Pricing and Risk Mitigation Analysis of a Cyber Liability Insurance using Gaussian, t and Gumbel Copulas – A case for Cyber Risk Index

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ABSTRACT

Cyber risk, a type of operational risk, is today considered a key component in the enterprise risk management framework. Under BASEL regulations, a bank could recognize the risk mitigating impact of the Cyber Liability Insurance (CLI) contract while calculating the minimum operational risk capital requirement. Despite this benefit and the onerous data protection acts, organizations are still reluctant to buy CLI contracts.

In this work, we price and analyze a CLI contract using Gaussian, *t* and Gumbel copulas and evaluate the contract's cyber risk mitigation effectiveness. We find that the current structure of the CLI contract with the limits and sub-limits may be inefficient at mitigating the cyber risk especially if the cyber risk losses were correlated and showed upper tail dependency. We then propose a case for a traded index for the cyber risk similar to the Property Claim Services (PCS) index for the catastrophic risk. A traded cyber risk index could offer wider cyber risk hedging alternatives to the insurers. Given such risk hedging alternatives, the insurers may have lower impetus to set conservative limits in the CLI contracts thus making the contracts more effective in mitigating the cyber risk of the organizations.

KEYWORDS

Interplay between finance and Insurance, cyber risk index, cyber liability insurance pricing, Gaussian, t and Gumbel copulas, operational risk, value at risk (VaR), conditional tail expectation (CTE), BASEL regulations, pricing of contingent claims in incomplete markets, Monte Carlo simulations

JEL Classification: G13, G21, G22, G28

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1. INTRODUCTION

Despite a plethora of cyber liability insurance (CLI) products in the market, organizations are still reluctant to buy them largely due to the lack of standardization and an inadequate coverage [Scharf (2014), Richards (2014)]. The randomness of loss occurrence, information asymmetries and cover limits are the main factors that impede the market development of the cyber liability insurances [Bandyopadhyay, Mookerjee, Rao (2009), Biener et. al. (2015)]. Despite these factors the cyber liability insurance market could grow as the insurance risk pool becomes larger and more data becomes available. This issue of limited success of the CLI contracts is pertinent for the banking sector as the cyber risk is a type of operational risk[3]. Under one of the prescribed approaches for the operational risk measurement (Advanced Measurement Approach) in BASEL regulations, a bank is allowed to recognize the risk mitigating impact of the insurance while calculating the minimum regulatory capital requirement [Basel Committee (2006)]. Furthermore, today cyber risk is also considered as a key component in the enterprise risk management frameworks and the data protection acts and corresponding fines are getting more onerous [World Economic Forum (2015), IT Governance UK (2015)]. Thus it is important to analyze the effectiveness of the CLI contract in mitigating the cyber risk losses.

In this paper, we price a CLI contract and analyze its effectiveness in mitigating the cyber risk. We demonstrate our method by providing a numerical example based on simple Monte Carlo simulations. Our objective though is not to undertake a survey of various CLI contract structures available in the market today. We then propose a case for a cyber risk index – an index similar to Property Claim Services (PCS) index for catastrophic risk. If an index were to track the cyber risk claims filed by the CLI contract holders with the insurers then this index could be used to develop derivatives that could serve as hedging tools for the insurers.

The paper is structured as follows: In section 2 we describe and price a CLI contract and measure the effectiveness of this contract structure in mitigating the cyber risk losses under the assumption that the sub-cyber risks are independent. Section 3 performs this same analysis under the assumption that the sub-cyber risks are correlated and exhibit upper tail dependency and we use Gaussian, *t* and Gumbel copulas to capture this dependency. We then take insurer's perspective and discuss a possible rationale in adopting the discussed CLI contract structure. In section 4 we propose a case for a cyber risk index by showing that such an index could help reduce the premium for the CLI contract and provide an impetus to the insurers to design less conservative CLI contracts. Section 5 concludes the paper.

[3] *Basel regulations define operational risk as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events [Basel Committee (2006)].*

2. PRICING AND ANALYSIS OF A CLI CONTRACT WHEN SUB – CYBER RISKS ARE INDEPENDENT

CLI contracts are largely stop loss insurances. Under an example CLI contract that we consider in this work, a claim is causally attributed to a sub-cyber risk and a sub limit (l_i) i.e. limit on the liability of the insurer, is imposed on each individual claim under a sub-cyber risk category[4]. Furthermore, there is also an aggregate limit of liability (L) imposed per policy period on the aggregate claims under all the sub-cyber risks covered under the CLI contract. Note: retention is not applied to the sub limits[4]. The table below lists six sub-cyber risks in our example CLI contract.

Table 1: Structure of an example CLI contract[4]

Sub-cyber risk Loss, (X_i)	**Cause of the Claim/ Loss (Sub-cyber risks)**	**Sub Limit, USD,** l_i
X_1	Data administration related investigations	l_1
X_2	Data administration related fines	l_2
X_3	Pro-active forensic services	l_3
X_4	Company's reputation	l_4
X_5	Individual's reputation	l_5
X_6	Restoring, recreating, or recollecting electronic data	l_6

Each sub-cyber risk loss process with limits ($X_i^l(t)$) (limits denoted by superscript – l) and aggregate cyber risk loss process with limits ($Y^l(t)$) for our example CLI contract can be expressed as:

$$X_i^l(t) = \sum_{j=1}^{N_i(t)} \min\left(l_i, S_{ij}\right) = \sum_{j=1}^{N_i(t)} \left(l_i - max\left(\left(l_i - S_{ij}\right), 0\right)\right), \ t \geq 0 \tag{1}$$

$$\begin{aligned} Y^l(t) &= min\left(L, \textstyle\sum_{i=1}^{n} X_i^l(t)\right) \\ &= L - \max\left(\left(L - \sum_{i=1}^{n} \sum_{j=1}^{N_i(t)} \left(l_i - max\left(\left(l_i - S_{ij}\right), 0\right)\right)\right), 0\right), \ t \geq 0 \end{aligned} \tag{2}$$

Where (S_{ij}) is the severity of the j^{th} claim pertaining to the i^{th} sub-cyber risk (as defined in the Table 1), ($N_i(t)$) is the number (frequency) of claims pertaining to the i^{th} sub-cyber risk and n is the number of the sub-cyber risks, in our case $n = 6$. We wish to analyze if the aggregate limit and the sub limits in the structure of the CLI contract affect the contract's effectiveness as a tool to mitigate cyber risk losses. For the comparison, we also analyze a CLI contract without the aggregate limit and the sub limits (no limits denoted by superscript - *nl*) and juxtapose its results with our example CLI contract with limits. Each sub-cyber risk loss process ($X_i^{nl}(t)$) and aggregate cyber risk loss process ($Y^{nl}(t)$) without the limits can be expressed as:

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[4] *http://www.aig.com/chartis/internet/uk/eni/AIGPROFCYBER%20CyberEdge%20Cyber%20Liability%20Insurance_tcm2538-409823.pdf*

$$X_i^{nl}(t) = \sum_{j=1}^{N_i(t)} S_{ij}\,,\ t \geq 0 \tag{3}$$

$$Y^{nl}(t) = \sum_{i=1}^{n} \sum_{j=1}^{N_i(t)} S_{ij}\,,\ t \geq 0 \tag{4}$$

Any CLI contract with the cyber risk loss as an underlying is currently priced in the incomplete markets because the underlying cyber loss is not a tradable asset and hence a replicating portfolio cannot be built using such an underlying. Thus there is no unique price for a CLI contract. But if the markets were liquid, in order to avoid arbitrage opportunities, the CLI contracts in the market may satisfy some internal consistency relationship between them. But currently the CLI contracts are tailored to the client's need and are thus not liquid. Hence the market price of cyber risk is not easily known. If the cyber risk were assumed to be not diversifyable, the insurer could price the CLI contract using the standard Utility Principle[5] [Delbaen and Haezendonck (1989), Embrechts (1996), Gerber and Pafumi (1998), Gritzalis et. al. (2007), Yannacopoulos et. al. (2008)]. Under this principle the total premium for underwriting a CLI contract will be such that the utility of the initial wealth of the insurer is same as the expectation of the utility of the summation of the initial wealth and the premium charged less the expected payoff under the contract [Yannacopoulos et. al. (2008), Shah, Dahake and Sri Hari Haran (2015)]. In this and the following section, we assume that the cyber risk is idiosyncratic and diversifyable and thus the market price of risk is zero. This assumption implies that the risk neutral distribution (***Q*** measure) of the cyber risk coincides with the real life distribution (***P*** measure). Thus the premium charged by an insurer at the time *t*, ($\Pi(t)$) for underwriting a CLI contract is equal to the expectation of the discounted cyber risk loss *(X)* at time *T* under the ***P*** measure. For a constant interest rate ($\Pi(t)$) is given as [Bjork (2009)]:

$$\Pi(t) = e^{-r(T-t)} E_t^P[X(T)] \tag{5}$$

This financial pricing is now equivalent to the pure premium or equivalence premium actuarial pricing principle [Bühlmann, (1980), Mikosch (2009),]. For zero interest rate, the premium charged ($\Pi(t)$) is given as:

$$\Pi(t) = E_t^P[X(T)] \tag{6}$$

We use variance, value and risk (VaR) and conditional tail expectation (CTE) as risk measures for the cyber risk loss that the CLI contract is supposed to mitigate. VaR is a α^{th} quantile of a cyber risk loss *(X)* distribution and CTE is average loss if VaR were exceeded. Formally, VaR and CTE of a loss variable *(X)* for a given quantile (α) at given time t are defined as [McNeil, Frey and Embrechts, (2005), Panjer (2006)]:

$VaR_\alpha(X) = F_X^{-1}(\alpha)$, for $VaR_\alpha(X) = x_\alpha$

5 *The applicability of the Utility principle is plagued with the difficulty of choosing an appropriate utility function. The standard deviation principle is more frequently used for pricing the insurance contracts in the property and casualty insurance [Bühlmann (2005)].*

$$CTE_\alpha(X) = E[X \mid X > x_\alpha] = \frac{\int_{x_\alpha}^{\infty} x f(x) dx}{1-\alpha}$$

Here we refrain from further discussion on these risk measures and direct the reader to the work by McNeil, Frey and Embrechts (2005). We measure the effectiveness of a CLI contract in mitigating cyber risk using a ratio of the extent of risk covered by the CLI contract (measured using a risk measure) to the premium charged by the insurer to underwrite the CLI contract. Following ratios are similar to the coefficient of variation which is a standardized measure of dispersion in the probability theory:

$$\frac{\sqrt{Var(X)}}{\Pi(t)}, \frac{VaR_\alpha(X)}{\Pi(t)} \; and \; \frac{CTE_\alpha(X)}{\Pi(t)}$$

We assume that the sub-cyber risk loss process without limits, is a compound Poisson[6] [Bowers et. al. (1996), Mikosch (2009)], cyber loss severities (S_{ij}) for each sub-cyber risk loss (X_i) are independent and identically Gamma distributed i.e. $(S_{i1}, S_{i2}, \ldots)$ severities are independent and identically distributed with Gamma distribution and the frequency $N_i(t)$ is independent of the severities and follows a homogenous Poisson process as given below:

$$S_{ij} \sim GAM(\theta_i, \kappa_i)\,, \qquad s_{ij} > 0,\ \theta_i > 0, \kappa_i > 0$$
$$where\ \theta_i\ is\ the\ scale\ parameter\ and\ \kappa_i\ is\ the\ shape\ parameter$$

$$N_i(t) \sim POI(\lambda_i t), \qquad \lambda_i > 0$$
$$where\ \lambda_i is\ the\ intensity\ of\ a\ homogeneous\ Poisson\ process\ N_i(t)$$

The literature is divided on the nature of the severity distribution of the actuarial losses and assumptions of log-normality or Gamma are more common [Fu and Moncher (2004)]. We adopt a Gamma distribution in this analysis because Gamma distribution is closed under convolutions and such an assumption would make the analysis more simple and transparent without affecting our conclusions qualitatively. Furthermore, in this section the sub-cyber risk losses are assumed to be independent but we relax this assumption in the next section. It is trivial to derive the following expressions for the expectation, the variance, VaR and CTE of various cyber risk loss processes [Mack (1984), Tan and Cai (2008)]:

$$E\left(X_i^l\right) = \lambda_i l_i t[1 - F(l_i;\ \theta_i, \kappa_i)] + \lambda_i \theta_i \kappa_i t F(l_i;\ \theta_i, \kappa_i + 1) \tag{7}$$

$$E(X_i^{nl}) = \lambda_i \theta_i \kappa_i t \tag{8}$$

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[6] *The compound Poisson processes that we simulate in this work have high kurtosis.*

$$E(Y^{nl}) = \sum_{i=1}^{n} \lambda_i \theta_i \kappa_i t \tag{9}$$

$$Var(X_i^l) = \lambda_i t E\left(\left(X_i^l\right)^2\right) = \lambda_i t \left(l_i^2\left(1 - F(l_i;\ \theta_i, \kappa_i)\right) + \theta_i^2 \kappa_i(\kappa_i + 1)F(l_i;\ \theta_i, \kappa_i + 2)\right) \tag{10}$$

$$Var(X_i^{nl}) = \lambda_i(\kappa_i \theta_i^2 + \kappa_i^2 \theta_i^2)t \tag{11}$$

$$Var(Y^{nl}) = \sum_{i=1}^{n} \lambda_i(\kappa_i \theta_i^2 + \kappa_i^2 \theta_i^2)t \tag{12}$$

For $VaR_\alpha(X_i^{nl}) = x_{i,\alpha}^{nl}$

$$\alpha = F_{X_i^{nl}}(x_{i,\alpha}^{nl}) = e^{-\lambda_i t}\left(1 + \sum_{j=1}^{\infty} \frac{(\lambda_i t)^j}{j!} F\left(x_{i,\alpha}^{nl};\ \theta_i, j\kappa_i\right)\right) \tag{13}$$

$$CTE_\alpha(X_i^{nl}) = \frac{\sum_{j=1}^{\infty} \frac{(\lambda_i t)^j}{j!} e^{-\lambda_i t}\left[j\kappa_i \theta_i\left(1 - F\left(x_{i,\alpha}^{nl};\ \theta_i, j\kappa_i + 1\right)\right)\right]}{(1-\alpha)} \tag{14}$$

The expectation, variance, VaR and CTE of the aggregate cyber risk loss process with limits (Y^l) and VaR and CTE of the no limit aggregate cyber risk loss process (Y^{nl}) can be computed numerically using Monte Carlo simulations. We illustrate the methodology to compare the effectiveness of the two CLI contract structures, with and without the limits using a simple numerical example. The table below summarizes the limits and the parameter values used in our example (Table 2):

Table 2: Limits and the parameter values used in the numerical simulation

	(X_1)	(X_2)	(X_3)	(X_4)	(X_5)	(X_6)
Sub limit per claim, '00,000 USD, l_i	5.00	4.00	2.00	1.50	2.00	3.00
Poisson distribution parameter, λ_i	0.05	0.03	0.10	0.10	0.10	0.05
Gamma distribution, Shape parameter, κ_i	20.26	35.16	34.06	14.29	126.18	14.55
Gamma distribution, Scale parameter, θ_i	0.17	0.09	0.04	0.07	0.01	0.14

Aggregate limit of liability, 00,000' USD = 7.5

The VaR and CTE are calculated for α = 99.9% and t = 1 year as these values are stipulated under BASEL regulations for calculating the operational risk capital requirement. Following are the results (Table 3):

Table 3: Simulation results for the independent sub-cyber risk losses

	Values for aggregate cyber risk loss with limits (Y^l)	Values for aggregate cyber risk loss - no limits, (Y^{nl})
Expectation, $\Pi(t)$ '00,000 USD	0.8	0.8
Standard deviation	1.3	1.3
$VaR_\alpha(X)$, '00,000 USD	7.5	8.1
$CTE_\alpha(X)$, '00,000 USD	7.5	9.1
$\frac{\sqrt{Var(X)}}{\Pi(t)}$,	1.7	1.7
$\frac{VaR_\alpha(X)}{\Pi(t)}$	9.6	10.5
$\frac{CTE_\alpha(X)}{\Pi(t)}$	9.6	11.8

For the set of parameter values that we have selected (table 2), the effectiveness of a CLI contract without limits is better than that of a CLI contract with limits i.e. the loss protection offered by a CLI contract without limits for a dollar of premium paid is better, especially for the tail risk measures such as VaR and CTE.

3. ANALYSIS OF A CLI CONTRACT WHEN SUB – CYBER RISKS ARE CORRELATED AND SHOW UPPER TAIL DEPENDENCE

Maillart and Sornette (2009) find that the tail of personal identity losses per event exhibit power law, suggesting that cyber risk losses are heavy tailed. In this section we repeat the analysis in the previous section albeit under the assumption that the sub-cyber risks (X_i) are now correlated and exhibit upper tail dependence. By upper tail dependence we mean a higher tendency for the sub-cyber risk X_i to be extreme when the sub-cyber risk X_j is extreme. We model the dependence between the sub-cyber risks using the concept of a copula. Simply put a copula is a d-dimensional distribution function on $[\mathbf{0},\mathbf{1}]^{\boldsymbol{d}}$ with standard uniform marginal distributions and a copula joins univariate probability distributions to form a multivariate probability distribution. Under the condition of continuity of the marginals, the famous theorem due to Sklar guarantees the uniqueness of a copula. We suggest works by Genest and MacKay (1986), Frees and Valdez (1997), and Emberechts (2009) for further reference. Copulas have been used before to model the cyber risk; Herath et. al. (2011) model the loss pertaining to a breach event as a function of number of affected computers and observed dollar losses. They use copulas to couple the marginal distributions of these two factors and obtain a joint distribution for the loss pertaining to a breach event. They do not use a compound Poisson distribution though to model the cyber losses.

We could model the dependence between the sub-cyber risk losses using a common shock model where the claims due to multiple sub risks occur at the same time due to some common event. The dependence between the severity of these claims could be modeled using copulas [Linskog, McNeil (2003), Avanzi, Cassar and Wong (2011)]. But despite the flexibility offered under this approach we decide against the common shock model due to its lack of parameter parsimony. For example our six sub-cyber risks could require specification of up to 63 independent Poisson arrival processes (combinations of $\boldsymbol{\lambda_i}$ of each sub-cyber risk) and 15 bivariate, 20 trivariate, 15 quadvariate, 6 pentavariate and 1 hexavariate copulas for describing dependence between the sub-cyber loss severity distributions [Avanzi, Cassar and Wong (2011)].

Hence instead of the common shock model we simply use a six dimensional copula (C) to couple marginal sub-cyber risk loss (X_i) distributions $\boldsymbol{F_{X_i}(x_i)}$ to obtain a multivariate aggregate cyber risk loss ($\boldsymbol{Y}$) distribution $\boldsymbol{F_Y(y)}$ at a specific time, $t = 1$ [McNeil, Frey and Embrechts (2005)]. The multivariate aggregate cyber risk distribution is defined below:

Let $\boldsymbol{Y} = (X_1, X_2 \dots, X_6)^T$ be 6 dimensional vector of sub-cyber risk loss random variables.

$\forall(\boldsymbol{x_1}, \boldsymbol{x_2} \cdots \boldsymbol{x_6}) \in [\boldsymbol{0}, \infty)^{\boldsymbol{6}}$, hence the joint cumulative distribution function of $\boldsymbol{Y}$ is given by:

$$\boldsymbol{F_Y(y)} = F_{X_1, X_2 \cdots X_6}(x_1, x_2 \cdots x_6) = C\left(F_{X_1}(x_1), F_{X_2}(x_2) \cdots F_{X_6}(x_6)\right) \tag{15}$$

Where $\boldsymbol{F_{X_i}(x_i)}$ is the marginal distribution of the sub-cyber risk loss X_i

Thus we are now treating the sub-cyber risk losses as the static random variables at $t = 1$ instead of the stochastic processes evolving over the time t. This approach suffices our present goal; alternatively if one wished to model the aggregate cyber risk loss (Y) over time while preserving its compound Poisson property (in no limits case), Levy copulas as proposed by Tankon and Cont (2004) and Böcker and Klüppelberg (2010) could be used.

We perform the analysis using Gaussian, t and Gumbel copulas as these copulas exhibit differences in the type of upper tail dependence. A Gaussian copula in our case is defined as below [McNiel et. al. (2005), Alexander (2008), Fusai (2008)]:

$$C_P^{Ga}(u_1, u_2, \dots, u_6) = \boldsymbol{\Phi}_P\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \dots, \Phi^{-1}(u_6)\right) \tag{16}$$

$$u_i = F_{X_i}(x_i);\ u_i \in [0,1]$$

Where $\boldsymbol{\Phi_P}$ denotes the joint distribution function of the 6-variate standard normal distribution with linear correlation matrix P and Φ^{-1}denotes the inverse of the distribution function of the univariate standard normal distribution. Gaussian copulas are symmetric and do not exhibit upper tail dependence [Embrechts, Lindskog and McNeil (2001)]. In the appendix, we define and illustrate the upper tail dependence in the context of the copulas discussed in this work.

A t copula is defined as below [McNiel et. al. (2005), Alexander (2008), Fusai (2008)]:

$$C^t_{\upsilon,P}(u_1, u_2, \dots, u_6) = \boldsymbol{t}_{\upsilon,P}\left(t_\upsilon^{-1}(u_1), t_\upsilon^{-1}(u_2), \dots, t_\upsilon^{-1}(u_6)\right) \quad (17)$$

Where $\boldsymbol{t}_{\upsilon,P}$ and t_υ are multivariate and univariate student t distributions respectively with υ degree of freedom and P correlation matrix. As the degree of freedom, $\upsilon \to \infty$, t copula approaches the corresponding Gaussian copula [Demarta et. al. (2004)]. Also a t copula is symmetric i.e. its upper and lower tail dependence are equal and its upper tail dependence increases with increasing correlation and decreasing degree of freedom. The upper tail dependence tends to zero as $\upsilon \to \infty$.

If the simultaneous occurrence of high sub-cyber risk losses were more likely than the lower sub-cyber risk losses, we could use Gumbel copula to model this asymmetric dependence. The Gumbel copula in our case is defined as:

$$C^{Gum}_\delta(u_1, u_2, \dots, u_6) = exp\left\{-\left[\sum_{i=1}^{6}\left(-ln(u_i)\right)^\delta\right]^{\frac{1}{\delta}}\right\}, 1 \le \delta < \infty \quad (18)$$

The parameter δ captures the extent of dependence between the sub-cyber risk losses. If $\delta = 1$ the sub-cyber risk losses are independent and as $\delta \to \infty$ the sub-cyber risks show perfect positive dependence [Alexander (2008), Fusai and Roncoroni (2008)]. We demonstrate the impact of the correlation and upper tail dependence between the sub-cyber risks on the CLI contract by continuing the numerical example provided in the section 2. Note: the values reported in the table 3 in the section 2 were under the assumption that the sub-cyber risk losses are independent. We report the effectiveness ratios of each CLI contract structure i.e. the one with and the other without limits in Table 4 and 5 respectively. The analysis is performed using Gaussian, t and Gumbel copulas.

The parameter sets used in the analysis are generated using the following procedure:

1. We select (δ), the Gumbel copula parameter[7] and compute the corresponding Kendall's tau, $\tau = 1 - \delta^{-1}$
2. Then we use the Kendall's tau from step 1 to estimate the correlation coefficient (ρ) between the sub-cyber risks in a t copula using the following expression:
   $\tau = \frac{2}{\pi}\arcsin(\rho)$
   The correlation coefficients between various sub-cyber risks are assumed to be equal. Thus once the correlation matrix, P is calibrated using Kendall's tau, we estimate the degree of freedom υ of the t copula from the Gumbel copula data[8] using maximum likelihood method [Alexander (2008)].
3. The correlation matrix P for the Gaussian copula is assumed to be equal to the correlation matrix of the t copula.

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[7] *The selection of the Gumbel copual parameter δ in this analysis is arbitrary. Our choice of $\delta = 1.3$ and 3.0 is motivated to qualitatively demonstrate the impact of differing correlations and upper tail dependence on the effectiveness ratios.*

[8] *Once the assumption of parameter δ is made in step 1, we simulate the Gumbel Copula data.*

Table 4: Simulation results for the dependent sub-cyber risk losses for the CLI contract with limits

	Parameter Set 1			Parameter Set 2		
Copulas	Gaussian Copula, $\rho_{ij} = 0.355$	t – Copula, $\rho_{ij} = 0.355$, $\nu = 20$	Gumbel Copula parameter, $\delta = 1.3$	Gaussian Copula, $\rho_{ij} = 0.866$	t – Copula, $\rho_{ij} = 0.866$, $\nu = 6$	Gumbel Copula parameter, $\delta = 3.0$
Expectation, $\Pi(t)$ '00,000 USD	0.7	0.8	0.7	0.6	0.6	0.6
Standard deviation	1.5	1.6	1.5	1.7	1.7	1.7
$VaR_\alpha(X)$, '00,000 USD	7.5	7.5	7.5	7.5	7.5	7.5
$CTE_\alpha(X)$, '00,000 USD	7.5	7.5	7.5	7.5	7.5	7.5
$\frac{\sqrt{Var(X)}}{\Pi(t)}$,	2.0	2.1	2.2	2.8	2.9	3.1
$\frac{VaR_\alpha(X)}{\Pi(t)}$	10.0	10.0	10.8	12.3	12.4	13.4
$\frac{CTE_\alpha(X)}{\Pi(t)}$	10.0	10.0	10.8	12.3	12.4	13.4

For our current choice of parameters under the table 2 in the section 2, in case of the CLI contracts with limits, the VaR and CTE for the varying degree of dependency are constant and equal to the gross aggregate limit of 0.75 million USD. Thus the limits and sub-limits could be set by the insurers such that VaR and CTE are not affected by the levels of correlation and upper tail dependence. As the expectation of the payoff decreases with the increasing sub-cyber risk correlation, the effectiveness ratios marginally increase with increasing correlation and upper tail dependence.

Table 5: Simulation results for the dependent sub-cyber risk losses for the CLI contract - No limits

	Parameter Set 1			Parameter Set 2		
Copulas	Gaussian Copula, $\rho_{ij} = 0.355$	t – Copula, $\rho_{ij} = 0.355$, $\nu = 20$	Gumbel Copula parameter, $\delta = 1.3$	Gaussian Copula, $\rho_{ij} = 0.866$	t – Copula, $\rho_{ij} = 0.866$, $\nu = 6$	Gumbel Copula parameter, $\delta = 3.0$
Expectation, $\Pi(t)$ '00,000 USD	0.8	0.8	0.8	0.8	0.8	0.8
Standard deviation	1.7	1.8	2.1	2.5	2.6	2.8
$VaR_\alpha(X)$, '00,000 USD	14.1	14.8	19.8	21.6	20.9	22.7
$CTE_\alpha(X)$, '00,000 USD	15.8	17.1	23.2	23.7	25.1	24.7
$\frac{\sqrt{Var(X)}}{\Pi(t)}$,	2.2	2.3	2.7	3.3	3.3	3.6
$\frac{VaR_\alpha(X)}{\Pi(t)}$	18.4	18.9	25.3	28.2	27.2	29.3
$\frac{CTE_\alpha(X)}{\Pi(t)}$	20.6	21.7	29.7	30.9	32.7	32.0

In the case of a CLI contract without any limits all the three risk measures namely standard deviation, VaR and CTE and all the corresponding effectiveness ratios increase with increasing correlation and increasing upper tail dependency. All the risk measures and all the effectiveness ratios for CLI contracts with limit are lower than those for the CLI contracts without limits. For example consider Gumbel copula with $\delta = 3.0$, the VaR (USD 2.27 million) and CTE (USD 2.47 million) of a CLI contract without limit reduces to USD 0.75 million of VaR and CTE of a CLI contract with limit. This is a reduction of 67% and 70% respectively if a CLI contract with the limits were used. Table 6 below summarizes the percentage reduction in the effectiveness ratios if a CLI contract with limits were to be underwritten by an insurer. The reduction in the effectiveness ratios is lowest for uncorrelated sub-cyber risks and increases with increasing dependence in the sub-cyber risks except for the Gumbel copula which captures an asymmetric upper tail dependence.

Table 6: Reduction in VaR, CTE and effectiveness ratios for CLI contracts with Limit in comparison with CLI contract - No limit.

		Parameter Set 1			Parameter Set 2		
Copulas	Un-correlated Risks	Gaussian Copula, $\rho_{ij} = 0.355$	t – Copula, $\rho_{ij} = 0.355$, $\nu = 20$	Gumbel Copula parameter , $\delta = 1.3$	Gaussian Copula, $\rho_{ij} = 0.866$	t – Copula, $\rho_{ij} = 0.866$, $\nu = 6$	Gumbel Copula parameter , $\delta = 3.0$
$VaR_\alpha(X)$, ’00,000 USD	7%	47%	49%	62%	65%	64%	67%
$CTE_\alpha(X)$, ’00,000 USD	18%	53%	56%	68%	68%	70%	70%
$\frac{VaR_\alpha(X)}{\Pi(t)}$	8%	45%	47%	57%	56%	55%	54%
$\frac{CTE_\alpha(X)}{\Pi(t)}$	18%	51%	54%	64%	60%	62%	58%

Figure 1, 2 and 3 compares VaR, CTE and corresponding effectiveness ratios for the CLI contracts with and without limits for various copulas. In the case of Gaussian and *t* copulas, for the CLI contracts without limits, the effectiveness ratios increase linearly with the increasing dependence. But in the case of Gumbel copula this increase does not seem to be linear; a “kink” is visible in the figure 3. Thus given limits in the CLI structure, if the sub-cyber risks were to exhibit even moderate correlation and upper tail dependence (Gumbel copula), then the effectiveness of a CLI contracts to mitigate cyber risk could deteriorate sharply.

Figure 1: Comparison of VaR, CTE and effectiveness ratios for the CLI contracts with and without limits (Gaussian Copula)[9]

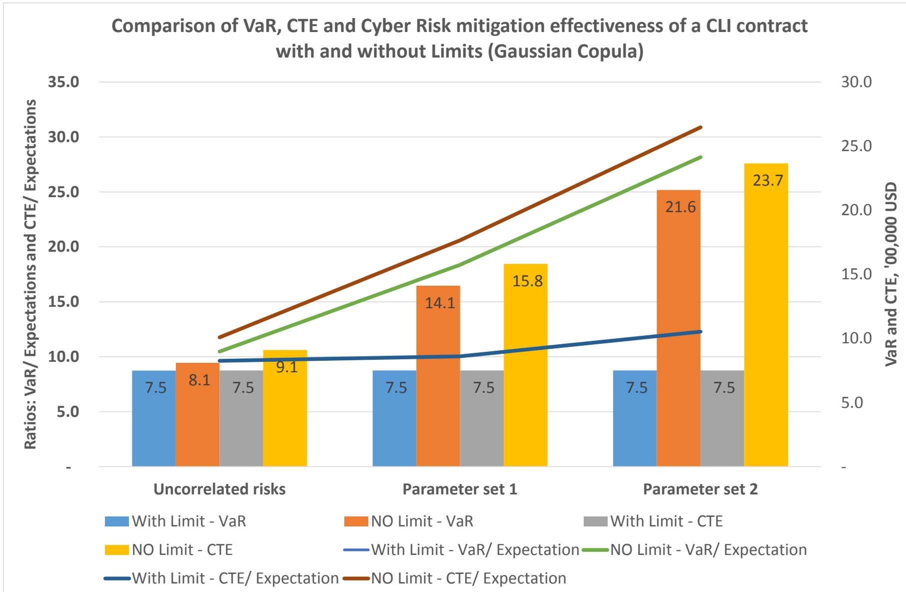
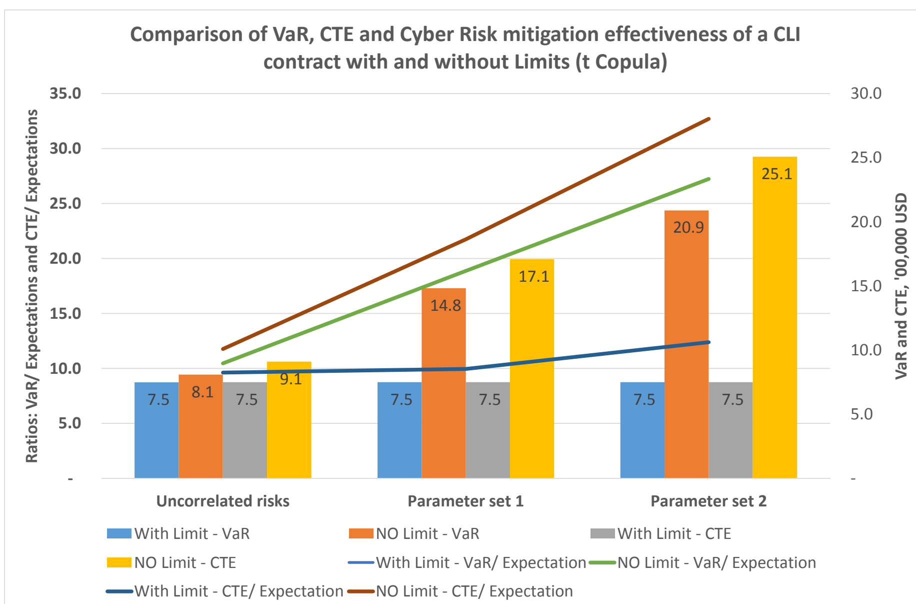
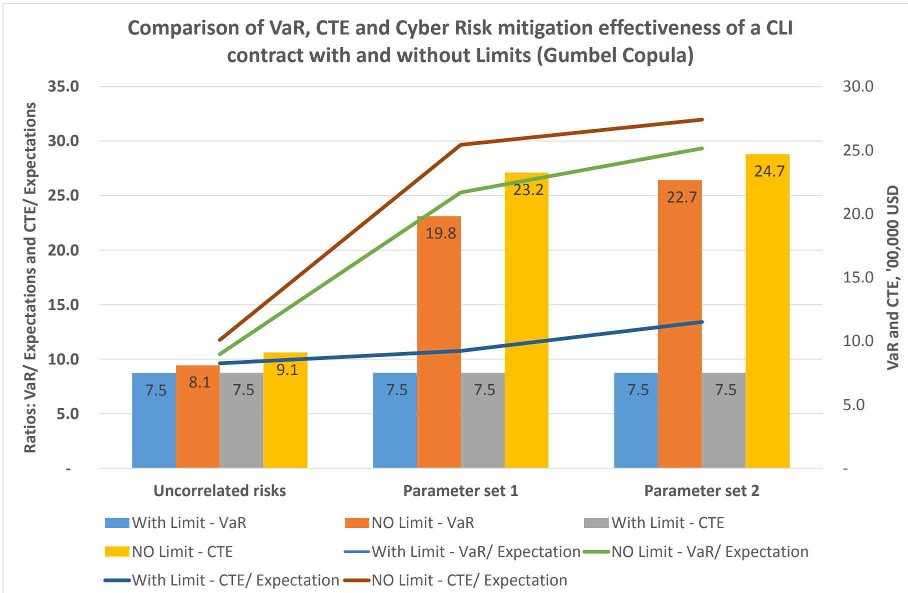


Figure 2: Comparison of VaR, CTE and effectiveness ratios for the CLI contracts with and without limits (*t* Copula)



[9] *Note: In figures 1,2 and 3 the lines for "with Limit VaR/ Expectation" and "with Limit CTE/ Expectation" coincide hence only a single line is visible.*

Figure 3: Comparison of VaR, CTE and effectiveness ratios for the CLI contracts with and without limits (Gumbel Copula)



We first analyzed a single CLI contract to understand the impact of the limits and the sub limits on the CLI contract's cyber risk mitigation effectiveness. This is important for BASEL regulators especially when a bank after purchasing a CLI contract claims an offset towards the operational risk capital charge. In our chosen case the benefit of a CLI contract in mitigating the cyber risk and consequently the operational risk is quite limited as the tail risks i.e. the low probability high severity risks losses, measured using VaR and CTE are capped by the aggregate limit (L). We acknowledge that these results are dependent on the parameter choices.

Now we take an insurer's perspective and analyze a possible rationale for incorporating these limits into the CLI contract structure. Consider a portfolio of underwritten CLI contracts where the individual aggregate cyber risk losses (Y) are correlated across the CLI contract portfolio. The total cyber risk loss (X_{TOT}) to an insurer is given by:

$$X_{TOT}(t) = \sum_{k=1}^{q} \min\big(L_k, Y_k(t)\big) = \sum_{k=1}^{q} \Big(L_k - max\Big(\big(L_k - Y_k(t)\big), 0\Big)\Big), \; t \geq 0 \tag{19}$$

where q is the number of CLI contracts in the portfolio, (L_k) is the set aggregate limit and (Y_k) is the aggregate cyber risk loss for a CLI contract k. If the sub limits in each CLI contract are ignored, it is easy to see that the maximum total cyber risk loss that could occur to an insurer is equal to the sum of the individual aggregate limit for each CLI contract.

$$Max\big(X_{TOT}(t)\big) = \sum_{k=1}^{q} L_k \ , \qquad t > 0 \tag{20}$$

Thus setting the aggregate liability limit for each CLI contract could help the insurer in capping the tail risk losses i.e. VaR and CTE. Extreme dependence between the aggregate cyber risk losses across the portfolio of the CLI contract is plausible due to the network type effects in the cyber world given the uniformity in the security technologies [Böhme (2005), Berthold and Böhme (2009), Böhme and Schwartz (2010)]. Bohme and Kataria (2006) study the correlation of the cyber risk properties at both firm and global level and find the evidence of correlation at both the levels. Given no hedging alternatives, an insurer may set the limits to the liability in the CLI contract structure to limit its own probability of ruin. In the next section we propose a cyber risk index – an index similar to Property Claim Services (PCS) for the catastrophic risk. Such a cyber risk index may help insurer hedge the impact of tail cyber risks and reduce the CLI contract premium.

4. A CASE FOR CYBER RISK INDEX

In this section we assume that the cyber risk is not fully diversifyable at an insurer level using insurer's CLI contract portfolio and the cyber risk is fully diversifyable at the capital market level only if a cyber risk index were to exist that facilitated the trading of the cyber risk[10]. Furthermore, we assume that the total cyber risk loss (X_{TOT}) for an insurer could be separated into an independent (orthogonal) local systematic (X_{lsys}) cyber risk loss and an idiosyncratic (X_{id}) cyber risk loss[11] [Boucher and Delpierre (2014)].

$$X_{TOT} = X_{lsys} + X_{id} \tag{21}$$

This separation of risk losses is plausible because unlike the idiosyncratic cyber risk the local systematic cyber risk could be low frequency and high severity event hence a tail risk. For example, in housing insurance policies such a local systematic risk loss is similar to a local flood or a hurricane and an idiosyncratic risk loss is similar to a possible damage to a single house due to a broken electricity pole (not caused by any common disaster such as a flood or a hurricane). Idiosyncratic cyber risk is assumed to be fully diversifyable at an insurer level using insurer's CLI contract portfolio and the local systematic cyber risk is assumed to be diversifyable at the capital market level only if a cyber risk index were to exist that facilitated the trading of the cyber risk.

It is simple to qualitatively demonstrate the possible benefit that could occur to the CLI contract holders in terms of the lower premium if a cyber loss index that facilitated the trading of the cyber risk in the capital market were to exist. We assume that an insurer uses the classical variance principle to arrive at a CLI contract premium [Venter (1991), Mikosch (2009)] and the premiums charged to cover (X_{lsys}) and (X_{id}),

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[10] *To keep the analysis simple we ignore the reinsurers*

[11] *The approach that we have adopted here is similar to that used by Boucher and Delpierre (2014) in analyzing the impact of the agricultural indices,*

$\Pi(X_{lsys})$ and $\Pi(X_{id})$ respectively are additive. The variance principle applied to calculate the premium to cover (X_{TOT}) is given below:

$\Pi(X_{TOT}) = E(X_{TOT}) + \zeta\, Var(X_{TOT})$, where ζ is a positive constant.

$$\Pi(X_{TOT}) = \Pi(X_{lsys} + X_{id}) = \Pi(X_{lsys}) + \Pi(X_{id}) \tag{22}$$

The premium charged for a CLI contract portfolio to cover the total cyber risk loss (X_{TOT}) given that there were no cyber index to trade the local systematic cyber risk ($\Pi_{NI}(t)$, "$No\ Index$" $denoted\ by\ subscript - NI$) is given by:

$$\Pi_{NI}(t) = \left[E\left(X_{lsys}(T)\right) + E(X_{id}(T)) + \zeta\left(Var\left(X_{lsys}(T)\right) + Var(X_{id}(T))\right)\right] \tag{23}$$

Here t = 1 year and the interest rate is assumed to be zero. As idosyncratic cyber risk is assumed to be fully diversifyable at an insurer level we get:

$$\Pi_{NI}(t) = \left[E\left(X_{lsys}(T)\right) + E(X_{id}(T)) + \zeta\left(Var\left(X_{lsys}(T)\right)\right)\right] \tag{24}$$

Given a cyber risk index to trade the local systematic cyber risk, an insurer could buy a suitable derivative contract from the market by paying a premium (P_I). As the local systematic cyber risk loss is assumed to be fully diversifyable in the capital markets given a cyber risk index, the corresponding premium is simply equal to the expectation of the local systematic cyber risk loss under a $\boldsymbol{P}$ measure.

$$P_I(t) = E_t^P\left(X_{lsys}(T)\right) \tag{25}$$

Thus the premium ($\Pi_I(t), I - Index$) charged to the portfolio given a cyber risk index is given as:

$$\Pi_I(t) = [E(X_{id}(T))] + P_I(t) \tag{26}$$

$$\Pi_I(t) = \left[E(X_{id}(T)) + E\left(X_{lsys}(T)\right)\right] \tag{27}$$

Hence the total savings that could occur to all CLI contract holders given a cyber risk index is given as:

$$\Pi_{NI}(\mathrm{t}) - \Pi_I(t) = \zeta\left(Var\big(X_{lsys}(T)\big)\right) > 0 \tag{28}$$

We leave the ponderous question of the exact constitution of the cyber risk index and the corresponding hedging methodology to the future work but propose a hedging outline based on the common approaches found in the catastrophic insurance literature. A possible model for cyber risk loss index *I(t)* could be [Gerber and Shiu (1994, 1995), Christensen (1999), Abdessalem and Ohnishi (2014)]:

$I(t) = I(0)exp\big(X(t)\big)$ and

$$X(t) = \sum_{i=1}^{N(t)} S_i\,,\ t \geq 0 \tag{29}$$

Where *(X(t))* is the index cyber risk loss process based on the gathered cyber risk claim data by an agency, *(N(t))* is the cyber loss frequency (homogenous Poisson process) and (S_i) is the reported cyber loss severity. But this model has a tendency to exhibit increasing severity after each loss event and therefore we propose the following simple model and ignore any index re-estimation issues at this juncture [Biagini, Bregman and Meyer-Brandis (2008)]. In this model the index cyber risk loss process is a sum of the cyber loss severities.

$$L(t) = X(t) = \sum_{i=1}^{N(t)} S_i\,,\ t \geq 0 \tag{30}$$

Insurers could hedge on such a cyber risk loss index using a stop loss type insurance contract (*C(t, X)*) which is similar to a European call option [Mikosch (2009)] or a contract similar to PCS type option ($C_P(t, X)$) which has the flexibility of a selecting a cap (L_{up}) on the losses [Christensen (1998)]. We had assumed earlier in this section that the local systematic cyber risk was fully diversifyable in the capital markets given a cyber risk index. This may not be the case and hence the pricing of these derivative contracts could occur under a market chosen ***Q*** measure. Thus value of these contracts at time *t* for strike *K* would be:

$$C(t, X) = e^{-r(T-t)} E_t^Q[max(X(T) - K, 0)] \tag{31}$$

$$C_P(t, X) = e^{-r(T-t)} E_t^Q\big[min\big(max(X(T) - K, 0), L_{up} - K\big)\big], \quad L_{up} > K \tag{32}$$

We assume that the index cyber risk loss process *X(t)* retains compound Poisson form under the ***Q*** measure and the distribution parameters could be estimated under the observed prices of traded derivative securities. [Christensen (2001), Lane and Movchan (1998)]. We assume that the severities $(S_i), i = 1,2,3$ are independent and identically Gamma distributed and *N*, frequency is independent of severity and follows a homogenous Poisson process.

$$S \sim GAM(\theta, \kappa)\,, \qquad s > 0, \qquad \theta > 0, \qquad \kappa > 0$$

$where\ \ is\ \theta\ the\ scale\ parameter\ and\ \kappa\ is\ the\ shape\ parameter$

$N(t) \sim POI(\lambda t), \qquad \lambda > 0$
$where\ \lambda\ is\ the\ intensity\ of\ a\ homogeneous\ Poisson\ process\ N(t)$

It is easy to show that the ideal price of such contracts under the implied parameters *($\boldsymbol{Q}$ measure)* would be [Embrechts and Meister (1997), Lane and Movchan (1998), Muermann (2003)]:

$For\ \tau = T - t$

$$C(t,X) = e^{-r(\tau)} \sum_{n=1}^{\infty} \frac{(\lambda\tau)^n e^{-\lambda\tau}}{n!} \left[n\kappa\theta\big(1 - F(K;\ \theta, n\kappa + 1)\big) - K\big(1 - F(K;\ \theta, n\kappa)\big)\right] \tag{33}$$

$$C_P(t,X) = e^{-r(\tau)} \sum_{n=1}^{\infty} \frac{(\lambda\tau)^n e^{-\lambda\tau}}{n!} \Big[n\kappa\theta\left(F\big(L_{up};\ \theta, n\kappa + 1\big) - F(K;\ \theta, n\kappa + 1)\right) - K\big(1 - F(K;\ \theta, n\kappa)\big) + L_{up}\left(1 - F\big(L_{up};\ \theta, n\kappa\big)\right)\Big] \tag{34}$$

An insurer could take a position in the derivatives to hedge the extent of cyber risk loss that constitutes tail risk for the insurer. In the earlier section we showed that a maximum total cyber risk loss $(max(X_{TOT}))$ for a CLI contract portfolio was equal to the sum of the set aggregate limit of each underwritten CLI contract $(\sum_{k=1}^{q} L_k)$. Thus an insurer could now select the strike price (K) based on the sum of set aggregate limits and thus ameliorate the need for setting the conservative limits in the CLI contract structure. If the total cyber risk loss of an underwritten CLI contract portfolio crosses the total of the aggregate limit of each CLI contract, the insurer would be compensated by a positive payoff from the long call position that the insurer has taken on the cyber index. Thus an insurer now has less impetus to set conservative limits in the CLI contract structure as the cyber risk losses above an insurer's risk appetite are hedged on a cyber risk index using the derivative contracts.

5. CONCLUSION

In this work we price a CLI contract using Gaussian, *t* and Gumbel copulas. When we analyze the structure of a CLI contracts for its cyber risk mitigation effectiveness, we find that the effectiveness is dependent on the limits and the sub limits set in the CLI contract structure especially if the sub-cyber risk losses were correlated and showed upper tail dependency. The limits in the CLI contract could help minimize the extreme payouts, thus setting up of the limits could be detrimental to the popularity of the CLI contract. The BASEL regulators under the supervisory review process (Pillar 2) may find it difficult to evaluate the impact of these limits on the effectiveness of the CLI contract and the pertinence of the offsets granted towards the operational risk capital. It is plausible that these limits guard the insurer against the systematic cyber risks. We contend that if a cyber risk index were constituted, it could help in pricing the cyber risk by the capital markets and lowering the CLI contract premiums. The derivatives with such an index as an underlying could offer wider risk hedging alternatives to the insurers, thus ameliorating the need for setting conservative limits in the CLI contract structure. Questions on the utility and the effectiveness of a cyber risk index are surely valid, but adhering to the classic tradition of financial economists we suggest that the final judgment be best left to the capital markets.

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APPENDIX

Given a $\left(X_i, X_j\right)^T$ vector of continuous random variables with marginal distribution functions $F_i\ and\ F_j$ respectively, the coefficient of upper tail dependence of $\left(X_i, X_j\right)^T$ is defined as below [Embrechts, Lindskog and McNeil (2001)]:

$$\lim_{u \to 1} \mathbb{P}\{X_i > F_i^{-1}(u) | X_j > F_j^{-1}(u)\} = \lambda_U \qquad \lambda_U \in [0,1]$$

λ_U could be thought of as a conditional probability that X_i takes a value on the upper tail given that X_j takes a value in the upper tail. Lower tail dependence is defined in an analogous way.

Figure 4 below depicts ten thousand simulated points from each of the six distributions with standard normal margins. The distributions are constructed using Gaussian, t and Gumbel copulas with the parameters that are used in this work.

For $\rho < 1$, the Gaussian copula does not have tail dependence asymptotically, regardless of how high the correlation. In the corresponding part of the figure 4 for Gaussian copula - $\rho = 0.866$ there seems to be no change in the correlation between X_i and X_j as any of the variables reaches large or small values. This can be seen in the upper right and lower left corners of the figure respectively.

t Copula exhibits both upper and lower tail dependence; consider a *t* copula with the parameters $\rho = 0.866, \upsilon = 6$, we can see that when both the variables X_i and X_j attain very high or very low values (seen in the upper right and lower left corners in the figure respectively) the correlation between X_i and X_j seem to increase.

While a *t* copula is symmetric i.e. its upper and lower tail dependence are equal, a Gumbel copula exhibits upper tail dependence. Consider Gumbel copula with $\delta = 3.0$, in the figure below, as both the variables X_i and X_j attain very high values (seen in the upper right corner), the correlation between the two variables tends to increase.

Figure 4 Ten thousand simulated points from the six distributions with standard normal margins, constructed using Gaussian, t and Gumbel copulas.