Commuting to Work in Cities: Bus, Car, or Train?

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by

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Abstract

In this paper, we study the commuting behavior of citizens living in or near a city who must decide how to get to work. Such citizens can always use their own car and drive to work. However, they can also take public transport to work. The two public transport options we consider involve taking either a bus or a train to work. In this setting, we perform two broad tasks. First, we analyze the car versus train choice. We compute the deadweight loss from the negative externality generated by car travel, i.e., the traffic congestion, and then discuss how a toll can achieve the efficient allocation of commuters between the car and the train modes of transport. Second, we analyze the car versus bus choice. Once again, we calculate the deadweight loss from the traffic congestion resulting from car travel and then discuss how a toll can achieve the efficient allocation of commuters between the car and the bus modes of transport that would be beneficial for all commuters.

Keywords: Bus, Car, Toll, Traffic Congestion, Train, Travel Time

JEL Codes: R41, D62
1. Introduction

1.1. Setting the scene

The prominent Brundtland Report (1987) not only introduced the now widely known notion of “sustainable development” but it also gave rise to new thinking on a number of related concepts such as the notion of “sustainable mobility.” Sustainable mobility has now become a multi-pronged concept in the sense that it encompasses freight transport, logistics and distribution, private transport, mass transit, and individual modes of mobility such as bicycling and walking. Both institutional and technological incentives have been provided to alter the spatial and behavioral patterns of contemporary humans. In a densely populated country like the Netherlands, this has led to a large number of policy initiatives including the upgrading of public transport quality, the creation of dedicated bicycle lanes, and the implementation of priority rules for cyclists.4

In addition to the Netherlands, regulatory authorities in many other countries of the world today have begun to concentrate on the ways in which they might encourage the use of sustainable modes of transport. As pointed out by Pucher and Buehler (2009) and Buehler (2010), this concentration has frequently led these authorities to discourage the use of private automobiles and encourage the use of public transport and other forms of “green” or environmentally friendly transport such as bicycling.

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It is now widely understood that in most cities in the world, commuting to work using public transport is “greener” than using a private car. That said, there are many modes of public transport such as buses and trains. Therefore, to compare properly the relative social benefit of commuting to work using public transport as opposed to using a private car, it is necessary to make bilateral comparisons between, for instance, a private car and a public train or a private car and a public bus. We now review the literature about alternate ways of commuting to work using both private and public options.

1.2. Literature review

Van Exel and Rietveld (2009) use survey data and compare the behavior of individuals traveling to Amsterdam by car and by train. They demonstrate that those traveling by car erroneously believe that travel by train will take much longer than what the “objective values” show. Therefore, if the perceptions of such travelers about travel time using trains are more accurate then two out of three car travelers will include train travel in their choice set and travel by train from time to time. Rizzi and De La Maza (2017) estimate the marginal external costs per kilometer for cars and buses in Santiago, Chile, in terms of congestion, road damage, accidents, air pollution, and noise. Their analysis shows that the marginal external costs per passenger-kilometer are highest for petrol-based cars, intermediate for diesel-based cars, and lowest for buses. Korsu and Le Nechet (2017) focus on Paris and conduct empirical simulations to see how the travel behavior of individuals changes when workers and jobs are rematched to minimize the average commute distance.

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Bergantino and Madio (2020) study the optimal transport mode choice question in the context of travel between Bari and Rome and Brindisi and Rome. Their empirical analysis shows that the likelihood of switching the mode of transport from the available alternatives to high-speed rail (HSR) depends greatly on a commuter’s age, income, education, and trip purpose. Batabyal and Nijkamp (2020) show how commuting costs affect the equilibrium distribution of workers in an aggregate economy consisting of an urban and an adjacent rural region.

The studies discussed thus far in this section have certainly advanced our understanding of the factors that influence the decision to adopt more environmentally friendly modes of transport by citizens. This notwithstanding, it is worth noting that many of the existing studies that look at the pros and cons of alternate ways of commuting to work are empirical in nature. That said, we would like to point out that there are some theoretical studies in the literature that analyze the transport mode choice question. These studies are complementary to the analysis we undertake in our paper. By “complementary,” we mean that although there is some overlap between the contents of these studies and the analysis we undertake in our paper, there are key differences as well that have not been studied adequately. We now concentrate on five representative theoretical studies and comment on the differences between these studies and the analysis we undertake in the present paper.

Basso and Jara-Diaz (2012) use a theoretical model to compute the optimal pricing and design of transport services in a setting in which consumers are able to choose between two modes of transport. The two modes of transport considered are a private automobile and a generic public transport option. In contrast to this setup, in our paper, we do not study a generic public transport option. Instead, we explicitly focus on the two most common public transport options, namely, a
train and a bus. Second, unlike these authors, we provide precise formulae for computing the deadweight loss from traffic congestion, first, when the transport mode choices being compared are a private automobile and a public train and, second, when the comparison is between the private automobile and a public bus. Holden (1989) provides a largely verbal discussion of theoretical issues concerning urban traffic congestion and traffic policy. However, in contrast with the kind of analysis we undertake in the present paper, he does not actually analyze any models, he also does not conduct bilateral comparisons between a private car and a public train or a private car and a public bus, and, finally, he does not compute expressions for the deadweight loss from traffic congestion.

Huang (2002) sheds light into aspects of transport pricing and the related mode choice question in a setting in which there are two modes of transport and demand is elastic. A key contribution of this paper is that it focuses on three pricing models that use the so-called “logit-based equilibrium concept” to study the implications of elastic demand in a bi-modal transportation system. However, as in Basso and Jara-Diaz (2012), the public transport option that is modeled is generic and not specific and there is no specialized discussion of either a public train or a bus option. In addition, there is also no computation of explicit formulae for the deadweight loss from traffic congestion. Parry and Small (2009) provide a fine theoretical and empirical analysis of urban transit subsidies and whether such subsidies ought to be reduced. That said, we note that the objectives of this paper are unrelated to our objectives in the present paper. As such, it is unsurprising to find that Parry and Small do not compute expressions for the deadweight loss from traffic congestion.
Finally, Verhoef et al. (1996) provide a theoretical analysis of the relative efficiency of second-best congestion pricing in a setting in which road users can choose between a tolled and an untolled road. As with the Parry and Small (2009) paper, once again, the objectives of this paper are distinct from our objectives in the current paper. Therefore, once again, we predictably find that this paper does not contain bilateral comparisons of, first, the impacts of commuting to work either by a private car or a public train and, second, a private car and a public bus. In addition, this Verhoef et al. (1996) paper contains no calculations of the deadweight loss from traffic congestion.

Now that we have distinguished our paper from the pertinent theoretical contributions in the literature, let us proceed to state the central objective of our paper. We wish to study the following question: Focusing on travel time and a congestion externality, what are the impacts of commuting to work in cities using a bus, a car, or a train? To answer this question, section 2 presents our linear model. Section 3 analyzes the car versus train choice in detail and then discusses the policy implications stemming from this comparative exercise. Section 4 studies the car versus bus choice expansively and then comments on the policy consequences arising from this comparative undertaking. Section 5 concludes and then discusses four ways in which the research delineated in this paper might be extended.

In a paper that is related to ours, Batabyal and Nijkamp (2013) use a probabilistic model to analyze the commuting decision of a green citizen. They study whether such a citizen ought to bicycle or take public transport to work. Instead of considering a generic form of public transport, we focus on two specific public transport options—commuting by bus or train—and hence our analysis is more general than the analysis of these researchers. In addition, in many cities in North America and in Northern Europe, bicycling to work is not a year-round commuting option because of cold weather. Therefore, we believe that a more reasonable way to compare alternate transport options is to compare the pros and cons of commuting to work by car with the pros and cons of specific public transport options such as a bus or a train.

The reader should note well the basic objective of our paper. That said, we acknowledge that in addition to the congestion externality, car travel can involve other costs because, for instance, cars occupy a non-trivial amount of urban space. However, an analysis of these other costs is beyond the scope of this paper.
2. The Theoretical Framework

Consider an arbitrary city in either North America or Northern Europe. There are a large number of citizens living in or near this city and these citizens must decide how to commute to work. The first option is to take the train and we suppose that it takes $T > 0$ minutes to get to work irrespective of the number of commuters that take the train. The second option is to take one’s own car and drive oneself to work. We assume that commuting by car takes

$$C(x) = \alpha + 3\alpha x$$  \hspace{1cm} (1)

minutes, where the intercept term $\alpha > 0$ and $x \in [0,1]$ is the fraction of all commuters that are taking their cars and driving to work. The third and final option is to take the bus to work. The commute time on a bus is increasing in the fraction of all commuters who are driving to work. As such, the time it takes in minutes to get to work using the bus is given by

$$B(x) = 2\alpha + \alpha x,$$  \hspace{1cm} (2)

where $2\alpha > 0$ is now the intercept term and $x$ is as described above.

Three assumptions we have made in our modeling thus far in this section deserve additional commentary. In this regard, observe that the commute time functions $C(x)$ and $B(x)$ in equations (1) and (2) are linear by assumption. There are two reasons for working with linear functional forms. First, Colak et al. (2016) have analyzed the nature of congested travel in five cities. Their empirical analysis shows that the relationship between experienced travel times and distance

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8 Note that $x$ is an element of the closed interval $[0,1]$ and not the open interval $(0,1)$ because we want to allow for the two possibilities that, at least in principle, no commuter drives to work and hence $x = 0$ and that all commuters drive to work and therefore $x = 1$.

9 Direct substitution in equations (1) and (2) shows that when $x = 1/2$, we get $C(x) = B(x) = 5\alpha/2$. In other words, in this knife-edge case, the time taken to get to work by using either the car or the bus is identical.
travelled in Rio de Janeiro, the San Francisco Bay area, and Porto is generally linear. Second, in their empirical analysis of traffic jams in urban networks, Saberi et al. (2020) focus on the relationship between $R_0$, a variable that describes how congestion spreads in an urban network and $\rho$ which is a threshold that measures alternate congestion levels. These researchers show that the relationship between $R_0$ and $\rho$ is also linear.

Our second assumption is that the two commute time functions in equations (1) and (2) are different for cars and for buses. In this regard, recent empirical work by Liao et al. (2020) points out that in general, travel times by car and by public transport (which includes buses) are different. In addition, observe that the starting point and the concluding point of the commute to work in our model are the same irrespective of whether a citizen drives his or her own car to work or takes a bus to work. That said, nothing in our model says that the road taken by a citizen driving his or her own car must be the same as the road that is taken by the bus. This road may or may not be the same for both modes of travel. Therefore, because of the above-mentioned empirical result and to allow for both possibilities---same or different road---in a straightforward manner, we have assumed that the commute time functions for the car and for the bus are dissimilar.

Our third assumption is that the commute time by train is fixed. Now, in general, the travel time by train consists of the on-train travel time plus the time spent waiting for the train which itself depends on the frequency with which trains are dispatched and hence arrive at a particular station. In our paper, to keep the subsequent mathematical analysis straightforward and tractable and because we believe that this additional wait time in general is likely to be small relative to the on-train travel time, we are abstracting away from the time that is not spent actually traveling on the train. That said, our assumption that the on-train travel time is fixed has definite precedents in
the literature. For instance, in their analyses of alternate aspects of train travel, both Zhu et al. (2020) and Huang et al. (2021) have pointed out that the on-train travel time can be regarded as being *fixed*. In addition, this fixity assumption concerning on-train travel times has also been made by Holden (1989). With this description of the linear model\(^\text{10}\) in place, let us proceed to analyze the car versus train commute choice in detail.\(^\text{11}\)

3. Car versus Train

3.1. Free market solution

Let us begin by determining the fraction of all commuters that will take their cars to commute to work on the assumption that all commuters are making their decisions freely and independently with the objective of minimizing their *individual* commute times. Clearly, as long as the commute choices are made independently, the fraction of all car users that we seek is given by equating the travel times by train and by car. This gives us

\[
T = \alpha + 3\alpha x. \tag{3}
\]

Solving the above equation for \(x_m\), the fraction of all car drivers that equates the train and car commute times in a free market, we get

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\(^{10}\)

We contend that the linear model we work with in this paper is simple but *not* simplistic. We say this because linear models have been regularly used to study a variety of transportation phenomena and, therefore, we believe it is fair to say that there is a long tradition of using linear models and their variants in the transportation literature. For a more detailed corroboration of this claim, see Abdalla and Abdel-Aty (2006), Bovy (2006), Schmitt and Jula (2007), Chou (2009), and Pariota *et al.* (2016).

\(^{11}\)

When studying the pros and cons of alternate modes of transport, Stopher (1969) observed relatively early that it makes sense to compare private car commutes with commutes using the *best available* public transport option. This suggestion makes obvious sense and therefore it is now common in the transportation literature—see Basso and Jara-Diaz (2012) and Huang (2002)—to make bilateral comparisons between two specific modes of transport. We follow this modeling strategy in our paper. As such, we make bilateral comparisons between, first, a private car and a public train, and, second, a private car and a public bus. In this regard, a Reviewer has rightly noted that it does not make sense to equate the travel time across all modes of transport because different modes provide different transportation services. To see this last point clearly, note that relative to commuting by car, an individual might prefer to commute by train because this choice permits this individual to read a newspaper and/or check e-mail.
Looking at figure 1, we see that the solution given in equation (4) corresponds to the intersection of the train and car commute time curves shown in this figure.

**Figure 1 about here**

of the train and car commute time curves shown in this figure.

### 3.2. Optimal solution

Next, let us compute the fraction of commuters by car that will minimize the total commute time for our citizens. To answer this question, observe that the total commute time or $T_{ct}$ is given by

$$T_{ct} = (\alpha + 3\alpha x)x + T(1-x).$$

Differentiating equation (5) with respect to $x$ and then setting the resulting derivative equal to zero gives us

$$\alpha + 6\alpha x - T = 0.$$  \hspace{1cm} (6)

Solving equation (6) for $x_o$ or the fraction of car users that minimizes the total commute time, we get

$$x_o = \frac{T - \alpha}{6\alpha}. $$  \hspace{1cm} (7)

### 3.3. Deadweight loss

Our next task is to compare the two solutions for $x_m$ and $x_o$ respectively in equations (4) and (7). It is clear that the free market fraction of all commuters who use their cars to get to work
or $x_m$ is higher than the socially optimal fraction given by $x_0$. This happens because the individual commuters do not take into account the negative externality or the traffic congestion that is generated by car travel.

We can compute the deadweight loss to all commuters in our city from the unaccounted-for congestion externality that we have just identified. Specifically, this loss is given by the difference between the two total commute times $T_m$ and $T_o$ associated with the fractions $x_m$ and $x_o$. Now some thought tells us that $T_m$ and $T_o$ are given by

$$T_m = (\alpha + 3\alpha x_m)x_m + T(1 - x_m)$$

and

$$T_o = (\alpha + 3\alpha x_o)x_o + T(1 - x_o)$$

respectively. Therefore, substituting the values of $x_m$ and $x_o$ from equations (4) and (7) into equations (8) and (9) and then simplifying, we get

$$T_m = T \text{ and } T_o = \frac{14\alpha T - \alpha^2 - T^2}{12\alpha}.$$ 

The deadweight loss itself is given by

$$T_m - T_o = T - \left(\frac{14\alpha T - \alpha^2 - T^2}{12\alpha}\right).$$

Simplifying the right-hand-side (RHS) of equation (11), we get
Inspecting equation (12), we see that the deadweight loss from the traffic congestion externality is a function of the intercept term \( \alpha \) and the amount of time it takes to commute to work using the train \( T \). In addition, this deadweight loss is always positive.\(^{12}\)

We now illustrate the results we have obtained thus far by working with a numerical example. Then, we discuss the policy consequences of our findings.

3.4. Numerical analysis and policy

Suppose that \( \alpha = 20 \) and that \( T = 70 \). Then, straightforward analysis with equations (1) through (12) demonstrates that the free market and the socially optimal solutions for the fraction of commuters driving to work are \( x_m = 0.83 \) and \( x_o = 0.42 \). Using these two fractional values, it is straightforward to compute the two total commute times \( T_m = 70 \) and \( T_o = 59.58 \). Putting these last two results together, the deadweight loss from the traffic congestion externality is \( 70 - 59.58 = 10.42 \) minutes.

Moving on to policy, let us assume that all the commuters in the city under study place a monetary value on their commute time. In this case, an appropriately designed congestion toll or charge is likely to induce many of them to switch from commuting by car to commuting by train. Further, given information on the monetary value of the commute time, the magnitude of this toll can be computed so that the fraction of all commuters that still finds it beneficial to commute by

\[
T_m - T_o = \frac{\alpha^2 + T^2 - 2\alpha T}{12\alpha} = \frac{(\alpha - T)^2}{12\alpha} > 0. \quad (12)
\]

There are two ways to understand why this deadweight loss is always positive. First, from a theoretical standpoint, \( x_m > x_o \) and therefore there clearly exists an unaccounted for congestion externality. This externality makes the deadweight loss always positive. Second, from a mathematical perspective, equation (12) tells us that the deadweight loss is the ratio of two terms, \( (\alpha - T)^2 \) and \( 12\alpha \), that are themselves always positive and thus the deadweight loss itself is always positive.
car is equal to the socially optimal level. We now proceed to analyze the car versus bus commute choice in detail.

4. Car versus Bus

4.1. Free market solution

Our first task is to ascertain the fraction of all commuters that will take their cars to commute to work on the supposition that all commuters are making their decisions freely and independently with the goal of minimizing their commute times. As long as the commuting choices are made independently, it should be clear to the reader that the fraction of all car users that we wish to identify is given by equating the travel times by bus and by car. Using equations (1) and (2), we get

\[ 2\alpha + \alpha x = \alpha + 3\alpha x. \]  

(13)

Solving the above equation for \( x_m \), the fraction of all car drivers that equates the bus and car commute times in a free market, we get

\[ x_m = \frac{1}{2}. \]  

(14)

Inspecting figure 2, we see that the solution given in equation (14) corresponds to the intersection

Figure 2 about here

of the bus and car commute time functions shown in this figure.

4.2. Optimal solution

Let us now calculate the fraction of commuters by car that will minimize the total commute time for our citizens. To address this issue, observe that the total commute time or \( T_{ct} \) is given by
the equivalent of equation (5) or

\[ T_{ct} = (\alpha + 3\alpha x) x + (2\alpha + \alpha x) (1 - x). \]  \hspace{1cm} (15)

Differentiating equation (15) with respect to \( x \) and then setting the resulting derivative equal to zero gives us

\[ 6\alpha x - 2\alpha x = 0. \]  \hspace{1cm} (16)

Solving equation (16) for \( x_o \) or the fraction of car users that minimizes the total commute time, we get

\[ x_o = 0. \]  \hspace{1cm} (17)

Equation (17) tells us that we now have a so-called corner solution.\(^\text{13}\) In other words, in the bus versus car commute choice question that we are analyzing in this section, the socially optimal fraction of commuters by car is zero. Put differently, no citizen ought to be commuting to work by driving himself or herself to work and all commuters ought to be getting to work by bus.

4.3. Deadweight loss

Our next task is to compare the two solutions in equations (14) and (17). As in section 3, it is once again the case that the free market fraction of all commuters who use their cars to get to work or \( x_m = 1/2 \) is higher than the socially optimal fraction given by \( x_o = 0 \). This happens because the individual commuters do not take into consideration the external diseconomy or the traffic congestion generated by car travel.

We can, as in section 3, compute the deadweight loss to all commuters in our city from the congestion externality. In particular, this loss is given by the difference between the two total

\(^{13}\)See Hirshleifer et al. (2005, pp. 96-97) for a textbook exposition of corner solutions.
commute times $T_m$ and $T_o$ associated with the fractions $x_m$ and $x_o$. The travel times $T_m$ and $T_o$ are given by

$$T_m = (\alpha + 3\alpha x_m)x_m + (2\alpha + \alpha x_m)(1 - x_m)$$  \hspace{1cm} (18)

and

$$T_o = (\alpha + 3\alpha x_o)x_o + (2\alpha + \alpha x_o)(1 - x_o)$$  \hspace{1cm} (19)

respectively. As such, substituting the values of $x_m$ and $x_o$ from equations (14) and (17) into equations (18) and (19) and then simplifying, we get

$$T_m = \frac{5\alpha}{2} \text{ and } T_o = 2\alpha.$$  \hspace{1cm} (20)

Using equation (20), the deadweight loss is given by

$$T_m - T_o = \frac{\alpha}{2} > 0.$$  \hspace{1cm} (21)

Inspecting equation (21), we see that the deadweight loss from the traffic congestion externality is a function of the intercept term ($\alpha$) only and it is, as in section 3, unambiguously positive. We now explicate the results obtained in this section by working with a numerical example. We then address the policy implications of our findings.

4.4. Numerical analysis and policy

Suppose that $\alpha = 20$. In this case, clear-cut computations with equations (13) through (21) demonstrate that the free market and the optimal solutions for the fraction of commuters driving to work are $x_m = 0.50$ and $x_o = 0$. Using these two fractional values, it is straightforward to
compute the two total commute times $T_m = 50$ and $T_o = 40$. Putting these last two results together, the deadweight loss from the traffic congestion externality is $50 - 40 = 10$ minutes. Comparing this last number with the corresponding number (10.42 minutes) from the car versus train choice studied in section 3, we see that the deadweight loss is smallest when all citizens leave their private cars at home and commute to work by train.

As far as policy is concerned, once again let us suppose that all the commuters in the city under consideration place a monetary value on their commute time. When this is the case, a suitably designed congestion toll is best placed to get commuters in our city to switch from commuting by car to commuting by bus. In addition, given information about the monetary value of the commute time, the amount of this toll can be calculated so that the fraction of all commuters that still finds it beneficial to commute by car is equal to the socially optimal level which equals zero. This concludes our discussion of the impacts of commuting to work using a bus, car, or train.

5. Conclusions

In this paper, we theoretically analyzed the commuting behavior of citizens living in or near a city who needed to decide how to commute to work. These citizens could always use their own car to drive to work. However, they could also take public transport to work. The two public transport options we considered involved taking either a bus or a train to work. In this setting, we performed two broad tasks. First, we studied the car versus train choice. We computed the deadweight loss from the negative externality generated by car travel, i.e., the traffic congestion, and then discussed how a toll could achieve the efficient allocation of commuters between the car and the train modes of transport. Second, we studied the car versus bus choice. Once again, we calculated the deadweight loss from the traffic congestion resulting from car travel and then
discussed how a toll could achieve the efficient allocation of commuters between the car and the bus modes of transport that would be beneficial for all the commuters under consideration.

The analysis in this paper with a linear model can be extended in a number of directions. Here are four potential extensions. First, it would be useful to study a scenario in which in addition to the travel time, a commuting citizen also cares about other factors such as the quality of the commute to determine which mode of transport (s)he would like to choose. Second, it would be instructive to generalize the analysis here by analyzing scenarios in which the length of the bus, car, or train commute time is a random variable. Third, following Beladi et al. (2013), it would be informative to model a scenario in which the commute choice decision generates negative environmental impacts that are regulated using, for instance, a tax and the tax proceeds are then used to support alternate public investment objectives. Finally, additional insights into the transport mode choice question can be gained by simultaneously comparing the pros and cons of commuting to work using either one’s own automobile, or a bus or a train. Studies that incorporate these features of the problem into the analysis will increase our understanding of the many factors that are germane in ascertaining a commuting citizen’s choice between alternate modes of transport.
Figure 1: Car vs. Train
Commuting Times
Figure 2: Car vs. Bus Commuting Times
References


