Hall and Taylors and John Taylors Model in DUALI

Mercado, Ruben and Kendrick, David

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Hall and Taylor´s and John Taylor´s Model in DUALI

By

P.Ruben Mercado
David A. Kendrick

University of Texas

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This paper,[1] and its accompanying programs,[2] provide a practical introduction to macroeconomic policy analysis methods and show how to implement in DUALI deterministic and stochastic simulations with standard[3] and rational expectations models.

The analysis of the *general properties of dynamic economic systems* is a complex task, facilitated by the application of some theoretical results and relatively simple simulation techniques. *Dynamic optimal policy* analysis is more demanding, usually requiring of specialized software. DUALI[4] is an optimal control software which is able to generate sophisticated deterministic and stochastic simulation environments and to compute, among other things, the optimal feedback rule and the implied optimal paths for target variables and policy tools.

Our general goals here are:

a) to introduce the use of some concepts for the analysis of dynamic properties of economic systems

b) to introduce the use of DUALI to perform deterministic and stochastic dynamic optimal policy analysis.

As a practical illustration of solution concepts and computational techniques, we use Robert Hall and John Taylor's open economy-flexible exchange rate model[5], and John Taylor's closed economy model with rational expectations and staggered contracts.[6]

Part A: Policy Analysis with Standard Macroeconomic Models

1) Hall and Taylor's Open Economy Model

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[1] These materials were developed at the Department of Economics, The University of Texas at Austin, as a part of the Project "Computational Experiments with John Taylor's US Macroeconomic Models". The Project Director was Professor David Kendrick.

[2] The programs are: htdua01.dui, htdua02.dui, htdua03.dui, taydua01.dui and taydua02.dui. They can be downloaded from David Kendrick's home page at the Department of Economics, The University of Texas at Austin:

http://www.eco.utexas.edu/

[3] By standard macroeconomic models we mean models that can be solved, in principle, in a forward-recursive way. That is, they do not involve two-point boundary problems as it is the case of models containing forward-looking variables. For an elementary introduction to solution methods for models with forward variables, see Mercado and Kendrick (1997b).

[4] If you are not familiar with this software, see Amman and Kendrick (1996b) and (1997c).

[5] See Hall and Taylor (1993). Though its building blocks are developed throughout the book, the whole model is presented only in MACROSOLVE, the software accompanying Hall and Taylor's book.

[6] See Taylor (1993), Chapter 1.

This is a twelve-equation nonlinear dynamic model for an open economy with flexible exchange rates which generates interesting and realistic patterns of macroeconomic behavior.

Hall and Taylor's model contains the equations, variables and parameters listed below. Equations *i-v* and *ix-x* can be seen as a standard IS-LM-Open Economy sub-model for the aggregate demand of the economy. Equations *vi-viii* define an "expectations augmented" Phillips Curve, that is, the aggregate supply. Finally, equations *xi* and *xii* are definitions for the government deficit and the unemployment rate.

The model is dynamic - all variables without subscripts correspond to time "t", those with "-1" subscript correspond to "t-1", and so on. Also the model is nonlinear - nonlinearities appear in equations *v*, *viii*, *ix* and *x*. Its dynamic behavior displays the "natural rate" property: nominal shocks may affect real variables in the short-run, but not in the long run.

Equations

i)	GDP identity	$Y = C + I + G + X$
ii)	Disposable Income	$Y^d = (1- t)\, Y$
iii)	Consumption	$C = a + b\, Y^d$
iv)	Investment	$I = e - d\, R$
v)	Money Demand	$M/P = k\, Y - h\, R$
vi)	Expected Inflation	$\pi^e = \alpha\, \pi_{-1} + \beta\, \pi_{-2}$
vii)	Inflation Rate	$\pi = \pi^e + f\{(Y_{-1} - Y_N) / Y_N\}$
viii)	Price Level	$P = P_{-1}\, (1 + \pi)$
ix)	Real Exchange Rate	$E\, P / P_w = q + v\, R$
x)	Net Exports	$X = g - m\, Y - n\, E\, P / P_w$
xi)	Government Deficit	$G_d = G - t\, Y$
xii)	Unemployment Rate	$U = U_N - \mu\{(Y - Y_N) / Y_N\}$

Endogenous Variables

C : Consumption
E : Nominal Exchange Rate
G_d : Government Deficit
I : Investment
P : Domestic Price Level
R : Real Interest Rate
U : Unemployment Rate
X : Net Exports
Y : GDP
Y^d : Disposable Income

Policy Variables

G : Government Expenditure
M : Money Stock

Exogenous Variables

P_w : Foreign Price Level
U_N : "Natural" Rate of Unemployment
Y_N : Potential GDP

π : Inflation Rate
π^e : Expected Inflation

Parameters

a = 220; b = 0.7754; d = 2000; e = 1000; f = 0.8; g = 600; h = 1000; k = 0.1583;

m = 0.1; n = 100; q = 0.75; t = 0.1875; v = 5; α =0.4; β =0.2; μ = 0.33;

To make use of theoretical results from the analysis of dynamic systems and from the optimal control literature, and to be able to perform policy analysis with DUALI, we need to obtain the state-space representation of Hall and Taylor's model, that is, to transform the model into a system of first order difference equations. To do this, we first linearize the model, then obtain its reduce form representation, and finally transform the reduce form into the state-space form.

Detailed steps to transform Hall and Taylor's nonlinear model into its state-space representation can be found elsewhere.[7] The linearization technique chosen for this model is that known as the Johansen's method, in which all the variables in the model are expressed as percent deviations from the model's steady-state solution. Without loss of generality, and to make the analytical and computational work easier, the original twelve-endogenous variables model was collapsed into a four-endogenous variables model involving GDP, the real interest rate, the nominal exchange rate and the price level as endogenous variables, the money supply and government expenditure as policy variables, and potential GDP and foreign prices as exogenous variables.[8]

The state-space representation of Hall and Taylor's model when collapsed into a four-endogenous variables model in which all the variables are percent deviations from the steady-state is given below.[9, 10]

1.1) $Y^{*} = A_{11}Y^{*}_{-1} + A_{13}\,\mathrm{plev}^{*}_{-1} + A_{17}\,\mathrm{xlplev}^{*}_{-1} + A_{1.11}\,\mathrm{xllplev}^{*}_{-1} + B_{11}M^{*}_{-1} + B_{12}\,G^{*}_{-1} + C_{11}YN^{*}_{-1} + C_{12}\,\mathrm{plevw}^{*}_{-1}$

1.2) $R^{*} = A_{21}Y^{*}_{-1} + A_{23}\,\mathrm{plev}^{*}_{-1} + A_{27}\,\mathrm{xlplev}^{*}_{-1} + A_{2.11}\,\mathrm{xllplev}^{*}_{-1} + B_{21}M^{*}_{-1} + B_{22}\,G^{*}_{-1} + C_{21}YN^{*}_{-1} + C_{22}\,\mathrm{plevw}^{*}_{-1}$

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7 See Mercado and Kendrick (1997a).

8 This variable structure is one of the most common ways in which textbook macroeconomic models are presented.

9 The steady-state solution for Hall and Taylor's original nonlinear model in levels is: Y = 6000, R = 0.05, plev = 1 and E =1. These steady-state values correspond to the following values for policy and exogenous variables: M = 900, G = 1200, YN = 6000 and plevw = 1. Since in the linearized state-space representation all variables are in percent deviations, their steady-state values are all zeroes.

10 In Hall and Taylor's model, the policy variables contemporaneously affect the model's endogenous variables, and this is also true for its "state-space" representation. In order to obtain a proper state-state representation, that is, one in which the control variables also appear with one lag, we have to assume that there is one lag of delay between a policy decision and its implementation (see Kendrick (1981), p. 10). Then, we can substitute M_{-1}^{*} for M^{*}, and G_{-1}^{*} for G^{*}. We will also assume that the exogenous variables YN^{*} and plevw^{*} affect the system with one lag instead of contemporaneously. Expressing the model in this way, we can make use of many results from the optimal control literature, which works with models with one-lag controls. Also, the DUALI software works in this way.

1.3) $plev^* = A_{31}\, Y^*_{-1} + A_{33}\ plev^*_{-1} + A_{37}\ xlplev^*_{-1} + A_{3.11}\ xllplev^*_{-1} + C_{31}\ YN^*_{-1}$

1.4) $E^* = A_{41} Y^*_{-1} + A_{43}\ plev^*_{-1} + A_{47}\ xlplev^*_{-1} + A_{4.11}\ xllplev^*_{-1} + B_{41} M^*_{-1} + B_{42}\ G^*_{-1} + C_{41} YN^*_{-1} + C_{42}\ plevw^*_{-1}$

1.5) $xlY^* = Y^*_{-1}$

1.6) $xlR^* = R^*_{-1}$

1.7) $xlplev^* = plev^*_{-1}$

1.8) $xlE^* = E^*_{-1}$

1.9) $xllY^* = xlY^*_{-1}$

1.10) $xllR^* = xlR^*_{-1}$

1.11) $xllplev^* = xlplev^*_{-1}$

1.12) $xllE^* = xlE^*_{-1}$

where:

Endogenous Variables

Y^* = GDP
R^* = Real Interest Rate
$plev^*$ = Domestic Price Level
E^* = Nominal Exchange Rate

Policy Variables

M^* = Money Stock
G^* = Government Expenditure

Exogenous Variables

$plevw^*$ = foreign Price Level
YN^* = Potential GDP

where the remaining "xl…" and "xll…" variables come from the re-labeling of the endogenous variables with lags greater than one, and where:

$A_{11} = -0.346$, $A_{13} = -0.606$, $A_{17} = 0.087$, $A_{1.11} = 0.087$,
$A_{21} = 7.811$, $A_{23} = 13.669$, $A_{27} = -1.953$, $A_{2.11} = -1.953$,
$A_{31} = 0.800$, $A_{33} = 1.400$, $A_{37} = -0.200$, $A_{3.11} = -0.200$,
$A_{41} = 1.154$, $A_{43} = 2.019$, $A_{47} = -0.288$, $A_{4.11} = -0.288$,

$B_{11} = 0.433$, $B_{12} = 0.231$, $B_{21} = -9.763$, $B_{22} = 4.386$,
$B_{41} = -2.442$, $B_{42} = 1.097$,

$C_{11} = 0.346$, $C_{12} = 0.000$, $C_{21} = -7.811$, $C_{22} = 0.000$,
$C_{31} = -0.800$, $C_{41} = -1.154$, $C_{42} = 1.000$.

In matrix notation, the state-space representation of Hall and Taylor's model can be written as:

1.13) $$x = A\, x_{-1} + B\, u_{-1} + C\, z_{-1}$$

where x is an augmented state vector defined as:

1.14) $$x = \begin{bmatrix} X \\ XL \\ XLL \end{bmatrix},$$

where:

1.15) $$X = \begin{bmatrix} Y^* \\ R^* \\ plev^* \\ E^* \end{bmatrix}, \qquad XL = X_{-1}, \qquad XLL = XL_{-1} = X_{-2},$$

1.16) $$u = \begin{bmatrix} M^*_{-1} \\ G^*_{-1} \end{bmatrix}, \qquad z = \begin{bmatrix} YN^*_{-1} \\ plevw^*_{-1} \end{bmatrix},$$

and where:

1.17) $$A = \begin{bmatrix} -0.346 & 0 & -0.606 & 0 & 0 & 0 & 0.087 & 0 & 0 & 0 & 0.087 & 0 \\ 7.811 & 0 & 13.669 & 0 & 0 & 0 & -1.953 & 0 & 0 & 0 & -1.953 & 0 \\ 0.8 & 0 & 1.4 & 0 & 0 & 0 & -0.2 & 0 & 0 & 0 & -0.2 & 0 \\ 1.154 & 0 & 2.019 & 0 & 0 & 0 & -0.288 & 0 & 0 & 0 & -0.288 & 0 \\ 1 & & & & & & & & & & & \\ & 1 & & & & & & & & & & \\ & & 1 & & & & & & & & & \\ & & & 1 & & & & & & & & \\ & & & & 1 & & & & & & & \\ & & & & & 1 & & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & & 1 & & & & \end{bmatrix},$$

1.18) $$B = \begin{bmatrix} 0.433 & 0.231 \\ -9.763 & 4.386 \\ 0 & 0 \\ -2.442 & 1.097 \end{bmatrix}, C = \begin{bmatrix} 0.346 & 0 \\ -7.811 & 0 \\ -0.8 & 0 \\ -1.154 & 1 \end{bmatrix}$$

2) Introduction to Dynamic Analysis Methods

Before beginning to perform optimal policy experiments with a given model, it is important to analyze its basic dynamic behavior. Here, we will introduce the most common theoretical results and simulation procedures to that end.

2.a) eigenvalues: computation and use

The eigenvalues of matrix A convey useful information about the dynamic properties of the model. [11] They can be easily computed with specialized software such as Matlab, Mathematica, etc.

Let's assume that the model has a steady-state. Then, depending on the magnitude of those eigenvalues, the system will be stable, unstable, or it will display the saddle-point property:

.. if they all lie within the unit circle[12], the model is globally stable. It will converge to its steady-state from any initial conditions

.. if they all lie outside the unit circle, the model is dynamically unstable. Unless it starts from the steady-state itself, it will diverge from it for any other set of initial conditions

.. if some lie within the unit circle, while others lie outside the unit circle, the steady-state is a saddle point. The system will converge towards the steady-state from some initial conditions, and will diverge from other.

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[11] For an extended treatment of the analysis of dynamic systems related to economics, see Chiang (1984), and Azariadis (1993).

[12] If the eigenvalues are complex numbers, this means that their modulus is smaller than 1. If they are real number, it means that their absolute value is smaller than one.

The speed of convergence-divergence is also determined by the magnitude of the eigenvalues. For instance, a modulus smaller than one but near one will indicate a slow adjustment towards the steady-state, while a modulus near zero will imply a faster convergence. A modulus greater than one but near one will indicate that anticipated changes in exogenous variables that will take place in the future can have large effects today. Finally, the presence of complex eigenvalues will imply cyclical behavior for some or all of the system variables.

For the linearized version of Hall and Taylor's model, we have the following eigenvalues:[13]

$$\lambda_1, \lambda_2 = 0.68431{+}0.4042i, \quad \lambda_3 = -0.31663, \quad \lambda_4 = 0.002, \quad \lambda_5 \text{ to } \lambda_{12} = \text{all near zero.}$$

There are two complex eigenvalues, with modulus less than one, while the remaining ones are all real and smaller than one in absolute value. Thus, Hall and Taylor's linearized model is stable and present cyclical behavior. That is, its convergence toward the steady-state will be in the form of damped oscillations.

2.b) dynamic paths

The next step in the analysis of the model is to visualize its dynamic evolution for given changes in policy variables, as a way of detecting implausible patterns of behavior.[14,15] The graphs below shows the results of two experiments: a 10% permanent increase in the money supply (M) and a 10% permanent increase in government expenditure (G).[16] On the vertical axes are the percent deviations from steady-state values while on the horizontal axes are the time periods (e.g.: a value of 0.02 means "2% above steady-state". It does not mean "2% increase with respect to the previous period". Thus, a 10% permanent increase in M means that the money stock is increased by 0.1 at the initial period and kept constant at the new level from then on).[17] Since all variables (endogenous, policy and exogenous) are in percent deviations, their steady-state values are all zeroes.

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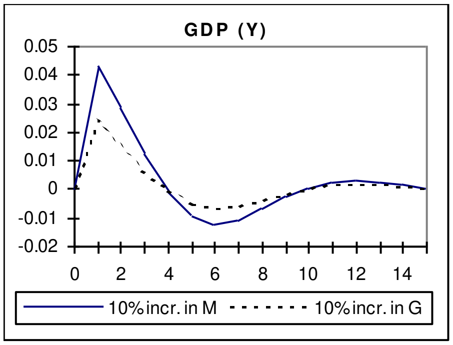
[13] There are many software packages capable of computing eigenvalues. These were computed with Matlab (see the Appendix).

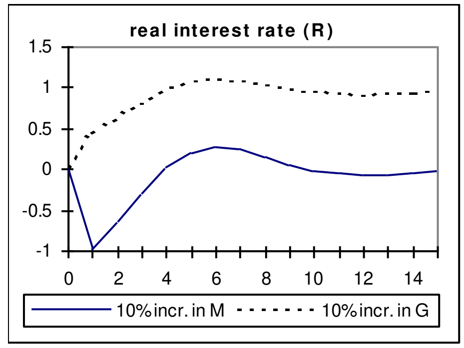
[14] These simulations can be easily implemented in software with standard simulation capabilities, such as, for example, GAMS (see Mercado, Kendrick and Amman (1997), and Mercado and Kendrick (1997a)). Though DUALI is a software oriented toward deterministic and stochastic control applications, it can also handle standard simulations. To do so, just set to zero the weights on the state variables (W matrix) and set to the maximum possible value the weights on the controls (Lambda matrix). Then, define the desired path for the controls as equal to the policy change to be introduced, and solve the as a Deterministic QLP problem (see Amman and Kendrick (1996b), Chapter 1). Simulations of shocks to the exogenous variables and to the initial values for the endogenous variables can be implemented in an analogous way.

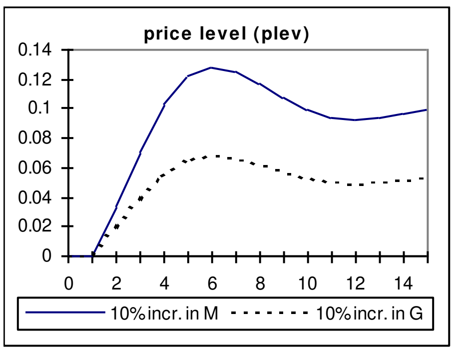
[15] To run these simulations, use program htdua01.dui (making the appropriate changes. See the "description" section in the "data" menu).

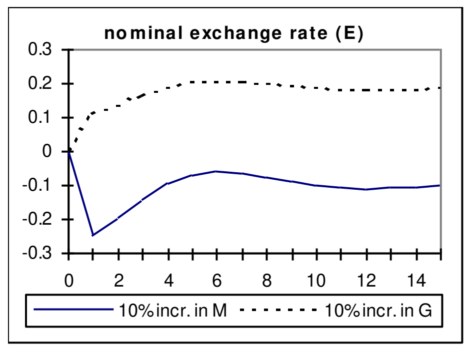
[16] To save notation, from now on we will obviate the " * " on the model's variables, but it should be clear that we will still be making reference to percent deviations from baseline.

[17] See Mercado and Kendrick (1997a).









As expected, the increase in M causes an increase in Y during the first periods, together with a significant drop in R. The value of 0.04 on the vertical axes of the GDP graph means that Y went up by 4% in the first quarter, while the value of -1 in the grapth for the real interest rate means that R has fallen by 100% (i.e from, say, 6% to 3%). However, in the long-run, real variables (Y and R) come back to their steady-state values, while nominal variables experience permanent changes (plev increases 10%, the same amount as M, while E decreases 10%) of equal magnitude to the change in M.

The increase in G has a smaller effect on Y, which is also neutral in the long-run. However, there is a strong impact on R, which after five periods increases by more than 100% with respect to its previous steady-state value, due to the crowding-out effect of government expenditure on private expenditure. Meanwhile, plev and E both increase and then stabilize on a new and higher steady-state value (around 6% higher for plev and almost 20% higher for E).

2.c) dynamic controllability: computation and use

Once we have studied the dynamic properties of our economic model, the next step is analyze its controllability, that is, the power of the available policy tools to drive the system towards pre-specified desired paths.

Jan Tinbergen[18] established the conditions for static controllability. In order to hit a number "n" of targets, we need at least an equal number of independent policy instruments. However, this condition can be overcome in a dynamic context.

We may start by asking if it is possible to transfer the system from any given state at time "0" to any other state at time "0 + t" through a suitable choice of values of the policy tools. This is the so called condition of dynamic controllability. For a system to be dynamically controllable, it has to be true that:

2.1) $$\mathrm{rank}(R_1, \ldots, R_m) = m$$

where "m" is the number of target variables, where "t" (the time horizon) is greater than "m", where:

2.2) $$R_i = S\,A^{i-1}\,B$$

and where A and B are respectively the state and control matrices of the model, and S is a matrix to select, from the set of state variables, those that will be the targets of policy. That is:

2.3) $$S\,Z = Y$$

where Z is the vector of state variables and Y is the vector of target variables.

For the state-state representation of Hall and Taylor's model, we know that A is a (12 x 12) matrix. However, 8 out of the 12 state variables are in fact lagged endogenous variables re-defined just for convenience. Thus, for instance, we may just be interested in controlling 4 variables only (Y, R, plev and E), or even a smaller subset. Assuming that we want to control all the four variables, we will have:

2.4) $$S = \begin{bmatrix} 1 & 0 & . & . & . & . & . & . & . & . & . & 0 \\ 0 & 1 & 0 & . & . & . & . & . & . & . & . & 0 \\ 0 & 0 & 1 & 0 & . & . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & 1 & 0 & . & . & . & . & . & . & 0 \end{bmatrix}$$

Thus, the dynamic controllability condition is:

[18] See Tinbergen (1956).

2.5) $$\mathrm{rank}(R_1, \ldots, R_4) = 4$$

which is effectively met by the linearized Hall and Taylor's model.[19]

There are many more theoretical results in connection with the controllability properties of a system in both deterministic and stochastic settings. The one presented here is one of the most intuitive and relatively easy to check.[20]

3) Introduction to optimal policy analysis methods with DUALI

In the previous section, we presented the responses of Hall and Taylor's model to changes in the policy variables. Optimal policy analysis is interested in a sort of "reverse" analysis. It begins by posing this question: how should policy variables be set in order for the target variables to follow pre-specified paths? [21]

The most popular way of stating this problem is as a Quadratic Linear Problem (QLP). In formal terms, the problem is expressed as one of finding the controls $(u)_{t=0}^{N}$ to minimize a quadratic "tracking" criterion function J of the form:

3.1) $$J = E\left\{\frac{1}{2}\left[x_N - x_N^{\#}\right]' W_N^{\#}\left[x_N - x_N^{\#}\right] + \frac{1}{2}\sum_{t=0}^{N-1}\left(\left[x_t - x_t^{\#}\right]' W_t^{\#}\left[x_t - x_t^{\#}\right] + \left[u_t - u_t^{\#}\right]' \Lambda_t^{\#}\left[u_t - u_t^{\#}\right]\right)\right\}$$

subject, as a constraint, to the state-state representation of the economic model:

3.2) $$x_t = A\, x_{t-1} + B\, u_{t-1} + C\, z_{t-1} + \varepsilon_{t-1}$$

where E is the expectation operator, $x^{\#}$ and $u^{\#}$ are desired paths for the state and controls variables respectively, W and Λ are weighting matrices for states and controls respectively, ε is a vector of random disturbances, and where all the other variables were defined above.

The quadratic nature of the criterion function implies that deviations above and below target are penalized equally, and that large deviations are more than proportionally

[19] For a program to compute this condition, see the TSP statement in the Appendix.
[20] For example, there are also uniqueness, stabilizability and instrument stability conditions for a dynamic system when it is put, as we will see below, within a control framework. For an introductory presentation of these conditions, see Turnorvsky (1977) and Holly and Hughes-Hallett (1989). For an advanced treatment, see Aoki (1976).
[21] For an introductory presentation of optimal control for economic models, see Turnovsky (1977). For a more advanced treatment, see Chow (1975), Holly and Hughes-Hallett (1989), and Kendrick (1981).

penalized than small deviations. This particular form of the criterion function is not the only possible one, but is the most popular.[22]

The way in which we treat uncertainty has important implications for the solution methods of this problem, as well as on the simulation techniques. If we completely ignore the presence of uncertainty - which may arise, for example, from additive noise, parameter uncertainty and/ or measurement error - we are left with a deterministic control problem. If we account for some or all of the possible forms of uncertainty, we face a stochastic control problem.

The solution of deterministic and / or stochastic control problems, even when they are of the Quadratic Linear form, quickly becomes very involved. Thus, to make our task feasible, we have to rely on computational methods and specialized software.

DUALI is a specialized software that can receive as inputs the desired paths for target and control variables, weighting matrices, and the state-space representation of the economic model with or without its stochastic specifications, and which is able to generate sophisticated simulation environments and to compute, among other things, the optimal feedback rule and the implied solution paths for states and controls.[23]

In what follows, we will use DUALI to perform deterministic and stochastic experiments with the state-space representation of Hall and Taylor's model. We will assume that the policy goal is to stabilize Y, R, plev and E around steady-state values (that is, around zero). High and equal weights[24] will be put on stabilizing Y and plev, lower and equal on R and E, and even lower and equal on the policy variables M and G. Neither the desired paths nor the weighting matrices (shown below) will vary with time.

---

[22] For a discussion of the properties of different criterion functions, see Blanchard and Fischer (1989), Chapter 11.

[23] See Amman and Kendrick (1996b) and (1997c).

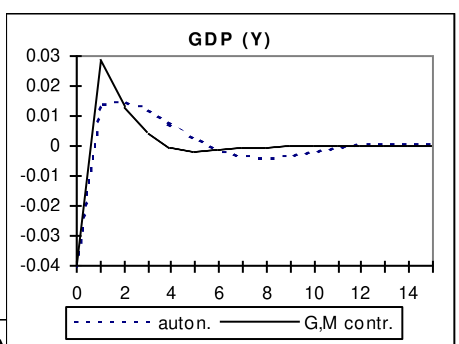
[24] There is a conceptual difference between "weights" and "priorities" which arises when the variables of interest are in levels and also expressed in different units of measurement. For instance, if GDP is measured in dollars and prices are measured by an arbitrary price index, equal weights on these two variables will probably imply different policy priorities and vice versa. Since all variables in the state-space representation of Hall and Taylor's model are in percent deviations from steady-state, weighs and priorities can be considered as equivalent within certain limits. However, it should be clear that, for example, an interest rate 50% below steady-state values is something feasible, while a level of GDP 50% below steady-state is not. In such a case, there is not an analogy between weights and priorities. See Park (1997).

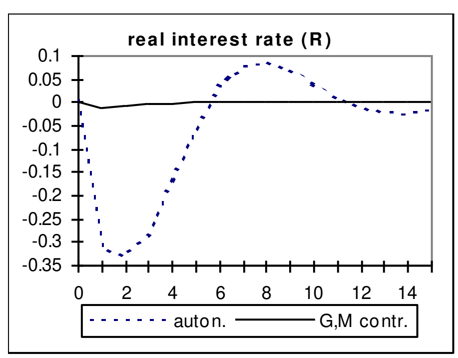
3.3) $$W = \begin{bmatrix} 100 &&&&&&&&&&& \\ & 50 &&&&&&&&&& \\ && 100 &&&&&&&&& \\ &&& 50 &&&&&&&& \\ &&&& 0 &&&&&&& \\ &&&&& 0 &&&&&& \\ &&&&&& 0 &&&&& \\ &&&&&&& 0 &&&& \\ &&&&&&&& 0 &&& \\ &&&&&&&&& 0 && \\ &&&&&&&&&& 0 & \\ &&&&&&&&&&& 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 25 & \\ & 25 \end{bmatrix}$$

3.a) deterministic control

In this section, we will ignore all possible sources of uncertainty. Let's assume, for example, that the economy is going through a recession provoked by a temporary adverse shock to net exports which causes Y to be 4% below its steady-state value. Given the weight structure adopted in the previous section, what would be the optimal paths for government expenditure (G) and the money supply (M) in order to bring the economy back to its steady-state? How do the optimal paths for the state variables compare against what would be the autonomous response of the system to that kind of shock?

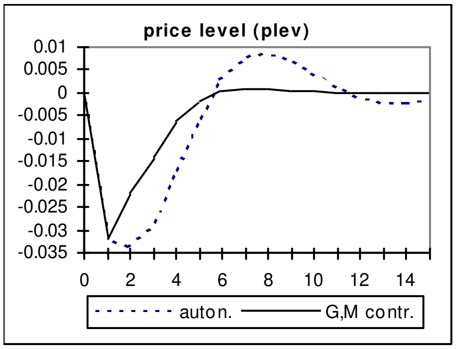
To implement this experiment in DUALI, we have to set the problem as a deterministic one, set all the desired paths for states and controls equal to zero, impose the corresponding weights on states and controls, set an initial value for Y equal to -0.04, and chose the option "Solve: QLP". [25] To obtain the autonomous path of the system, we have to proceed in an analogous way as we did in the previous section to simulate the effects of changes in policy variables. That is, we have to impose zero weights on the state variables, very high and equal weights on the controls and, as above, set an initial value for Y equal to -0.04. The results are shown in the graphs below.[26]

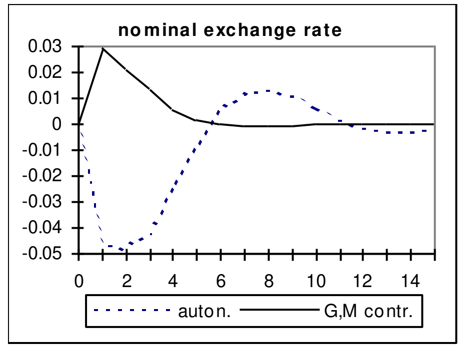


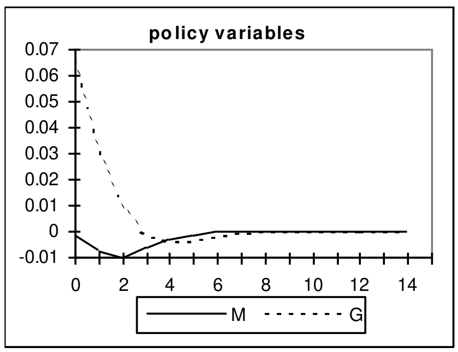


[25] See A

[26] To run this simulations, use program htdua01.dui (making the appropriate changes. See the "description" section in the "data" menu).







The optimal solution paths for the states outperform the autonomous responses of the system for all the four target variables. This comes as no surprise, though it may not always be the case. Indeed, remember that the optimal solutions are obtained from the minimization of an overall loss function. On some occasions, depending on the weight structure, it may be better to do worse than the autonomous response for some targets in order to obtain more valuable gains from others.

Why does the autonomous path of the economy display the observed behavior? Here is how Hall and Taylor explain it:

"With real GDP below potential GDP after the drop in net exports, the price level will begin to fall. Firms have found that the demand for their products has fallen off and they will start to cut their prices (...). The lower price level causes the interest rate to fall.[27] With a lower interest rate, investment spending and net exports will increase.[28] The

[27] Since less money is demanded by people for transactions purposes. See Hall and Taylor (1993), Chapter 8.3.
[28] Since the price level falls much less than the real interest rate during the first periods of the adjustment, the nominal exchange rate has to fall too, as can be derived from equation "*ix*" in the original Hall and Taylor's model. This implies that the real exchange rate will fall, then causing net exports (see equation "*x*") to raise.

increase in investment and net exports will tend to offset the original decline in net exports. This process of gradual price adjustment will continue as long as real GDP is below potential GDP." [29]

What does explain the observed optimal path of the four variables of interest? We can see that Y is brought up very quickly, going from 4% below steady-state to a 3% above steady-state, to then decay slowly to its steady-state value. This performance could be attributed to the more than 6% increase in G that can be observed in the policy variables' graph. Meanwhile, R experiences almost no variation when compared to the big drop - almost 35% - implied by the autonomous behavior of the system. Once again, the increase in G, which crowds out investment, puts an upward pressure on the interest rate, thud keeping it from falling. Finally, the nominal exchange rate has to go up to compensate for the fall in prices, given that the real interest rate does not change much.

We can also see that monetary policy plays a minor role when compared against fiscal policy.[30] Even though we put the same weights on both variables, government expenditure appears to be more effective to bring the economy out of its recession given the weight structure we put on the target variables.

It is interesting to analyze the different combinations of behavior of variables that the policy maker can achieve given a model and a criterion function. The curve showing those combinations is known as the policy frontier.[31]

For instance, we may want to depict the trade-off between the standard deviations of Y and plev in Hall and Taylor's model when, as above, Y is shocked by a negative 4% in period zero. To obtain the corresponding policy frontier, we have to vary the relative weights on Y and plev, perform one simulation for each weight combination and compute the corresponding standard deviations. The results of six of such experiments, keeping the same weights on the remaining states and controls as in the above simulation, are shown in the table and graph below.[32]

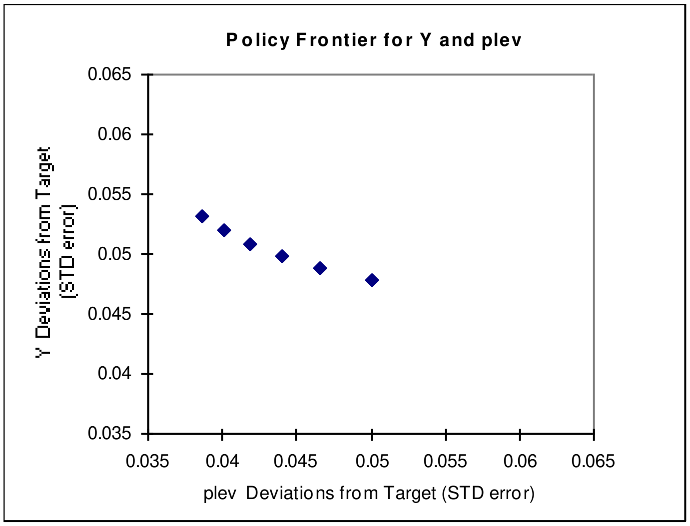
Experiment	**Weight on Y**	**Weigh on plev**	**STD Y**	**STD plev**
1	100	0	0.0479	0.0500
2	80	20	0.0489	0.0466
3	60	40	0.0499	0.0440
4	40	60	0.0509	0.0419
5	20	80	0.0520	0.0401
6	0	100	0.0531	0.0386

[29] Hall and Taylor (1993), page 232.

[30] Notice that the optimal values for the policy variables are computed for periods 0 to 14 only. Given that we are working with a state-space representation of the model, policy variables can only influence the next period state variables. That is, the controls at period 0 are chosen, with a feedback-rule, as a function of period 0 states, but they determine period 1 states, and so on. See Kendrick (1981).

[31] See Hall and Taylor (1993), Chapter 18.

[32] To run these simulations, use program htdua01.dui, changing the weights on Y and plev.



The policy frontier for Y and plev is clearly shown in the graph above, where each diamond represents the result of an experiment. The higher the weight on Y relative to that of plev, the lower its standard deviation, and vice versa. The flatness of the curve indicates that it is easier the achieve a reduction in the percent deviation from target for plev than for Y. Of course, shape and location of this particular policy frontier are conditional to the weight structure imposed on the other model's variables. For example, if we increase the weigh on the policy variables, the policy frontier will shift up and to the right, farther away from the origin (the (0,0) point of zero deviations for Y and plev). This will be due to the more restricted possibilities for actively using the policy variables to reach the targets for Y and plev.

3.b) stochastic control

In this section, we will begin to take uncertainty into account. Indeed, macroeconomic models are only empirical approximations to reality. Thus, we have to take into account that there are random shocks almost permanently hitting the economy (additive uncertainty), that the model parameters are just estimated values with associates variances and covariances (multiplicative uncertainty), and that the actual values of the model's variables and initial conditions are never known with certainty (measurement error). [33]

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[33] See Kendrick (1981). As we said above, CE presupposes additive uncertainty only, while QLP is deterministic (no uncertainty). However, the presence of additive uncertainty does not affect the form of the solution procedure for choosing the optimal controls (of course, it implies a different simulation method in order to generate additive uncertainty). In this sense, QLP and CE are equivalent.

Stochastic control methods artificially generate a dynamic stochastic environment through random shocks generation. They use specific procedures for choosing the optimal values for each period policy variables: Certainty Equivalence (CE) when there is additive uncertainty only, Open Loop Feedback (OLF) when there is parameter uncertainty, and DUAL when there is active learning. Also there are specific mechanisms of projection-updating of parameters and variables. In that way, these methods allow us to perform sophisticated simulations.

In what follows, we will perform experiments incorporating some forms of additive and multiplicative uncertainty into Hall and Taylor's model. We will proceed in three steps. First, we will analyze the differences in qualitative behavior of the policy variables when some different procedures for choosing their optimal values are used (specifically, (CE) versus (OLF)). Second, we will compare the quantitative performances of CE and OLF procedures within artificially generated stochastic environments including passive learning mechanisms. Finally, we will compute an optimal policy frontier.

3.b.1) qualitative comparison between CE and OLF: control without parameter updating

Some years ago, William Brainard[34] showed that, for a static model, the existence of parameter uncertainty causes the optimal policy variables to be used in a more conservative way as compared to the case of no parameter uncertainty. However, as with the famous Tinbergen result, this finding can not be completely translated into a dynamic setting. Once again, the existence of dynamics implies considerably changes and at the same time opens new possibilities for policy management.

The procedure for choosing the controls in the presence of parameter uncertainty (OLF) differs from the standard deterministic QLP procedure or its "certainty equivalent" (CE).[35] Some analytical results have been provided by Franklin Shupp[36] in connection with the qualitative behavior of the policy variables when the OLF procedure is used in a model with one state and one control. He found that when uncertainty concerns the control parameters only, Brainard's result still holds: a more conservative use of the controls will be the optimal policy. However, he also found that the reverse is true when the uncertainty is in the state parameters only. Finally, he found that when uncertainty is in both the control and the state parameters, no general results can be obtained.

There are not straightforward theoretical results for the case of models with several states and controls. To illustrate some possible outcomes, and to show a first contrast between patterns of behavior generated by CE and OLF procedures, we will perform an experiment with Hall and Taylor's model. As in the previous section, we will assume that Y is 4% below its steady-state value at time zero and we will keep the same weight structure and desired paths. We will also assume that there is uncertainty in

[34] See Brainard (1969).
[35] See Kendrick (1981).
[36] See Shupp (1976).

connection with six out of the eight control parameters in the B matrix, and that the standard deviation of each of these parameters is equal to 20%. The vector of the initial values of uncertain parameters (TH0), the matrix that indicates which parameters in the model are treated as uncertain (ITHN), and the variance-covariance matrix of uncertain parameters (SITT0) will be as follows:

3.4) $$TH0 = \begin{bmatrix} b_{11} = 0.433 \\ b_{12} = 0.231 \\ b_{21} = -9.763 \\ b_{22} = 4.386 \\ b_{41} = -2.442 \\ b_{42} = 1.097 \end{bmatrix}, ITHN = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 4 & 1 \\ 1 & 4 & 2 \end{bmatrix},$$

$$SITT0 = \begin{bmatrix} 0.00749 & & & & & \\ & 0.00213 & & & & \\ & & 3.81264 & & & \\ & & & 0.76947 & & \\ & & & & 0.23853 & \\ & & & & & 0.04813 \end{bmatrix},$$

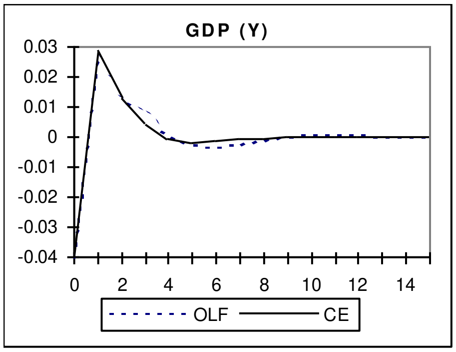
All three matrices will remain constant during the simulation. The elements in SITT0 are computed by taking 20% of the corresponding element in TH0 and then squaring the result. Thus, for the b_{11} coefficient this is

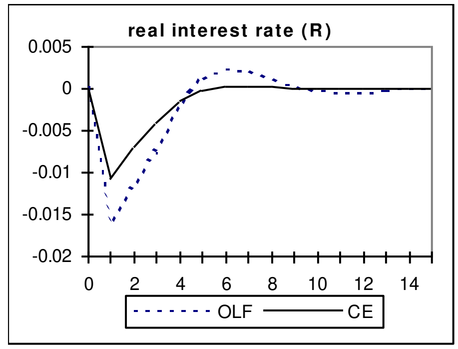
$$[(0.2)\ (0.433)]^2 = 0.00749.$$

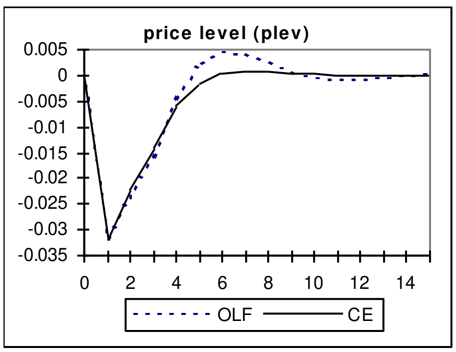
To carry out the experiment, we will select the following DUALI options: *complexity*: stochastic without measurement error; *model size*: 6 uncertain parameters, 1 Monte Carlo run*; options stochastic*: read in random terms, but set them (i.e. the XSIS) all equal to zero.[37]

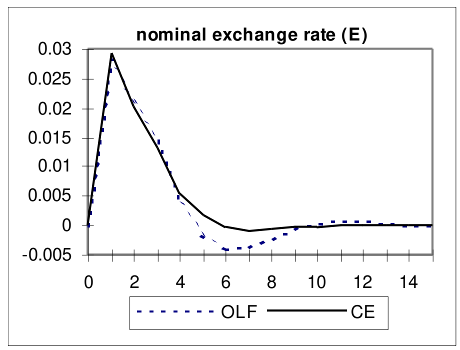
The graphs below show the results for both the CE[38] solution obtained with "Solve: QLP" and the OLF solution obtained with "Solve: OLF".[39]

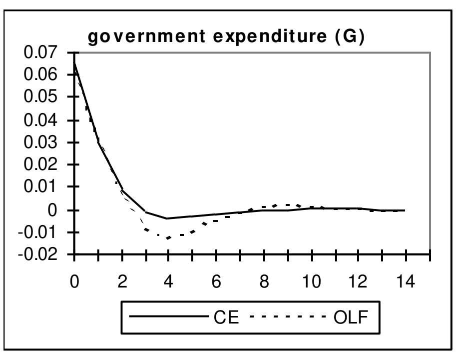
[37] See Amman and Kendrick (1996b), Chapter 2.
[38] Since the only disturbance is the off-steady state initial Y value (equal to -0.04), the CE solution and the deterministic solution are completely equivalent.
[39] To run these simulations, use program htdua01.dui for the CE procedure (making the appropriate changes. See the "description" section in the "data" menu) and, for the OLF procedure, use program htdua02.dui.

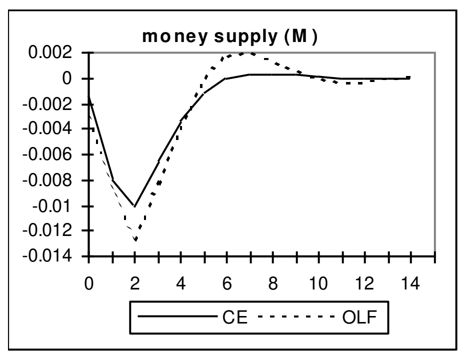












Qualitatively, the patterns of behavior for both the states and policy variables appear quite similar, though overall results are worse in the OLF case. This is not surprising, since the "quasi-deterministic" environment within which we performed the experiment does not allow the exploitation of the knowledge of the variance-covariance parameter matrix through a learning process.[40] In fact, the interest of this experiment resides in the comparison between the behavior of the policy variables across different procedures.

As can be seen in the graphs above, the use of government expenditure is "more cautious" with the OLF procedure and for the first periods. This seems to be in line of

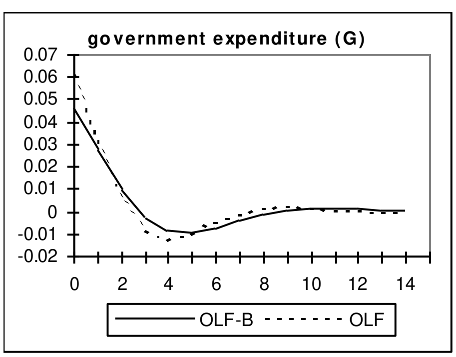
[40] This point will become more clear in the next section, when we compare CE versus OLF across Monte Carlo simulations with a projection-updating mechanism

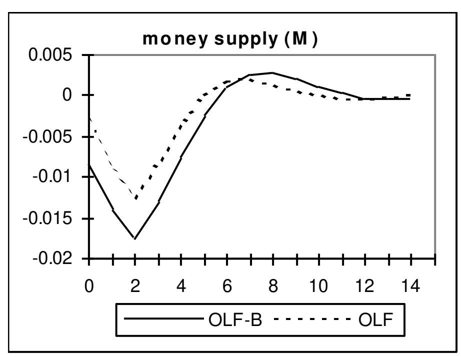
the Brainard-Shupp results mentioned before. However, the reverse is true for the case of the money supply, which is used "more aggressively" with OLF. Thus, we can see how going from a univariate to a multivariate setting may have important consequences, as is also the case of a change from static to dynamic models.[41]

It is interesting to explore the consequences of increasing the level of uncertainty of the model parameter's corresponding to one of the policy variables. For example, let's assume that we now double the standard deviation of the parameters corresponding to government expenditure (parameters b_{12}, b_{22} and b_{42}) from 20% to 40%. Then, the SITT0 matrix becomes:

3.5)
$$SITT0 = \begin{bmatrix} 0.00749 & & & & & \\ & 0.00853 & & & & \\ & & 3.81264 & & & \\ & & & 3.07791 & & \\ & & & & 0.23853 & \\ & & & & & 0.19254 \end{bmatrix}$$

The graphs below contrast the behavior of the policy variables for this experiment[42] (named OLF-B) against their behavior showed by the same variables in the experiment analyzed above (named, as above, OLF).





[41] Remember, for example, the case of the "Tinbergen results" that we analyzed before, when we presented some results of dynamic controllability.
[42] To run this experiment, use program htdua02.dui, introducing the corresponding changes in the SITT0 matrix.

As one could expect, the increase in the relative uncertainty of government expenditure parameters induces a more cautious use of that policy variable, at least during the first periods. At the same time the money supply, now with a relatively lower associated uncertainty, is used more actively, also during the first periods. Though these findings seem plausible, they do not reflect any theoretical result, not yet available for this kind of problem. As with the previous experiments, we could perhaps find different results for a different model.

3.b.2) quantitative performance comparison between CE and OLF: control with parameter updating

We will now move towards a more complex stochastic environment. As in the previous section, we will assume that that some of the model parameters are uncertain, but now we will also assume that the model is constantly shocked by additive noise, that the true model is not known to the policy maker, and also that a passive-learning process takes place. We will perform several Monte Carlo runs for each of the procedures (CE and OLF).[43]

The general structure of each Monte Carlo run will be as follows. At time zero, a vector of model parameters will be drawn from a normal distribution whose mean and variances are those of matrices TH0 and SITT0. Then, at each time “t”, we will have:

1) random generation of a vector of an additive shocks
2) computation the optimal controls for periods t to N (terminal period)
3) propagation of the system one period forward (from period t to period t+1) applying the vector of controls (for period t only) computed in step 2.
4) projection-updating of next period parameters and variance-covariance matrix

For choosing the optimal control at each period (step 2) we will use either a Certainty Equivalence (CE) procedure or, alternatively, an Open Loop Feedback procedure (OLF). For the projection-updating mechanism (step 4) we will use a Kalman filter.

Thus, each Monte Carlo run begins with a vector of parameter estimates which is different from their “true” value. Using this parameter vector, the policy maker computes (with a CE or an OLF procedure) the optimal values of the controls, and then she “applies” those values corresponding to time “t” only. However, the response of the economic system (its forward movement from time “t “to time “t+1”) will be generated by “the computer” using the “true” parameter values which are unknown to the policy maker. Then, at period “t+1” a new observation is made of the state vector, which is used to compute updated parameter estimates with a Kalman filter. After a number of time periods, the sequence of updated estimates should begin to converge to their “true” value.

[43] See Amman and Kendrick (1996b), Chapter 4.

As in the previous section, we will assume that there is uncertainty in connection with six out of the eight control parameters in the B matrix, and that the standard deviation of each of these parameters is equal to 20%. Then, matrices TH0, SITT0 and ITHN will be the same as in 3.5. We will also assume that GDP (Y) and the price level (plev) are hit by additive shocks of 2% standard deviation, while the real interest rate (R) and the nominal exchange rate (E) experience shocks of 5% standard deviation. Thus, the variance-covariance matrix of additive noises (Q), will be as follows:[44]

(3.6)

$$Q = \begin{bmatrix} 0.0004 &&&&&&&&&&& \\ & 0.0025 &&&&&&&&&& \\ && 0.0004 &&&&&&&&& \\ &&& 0.0025 &&&&&&&& \\ &&&& 0.1^{(-9)} &&&&&&& \\ &&&&& 0.1^{(-9)} &&&&&& \\ &&&&&& 0.1^{(-9)} &&&&& \\ &&&&&&& 0.1^{(-9)} &&&& \\ &&&&&&&& 0.1^{(-9)} &&& \\ &&&&&&&&& 0.1^{(-9)} && \\ &&&&&&&&&& 0.1^{(-9)} & \\ &&&&&&&&&&& 0.1^{(-9)} \end{bmatrix}$$

The results of 100 Monte Carlo runs are shown in the table below.[45]

	CE	OLF
Average Criterion Value	5.60	5.59
Runs with Lowest Criterion	47	53

The Open Loop Feedback procedure does slightly better than the Certainty Equivalence, not only in connection with the average criterion value, but also in terms of the number of Monte Carlo runs with the lowest criterion. As can be appreciated in the graph below, where each diamond represents the value of the criterion function for one Monte Carlo run, most of the diamonds are close to the 45 degree line, indicating a similar performance for both procedures. There are not significant outliers that could be introducing a bias in the computed average criterion values.

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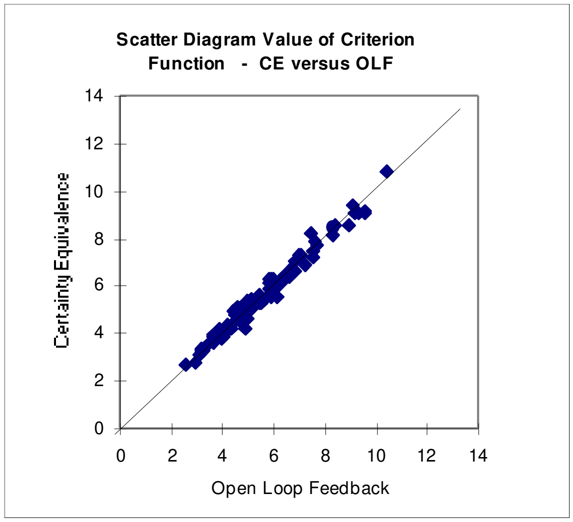
[44] We want the shocks to affect contemporaneous variables only, and not their lagged values. However, if we set to zero the elements of the Q matrix corresponding to lagged variables, DUALI will give us an error message. That is why we set those elements equal to the minimum possible value (0.000000001).

[45] To run this simulation, use program htdua03.dui (see the “description” section in the “data” menu).



These results are against what one would intuitively expect, since in the presence of parameter uncertainty OLF should do much better than CE. However, we have to mention that there are not theoretical results yet developed in connection with the relative performance of CE versus OLF. This experiment results are conditioned on the model structure, its parameter and parameter variances values, and may well change (in any direction) in a different context.[46]

Part B: Policy Analysis with Rational Expectations Macroeconomic Models

4) John Taylor's Closed Economy Model

John Taylor's closed economy model is a small prototype model with staggered contracts and rational expectations variables which generates a rich pattern of dynamic behavior.[47] It contains the equations, variables and parameters listed below.[48]

Equations

$$1A) \qquad x_t = \tfrac{\delta}{3} \sum_{i=0}^{2} \hat{w}_{t+i} + \tfrac{1-\delta}{3} \sum_{i=0}^{2} \hat{p}_{t+i} + \tfrac{\gamma}{3} \sum_{i=0}^{2} \hat{y}_{t+i}$$

[46] See Amman and Kendrick (1997b). Working with a different model, they find a better performance of OLF with respect to CE.

[47] See Taylor (1993), Chapter 1. Earlier versions of this model can be found in Carlozzi and Taylor (1985), which contains extended explanations of its inner workings, and in Taylor (1985), which also contains policy coordination experiments in a difference games framework.

[48] For a detailed presentation of each of the equations you are referred to Taylor (1993).

$$2A) \qquad w_t = \tfrac{1}{3} \sum_{i=0}^{2} x_{t-i}$$
$$3A) \qquad p_t = \theta\, w_t$$
$$4A) \qquad y_t = -d\, r_t$$
$$5A) \qquad m_t - p_t = -b\, i_t + a\, y_t$$
$$6A) \qquad r_t = i_t - \hat{p}_{t+1} + p_t$$

Variables (all except p_t, i_t and r_t, are logarithms and are deviations from means or secular trends) [49]

x_t = contract wage
w_t = average wage
p_t = price level (the expected inflation rate is defined as $\hat{\Pi}_t = \hat{p}_{t+1} - p_t$)
y_t = output
i_t = nominal interest rate
r_t = real interest rate
m_t = money stock

where " ^ " means "expectation through period t".

Parameters

$\delta = 0.5; \quad \gamma = 1; \quad \theta = 1; \quad a = 1; \quad b = 4; \quad d = 1.2;$

Equation 1A is a staggered-wage setting equation. A wage decision lasts three years, with one third of the wages being negotiated each year. The contract wage at time "t" depends on expectations of future wages paid to other workers, expectations of prices, and expectations of real output as a proxy for future demand conditions. Equation 2A gives the average wage in the economy, while equation 3A reflects mark-up pricing behavior from the part of firms. Equations 4A and 5A define, respectively, standard IS and LM schedules. Finally, equation 6A gives the real interest rate as the nominal interest rate deflated by the rationally expected inflation rate.

The model has 6 equations, 6 endogenous variables, 1 "explicit" policy variable (m = money stock) and 1 "implicit" (Government expenditure) which will be appended to equation 4A as a shift factor. This model is dynamic, linear, and has the "natural rate" property: nominal shocks may affect real variables in the short-run, but not in the long run.

As a rational expectations model, Taylor's model requires of specific solution methods different from those applied to standard models.[50] Also, the analysis of its

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[49] For a basic introduction to this way of expressing a model's variables, see Mercado and Kendrick (1997a). For an extended treatment of this topic, see Amman and Kendrick (1996a); Dixon et al. (1992), Chapter 3; and Kendrick (1990), Chapter 4.

[50] For a practical introduction to these solution methods, with applications to John Taylor's closed economy model and its extension to a two-country model, see Mercado and Kendrick (1997b).

dynamic properties such as the computation of eigenvalues and the condition of dynamic controllability become more involved. We will not deal with these issues here.[51] In what follows, we will focus on the implementation of Taylor's model in DUALI to perform simulations and optimal policy analysis.

In matrix notation, Taylor's model can be written as:[52]

4.1) $$x_t = A_0 x_t + A_1 x_{t-1} + B_1 u_{t-1} + C_1 z_{t-1} + \hat{D}_1 x^e_{t/t} + \hat{D}_2 x^e_{t+1/t} + \hat{D}_3 x^e_{t+2/t} + \varepsilon_t$$

where:

4.2) $$x_t = \begin{bmatrix} x_t^{cw} \\ w_t \\ p_t \\ y_t \\ i_t \\ r_t \\ xl_t^{cw} \end{bmatrix}, \quad u_{t-1} = \begin{bmatrix} m_{t-1} \\ g_{t-1} \end{bmatrix},$$

$$A_0 = \begin{bmatrix} 0 & 0.1666 & 0.1666 & 0.3333 & 0 & 0 & 0 \\ 0.3333 & 0 & & & & & 0.3333 \\ & 1 & 0 & & & & \\ & & & 0 & & -1.2 & \\ & & 0.25 & 0.25 & 0 & & \\ & & 1 & & 1 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & & & & & 0.3333 \\ & & 0 & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ -0.25 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \hat{D}_2 = \begin{bmatrix} 0 & 0.1666 & 0.1666 & 0.1666 & 0 & 0 & 0 \\ & 0 & & & & & \\ & & 0 & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & & -1 & & & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

[51] To learn more about these computations, see Holly and Hughes-Hallett (1989), Chapter 7.

[52] This particular matrix notation is known as "Pindyck Form" or "I-A" form (see Amman and Kendrick (1996), Chapter 1. In Amman and Kendrick (1996b), expectations are conditioned on the information available at "t-1", while in Taylor's model they are conditioned at "t".

$$\hat{D}_3 = \begin{bmatrix} 0 & 0.1666 & 0.1666 & 0.1666 & 0 & 0 & 0 \\ & 0 & & & & & \\ & & 0 & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and where x_t^{cw} is the contract wage in Taylor's model, which we re-labeled here to avoid notational confusion with x_t , which is the vector of stacked variables of the model. xl_t^{cw} is equal to lagged x_t^{cw}, that is, x_{t-1}^{cw}.

$x_{t/t}^{e}$ is the expectation of x_t conditional on the information available at time "t", and so on. Notice that $\hat{D}_1$ is set equal to zero, since in Taylor's model expectations are conditional on the information available at time "t". Thus,

4.3) $$x_{t/t}^{e} = x_t .$$

z_{t-1} is a vector of exogenous variables, while C_1 is a matrix. For Taylor's model, they are both equal to zero. In the model, m and g appear (this one implicitly, as a shift factor in equation 4A) as contemporaneous to the endogenous variables. By assuming that there is one lag between a policy decision and its implementation, we can redefine them as m_{t-1} and g_{t-1}, since DUALI, as well as the optimal control literature, works with one-lag policy variables. Finally, ε_t is a vector of additive noise.

5) Dynamic analysis

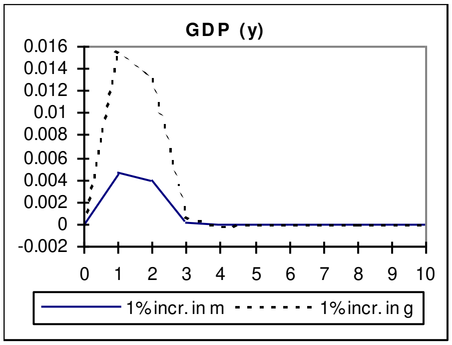
As we did above in section 2.b with Hall and Taylor's model, here we will analyze the dynamic evolution of John Taylor's closed economy model for given changes in its policy variables. The graphs below show the results of two experiments: a 1% unanticipated permanent increase in the money supply (m) and a 1% unanticipated permanent increase in government expenditure (g).[53] That is, m and g increase by 0.01 at the first period of each of the two experiments, and stay at their new value from the second period onwards.

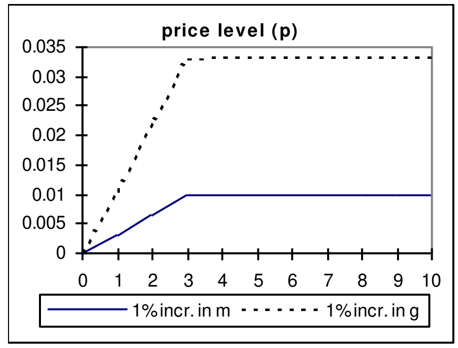
On the horizontal axes are the time periods. For y and p, the vertical axes correspond to percent deviations from steady-state values, while for i and r the vertical axes show percent points.[54] This is, a value of 0.01 in the GDP graph means that GDP is

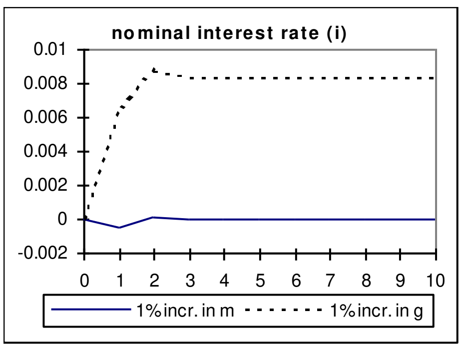
[53] To learn how to implement these simulations, see note 14. To run the simulations, use program taydua01.dui.

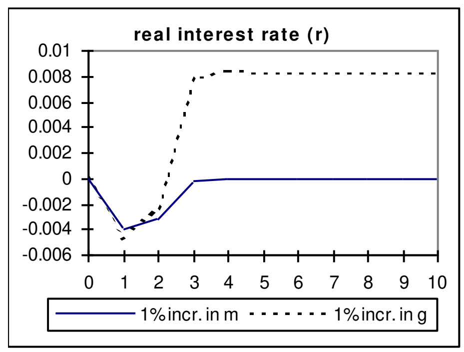
[54] Remember that in Taylor's model, y and p are in logs, which is equivalent to percent deviations from steady-state (see Mercado and Kendrick (1997a)), while i and r are not.

goes from, for example, 600 to 606 trillion dollars, while a value of 0.01 in the nominal interest rate graph means that that rate goes from, for example, a 5% to a 6% level.[55]









Here is how John Taylor explains the observed behavior of the model for the two experiments:

"Monetary policy has an expected positive effect on output that dies out as prices rise and real-money balances fall back to where they were at the start. Note that the real interest rate drops more than the nominal rate because of the increase in expected inflation that occurs at the time of the monetary stimulus. For this set of parameters the nominal interest hardly drops at all; all the effect of monetary policy shows up in the real interest rate.

Fiscal policy creates a similar dynamic pattern for real output and for the price level. Note, however, that there is a surprising "crowding-in" effect of fiscal policy in the short run as the increase in the expectation of inflation causes a drop in the real interest rate. Eventually the expected rate of inflation declines and the real interest rate rises; in the long run, private spending in completely crowded out by government spending."[56]

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[55] Taylor (1993), Chapter 1, present graphs conveying the same information as the ones we show here. However, he presents the results in levels.

[56] Taylor (1993), page 25.

6) A primer on rational expectations and optimal policy analysis with DUALI

We will now extend the analysis of section 3 to models with rational expectations. The problem is to find the optimal paths for the policy variables given desired paths for the target variables, and it can be stated in the same form as we did before. That is, the problem is expressed as one of finding the controls $(u)_{t=0}^{N}$ to minimize a quadratic "tracking" criterion function J of the form:

3.1)

$$J = E\left\{\frac{1}{2}\left[x_N - x_N^{\#}\right]' W_N^{\#}\left[x_N - x_N^{\#}\right] + \frac{1}{2}\sum_{t=0}^{N-1}\left(\left[x_t - x_t^{\#}\right]' W_t^{\#}\left[x_t - x_t^{\#}\right] + \left[u_t - u_t^{\#}\right]' \Lambda_t^{\#}\left[u_t - u_t^{\#}\right]\right)\right\}$$

subject, as a constraint, to the state-state or, as we did with Taylor's model, to the "Pindyck form" representation of the economic model:

4.1)

$$x_t = A_0 x_t + A_1 x_{t-1} + B_1 u_{t-1} + C_1 z_{t-1} + \hat{D}_1 x^e_{t/t} + \hat{D}_2 x^e_{t+1/t} + \hat{D}_3 x^e_{t+2/t} + \varepsilon_t$$

where E is the expectation operator, $x^{\#}$ and $u^{\#}$ are desired paths for the state and controls variables respectively, W and Λ are weighting matrices for states and controls respectively, and where all the other variables were defined above.

We will assume that the policy goal is to stabilize y, p, i and r around steady-state values (that is, around zero). We will put high and equal weights on stabilizing y and p, lower and equal on i and r, and even lower on the policy variables m and g. The corresponding weighting matrices, shown below, will remain constant through time.

6.1)

$$W = \begin{bmatrix} 0 & & & & & & \\ & 0 & & & & & \\ & & 100 & & & & \\ & & & 100 & & & \\ & & & & 50 & & \\ & & & & & 50 & \\ & & & & & & 0 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 25 & \\ & 25 \end{bmatrix}$$

To perform a deterministic experiment, we will assume that the economy is going through a recession provoked by a temporary adverse shock to y which brings it 4% below its steady-state value. What would be, in this situation, the optimal paths for m and g ? What would be the optimal path for the state variables as compared against the autonomous response of the system?

To implement this experiment in DUALI[57], we have to select the following DUALI options: *complexity*: deterministic; *model size*: 7 states, 2 controls, 1 exogenous variable, with a maximum lead of 3 for the forward variables and an arbitrary iteration limit.[58] In *options-deterministic* we select *criterion function*: quadratic tracking; *system equations*: Pindyck form; *forward variables*: yes; *time-varying elements-z exog. var.*: time varying, while all the remaining elements are set to constant (no time varying).

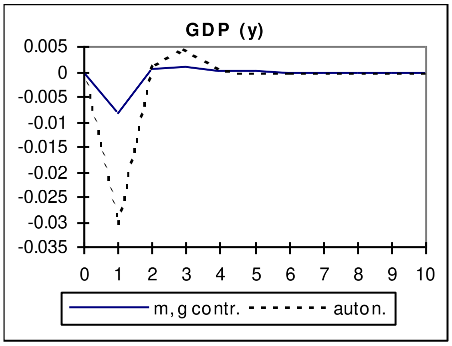
Notice that we introduced an artificial time-varying exogenous variable. We need to do so in order to implement the shock to y in the first simulation period. That is, that shock will be defined as a first-period change in an arbitrary exogenous variable affecting the state variable y only. To do so, in the *system equations* option we set equal to 1 the fourth element of the matrix C_1, and set equal to -0.04 the first element of z.

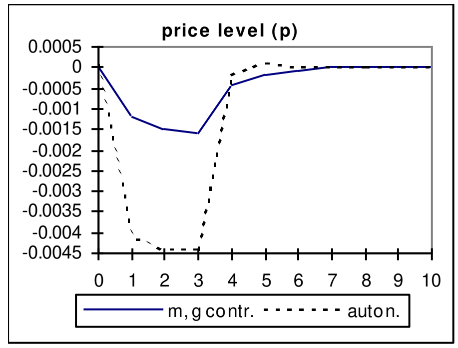
Notice also that this procedure is different from the one we use to implement an analogous shock in Hall and Taylor's model in section 3.a. There, we applied the shock to the initial value of the shocked variable, that is, we defined the shock in the DUALI *option system equations-x0*. We can not do that here, since the variable of interest (y) does not appear with lagged values in Taylor's model.[59]

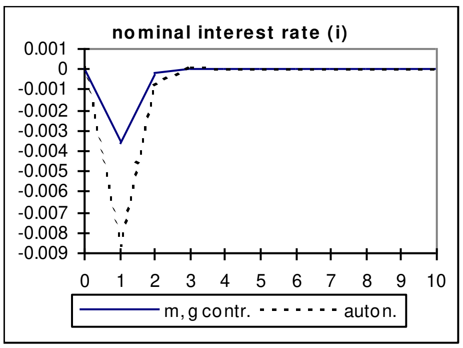
Finally, to complete the specification of the problem, we have to set all the desired paths for states and controls equal to zero and impose the corresponding weights on states and controls.

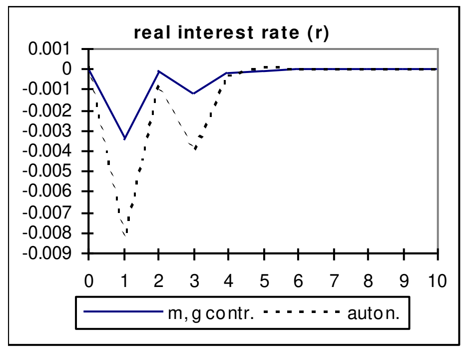
The graphs below show the autonomous response of the system to a -0.04 unanticipated transitory shock to y, and the behavior obtained when applying deterministic optimal control (QLP) to face the same shock, that is, when actively using m and g as controls.[60]

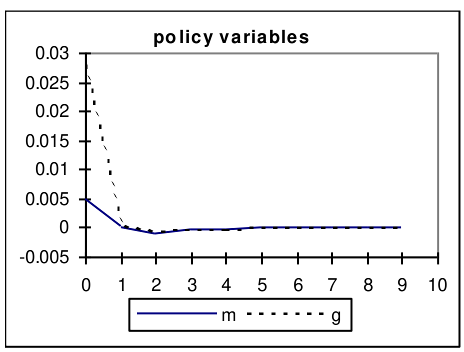
[57] See program taydua02.dui.
[58] To solve optimal control problems involving rational expectations models, DUALI uses an adaptation of the iterative Fair-Taylor's method. To learn about the way this method is implemented to solve deterministic problems, see Amman and Kendrick (1996a). There are other methods that can be applied to the same kind of problems. See, for example, Amman and Kendrick (1997a), where they apply Sims "generalized eigenvalue" method.
[59] We could use the option "system equations-x0" if, instead of shocking the variable "y", we decide to shock the contract wage, since the contract wage is the only variable with lagged values in Taylor's model.
[60] To run these experiments, use program taydua02.dui (making the appropriate changes. See the "description" section in the "data" menu). Both experiments are run with the option "Solve:QLP".











We can observe how the behavior of the state variables under the optimal control solution outperforms notoriously the autonomous response of the system, reducing by any measure the costs of getting the economy out of the recession. In order to generate that behavior, as can be seen in the policy variables graph, the optimal policy mix relies on a 2.5% transitory expansion in government expenditure during the first period, at the same time that is also requires a small transitory increase (0.5%) of the money supply also during the first period.

It may be surprising to find such a positive active role in the presence of rational expectations, being these usually identified with the idea of "policy neutrality". However, we have to remember that Taylor's model contains a built-in "rigidity" (a staggered contracts mechanism) which breaks down the neutrality of policy in the short-run.[61]

More generally, rational expectations will tend to increase the degree of controllability of an economic system, unless the particular structure and/or parameter values of the model implies a complete neutralization of the policy variables effects.[62] Indeed, not only can the policy-maker influence the economy through past and current controls, but she can also affect the economic system through the pre-announcement of future control values. However, for these announcements to have a positive effect on the economic performance, they have to be credible, that is, the policy-maker has to be committed to carry them on.[63]

In this section, we have presented a deterministic (QLP) experiment with Taylor's rational expectations-staggered contracts model. The next natural steps in the analysis of its dynamic performance would be to move on towards stochastic experiments of increasing complexity. These experiments can be also be implemented in DUALI, in an analogous way to what we did in part 1 of this paper with Hall and Taylor's model.[64]

[[**DR. KENDRICK**: in connection with the previous-to-the-last paragraph above, I have a doubt. There is a difference between solving a rational expectations model in a "stacked way" or by using dynamic programming. (The best explanations of this issue are, in my opinion, Holly and Hughes-Hallett (1989), Ch. 8, and Blanchard and Fisher (1989), Ch. 11). The stacking method yields the optimal solution though, if the policy-maker is not committed to carry that solution out without revisions, it will be time-inconsistent (at time t, there will be a temptation to revise the solution path computed at t-1 even with no arrival of new information in the form of shocks, etc.). On the other hand, the dynamic programming method yields a time-consistent solution (since it solves backwards, previous decisions are always optimal even from a future point of view), though it is sub-optimal when compared against the stacking method.
I am not sure how to label the solution obtained with DUALI, since it appears to be a combination of a stacking procedure (the Fair-Taylor's method, I think, falls within this category, since it solves iterative but simultaneously for the whole path) and a dynamic programming procedure to get each period solution.]]

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[61] To learn about the role of nominal and real "rigidities" in macroeconomic models, see Blanchard and Fischer (1989).
[62] See Holly and Hughes-Hallett (1989), Chapter 7.
[63] Lack of credibility may lead to problems of "time inconsistency". See Holly and Hughes-Hallett (1989), Chapter 8; and Blanchard and Fischer (1989), Chapter 11. For a recent critical appraisal of the practical importance of this issue, see Blinder (1997).
[64] To learn about the solution method for stochastic control problems involving forward-looking variables which is implemented in DUALI (an adaptation of the Fair-Taylor method) see Amman and Kendrick (1993). For an alternative solution method using the Blanchard and Khan approach, see Amman and Kendrick (1995). For an application of this method to an early econometric version of Taylor's model, see Achath, Amman and Kendrick (1994).

Appendix

A.1) Matlab Program to Compute Eigenvalues

```
% Computes the eigenvalues for the A matrix for
% the linearized version of Hall and Taylor's (1993)
% macroeconomic model.

echo on;
clear;

A = [ -0.346 0  -0.606  0   0  0  0.087  0   0  0  0.087 0 ; ...
     7.811  0  13.669  0   0  0 -1.953  0   0  0 -1.953 0 ; ...
     0.8    0   1.4    0   0  0 -0.2    0   0  0 -0.2   0 ; ...
     1.154  0   2.019  0   0  0 -0.288  0   0  0 -0.288 0 ; ...
     1      0   0      0   0  0  0      0   0  0  0     0 ; ...
     0      1   0      0   0  0  0      0   0  0  0     0 ; ...
     0      0   1      0   0  0  0      0   0  0  0     0 ; ...
     0      0   0      1   0  0  0      0   0  0  0     0 ; ...
     0      0   0      0   1  0  0      0   0  0  0     0 ; ...
     0      0   0      0   0  1  0      0   0  0  0     0 ; ...
     0      0   0      0   0  0  1      0   0  0  0     0 ; ...
     0      0   0      0   0  0  0      1   0  0  0     0 ]

lambda = eig(A)
```

A.2) TSP Program to Compute Dynamic Controllabity Conditions

```
? Computes dynamic controllability for both the full
? state vector and a subset of target variables for
? the linearized version of Hall and Taylor's (1993)
?  macroeconomic model.

load (nrow=12, ncol=12,  type=general) A;
-0.346    0  -0.606   0    0    0  0.087   0    0    0  0.087   0
 7.811    0  13.669   0    0    0  -1.953  0    0    0  -1.953  0
   0.8    0   1.4     0    0    0  -0.2    0    0    0  -0.2    0
 1.154    0   2.019   0    0    0  -0.288  0    0    0  -0.288  0
    1     0    0      0    0    0    0     0    0    0    0     0
    0     1    0      0    0    0    0     0    0    0    0     0
    0     0    1      0    0    0    0     0    0    0    0     0
    0     0    0      1    0    0    0     0    0    0    0     0
    0     0    0      0    1    0    0     0    0    0    0     0
    0     0    0      0    0    1    0     0    0    0    0     0
    0     0    0      0    0    0    1     0    0    0    0     0
    0     0    0      0    0    0    0     1    0    0    0     0
;



load (nrow=12, ncol=2,type=general) B;
```

```
 0.433 0.231
-9.763 4.386
    0     0
-2.442 1.097
    0     0
    0     0
    0     0
    0     0
    0     0
    0     0
    0     0
    0     0
;

load (nrow=4, ncol=12, type=general) S;
    1    0    0    0    0    0    0    0    0    0    0    0
    0    1    0    0    0    0    0    0    0    0    0    0
    0    0    1    0    0    0    0    0    0    0    0    0
    0    0    0    1    0    0    0    0    0    0    0    0

print A, B, S;

mat AB = A*B;
mat A2B = (A**2)*B;
mat A3B = (A**3)*B;
mat A4B = (A**4)*B;
mat A5B = (A**5)*B;
mat A6B = (A**6)*B;
mat A7B = (A**7)*B;
mat A8B = (A**8)*B;
mat A9B = (A**9)*B;
mat A10B = (A**10)*B;
mat A11B = (A**11)*B;

mmake  P  B, AB, A2B, A3B, A4B,A5B,A6B,A7B,A8B,A9B,A10B, A11B;

mat G = rank(P); print G;

mat R1 = S*B;
mat R2 = S*A*B;
mat R3 = S*(A**2)*B;
mat R4 = S*(A**3)*B;

mmake  R   R1, R2, R3, R4;

mat M = rank(R); print M;

end;
```

Bibliographical References

Achath, S. , H. Amman and D. Kendrick (1994), *Soution of the Taylor Macro-Model with the Blanchard and Khan Procedure*, Department of Economics, The University of Texas at Austin, Working Paper 9408.

Amman, H. and D. Kendrick (1993), "Forward Looking Variables in Stochastic Control", *The International Journal of Supercomputer Applications,* 7, pp. 201-211.

Amman, H. and D. Kendrick (1995), "Solving Stochastic Optimization Models with Learning and Rational Expectations", *Economic Letters*, 48, pp. 9-13.

Amman, H. and D. Kendrick (1996a), "Forward Looking Variables in Deterministic Control", *Annals of Operations Research*, 68, pp. 141-159.

Amman, H. and D. Kendrick (1996b), *The DUALI/DUALPC Software for Optimal Control Models: User's Guide*, Center for Applied Research in Economics, The University of Texas, TP 92-03, (revised September 1996).

Amman, H. and D. Kendrick (1997a), *Linear Quadratic Optimization for Models with Rational Expectations*, Department of Economics, The University of Texas at Austin, Working Paper 9708.

Amman, H. and D. Kendrick (1997b), *Should Macroeconomic Policy Makers Consider Parameter Covariances?*, mimeo.

Amman, H. and D. Kendrick (1997c), "The DUALI/DUALPC Software for Optimal Control Models: Introduction", in Amman, H. et al, *Computational Approaches to Economic Problems*, Kluwer Academic Publishers, Dordrecht, The Netherlands, pp. 363-372.

Aoki, M. (1976), *Optimal Control and System Theory in Dynamic Economic Analysis*, North Holland.

Azariadis, C. (1993), *Intertemporal Macroeconomics*, Blackwell Publishers Inc., Cambridge, MA.

Blanchard, O and S. Fischer (1989), *Lectures on Macroeconomics*, The MIT Press.

Blinder, A. (1997), "Distinguished Lecture on Economics in Government: What Central Bankers Could Learn from Academics - and Vice Versa", *Journal of Economic Perspectives*, V. 11, N. 2, Spring.

Brainard, W. (1969), "Uncertainty and the Effectiveness of Policy", *American Economic Review*, vol. 57, May, pp. 411-425.

Chiang, A. (1984), *Fundamental Methods of Mathematical Economics*, McGraw-Hill.

Chow, G. (1975), *Analysis and Control of Dynamic Economic Systems*, John Wiley & Sons.

Dixon, P., B. Parmenter, A. Powel and P. Wilcoxen (1992), *Notes and Problems in Applied General Equilibrium Analysis*, North Holland, Amsterdam..

Hall, R. and J. Taylor (1993), *Macroeconomics*, fourth edition, W.W. Norton & Company, New York.

Holly, S. and A. Hughes-Hallett, (1989), *Optimal Control, Expectations and Uncertainty*, Cambridge University Press.

Kendrick, D. (1990), *Models for Analyzing Comparative Advantage*, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Kendrick, D. (1981), *Stochastic Control for Economic Models*, McGraw-Hill Book Company, New York.

Mercado, P. R. and D. Kendrick (1997a), *HTGAMS Hall and Taylor's Model in GAMS*, Department of Economics, The University of Texas at Austin, Working Paper 9704.

Mercado, P. R. and D. Kendrick (1997b), *TAYGAMS John Taylor's Two-Country Model in GAMS*, Department of Economics, The University of Texas at Austin, Working Paper 9703.

Mercado, P. R., D. Kendrick and H. Amman (1997), "Teaching Macroeconomics with GAMS", *Computational Economics*, (forthcoming).

Park, H. J. (1997), *A Control Theory Analysis of Macroeconomic Policy Coordination by the US, Japan and Korea*, Ph.D. Dissertation, The University of Texas at Austin.

Shupp, F. (1976), "Uncertainty and Optimal Stabilization Policy", *Journal of Public Economics*, 6, pp. 243-253.

Tinbergen, J. (1956), *Economic Policy: principles and design*, North-Holland Pub.Co., Amsterdam.

Turnovsky, S. (1977), *Macroeconomic Analysis and Stabilization Policy*, Cambridge University Press.