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Monopoly Persistence under the Threat of Supply Function Competition

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Abstract. Can a monopoly persist by expanding its operation to a new market after strategically bidding for an exclusive license under the threat of supply function competition with a potential entrant? The answer may be yes or no depending on how the monopolist's existing product and the new product are related. The monopolist can win the bidding for the new market and thus expand its operation if the marginal cost (to produce a unit output) is sufficiently low with respect to the degree of product differentiation, while its likelihood of winning is higher if the two products are substitutes than if they are complements.

Keywords: Monopoly persistence; supply function competition; strategic bidding.

JEL Codes: D42; D43; L13.

1 Introduction

In this paper, we aim to answer whether a monopoly can persist by expanding its operation to a new market after strategically bidding for an exclusive license under the threat of supply function competition with a potential entrant. To this aim, we consider a model involving two markets: an existing market where an established firm (the incumbent) has a monopoly position to produce the existing good and a new market where the incumbent and a potential entrant bid for the exclusive license –offered by an outside inventor/licensor– to produce the new good. The existing and new goods are differentiated products (substitutes or complements) that are distinguished with respect to differences in design (leading to no difference in costs). Also, they are produced under uncertain demand of a continuum of identical consumers. Moreover, the incumbent is technologically constrained so that whenever it dominates both markets, it has to produce the two goods simultaneously and in the same plant. We assume that the identity of the producer in the new market as well as the outputs in the established and new markets are determined in the equilibrium of a two-stage strategic game played by the incumbent and the potential entrant. In the first stage of this game, the firms bid for the license of the new good and in the second stage the production of the existing and new goods takes place. The bidding in the first stage is in the form of a first-price auction under complete information where the revenues accrue to the auctioneer (the outside inventor/licensor). After the bidding is over, the firm with the highest bid wins; it pays its bid and obtains the license. If the incumbent wins the bidding in the first stage, then it acts, in the second stage, as a monopolist in both existing and new markets. On the other hand, if the potential entrant wins the bidding in the first stage, then in the second stage the incumbent and the entrant produce the existing and new goods respectively by engaging in supply function competition with each other.

After theoretically calculating the solution of the described two-stage game using the notion of subgame perfect Nash equilibrium introduced by Selten (1965), we conduct several numerical computations which show that in equilibrium the existing firm can win the bidding for the new market and thus expand its operation if the marginal cost (to produce a unit output) is sufficiently low with respect to the degree of product differentiation, while its likelihood of winning is higher if the two products are substitutes than if they are complements. Moreover, the expected price can be lower, the expected industry profits can be higher, and the welfare distribution between consumers and producers can become more equitable in the former case than in the latter if the marginal cost of unit output is very close to zero and the two products are almost perfect complements. The reason underlying these findings is that in the extreme cases of cost and product complementarity, the monopolization of both markets allows the established firm to fully internalize the effects of product differentiation.

Our paper can be closely related to two distinct strands of literature: one on monopolistic expansion and the other on supply function competition in oligopolistic markets. Regarding the first literature, a pioneering work of Gilbert and Newbery (1982) showed that a monopolistic firm has, under certain conditions, an incentive to invent and patent (possibly without using) a new technology before potential competitors to maintain its monopoly power. However, this result may not hold if there are uncertainties in the invention process of the new technology, as shown by Reinganum (1983), or if there exist multiple new resources available to be acquired simultaneously or sequentially by the monopolist and its potential competitors, as extensively shown by Lewis (1983), Kamien and Zang (1990), and Krishna (1993). More recently, Chen (2000) studied the monopoly persistence when its existing product and a new product –for the license of which a monopoly and a potential entrant bid in a first-price auction– are related as substitutes or complements. There are four possible outcomes depending on whether the monopoly or the entrant wins the bidding and whether in each of these two cases the entrant stays out of, or enters, the existing market of the monopoly. Chen (2000) showed that if the cost of entry to the existing product market is not extremely high or low, then the entrant wins the bidding for the new product but does not enter the monopoly's existing market if the two products are strategic substitutes and the monopolist wins the bidding and monopolizes both product markets if the two products are strategic complements.

Our paper is different from Chen (2000) in two important aspects. While he allows potential entrance to the monopoly's existing product market, we do not. We assume that the monopoly power in the existing market is independent of the bidding strategies and fixed throughout the analysis. Second, Chen (2000) assumes that the potential entrant, if it wins the bidding for the new product market, competes with the monopoly in quantities á la Cournot (1838), whereas in our model the duopolists compete in supply functions in such a case. The differences in our models also yield differences in our results. Chen (2000) shows that when entry cost to the existing market of the monopoly is sufficiently high, the monopoly always wins the bidding for the new market independent of the degree of product substitution (complementarity) while the case where the entrant solely produces the new product arises if and only if the new and existing products are complements. However, in our case (where entry to the existing market of the monopoly is never allowed by assumption) the monopoly firm can dominate both markets only if the marginal cost of unit output is sufficiently low with respect to the degree of product differentiation, while the likelihood of this complete domination is higher if the two products are substitutes than if they are complements. Our theoretical results show that the main element of our model that distinguishes our results from Chen (2000) is the form of competition, supply function competition, which we restrict the established firm and the entrant to engage in, in case the bidding for the new product is won by the entrant. There is an extensive literature on this relatively new form of competition. Below, we will briefly address some of the works in this literature in order to position therein our contribution more precisely.

Supply function competition was first established by Grossman (1981) and extended for economic applications by Klemperer and Meyer (1989) who eliminated, using exogenous shocks to demand functions, the problems caused by the multiplicity of supply function equilibria. In the last three decades, supply function competition has been widely applied in wholesale electricity markets to model the strategic interaction among electricity generators (Green and Newbery, 1992; Rudkevich and Duckworth, 1998; Rudkevich et al., 1998; Green, 1999; Day et al., 2002; Newbery and Greve, 2017; Escrihuela-Villar et al. 2020) and also applied to model bidding behavior in many economic settings, such as government procurement contracts, treasury auctions, management consulting, strategic agency and trade policy, and airline pricing reservation systems (Vives, 2011). Given the increasing popularity of supply function competition, many works in the literature have studied, in depth, its equilibrium implications in comparison to price and, especially, quantity competitions.

Among these works, Klemperer and Meyer (1989) showed that when demand uncertainty is absent, oligopolists' profits under supply function competition are intermediate between what they would obtain under price and quantity competitions. Delbono and Lambertini (2016) showed that this result substantially changes with quadratic costs. Monden (2017) established that supply function competition may yield lower social welfare in a vertical market than quantity and price competitions. In contrast, Saglam (2018a) showed that in a non-differentiated oligopolistic industry with demand uncertainty, supply function competition always yields higher ex-ante welfare to consumers than quantity competition, while it can also yield higher ex-ante welfare to producers if the demand uncertainty is sufficiently large. Saglam (2018b) extended the work of Saglam (2018a) to a differentiated product duopoly relevant for electricity markets under the motivation provided earlier by Woo et al. (2014), arguing that many features of electricity –involving quality, consumption volume (kWh), maximum demand (kW), reliability, time of use, and environmental impact– allows it to be packaged as a differentiated product. Saglam (2018b) showed that under product differentiation the expected consumer welfare is larger under supply function competition than under quantity competition, and in addition, this is also true for the producer welfare if the products are complements, independent, or only poor substitutes. Saglam (2018b) also showed that even if the last condition does not hold, supply function competition can Pareto dominate quantity competition if the demand uncertainty is sufficiently large. While this work is partly related to ours regarding the use of product differentiation and supply function competition, there is an important difference between the two works: In Saglam (2018b) the industry structure is fixed: it is assumed to be a duopoly and there is no room for strategic issues that we study, such as the incentives for monopolistic expansion from the viewpoint of a monopolist or incentives for entrance to the industry from the viewpoint of a potential entrant.

Very recently, a number of papers also integrated technology licensing with supply function competition or collusion. For instance, Saglam (2021) deals with licensing cost-reducing innovations in a duopolistic industry under supply function competition and evaluates fixed-fee licensing, revenue-royalty licensing, and mixed licensing from the viewpoints of innovator and consumers under alternative forms of cost functions and royalty payments. Celen and Saglam (2022) study an infinite-horizon duopoly under asymmetric costs and find that licensing makes collusion in supply functions more unlikely; but it always increases the welfares of both consumers and the less efficient one of the duopolists. These works all assume that the licensor of the new product is an insider, one of the duopolistic firms, unlike in our paper where we assume that the licensor is an outsider (i.e., a research lab that has developed or innovated the product to be licensed but has no intention or means to produce it).

The remainder of our paper is organized as follows: Section 2 introduces the model. Section 3 characterizes the equilibrium of the strategic two-stage game between the monopolist and the potential entrant and also presents some comparative static results. These results are further extended by computational results in Section 4. Finally, Section 5 contains some concluding remarks.

2 Model

Two firms, named I (incumbent) and E (potential entrant), bid for the exclusive license to produce a good named Y. Firm I already has a monopoly power to produce a good named X, possibly due to unique access to distribution or factors of production of this good or due to an earlier patent invented or an exclusive license purchased. The goods X and Y are differentiated products each produced under an uncertain demand by a continuum of identical consumers. We call X and Y to denote these products as well as the markets where they are produced. As in Singh and Vives (1984), we assume that a representative consumer obtains the demands for X and Y by maximizing the surplus

$$U(q_X, q_Y) - p_X q_X - p_Y q_Y, \tag{1}$$

where q_X and q_Y denote the quantities of goods X and Y, while p_X and p_Y denote their prices. The utility function U is of the form

$$U(q_X, q_Y) = \alpha(q_X + q_Y) - \frac{1}{2}(q_X^2 + 2\delta q_X q_Y + q_Y^2), \tag{2}$$

where α denotes a scalar random variable which represents an ex-ante unobservable shock to consumers' utility and $\delta \in (-1, 1)$ denotes a parameter measuring how goods X and Y are related. We assume that the mean and the standard deviation of α are both positive and denoted by $E[\alpha] = \mu$ and σ respectively.

It should be clear that the solution to the consumer's maximization problem yields the inverse demands

$$P_X(q_X, q_Y) = \alpha - q_X - \delta\, q_Y \tag{3}$$

and

$$P_Y(q_X, q_Y) = \alpha - q_Y - \delta\, q_X. \tag{4}$$

These inverse demands show that goods X and Y are independent if $\delta = 0$, substitutes if $\delta > 0$, and complements if $\delta < 0$. Over the set of quantities where the prices p_X and p_Y are non-negative, the demand curves can be derived as follows:

$$D_X(p_X, p_Y) = a - b\, p_X + d\, p_Y, \tag{5}$$

and

$$D_Y(p_X, p_Y) = a - b\, p_Y + d\, p_X, \tag{6}$$

where $a = \alpha/(1+\delta)$, $b = 1/(1-\delta^2)$, and $d = \delta/(1-\delta^2)$.

Whenever any firm produces $q \geq 0$ units of any good, it faces the cost function

$$C(q) = \frac{c}{2} q^2, \tag{7}$$

where $c \geq 0$ is the marginal cost of a unit output. Here, we assume that the products X and Y are distinguished merely with respect to differences in design without leading to any cost differences. We also assume that the incumbent, whenever it can produce in both markets X and Y, is technologically constrained to produce the two goods simultaneously and in the same plant. The functional forms of the demand, inverse demand, and cost curves as well as the parameters δ, b, d, c, μ, and σ are common knowledge.

Given the structures described above, we assume that the identity of the producer in the new market as well as the outputs in the established and new markets are determined according to the equilibrium solution of a two-stage strategic game played by the incumbent and the potential entrant. In the first stage the firms bid for the license of good Y and in the second stage the production of goods X and Y takes place. If firm I wins the bidding in the first stage, then it has two choices in the second stage: It can either use its new license for Y to act as a monopolist in both market X and market Y and to obtain the profit $\pi_I^{M,XY}$ or disuse its new license to act as a monopolist only in market X and to obtain the profit $\pi_I^{M,X}$. On the other hand, if firm E wins in the first stage the bidding for market Y, then in the second stage firms I and E produce in markets X and Y by competing in supply functions and they obtain the profits $\pi_I^{SF,X}$ and $\pi_E^{SF,Y}$ respectively.

We finally assume that the bidding for market Y in the first stage of the game is in the form of a first-price auction under complete information where the revenues accrue to an outside auctioneer (the licensor/innovator of the new product). To eliminate an inessential multiplicity of equilibrium in the auction, we also assume that whenever the auctioneer becomes indifferent between any two alternatives, it always chooses the one that is more desirable for consumers. We let the bids of the incumbent and the entrant be denoted by b_I and b_E, respectively. After the bidding is over, the firm with the highest bid wins; it pays its bid and obtains the license. If the firms make the same bids, then the winner is announced to be the incumbent if and only if the total consumer surplus generated in markets X and Y is higher when good Y is produced by the incumbent than when it is produced by the entrant.

3 Characterization of Equilibrium

We will solve the game described in the previous section using the notion of subgame-perfect Nash equilibrium. Thus, a strategy profile for our game will involve a pair of bids in the first stage along with production plans represented by supply functions at each possible subgame in the second stage. Using the idea of subgame perfection, we will first analyze the game in the last stage.

3.1 Second Stage: Production

There are two possible subgames (and outcomes) in the second stage depending upon who becomes the winner of the first-stage auction. Below, we will calculate the profits and the consumer surplus for each possible subgame.

A. Incumbent Wins the Auction for Market Y.

Here, we consider the subgame (starting in the second stage) where the auction in the first stage was won by firm I. The problem of firm I is then to decide which of the two licences to use. It has three alternatives: (i) to use the licences for both goods X and Y and produce both of them, (ii) to use the license for good X only (and only produce it), or (iii) to use the license for good Y only (and only produce it). Of these, the last two alternatives are essentially non-distinguishable from each other since goods X and Y are symmetrically differentiated (i.e., the two goods are equally substitutable between themselves) and can be produced under identical cost functions. Thus, we will assume without loss of generality that if firm I wins the auction for market Y and finds it optimal to produce in a single market only, it will always produce in the established market X. So, we have to only consider alternatives (i) and (ii) to solve the decision problem faced by firm I if it wins the auction for market Y.

Let us first consider alternative (i), where firm I uses the newly acquired license for product Y in addition to its license for the product X, and monopolizes the production of both goods. Its problem is to decide how much to produce in markets X and Y for each value of the *ex-ante* unknown demand variable α. By rationality, firm I will solve this problem by choosing the output plans for goods X and Y, $q_X(\alpha)$ and $q_Y(\alpha)$, that maximize –at each possible realization of α– the profit given by

$$\begin{aligned}\pi_I^{M,XY}(q_X, q_Y) &= P_X(q_X, q_Y)q_X + P_Y(q_X, q_Y)q_Y - \frac{c}{2}(q_X + q_Y)^2 \\ &= (\alpha - q_X - \delta\, q_Y)\, q_X + (\alpha - q_Y - \delta\, q_X)\, q_Y - \frac{c}{2}\,(q_X + q_Y)^2. \quad (8)\end{aligned}$$

(Notice that firm I cannot gain anything by choosing supply functions instead of fixed quantities whenever it faces no competitor and monopolizes markets X and Y.) Once firm I chooses $q_X(\alpha)$ and $q_Y(\alpha)$, the realization of α will be known *ex-post*, and the outputs $q_X(\alpha)$ and $q_Y(\alpha)$ will be produced accordingly. Notice from the above profit expression that the argument of the cost function is the total production q_X+q_Y, due to our earlier assumption that the incumbent is technologically constrained to produce good X and Y simultaneously and in the same plant. As the cost function is quadratic, the assumed inability of the incumbent to produce the goods X and Y separately in different plants or sequentially in the same plant will affect its cost and profit calculations and consequently its ability to win the bidding for market Y, as we will show in the next section.

Given the monopoly profit in (8) to be maximized, the first-order condition with respect to q_i is given by

$$q_i = \frac{\alpha - (2\delta + c)q_j}{2 + c} \tag{9}$$

for any $i, j \in \{X, Y\}$ with $i \neq j$. Solving the first-order conditions for q_X and q_Y together directly implies that if the incumbent monopolizes both markets, then it should optimally set the production of goods X and Y at the same level $q_X^{M,XY}(\alpha) = q_Y^{M,XY}(\alpha) \equiv q^{M,XY}(\alpha)$ where

$$q^{M,XY}(\alpha) = \frac{\alpha}{2(1 + \delta + c)}. \tag{10}$$

From (10) and equations (3)-(4) it follows that the optimal monopoly prices set by the incumbent for goods X and Y should be equal, i.e., $p_X^{M,XY}(\alpha) = p_Y^{M,XY}(\alpha) \equiv p^{M,XY}(\alpha)$, where

$$p^{M,XY}(\alpha) \;=\; \alpha - q^{M,XY}(\alpha) - \delta q^{M,XY}(\alpha) = \frac{(1 + \delta + 2c)\alpha}{2(1 + \delta + c)}. \tag{11}$$

We can thus calculate the equilibrium profit of the incumbent as

$$\begin{aligned} \pi_I^{M,XY}(\alpha) \;=\; & p_X^{M,XY}(\alpha) q_X^{M,XY}(\alpha) + p_Y^{M,XY}(\alpha) q_Y^{M,XY}(\alpha) \\ & - \frac{c}{2} \left(q_X^{M,XY}(\alpha) + q_Y^{M,XY}(\alpha) \right)^2 = \frac{\alpha^2}{2(1 + \delta + c)}. \end{aligned} \tag{12}$$

Also, using (1), (2), (10), and (11) we can calculate the equilibrium consumer surplus as

$$\begin{aligned} CS^{M,XY}(\alpha) \;=\; & \alpha \left(q_X^{M,XY}(\alpha) + q_Y^{M,XY}(\alpha) \right) \\ & - \frac{1}{2} \left[\left(q_X^{M,XY}(\alpha) \right)^2 + 2\delta q_X^{M,XY}(\alpha) q_Y^{M,XY}(\alpha) + \left(q_Y^{M,XY}(\alpha) \right)^2 \right] \\ & - p_X^{M,XY}(\alpha) q_X^{M,XY}(\alpha) - p_Y^{M,XY}(\alpha) q_Y^{M,XY}(\alpha) \\ =\; & \frac{(1 + \delta)\alpha^2}{4(1 + \delta + c)^2}. \end{aligned} \tag{13}$$

Now, we can consider the second alternative faced by firm I if it wins the auction for market Y. According to this alternative, firm I should use the license only for the established market X (and produce good X only). Then, setting $q_Y = 0$, we can write –at each possible realization of α– the profit of firm I as follows:

$$\pi_I^{M,X}(q_X) \;=\; P_X(q_X)q_X - \frac{c}{2}\,(q_X)^2 = (\alpha - q_X)\,q_X - \frac{c}{2}\,(q_X)^2\,. \tag{14}$$

Once firm I chooses $q_X(\alpha)$, the realization of α will be known *ex-post*, and the output $q_X(\alpha)$ will be produced accordingly. Given the objective in (14) to be maximized, the first-order condition with respect to q_X implies that if the incumbent wins the auction for market Y but produces in market X only, then it should optimally set the production of good X as

$$q^{M,X}(\alpha) = \frac{\alpha}{2+c}. \tag{15}$$

From equations (15) and (3) and the fact that firm I does not produce good Y, it follows that the optimal price set by the incumbent is equal to

$$p^{M,X}(\alpha) \;=\; \alpha - q^{M,X}(\alpha) = \frac{(1+c)\alpha}{2+c}. \tag{16}$$

We can then calculate the equilibrium profit of the incumbent as

$$\pi_I^{M,X}(\alpha) \;=\; p^{M,X}(\alpha)q^{M,X}(\alpha) - \frac{c}{2}\,\left(q^{M,X}(\alpha)\right)^2 = \frac{\alpha^2}{2(2+c)}. \tag{17}$$

Having calculated $\pi_I^{M,XY}(\alpha)$ and $\pi_I^{M,X}(\alpha)$, we are ready to write the equilibrium strategy of firm I for the studied subgame where it wins the auction for market Y. We should notice that firm I should produce in both markets X and Y if $E[\pi_I^{M,XY}(\alpha)] > E[\pi_I^{M,X}(\alpha)]$. Checking this inequality using equations (12) and (17) implies the following result.

Proposition 1. *If the incumbent wins the auction in the first stage, then it should produce, as a monopoly, in both markets X and Y.*

(We relegate the proofs of our theoretical results to Appendix A.) Proposition 1 trivially holds when the goods X and Y are complements, as each good reinforces, in this case, the demand for the other. The same result holds even when the two goods are substitutes and weaken the demand of each other. This is because the goods are, by assumption, never perfectly substitutable, i.e., δ is always less than 1. Now, we should notice from Proposition 1 that the expected profit of firm I in the studied subgame will always be $E[\pi_I^{M,XY}(\alpha)]$. We can also calculate the implied expected consumer surplus. Inserting $q_X = q^{M,X}(\alpha)$ and $q_Y = 0$ into equations (1) and (2), we can obtain the equilibrium consumer surplus as

$$CS^{M,XY}(\alpha) = \alpha q^{M,X}(\alpha) - \frac{1}{2}(q^{M,X}(\alpha))^2 - p^{M,X}(\alpha)q^{M,X}(\alpha) = \frac{\alpha^2}{2(2+c)^2}. \quad (18)$$

Inspecting equations (15)-(18), we should simply observe the following.

Proposition 2. *If the incumbent wins the auction for market Y, then*

(i) its expected optimal output $E[q^{M,XY}(\alpha)]$ in each market is decreasing in both c and δ,

(ii) the expected optimal price $E[p^{M,XY}(\alpha)]$ in each market is increasing in c whenever c is positive and decreasing in δ,

(iii) the expected optimal total profit $E[\pi^{M,XY}(\alpha)]$ obtained from the two markets is decreasing in both δ and c,

(iv) the expected total consumer surplus $E[CS^{M,XY}(\alpha)]$ obtained from the two markets is decreasing in c, whereas it is decreasing in δ if $c < (1+\delta)$ and increasing in δ if $c > (1+\delta)$.

We should observe from the proof of Proposition 2 that all the results which are expressed for expected (*ex-ante*) values of the model variables are valid for the actual (*ex-post*) values, as well. That is, the effects of c and δ on the actual values of the price, output, profit, and consumer surplus are in the same directions as the effects on the expected (*ex-ante*) values of these variables, since α and α^2 as well as $E[\alpha]$ and $E[\alpha^2]$ turn out to be merely scaling parameters. Proposition 2 shows that an increase in c, the marginal cost of the unit output, increases the expected monopoly price (markup over the marginal cost) and decreases the expected monopoly output in each market as well as the monopoly profit and consumer surplus, as one should expect. Regarding the effect of the demand relation parameter δ, we have almost similar effects. As δ increases and goods X and Y become more substitutable, the demand curve becomes steeper with respect to both the product's own price and its substitute's price. Since the former effect is larger than, and in the opposite direction to, the second effect, the optimal monopoly output becomes smaller with an increase in δ. Oppositely, an increase in δ makes the inverse demand curve for each good shift down in its quantity-price plane. On the other side, the negative effect of δ on the output of the substitute good shifts the aforementioned curve up. However, this negative effect turns out to be weaker; hence the optimal monopoly price becomes smaller with an increase in δ. It also turns out that the reduction in the costs of the monopoly firm (the incumbent) due to an increase in δ is not sufficiently high as compared to the reduction in its revenues, and therefore the profit of the incumbent becomes lower. The final part of Proposition 2 deals with the effect of δ on the consumer surplus. An increase in the substitution parameter raises the consumer surplus if and only if its level is sufficiently smaller than the marginal cost of the unit output. The intuition underlying this result is that the expected consumer surplus can be written as $E[CS^{M,XY}(\alpha) = (1+\delta)E[(q^{M,XY}(\alpha))^2]$. An increase in δ reduces $E[(q^{M,XY}(\alpha))^2]$ and this negative effect is offset by its positive direct effect if and only if c is sufficiently small, making $q^{M,XY}(\alpha)$ and $E[(q^{M,XY}(\alpha))^2]$ sufficiently small.

B. Entrant Wins the Auction for Market Y.

Now, we consider the subgame where firm E is the winner of the auction in the first stage. In this subgame, firm I and firm E produce goods X and Y respectively, competing in supply functions. Under this form of competition, the two firms specify their supply functions simultaneously, without observing the demand shock α. The supply function of each firm maps a non-negative price for its product into a non-negative quantity of output. Formally, it is assumed to be a linear function given by $S_X = \eta_X p_X$ for the incumbent and $S_Y = \eta_Y p_Y$ for the entrant, where $p_X, p_Y \geq 0$ denote the prices of goods X and Y while $\eta_X, \eta_Y \geq 0$ denote the slope parameters. Given the strategies S_X and S_Y, the markets X and Y clear simultaneously if $D_X(p_X, p_Y) = S_X(p_X)$ and $D_Y(p_X, p_Y) = S_Y(p_X)$, implying the equilibrium prices

$$p_X^{SF} = \frac{(b+d+\eta_Y)a}{(b+\eta_X)(b+\eta_Y)-d^2} \tag{19}$$

and

$$p_Y^{SF} = \frac{(b+d+\eta_X)a}{(b+\eta_X)(b+\eta_Y)-d^2}. \tag{20}$$

A pair of supply functions $(S_X^*(p_X), S_Y^*(p_Y)) = (\eta_X^* p_X, \eta_Y^* p_Y)$ constitutes a Nash equilibrium (Nash, 1950) if for each $i, j \in \{X, Y\}$ with $j \neq i$ the function $S_i^*(p_i)$ maximizes the expected profit of the firm that produces the good i when the other firm produces according to the supply function $S_j^*(p_j)$. That is, $(\eta_X^* p_X, \eta_Y^* p_Y)$ constitutes a Nash equilibrium if for each $i, j \in \{X, Y\}$ with $j \neq i$ the parameter η_i^* solves

$$\max_{\eta_i \geq 0} E\left[p_i^{SF}\left(\eta_i, \eta_j^*\right) S_i^*\left(p_i^{SF}\left(\eta_i, \eta_j^*\right)\right) - \frac{c}{2}[S_i^*(p_i^{SF}\left(\eta_i, \eta_j^*\right))]^2\right], \tag{21}$$

or more explicitly

$$\max_{\eta_i \geq 0} \left(\eta_i - \frac{c\eta_i^2}{2}\right)\left(\frac{b+d+\eta_j^*}{(b+\eta_i)(b+\eta_j^*)-d^2}\right)^2 E[a^2]. \tag{22}$$

From Klemperer and Meyer (1989), we know the solution to this problem.

Proposition 3. *If firms I and E produce in markets X and Y respectively and compete in supply functions, then there exists a unique symmetric Nash equilibrium with $S_X^{SF}(p_X), S_Y^{SF}(p_Y)$ such that for each $i \in \{X, Y\}$*

$$S_i^{SF}(p_i) = \eta^* p_i, \tag{23}$$

where

$$\eta^* = \frac{-1 + \sqrt{1 + \frac{4}{c}\left(1 + \frac{1}{bc}\right)}}{2\left(1 + \frac{1}{bc}\right)}. \tag{24}$$

Given the equilibrium supply functions in (24), at each realization of the demand parameter α the equilibrium prices of the two firms become identical, i.e., $p_X^{SF}(\alpha) = p_X^{SF}(\alpha) \equiv p^{SF}(\alpha)$, where

$$p^{SF}(\alpha) = \frac{\alpha}{(1+\delta)(b+\eta^* - d)}. \tag{25}$$

Consequently, the firms would always produce the same equilibrium quantity, i.e., $q_X^{SF}(\alpha) = q_X^{SF}(\alpha) \equiv q^{SF}(\alpha)$ for all $\alpha \geq 0$, where

$$q^{SF}(\alpha) = \eta^* p^{SF}(\alpha) = \frac{\alpha\eta^*}{(1+\delta)(b+\eta^* - d)}. \tag{26}$$

Once the supply functions are chosen by the firms, the demand uncertainty α is realized *ex-post*, and the firms would each produce the output $q^{SF}(\alpha)$ and sell it at $p^{SF}(\alpha)$. Then, we can calculate the equilibrium profits of firm I and E as

$$\pi_I^{SF,X}(\alpha) = \pi_E^{SF,Y}(\alpha) = \left(\eta^* - \frac{c}{2}(\eta^*)^2\right)\left(\frac{1}{b+\eta^*-d}\right)^2 \frac{\alpha^2}{(1+\delta)^2}. \tag{27}$$

Also, from (1), (2), (25), and (26), we can obtain the equilibrium consumer surplus as

$$\begin{aligned} CS^{SF}(\alpha) &= 2\,\alpha\, q^{SF}(\alpha) - \frac{1}{2}(2+2\delta)\left(q^{SF}(\alpha)\right)^2 - \frac{2}{\eta^*}\left(q^{SF}(\alpha)\right)^2 \\ &= \frac{[(1+\delta)(2b+\eta^*-2d)-2]\eta^*}{(b+\eta^*-d)^2}\frac{\alpha^2}{(1+\delta)^2}. \end{aligned} \tag{28}$$

Now, we are ready to present some comparative statics.

Proposition 4. *If the incumbent and the entrant produce goods X and Y respectively and compete in supply functions, then*

(i) the slope, η^*, of the equilibrium supply function of each firm is decreasing in c,

(ii) the expected equilibrium output $E[q^{SF}(\alpha)]$ of each firm is decreasing in c,

(iii) the expected equilibrium price $E[p^{SF}(\alpha)]$ charged by each firm is increasing in c.

Findings similar to those in Proposition 4 were earlier obtained in a different context by Saglam (2018b). Unfortunately, the mathematical complexity of the supply function equilibrium does not allow us to obtain the welfare effects of c and the price/quantity/welfare effects of δ analytically. We will obtain these missing results as well as the results on optimal bidding of the firms by numerical computations in Section 4.

3.2 First Stage: Bidding

In the first stage, the demand uncertainty is not resolved (α is not realized) yet. Therefore, the firms can bid for market Y taking into consideration only the expected values of the *ex-post* equilibrium profits we have calculated above for two possible market organizations (subgames). We should also notice that the expected equilibrium profit of the incumbent is $E[\pi_I^{M,XY}(\alpha)]$ if it wins the bidding for market Y and $E[\pi_I^{SF,X}(\alpha)]$ otherwise. So, the incumbent's value of winning the bidding is equal to

$$V_I = E[\pi_I^{M,XY}(\alpha)] - E[\pi_I^{SF,X}(\alpha)]. \tag{29}$$

On the other hand, the expected equilibrium profit of the entrant is $E[\pi_E^{SF,Y}(\alpha)]$ if it wins the bidding for market Y and zero otherwise. Hence, the entrant's value of winning the bidding is

$$V_E = E[\pi_E^{SF,Y}(\alpha)]. \tag{30}$$

As the bidding is in the form of a first-price auction with complete information, the optimal bids of the incumbent and the entrant will be equal to the minimum of V_I and V_E, i.e., $b^* = \min\{V_E, V_I\}$. In reality, whenever $V_E \neq V_I$, the bidding process will drive b^* to the level $\min\{V_E, V_I\} + \epsilon$, where ϵ is an arbitrarily (or admissibly) small positive real number. Thus, the winner, who has the valuation equal to $\max\{V_E, V_I\}$, will be uniquely identified by the bidding process whenever $V_E \neq V_I$.

To formalize the winner of the auction, let us define $\Delta V = V_E - V_I$. Using the expected values of the profits in (12) and (27) along with the fact that $\pi_E^{SF,Y}(\alpha) = \pi_I^{SF,X}(\alpha)$ for all $\alpha \geq 0$, we can obtain

$$\begin{aligned}
\Delta V &= E[\pi_E^{SF,Y}(\alpha)] - \Big(E[\pi_I^{M,XY}(\alpha)] - E[\pi_I^{SF,X}(\alpha)]\Big) \\
&= 2E[\pi_I^{SF,X}(\alpha)] - E[\pi_I^{M,XY}(\alpha)] \\
&= \left(\frac{2\eta^* - c(\eta^*)^2}{(b+\eta^*-d)^2(1+\delta)^2} - \frac{1}{2(1+\delta+c)}\right) E[\alpha^2]. \qquad (31)
\end{aligned}$$

Clearly, the bidding is won by the entrant if $\Delta V > 0$ and by the incumbent if $\Delta V < 0$. When $\Delta V = 0$, the bids of the firms become equal, i.e., $b_I = b_E = b^* = V_E = V_I$. For this particular case, we assume that the winner is determined by comparing the consumer surplus generated when the winner is the entrant (and the two firms engage in supply function competition) and when the winner is the incumbent (and acts as a monopoly in both markets). So, let us define $\Delta CS = E[CS^{SF}(\alpha)] - CS^{M,XY}(\alpha)]$. Using the expected values of consumer surplus expressions (13) and (28), we can calculate

$$\Delta CS = \left(\frac{[(1+\delta)(2b+\eta^*-2d)-2]\eta^*}{(b+\eta^*-d)^2(1+\delta)^2} - \frac{(1+\delta)}{4(1+\delta+c)^2}\right) E[\alpha^2]. \qquad (32)$$

Whenever $\Delta V = 0$, the bidding is won by the entrant if $\Delta CS \geq 0$ and by the incumbent if $\Delta CS < 0$.

3.3 Two Stages Combined: Equilibrium

Combining the equilibrium strategies calculated for the first and second stages, we are ready to state the subgame-perfect Nash equilibrium of the two-stage game.

Proposition 5. *The described strategic game between the incumbent and the potential entrant has a unique subgame perfect Nash equilibrium (SPNE) such that*

(i) the incumbent and the potential entrant both plan to bid, in the first stage, the amount $b^* = \min\{V_I, V_E\}$, where V_I and V_E are respectively their valuations satisfying (29) and (30);

(ii) the incumbent plans to produce, at each realization of α, in both markets the monopoly output $q^{M,XY}(\alpha)$ in case it wins the bidding and to produce according to the supply function $S_X^{SF}(p_X) = \eta^* p_X$ in case the potential entrant

wins, whereas the potential entrant plans to produce according to the supply function $S_Y^{SF}(p_Y) = \eta^* p_Y$ in case it wins the bidding and not to produce if the incumbent wins.

We should note that in the SPNE the incumbent's expected net profit is equal to $E[\pi_I^{M,XY}(\alpha)] - b^*$ if it wins the bidding for market Y and equal to $E[\pi_I^{SF,X}(\alpha)]$ if it loses. On the other hand, the entrant's expected net profit is equal to $E[\pi_E^{SF,Y}(\alpha)] - b^*$ if it wins the bidding for market Y and equal to zero if it loses. Finally, the net welfare of consumers plus the welfare of the outsider licensor (auctioneer) can be written as $E[CS^{M,XY}(\alpha)] + b^*$ if the incumbent wins the bidding and as $E[CS^{SF}(\alpha)] + b^*$ otherwise. Below, we will compute the SPNE of the described production game for a wide range of cost and demand parameters and present various comparative statics results.

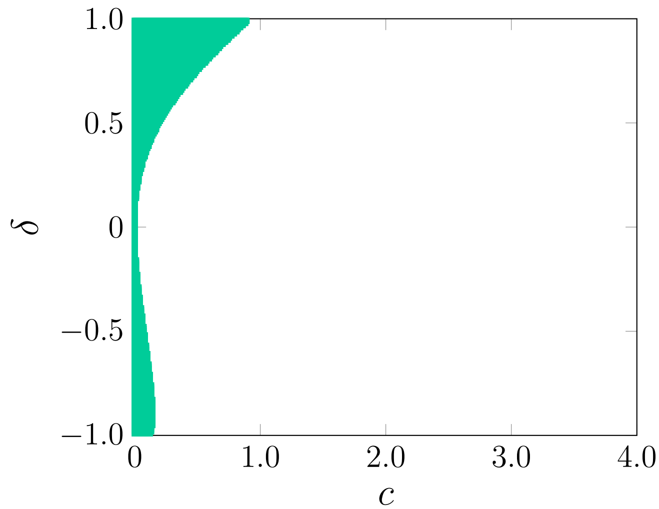
4 Computational Results

We have performed all numerical computations using MATLAB, Release 2021a. The source code and the data generated are available from the author upon request.

For our computations, we vary the cost parameter c in the set $\{0, \ldots, 3.99\}$ with increments of 0.01 and the demand parameter δ in the set $\{-0.99, \ldots, 0.99\}$ with increments of 0.01. Notice that the mean (μ) of the demand shock α and its standard deviation (σ) are only scale parameters in our model. Their values do not have any impact on the identity of the winner and the market organization. In more detail, the parameter $\mu = E[\alpha]$ scales the expected equilibrium prices and quantities in our model uniformly. Likewise, $E[\alpha^2] = \mu^2 + \sigma^2$ scales the expected profits and consumer surplus. Thus, we set both μ and σ to 1 for simplicity and do not vary them in our computations. Given the described setting, we consider $400 \times 199 = 79600$ distinct pairs of (c, δ) values in our analysis.

We first compute the set of (c, δ) values under which the incumbent wins the bidding for market Y and monopolize both markets X and Y. Our results illustrated in Figure 1 shows that the likelihood of monopolization (the area of the green shaded region in Figure 1) is rather small within the simulated range of parameters and this likelihood is even smaller when the products are complements (i.e., when δ is negative). In more detail, we show that the bidding for market Y is always won by the entrant if either (i) $\delta > 0$ and $c > 0.89$ or (ii) $\delta < 0$ and $c > 0.14$. For all other cases, the bidding is won by the entrant if $|\delta|$ is sufficiently small and it is won by the incumbent otherwise.

Figure 1. The Set of (c, δ) Pairs Supporting Monopolization of Both Markets by the Incumbent (Green Shaded Area)



The finding in Figure 1 depends on the comparison of the values from winning the bidding for the entrant and the incumbent, while these values depend on their expected profits under two possible market organizations that may arise in a SPNE. So, to better understand the above finding, we have to study how cost and demand parameters affect the expected values of price, output, and profits. To that account, we make additional computations and present the results graphically in Figures 2-11 located in Appendix B. In Figures 2-6 we fix the parameter δ to each value in $\{-0.67, -0.33, 0, 0.33, 0.67\}$ consecutively and study the effects of c on model variables, whereas in Figures 7-11 we fix the parameter c to each value in $\{0.25, 0.50, 1.00, 2.00, 3.99\}$ consecutively and study the effects of δ.

Panel (i) of Figures 2-6 shows that the slope of the equilibrium supply function chosen by the firms when they compete is decreasing in c, as already proved in Proposition 4-(ii). We should also observe that the investigated slope becomes smaller as δ becomes smaller in absolute value. We can verify this more closely in panel (i) of Figures 7-11. In fact, we observe that the slope of the equilibrium supply function depends not on whether the goods are complements or substitutes but solely on $|\delta|$, the absolute intensity of the level of substitution/complementarity. This can be theoretically predicted as well, since equation (24) shows that δ enters into η^* through the direct demand parameter b which is equal to $1-\delta^2$.

Next, we will present the comparative statics results on the expected values of the monopoly price, $E[p^{M,XY}(\alpha)] \equiv p^{M,e}$, and the supply function equilibrium price (or simply the SFE price) $E[p^{SF}(\alpha)] \equiv p^{SF,e}$. We observe from panel (ii) Figures 2-11 that the expected values of both prices increase in c for all sample values of δ and decrease in δ for all sample values of c, some of which results were also theoretically predicted by Propositions 2 and 4. From our computations we also observe that the expected monopoly price can be below the expected SFE price if and only if c is sufficiently close to zero and δ is sufficiently close to -1.

Panel (iii) of Figures 2-11 illustrates that the expected values of outputs and prices in each market always move in opposite directions when they are affected by a change in c. In contrast to the expected prices, the expected values of both monopoly and SFE outputs in each market (i.e., $E[q^{M,XY}(\alpha)] \equiv q^{M,e}$ and $E[q^{SF}(\alpha)] \equiv q^{SF,e}$) decrease in c for all sample values of δ, as theoretically expected. However, a change in δ affects the expected price and outputs in the same direction. Like the expected prices, the expected outputs always decrease with an increase in δ.

In panels (iv) and (v) of Figures 2-11, we report the expected potential profits and consumer surplus under each studied industry organization. In these panels, we denote by $\pi_{E,I}^{SF,e}$, $\pi_{E+I}^{SF,e}$, and $\pi_I^{M,e}$ respectively the expected profit of each firm under SFE, the expected industry profit under SFE, and the expected profit of the incumbent when it monopolizes both markets X and Y. Similarly, by $CS^{SF,e}$ and $CS^{M,e}$ we respectively denote the expected consumer surplus under SFE and under the case of the incumbent's monopolization of both markets. Panel (iv) illustrates that the expected potential profits of the incumbent and the entrant are always decreasing in both c and δ irrespective of the winner of the bidding. Results also show that the expected industry profits under supply function competition, $\pi_{E+I}^{SF,e}$, are higher than the expected industry profits under the incumbent's monopolization, $\pi_I^{M,e}$ unless c is sufficiently close to zero and δ is sufficiently close to 1. The reason is that in these extreme cases, the monopolization of the incumbent enables it to fully internalize the adverse effects of the substitutability of the goods X and Y on the industry profits. However, this positive effect of monopolization which is missing under supply function (or any form of) competition is countered by a negative effect stemming from the monopolist's inefficiency in the cost structure (due to its inability to produce the outputs of X and Y in different plants). This negative effect outweighs the positive effect of monopolization in situations where the marginal cost of the unit output c is not very low and the demand substitution parameter δ is not very high. We should recall from (the unshaded area in) Figure 1 that these situations characterize when the entrant wins the bidding for market Y, or analytically when the entrant's excess value of winning the bidding, $\Delta V = V_E - V_I = \pi_{E+I}^{SF,e} - \pi_I^{M,e}$, is positive.

In panel (v) of Figures 2-11, we picture the potential consumer surplus in each possible (but not necessarily realized) industry organization along with the corresponding industry profits. We can see that the potential consumer surplus in each industry organization is always decreasing in c and nondecreasing in δ. We also observe that when c is sufficiently close to zero and δ is sufficiently close to 1, there is a more equitable welfare distribution between the producer(s) and consumers especially when the incumbent monopolizes the whole industry.

Finally, in panel (vi), we expected SPNE welfares taking into account the identity of the winner of the bidding and thus the realized form of industrial organization. (Here, CS^e is the 'expected' consumer surplus in SPNE. It is equal to $CS^{M,e}$ if the incumbent is the winner of the first-stage auction and equal to $CS^{SF,e}$ otherwise. Consequently, the profit earned by the winner of the bidding is calculated to be the operating profit of the winner net of the equilibrium bid b^*, whereas the consumer welfare plus the auctioneer's revenue can be calculated as $CS^e + b^*$. The observed jumps in the firms' expected operating profits net of equilibrium bid (as well as in the net consumer welfare) occur when the winner of the bidding changes between the firms (depending upon the values of c and δ). Recall that the incumbent obtains operating profit from both markets if it wins the bidding and only from market X otherwise, whereas the entrant obtains operating profit only from market Y if and only if it wins the bidding. Thus, in line with Figure 1, we see that the entrant's net welfare is zero when c is very low and δ is sufficiently high in absolute value. We also observe that an increase in δ reduces each firm's net welfare, while increasing consumer net welfare. In fact, consumer net welfare is found to exceed each firm's net welfare when δ is above a certain threshold, while this threshold is increasing with c.

5 Conclusion

In this paper, we have investigated whether an incumbent firm that has a monopoly position in an established market can persist by expanding its operation to a new market licensed by an outside inventor under the threat of supply function competition with a potential entrant. To study this question, we have considered a two-stage strategic game between by the incumbent and the potential entrant. The first stage of this game involves an auction (bidding) between the two players for the new market, and the second stage requires the production of the two goods. After characterizing the (subgame perfect Nash) equilibrium of this strategic game and presenting some theoretical analysis, we have conducted numerical computations to obtain several insights about the equilibrium.

Our main result is that in equilibrium the incumbent firm can win the bidding for the new market and expand its operation there if the marginal cost (to produce a unit output) is sufficiently low with respect to the degree of product differentiation, while its likelihood of winning is higher if the two products are substitutes than if they are complements. Our results also show that the expected industry profits can be higher, and the welfare distribution between consumers and producers can become more equitable in the former case than in the latter if the marginal cost of unit output is very close to zero and the two products are almost perfect complements. The reason is that in these extreme cases of cost and product complementarity, the monopolization of both markets allows the established firm to fully internalize the effects of product differentiation.

Our main result is different from an earlier result of Chen (2000) that studied the problem of monopoly expansion in a partly different model where the incumbent faces the threat of Cournot competition with a potential entrant not only in the new market but also in the established market. Chen (2000) basically showed that when entry cost to the existing market of the monopoly is sufficiently high, the monopoly always wins the bidding for the new market independent of the degree of product substitution or complementarity while the case where the entrant produces only the new product arises if and only if the new and existing products are complements. On the other hand, our paper, which assumes away entry to the existing market of the monopoly, shows that the case where the new product is produced (only) by the entrant can arise both when the new and established products are substitutes and when they are complements, and this is mainly due to the nature of the equilibrium under supply function competition.

Our main result provides a practical policy suggestion, especially for regulatory authorities in power (electricity) markets that are dominated by a small number of firms (generators). Our result shows that the existing monopoly (or in a more general setting, the existing set of dominating firms) is more likely to expand when the production costs in the established and new markets are sufficiently low whereas duopolistic (or in general oligopolistic) competition is more likely to arise otherwise. In the former case, the need for regulation is obvious, despite the additional economic efficiency (even in the presence of monopoly) created by the low costs of production that incidentally support the expansion of the monopoly. More interestingly, a regulator may have incentives to regulate the industry even when the incumbent and entrant engage in a duopolistic competition which arises when costs of production are sufficiently high in our model. In such a case, the regulator may consider, instead of controlling the prices or quantities of the regulated firms, to design the mode of competition the firms will follow whenever the potential entrant wins the bidding for the new market. That is, the regulator may enforce or incentivize the firms to compete in fixed quantities instead of supply functions, or vice versa, whenever it is more beneficial from the viewpoint of consumers (or the society as a whole). In fact, an earlier work of Saglam (2018a) sheds light on this matter by showing that regulators in power markets should not enforce quantity competition –over supply function competition– except when the demand uncertainty is estimated to be sufficiently low. Clearly, one may fruitfully integrate the results of Saglam (2018a) with our work to investigate how the incumbent's incentive to dominate the new market changes when it faces a regulatory threat in the established and/or the new market.

We should notice that both Chen (2000) and our current work assume that the incumbent and the potential entrant compete with each other whenever the incumbent loses the bidding for the new product. A relevant alternative is to consider the possibility of collusion between the firms. A small literature pioneered by Ciarreta and Gutiérrez-Hita (2012) studies how collusion arises in industries under supply function competition. Ciarreta and Gutiérrez-Hita (2012) show that an increase in cost differences reduces the likelihood and sustainability of cartels, whereas Saglam (2020) studies the effects of several profit-sharing rules on the incentives to collude in supply functions under cost asymmetry and demand uncertainty. More recently, Celen and Saglam (2021) study the incentives of duopolistic firms to compete or collude in supply functions under technology licensing. Integrating these studies with our work, future research may fruitfully study monopolistic expansion under the possibility of duopolistic collusion (in supply functions) for the production of a newly introduced good, licensed by an outsider or insider inventor.

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Appendix A.

Proof of Proposition 1. Producing in both markets X and Y is more profitable for firm I than producing only in market X if and $E[\pi_I^{M,XY}(\alpha)] > E[\pi_I^{M,X}(\alpha)]$. From equations (12) and (17), it follows that the above inequality holds if and only if

$$\frac{\alpha^2}{2(1+\delta+c)} > \frac{\alpha^2}{2(2+c)}, \tag{33}$$

which is always true since $\delta < 1$ by assumption. □

Proof of Proposition 2. Below, we prove each part separately.

(i) Differentiating $E[q^{M,XY}(\alpha)]$ with respect to c and δ, we respectively obtain

$$\frac{\partial E[q^{M,XY}(\alpha)]}{\partial c} = \frac{\partial E[q^{M,XY}(\alpha)]}{\partial \delta} = -\frac{E[\alpha]}{2(1+\delta+c)^2} \tag{34}$$

which is negative for all $c \geq 0$ and $\delta \in (-1, 1)$ since $E[\alpha] = \mu > 0$.

(ii) Differentiating $E[p^{M,XY}(\alpha)]$ with respect to c, we obtain

$$\frac{\partial E[p^{M,XY}(\alpha)]}{\partial c} = \frac{(1+\delta)E[\alpha]}{2(1+\delta+c)^2} \tag{35}$$

which is positive for all $c \geq 0$ and $\delta \in (-1, 1)$ since $E[\alpha] > 0$. Also, differentiating $E[p^{M,XY}(\alpha)]$ with respect to δ, we obtain

$$\frac{\partial E[p^{M}(\alpha)]}{\partial \delta} = -\frac{cE[\alpha]}{2(1+\delta+c)^2} \tag{36}$$

which is negative for all $c > 0$ and $\delta \in (-1, 1)$ since $E[\alpha] = \mu > 0$.

(iii) Differentiating $E[\pi^{M,XY}(\alpha)]$ with respect to c and δ, we respectively obtain

$$\frac{\partial E[\pi^{M,XY}(\alpha)]}{\partial c} = \frac{\partial E[\pi^{M,XY}(\alpha)]}{\partial \delta} = -\frac{E[\alpha^2]}{2(1+\delta+c)^2} \tag{37}$$

which is negative for all $c \geq 0$ and $\delta \in (-1, 1)$ since $E[\alpha^2] = \mu^2 + \sigma^2 > 0$.

(iv) Differentiating $E[CS^M(\alpha)]$ with respect to c, we obtain

$$\frac{\partial E[CS^{M,XY}(\alpha)]}{\partial c} = -\frac{(1+\delta)E[\alpha^2]}{2(1+\delta+c)^3} \tag{38}$$

which is negative for all $c \geq 0$ and $\delta \in (-1, 1)$ since $E[\alpha^2] = \mu^2 + \sigma^2 > 0$. Also, differentiating $E[CS^{M,XY}(\alpha)]$ with respect to δ, we obtain

$$\frac{\partial E[CS^{M,XY}(\alpha)]}{\partial \delta} = \frac{(c-1-\delta)E[\alpha^2]}{4(1+\delta+c)^3} \tag{39}$$

which is negative if $c < 1+\delta$ and positive if $c > 1+\delta$ as $E[\alpha^2] = \mu^2+\sigma^2 > 0$. □

Proof of Proposition 3. See Klemperer and Meyer (1989, pp. 1267-1270). □

Proof of Proposition 4. Below, we will prove each part separately.

(i) Differentiating η^* with respect to c, we obtain

$$\frac{\partial \eta^*}{\partial c} = \frac{K_c}{\sqrt{A}K^2}\left(bK + \sqrt{A} - 1\right) \tag{40}$$

where $K = 2(1 + (bc)^{-1})$ and $A = 1 + (2/c)K$. It is easy to see that $K_c = -2/(bc^2) < 0$ and $\sqrt{A} < 1$ for all $c \geq 0$ and $\delta \in (-1, 1)$. Therefore, $\partial\eta^*/\partial c$ is always negative.

(ii) Differentiating $E[q^{SF}(\alpha)]$ with respect to c and δ, we respectively obtain

$$\frac{\partial E[q^{SF}(\alpha)]}{\partial c} = \frac{(b-d)E[\alpha]}{(1+\delta)(b+\eta^*-d)^2}\frac{\partial \eta^*}{\partial c} \tag{41}$$

which is negative for all $c \geq 0$ and $\delta \in (-1, 1)$ since $b - d = 1/(1 + \delta) > 0$, $E[\alpha] > 0$, and $\partial\eta^*/\partial c < 0$.

(iii) Differentiating $E[p^{SF}(\alpha)]$ with respect to c, we obtain

$$\frac{\partial E[p^{SF}(\alpha)]}{\partial c} = -\frac{E[\alpha]}{(1+\delta)(b+\eta^*-d)^2}\frac{\partial \eta^*}{\partial c} \tag{42}$$

which is positive for all $c \geq 0$ and all $\delta \in (-1, 1)$ since $E[\alpha] > 0$ and $\partial\eta^*/\partial c < 0$. □

Proof of Proposition 5. Directly follow from Propositions 1 and 3 and equations (29)-(32).

Appendix B.

Figure 2. The Effects of c on Various Outcomes ($\delta = -0.67$)

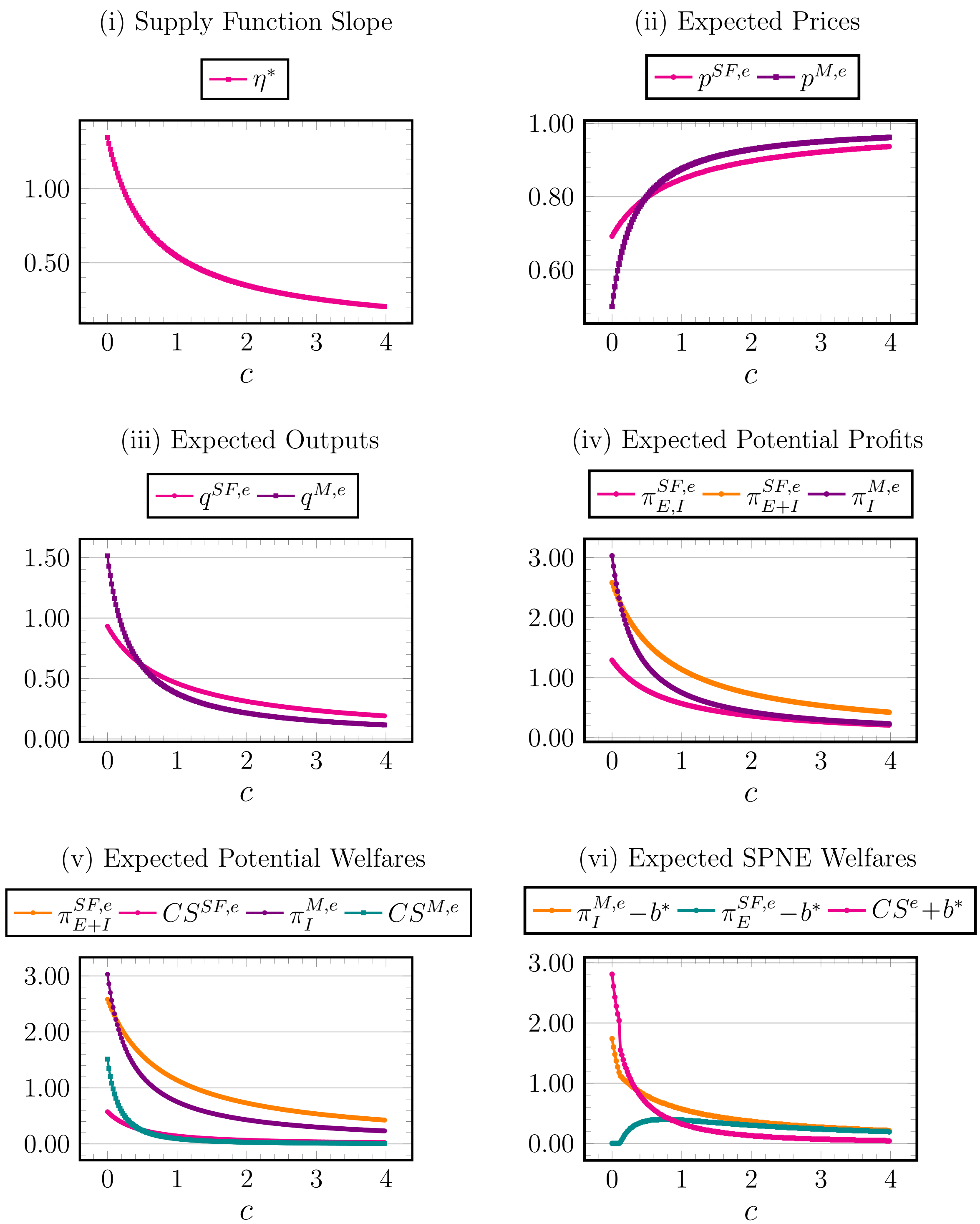

Figure 3. The Effects of c on Various Outcomes ($\delta = -0.33$)

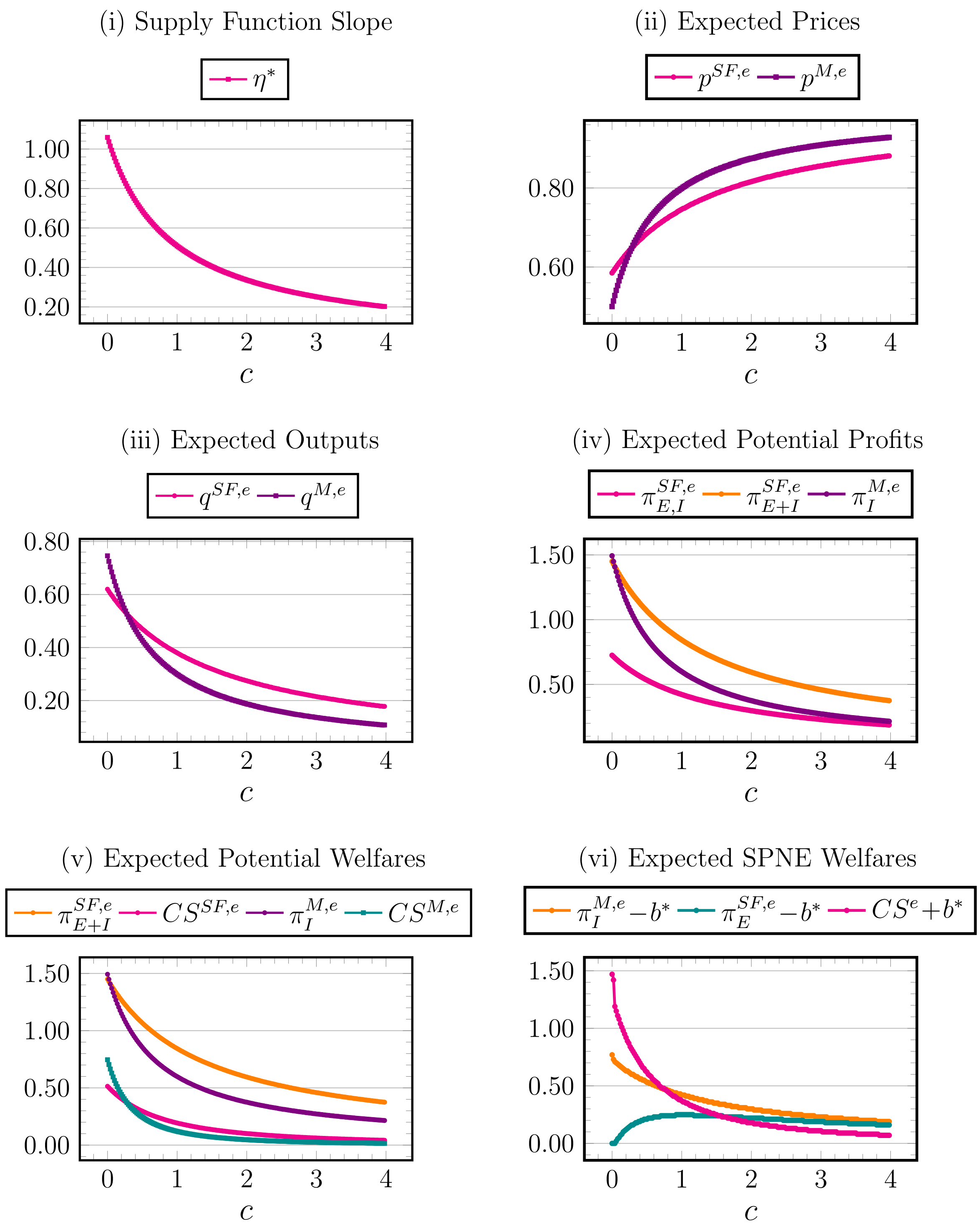

Figure 4. The Effects of c on Various Outcomes ($\delta = 0$)

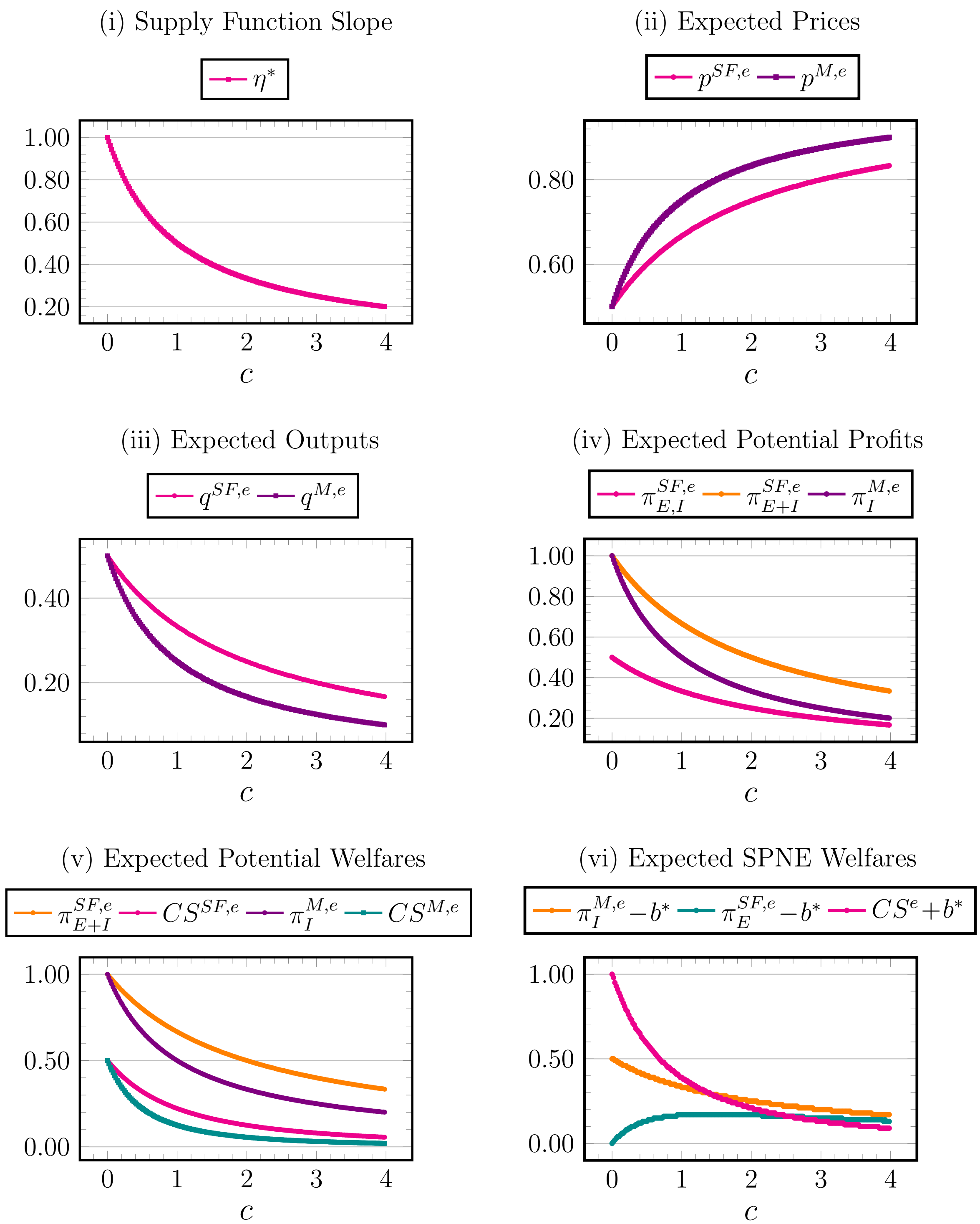

Figure 5. The Effects of c on Various Outcomes ($\delta = 0.33$)

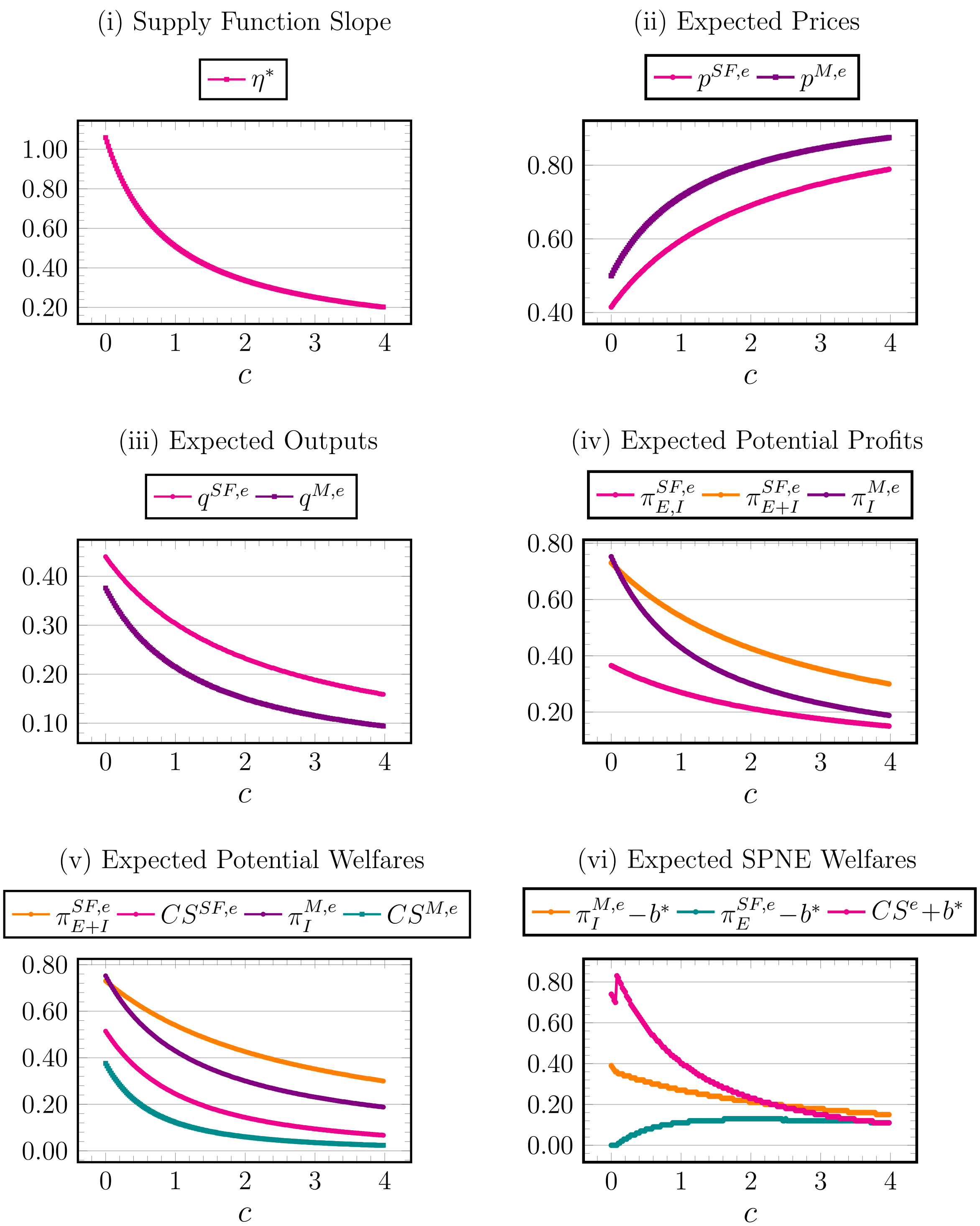

Figure 6. The Effects of c on Various Outcomes ($\delta = 0.67$)

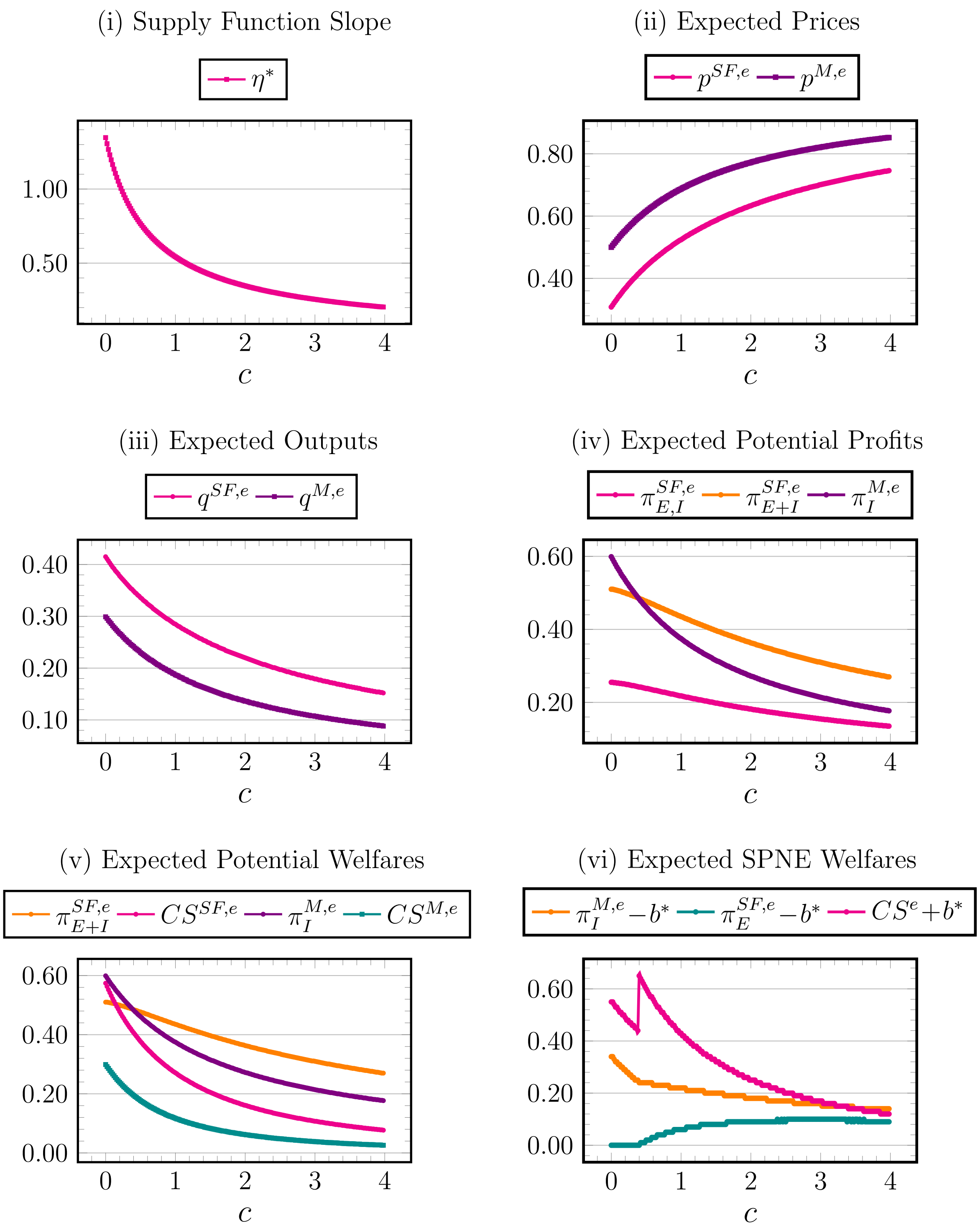

Figure 7. The Effects of δ on Various Outcomes ($c = 0.25$)

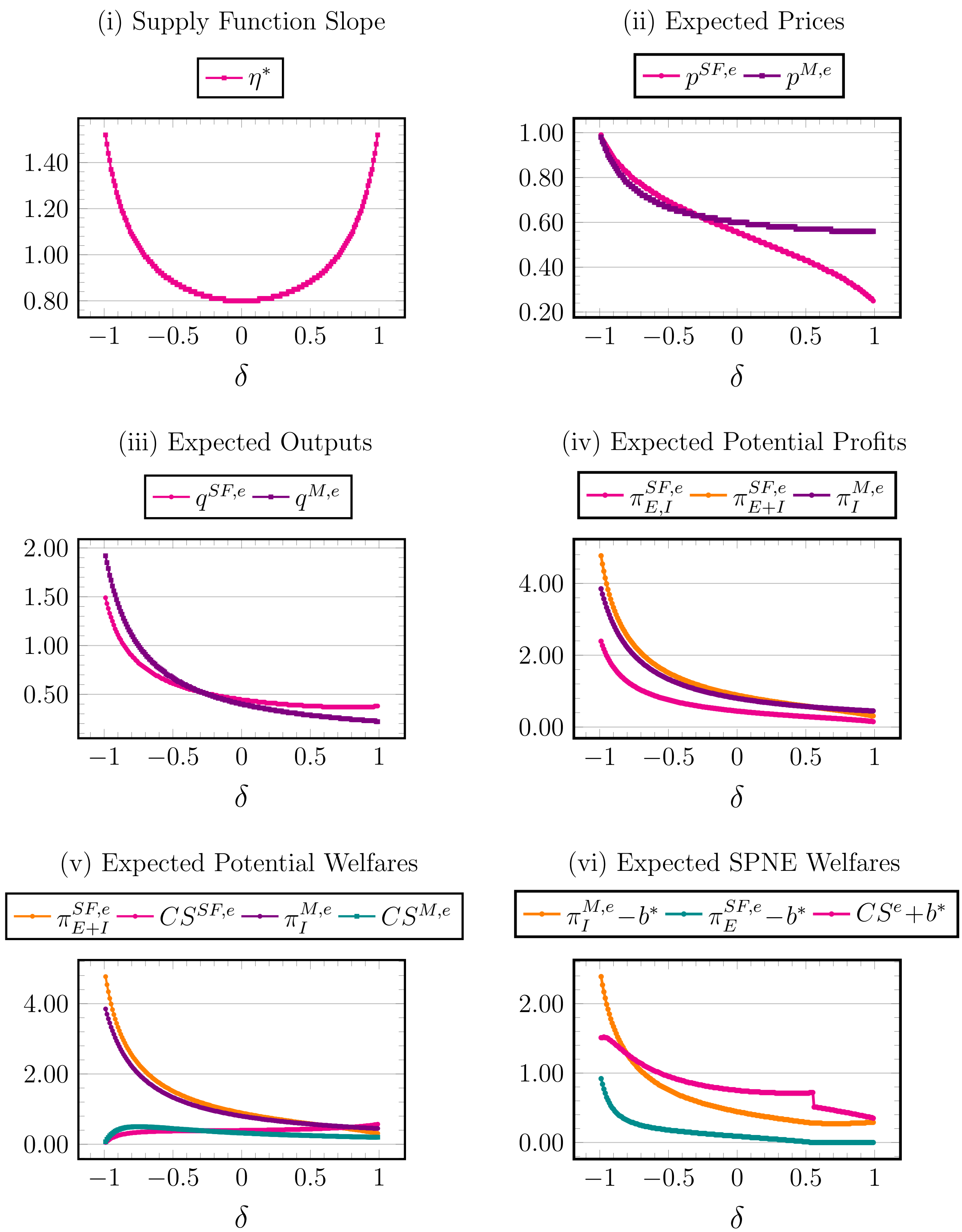

Figure 8. The Effects of δ on Various Outcomes ($c = 0.50$)

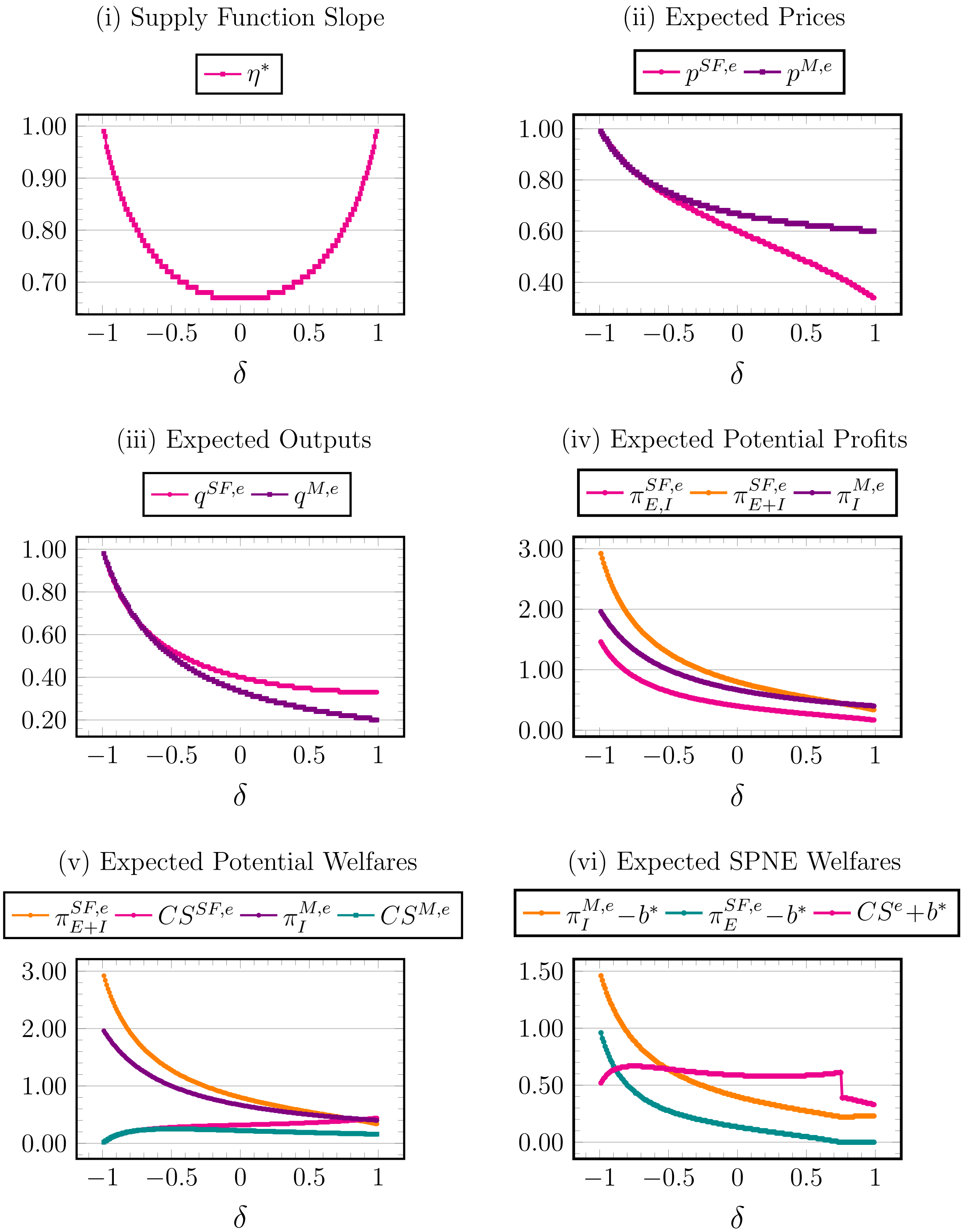

Figure 9. The Effects of δ on Various Outcomes ($c = 1.00$)

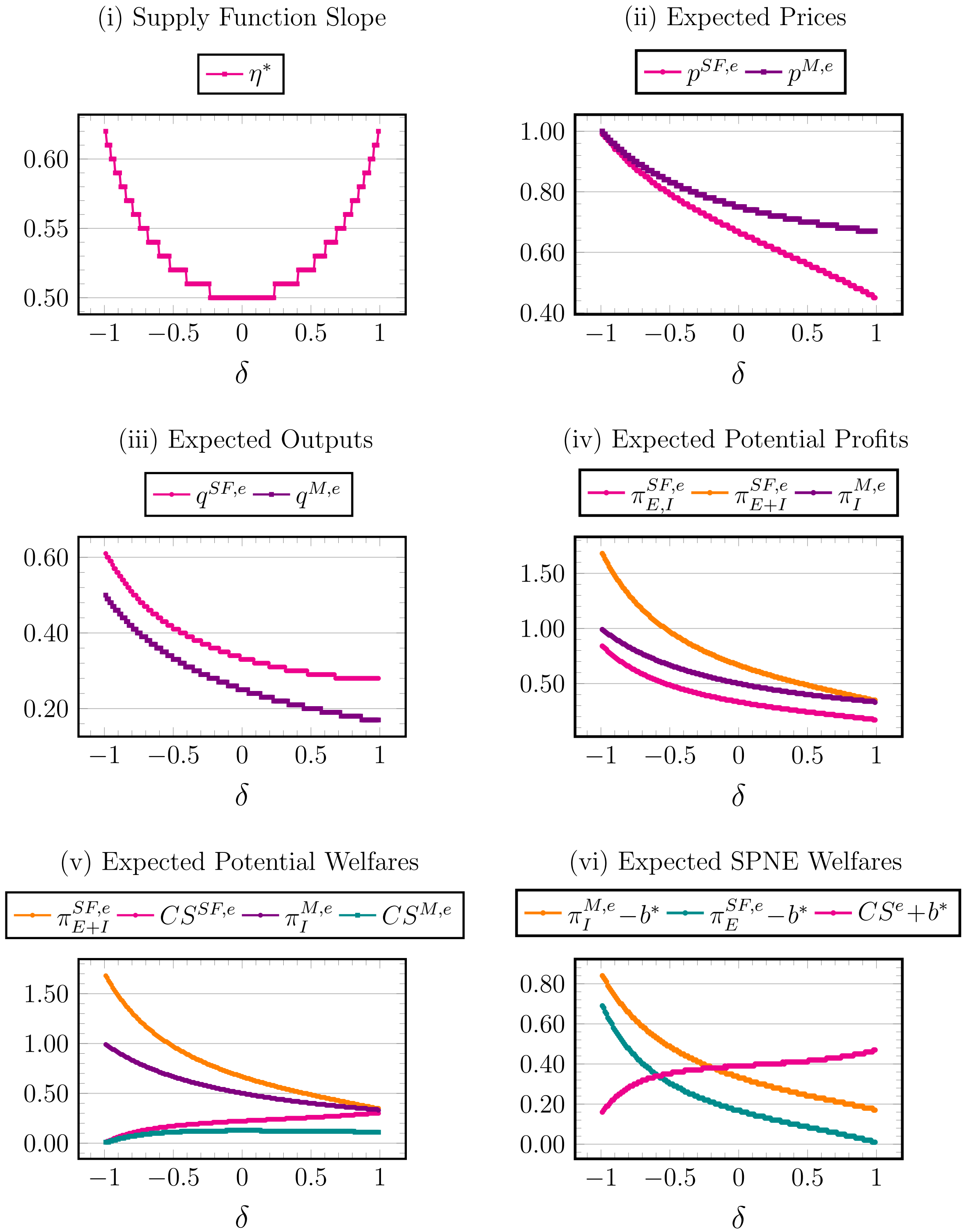

Figure 10. The Effects of δ on Various Outcomes ($c = 2.00$)

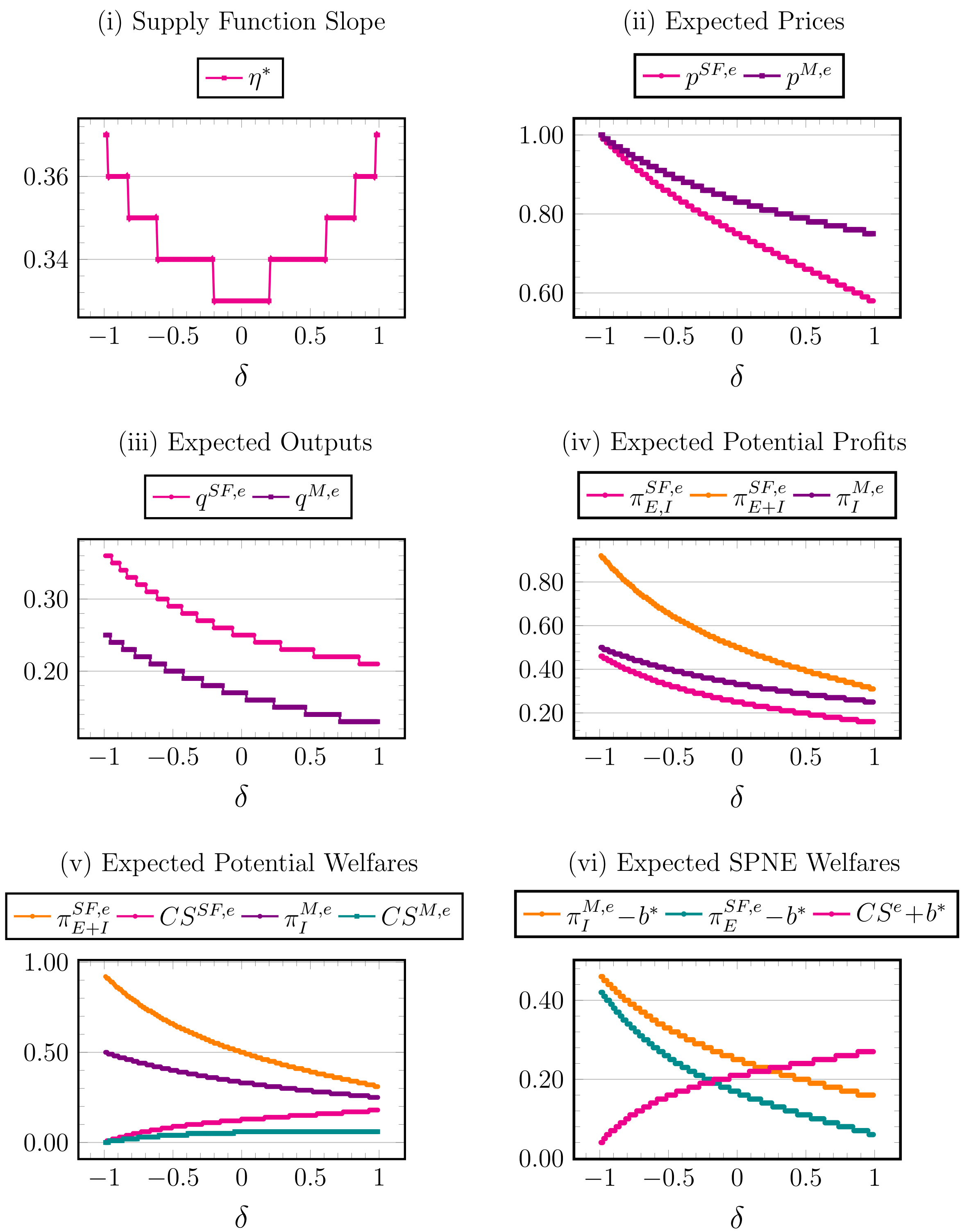

Figure 11. The Effects of δ on Various Outcomes ($c = 3.99$)