The Marshall Lerner condition and money demand: a note

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Abstract

What are the respective effects of a unit increase in money demand on the real exchange rate and on the current account, all else equal? The real exchange rate is known to appreciate, but the current account need not deteriorate, as the canonical Marshall Lerner condition instead seems to suggest. As this work presents, the current account deteriorates by virtue of a real exchange appreciation due to a fall in the real money supply, all else equal, and *vice versa;* it further specifies that the current account improves by virtue of a real exchange rate appreciation due to a rise in money demand, all else equal, and *vice versa.*

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1. Introduction

The central contribution of this work is the extension of the Marshall Lerner condition in order to distinguish between (A) a current account *improvement* due to a real exchange rate *depreciation* and (B) a current account *improvement* due to a real exchange rate *appreciation.*

Such seems contradictory because the canonical Marshall Lerner condition only conveys the former (A). Now, the former (A) is due to a rise in the *real money supply,* while the latter (B) is due to a rise in *money demand.* This work proves precisely such.

The linchpin of the proof is that the real exchange rate is an increasing function (sc. depreciation) of the real money supply and a decreasing one (sc. appreciation) of money demand: $\forall m_S,\ m_D \in \mathbb{R}_{++},\ e = e(\overset{+}{m_S},\ \overset{-}{m_D})$. Such is conventional wisdom, at least indirectly, as discerned in Krugman *et al.* (2018): graphing money against the interest rate, in real terms, yields the usual money supply and demand curves (sc. vertical and downward), whose dynamics are projected on to the real exchange rate market as outlined above.

Specifically, a rise in money demand, all else equal, causes a rise in the real interest rate within the real money market, which, net of undershooting, translates into a real exchange rate appreciation; real output is known to rise, but what exactly happens to the current account? The answer is that it improves if and only if the Marshall Lerner condition for money demand derived below is satisfied.

Once such is acknowledged the derivations are sheerly mechanical work. For simplicity, the economy can be envisaged as purely external, whereby money demand is merely a function of export and import demand, being respectively increasing and decreasing therein: $\forall ed,\ id \in \mathbb{R}_{++},\ m_D = m_D(\overset{+}{ed},\ \overset{-}{id})$. In fact, the changes in money demand are to be attributed to export or import demand for scopes of intuition, but a dummy variable (autonomous money demand, say) would equally work.

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A standard reference for the derivation of the canonical Marshall Lerner condition is Krugman *et al.* (2018). Salient references of the canonical Marshall Lerner condition and of its generalisation within partial and general equilibrium environments are Devereux (2000), Lombardo (2001), Carnevali *et al.* (2020) and Özçam (2020).

Devereux (2000) analysed the effects of an exchange rate devaluation on the current account within a dynamic environment featuring price rigidity. In detail, he found that the canonical Marshall Lerner condition is satisfied for prices set in producer currency; for prices set in consumer currency the current account response depended on the size of the inter-temporal elasticity of consumption substitution; partial pricing to market instead determined current account responses based on the ratio between non-temporal and inter-temporal elasticity.

Lombardo (2001) studied monopolistic distortion in relation to the elasticity of substitution between domestic output and imports. He specifically disentangled the former from the latter in order to derive a negative response in the trade balance in the face of monetary shocks, absent the introduction of capital accumulation, thereby confirming the stylised fact of anti-cyclical trade balances.

Carnevali *et al.* (2020) derived a general equilibrium condition with regard to the terms of trade of a bi-regional economy. In particular, they discerned the canonical Marshall Lerner condition as being a mere declension thereof, of little approximative use. In their stock and flow setting the canonical Marshall Lerner condition hinges on the assumption of a full exchange rate pass through to import prices, weighing upon the economy's convergence towards efficiency; in other words, readjustment is hampered by exporters who preserve market power by maintaining their prices denominated in foreign currency unvaried.

Özçam (2020) elaborated the canonical Marshall Lerner condition through a partial equilibrium analysis of a region's imports market and exports market, considering both demand and supply. The volume and value effects of a nominal depreciation upon the trade balance were particularly studied, alongside their interaction. In the presence of an initial deficit and surplus, respectively, the canonical Marshall Lerner condition was especially found to be both insufficient and unnecessary for positive changes within the trade balance.

The broader contribution of this work could be thus understood as a generalisation of the canonical Marshall Lerner condition within a static macroeconomic equilibrium framework, merging the elasticity approach to the balance of payments with the monetary approach to it (e.g. Johnson (1972)).

2. Marshall Lerner conditions

The Marshall Lerner condition is to be first derived with regard to case (A) and then case (B). If the real money supply and money demand were treated autonomously there would then emerge two Marshall Lerner conditions in total, one for each, but since the real money supply and money demand are to be expressed in terms of their outlined components there will emerge four Marshall Lerner conditions, for (i) the nominal money supply, (ii) prices, (iii) export demand and (iv) import demand, whereof the last two are especial contributions of this work.

The real money supply is to be expressed in terms of nominal money supply and prices: $\forall M_S,\ p \in \mathbb{R}_{++},\ m_S = \frac{M_S}{p}$. Real exports are an increasing function of the real exchange rate and of export demand and real imports are a decreasing function of the real exchange rate and an increasing function of import demand: $ex = ex(\overset{+}{e},\ \overset{+}{ed})$ and $im = im(\overset{-}{e},\ \overset{+}{id})$. Notice that real exports and real imports are thus specified *qua* partial inverses of the real exchange rate equation in the real exports market and the real imports market, respectively: $e = e(\overset{+}{ex},\ \overset{-}{ed})$ and $e = e(\overset{-}{im},\ \overset{+}{id})$.

Notice further that in the real exports market export demand figures as the upward (or vertical) supply curve and real exports figure as the downward demand curve, counterintuitively, but precisely as the DD curve, which speaks to money demand, figures as the upward supply curve and the AA curve, which speaks to the real money supply, figures as the downward demand curve in the real output market: $ed \equiv q_S,\ ex \equiv q_D$ and $e \equiv p$ such that $p = p(\overset{-}{q_S},\ \overset{+}{q_D})$. In fact, the expression of the real exchange rate in terms of the real money supply and money demand above behaves analogously; money demand figures as the vertical supply curve and the real money supply figures as the downward demand curve: $m_D \equiv q_S,\ m_S \equiv q_D$ and $e \equiv p$, such that $p = p(\overset{-}{q_S},\ \overset{+}{q_D})$.

For completeness, in the real imports market import demand figures as the downward demand curve and real imports figure as the upward (or vertical) supply curve: $id \equiv q_D$, $im \equiv q_S$ and $e \equiv p$ such that $p = p(\overset{-}{q_S},\ \overset{+}{q_D})$. This is because real imports are none other than foreign real exports; specifically, import demand is foreign export demand: $im = im(\overset{-}{e},\ \overset{+}{id}) = ex^* = ex^*(\overset{+}{e^*},\ \overset{+}{ed^*})$, whereby $id = ed^*$.

Yea, because the economy is effectively envisaged as being purely external the above exposition of the real exports market is the very AA DD model, wherein export demand would be driven by money demand dynamics and real exports by those of the real money supply, reverse dynamics accounting for the real imports market: $DD \equiv ed$ and $AA \equiv ex$, *ceteris paribus.* In detail, real exports $ex = ex_2(\overset{+}{e},\ \overset{+}{ed})$ can be simplified to equation $ex = ex_2(\overset{+}{e},\ \overset{+}{am_D})$, since money demand $m_D = m_D(\overset{+}{ed},\ \overset{-}{id})$ itself can be simplified to equation $m_D = m_D(\overset{+}{am_D})$, for any $am_D \in \mathbb{R}_{++}$, that is, by anonymising export and import demand into autonomous money demand, as mentioned above. Now, real exports are also an increasing function of the real money supply: $ex = ex_1(\overset{+}{m_S})$. Consequently, equation $ex_1(\overset{+}{m_S}) = ex_2(\overset{+}{e},\ \overset{+}{am_D})$ gives rise to real exchange rate $e = e(\overset{+}{m_S},\ \overset{-}{am_D}) = e(\overset{+}{M_S},\ \overset{-}{p},\ \overset{-}{am_D})$ by means of its partial inverse. Consult Krugman *et al.* (2018)'s illustration of the AA DD model for an overall picture.

Claim 2.1 (Marshall Lerner condition for the nominal money supply) *The first derivative of the current account with respect to the nominal money supply is positive if and only if the sum of (i) the elasticity of real exports to the real exchange rate and (ii) the modulus elasticity of real imports to the real exchange rate is greater than one. Formally:*

$$ca_{M_S} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| > 1. \tag{1}$$

Proof. Consider the current account equation expressed in real terms: $ca = ex - e \cdot im$. Differentiate it with respect to the nominal money supply: $ca_{M_S} = ex_e e_{m_S} m_{S_{M_S}} - \left(e_{m_S} m_{S_{M_S}} im + e \cdot im_e e_{m_S} m_{S_{M_S}}\right)$. Divide it by $e_{m_S} m_{S_{M_S}}$: $\frac{ca_{M_S}}{e_{m_S} m_{S_{M_S}}} = ex_e - (im + e \cdot im_e)$. Multiply it by $\frac{e}{ex}$: $\frac{ca_{M_S} e}{e_{m_S} m_{S_{M_S}} ex} = \frac{e}{ex}(ex_e - im - e \cdot im_e)$. Now, at efficiency the current account is null: $ca = 0 \longleftrightarrow ex = e \cdot im$. Moreover, recall the definition of elasticities: $\eta_q := \frac{q_p p}{q}$. Therefrom, substitute $\frac{e}{ex}$ with $\frac{1}{im}$ and apply the definition of elasticities: $\frac{ca_{M_S}}{e_{m_S} m_{S_{M_S}} im} = \eta_{ex_e} - \frac{1}{im}(im + e \cdot im_e) \longrightarrow \frac{ca_{M_S}}{e_{m_S} m_{S_{M_S}} im} = \eta_{ex_e} - 1 - \eta_{im_e}$. It follows that ca_{M_S} is positive if and only if $\eta_{ex_e} - 1 - \eta_{im_e}$ is positive: $ca_{M_S} > 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} > 0$. Finally, recall that price elasticities of demand are negative and that real exports are supplied and real imports demanded; the elasticity of real imports to the real exchange rate is therefore negative: $\eta_{im_e} < 0$ and $|\eta_{im_e}| = -\eta_{im_e}$. Consequently, ca_{M_S} is positive if and only if $\eta_{ex_e} + |\eta_{im_e}|$ is greater than one: $ca_{M_S} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| > 1$. *QED*

Such is a Marshall Lerner condition for an increase in the current account given an increase in the real exchange rate (sc. depreciation) due to an increase in the nominal money supply.

Claim 2.2 (Marshall Lerner condition for prices) *The first derivative of the current account with respect to prices is negative if and only if the sum of (i) the elasticity of real exports to the real exchange rate and (ii) the modulus elasticity of real imports to the real exchange rate is smaller than one. Formally:*

$$ca_p < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| < 1. \tag{2}$$

Proof. Consider the current account equation expressed in real terms: $ca = ex - e \cdot im$. Differentiate it with respect to prices: $ca_p = ex_e e_{m_S} m_{S_p} - \left(e_{m_S} m_{S_p} im + e \cdot im_e e_{m_S} m_{S_p}\right)$. Divide it by $e_{m_S} m_{S_p}$: $\frac{ca_p}{e_{m_S} m_{S_p}} = ex_e - (im + e \cdot im_e)$. Multiply it by $\frac{e}{ex}$: $\frac{ca_p e}{e_{m_S} m_{S_p} ex} = \frac{e}{ex}(ex_e - im - e \cdot im_e)$. Substitute $\frac{e}{ex}$ with $\frac{1}{im}$ and apply the definition of elasticities: $\frac{ca_p}{e_{m_S} m_{S_p} im} = \eta_{ex_e} - \frac{1}{im}(im + e \cdot im_e) \longrightarrow \frac{ca_p}{e_{m_S} m_{S_p} im} = \eta_{ex_e} - 1 - \eta_{im_e}$. It follows that ca_p is negative if and only if $\eta_{ex_e} - 1 - \eta_{im_e}$ is negative, namely, if and only if $\eta_{ex_e} + |\eta_{im_e}|$ is smaller than one: $ca_p < 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| < 1$. *QED*

Such is a Marshall Lerner condition for a decrease in the current account given a decrease in the real exchange rate (sc. appreciation) due to an increase in prices.

Claim 2.3 (Marshall Lerner condition for export demand) *The first derivative of the current account with respect to export demand is positive if and only if the sum of (i) the elasticity of real exports to the real exchange rate, (ii) the modulus elasticity of real imports to the real exchange rate and (iii) the quotient of the elasticity of real exports to export demand, the elasticity of the real exchange rate to money demand and the elasticity of money demand to export demand is greater than one. Formally:*

$$ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} > 1. \tag{3}$$

Proof. Consider the current account equation expressed in real terms: $ca = ex - e \cdot im$. Differentiate it with respect to export demand: $ca_{ed} = ex_e e_{m_D} m_{D_{ed}} + ex_{ed} - (e_{m_D} m_{D_{ed}} im + e \cdot im_e e_{m_D} m_{D_{ed}})$. Divide it by $e_{m_D} m_{D_{ed}}$: $\frac{ca_{ed}}{e_{m_D} m_{D_{ed}}} = ex_e + \frac{ex_{ed}}{e_{m_D} m_{D_{ed}}} - (im + e \cdot im_e)$. Multiply it by $\frac{e}{ex}$, substitute it with $\frac{1}{im}$ where needed and apply the definition of elasticities: $\frac{ca_{ed} e}{e_{m_D} m_{D_{ed}} ex} = \frac{ex_e e}{ex} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - \frac{e}{ex}(im + e \cdot im_e) \longrightarrow \frac{ca_{ed}}{e_{m_D} m_{D_{ed}} im} = \eta_{ex_e} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - (1 + \eta_{im_e})$. Multiply it by $\frac{m_D ed}{m_D ed}$ and apply the definition of elasticities: $\left(\frac{m_D ed}{m_D ed}\right) \frac{ca_{ed}}{e_{m_D} m_{D_{ed}} im} = \frac{m_D ed}{m_D ed}\left(\eta_{ex_e} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - 1 - \eta_{im_e}\right) \longrightarrow \frac{ca_{ed}}{e_{m_D} m_{D_{ed}} im} = \eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} - 1 - \eta_{im_e}$. It follows that ca_{ed} is positive if and only if $\eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} - 1 - \eta_{im_e}$ is positive, namely, if and only if $\eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}}$ is greater than one: $ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} - 1 - \eta_{im_e} > 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} > 1$. *QED*

Such is a Marshall Lerner condition for an increase in the current account given a decrease in the real exchange rate (sc. appreciation) due to an increase in export demand.

Claim 2.4 (Marshall Lerner condition for import demand) *The first derivative of the current account with respect to import demand is negative if and only if the sum of (i) the elasticity of real exports to the real exchange rate, (ii) the modulus elasticity of real imports to the real exchange rate and (iii) the quotient of the elasticity of real imports to import demand, the modulus elasticity of the real exchange rate to money demand and the elasticity of money demand to import demand is smaller than one. Formally:*

$$ca_{id} < 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}} < 1. \tag{4}$$

Proof. Consider the current account equation expressed in real terms: $ca = ex - e \cdot im$. Differentiate it with respect to import demand: $ca_{id} = ex_e e_{m_D} m_{D_{id}} - [e_{m_D} m_{D_{id}} im + e(im_e e_{m_D} m_{D_{id}} + im_{id})]$. Divide it by $e_{m_D} m_{D_{id}}$: $\frac{ca_{id}}{e_{m_D} m_{D_{id}}} = ex_e - \left[im + e\left(im_e + \frac{im_{id}}{e_{m_D} m_{D_{id}}}\right)\right]$. Multiply it by $\frac{e}{ex}$, substitute it with $\frac{1}{im}$ where needed and apply the definition of elasticities: $\frac{ca_{id} e}{e_{m_D} m_{D_{id}} ex} = \frac{ex_e e}{ex} - \frac{e}{ex}\left[im + e\left(im_e + \frac{im_{id}}{e_{m_D} m_{D_{id}}}\right)\right] \longrightarrow \frac{ca_{id}}{e_{m_D} m_{D_{id}} im} = \eta_{ex_e} - \left(1 + \eta_{im_e} + \frac{e \cdot im_{id}}{e_{m_D} m_{D_{id}} im}\right)$. Multiply it by $\frac{m_D id}{m_D id}$ and apply the definition of elasticities: $\left(\frac{m_D id}{m_D id}\right) \frac{ca_{id}}{e_{m_D} m_{D_{id}} im} = \frac{m_D id}{m_D id}\left[\eta_{ex_e} - \left(1 + \eta_{im_e} + \frac{e \cdot im_{id}}{e_{m_D} m_{D_{id}} im}\right)\right] \longrightarrow \frac{ca_{id}}{e_{m_D} m_{D_{id}} im} = \eta_{ex_e} - 1 - \eta_{im_e} - \frac{\eta_{im_{id}}}{\eta_{e_{m_D}}\eta_{m_{D_{id}}}}$. Notice that the elasticity of the real exchange rate to money demand is negative: $\eta_{e_{m_D}} < 0$ and $|\eta_{e_{m_D}}| = -\eta_{e_{m_D}}$. It follows that ca_{id} is negative if and only if $\eta_{ex_e} - 1 - \eta_{im_e} - \frac{\eta_{im_{id}}}{\eta_{e_{m_D}}\eta_{m_{D_{id}}}}$ is negative, namely, if and only if $\eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}}$ is smaller than one: $ca_{id} < 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} - \frac{\eta_{im_{id}}}{\eta_{e_{m_D}}\eta_{m_{D_{id}}}} < 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}} < 1$. *QED*

Such is a Marshall Lerner condition for a decrease in the current account given an increase in the real exchange rate (sc. depreciation) due to an increase in import demand.

Since money demand is complex to measure the elasticity of the real exchange rate to money demand, $\eta_{e_{m_D}}$, that is to say, is not easily calculable, thus, for empirical testing one can specifically express the real exchange rate in terms of the real money supply and of export and import demand: $e = f(\overset{+}{m_S}, \overset{-}{ed}, \overset{+}{id})$, *ceteris paribus.* Export demand can be proxied via tariffs, quotas or even confidence and import demand is again foreign export demand: $id = ed^*$. It follows that the twofold Marshall Lerner condition for money demand is accordingly simplified: (i) $ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{ed}}} > 1$; (ii) $ca_{id} < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| - \frac{\eta_{im_{id}}}{\eta_{e_{id}}} < 1$.

Nevertheless, misspecification problems suggest the regression of the real exchange rate on all independent variables of money demand. Identical issues in fact suggest the same for the calculation of the first derivative of money demand with respect to (i) export demand and to (ii) import demand within the orbit of the attendant elasticity measures, that is, of $m_{D_{ed}}$ in $\eta_{m_{D_{ed}}}$ and of $m_{D_{id}}$ in $\eta_{m_{D_{id}}}$.

3. Conclusion

What are the respective effects of a unit increase in money demand on the real exchange rate and on the current account, all else equal? The real exchange rate is known to appreciate, but the current account need not deteriorate, as the canonical Marshall Lerner condition instead seems to suggest. Indeed, an outward application thereof dictates a current account deterioration by virtue of a real exchange rate appreciation. A noumenal application thereof, which this work has presented, by contrast clarifies that the current account deteriorates by virtue of a real exchange appreciation due to a fall in the real money supply, all else equal, and *vice versa;* it further specifies that the current account improves by virtue of a real exchange rate appreciation due to a rise in money demand, all else equal, and *vice versa.*

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