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CAN RIVALRY IN R&D BE HARMFUL TO DUOPOLISTS UNDER SUPPLY FUNCTION COMPETITION?

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Abstract. We consider a duopoly with cost asymmetry and demand uncertainty and show that rivalry in (process) R&D can be ex-ante harmful to both firms if they produce under supply function competition. However, if the firms produce under Cournot competition, only the efficient firm ex-ante suffers from R&D rivalry. Moreover, this rivalry always narrows down the efficiency gap between the duopolists, and more visibly so under Cournot competition. On the other hand, we find that consumers always ex-ante benefit from R&D rivalry, both under Cournot and supply function competitions.

Keywords: Duopoly; process R&D; supply function competition; Cournot competition.

JEL Codes: D43; L13; O30.

1 Introduction

In this paper, we explore the welfare effects of process (cost-reducing) R&D rivalry in a homogeneous-product duopoly with cost asymmetry and demand uncertainty when the firms either compete in either supply functions or compete in (fixed) quantities. The welfare effects of R&D rivalry under quantity competition of Cournot (1838) and under price competition of Bertrand (1883) have been extensively studied by many works in the oligopoly literature. A fundamental question investigated by most of these works is whether the celebrated results of Singh and Vives (1984) and Vives (1985) about the superiority of Bertrand competition over Cournot competition in terms of efficiency (in consumer surplus and total surplus) remains to hold when the duopolistic firms compete in process or product R&D before production occurs. A surprising answer to this question was provided by Qiu (1997), who showed that in a differentiated duopoly with process R&D, Cournot competition in the output market always induces a higher R&D effort than Bertrand competition, while the outcome of Cournot competition can become more efficient than the outcome of Bertrand competition if the duopolistic products are close substitutes and if R&D productivity and spillovers in the output of R&D are sufficiently high. In fact, the same results were later shown to also hold when the model of Qiu (1997) is modified to involve product R&D (as in Symeonidis, 2003) instead of process R&D or modified to involve input spillovers in R&D (as in Hinloopen and Vandekerckhove, 2009) instead of output spillovers.

To the best of our knowledge, the investigation of welfare effects of R&D rivalry under supply function competition is novel to our study. In fact, the theory of supply function competition is itself relatively new in the economic literature. This theory, which was first introduced by Grossman (1981), faced problems with indeterminacy/multiplicity of equilibria until they were eliminated by the extension of Klemperer and Meyer (1989), who allowed for demand uncertainties. This extension was applied especially to electricity markets following the pioneering work of Green and Newbery (1992) and also to several other economic problems including airline pricing reservation systems, spectrum access procurement auctions, management consulting, treasury auctions, and strategic agency and trade policy.¹

¹See Vives (2011) and Correa et al. (2014) for more on these applications.

In our model, where the main element of interest is supply function competition (in addition to Cournot competition) in the output market, we assume the existence of a process R&D as in Qiu (1997) and Hinloopen and Vandekerckhove (2009). However, unlike these two works, we allow for neither spillovers in R&D nor differentiation in the output market. Our model (and our main problem of interest) also differs from Saglam (2021), studying licensing of a cost-reducing (superior) production technology (or innovation) in a duopolistic industry under supply function competition. Saglam (2021) is not interested in the development of superior technology. He rather assumes that one of the duopolists has asymmetric access to it and studies how different kinds of licensing (fixed-fee, revenue-royalty, and mixed licensing) contracts can be ranked by the licensor, licensee, and consumers in the equilibrium of supply function competition. On the other hand, we endogenize the development of cost-reducing innovations, by allowing each firm in the duopoly to engage in R&D, after taking into account the possible effects of firms' R&D investments on the equilibrium allocations in the product market and eventually on their profits.

To give more details, we model the R&D and production process as a two-stage perfect-information game where the duopolists non-cooperatively choose (cost-reducing) R&D investments in the first stage and compete either in supply functions or in fixed quantities in the second stage. Computing the subgame-perfect Nash equilibrium (Selten, 1965) of this game numerically for a wide range of initial cost parameters and comparing it to the equilibrium with no R&D, we show that rivalry in (process) R&D can be ex-ante harmful to both firms in the duopoly under supply function competition. In contrast, when the firms produce under Cournot competition, only the efficient firm suffers from R&D rivalry. Moreover, this rivalry always narrows down the efficiency gap between the duopolists, and more visibly so under Cournot competition.

Our results allow us to investigate whether there exists a Pareto superior mode of competition in the product market, with or without R&D. Results in the absence of R&D were earlier provided by Saglam (2018a) and (2018b). Unlike our paper, both of these works consider a symmetric duopoly. Saglam (2018a) shows that when the duopolists produce a single homogeneous product supply function competition can Pareto dominate Cournot competition if and only if the size of demand uncertainty

is sufficiently large, whereas Saglam (2018b) finds that under product differentiation the dominance relation in Saglam (2018a) remains to hold irrespective of the size of demand uncertainty if the degree of product substitution is extremely low. On the other hand, our paper shows that irrespective of the presence of R&D rivalry, no form of competition is ex-ante Pareto superior to the other from the viewpoint of the whole society. Whereas consumers always ex-ante prefer supply function competition to Cournot competition, the opposite is true for the inefficient firm. The efficient firm, on the other hand, ex-ante prefers Cournot competition if the size of cost asymmetry is small and ex-ante prefers supply function competition otherwise.

The rest of the paper is organized as follows: Section 2 presents basic structures, Section 3 contains our theoretical results, and Section 4 contains our numerical computations dealing with the output and welfare effects of R&D rivalry. Finally, Section 5 concludes.

2 Basic Structures

We consider a duopolistic industry where a single homogeneous good is produced under cost asymmetry. Firm $i = 1, 2$ faces the cost function

$$C_i(q_i) = \frac{c_i(x_i)}{2} q_i^2 \tag{1}$$

where q_i is the quantity produced by firm i and $c_i(x_i) > 0$ is its unitary marginal cost that is affected by the variable $x_i \geq 0$, denoting the investment in process R&D (hereafter, simply R&D) by firm i . We assume that the common R&D technology of the firms is such that for each $i = 1, 2$ the unitary marginal cost of firm i satisfies

$$c_i(x_i) = c_{i,0} e^{-x_i} \tag{2}$$

where $x_i \geq 0$ and $c_{2,0} > c_{1,0} > 0$, i.e., before any R&D takes place in the industry, firm 1 has a lower unitary marginal cost than firm 2. Therefore, firms 1 and 2 will be called the efficient and inefficient firms, respectively. For simplicity, we also set $c_{1,0} = 1$ and $c_{2,0} = c > 1$. Note that the cost asymmetry between the duopolists becomes larger as c increases; thus we will hereafter refer to c as the size of the cost asymmetry.

We should notice that the technology in (2) implies no R&D spillovers, i.e., $\partial c_i(x_i)/\partial x_j = 0$ for any $i, j \in \{1, 2\}$ with $j \neq i$. We assume that any firm investing in $x \geq 0$ units of R&D incurs a quadratic cost (as in d'Aspremont and Jacquemin, 1988):

$$z(x) = \frac{\delta}{2}x^2 \tag{3}$$

where δ is a positive parameter. Note that according to (3), the marginal cost of R&D is increasing and independent of the output size of the firm.

Also, we assume that the demand curve faced by the duopolistic firms is given by

$$D(p) = \alpha - bp \tag{4}$$

where p denotes the product price, b is a positive constant scalar denoting the slope of the demand curve, and α is a positive-valued scalar random variable capturing unobservable shocks to the size of demand. We denote the mean and the variance of α by $E[\alpha]$ and $Var[\alpha]$ respectively, and assume that they are both positive. We assume that all equations above and the parameters $b, c, \delta, E[\alpha]$, and $Var[\alpha]$ are common knowledge.

Finally, we let Q denote the industry output, i.e., $Q = q_1 + q_2$. Given the demand curve in (4), the consumer surplus at an industry output $Q \geq 0$ can be calculated as

$$CS(Q) = \frac{Q^2}{2b}. \tag{5}$$

We will define the profit functions of the firms in the following section.

3 Theoretical Results

For the duopolistic industry described above, we will consider a two-stage perfect-information game where the duopolists non-cooperatively determine their R&D investments in stage one and then non-cooperatively determine their outputs, and consequently the market price, in stage two. Using backward induction, we will solve this game starting from the second stage where the duopolists will either compete in supply functions or compete in fixed quantities. Using the equilibrium

strategies calculated for the second stage, we will then solve the equilibrium of the R&D rivalry in the first stage.

3.1 Supply Function Competition with R&D Investment

Here, we will consider the case where the duopolistic firms compete in supply functions in the second-stage game.² Under this form of competition, the two firms specify their supply functions simultaneously, without observing the demand shock α . Formally, a stage-two strategy for firm $i = 1, 2$ is a linear function mapping prices into quantities, i.e., $S_i = \eta_i p$ where $\eta_i \geq 0$. Given the strategies S_1 and S_2 , the product market clears if

$$D(p) = S_1(p) + S_2(p) \tag{6}$$

or

$$\alpha - bp = \eta_1 p + \eta_2 p, \tag{7}$$

implying an equilibrium price given by

$$p(\eta_1, \eta_2, \alpha) = \frac{\alpha}{b + \eta_1 + \eta_2}. \tag{8}$$

Using this price, we can calculate the realized (ex-post) profit of firm i as

$$\pi_i(\eta_i, \eta_j, \alpha) = p(\eta_i, \eta_j, \alpha) S_i(p(\eta_i, \eta_j, \alpha)) - \frac{c_i(x_i)}{2} S_i(p(\eta_i, \eta_j, \alpha))^2 - z(x_i). \tag{9}$$

A pair of supply functions $(S_1^*(p), S_2^*(p)) = (\eta_1^* p, \eta_2^* p)$ form a Nash (1950) equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$ the supply function $S_i^*(p)$ maximizes the expected (ex-ante) profit of firm i when firm j produces according to the supply function $S_j^*(p)$. That is, the profile $(\eta_1^* p, \eta_2^* p)$ forms a Nash equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$ the (slope) parameter η_i^* solves

$$\max_{\eta_i \geq 0} E_\alpha [\pi_i(\eta_i, \eta_j^*, \alpha)], \tag{10}$$

²The supply function competition model we consider here is an adaptation of the symmetric oligopoly model of Klemperer and Meyer (1989) to an asymmetric duopoly, like in Green (1999). However, we cannot borrow our related characterization result (Proposition 1) from Green (1999), as he did not need to explicitly characterize the equilibrium supply functions.

where E_α is the expectations operator with respect to α .

Proposition 1. *Given the R&D levels x_1 and x_2 determined in the first stage of the duopolistic game, the stage-two competition in linear supply functions has a unique Nash equilibrium characterized by $S_i^{SF}(p) = \eta_i^{SF}(x_i, x_j)p$ for each $i, j \in \{1, 2\}$ with $j \neq i$, where*

$$\eta_i^{SF}(x_i, x_j) = \frac{2}{c_i(x_i) + \sqrt{c_i(x_i)^2 + \frac{4}{b} \left(\frac{c_i(x_i) + c_j(x_j) + bc_i(x_i)c_j(x_j)}{2 + bc_j(x_j)} \right)}}. \quad (11)$$

Proof. See Appendix.

Note from Proposition 1 that the equilibrium supply functions are independent of the demand shock α (and also independent of its mean and variance). However, the equilibrium price will be different before and after α is realized. In fact, taking the expected value of the equilibrium price in (8), we should easily observe that the expected equilibrium price depends on $E[\alpha]$. Once the value of α is realized, the equilibrium price will be completely known ex-post and equal to $p^{SF}(x_1, x_2, \alpha) \equiv p(\eta_1^{SF}(x_1, x_2), \eta_2^{SF}(x_2, x_1), \alpha)$ for any x_1 and x_2 . Consequently, the ex-post equilibrium outputs of the firms can be calculated as $q_1^{SF}(x_1, x_2, \alpha) \equiv \eta_1^{SF}(x_1, x_2)p^{SF}(x_1, x_2, \alpha)$ and $q_2^{SF}(x_2, x_1, \alpha) \equiv \eta_2^{SF}(x_2, x_1)p^{SF}(x_1, x_2, \alpha)$.

Because firm i can perfectly anticipate –in the first stage of the strategic game– the equilibrium supply functions that will be chosen in the second stage, it can calculate, for each possible investment pair (x_i, x_j) , its expected profit $E_\alpha [\pi_i^{SF}(x_i, x_j, \alpha)] \equiv E_\alpha [\pi_i(\eta_i^{SF}(x_i, x_j), \eta_j^{SF}(x_j, x_i), \alpha)]$, which is the expected payoff of the reduced game in stage one. The ability of the two firms to calculate these expected payoffs allows them to enter into a rivalry where each of the firms aims to choose the best investment strategy given its conjecture about the strategy of the other. We say that a pair of R&D investment strategies (x_1^{SF}, x_2^{SF}) forms a Nash equilibrium of the mentioned R&D rivalry (or reduced normal-form game) in stage one if for each $i, j \in \{1, 2\}$ with $j \neq i$, x_i^{SF} maximizes the expected profit of firm i when firm j invests x_j^{SF} . That is, for each $i, j \in \{1, 2\}$ with $j \neq i$, the R&D level x_i^{SF} solves

$$\max_{x_i \geq 0} E_\alpha [\pi_i^{SF}(x_i, x_j^{SF}, \alpha)]. \quad (12)$$

Consequently, the strategy profile consisting of the plans $\langle (x_1^{SF}, \eta_1^{SF}(x_1, x_2))$ for any $(x_1, x_2) \rangle$ and $\langle (x_2^{SF}, \eta_2^{SF}(x_2, x_1))$ for any $(x_2, x_1) \rangle$ constitutes a subgame-perfect Nash equilibrium (SPNE) of the two-stage strategic game whenever the duopolists compete in supply functions. In such an equilibrium, the expected profit obtained by firm i will equal to $E_\alpha [\pi_i^{SF}(x_i^{SF}, x_j^{SF}, \alpha)]$.

Let $Q^{SF}(x_1^{SF}, x_2^{SF}, \alpha)$ denote the equilibrium value of the realized (ex-post) industry output; i.e.

$$Q^{SF}(x_1^{SF}, x_2^{SF}, \alpha) = q_1^{SF}(x_1^{SF}, x_2^{SF}, \alpha) + q_2^{SF}(x_2^{SF}, x_1^{SF}, \alpha). \quad (13)$$

Using equation (5), we can calculate the ex-post consumer surplus in a SPNE, i.e., $CS(Q^{SF}(x_1^{SF}, x_2^{SF}, \alpha))$, and also its ex-ante value $E_\alpha [CS(Q^{SF}(x_1^{SF}, x_2^{SF}, \alpha))]$. We leave the calculation of x_1^{SF} and x_2^{SF} as well as the corresponding equilibrium outputs and welfares to Section 4.

3.2 Cournot Competition with R&D Investment

Here, we assume that the duopolistic firms compete in quantities à la Cournot in the second stage of their game. A strategy for firm $i = 1, 2$ is a fixed quantity $q_i \geq 0$ that needs to be chosen without observing the demand shock α . Given strategies q_i and q_j simultaneously chosen by firms i and j , the product market clears if

$$D(p) = q_i + q_j, \quad (14)$$

implying an equilibrium price given by

$$p(q_i, q_j, \alpha) = \frac{\alpha - q_i - q_j}{b}. \quad (15)$$

Using this price, we can calculate the realized (ex-post) profit of firm i as

$$\pi_i(q_i, q_j, \alpha) = p(q_i, q_j, \alpha) q_i - \frac{c_i(x_i)}{2} q_i^2 - z(x_i). \quad (16)$$

A pair of quantities (q_1^*, q_2^*) forms a (Cournot) Nash equilibrium in the second stage game if for each $i, j \in \{1, 2\}$ with $j \neq i$ the quantity q_i^* maximizes the expected

profit of firm i when firm j produces the quantity q_j^* . That is, the profile (q_1^*, q_2^*) forms a Nash equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$ the quantity q_i^* solves

$$\max_{q_i \geq 0} E_\alpha [\pi_i(q_i, q_j, \alpha)]. \quad (17)$$

Proposition 2. *Given the R&D levels x_1 and x_2 determined in the first stage of the duopolistic game, the stage-two competition in quantities has a unique Nash equilibrium characterized by $\langle q_1^C(x_1, x_2), q_2^C(x_2, x_1) \rangle$ such that for each $i, j \in \{1, 2\}$ with $j \neq i$,*

$$q_i^C(x_i, x_j) = \frac{[1 + bc_j(x_j)] E[\alpha]}{(2 + bc_i(x_i))(2 + bc_j(x_j)) - 1}. \quad (18)$$

Proof. See Appendix.

Notice from Proposition 2 that the equilibrium output of each firm is the same both ex-ante and ex-post, as it is independent of the realization of the demand shock α (while it positively depends on its mean $E[\alpha]$). On the other hand, the equilibrium price may be different in the ex-ante and ex-post states. Notice that the ex-post price, which is known after α is realized, is equal to $p^C(x_i, x_j, \alpha) \equiv p(q_i^C(x_i, x_j), q_j^C(x_j, x_i), \alpha)$, as suggested by (15) and (18).

Because firm i can perfectly anticipate—in the first stage of the strategic game—the equilibrium outputs that will be chosen in the second stage, it can calculate, for each possible investment pair (x_i, x_j) , its expected profit $E_\alpha [\pi_i^C(x_i, x_j, \alpha)] \equiv E_\alpha [\pi_i(q_i^C(x_i, x_j), q_j^C(x_j, x_i), \alpha)]$. The ability of the two firms to calculate these expected payoffs induces a rivalry (a reduced form game) in the first stage. Given this rivalry, we say that a pair of R&D investment strategies (x_1^C, x_2^C) forms a Nash equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$, x_i^C maximizes the expected profit of firm i when the R&D level of firm j is x_j^C . That is, x_i^C solves

$$\max_{x_i \geq 0} E_\alpha [\pi_i^C(x_i, x_j^C, \alpha)]. \quad (19)$$

Consequently, the strategy profile consisting of the plans $\langle (x_1^C, q_1^C(x_1, x_2))$ for any $(x_1, x_2) \rangle$ and $\langle (x_2^C, q_2^C(x_2, x_1))$ for any $(x_2, x_1) \rangle$ constitutes a subgame-perfect Nash

equilibrium (SPNE) of the two-stage strategic game whenever the duopolists compete in quantities. In such an equilibrium, the expected profit of firm i will be equal to $E_\alpha [\pi_i^C(x_i^C, x_j^C, \alpha)]$.

Finally, we let $Q^C(x_1^C, x_2^C)$ denote the equilibrium value of the ex-post and ex-ante industry output; i.e.

$$Q^C(x_1^C, x_2^C) = q_1^C(x_1^C, x_2^C) + q_2^C(x_2^C, x_1^C). \quad (20)$$

Using equation (5), we can calculate the ex-post consumer surplus in a SPNE as $CS(Q^C(x_1^C, x_2^C))$. Notice that the ex-post industry output is independent of the demand shock α ; therefore the ex-post consumer surplus is equal to the ex-ante consumer surplus, i.e., $E_\alpha [CS(Q^C(x_1^C, x_2^C))] = CS(Q^C(x_1^C, x_2^C))$. We leave the calculation of x_1^C and x_2^C as well as the corresponding equilibrium outputs and welfares to Section 4.

4 Computational Results

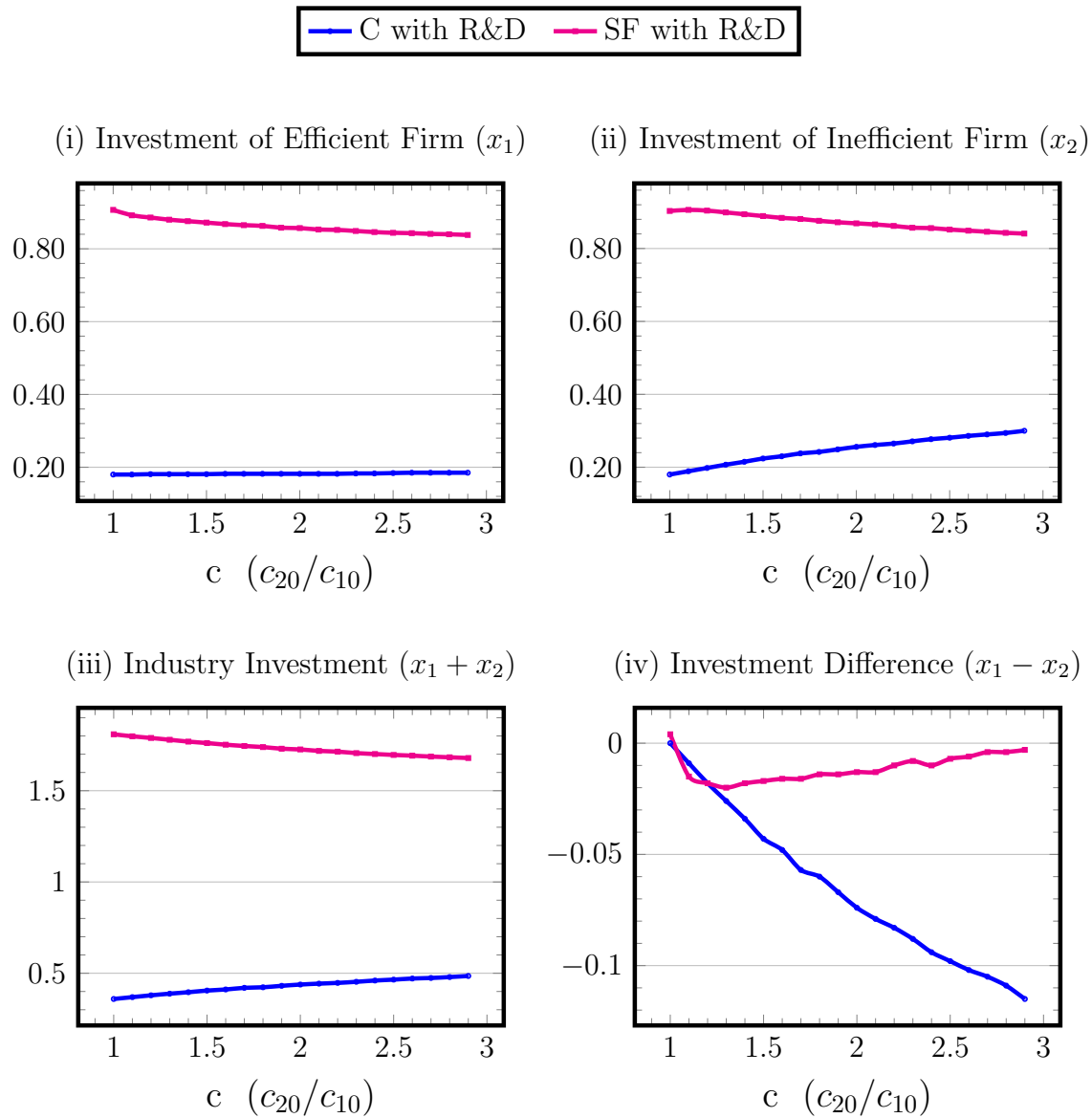
Because of the functional complexity of the optimization programs in (12) and (19), we cannot analytically calculate the subgame-perfect Nash equilibria of the two-stage strategic game played by the duopolists. However, we will be able to compute these equilibria numerically with the help of a computer, using the programming package Gauss Version 3.2.34 (Aptech Systems, 1998). The source code and the simulated data are available from the author upon request.

For our computations, we set $a = 3$ and $b = 0.25$, while we vary the cost parameter $c \equiv c_{2,0}/c_{1,0}$ from 1.0 to 2.9 with increments 0.1 and vary the parameter δ from 0.1 to 9.6 with increments 0.5. At each parameter set, we compute the Nash equilibrium in R&D investments with a grid search technique. Basically, given a competition type $t \in \{SF, C\}$ that we have considered in Sections 3.1 and 3.2, we change both $\exp(-x_i)$ and $\exp(-x_j)$ from 0.005 to 0.995 with increments of 0.005, and compute all possible $\pi_i^t(x_i, x_j)$ and $\pi_j^t(x_i, x_j)$ values. Given these computations, we pick a pair (x_i^t, x_j^t) of R&D investments to be a Nash equilibrium for the competition type t if $\pi_i^t(x_i^t, x_j^t) \geq \pi_i^t(x_i, x_j^t)$ for all x_i such that $\exp(-x_i) \in \{0.005, 0.010, \dots, 0.995\}$ and $\pi_j^t(x_j^t, x_i^t) \geq \pi_j^t(x_j, x_i^t)$ for all x_j such that $\exp(-x_j) \in$

$\{0.05, 0.010, \dots, 0.995\}$. If there exist multiple Nash equilibria, we pick the Nash equilibrium (x_i^t, x_j^t) with the highest $x_i^t + x_j^t$ value.

In Figures 1-3 below, we compare several outcomes obtained in Sections 3.1 and 3.2. Note that all graphs in all three figures plot at each simulated value of c the average value of a relevant model outcome, corresponding to the 20 distinct simulation values of δ between 0.1 and 9.6.

Figure 1. Investments Under the Two Forms of Competition (C & SF)

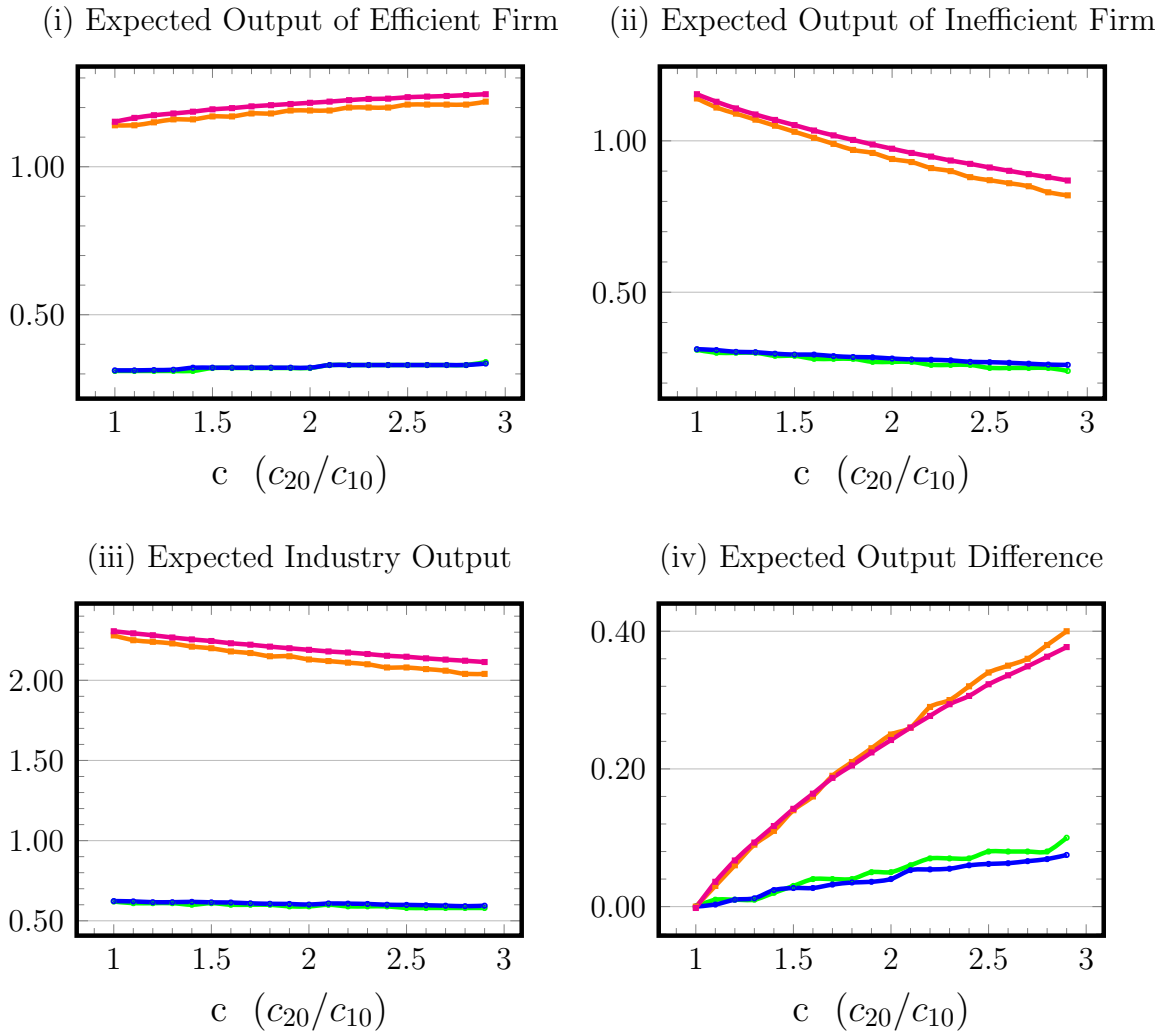


In Figure 1, we can observe that the R&D investment of both firms, and their sum, are higher under supply function competition than under Cournot competition at all levels of cost asymmetry. Moreover, under supply function competition the R&D investments of both firms become lower as the size of the cost asymmetry (c) increases. Under Cournot competition, this dependence no longer exists for the efficient firm whereas it is reversed for the inefficient firm. Also, panel (iv) of Figure 1 shows that the inefficient firm always invests more than the efficient firm, and the investment gap in favor of the inefficient firm is higher (lower) under Cournot (supply function) competition when the cost asymmetry is larger. These results suggest that R&D rivalry narrows down the efficiency gap (the difference in the marginal production costs of a unit output) between the duopolists, and this can be observed more clearly under Cournot competition.

In Figure 2, we plot the expected equilibrium outputs both in the presence and absence of R&D. (Notice that an equilibrium with no R&D arises when the firms have no access to the R&D technology in equation (2) or when R&D is infinitely costly, i.e. $\delta = \infty$ in equation (3).) As shown by the first three panels, the expected outputs of both firms, and consequently the expected industry output, are higher when investment in R&D is present than when it is not. However, the positive effect of R&D seems to be much more significant under supply function competition than under Cournot competition. In panels (i) and (ii) of Figure 2, we also observe that the effect of cost asymmetry on the expected output is different for the two firms. For both types of competition, this effect is positive for the efficient firm and negative (and much larger in magnitude) for the inefficient firm, irrespective of the presence of R&D possibility. In fact, the above negative effect on the expected output of the inefficient firm is so large, especially under supply function competition, that the expected industry output is always decreasing in the level of cost asymmetry, as we observe in panel (iii). In addition, we observe in the first three panels that the effect of cost asymmetry becomes always more pronounced when the firms compete in supply functions. Finally, panel (iv) shows that the efficient firm is always expected to produce more than the inefficient firm under both types of competition irrespective of whether the two firms can engage in R&D or not. Moreover, the difference between the firms' expected outputs is always higher under supply function competition than under Cournot competition, while this difference is not

affected by the possibility of R&D investment much.

Figure 2. Expected Outputs Under the Two Forms of Competition (C & SF)

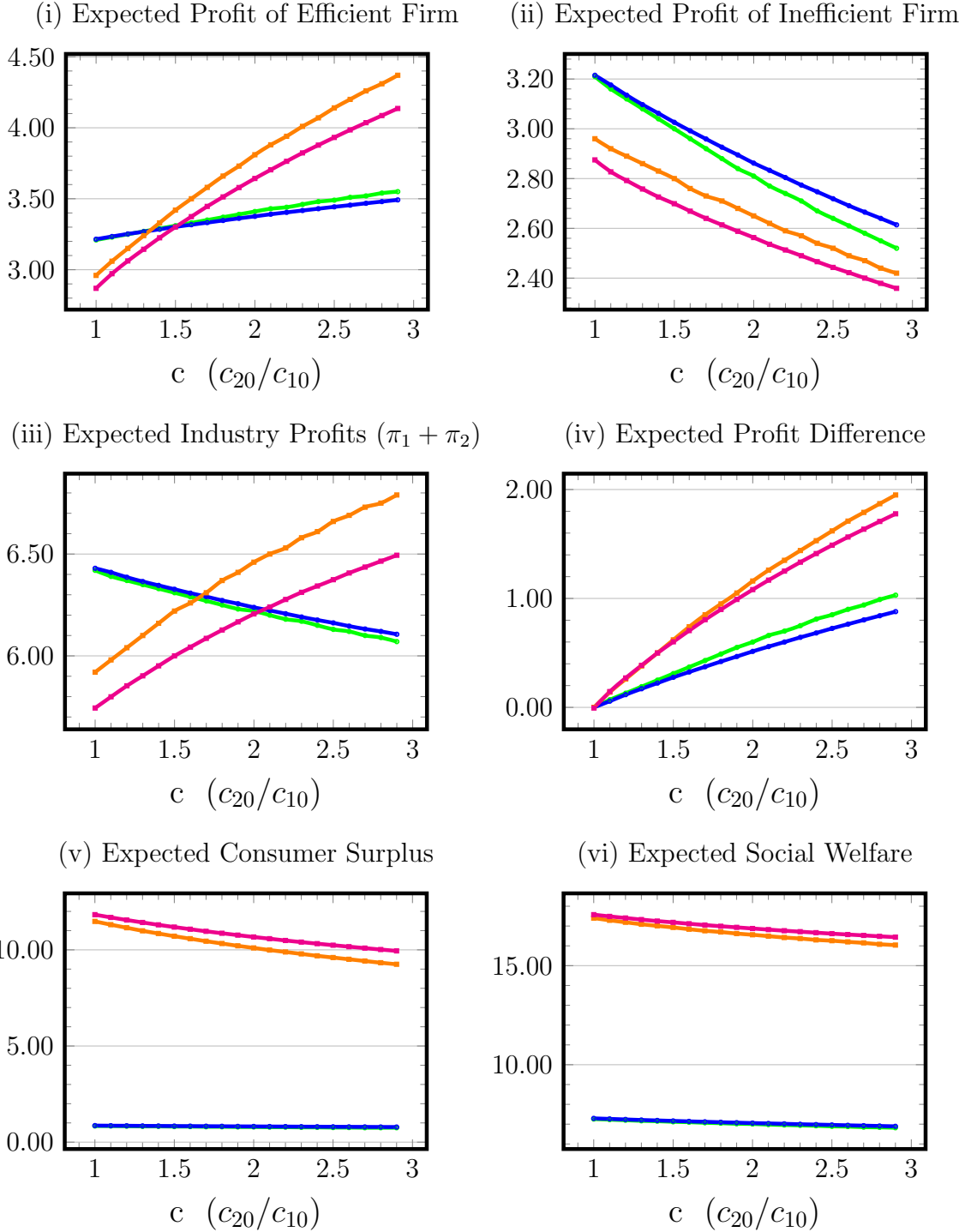


Next, in Figure 3 we plot the expected equilibrium welfares. Panel (i) shows that rivalry in R&D is ex-ante harmful to the efficient firm, especially under supply function competition. On the other hand, panel (ii) shows that the inefficient firm ex-ante suffers from R&D rivalry only if the firms compete in supply functions in the product market. If the firms compete in quantities, then R&D rivalry always

ex-ante benefits the inefficient firm. Moreover, as expected, an increase in the cost asymmetry has a positive effect on the expected welfare of the efficient firm and a negative effect on the expected welfare of the inefficient firm under both types of competition, irrespective of the presence of R&D rivalry. However, which of these opposite effects becomes dominant on the expected industry profits is more involved. As illustrated in panel (iii) of Figure 3, when the firms engage in Cournot competition in the product market, the expected industry profits are decreasing at all levels of cost asymmetry irrespective of the presence of R&D rivalry. On the other hand, when the firms compete in supply functions in the product market, the expected industry profits are always increasing with respect to the cost asymmetry both in the presence and absence of R&D rivalry. Also, we observe in panel (iv) that the firm that has an initial cost advantage in production can always obtain higher expected profit than the other firm; whereas the expected profit gap is higher under supply function competition, and especially so if the size of cost asymmetry is sufficiently large.

Finally, panels (v) and (vi) of Figure 3 illustrate that the effects of R&D rivalry on the expected consumer surplus and the expected social welfare (which we define to be the sum of expected consumer surplus and the expected industry profits) are significant and positive under supply function competition and extremely small, yet positive, under Cournot competition. Moreover, irrespective of the presence or absence of R&D rivalry, the expected welfares of both consumers and the society as a whole are always higher under supply function competition. We can also observe that the size of cost asymmetry has, in general, negative effects on the expected consumer surplus and expected social welfare, especially under supply function competition. Interestingly, panels (i), (ii), and (v) together show that there exists no ex-ante Pareto superior mode of competition from the viewpoint of the society. Whereas consumers always ex-ante prefer supply competition (with or without R&D) to Cournot competition, the opposite is true for the inefficient firm. On the other hand, the efficient firm has mixed preferences: it ex-ante prefers Cournot competition (with or without R&D) if the size of the cost asymmetry is small and ex-ante prefers supply function competition otherwise. Thus, the duopolists can agree on the name of an ex-ante superior mode of competition only if the size of the cost asymmetry is small.

Figure 3. Expected Welfares Under the Two Forms of Competition (C & SF)



5 Conclusion

In this paper, we have considered a duopolistic model with cost asymmetry and demand uncertainty and studied how rivalry in process R&D may affect the welfares of producers, consumers, and the society as a whole when firms either compete in supply functions or compete in quantities in the product market. To this end, we have constructed a two-stage perfect-information game where the duopolistic firms non-cooperatively choose in the first stage their R&D investments and in the second stage their productions (according to supply function or quantity competition). Solving the subgame-perfect Nash equilibrium of this game numerically for a wide range of model parameters, we have found that under both types of competition the outputs of both firms, and resultingly the industry output, are always higher when they both invest in R&D than when neither of them makes any investment.

We have also observed that the output expansion due to the aggressive R&D investment under supply function competition occurs at such a level that the direct cost of this expansion on the profits of the firms and the whole industry outweighs the benefit of R&D channeling through a reduction in the unitary marginal costs of the firms. Consequently, under supply function competition with process R&D the duopolistic firms find themselves trapped in a situation like the Prisoners' Dilemma. Even though R&D can be ex-ante beneficial to any firm when the rival firm has no access to R&D, it becomes ex-ante harmful to each firm under supply function competition when both firms non-cooperatively engage in R&D. In contrast, R&D rivalry before Cournot competition in the product market has asymmetric welfare effects: R&D rivalry is always ex-ante beneficial to the inefficient firm, while it is always ex-ante harmful to the efficient firm. Regarding the welfares of consumers and the society as a whole, we have found that R&D rivalry has always a positive effect under both types of competition, whereas this effect is incomparably larger under supply function.

Our simulations have also shown that no mode of competition is ex-ante Pareto superior to the other from the viewpoint of the whole society. The efficient firm ex-ante prefers Cournot competition if the size of cost asymmetry is small and ex-ante prefers supply function competition otherwise. On the other hand, the inefficient firm always ex-ante prefers Cournot competition to supply function competition. In

contrast, for consumers the opposite is true.

Our main result that the rivalry in R&D can be harmful to duopolists under supply function competition certainly calls for the need of public intervention. Public authorities may use subsidies for incentivizing firms, especially those in power industries, to make them invest in the socially optimal level of R&D under supply function competition. Very recently, Chen and Lee (2022) study the welfare implications of government subsidies for R&D investment (and output) in a duopoly (with differentiated products) under Cournot and Bertrand competition. Future research can fruitfully integrate our model with their work to explore the effects of R&D subsidies under supply function competition and how these effects change with the degree of product substitutability (or complementarity).

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Appendix.

Proof of Proposition 1. If the pair of supply functions $\langle \eta_1^{SF}(x_1, x_2)p, \eta_2^{SF}(x_2, x_1)p \rangle$ forms a Nash equilibrium, then the price $p(\eta_1^{SF}(x_1, x_2), \eta_2^{SF}(x_2, x_1))$ must solve

$$\max_{p \geq 0} E_\alpha \left[p(\alpha - bp - S_j^{SF}(p)) - \frac{c_i(x_i)}{2} (\alpha - bp - S_j^{SF}(p))^2 - z(x_i) \right] \quad (21)$$

for each $i, j \in \{1, 2\}$ with $j \neq i$. The first-order necessary condition for the above maximization implies

$$\begin{aligned} 0 &= E_\alpha [S_i^{SF}(p) + (p - c_i(x_i)S_i^{SF}(p))(-b - \eta_j^{SF}(x_j, x_i))] \\ &= E_\alpha [\eta_i^{SF}(x_i, x_j)p + (p - c_i(x_i)\eta_i^{SF}(x_i, x_j)p)(-b - \eta_j^{SF}(x_j, x_i))], \end{aligned} \quad (22)$$

implying

$$\eta_i^{SF}(x_i, x_j) = \frac{b + \eta_j^{SF}(x_j, x_i)}{1 + c_i(x_i)(b + \eta_j^{SF}(x_j, x_i))}. \quad (23)$$

Let $\eta_i^{SF} \equiv \eta_i^{SF}(x_i, x_j)$ and $\eta_j^{SF} \equiv \eta_j^{SF}(x_j, x_i)$. Then, equation (23) implies

$$\frac{1}{\eta_i^{SF}} = \frac{1}{b + \eta_j^{SF}} + c_i(x_i) \quad (24)$$

and

$$\frac{1}{\eta_j^{SF}} = \frac{1}{b + \eta_i^{SF}} + c_j(x_j). \quad (25)$$

Define $\varphi_i = 1/\eta_i^{SF}$ and $\varphi_j = 1/(b + \eta_j^{SF})$. Then, (24) and (25) can be rewritten as

$$\varphi_i = \varphi_j + c_i(x_i) \quad (26)$$

and

$$\frac{1}{\varphi_j - b} = \frac{1}{\varphi_i + b} + c_j(x_j). \quad (27)$$

Inserting (27) into (26) and with the help of some arrangements we obtain

$$\frac{\varphi_j}{1 - b_j} = \frac{(1 + bc_j(x_j))\varphi_j + c_i(x_i) + c_j(x_i) + bc_i(x_i)c_j(x_j)}{1 + b\varphi_j + bc_i(x_i)}. \quad (28)$$

It follows from (28) that

$$\varphi_j^2 + c_i(x_i)\varphi_j - \frac{c_i(x_i) + c_j(x_j) + bc_i(x_i)c_j(x_j)}{b(2 + bc_j(x_j))} = 0. \quad (29)$$

The positive-valued solution to the above quadratic equation can be calculated as

$$\varphi_j = \frac{-c_i(x_i) + \sqrt{c_i(x_i)^2 + \frac{4}{b} \left(\frac{c_i(x_i) + c_j(x_j) + bc_i(x_i)c_j(x_j)}{2 + bc_j(x_j)} \right)}}{2}. \quad (30)$$

Then using (26) and $\varphi_i = 1/\eta_i^{SF}$, we obtain (11). To check the second-order sufficiency condition, we differentiate the right-hand side of (22) with respect to p to obtain $(-b - \eta_j^{SF}(x_j, x_i)) + (1 + c_i(x_i)(b + \eta_j^{SF}(x_j, x_i)))(-b - \eta_j^{SF}(x_j, x_i)) < 0$ for all $p \geq 0$. So, $p(\eta_1^{SF}(x_1, x_2), \eta_2^{SF}(x_2, x_1))$ solves the problem in (21), implying that the supply functions $\eta_1^{SF}(x_1, x_2)p$ and $\eta_2^{SF}(x_2, x_1)p$ form a Nash equilibrium in the second-stage game. \square

Proof of Proposition 2. The first-order necessary condition associated with the maximization problem in (17) is given by

$$-\frac{1}{b}q_i + \frac{E[\alpha] - q_i - q_j^*}{b} - c_i(x_i)q_i = 0. \quad (31)$$

If $(q_1^*, q_2^*) = (q_1^C, q_2^C)$ forms a Nash equilibrium, then for each $i, j \in \{1, 2\}$ with $j \neq i$ the quantity $q_i = q_i^C$ must satisfy the above first-order condition when $q_j^* = q_j^C$, implying

$$q_i^C = \frac{E[\alpha] - q_j^C}{2 + bc_i(x_i)}. \quad (32)$$

Changing the role of i and j in (31), we can also get

$$q_j^C = \frac{E[\alpha] - q_i^C}{2 + bc_j(x_j)}. \quad (33)$$

Then, solving (32) and (33) together, we can obtain $q_i^C(x_1, x_2)$ as in (18). To check the second-order sufficiency condition, we differentiate the left-hand side of (31) with respect to q_i to obtain $-(2/b) - c_i(x_i) < 0$ for all $q_i \geq 0$. So, the quantity $q_i^C(x_i, x_j)$ solves the problem in (17), implying that the strategies $q_1^C(x_1, x_2)$ and $q_2^C(x_2, x_1)$ form a Nash equilibrium in the second-stage game. \square