A statistical foundation for the measurement of managerial ability

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Abstract: Demerjian, Lev, and McVay (2012) (DLM) provide a conceptual framework for the measurement of managerial ability using data envelopment analysis (DEA). We show that the DLM method provides a consistent estimator of managerial ability. The DLM approach to measuring managerial ability begins with the first stage estimation of firm efficiency in transforming inputs into outputs. The second stage removes the impact of contextual variables on the firm efficiency so that the residuals measure the impact of unobserved managerial ability. We leverage the properties of the DEA estimator (Banker and Natarajan 2008) to show that the DLM approach provides a statistically consistent estimator of the managerial ability's impact on firm efficiency.

Keywords: DEA, Efficiency, Managerial Ability, Simulation

1. Introduction

In their seminal work (2012), Demerjian, Lev, and McVay (hereafter DLM) provide a conceptual framework for the measurement of managerial ability using data envelopment analysis (hereafter DEA) relating input and output data. Managerial ability is an important input in the transformation of inputs to outputs that is likely to persist over time. However, the ability is hard to observe directly and incorporate in empirical research. Ever since the publication of Demerjian, Lev, Lewis, and McVay's (2013) paper in The Accounting Review, which first showed the connection between managerial ability and economic outcomes, the economic usefulness of the DLM measure has been manifested in several studies.[1] Despite the widespread use of this measure in empirical research, the extant literature has not yet established the statistical properties of the impact of the DLM measure of managerial ability on firm efficiency.

The neoclassical economics tradition conceptualizes a firm as transforming multiple inputs into multiple outputs. In addition to inputs and outputs, two types of additional factors describe this transformation process: contextual variables and managerial ability. The contextual factors include variables such as firm size, market share, cash availability, life cycle, operational complexity, foreign operations, which are not under the discretionary control of managers. Managerial ability, however, reflects manager-specific features such as ability, talent, or style (Demerjian, Lev, and McVay 2012).

The DLM method is a two-stage approach to measuring managerial ability wherein the first stage involves using DEA for the estimation of firm efficiency relating input measures to output measures. The second step of the DLM method is to remove the impacts of contextual variables[2] on the efficiency in transforming inputs into outputs. The residual obtained from the second-stage regression yields a noisy measure of managerial ability as it contains other factors on unobserved variables that are not included in the earlier stages.

Since its inception by Demerjian et al. (2012), the DLM approach has been gaining increasing popularity for a wide range of applications. In the literature, the DLM measure has been employed,

[1] A simple search in Google Scholar shows that there have been more than 1,130 and 850 citations for the Demerjian et al.' (2012) paper in Management Science and Demerjian et al.'s (2013) paper in The Accounting Review, respectively.

[2] Contextual variables are not the proxies for ability but are known to affect firm efficiency.

for instance, to explain outcomes such as earnings quality (Demerjian et al. 2013), income smoothing (Baik, Choi, and Farber 2020), credit risk assessment (Bonsall, Holzman, and Miller 2017; Cornaggia, Krishnan, and Wang 2017), internal control effectiveness (Ge, Koester, and McVay 2017), and corporate social responsibility (Yuan, Tian, Lu, and Yu 2019). The standard regression model employed for this exercise presumes that managerial ability and all other explanatory variables have all been measured with no error. However, managerial ability measure is a residual which includes other statistical noise. Even a correctly specified measurement of managerial ability results in attenuation bias. This should raise serious concerns among empirical researchers on drawing valid inferences with this inconsistent managerial ability estimate.

Despite the wide use of the DLM measure of managerial ability in various application areas, the existing literature has not yet established the statistical properties of the impact of this measure on firm efficiency. By leveraging the properties of the DEA estimator (Banker and Natarajan 2008), the present paper provides a rigorous statistical foundation of the impact of the DLM measure of managerial ability on firm efficiency and explores some implications for accounting research.

We begin developing our theoretical model by using a monotone increasing and concave production function where output is specified as a general function of multiple inputs and an error term. We then model the error term as consisting of four distinct components: a linear function of multiple contextual variables, a managerial ability term, a one-sided inefficiency term, and a two-sided random noise term. Except for the additional component involving a managerial ability term, our treatment of the error term is that of the composed error term by Banker and Natarajan (2008). This formulation allows us to develop a consistent estimator of the impact of the DLM measure managerial ability on firm efficiency.

To demonstrate the statistical consistency of the DLM measure of managerial ability, in the spirit of Banker and Natarajan (2008), we conduct simulations by generating observed data of 10 different sample sizes using one-output-two inputs Cobb-Douglas (C-D) production function. First, we use one contextual variable and one managerial ability variable. Second, we combine the realization of two inputs, a contextual variable, a managerial ability variable, a one-sided inefficiency, and a two-sided noise from known distributions with the C-D production function to generate values of the output. We then find that the coefficient of DLM measure of managerial ability possesses the nice desirable property of statistical consistency, which we have demonstrated using our focused simulation.

We organize the remainder of the paper as follows. We present the DLM model of managerial ability in §2 where we begin with the discussion of our general economic model followed by the associated data generating process to demonstrate the statistical consistency property of the impact of the DLM measure of managerial ability on firm efficiency. We discuss the caveats associated with the use of the DLM measure in §3 followed by some concluding remarks in §4.

2. DLM Model of Managerial Ability

2.1 Economic Model

Consider observations on $j = 1, \dots, N$ decision-making units (DMUs) or firms with each observation comprising a vector of M inputs $X_j \equiv (x_{1j}, \dots y_{Mj}) \in R_+^M$, a vector of S outputs $Y_j \equiv (y_{1j}, \dots y_{Sj}) \in R_+^S$, a vector of R contextual variables $Z_j \equiv (z_{1j}, \dots z_{Rj}) \in R_+^R$, and one managerial ability variable A_j that may affect the overall efficiency in transforming the inputs into the outputs. Notice while Y, X, and Z are all observed for each period t and may change from period to period, A is not observed but conceptualized as a persistent parameter.

To compute the impact of the DLM measure of managerial ability on firm performance, we follow the two-stage approach of Banker and Natarajan (2008) (hereafter BN). We start with an economic model using a single output[3] and multiple inputs where the data on N observations are generated from the true production function $\phi(X)$ and a composite error term ε^*. Furthermore, $\phi(X)$ is continuous, monotone increasing and concave in X, and is specified by the following equation:

$$y = \phi(X)e^{\varepsilon^*} \tag{1}$$

where the random variable representing the error ε^* is generated by the following process:

$$\varepsilon^* = v - u - \sum_{r=1}^{R} \beta_r z_r - \gamma A \tag{2}$$

Here, v represents random noise and has a two-sided distribution; u represents technical

[3] For expositional convenience, we represent the production correspondence using a single output. However, it is easy to see that the idea presented here carries over readily to the case of multiple outputs by considering expansion or contraction along the radial.

inefficiency[4] and has a one-sided distribution; the contextual variables $z_r, r \in \{1, \ldots, R\}$ are all non-negative; and the managerial ability measure A has a two-sided distribution. Except for the additional component involving managerial ability measure A, the specification of the composite error term in (2) is analogous to that of the DEA-based stochastic frontier framework by the BN approach.

Like in the BN approach, we impose the same structure on the probability density functions[5] to generate the input variable vector X, the contextual variable vector Z, the managerial ability measure A, the technical inefficiency u, and the noise v, which are all assumed to be independently distributed. And, we impose no restrictions on the joint distribution of the component random variables within the input vector X or the contextual variable vector Z. Stochastic variables have all each finite variance and the noise variable v has zero mean.

2.2 Estimation of the Impact of Managerial Ability on Firm Efficiency

If we specify $\phi(X)$ as $\phi(X; \mu)$ where μ represents the parameter vector determining the structure of production technology, we rewrite the production relationship in (1) as

$$\ln y = \ln \phi(X; \mu) - \sum_{r=1}^{R} \beta_r z_r - \gamma A + \varepsilon \quad (3)$$

where ε represents the error, attributed to only random noise v and technical inefficiency u and is defined as $\varepsilon = v - u$.

One can compute the impact of managerial ability on firm efficiency using either a parametric or non-parametric approach depending on whether the parametric specification of $\phi(X; \mu)$ is a priori known or not. In parametric approach, if any specific functional form (e.g., C-D, translog, CES, etc.) is assumed for $\phi(X; \mu)$, one can employ the method of ordinary least squares (OLS)[6] or maximum likelihood (ML)[7] estimation for regression equation (3) to generate a consistent

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[4] In the economics literature, the ratio of observed output y to potential output $\phi(X)$, i.e., $y/\phi(X)$ is referred to as technical efficiency, and the ratio of potential output $\phi(X)$ to observed output y minus one, i.e., $(\phi(X)/y) - 1$ is as technical inefficiency.

[5] $f_{x_i}(x_i) = 0 \; \forall \, x_i < 0$, $f_{z_r}(z_r) = 0 \; \forall \, z_r < 0$, $f_u(u) = 0 \; \forall \, u < 0$, $f_v(v) = 0 \; \forall \, |v| > V^M$ and $f_A(A) = 0 \; \forall \, |A| > A^M$ where V^M and A^M each represents the finite upper bound of v and A, respectively.

[6] It is because ε has finite variance (Schmidt 1976).

[7] The use of ML estimation additionally requires a specific parametric form for the probability density

estimate of managerial ability.[8]

In non-parametric DEA approach (Banker 1993), no prior information on the parametric specification of $\phi(X;\mu)$ is required except that is it is monotone increasing and concave in X. In this case, the BN approach suggests making use of any DEA model in the first stage to estimate individual firm efficiency, followed by OLS or maximum likelihood (ML) estimation method in the second stage regression to evaluate the managerial ability measure affecting firm efficiency.

In the next section, we turn to demonstrate how the two-stage BN approach provides a statistically consistent estimator of the managerial ability's impact on firm efficiency.

2.3 Statistical Consistency of the Measure of Managerial Ability

2.3.1 First Stage of the BN Approach

To estimate the efficiency ($\tilde{\varphi}_k$) of firm k, $k \in \{1, \dots, N\}$, we employ the following model (Banker, Charnes, and Cooper 1984):

$$\tilde{\theta}_k = \max \theta_k \tag{4}$$

$$\begin{aligned} \text{s.t.} \quad & \textstyle\sum_{j=1}^{N} \lambda_j x_{mj} \leq x_{mk} \ \ (m = 1, \dots, M), \\ & -\textstyle\sum_{j=1}^{N} \lambda_j y_{sj} + \theta_k y_{sk} \leq 0 \ \ (s = 1, \dots, S), \\ & \textstyle\sum_{j=1}^{N} \lambda_j = 1, \\ & \lambda_j \geq 0 \ (j = 1, \dots, N), \theta_k \geq 0. \end{aligned}$$

The idea underlying the firm efficiency evaluation program (4) is to reallocate the input vector X_k over all the N firms, and then run them with the respective intensities $\lambda_1, \lambda_2, \dots, \lambda_N$ to expand the observed output vector Y_k by an expansion factor θ_k. Let $\tilde{\theta}_k$ be the optimal value of program (4). The expanded $\tilde{\theta}_k Y_k$ is the potential output of firm k using the benchmark (best-

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function of ε. For instance, one can assume a normal distribution for v and half-normal (Aigner, Lovell, and Schmidt 1977) or exponential distribution (Meeusen and van den Broeck 1977) for u.

[8] Alternatively, one can employ a two-stage parametric approach (Pitt and Lee 1981, Kalirajan 1989), where one can estimate the production function $\ln y = \ln \phi(X;\mu) + \varepsilon^*$ in the first stage, and then regress the estimated residual with contextual variables and managerial ability measure, $\hat{\varepsilon}^* = -\sum_{r=1}^{R} \beta_r z_r - \gamma A + \varepsilon$ in the second stage. However, such a two-stage parametric approach generates estimates $\beta_r, r \in \{1, \dots, R\}$ and γ that are biased downward (Wang and Schmidt 2002).

practice) technologies that are signaled by $\lambda_j > 0$ in the program (4). The firm k's efficiency ($\tilde{\varphi}_k$) is then computed as the ratio of observed output Y_k to potential output vector $\tilde{\theta}_k Y_k$, i.e., $\tilde{\varphi}_k = 1/\tilde{\theta}_k$, which lies between 0 and 1.[9]

2.3.2 Second Stage of the BN Approach

To demonstrate the statistical consistency property of the impact of the DLM measure of managerial ability on firm efficiency, following the BN approach, we regress $\ln \tilde{\theta}$ with contextual variables (Z) and managerial ability measure (A) in the second stage, i.e.,

$$\ln \tilde{\theta} = \beta_o - \sum_{r=1}^{R} \beta_r z_r - \gamma A + \delta \qquad (5)$$

Except for the additional component, i.e., managerial ability measure (A), our specification of the regression equation (5) is analogous to the second-stage regression equation (11) of the BN approach. Since error term δ has zero mean and finite variance, OLS estimation of equation (6) yields a consistent estimate $\hat{\gamma}$ of managerial ability's impact on firm efficiency.[10]

We now turn to conduct a simple simulation to show the consistency property of the estimator of managerial ability's impact on firm efficiency.

3. Simulation Study

To generate the 'observed' sample data for the simulations, we represent the production technology $\phi(X;\mu)$ by the following two-input-one-output Cobb-Douglas production function:

$$\phi(X;\mu) = \mu_o x_1^{\mu_1} x_2^{\mu_2}$$

We generate input variables x_1 and x_2 from the uniform distribution over the interval $[5, 15]$. The coefficients μ_o, μ_1, and μ_2 determine the properties of production technology. We present the

[9] The firm-efficiency measure ($\tilde{\varphi}$) captures the effects of both contextual variables (i.e., firm-specific factors) and management characteristics. To find the measure of managerial ability, it is necessary to remove from the firm efficiency measure the effects of contextual variables that are expected to aid or hinder managers' efforts. Therefore, to estimate managerial ability measure, we run the following regression: $\ln \tilde{\varphi}_k = \alpha_o + \sum_{r=1}^{R} \alpha_r z_{rk} + \epsilon_k$. The estimated residual $\tilde{\epsilon}_k$ $(i.e., \ln \tilde{\varphi}_k - \tilde{\alpha}_o - \sum_{r=1}^{R} \tilde{\alpha}_r z_{rk})$ obtained from the OLS estimation of this regression model represents a noisy measure of managerial ability for firm k relative to the sample mean where $\tilde{\alpha}_o$ and $\tilde{\alpha}_r$ $(r = 1, \dots, R)$ represent the estimated values of α_o and α_r, respectively.

[10] See Appendix A for the proof that the BN approach yields a consistent estimator of managerial ability.

results for a set of coefficient values: $\mu_o = 1.5$, $\mu_1 = 0.4$, and $\mu_2 = 0.35$ (Banker, Gadh, and Gorr 1993) for $\phi(X; \mu)$ that is continuous, monotone increasing, and concave in the range $[5, 15]$.

Our experimental design also includes a contextual variable z and managerial ability measure A, which are generated from the uniform distribution in the ranges $[0, 1]$ (Banker and Natarajan 2008) and $[-0.415, 0.557]$ (Demerjian et al. 2012), respectively. We also assign values of 0.2 to β (Banker and Natarajan, 2008) and 0.6 to γ to capture the impacts of contextual variable and managerial ability measure, respectively. We generate the noise variable v from a mean of zero and standard deviation σ_v of 0.04 of a two-sided normal distribution (Banker and Natarajan 2008). Similarly, we also generate the inefficiency variable u from a mean of 0.12 (Banker 1993) and standard deviation σ_u of 0.15 of a one-sided half-normal distribution (Banker and Natarajan 2008).

For our simulation experiment, we consider observations of ten different sample sizes 10, 20, 20, 40, 50, 60, 70, 80, 90, and 100 to know at sample size the estimated managerial ability coefficient converges to its true value. For each of these sample sizes, we generated 20 sets of sample observations. In each of the 20 iterations, we estimated the impact of the managerial ability measure on firm efficiency for each of our sample sizes. We thus estimated 20 values for γ, the weight on the managerial ability measure A for each sample size. We use two performance measures - mean absolute deviation (MAD) percentage and root mean squared deviation (RMSD) percentage, which are, respectively, defined as

$$\frac{1}{0.6}\left(\frac{1}{20}\sum_{t=1}^{20}|\hat{\gamma}_t - 0.6|\right) \times 100$$

and

$$\frac{1}{0.6}\sqrt{\left(\frac{1}{20}\sum_{t=1}^{20}(\hat{\gamma}_t - 0.6)^2\right)} \times 100$$

where $\hat{\gamma}_t$ is the estimated value of γ_t for iteration t. Table 1 reports the performance scores of ten different sample sizes. As one can see, as the sample size increases, the average MAD% shows a monotonically declining trend, implying a consistent increase in the performance of the coefficient of our managerial ability estimator. One can also see in Fig.1 the spread of the MAD% score over 20 iterations for each sample size. The simulation evidence shows that the coefficient of the estimated managerial ability measure converges to its true value at the sample size of 60 if less than 10% error is considered as the norm. Remark that the performance of our managerial ability estimator is the worst at the sample size of 10 as this sample size is considered too small to estimate efficiency, given the number of inputs and outputs (two inputs and one output in our case).

Table 1. Performance comparison over different sample sizes

Sample Size (N)	Mean absolute deviation (%)	Root mean squared deviation (%)
N = 10	38.64	44.65
N = 20	20.71	25.41
N = 30	15.71	21.32
N = 40	18.33	20.63
N = 50	13.60	17.09
N = 60	9.89	12.03
N = 70	9.13	12.13
N = 80	6.37	8.17
N = 90	7.33	8.90
N = 100	5.32	6.69

Notes. Base case: β = 0.2, γ = 0.6, σ_u= 0.15, σ_v = 0.04, $E(e^u)$ = 0.89, number of iterations = 20 for each sample size.

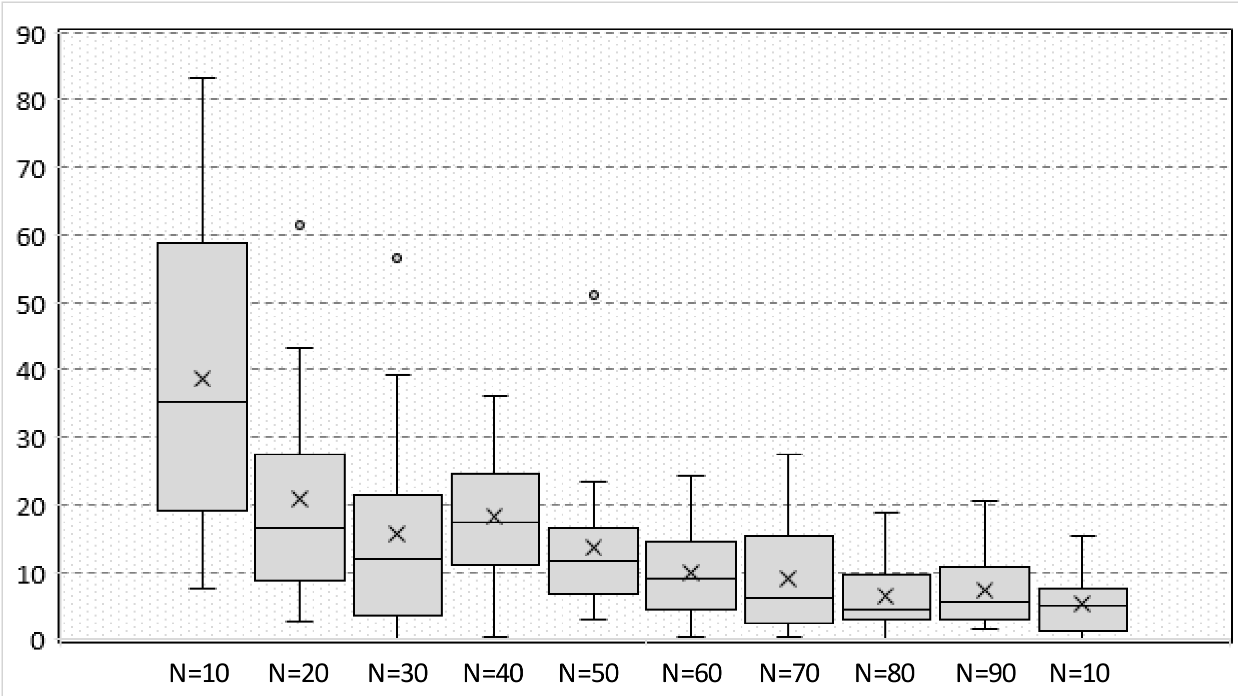

Fig. 1. Box & Whisker plot of MAD (%) scores over various sample sizes

The next section discusses some caveats to be considered when evaluating the two-stage approach of estimation of the DLM measure of managerial ability.

4. Caveats of the DLM Measure

Evaluation of managerial ability involves the estimation of firm technical efficiency using DEA in the first stage and the removal of the impacts of the contextual variables from firm efficiency using regression analysis in the second stage. For each stage, we suggest best practices to be followed when carrying out efficiency analysis by empirical researchers.

4.1 First-Stage Estimation of Firm Efficiency

4.1.1 Postulates on Returns to Scale

Though alternative returns to scale postulates[11] are available for modeling DEA technology, variable returns to scale (VRS) postulate is preferable as the efficiency frontier with VRS converges faster than efficiency frontiers with other returns to scale postulates as sample sizes

[11] These include constant returns to scale (CRS), non-increasing returns to scale (NIRS), non-decreasing returns to scale (NDRS), free disposal hull (FDH), among others.

increases (Banker and Park 2021).[12] Furthermore, the VRS postulate is more realistic in modeling multi-input and multi-output technology as it allows for the most efficient scale of operation (most productive scale size (MPSS) in the terminology of Banker, Charnes, and Cooper 1984), as has been mandated by economic theory. Therefore, due to the ability to incorporate standard production features and favorable finite sample properties, researchers may choose the VRS postulate in most cases.

4.1.2 Number of Inputs and Outputs

The number of inputs and outputs in DEA should be kept moderate. Prior literature suggests that DEA requires a larger number of observations than other parametric methods. The number of observations required for DEA to provide the same degree of confidence as regression is $N^{(d+1)/4}$ where d represents the sum of the number of inputs and outputs and N is the number of observations required for regression (Wilson 2018).[13] However, as the recommendations on the number of observations are based on analytic assumptions, they cannot diagnose the problem. Researchers can empirically diagnose a slow convergence rate problem by examining the distribution of efficiency scores. Even if the VRS estimator exhibits a faster convergence than other alternatives, it may have insufficient data to recover the efficiency frontier, introducing thus a significant upward bias in firm-efficiency estimates.

4.1.3 Industry-Specific Inputs and Outputs

Researchers may rely on industry-specific inputs and outputs rather than general inputs (costs) and outputs (revenue) while estimating industry-specific production function as they allow them to adjust prices in the second-stage estimation. On the contrary, while dealing with industry production function comprising multiple industries with many firms, researchers can consider using general inputs and outputs as they allow comparison over multiple industries.

4.1.4 Outliers

[12] Even if sample data exhibits CRS, efficiency frontier estimation with VRS exhibits a faster convergence (Kneip, Simar, and Wilson 2016).

[13] The literature (Cooper, Seiford, and Zhu 2004) also suggests a different number of observations for a given number of inputs and outputs. The number of observations N should be greater than $2MS$, where M and S are the number of inputs and outputs, respectively.

DEA efficiency estimates can be sensitive to outliers. The outlier efficient units are the units exhibiting unusually high-efficiency scores by having extremely high outputs and low inputs. If the efficiency frontier consists of these outlier units, the efficiency scores of all other units are biased downward. Researchers can therefore consider removing outlier efficient units while carrying efficiency analysis by using the super-efficiency estimation method of Banker and Chang (2006).

4.1.5 Choice of Estimation Sample and Reference Groups

In any benchmarking exercise, researchers often face a trade-off between a fair comparison by selecting a homogenous sample in terms of production function (e.g., industry-year sample) and ensuring the reliability of estimates by choosing a broad reference group (e.g., pooled, year or industry sample). Under an ideal condition of a sufficiently large sample, estimation by industry-year is most desirable. Each industry may have a unique production process. The industry-specific properties (such as factor productivity and MPSS) of a production function define the efficiency frontier against which firms are benchmarked. Industry-year estimation can also account for time-varying productivity change unique to each industry.

However, variations in the sample size among the industry-years result in a systematic bias in firm-efficiency estimates. Any noise in the data can bias the efficiency frontier upward. As a result, firm-efficiency scores are biased downward, on average, in a finite sample (Banker 1993; Gstach 1998). If the sample size is too small to satisfy the minimum requirements for the number of inputs and outputs, systematic bias in firm efficiency is upward. Industry-year estimation in such a case is likely to overestimate industry-year groups' firm efficiency.

While researchers estimate firm efficiency in the first stage by year or industry, they might face the risk of having inconsistent reference groups. Estimation by year may overestimate firm efficiencies in a productive industry, which in turn carries over to the second-stage estimation of managerial ability unless an adjustment for industry heterogeneity is made. Similarly, the industry-by-industry estimation may overestimate firms' managerial ability in periods of favorable economic conditions and underestimate in the recession periods. It can therefore be advantageous to maintain similar sample sizes across sub-samples of the first-stage estimation in which case year-by-year estimation is advantageous over industry-by-industry estimation. On the other hand, the industry-by-industry estimation may be advantageous if the industry-year production function follows VRS with MPSS unique to each industry.

4.2 Second-Stage Estimation of Managerial Ability

In this section, we discuss how the firm efficiency with adjustments for the potential biases in the second stage may yield valid managerial ability estimates.

4.2.1 Consistency of the DLM Estimator of Managerial Ability

Estimation of managerial ability requires the removal of the impacts of contextual variables on the firm-efficiency measure. Following Banker and Natarajan (2008), one can do this by regressing firm-efficiency score on the contextual variables and taking residuals of the regression as the measure of unobservable managerial ability on firm efficiency. By leveraging the properties of the DEA estimator (Banker and Natarajan 2008), we have shown in our simulation experiment that the DLM approach provides a statistically consistent estimator of managerial ability.

4.2.2 Adjustments for Reference Groups Heterogeneity

It is necessary to appropriately adjust the first-stage firm-efficiency score to account for industry and year's unmodeled heterogeneity in the second stage to obtain a valid measure of managerial ability.

Fixed effects account for heterogeneity not modeled in the first stage. The second-stage regression can consider including industry effects or year-effects depending on how the first stage estimates firm efficiency. Year-by-year estimation of firm-efficiency score in the first stage yields an efficiency score over the yearly average. Without an average industry adjustment, firm efficiencies in more productive industries appear excessively higher, biasing thus their managerial ability measure upward. Industry-fixed effects remove these potential biases from the managerial ability measure. Similarly, researchers can include year-effects in the second stage for each industry using firm efficiency from industry-by-industry estimation (Demerjian, Lev, and McVay 2012). The fixed effects adjust the mean-efficiency score for each year. However, if researchers decide to implement a pooled estimation in the first stage, they can include industry-year fixed effects. The resulting managerial ability represents the incremental managerial ability relative to the year and industry average. Any inference becomes then most precise when researchers examine the managerial ability measure and outcomes within industry-year.

4.2.3 Selection of Contextual Variables

Researchers need to be careful enough not to mix up managerial-ability proxies with the set of contextual variables used in the second-stage regression. The failure to do so prevents researchers from detecting the relationship between managerial ability and outcomes. As a result, the managerial ability measure becomes more like a statistical noise and is hence not appropriate for drawing valid inferences.

Nonetheless, researchers need to include a comprehensive set of identifiable variables affecting firm efficiency to avoid Type-I-Error in the inference with managerial ability. Since managerial ability is a residual, adding a variable makes the managerial ability independent of the included variable. As a result, when the managerial ability is used to predict any outcome variable, its coefficient is not affected by the included variable. However, not including the variable may result in the coefficient of managerial ability measure in picking up the relationship between the excluded variable and an outcome, resulting thus in a Type-I-Error.

4.2.4 Functional Form and Distributional Assumption of the Second-Stage Regression

As regards the choice over the functional forms of the second-stage regression, the literature recommends the use of OLS regression while suggesting some alternatives. Given that the firm-efficiency scores lie between zero and one, researchers can consider the logarithmic transformation of firm efficiency in the second-stage regression as it may improve the distributional fit (Wooldridge 2010). Alternatively, researchers can estimate fractional logit for variables ranging between zero and one (Papke and Woolddridge 1996).

Literature also suggests that Tobit regression may be an alternative but OLS regression is more robust than Tobit. Although firm efficiency score lies between zero and one naturally and not censored or truncated, Tobit regression may perform well if the distributional assumption is satisfied with the data (Simar and Wilson 2007). The evidence indeed suggests that a functional form with a specific distributional assumption performs better when the data precisely satisfy the assumption. Still, the OLS performs robustly when the distributional assumptions are not satisfied (Banker, Natarajan, and Zhang 2019). Researchers may therefore use OLS for robustness. Finally, taking log or not, or using OLS is an empirical choice. We however suggest OLS for robustness. Researchers can decide on whether to take a log or not depending on the model fit.

4.3 Managerial Ability as a Dependent or Independent Variable

While examining the determinants of managerial ability, researchers should be careful enough not to consider managerial ability as a dependent variable in the regression analysis.[14] It is because any inference drawn based on this regression is sensitive to noise in the estimation of managerial ability. For instance, a contextual variable omitted in the second-stage regression leads to residuals having information on that omitted contextual variable. The estimated managerial ability measure represents then a linear combination of true managerial ability and the omitted contextual variable. In such a case, researchers are more likely to find a relationship between the omitted contextual variable and a presumed managerial ability determinant,[15] which will result in the regression of the managerial ability estimate prone to Type-I-Error. However, research can consider using managerial ability as a dependent variable only when their second-stage regression model is well specified.

In most empirical studies, it is a common practice to use the DEA-based measure of managerial ability as an independent variable to explain an outcome variable. In the pursuit of further empirical studies in the future, researchers should however be careful enough not to consider ability proxies with managerial ability to explain an outcome variable as they may confound the coefficient of managerial ability. Furthermore, even if second-stage regression is correctly specified, the managerial ability coefficient may suffer from attenuation bias in which case the quantitative interpretation based on the magnitude of this coefficient becomes problematic. To draw a valid inference based on quantitative interpretation, one may consider calculating the lower and upper bounds of managerial ability coefficient by using a reverse regression of managerial ability on the dependent variable and other control variables (Klepper and Leamer 1984).

5. Concluding Remarks

The DLM approach has become a well-grounded tool both empirically and theoretically for

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[14] To our knowledge, the exception is due to Baik, Chae, Choi, and Farber (2013) who use the Malmquist index as a dependent variable.

[15] The estimation problem is not that serious when a mis-specified managerial ability measure is used as an independent variable as the other control variables in the regression analysis of determinants of managerial ability are like to minimize the errors due to misspecification. However, control variables do not offer any respite when misspecification occurs in the measurement of the dependent variable.

measuring managerial ability, given its vast use in the literature. In this paper, we leverage the statistical properties of DEA estimators (Banker and Natarajan 2008) to demonstrate that the DLM approach provides a statistically consistent estimator of managerial ability.

We point to some avenues for future research. First, an extensive simulation study is required to measure the impact of errors-in-variables in the second-stage estimation while drawing an inference in the regression analysis of using managerial ability to explain outcomes. This will help researchers build a comprehensive set of contextual variables in the second stage so that they can mitigate the potential errors while drawing inference. Second, to the extent that there is a process by which managerial ability persists over time, it necessitates a simulation study on forecasting the managerial ability of a firm based on its past managerial abilities. Finally, the question that should be of interest is whether the convergence of managerial ability estimator to its true value may depend on the sample size (N) growing large or the number of time periods (T) for which a firm is observed is large. Simulation exercises reported in future research should examine this issue whether it depends on the speed of convergence.

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Appendix A

The proof is like that of Banker and Natarajan (2008) except that we have an additional component involving managerial ability measure A in our composed error term.

The first stage of the BN approach involves the use of the BCC model to estimate firm efficiency. Following the BN approach, we define the frontier by shifting the production function $\phi(X;\mu)$ by the finite upper bound value of random noise V^M, i.e.,

$$\tilde{\phi}(X;\mu) = \phi(X;\mu)e^{V^M} \text{ or } \ln\tilde{\phi}(X;\mu) = \ln\phi(X;\mu) + V^M \quad \text{and} \tag{A.1}$$

$$\ln\tau = (\varepsilon - V^M) - \sum_{r=1}^{R}\beta_r z_r - \gamma A$$

$$= (v - V^M) - u - \sum_{r=1}^{R}\beta_r z_r - \gamma A \tag{A.2}$$

Substitution of (A.1) and (A.2) into (1) yields

$$\ln y = \ln\tilde{\phi}(X;\mu) + \ln\tau \tag{A.3}$$

Since $\tau \leq 1$, equation (A.3) resembles the usual DEA model where the deviation of observed output y from the frontier output $\tilde{\phi}(X;\mu)$ is described as efficiency τ.

On comparison between (1), (A.2) and (A.3), one can see that

$$\ln\tau = -\sum_{r=1}^{R}\beta_r z_r - \gamma A - \bar{\varepsilon} \tag{A.4}$$

where $\bar{\varepsilon} = V^M - \varepsilon \geq 0$. One can clearly see that equation (A.4) resembles with the traditional parametric specification of production function where one can interpret τ as output and z_r and A as inputs.

By defining $\beta_o = E(\varepsilon) - V^M$ and $\delta = \varepsilon - E(\varepsilon)$, we rewrite (A.4) as

$$\ln\tau = \beta_o - \sum_{r=1}^{R}\beta_r z_r - \gamma A + \delta \tag{A.5}$$

Clearly, the error term δ has zero mean and a finite variance. Since the true efficiency τ is not known, we can replace it with DEA estimator $\tilde{\theta}$, and then run the regression equation (A.5) with OLS in the second stage to obtain consistent estimators of contextual variable coefficient $\beta_r, r \in \{1, \dots, R\}$ and managerial ability coefficient γ.