



Munich Personal RePEc Archive

Using the asymptotic approximation of the maxwell element model for the analysis of stress in a conveyor belt

Pihnastyi, Oleh and Chernavska, Svtlana

National Technical University "Kharkiv Polytechnic Institute",
National Technical University "Kharkiv Polytechnic Institute"

22 December 2021

Online at <https://mpra.ub.uni-muenchen.de/111831/>
MPRA Paper No. 111831, posted 06 Feb 2022 09:38 UTC

USING THE ASYMPTOTIC APPROXIMATION OF THE MAXWELL ELEMENT MODEL FOR ANALYSIS OF STRESS IN CONVEYOR BELT

Oleh Pihnastyi, Svitlana Cherniavska

The features of the propagation of dynamic stresses in a conveyor belt, the material properties of which correspond to the Maxwell element model, are considered. Analytical expressions are presented for calculating the dynamic modulus of elasticity, the modulus of loss, and the angle of mechanical loss depending on the frequency of longitudinal vibrations in the belt of an extended transport conveyor. To analyze the dynamic stress propagation process, dimensionless parameters are introduced that characterize the specific features of the viscoelastic process in a conveyor belt, the material properties of which correspond to the Maxwell element model. The transition to the dimensionless Maxwell element model is made and the analysis of the relationship between stress and deformation of a conveyor belt element for extremely large and small values of dimensionless parameters is made. The substantiation of the scope of the Maxwell element model is given. It is shown that at sufficiently high frequencies of longitudinal voltage oscillations in a conveyor belt, at which the oscillation period is much less than the characteristic time of oscillation damping, the relationship between stress and deformation of the conveyor belt element corresponds to Hooke's law. A qualitative analysis of the relaxation time was carried out for a conveyor belt material, the properties of which correspond to the Maxwell element model. The analysis of the propagation of dynamic stresses in the conveyor belt for the characteristic modes of operation of the transport conveyor is carried out. The mode of operation of the conveyor with a constant deformation speed of the belt element has been investigated; a mode in which a constant load is suddenly applied to a belt element; mode of operation of the conveyor with an instantly applied load to the belt element. It was determined that in cases where the characteristic process time significantly exceeds the stress relaxation time in the conveyor belt or the longitudinal oscillation period is much less than the stress relaxation time in the conveyor belt, the Maxwell element model can be replaced with a sufficient degree of accuracy by the Hooke element model.

Key words: viscoelastic process, Maxwell element, Hooke element, transport conveyor, dynamic modulus of elasticity

1. Introduction

One of the characteristics of a transport conveyor, which determines its operational capabilities, is the strength of the conveyor belt [1, 2]. The mechanical strength of the belt is the ability of the belt to resist destruction under the action of the dynamic Maxwell element loads that occur during material transport. The tensile strength of the belt depends on the properties of the material, the speed of deformation, and the temperature of the tape material [3, 4]. For transport conveyors that operate continuously for a long time, belt failure occurs at stresses that are significantly lower than the ultimate strength of the belt material. The behavior of the material of the belt at the moment of destruction is determined by the relaxation and strength properties of

the material of the conveyor belt, between which there is a relationship [5]. The viscoelastic properties of the material of the conveyor belt, caused by relaxation processes, affect the speed of the destruction of the conveyor belt [6, 7], are closely related to the problem of saving costs for transportation of the material. One of the ways to reduce transport costs, which occupy a significant part in the cost of material extraction, is the use of a conveyor belt speed control system [8]. Switching speed modes leads to acceleration or deceleration of the conveyor belt, and, accordingly, to the emergence of dynamic stresses in the belt. This imposes additional restrictions on the speed limits of the transport system. The proposed algorithms for stepwise regulation of the belt speed imply instant switching of speed modes, and algorithms for smooth speed control do not take into account the propagation of dynamic disturbances in the conveyor belt. At the same time, with a non-stationary incoming of material at the input of the transport system, the duration of the transient process takes up a fairly large part of the total time of the control process. For an in-depth analysis of these limitations, which consists in determining the dependence of the stresses value on the magnitude of the belt speed and acceleration, a solution of the wave equation is required. For the Hooke-element model, the solution to the wave equation is obtained in an analytical form. For more complex elastic element models, among which the Maxwell element model should be distinguished, the solution of the wave equation is associated with additional difficulties. In this regard, the problem of constructing simple analytical dependencies between the stress and deformation of a belt element is urgent, which would simplify the solution of the wave equation and make it possible to form constraints on the phase coordinates when designing algorithms for optimal control of the belt speed, the material properties of which correspond to the Maxwell element model.

2. Literature analysis and problem statement

In [9], the results of studies of the propagation of dynamic stress disturbances in conveyor belts made of a material whose characteristics correspond to the Voigt element model are presented. It is shown that the materials of the conveyor belt have pronounced viscoelastic properties, which impose specific features on the process of propagation of disturbances [7]. For some modes of operation of the conveyor, a numerical calculation of the values of the conveyor belt speed, acceleration, and stress in the conveyor belt is carried out. However, the questions of constructing a solution to the problem in an analytical form remained open. The reason for this is the objective difficulties caused by the relationship between stress and deformation in the Voigt element model. The problem of constructing a solution to the problem in an analytical form remained open in the works [5, 10, 11]. In work [5] for the conveyor belt of the elastic model Kelvin-Voigt element, the tension of the belt in the steady-state and transient conditions is analyzed. For the numerical calculation, the Lagrange equation system was used. The work [10] presents an analysis of the main models of elastic elements for a conveyor belt: Hooke element, Newtonian element, Maxwell element, Kelvin element, Venant element, CDI geometric element, CDI five-element. The calculation of two transport systems (conveyor length nine kilometers) for start and stop modes with the CDI five-element composite model was performed using the finite

element method. The dynamic stress in a conveyor belt, the material characteristics of which correspond to the Kelvin-Voigt element model, the combination of Hooke and Kelvin-Voigt element, was investigated in [11]. The conveyor belt segment is represented by a two-parameter rheological model.

In some cases, these difficulties can be overcome if, during the operation of the conveyor, it is assumed that there are small deformations in the belt, at which the relationship between stress and deformation in an element can be approximately represented by Hooke's law

$$\sigma(t, S) = E\varepsilon(t, S), \quad (1)$$

where $\sigma(t, S)$, $\varepsilon(t, S)$ are stress and deformation of the belt at time t at point S of the conveyor section of length S_0 ; E is the modulus of elasticity of the material. This approach to the analysis of the propagation of long-wave oscillations in the conveyor belt of the transport system is implemented in [12]. Also, an attempt to construct an approximate analytical solution in a particular case was carried out in [13] for a transport system, the belt material of which corresponds to the Winkler foundation model. All this allows us to assert the feasibility of conducting a study on the construction of approximate Maxwell element models with their subsequent use to calculate the propagation of longitudinal vibrations in a conveyor belt.

3. The purpose and objectives of the study

The aim of the study is to develop asymptotic models of viscoelastic processes in a conveyor belt, the material of which corresponds to the model of the Maxwell element. This made it possible to synthesize algorithms for controlling the belt speed, taking into account the limitations associated with the presence of dynamic disturbances in the conveyor belt, and, accordingly, to further reduce the cost of transporting material by optimizing the speed modes.

To achieve this goal, the following tasks were set:

- perform an asymptotic analysis of the model of the Maxwell element and justify the scope of the model;
- to analyze the characteristic modes of operation of the transport conveyor, to carry out for each of the considered modes the transition from the general model of the Maxwell element to the asymptotic model of the viscoelastic element, justifying its use.

4. Materials and research methods

The materials from which the conveyor belts are made have both elastic and viscous properties, which leads to a rather complex relationship between the stress $\sigma(t, S)$ in the conveyor belt and its deformation $\varepsilon(t, S)$. One of the common models used to describe the viscoelastic properties of a conveyor belt material is the model of the Maxwell element. An analytical solution to the problem of the propagation of longitudinal dynamic stress disturbances in a conveyor belt, the material of which corresponds to the model of the Maxwell element, was obtained for a number of simple cases. A common way to solve this problem for more complex cases is to use numerical

methods [5, 10, 13]. However, a numerical experiment is not a sufficiently convenient tool for determining the general patterns between stress and deformation in a conveyor belt element, which creates the preconditions for the further development of analytical methods that allow, in a particular case, to study the propagation of longitudinal dynamic stress disturbances in a conveyor belt, the material of which corresponds to the Maxwell element model. An alternative way to solve the problem is that the analytical solution can be achieved by introducing simplifications in the formulation of the problem or in the course of its solution. The proposed simplification is based on the use of a small parameter, the presence of which reflects the essence of the formulation of the problem of determining the general patterns between stress and deformation in the model of the Maxwell element. In this regard, characteristic dimensionless numbers are introduced that determine the features of the viscoelastic process in the belt for studying the properties of the Maxwell element. The transition to dimensionless variables and the use of similarity criteria made it possible to generalize the research results for conveyor belts made of different materials. The next step of the study consisted in the fact that for the limiting values of the introduced similarity criteria, asymptotic models of the Maxwell element were obtained, which characterize the features of the viscoelastic process in a conveyor belt. Essentially, asymptotic Maxwell element models are represented as simple relationships between stress and deformation in a conveyor belt, some of which are consistent with Hooke's Law to the required degree of accuracy. The area of application of asymptotic models is determined in accordance with the characteristic values of the introduced similarity criteria for a viscoelastic process.

5. Study results from asymptotical approximation for the model of the Maxwell element

5. 1. Asymptotic analysis and justification for the application of the model of the Maxwell element

The relationship between stress $\sigma(t, S)$ and deformation $\varepsilon(t, S)$ can be represented in general form

$$\sigma(t, S) = E_c \varepsilon(t, S), \quad E_c = E_1 + iE_2,$$

$$|E_c| = \sqrt{E_1^2 + E_2^2}, \quad \operatorname{tg} \delta = E_2 / E_1, \quad (2)$$

where E_c is the complex modulus of elasticity of the material. The real part of the complex modulus of elasticity of the material $\operatorname{Re}(E_c) = E_1$ is the dynamic modulus of elasticity, which characterizes the process of energy transfer through the element of the conveyor belt. The imaginary part of the complex modulus of elasticity of the material $\operatorname{Im}(E_c) = E_2$ is the modulus of losses, which characterizes the process of dissipation of vibration energy in a viscoelastic body, during which the conveyor belt is heated. If at the point S_0 of the conveyor belt there is a dynamic, periodically varying stress $\sigma(t, S_0) = \varepsilon_0 \cos(\omega t)$, then taking into account the relations $E_c = |E_c| (\cos \delta + i \sin \delta)$, $\varepsilon(t, S_0) = \varepsilon_0 (\cos \phi + i \sin \phi)$ and, using the Moivre formula, the dependence follows:

$$\sigma(t, S_0) = \sigma_0 \cos(\omega t) = |E_c| \varepsilon_0 \cos(\delta + \varphi), \quad (3)$$

$$\varepsilon(t, S_0) = \frac{\sigma_0}{|E_c|} \cos(\omega t - \delta). \quad (4)$$

A viscoelastic element of a conveyor belt is characterized by a phase shift between stress and deformation, which is set by the tangent of the angle δ of mechanical losses. The dynamic modulus of elasticity and the modulus of loss are the main parameters that determine the propagation of longitudinal vibrations in a conveyor belt. One of the important problems in the analysis of the propagation of disturbances is to establish the dependence of the dynamic modulus of elasticity E_1 and the modulus of loss E_2 on the vibration frequency ω . Let's obtain the indicated dependencies for the models used to describe the process of propagation of longitudinal vibrations in a conveyor belt.

The equation defining the relationship between stress $\sigma(t, S)$ and deformation $\varepsilon(t, S)$ at point S_0 of the conveyor belt, the material of which corresponds to the model of the Maxwell element (Fig. 1), has the form

$$\frac{1}{E} \frac{d\sigma(t, S_0)}{dt} + \frac{\sigma(t, S_0)}{\eta} = \frac{d\varepsilon(t, S_0)}{dt}, \quad (5)$$

where E is the modulus of elasticity of the element; η is element viscosity.

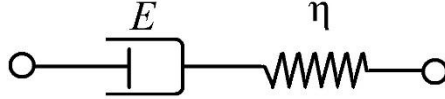


Fig. 1. Viscoelastic Maxwell element

Let us seek a solution to equation (5) in the form

$$\sigma(t, S_0) = \sigma_0 e^{i\omega t}. \quad (6)$$

Taking into account relations (2), after substituting expression (6) into (5), the equation is obtained

$$\frac{i\omega}{E} + \frac{1}{\eta} = \frac{i\omega}{E_c}, \quad (7)$$

which was used to determine the dynamic elastic modulus E_1 and loss modulus E_2 :

$$\begin{cases} \frac{\omega}{E} E_1 + \frac{1}{\eta} E_2 = \omega, \\ \frac{1}{\eta} E_1 - \frac{\omega}{E} E_2 = 0. \end{cases} \quad (8)$$

Introduced dimensionless parameters

$$\chi = \omega\eta / E, \quad \varepsilon_1 = E_1 / E, \quad \varepsilon_2 = E_2 / E, \quad (9)$$

are used to transform the system of equations (8) to the form

$$\begin{cases} \chi\varepsilon_1 + \varepsilon_2 = \chi, \\ -\varepsilon_1 + \chi\varepsilon_2 = 0. \end{cases} \quad (10)$$

The solution of the system of equations (10) made it possible to obtain the dependences of the dynamic modulus of elasticity ε_1 and the modulus of losses ε_2 on the parameter χ , which have the form (Fig. 2)

$$\varepsilon_1 = \frac{\chi^2}{1+\chi^2}, \quad \varepsilon_2 = \frac{\chi}{1+\chi^2}, \quad \text{tg}\delta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{1}{\chi}. \quad (11)$$

The parameter χ is the dimensionless vibration frequency

$$\chi = \omega \frac{\eta}{E} = \frac{\omega}{\omega_0}, \quad t_0 = \frac{\eta}{E} = 1/\omega_0. \quad (12)$$

where t_0 is the characteristic decay time of the oscillations. For a viscoelastic Maxwell element with parameters $E=2.5e8$ (Pa) and $\eta=1875$ (Nsec/m²), the characteristic decay time of oscillations is $t_0 \sim 10^{-5}$ (sec).

Using solution (11), the analysis of the dependence of the dynamic elastic modulus ε_1 and the loss modulus ε_2 on the dimensionless frequency χ of stresses fluctuations in the conveyor belt for the model of the Maxwell element is carried out. At large values of vibration frequencies ($\chi \gg 1$), the solution to equation (11) can be represented in the form

$$\varepsilon_1 \approx 1 - \frac{1}{\chi^2} + \dots \rightarrow 1, \quad \varepsilon_2 \approx \frac{1}{\chi} - \frac{1}{\chi^3} + \dots \rightarrow 0, \quad \text{tg}\delta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{1}{\chi} \rightarrow 0. \quad (13)$$

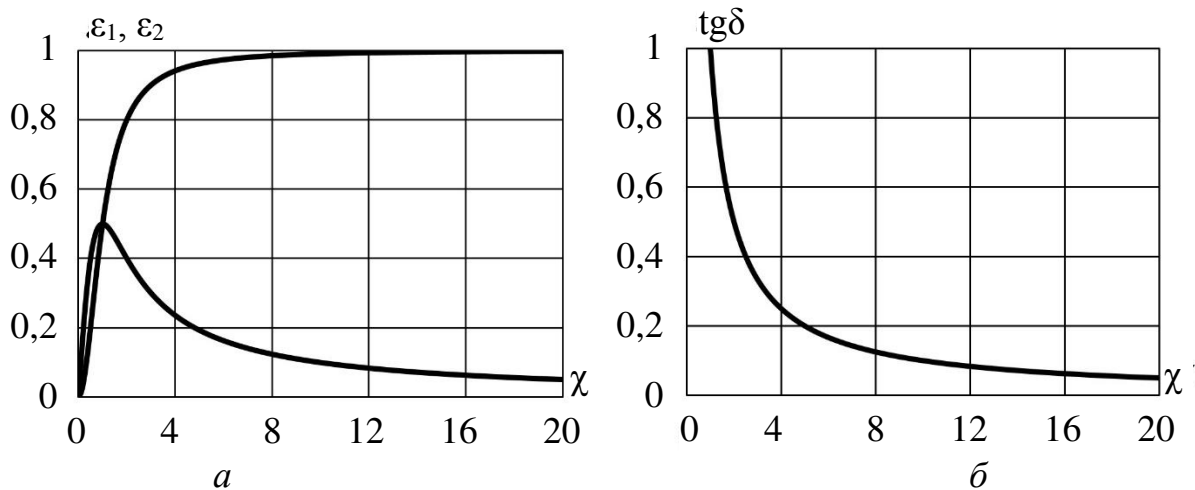


Рис. 2. Characteristics of a viscoelastic element depending on the frequency χ :

a – dynamic modulus of elasticity ε_1 , loss modulus ε_2 ; δ – angle of mechanical losses

At large values of dimensionless frequencies $\chi \gg 1$, the value of the dynamic modulus of elasticity ε_1 tends to the value of Young's modulus E . Losses that lead to the dissipation of elastic energy in the conveyor belt and are characterized by the modulus ε_2 , can be neglected (Fig. 2). There is no phase shift δ between stress $\sigma(t, S)$ and deformation $\varepsilon(t, S)$ at point S_0 of the conveyor belt. Thus, solution (2) for equation (5) is presented in the form of Hooke's law (1). For high vibration frequencies, the relationship between stress and strain in the model of the Maxwell element follows Hooke's law.

For small values of the parameter $\chi \ll 1$, the solution to equation (11) has the form

$$\varepsilon_1 \approx \chi^2 - \chi^4 + \dots \rightarrow 0, \quad \varepsilon_2 \approx \chi - \chi^3 + \dots \rightarrow 0,$$

$$\operatorname{tg} \delta = \varepsilon_2 / \varepsilon_1 = \chi^{-1} \rightarrow \infty. \quad (14)$$

The results obtained for the case of low vibration frequencies showed that the dynamic modulus of elasticity ε_1 does not have a nonzero value. The close-to-zero value of the elastic modulus does not agree with the experimental data. Therefore, to describe the dynamic viscoelastic properties at low vibration frequencies, the use of the model is incorrect due to the fact that the stress in the belt element tends to zero at an arbitrary value of the deformation of the viscoelastic element.

A qualitative assessment of the value of the characteristic damping time of oscillations t_0 in a viscoelastic element was made taking into account the results of studies of the experimental reference dependences storage $G_1(\omega) \sim E_1(\omega)/3$ and loss modules $G_2(\omega) \sim E_2(\omega)/3$ for materials of a conveyor belt, which were obtained in [14]. The results of the experiment [14] (Fig. 4), presented in the form of graphical dependencies $G_1(\omega)$, $G_2(\omega)$, , for the convenience of processing and perception of the results, are transformed into a tabular presentation (Table 1).

Table 1
Typical values of the parameters of a viscoelastic element [14]

$G_1(\omega)$, MPa	$E_1(\omega)$, MPa	$G_2(\omega)$, MPa	$E_2(\omega)$, MPa	f , Hz	ω , rad/sec
5,2	15,6	0,9	2,7	10^{-9}	$2\pi 10^{-9}$
11,2	33,6	1,4	4,2	10^{-5}	$2\pi 10^{-5}$
20,0	60,0	2,8	8,4	10^{-1}	$2\pi 10^{-1}$
22,0	66,0	3,0	9,0	$1/2\pi$	1
27,0	81,0	3,8	11,4	10^0	$2\pi 10^0$
33,0	99,0	5,0	15,0	10^1	$2\pi 10^1$
40,0	120,0	6,0	18,0	10^2	$2\pi 10^2$
170,0	510,0	90,0	270,0	10^5	$2\pi 10^5$
1700,0	5100,0	100,0	300,0	10^{10}	$2\pi 10^{10}$

1720,0	5160.0	20,0	60,0	10^{11}	$2\pi 10^{11}$
--------	--------	------	------	-----------	----------------

Taking into account the dimensionless expression (11), the characteristic damping time of oscillations t_0 , was determined by presenting the value of the dynamic elastic modulus $E_1(\omega)$ for an arbitrary value of the frequency ω and the dynamic modulus of elasticity $E_1(1)$ for the frequency $\omega=1$ in the following form:

$$E_1(\omega) = E \frac{\omega^2 t_0^2}{1 + \omega^2 t_0^2}, \quad E_1(1) = E \frac{t_0^2}{1 + t_0^2}. \quad (15)$$

Divide the first equation by the second equation, we get the equation

$$\frac{E_1(\omega)}{E_1(1)} = \omega^2 \frac{1 + t_0^2}{1 + \omega^2 t_0^2}, \quad (16)$$

which let's will solve for the characteristic time of damping of oscillations

$$t_0 = t_0(\omega) = \frac{1}{\omega} \sqrt{\frac{E_1(\omega)/E_1(1) - \omega^2}{1 - E_1(\omega)/E_1(1)}}. \quad (17)$$

Substitution of the values $E_1(\omega)$, $E_1(1)$, ω into the last equation made it possible to obtain a qualitative dependence of the characteristic decay time of oscillations $t_0(\omega)$ [sec] for the model of the viscoelastic Maxwell element on the frequency ω [rad/sec] (Fig. 3).

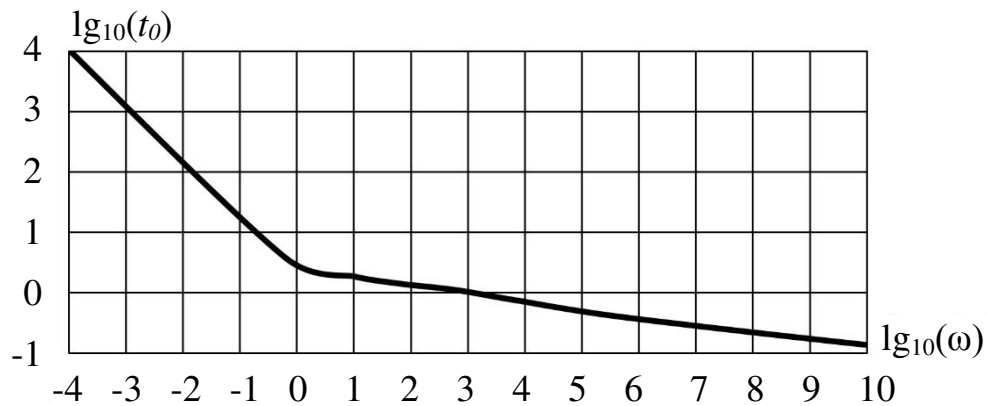


Fig. 3. The characteristic decay time of oscillations $t_0(\omega)$ [sec] for the model of viscoelastic Maxwell element

The length of time required for the transient mode of acceleration of the conveyor belt is several minutes [15–17], which significantly exceeds the characteristic decay time of high-frequency oscillations t_0 in a viscoelastic element. With a decrease in the frequency of oscillations, the characteristic damping time of oscillations t_0 in a viscoelastic element increases significantly (11), (17).

5. 2. Construction of asymptotic models for the main modes of operation of the transport conveyor

Let us consider the solution of equation (5) for some common cases of operation of a transport conveyor using conveyor belts made of a material whose characteristics correspond to the model of the viscoelastic element Maxwell element.

Let us introduce dimensionless parameters

$$\sigma_\chi = \sigma(t, S_0) / \sigma_0, \quad \varepsilon_\chi = \varepsilon(t, S_0) / \varepsilon_0, \quad \tau = t\omega. \quad (18)$$

Since the choice of the scale factors σ_0, ε_0 is arbitrary, then setting

$$\sigma_0 = E\varepsilon_0, \quad (19)$$

we obtain equation (5) in dimensionless form

$$\frac{d\sigma_\chi}{d\tau} + \frac{1}{\chi}\sigma_\chi = \frac{d\varepsilon_\chi}{d\tau}, \quad (20)$$

which we use to analyze the characteristic modes of operation of the transport conveyor. For extremely high vibration frequencies ($\chi \gg 1$), the solution to equation (20) is determined by the quasi-linear relationship between stress and deformation in a belt element (1).

5. 2. 1. The case of a constant deformation speed of a conveyor belt

This mode of operation of the transport system is characteristic of the initial tension of the conveyor belt at the start of the transport conveyor. In the presence of a constant deformation speed $d\varepsilon_\chi/d\tau = v_\varepsilon$, equation (16) can be represented as follows:

$$\begin{aligned} \frac{d\sigma_\chi}{d\tau} + \frac{1}{\chi}\sigma_\chi &= v_\varepsilon, \quad \sigma_\chi(0) = 0, \\ \frac{d\varepsilon_\chi}{d\tau} &= v_\varepsilon \equiv \text{const}, \quad \varepsilon_\chi(0) = 0. \end{aligned} \quad (21)$$

Let's write the solution of equations (21) in the form (Fig. 4):

$$\sigma_\chi = \chi v_\varepsilon (1 - \exp(-\tau / \chi)), \quad (22)$$

$$\varepsilon_\chi = v_\varepsilon \tau, \quad \tau / \chi = t / t_0. \quad (23)$$

At $\tau / \chi \gg 1$, it follows that the stress σ_χ for the case of a constant deformation speed tends to the value χv_ε . The deformation grows indefinitely (Fig. 4).

$$\varepsilon_\chi / \chi v_\varepsilon, \quad \sigma_\chi / \chi v_\varepsilon, \quad \varepsilon_1$$

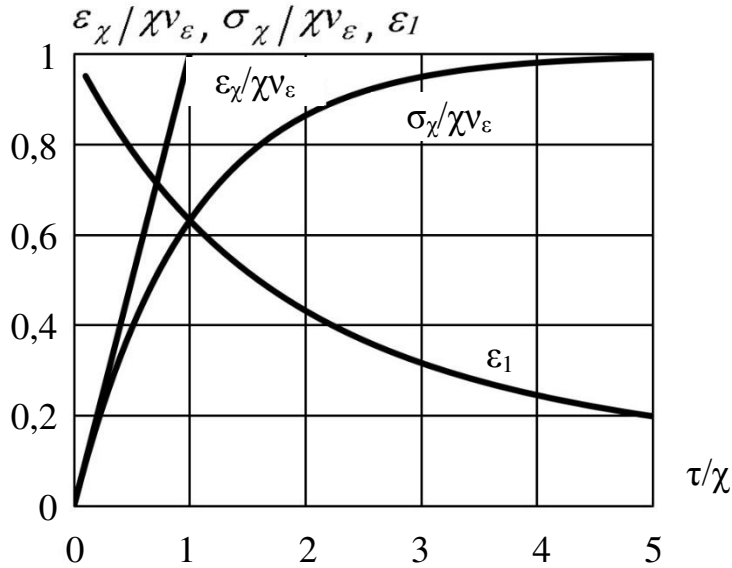


Fig. 4. Stress σ_χ , deformation ε_χ and dynamic modulus of elasticity ε_1 at a constant deformation speed of the conveyor belt

If the characteristic time of the deformation process t is much shorter than the relaxation time t_0 ($\tau/\chi \ll 1$), then

$$\lim_{\tau/\chi \rightarrow 0} (\sigma_\chi / \varepsilon_\chi) = \lim_{\tau/\chi \rightarrow 0} \varepsilon_1 = \lim_{\tau/\chi \rightarrow 0} \frac{(1 - \exp(-\tau/\chi))}{\tau/\chi} = 1. \quad (24)$$

At small values of the dimensionless parameter τ/χ the behavior of the viscoelastic Maxwell element of the conveyor belt obeys Hooke's law (1).

5. 2. 2. The case of a constant speed of stress change in the element of a conveyor belt

With a uniform distribution of material along the conveyor belt and a constant force of primary resistance, the stress in the belt changes linearly along the length of the section. If the conveyor belt moves at a constant speed, then the speed of change in the stress $d\sigma_\chi/d\tau = v_\sigma$ in the dS element during its movement as a result of material transportation will also be constant in magnitude. At a constant speed of stress change v_σ equation(16) takes the form:

$$v_\sigma + \frac{1}{\chi} \sigma_\chi = \frac{d\varepsilon_\chi}{d\tau}, \quad \varepsilon_\chi(0) = \varepsilon_{\chi 0},$$

$$\frac{d\sigma_\chi}{d\tau} = v_\sigma \approx \text{const}, \quad \sigma_\chi(0) = \sigma_{\chi 0}, \quad (25)$$

where the value of σ_χ is determined by the tension of the belt to eliminate its sagging during material transportation. The solution of equations (25) is presented in the form:

$$\varepsilon_\chi = \frac{V_\sigma}{2} \frac{\tau^2}{\chi} + \sigma_{\chi 0} \frac{\tau}{\chi} + V_\sigma \tau + \varepsilon_{\chi 0}, \quad \sigma_\chi = V_\sigma \tau + \sigma_{\chi 0}. \quad (26)$$

At $\tau/\chi \ll 1$ ($t \ll t_0$), the solution (26) implies a relationship between the magnitude of the stress and deformation of the belt element dS

$$\varepsilon_\chi \sim V_\sigma \tau + \varepsilon_{\chi 0} \sim \sigma_\chi - \sigma_{\chi 0} + \varepsilon_{\chi 0}. \quad (27)$$

The behavior of the viscoelastic Maxwell element of the conveyor belt follows Hooke's law. For the case $\tau/\chi \gg 1$, when the characteristic time of the process significantly exceeds the relaxation time ($t \gg t_0$)

$$\varepsilon_\chi \sim \frac{V_\sigma}{2} \frac{\tau^2}{\chi} + \sigma_{\chi 0} \frac{\tau}{\chi},$$

a nonlinear increase in the value of deformation ε_χ with time is observed.

5. 2. 3. The conveyor belt element is suddenly subjected to a constant load

The element of the conveyor belt is quite often subjected to a sudden change in the value of the load as a result of damage to the structural elements of the transport conveyor, which leads to a sharp increase in the value of the primary resistance to the movement of the belt. The constant stress $\sigma_{\chi 0}$, suddenly applied to the element dS of the conveyor belt, can be represented in the following form

$$\sigma_\chi(\tau) = H(\tau)\sigma_{\chi 0}, \quad H(\tau) = \int_{-\infty}^{\tau} \delta(\alpha) d\alpha,$$

$$H(\tau) = \begin{cases} 1, & \text{if } \tau \geq 0, \\ 0, & \text{if } \tau < 0, \end{cases} \quad (28)$$

where $H(\tau)$, $\delta(\tau)$ are the Heaviside function and the Dirac function, respectively. Substitution of the expression that determines the value of the suddenly applied stress (28) into equation (16) makes it possible to obtain an equation that determines the deformation of the belt element

$$\delta(\tau)\sigma_{\chi 0} + \frac{1}{\chi} H(\tau)\sigma_{\chi 0} = \frac{d\varepsilon_\chi}{d\tau}. \quad (29)$$

Having integrated the last equation, let's write down the solution for the load applied at the time $\tau=0$, in the form (Fig. 5):

$$\varepsilon_\chi(\tau) = H(\tau)\sigma_{\chi 0} + \int_{-\infty}^{\tau} \frac{1}{\chi} H(\alpha)\sigma_{\chi 0} d\alpha, \quad (30)$$

$$\varepsilon_\chi(\tau) = 0, \quad \tau < 0, \quad \varepsilon_\chi(\tau) = \sigma_{\chi 0}, \quad \tau = 0,$$

$$\varepsilon_{\chi}(\tau) = \sigma_{\chi 0} + \sigma_{\chi 0} \frac{\tau}{\chi}, \quad \tau > 0.$$

For an arbitrary moment of time when the load is applied $\tau = \tau_s$, the solution to equation (29) can be represented in the general form

$$\varepsilon_{\chi}(\tau) = H(\tau - \tau_s) \sigma_{\chi 0} + \int_{-\infty}^{\tau} \frac{1}{\chi} H(\alpha - \tau_s) \sigma_{\chi 0} d\alpha.$$

With an increase in the time of application of the load, the deformation increases linearly.

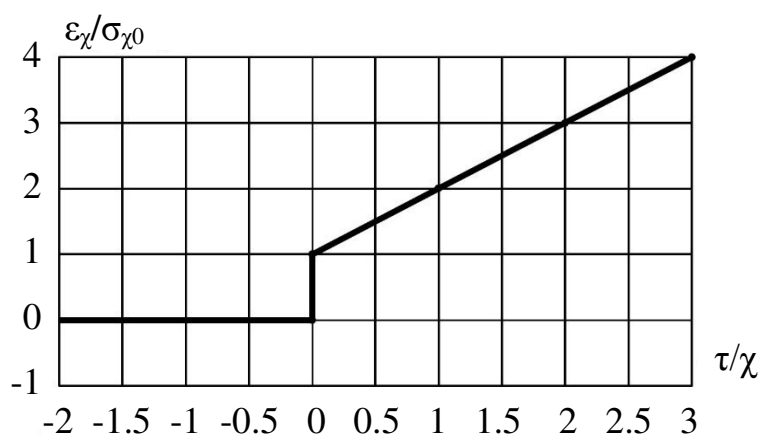


Fig. 5. Deformation of a conveyor belt element under a suddenly applied load $\sigma_{\chi 0}$

If the load is applied for a long time (even at small values), for the case $\tau/\chi \gg 1$, when the characteristic time of the process significantly exceeds the relaxation time ($t \gg t_0$), the deformation value exceeds the maximum permissible value. The deformation process becomes irreversible.

5. 2. 4. Element of a conveyor belt with an instantaneous force of resistance to the movement of the belt

Instantaneous loading can be caused by instant braking of a belt element of thickness b and width h as a result of instantaneous sharp jamming of moving or rotating parts of the conveyor structure. The resulting instantaneous stress $\sigma_{\chi}(\tau)$, caused by the instantaneously acting force of resistance to the movement of the belt $P_{\chi}(\tau)$ is determined in the following way

$$\sigma_{\chi}(\tau) = \delta(\tau) \sigma_{\chi t}, \quad \sigma_{\chi t} = \text{const}, \quad P_{\chi}(\tau) = \sigma_{\chi}(\tau) bh. \quad (31)$$

At the moment of jamming, a sharp increase in the force of resistance to the movement of the belt $P_{\chi}(\tau)$, is observed, the action of which is instantaneous, which leads to instant braking of the tape, followed by the restoration of the functioning of the transport system. Substitution of expression (31) into equation (16) allows one to obtain an equation for determining the change in the deformation of the element dS depending on time

$$\sigma_{\chi\tau} \frac{d\delta(\tau)}{d\tau} + \frac{1}{\chi} \delta(\tau) \sigma_{\chi\tau} = \frac{d\varepsilon_{\chi}}{d\tau}. \quad (32)$$

Having integrated the last equation, let's write the solution in the form (Fig. 6)

$$\varepsilon_{\chi}(\tau) = \int_{-\infty}^{\tau} \frac{1}{\chi} \delta(\alpha) \sigma_{\chi\alpha} d\alpha + \int_{-\infty}^{\tau} \sigma_{\chi\alpha} \frac{d\delta(\alpha)}{d\alpha} d\alpha, \quad (33)$$

$$\varepsilon_{\chi}(\tau) = \sigma_{\chi\tau} \delta(\tau) + \frac{1}{\chi} H(\tau) \sigma_{\chi\tau}, \quad \varepsilon_{\chi}(\tau) = 0,$$

$$\tau < 0, \quad \varepsilon_{\chi}(\tau) = \frac{1}{\chi} \sigma_{\chi\tau}, \quad \tau > 0, \quad (34)$$

where

$$\int_{-\infty}^{\tau} \sigma_{\chi\alpha} \frac{d\delta(\alpha)}{d\alpha} d\alpha = \sigma_{\chi\tau} \delta(\alpha) \Big|_{-\infty}^{\tau} - \int_{-\infty}^{\tau} \delta(\alpha) \frac{d\sigma_{\chi\alpha}}{d\alpha} d\alpha = \sigma_{\chi\tau} \delta(\tau),$$

$$\int_{-\infty}^{\tau} \frac{\delta(\alpha)}{\chi} \sigma_{\chi\alpha} d\alpha = \frac{H(\alpha)}{\chi} \sigma_{\chi\tau}.$$

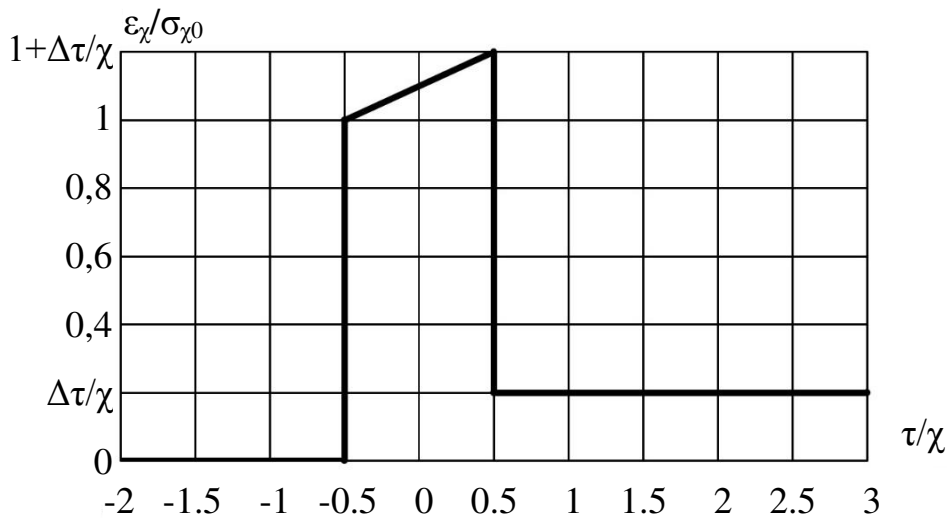


Fig. 6. Deformation of a belt element under an instantaneous applied load $\sigma_{\chi}(\tau) = \delta(\tau) \sigma_{\chi\tau}$

Analysis of solution (33) for values of time τ in the vicinity of zero using the Dirac function $\delta(\tau)$ and Heavisite $H(\tau)$ in the form

$$\delta(\tau) = \begin{cases} 1/\Delta\tau, & \text{if } |\tau| \leq \Delta\tau/2, \\ 0, & \text{otherwise,} \end{cases} \quad H(\tau) = \int_{-\infty}^{\tau} \frac{1}{\Delta\tau} d\alpha, \quad (35)$$

allowed to represent solution (33) in the form

$$\begin{aligned} \varepsilon_{\chi}(\tau) &= \frac{\sigma_{\chi\tau}}{\Delta\tau} + H(\tau) \frac{1}{\chi} \sigma_{\chi\tau} = \\ &= \frac{\sigma_{\chi\tau}}{\Delta\tau} \left(1 + H(\tau) \frac{\Delta\tau}{\chi} \right) = \sigma_{\chi} \left(1 + H(\tau) \frac{\Delta\tau}{\chi} \right), \\ \lim_{\Delta\tau \rightarrow 0} \varepsilon_{\chi}(\tau) &= \lim_{\Delta\tau \rightarrow 0} \sigma_{\chi} \left(1 + H(\tau) \frac{\Delta\tau}{\chi} \right) = \sigma_{\chi}. \end{aligned} \quad (36)$$

The behavior of the viscoelastic element for the considered case corresponds to the Hooke model.

6. Discussion of the results of the study of asymptotic models of the Maxwell element of the conveyor belt

The result of the study is an asymptotic analysis of the Maxwell element model. The peculiarities of the proposed method lie in the use of similarity criteria to describe viscoelastic processes in a belt, the material properties of which correspond to the model of the Maxwell element. Reducing the original model to a dimensionless form, the specific properties of which are characterized by the values of the similarity criteria of the process under consideration, making it possible to construct asymptotic models of the Maxwell element for some cases of the transport system functioning. Asymptotic models of a viscoelastic element in the form of simple analytical expressions that determine the relationship between stress and deformation of a conveyor belt element are convenient for practical use in solving specific engineering problems. The application of the proposed approach made it possible to replace the model of the Maxwell element with an asymptotic model close to the model of the Hooke element for individual modes of a transport conveyor operation.

The asymptotic analysis of the model of the Maxwell element clearly demonstrates that at sufficiently high frequencies of longitudinal stress fluctuations in the conveyor belt, at which the oscillation period is much less than the characteristic decay time of the oscillations, the model of the Maxwell element can be replaced with a sufficient degree of accuracy by the model of the Hooke element. This is due to the fact that over a period of time equal to the period of longitudinal oscillations, there is a slight change in stress associated with the component determined by the viscous properties of the model of the Maxwell element. In this case, the main contribution to the propagation of longitudinal vibrations along the conveyor belt is made, respectively, by the Hooke element (Fig. 1). A similar explanation can be given as a result of the analysis of the typical modes of operation of the transport conveyor. If the

characteristic time of the considered process of propagation of disturbances is much less than the characteristic time of damping of oscillations in the material of the conveyor belt, then the viscous properties of the Maxwell element can be neglected. In this case, as in the case of high frequencies of longitudinal stress fluctuations in the conveyor belt, the Maxwell element can be replaced with a sufficient degree of accuracy by the Hooke element. The analysis of the propagation of longitudinal disturbances in the material of a belt in the general case is based on the numerical solution of the wave equation, the foundation of which is the model of a viscoelastic element [5, 10, 11], in particular, the model of the Maxwell element in general form (5). This significantly complicates the construction of an analytical solution that determines the amount of dynamic stress in the material along the conveyor belt. An analytical solution can be obtained when using fairly simple models of a viscoelastic element [12], the model of the Hooke element. Thus, a natural step is to build simplified models of the viscoelastic Maxwell element for the main modes of operation of the transport system. The developed asymptotic models of the viscoelastic Maxwell element make it possible to construct an analytical solution of the wave equation for the corresponding operating modes of the transport conveyor, greatly simplifying the analysis of the propagation of longitudinal stresses in the material of the conveyor belt, which is a significant advantage of using the proposed approach. However, it should be noted that the area of application of asymptotic models is limited to the range of limiting values of the introduced similarity criteria for a viscoelastic process. A rather important result of the study is the fact that the use of the asymptotic approximation made it possible to substantiate and determine the area of application of the general original model of the Maxwell element (5). To construct asymptotic models of the Maxwell element, a linear approximation is used, which limits the accuracy of the model. Future research may be aimed at eliminating this drawback by replacing linear models with nonlinear models with a given degree of accuracy to solve the problem. The possibility and expediency of using nonlinear asymptotic models require additional research. The practical significance of the study lies in the use of the results obtained for the design of systems for optimal control of the conveyor belt speed, taking into account the restrictions on the phase coordinates and the propagation of dynamic stresses in the conveyor belt. A prospect for further research is the analysis of the propagation of stress disturbances in a conveyor belt for cases in which the properties of the conveyor belt material can be represented by asymptotic models of the Kelvin-Voigt element.

7. Conclusions

1. The use of the asymptotic approximation to analyze the characteristic modes of operation of an extended transport conveyor, the properties of the belt material of which corresponds to the Maxwell element, made it possible to:

a) determine, for the limiting values of similarity criteria, the relationship between stress and deformation;

б) justify the scope of the model of the Maxwell element.

It is shown that at high frequencies $\omega \gg \omega_0$, for which the oscillation period is much shorter than the relaxation time t_0 , the behavior of the viscoelastic Maxwell

element corresponds to Hooke's law. A qualitative analysis shows that for the oscillation frequencies $\omega=10^1\div 10^3$, the characteristic oscillation decay time is several seconds, $t_0=1\div 10$. Thus, in the case of a quasi-stationary mode of acceleration/deceleration of the belt during the transition period, which is several minutes, the disturbances that arise quickly damp out. The most dangerous is the initial moment of acceleration/deceleration of the belt. For small values of frequencies $\omega\ll\omega_0$ the application of the model of the Maxwell element requires additional justification.

2. Analysis of the main modes of operation of the transport conveyor made it possible to draw additional conclusions: for the case of a constant deformation rate of the conveyor belt, with a characteristic process time significantly exceeding the relaxation time $t\gg t_0$, the stress in the conveyor belt tends to a constant value. In this case, an increase in the magnitude of deformation reaches its limiting values. For small values of the characteristic time of the process in comparison with the relaxation time $t\ll t_0$ the model of the Maxwell element can be replaced by the model of the Hooke element. A similar situation is typical for the case of a constant speed of stress change in a conveyor belt element. Of practical interest is the analysis of the functioning of the transport conveyor for cases when a constant or instantaneous load is suddenly applied to a belt element. Of particular importance, in this case, is the analysis of the transient process with the characteristic time of the process $t\ll t_0$. In this case, the model of the Maxwell element can be replaced by the model of the Hooke element.

References

1. Zeng, F., Yan, C., Wu, Q., Wang, T. (2020). Dynamic behaviour of a conveyor belt considering non-uniform bulk material distribution for speed control. *Appl. Sci.*, 10, 4436, 1–19. <https://doi.org/10.3390/app10134436>
2. Yao Y, Zhang B. (2020) Influence of the elastic modulus of a conveyor belt on the power allocation of multi-drive conveyors. *PLoS ONE*, 15(7), e0235768, 1–19. <https://doi.org/10.1371/journal.pone.0235768>
3. Woźniak, Dariusz, (2020) Laboratory tests of indentation rolling resistance of conveyor belts. *Measurement*, 150, 107065, 1–19. <https://doi.org/10.1016/j.measurement.2019.107065>
4. Blazej, R., Jurdziak, L., Kirjanow-Blazej, A. et al. (2021). Identification of damage development in the core of steel cord belts with the diagnostic system. *Sci Rep* 11, 12349. <https://doi.org/10.1038/s41598-021-91538-z>
5. Sanjay G. Sakharwade, Shubharata Nagpal, (2019). Analysis of transient belt stretch for horizontal and inclined belt conveyor system, *International Journal of Mathematical, Engineering and Management Sciences*, 4(5), 1169–1179, <https://dx.doi.org/10.33889/IJMEMS.2019.4.5-092>
6. Manjgo, M., Piric, E., Vuherer, T., Burzic, M., (2018) Determination of mechanical properties of composite materials-the rubber conveyor belt with cartridges made of polyester and polyamide. *Annals of the Faculty of Engineering Hunedoara*, 16(1), 141–144. <http://annals.fih.upt.ro/pdf-full/2018/ANNALS-2018-1-22.pdf>
7. Ferry, J. (1980) *Viscoelastic Properties of Polymers*. Wiley, New York, p.672
8. Pihnastyi, O. (2019) Control of the belt speed at unbalanced loading of

the conveyor. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 6 122–129. <https://doi.org/10.29202/nvngu/2019-6/18>

9. Yang, G., (2014). Dynamics analysis and modeling of rubber belt in large mine belt conveyors. *Sensors & Transducers*, 81(10), 210–218, https://www.sensorsportal.com/HTML/DIGEST/P_2492.htm

10. Nordell, L., Ciozda, Z., (1984) Transient belt stresses during starting and stopping: elastic response simulated by finite element methods, *Bulk Solids Handling*, 4(1), 99–104. <http://www.ckit.co.za/secure/conveyor/papers/troughed/transient/transient-belt-stresses.htm>

11. Kulinowski, P., (2014) Simulation method of designing and selecting tensioning systems for mining belt conveyors. *Archives of Mining Sciences*, 59(1), 123–138. <https://doi.org/10.2478/amsc-2014-0009>

12. Pihnastyi, O., Khodusov, V. (2020). Hydrodynamic model of transport system. *East European Journal of Physics*, 1, 121-136. <https://doi.org/10.26565/2312-4334-2020-1-11>

13. Lu, Y., Lin, F., Wang, Yu., (2015). Investigation on influence of speed on rolling resistance of belt conveyor based on viscoelastic properties. *Journal of Theoretical and Applied Mechanics*, 45(3), 53–68. <https://doi.org/10.1108/JEDT-04-2015-0026>

14. Rudolphi, T., Reicks, A., (2006). Viscoelastic Indentation and Resistance to Motion of Conveyor Belts using a Generalized Maxwell Model of the Backing Material. *Rubber Chemistry and Technology*, 79(2), 307–319. <https://doi.org/10.5254/1.3547939>

15. Pihnastyi, O., Cherniavska, S., (2021). Analysis of stress in the conveyor belt (Maxwell–element model). *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 4 74–81. <https://doi.org/10.33271/nvngu/2021-4/074>

16. He, D., Pang, Y., Lodewijks, G., Liu, X., (2016). Determination of Acceleration for Belt Conveyor Speed Control in Transient Operation. *International Journal of Engineering and Technology* 8(3), 206–211. <http://dx.doi.org/10.7763/IJET.2016.V8.886>

17. Pascual, R. Meruane, V.& Barrientos, G. (2005). Analysis of transient loads on cable-reinforced conveyor belts with damping consideration, in: XXVI Iberian Latin-American Congress on Computational Methods in Engineering, CIL0620, 1–15, <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.494.34&rep=rep1&type=pdf>