.2% of CEO positions in the companies on the 2016 Fortune 500 list (Zarya, 2016). There were only four African-American CEOs in the US Fortune 500 companies in June 2020 (Yurkevich, 2020).

Asymmetry in the expected or perceived payoffs from entry across players may also result from a lack of knowledge about the contest among some players. Suppose the two players are ex ante identical in their characteristics, but player 2 may not be aware of the existence of the contest with a positive probability. If player 1 knows of this potential unawareness by player 2, she will be the advantaged one. In New York City, black and Hispanic middle-schoolers are much less likely to be aware of the city's specialized high schools and how the entrance exam in these schools work (Shapiro, 2019b). Consequently, black and Hispanic students make up slightly more than 10% of the student body in the city's eight specialized high schools, while they make up 70% of the City's public school students as a whole (Shapiro, 2019a). Hoxby and Turner (2015) find low-income students to be less informed about various aspects of college admission and college education. Such discrepancy in knowledge may provide another explanation for the finding by Hoxby and Avery (2013) that low-income students with strong academic preparations diverge from similar high-income students significantly at the college application stage.

In a famous study, Niederle and Vesterlund (2007) find men to be twice as likely as women to enter a tournament where the task was adding numbers. They ascribe this difference as "competition aversion," women simply dislike having to compete. Such competition aversion has been documented in many subsequent studies in different settings (Niederle and Vesterlund, 2010). If competition aversion is modeled as an additional cost for entering into a tournament, large differences in entry probabilities may result from miniscule competition aversion, consistent with empirical findings. Moreover, even if the competition-averse player suffers an additional cost or negative utility only in the case that the other player enters, the model will lead to exactly the same equilibrium.

In our model, ability of the winner can be any value in $[\underline{\alpha}, \overline{\alpha}]$ when it is player 1, but can be only above $F^{-1}\left(\frac{c_2}{V_2}\right)$ if it is player 2. Hence, the winner's expected ability is higher if the winner is player 2. This implies that managers from a disadvantaged group will be more capable than their advantaged counterparts, on average. There is some evidence that this is indeed the case.[7] Another implication is that if $\frac{c_1}{V1}$ is increased in a way that it approaches $\frac{c_2}{V_2}$ from below, the entry probabilities of either player does not change at all in the converging sequence. However, if $\frac{c_2}{V_2}$ is reduced to approach $\frac{c_1}{V_1}$ from above, the entry probability of player 2 increases while the entry

[7] See Martinsen and Glasø (2013) for a survey of personality traits of managers in Norway. A study of American managers by Gallup (2015) also has similar findings.

probability of player 1 remains at 1. This leads to the following corollary of our main result.

Corollary 2 *When the two players differ only in terms of the reward value or the entry cost, equilibrium entry probabilities of either player will not change if we decrease the advantaged player's reward value or increase her entry cost as long as she stays advantaged. However, the disadvantaged player's entry probability will increase without changing the advantaged player's entry probability if we increase the disadvantaged player's reward value or decrease her entry cost.*

3.2 Non-identical Ability Distributions

Consider the case where the two contestants have the same reward V and entry cost c, but their abilities are drawn from differing distributions F_1 and F_2. Player 1 being advantaged implies that F_1 reaches the $\frac{c}{V}$-th quantile at a greater ability level. That is, $F_2^{-1}\left(\frac{c}{V}\right) < F_1^{-1}\left(\frac{c}{V}\right)$. Thus, this relationship completely determines the advantaged player and player 1 enters for all ability levels no matter how small the difference is. The equilibrium probability of entry for the disadvantaged player, $F_2\left(F_1^{-1}\left(\frac{c}{V}\right)\right)$, depends on both distributions.

A natural ordering for comparing players in terms of ability is first-order stochastic dominance. It seems intuitive that the player whose ability is, on average, higher should be more willing to enter, and this proves to be the case. If F_1 first order stochastically dominates F_2 then player 1 enters with probability 1. Ordering of the players is more nuanced for second order stochastic dominance.

Suppose F_i and F_j are symmetric distributions and F_j is a mean-preserving spread of F_i, while intersecting each other only at the median. While neither player is more able than the other on average, advantage depends on the effect of dispersion. When players differ according to a dispersion ordering, the key to advantage is which tail determines relative strength in terms of the ability distributions. When the entry cost is relatively low ($\frac{c}{V} < \frac{1}{2}$), a player enters with probability greater than a half if the other player enters for sure. As a result, smaller dispersion is an advantage by being less likely to produce players of low ability. As a result, the player with the lower variance (player i) enters for sure. By contrast, when entry is relatively costly, the opposite holds. A distribution with larger dispersion is more likely to produce players of high ability. When $\frac{c}{V} > \frac{1}{2}$, then player j, who has the higher variance or heavier tails, is the advantaged player.

3.3 Sociocultural Perception

Suppose that the two players are inherently ex ante identical, but asymmetry arises between them due to misperception regarding ability. The misperception can arise at individual level where a player has incorrect self-perception of her own ability, but socially there is no misperception about players' common ex ante ability distribution. There can also be sociocultural misperception regarding the players' ex ante ability, instead. We show that the equilibrium of such a game depends critically on whether misperception is at the private level or at the social level, shared by both players.

Recall that when players are ex ante identical and this is common knowledge, the two players will enter using the cutoff $F^{-1}\left(\frac{c}{V}\right)$ in the unique symmetric equilibrium. First, consider the case where player 1 is privately overconfident. She incorrectly perceives her own ability to be $\alpha_1 + \varepsilon$ for some $\varepsilon > 0$. Suppose player 2 perceives α_2 correctly and both players correctly believe that the other player's ability is drawn from distribution F. In the equilibrium, players 1 and 2 will enter using the cutoffs $F^{-1}\left(\frac{c}{V}\right) - \varepsilon$ and $F^{-1}\left(\frac{c}{V}\right)$, respectively. While the entry probability will be different, the difference will be proportional to the level of overconfidence and will vanish as ε approaches zero. Similar results will happen if player 2 is privately underconfident.

Equilibrium outcome will, however, be different when the misperception is at the social level. While both α_1 and α_2 are drawn from the same distribution F, suppose it is commonly believed that α_1 is drawn from F_1 which first order stochastically dominates F. Then, player 2 will enter using the cutoff $F_1^{-1}\left(\frac{c}{V}\right)$, which is greater than $F^{-1}\left(\frac{c}{V}\right)$. As a result, player 1 will enter with probability 1 and player 2 will enter with a probability smaller than $1 - \frac{c}{V}$. As F_1 approaches F (while maintaining stochastic dominance), player 2's entry probability will approach $1 - \frac{c}{V}$, but player 1's entry probability will remain at 1. We will observe large differences in entry probability even for a very small level of social misperception. Connecting this to our analysis in Section 3.2, advantage to one player may also arise when only the variance, but not the mean, of a player's ability distribution is misperceived.

Lawless and Fox (2005) find that women are underconfident regarding their likelihood of winning is they ran from a political office. Our analysis suggests that such underconfidence would lead to a disproportionately lower level of female candidates running for political offices when such misperception is commonly shared by the society. If women are (incorrectly) stereotyped to be weaker in mathematics and sciences, they would participate in math based contests with a much lower probability even when the magnitude of the stereotype is very small. This also suggests that competition aversion may arise from stereotyping instead of an explicit cost of competing with another player.

4 Leveling the Playing Field

In this section, we investigate economic welfare of the contest designer. We illustrate two ways of improving welfare—by handicapping the advantaged player and by subsidizing the disadvantaged player using a surcharge from the advantaged player. Both of these schemes increase the likelihood of the disadvantaged entering without reducing the probability of entry by the advantaged player. Nonetheless, conditional on winning the contest, the average ability of the disadvantaged player would still be higher. As these schemes can be considered to be affirmative action policies, this is consistent with the finding by Holzer and Neumark (2000) that affirmative action hires typically do not exhibit weaker job performance.

4.1 Handicapping the Advantaged Player

For simplicity, let us assume that the two players have identical payoffs, but their ability distributions are different. Suppose that the designer is risk-neutral and gets a net payoff of α from hiring a player with ability α and 0 from not hiring anyone. This is consistent with our implicit assumption that the designer receives a (weakly) positive net payoff from hiring even the lowest-ability player.[8]

For $i, j \in \{1, 2\}$ and $j \neq i$ denote $F_j^{-1}\left(\frac{c}{V}\right) = \alpha_i^*$ and suppose $\alpha_1^* = \alpha_2^* - \eta$ for some $\eta > 0$, making player 1 advantaged under meritocracy. Now consider the γ-handicap rule where, when both players enter, player 1 wins if $\alpha_1 > \alpha_2 + \gamma$ for some $\gamma \in [0, \eta)$ and player 2 wins otherwise. Here $\gamma = 0$ representing meritocracy. Proposition 1 shows that the optimal handicapping level is strictly positive.

Proposition 1 *The contest designer's welfare can be improved over meritocracy by a γ-handicap rule for some $\gamma > 0$.*

Proof. As $\gamma < \eta$, player 1 is still advantaged under the γ-handicap rule. Player 1 enters for all α_1 and player 2 enters if and only if $\alpha_2 > \alpha_2^* - \gamma$ in the unique

[8] None of our results change if the net payoff for the contest designer from hiring a worker with ability α equals $W(\alpha)$ where $W(\alpha)$ is a strictly increasing, continuous, and differentiable function of $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, with $W(\underline{\alpha}) \geq 0$.

equilibrium. With a handicap of $\gamma \in [0, \eta)$, the contest designer's expected payoff is

$$\tilde{EU}(\gamma) = F_2(\alpha_2^* - \gamma) \int_{\underline{\alpha}}^{\bar{\alpha}} z dF_1(z) + \int_{\alpha_2^*-\gamma}^{\bar{\alpha}} \left(yF_1(y+\gamma) + \int_{y+\gamma}^{\bar{\alpha}} z dF_1(z) \right) dF_2(y).$$

The marginal change to the contest designer's payoff as γ changes,

$$\tilde{EU}'(\gamma) = f_2(\alpha_2^* - \gamma) \int_{\underline{\alpha}}^{\alpha_2^*} (\alpha_2^* - z) dF_1(z) - \gamma \left(f_2(\alpha_2^* - \gamma) F_1(\alpha_2^*) + \int_{\alpha_2^*-\gamma}^{\bar{\alpha}} f_1(y+\gamma) dF_2(y) \right).$$

Hence,

$$\lim_{\gamma \to 0} \tilde{EU}'(\gamma) = f_2(\alpha_2^*) \int_{\underline{\alpha}}^{\alpha_2^*} (\alpha_2^* - z) dF_1(z) > 0.$$

By continuity, the contest designer's payoff strictly increases for a positive measure of γ and the optimal handicap value is γ^* that solves

$$\gamma^* = \frac{f_2(\alpha_2^* - \gamma^*) \int_{\underline{\alpha}}^{\alpha_2^*} (\alpha_2^* - z) dF_1(z)}{f_2(\alpha_2^* - \gamma^*) F_1(\alpha^*) + \int_{\alpha_2^*-\gamma^*}^{\bar{\alpha}} f_1(y+\gamma^*) dF_2(y)}.$$

■

The discrepancy in the two players entry probability can result in selecting a player 1 of lower ability than player 2 when she does not enter. A policy that induces marginal player 2 types to enter more frequently can partially rectify this situation. By handicapping the advantaged player without completely eliminating her advantage, i.e. reducing her effective ability by γ, player 2 enters for more types on the margin, i.e. those within γ of the cutoff α^*. This produces two effects. In circumstances where player 2 is marginal, she now displaces low ability player 1 types from winning, a clear gain. By handicapping player 1, however, the "wrong" player is sometimes selected if the two entrants are close. The first effect represents a first order gain, lower ability player 1 types (i.e. $E[\alpha_1 | \alpha_1 < \alpha^*]$) are replaced by marginal player 2 types. The negative effect, which involves a small loss from choosing the wrong player, is a second order effect. Hence, some degree of handicapping is always optimal.[9]

Handicapping a player improves a designer's expected playoff in many mechanism design situations with asymmetric players. However, in those cases, the strategies of the players are typically strategic complements. In our game, on the other hand,

[9] Here we analyze how handicapping the advantaged player can improve the contest designer's welfare when the players differ in terms of ability distributions. Handicapping the advantaged player can also be useful even when the players differ in terms of payoffs or preferences.

entry decisions are not so. For example, the optimal probability of entry for player i is (weakly) decreasing in her belief about the entry probability of player j. This also illustrates the importance of considering entry decisions in contest designing. Meyer (1991) showed that in a multi-round contest to determine the worker with higher ability when ability is not perfectly observed, the leader in the earlier rounds should be favored. Our results suggest that optimality of such reinforcement of advantage may be reversed when participation is costly.

4.2 Surcharges and Subsidies

We consider another intervention where the contest is won by the entrant with the higher ability, but there is a transfer between the two players facilitated by the contest designer. Specifically, conditional on winning, the advantaged player is charged a surcharge and the disadvantaged player is provided with a subsidy. Suppose the two players vary in terms of rewards and entry costs, but their abilities are drawn from the same distribution F. Player 1 is advantaged as $\frac{c_1}{V_1} < \frac{c_2}{V_2}$. Now suppose that, conditional on winning, player 1 is charged a surcharge of d which the contest designer receives. This virtually reduces player 1's reward upon winning to $V_1 - d$. On the other hand, if player 2 wins, she is provided with a subsidy s by the contest designer, which virtually increases her reward upon winning to $V_2 + s$. We assume that s and d are such that $\frac{c_1}{V_1 - d} < \frac{c_2}{V_2 + s}$. This transfer scheme is *revenue neutral* if the expected sum of subsidy and surcharge is zero. The entry probability of the disadvantaged player will increase by $\frac{c_2}{V_2} - \frac{c_2}{V_2 + s}$. Additionally, this scheme also increases the contest designer's payoff as the player with the greater ability is chosen with a higher likelihood.

Proposition 2 *The contest designer can increase player 2's entry probability from* $1 - \frac{c_2}{V_2}$ *to* $1 - \frac{c_2}{V_2 + s}$ *while keeping player 1's entry probability at 1 by choosing surcharge of* d *for player 1 and a subsidy of* s *for player 2 such that* $\frac{c_1}{V_1 - d} < \frac{c_2}{V_2 + s}$. *This scheme is revenue neutral if* $d = \frac{(V_2+s)^2 - c_2^2}{(V_2+s)^2 + c_2^2} s$.

Proof. Since $\frac{c_1}{V_1 - d} < \frac{c_2}{V_2 + s}$, player 1 remains advantaged. Hence, player 1 enters for all α_1 and player 2 enters if $\alpha_2 > F^{-1}\left(\frac{c_2}{V_2 + s}\right)$ in equilibrium. Equilibrium entry probability for player 2 is $1 - \frac{c_2}{V_2 + s}$, which is increasing in s and independent of d.

Therefore, the expected amount of transfer is

$$\frac{c_2}{V_2+s}d+\left(1-\frac{c_2}{V_2+s}\right)\left(\frac{c_2}{V_2+s}(-s)+\left(1-\frac{c_2}{V_2+s}\right)\left(\frac{d}{2}+\frac{-s}{2}\right)\right)$$
$$= \frac{1}{2}\left(\left(1+\left(\frac{c_2}{V_2+s}\right)^2\right)d-\left(1-\left(\frac{c_2}{V_2+s}\right)^2\right)s\right)$$

Revenue-neutrality would require

$$d=\frac{(V_2+s)^2-c_2^2}{(V_2+s)^2+c_2^2}s.$$

■

As an example, suppose $V_1 = 5, V_2 = 3$, and $c_1 = c_2 = 1$. If $s = 1$ then the scheme is revenue-neutral if $d = \frac{15}{17}$ and player 1 remains the advantaged player. Entry probability of player 2 increases from 66.67% to 75% due to this scheme. This scheme utilizes the asymmetry mentioned in Corollary 2. A small surcharge on player 1 does not reduce her entry probability. On the other hand, a small subsidy to player 2 increases her entry probability, without affecting player 1's entry probability. Thus, the contest designer can design a revenue-neutral transfer scheme to increase the entry probability of player 2. In education, one may think of the reward as the return from earning a university degree. When the tuition fee is increased, that reduces the net return from the degree. Many private universities subsidize education of financially-challenged under-represented admits while charging a very high tuition to more financially solvent students. We show that this indeed can increase application by disadvantaged students. Such schemes can be also designed by using entry fees. It can also include a surcharge in one dimension but a subsidy in the other for the advantaged player. Thus, some subsidy can be provided to all students, not only to the disadvantaged students by appropriately choosing the surcharge for advantaged students. One example can be improving facilities for all students using surcharges from advantaged students.

5 Conclusion

While theoretical models of contests mainly focus on effort provision during the contest, many real life situations can be viewed as contests whose outcome depend more on cumulated past efforts and natural talent rather than contemporaneous effort.

Moreover, participation in many such contests is costly and not everyone who consider joining may ultimately participate. A small difference among players may lead to large discrepancies in entry probabilities in such contests. Theoretically, this implies that symmetry may not always be a harmless assumption when contestants are almost identical. This may explain the lack of diversity even in a perfect meritocracy well. While considering representation and diversity, one needs to seriously consider that some players may not even come to the playing field. The benefit of affirmative action policies become more obvious when we take into account that these interventions can expand the pool of participants in an economic situation, thus improving efficiency and overall welfare. This paper illustrates the benefits of leveling the playing field from a societal point of view.

Our model is very simple, which makes it easier to illustrate the main insights. The simplicity of the mechanism allows us to comprehensively consider heterogeneity across players. Moreover, the model is robust to various extensions. For example, the reader may worry that all our main insights hold only for a contest with one reward where all players are ex ante non-identical. However, suppose there are two ex ante non-identical types of players with multiple players of each type. Then all players of the advantaged type will enter for sure if the number of rewards is at least as large as the number of players from that type. When all players are of different types as in our model, but there are multiple rewards, then there will still be some players who enter for any ability level and other players will enter using a cutoff strategy. Another extension can be the case where ability is observed by the contest designer with some noise. Our main result of extreme asymmetry in entry probability will hold when the noise is small relative to the differences between the two players.

One implication of our analysis is that affirmative action policies that do not take potential applicants' decision to enter the playing field in the first place may not be very successful. A policy like the Rooney Rule in the NFL, may not increase representation if it does not decrease a disadvantaged candidate's entry costs relative to potential rewards. Such a rule may also come too late to create a large enough pool of disadvantaged candidates. Similarly, to increase diversity among assistant professors at the university level, one may need to increase diversity in the applicant pool at the undergraduate or high school (for specialized high schools) level. The main lesson from this paper is that we need to focus on potential applicants rather than only actual applicants when we want the workforce to be more inclusive and equitable.

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