Monetary Policy and Economic Growth in a Schumpeterian Model with Incumbents and Entrants

Lu, You-Xun and Chen, Shi-kuan and Lai, Ching-chong

National Taiwan University, National Taiwan University, Academia Sinica

18 January 2022
Monetary Policy and Economic Growth in a Schumpeterian Model with Incumbents and Entrants*

You-Xun Lu  
Department of International Business, National Taiwan University, Taiwan

Shi-kuan Chen  
SinoPac Financial Holdings Company Limited, Taiwan  
Department of International Business, National Taiwan University, Taiwan

Ching-chong Lai†  
Institute of Economics, Academia Sinica, Taiwan  
Department of Economics, National Cheng Chi University, Taiwan  
Institute of Economics, National Sun Yat-Sen University, Taiwan

January 2022

Abstract
An important aspect of economic growth is the interaction between incumbents and new firms. In this study, we develop a monetary Schumpeterian model with an endogenous market structure (EMS) and two types of quality improvements (the own-product improvements of incumbents and creative destruction of entrants) to analyze the effects of monetary policy. The key finding of our analysis is that an increase in the nominal interest rate importantly affects the composition of innovation that drives economic growth, stimulating the incumbents’ own-product improvements and reducing the entrants’ creative destruction. Therefore, the growth effect of monetary policy is ambiguous, and depends on the relative magnitudes of the incumbents’ and entrants’ contributions to R&D and growth. Finally, we provide a quantitative analysis of the growth and welfare effects of monetary policy and consider an extension of the benchmark model with an elastic labor supply and a CIA constraint on consumption.

JEL classification: E41, O31, O41  
Keywords: innovation, monetary policy, economic growth, endogenous market structure

* The authors are truly indebted to Juin-Jen Chang, Jui-Lin Chen, Kuan-Jen Chen, Ping-Ho Chen, Hsun Chu, Mei-Ying Hu, Wei-chi Huang, Shih-fu Liu, Po-Yang Yu for their helpful discussions and comments. Any errors or shortcomings are, of course, the authors’ responsibility.

† Corresponding author. Email address: cclai@econ.sinica.edu.tw. Institute of Economics, Academia Sinica, Nankang, Taipei 115, Taiwan.
1. Introduction

How monetary policy affects economic growth and social welfare is both a fundamental and important issue in macroeconomics that has been well studied in capital-driven growth models.\(^1\) However, endogenous growth models typically suggest that, in the long run, technological progress is the main engine of growth in the economy. Thus, influencing R&D investment is an important channel through which monetary policy affects growth. Recently, a growing literature has been developed to investigate the relationship between monetary policy and R&D-driven economic growth in variety-expanding models or quality-ladder models; see, for instance, Chu and Lai (2013), Chu and Cozzi (2014), Chu et al. (2017) and Arawatari et al. (2018).

This paper re-examines the growth and welfare effects of monetary policy in an R&D-driven growth model and seeks to shed some new light on this important issue. We develop a monetary Schumpeterian model in which the own-product improvements of incumbents and creative destruction of entrants jointly promote economic growth. The novelty of our analysis is that we not only consider simultaneous quality improvements (i.e., vertical innovation) by incumbents and entrants, but also highlight the crucial role of an endogenous market structure (EMS) on the growth and welfare effects of monetary policy. Specifically, in each industry, there is an incumbent firm (i.e., the monopolistic quality leader), which produces an intermediate good with the highest quality and invests in R&D to improve the quality of its product. Meanwhile, in the economy, there are many new firms actively innovating over existing goods in an attempt to become new intermediate goods producers (i.e., entrants threaten incumbents via Schumpeterian creative destruction). Through firm entry, market structure, measured by the mass of firms, is endogenously determined by the competition between incumbents and entrants. Obviously, the EMS approach in our model is based on imperfect competition and patent races among R&D firms. To generate an endogenous entry of new firms, just as in Melitz (2003) and elsewhere,\(^2\) we introduce a fixed entry cost into the model. Moreover, we assume that both incumbents and entrants face liquidity problems (i.e., cash constraints).\(^3\)

---

\(^1\) See Gillman and Kejak (2005) for a survey of this strand of the literature.

\(^2\) See also Baldwin and Robert-Nicoud (2008), Gustafsson and Segerstrom (2010), Chu and Ji (2016), and Chu et al. (2017) who incorporate a fixed entry cost into the R&D-based growth model, but do not consider this in a framework featuring both the incumbents’ quality improvements and entrants’ creative destruction.

\(^3\) The empirical evidence supports the view that R&D investment is severely affected by liquidity requirements. See Berentsen et al. (2012), Chu and Cozzi (2014) and Chu et al. (2015) for extensive discussions.
Therefore, following the standard treatment in the existing literature (e.g., Chu and Cozzi, 2014; Arawatari et al., 2018), we incorporate money demand into this monetary Schumpeterian growth model by imposing cash-in-advance (CIA) constraints on R&D and entry.⁴

One advantage of our model is that it allows us to investigate the mutual interaction between incumbents and entrants. Although the well-known Schumpeterian creative destruction is an important growth engine and has been well studied, existing evidence suggests that the own-product improvement of incumbents also contributes substantially to economic growth. For example, Bartelsman and Doms (2000) suggest that, in the US, 25% of productivity improvements are accounted for by the entry of new firms, with the remainder being accounted for by existing firms. Similarly, using Danish firm data, Lentz and Mortensen (2008) document that net entry accounts for only 21% of the aggregate growth rate. More recently, Garcia-Macia et al. (2019) have provided empirical evidence in support of the view that innovations by both incumbents and entrants are growth engines that cannot be ignored (incumbents’ quality improvements appear to be more important than entrants’ creative destruction).

Within our monetary growth-theoretic framework, the key prediction of our analysis is that an increase in the nominal interest rate crucially affects the composition of innovation (economic growth), stimulating incumbents’ own-product improvements and reducing entrants’ creative destruction. Intuitively, a higher nominal interest rate increases the R&D expenditure of entrants, thereby discouraging the entry of new firms. In other words, increasing the nominal interest rate deters the aggregate creative destruction by reducing the mass of new R&D firms. However, the decrease in creative destruction leads to incumbents facing less competitive pressure from entrants and thus being replaced less frequently, which in turn increases the value of in-house R&D. Therefore, although a higher nominal interest rate increases the costs of in-house R&D, incumbents still have incentives to set higher innovation rates. Accordingly, the relationship between the nominal interest rate and economic growth is ambiguous, depending upon the relative magnitude of the above two conflicting effects. Specifically, when the entry cost is sufficiently small, there are many entrants investing in R&D in the economy, and the aggregate creative destruction is sizable. Under such a situation, an increase in the nominal interest rate

---

leads to a sharp reduction in the entrants’ innovation. As a result, the negative effect of increasing the nominal interest rate on the entrants’ contribution to growth dominates the associated positive effect on the incumbents’ contribution. Consequently, in this case, a higher nominal interest rate decreases the economic growth rate. Conversely, in the case where the entry cost is sufficiently large, there are fewer entrants investing in R&D, and the aggregate creative destruction is limited. Under such a situation, in response to an increase in the nominal interest rate, the decrease in the entrants’ contribution is smaller than the increase in the incumbents’ contribution. Therefore, in this case, the economic growth rate is increasing in the nominal interest rate.

We also calibrate the model to provide a quantitative analysis of the growth and welfare effects of monetary policy. Under the baseline parameter values, we find that an increase in the nominal interest rate decreases the economic growth rate, which is consistent with empirical findings (Evers et al., 2007; Vaona, 2012; Chu and Lai, 2013; Chu et al., 2015) and previous studies (Marquis and Reffett, 1994; Chu and Lai, 2013; Chu and Cozzi, 2014). Moreover, our quantitative results also predict that social welfare increases with the nominal interest rate. Intuitively, a higher nominal interest rate reduces the incentives of entrants to invest in R&D, thereby leading entrants to reduce the resource usage for R&D and entry. With the economy’s resource constraint, the household tends to stimulate its consumption by taking away resources from entrants. Thus, an increase in the nominal interest rate contributes to raising the social welfare level. Under our parameter values, this positive reallocation effect on welfare dominates the negative effect resulting from the decline in the economic growth rate.

Furthermore, we consider an extension to the benchmark by imposing a more conventional CIA constraint on consumption and allowing for an elastic labor supply. With this extension, in response to an increase in the nominal interest rate, the decline in labor supply generates an additional negative effect on the entrants’ creative destruction. As a result, in this case, the negative growth effect of monetary policy becomes more significant than in the benchmark model. However, we find that the welfare effect of monetary policy remains positive and increases with the strength of the CIA constraint on consumption. The reason for this is that a fall in labor supply is associated with a rise in leisure, which generates an additional positive effect on welfare, and this positive effect becomes stronger when consumption is more cash-constrained.

1.1. Related literature
This study is related to the literature on R&D-based growth. Romer (1990) is the seminal study that develops a variety-expanding model in which growth is driven by the development of new products. Segerstrom et al. (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) develop the Schumpeterian quality-ladder models in which innovation and growth are driven by the quality improvements of existing products. Then, in their second-generation R&D-based growth models, Peretto (1998), Howitt (1999) and Segerstrom (2000) combine two dimensions of technological progress: variety expansion and quality improvements. However, none of these seminal studies is suited to investigating the mutual interaction between the simultaneous quality improvements of incumbents and entrants. Our paper is more related to a few growth models featuring the heterogeneous vertical innovations (i.e., quality improvements) of incumbents and entrants such as those of Klette and Kortum (2004), Acemoglu and Cao (2015), Acemoglu et al. (2018), Akcigit and Kerr (2018), and Iwaisako and Ohki (2019). Among them, Acemoglu and Cao (2015) extend the basic Schumpeterian model by allowing for the incremental innovations (own-product improvements) of existing firms, while new firms invest in radical innovations in an attempt to replace incumbents. Moreover, Akcigit and Kerr (2018) provide a tractable framework for the analysis of economic growth driven by heterogeneous innovations, in which incumbents invest in R&D to improve their products and acquire new product lines, while new firms innovate based on existing products in order to become intermediate producers on a successful innovation. Our paper complements this strand of the literature by incorporating money demand in a Schumpeterian growth model with two types of quality improvements (the own-product improvements of incumbents and creative destruction of entrants). Our reduced-form modeling of innovation by incumbents and entrants allows us to provide a tractable analysis of the effects of monetary policy on the market structure, economic growth, and social welfare.

This study also contributes to the literature on monetary policy (inflation) and R&D-based growth. An early study by Marquis and Reffett (1994) explores the effects of monetary policy on innovation by imposing a CIA constraint on consumption in the Romer variety-expanding model. Recently, Arawatari et al. (2018), Hori (2020), and Furukawa and Niwa (2021) consider monetary policy in

---

5 See Aghion et al. (2014) for a survey of Schumpeterian growth theory.

6 See Chu (2021) for a survey of the literature that explores the relationship between inflation and economic growth in R&D-based growth models.
variety-expanding models with heterogeneity in firms’ R&D productivity. In a distinct
the effects of inflation on growth in a Schumpeterian growth model with a
money-in-utility specification. Chu and Cozzi (2014) is the first study to introduce a
CIA constraint into a standard Schumpeterian model by formulating CIA constraints
on R&D investment and they find a negative relationship between inflation and
growth. Subsequent studies, such as He and Zou (2016), Chu et al. (2017), Huang et
al. (2017), Neto et al. (2017), He (2018), Chu et al. (2019), Lin et al. (2020), Zheng et
al. (2020), Chu et al. (2021), Huang et al. (2022), and Lu et al. (2022) also explore the
relationship between monetary policy and growth in the Schumpeterian growth model.
Moreover, Huang et al. (2021) and Zheng et al. (2021) consider a second-generation
R&D-based growth model with CIA constraints on the quality improving and
variety-expanding R&D of entrants. However, none of these studies consider CIA
constraints on two types of vertical innovation (i.e., quality improvements). Therefore,
the present paper contributes to this literature by allowing for two distinct CIA
constraints on own-product improvement and Schumpeterian creative destruction,
respectively. Since there are two growth engines in the economy, another novel
contribution of this study is that it analyzes how monetary policy affects the
composition of innovation that drives economic growth.

Studies by Chu and Ji (2016), He and Wang (2020), and Huang et al. (2021) also
examine the crucial role of the EMS in determining the growth and welfare effects of
monetary policy. In their models, incumbents invest in in-house R&D to improve their
products, while entrants create new products that are completely differentiated from
the incumbents’ products and compete with incumbents for market share. The present
paper complements these studies by allowing entrants to innovate based on existing
products and threaten incumbents with exiting the market through Schumpeterian
creative destruction, while also analyzing the effect of monetary policy on the
aggregate creative destruction and growth. Moreover, their models measure the
market structure based on the market size, whereas our paper measures the market
structure based on the mass of R&D firms. This new type of measurement gives rise
to some new predictions.

The rest of the paper is organized as follows. Section 2 presents the monetary
Schumpeterian model with incumbents and entrants. In Section 3, we analyze the
growth and welfare effects of monetary policy. Section 4 provides the quantitative
analysis and an extension of the benchmark model. The final section concludes.

2. A monetary Schumpeterian model with incumbents and entrants

In this section, we develop a monetary version of the Schumpeterian model to analyze how monetary policy affects economic growth, market structure, and the mutual interaction between heterogeneous innovations (own-product improvements and creative destruction). We extend the basic quality-ladder model by adding three features: (i) allowing incumbents to invest in in-house R&D to improve the quality of their products, (ii) incorporating a fixed entry cost to generate an endogenous entry of new firms, and (iii) introducing money demand via CIA constraints on R&D and entry. Our model is based on Acemoglu and Cao (2015) and Akcigit and Kerr (2018), in which both in-house R&D and creative destruction are growth engines of the economy. The economy consists of a representative household, firms (firms that compete for the final good, incumbent firms that engage in intermediate goods production and in-house R&D, and entrant firms that engage in creative destruction), and the monetary authority. The final good is either consumed by the household or used as an input for the production of intermediate goods, R&D and entry, and labor is used as an input for the final good production.

2.1. Household

In the economy, the representative household’s population size is normalized to unity. The lifetime utility function of the household is given by

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt,$$

where \(\rho > 0\) is the subjective discount rate and \(c_t\) denotes the consumption of the final good at time \(t\). The household maximizes its lifetime utility subject to the following asset-accumulation equation:

$$a_t + m_t = ra_t + w_t + \tau_t + i_t b_t - \pi_t m_t - c_t,$$

\(a_t\) refers to the real assets (in the form of equity issued by intermediate goods firms) owned by the household and \(r_t\) is the real interest rate. The household inelastically supplies one unit of labor to earn a real wage rate \(w_t\).\(^7\) \(b_t\) is the amount of household loans that are extended to R&D firms, and the return rate on \(b_t\) is the nominal interest rate \(i_t\). \(m_t\) is the real money balances held by the household, and

\(^7\) In Subsection 4.3 below, we will consider the case of an elastic labor supply in an extension to the benchmark model.
\( \pi_t \) is the inflation rate. The CIA constraint is given by \( b_t \leq m_t \). Finally, the household also receives a lump-sum transfer \( \tau_t \) from the government.

From standard dynamic optimization, the familiar Euler equation that governs the growth of consumption is

\[
\frac{\dot{c}_t}{c_t} = r_t - \rho, \quad (3)
\]

and the no-arbitrage condition between the real assets and real money balances is

\[
i_t = r_t + \pi_t. \quad (4)
\]

Equation (4) is the Fisher equation linking the nominal interest rate \( i_t \) and the real interest rate \( r_t \).

2.2. Final good

Following Acemoglu and Cao (2015) and Akcigit and Kerr (2018), we assume that the unique final good is produced by perfectly competitive firms using the following production technology:

\[
Y_t = \frac{1}{1-\beta} \left( \int_0^1 q_j^0 x_j^0 \beta^j d\beta \right) L_t^\beta, \quad (5)
\]

where \( \beta \in (0,1) \). The variable \( x_j^0 \) denotes the quantity of the intermediate good of type \( j \) used in the production, and \( q_j^0 \) is its quality. As mentioned above, since labor is supplied inelastically and normalized to unity (i.e., \( L_t = 1 \)), we therefore omit the parameter \( L_t \) in the benchmark model.

From the profit maximization of the final good firms, the equilibrium wage rate is

\[
w_t = \beta Y_t, \quad (6)
\]

and the conditional demand function for intermediate good \( j \) is determined by

\[
x_j^0 = \left( \frac{P_i}{P_j} \right)^{\frac{1}{\beta}} q_j^0, \quad (7)
\]

where \( P_i \) and \( P_j \) are the prices of the final good and intermediate good \( j \), respectively.

2.3. Intermediate goods and R&D

Differentiated intermediate goods are produced in a unit continuum of industries \( j \in [0,1] \). Each industry is temporarily dominated by an industry leader (i.e., the
incumbent firm) who enforces a patent on the highest-quality version of intermediate
good \( j \) until the arrival of the new innovation (i.e., creative destruction) of a
potential entrant. In line with Acemoglu and Cao (2015) and Akcigit and Kerr (2018),
we assume that only the highest-quality version of each intermediate good will be
used by the final good producers. In the economy, there are two types of innovation:
in-house R&D and creative destruction. In-house R&D involves own-product
improvements, carried out by incumbents in an attempt to improve the quality of their
existing products. Meanwhile, creative destruction takes place as a result of new firms
entering the market, innovating on intermediate goods that they do not currently own,
and replacing incumbent firms as the result of a successful innovation.

2.3.1. Incumbents

Without loss of any generality, we assume that each type of intermediate good
can be produced at \( 1 - \beta \) units of the final good to simplify expressions. Given this
constant marginal cost, the profit-maximization problem of each incumbent \( j \in [0,1] \)
gives its optimal price and quantity, which are respectively given by

\[
p_j = P, \quad (8)
\]
\[
x_j = q_j. \quad (9)
\]

Using (8) and (9), the monopolistic profit in industry \( j \) in terms of the final good is

\[
\Pi_j = \beta q_j, \quad (10)
\]

which is distributed to the household that owns the firm. As is obvious, the
monopolistic profit of an incumbent in (10) is proportional to the quality of its product.
Consequently, each incumbent firm has an incentive to engage in in-house R&D in
quality improvements. To achieve an instantaneous Poisson arrival rate of innovation
\( z_{j} \geq 0 \), we follow Akcigit and Kerr (2018) and assume that the flow R&D cost of an
incumbent firm in terms of the final good is

\[
C_m(z_j, q_j) = -\frac{1}{\alpha} \delta_m q_j z_j. \quad (11)
\]

The parameter \( \alpha > 1 \) captures the sensitivity of the incumbent’s in-house R&D cost
with respect to the successful innovation probability, and the parameter \( \delta_m > 0 \)
denotes the productivity of own-product improvement. Specifically, the flow
innovation cost \( C_m(z_j, q_j) \) is proportional to the quality level \( q_j \), which implies
that improving a higher-quality intermediate input is more expensive. An incumbent’s
successful innovation will lead to an increase in the quality of intermediate good $j$ from $q_{jt}$ to $(1+\lambda_m)q_{jt}$, where $\lambda_m > 0$ represents the step size of own-product improvement.

Since the outcome of innovation is uncertain, R&D investment has well-known liquidity problems. In the economy, each incumbent firm needs to borrow money from the household at the nominal interest rate $i$ to finance its innovation expenditure. To investigate the monetary effects of this CIA constraint on incumbents, we follow Chu and Cozzi (2014) and assume that a fraction $\xi_m \in [0,1]$ of the in-house R&D cost is constrained by the CIA constraint. Therefore, this CIA constraint forces each incumbent firm to borrow an amount of real money balances $b_{jt} = \frac{1}{1+\delta_m}q_{jt}q_{jt} \xi_m$ from the household.

2.3.2. Entrants

As in Klette and Kortum (2004), Acemoglu and Cao (2015), and Akcigit and Kerr (2018), new firms enter the market by engaging in innovation. An entrant targets an existing product and devotes resources to improve its quality. The entrant firm replaces the incumbent’s original product with a successful innovation and becomes the new industry leader. Consistent with the R&D cost function of an incumbent, an entrant firm chooses an instantaneous Poisson arrival rate of $z_{et} \geq 0$ with a flow R&D cost in terms of the final good:

$$
C_e(z_{et}, \bar{q}_t) = \frac{1}{\gamma} \delta_e z_{et}^{\vartheta} \bar{q}_t. \tag{12}
$$

The parameter $\gamma > 1$ captures the sensitivity of the entrant’s R&D cost with respect to the successful innovation probability, and the parameter $\delta_e > 0$ denotes the productivity of creative destruction. Following Akcigit and Kerr (2018), in (12) we set the flow R&D cost of entrants proportional to the average technology level $\bar{q}_t = \int_0^t q_{jt} dj$ in the economy. This specification implies that new firms that invest in innovation to replace incumbents need to pay higher flow costs in a technologically more advanced economy.\(^8\) Because potential new entrants seek to obtain leadership over products that they do not currently own, their innovations have wide and uncertain applications. To model this uncertainty, in line with Akcigit and Kerr (2018), we assume that each entrant achieves a breakthrough in any intermediate good industry $j \in [0,1]$ with equal probability (i.e., entrants’ R&D efforts are undirected).\(^9\)

---

\(^8\) In addition, this specification removes the dependence of entrants’ R&D efforts on the average quality level since the returns to creative destruction will be proportional to $\bar{q}_t$, which we will show later.

\(^9\) See Akcigit and Kerr (2018) for a more detailed discussion.
Thus, in accordance with expectations, when an innovation by an entrant firm is successful, the average quality level in the economy improves by a step size \( \lambda > 0 \) such that \( \bar{q}_{(t,\Delta t)} = (1 + \lambda_c) \bar{q}_t \).

In addition to the variable R&D expenditure, entrants are also required to face an entry cost to enter the market. Specifically, each entrant pays \( \psi \bar{q}_t \) units of the final good to set up the R&D equipment, where \( \psi > 0 \) is the cost parameter. As we will show later, this fixed cost ensures that our model has an endogenous entry and a unique balanced growth path (BGP). Through the entry and exit of firms, the mass of entrants is endogenous and we denote it by \( m_e \). Consequently, the creative destruction rate \( \phi_e \) is endogenously determined by aggregating the innovation flow rates across the mass of entrants. We consider that all entrants are homogeneous in the economy, and then optimal innovation flow rates are equal across entrant firms. Thus, the aggregate creative destruction rate is immediately given by

\[
\phi_e = m_e z_e. 
\]

Eq. (13) implies that the number of entrants actively investing in R&D \( m_e \) and the instantaneous Poisson arrival rate \( z_e \) (i.e., R&D effort) of new firms jointly determine the frequency of innovations coming from creative destruction \( \phi_e \).

As incumbent firms, new innovative firms also face liquidity problems and are subjected to cash constraints. To capture the monetary effect of the CIA constraint on the entrants’ innovation and entry, we assume that a fraction \( \xi_e \in [0,1] \) of the R&D investment and entry costs of entrant firms needs to be borrowed from the household at the nominal interest rate \( i \).\(^{11}\) Thus, this CIA constraint forces an entrant to borrow an amount of real money balances \( b_e = [(1/\gamma)\delta_e z_e^e + \psi] \bar{q}_e \bar{g}_e \) from the household to finance its variable and fixed costs.

As mentioned above, there are two types of innovative firms in the economy: incumbents and entrants. Potential entrants are new firms, and the empirical evidence indicates that new firms are more likely to engage in radical innovations, which are more radical than incumbents’ innovations.\(^{12}\) Janiak and Monteiro (2011) show that

---

\(^{10}\) In fact, \( \phi_e \) is the expected rate of creative destruction. In the rest of this study, \( \phi_e \) is simply called the rate of creative destruction since this will not cause any confusion.

\(^{11}\) To focus on the effect of monetary policy on the interaction between own-product improvements and creative destruction, we assume that entrants’ R&D investment and fixed entry costs are subjected to the same strength of CIA constraint. Our main results still hold without this assumption.

\(^{12}\) See, for example, Akcigit and Kerr (2018) for empirical evidence which supports the view that new firms engage in more radical innovations. Caggese (2019) also argues that young firms are much more likely to invest in radical innovation, while older firms are, on average, more likely to invest in incremental innovation.
new firms face a stronger cash constraint than older firms. Moreover, Akicigit (2009) and Caggese (2019) argue that radical R&D is more cash-constrained than incremental R&D (for example, own-product improvement in this paper).\(^{13}\) To capture these empirical findings, with regard to the relative extents of the CIA constraints on in-house R&D and creative destruction, we assume that \(\xi > \xi_w\) holds in our model. In other words, entrants face more serious liquidity problems and CIA constraints than incumbents.

2.4. Monetary authority

In the presence of the CIA constraints on R&D and entry, the monetary authority can affect economic growth via the nominal interest rate. Thus, we consider the nominal interest rate to be the monetary policy instrument in this economy, which is exogenously imposed by the monetary authority. The nominal money supply is denoted by \(M_t\), and its growth rate is \(\dot{M}_t / M_t = \mu_t\). Given that real money balances \(m_t = M_t / P_t\), the growth rate of real money balances is then given by \(\dot{m}_t / m_t = \mu_t - \pi_t\). Substituting this expression and the Euler equation (3) into the Fisher equation (4), we derive the growth rate of nominal money supply \(\mu_t = i_t - \rho\) along the balanced growth path, which is endogenously determined by the nominal interest rate \(i_t\).\(^{14}\) Thus, there is a one-by-one relationship between the nominal interest rate and the growth rate of nominal money supply.\(^{15}\) To balance the government budget, we follow the standard treatment in previous studies to assume that the seigniorage revenue will be distributed to the household as a lump-sum transfer by the monetary authority. Thus, the monetary authority’s budget constraint is given by \(\dot{M}_t / P_t = \mu_t m_t = \tau_t\).

2.5. Aggregations

Substituting (9) into (5) with \(L_t = 1\) yields the aggregate production function given by

\[
Y_t = \frac{1}{1 - \beta} q_t. \quad (14)
\]

Using (9), we obtain the aggregate expenditure on intermediate goods production

\(^{13}\) A recent study by Acemoglu and Cao (2015) regards the own-product improvements of incumbents as incremental innovation, and the creative destruction of entrants as radical innovation.

\(^{14}\) We use the fact that along the balanced growth path, \(m_t\) and \(c_t\) grow at the same rate.

\(^{15}\) Thus, one can also regard the growth rate of nominal money supply as the monetary policy instrument. See, for example, Chu and Lai (2013) who assume that the monetary authority implements its monetary policy by targeting the growth rate of money supply \(\mu_t\).
$X_i = (1 - \beta) \int_0^t x_j^i dj$ given by

$X_i = (1 - \beta) \bar{q}_i$.  

(15)

Substituting (14) into (6), the real wage rate is

$w_i = \frac{\beta}{1 - \beta} \bar{q}_i$.  

(16)

Moreover, based on (11) and (12), and imposing the assumption that new firms are homogeneous, the total R&D expenditure of incumbents $C_{Mt}$, the total R&D expenditure of entrants $C_{Et}$, and the total entry costs $C_{Fi}$ at time $t$ in terms of the final good are respectively given by

$C_{Mt} = \frac{1}{\alpha} \delta_m \left( \int_0^t z^\alpha \int_0^t d\bar{q}_i \right) \bar{q}_i$,  

(17)

$C_{Et} = - m_\alpha \delta_\gamma z^\gamma \bar{q}_i$,  

(18)

$C_{Fi} = m_\alpha y \bar{q}_i$.  

(19)

2.6. Decentralized equilibrium

This section defines the equilibrium and characterizes the balanced growth path. The equilibrium consists of a time path of prices $\{w_i, r_i, P_i, p_\beta, V(q_\beta)\}_{t=0}^\infty$, a time path of the mass of entrants, innovation flow rates, and aggregate creative destruction rate $\{m_{\alpha}, z_\beta, \gamma_{\alpha}, \phi_{\beta}\}_{t=0}^\infty$, a time path of policies $\{\mu, \tau_i\}_{t=0}^\infty$, and a time path of allocations $\{c, a, m, Y, X_i, b_i, b_{et}, b_t, C_{mt}(z_\beta, q_\beta), C_e(z_\alpha, \bar{q}_i), C_{Mt}, C_{Et}, C_{Fi}\}_{t=0}^\infty$. In addition, at each instant of time,

(a) the household maximizes utility taking $\{w_i, r_i, \pi_i, i, \tau_i\}$ as given;

(b) competitive final good firms maximize profits taking $\{w_i, P_i, p_\beta, q_\beta\}$ as given;

(c) incumbents produce $\{x_j\}$ and choose $\{z_\beta, p_\beta\}$ to maximize expected profits taking $\{i, r_i, \phi_{\beta}\}$ as given;

(d) entrants make entry decisions and choose $z_\alpha$ to maximize expected net returns taking $\{i, r_i\}$ as given;

(e) the final good market clears such that $c_i + X_i + C_{Mt} + C_{Et} + C_{Fi} = Y_i$;

(f) the asset market clears such that the value of monopolistic firms adds up to the value of the household’s asset: $\int_0^t V(q_\beta) dj = a_i$;

(g) the amount of money borrowed by incumbents and entrants is $b_i = \int_0^t b_\beta dj + m_\beta b_{et}$;

(h) the monetary authority balances its budget such that $\mu m_i = \tau_i$.

\[\text{\textsuperscript{16}} V(q) \text{ denotes the value of an incumbent that produces an intermediate good with quality } q, \text{ which we will discuss in detail in the next subsections.}\]
2.7. Optimal innovation rates

To fully understand how firms’ R&D decisions shape the decentralized equilibrium, we need to determine the R&D effort levels of incumbents and entrants. Thus, in this subsection, we consider the profit-maximization problem of these two types of R&D firms to obtain the optimal innovation flow rates, respectively.

2.7.1. Incumbents’ maximization problem

We focus on the symmetric equilibrium. Let $V(q)$ denote the value of an incumbent firm that produces an intermediate input with quality level $q$. For simplicity, henceforth, the firm subscript $j$ is suppressed. Under an optimal innovation decision, the value function $V(q)$ satisfies the following standard Hamilton-Jacobi-Bellman (HJB) equation:

$$rV(q) - \dot{V}(q) = \max_z \left\{ \Pi - C_m(z,q)(1+\xi_i) + z[V((1+\lambda_m)q) - V(q)] - \phi V(q) \right\}.$$  \hspace{1cm} (20)

The term $\dot{V}(q)$ on the left-hand side of (20) is the change in the incumbent firm value without any successful innovations (i.e., the quality level $q$ in the industry does not change). The right-hand side of (20) is the sum of four terms. $\Pi$ is the monopolistic profit of production given by (10), while $C_m(z,q)(1+\xi_i)$ is the total R&D expenditure of the incumbent for improving the quality of its product. The last two terms capture changes in the incumbent firm value due to innovation either by the incumbent or by an entrant. The term $z[V((1+\lambda_m)q) - V(q)]$ represents the probability-weighted change in the incumbent firm value due to quality improvements by itself (at the arrival rate $z$, and the step size of the quality improvements is $\lambda_m$). The last term $\phi V(q)$ is the expected value loss due to creative destruction (at the rate $\phi$), in which case the incumbent is replaced and exits the economy.\(^{17}\)

Solving the maximization problem in (20) yields the optimal innovation flow rate of incumbents:

$$z = \left[ \frac{V((1+\lambda_m)q) - V(q)}{\delta_nq(1+\xi_i)} \right]^{\alpha-1}.$$  \hspace{1cm} (21)

To solve for the incumbent value function $V(q)$, in line with Acemoglu and Cao (2015) and Akcigit and Kerr (2018), we conjecture $V(q) = vq$ (i.e., the value function of a monopolist incumbent firm with quality $q$ is linear in $q$), where $v > 0$ is a time-invariant scale parameter that denotes the marginal (and average) value of

\(^{17}\) Recall that $\phi$ is the rate of creative destruction reported in (13) at which an incumbent loses its market position.
quality. Given this linear structure, the optimal innovation flow rate of incumbents is given by

\[
z = \left[ \frac{v\lambda_m}{\delta_m (1 + \xi_m i)} \right]^{1/(1-\alpha)}.
\]  

(22)

Recalling that \( \alpha > 1 \), (22) clearly shows that an incumbent firm’s innovation effort level is positively related to the change in its (scaled) value due to in-house R&D \( v\lambda_m \) (i.e., the gain from own-product improvement). Moreover, the optimal innovation rate \( z \) decreases with the adjusted R&D cost coefficient \( \delta_m (1 + \xi_m i) \) in the presence of a CIA constraint on in-house R&D.

2.7.2. Entrants’ maximization problem

We denote \( V_e(\bar{q}) \) as the expected value of an entrant from entering the market but before innovating successfully (i.e., the ex-ante value of an innovation), which is a function of the average quality level \( \bar{q} \). The standard HJB equation for \( V_e(\bar{q}) \) is

\[
rV_e(\bar{q}) - \dot{V}_e(\bar{q}) = \max_{z_e} \left\{ z_e E_{\mu[q_t]} \left[ V \left( (1 + \lambda_m) q_t \right) - V_e(\bar{q}) \right] - C_e(z_e, \bar{q})(1 + \xi, i) \right\}.
\]

(23)

The term \( \dot{V}_e(\bar{q}) = \frac{\partial V_e(\bar{q})}{\partial t} \) represents the partial derivative of the ex-ante value with respect to time \( t \). The first term on the right-hand side of (23) is the probability-weighted expected change in firm value when an entrant attains a breakthrough and becomes the new incumbent of an input industry. The second term \( C_e(z_e, \bar{q})(1 + \xi, i) \) is the total variable R&D cost paid by the entrant at time \( t \) in the presence of a CIA constraint on creative destruction.

From (23), we can solve for the optimal innovation flow rate of entrants as

\[
z_e = \left[ E_{\mu[q_t]} \left[ V \left( (1 + \lambda_m) q_t \right) - V_e(\bar{q}) \right] / \delta_e(1 + \xi, i) \right]^{1/(1-\alpha)}.
\]

(24)

When there is a positive entry,\(^{19} \) the free entry condition implies that the ex-ante value of an entrant equals the fixed entry cost:

\[V_e(\bar{q}) = \psi \bar{q} (1 + \xi, i).\]

(25)

Substituting (25) into (24) and using the linear structure of the incumbent value function, the optimal innovation rate \( z_e \) can be rewritten as

\[\text{In the rest of the paper, we often refer to } v \text{ simply as the “value of quality” when causing no confusion.}\]

\[\text{When there is no new entry, the model will degenerate to a special case of in-house R&D only. We assume for now that there is a positive entry in the economy, and later impose a restriction on this condition.}\]
\[
 z_v = \left[ \frac{v(1+\lambda_v) - \psi(1+\xi_i)}{\delta_v(1+\xi_i)} \right]^{1/\gamma}.
\]  

(26)

Similar to incumbents, given that \( \gamma > 1 \), (26) shows that in the presence of CIA constraints on entrants' R&D and entry, a new firm's innovation effort level is positively related to the net (scaled) gain from becoming a new incumbent \( v(1+\lambda_v) - \psi(1+\xi_i) \), but decreases with the adjusted R&D cost coefficient \( \delta_v(1+\xi_i) \).

2.8. BGP properties and growth decomposition

It is useful to note that the interaction between in-house R&D and creative destruction is the core of our model. To ensure that the innovation rate of entrants \( z_v > 0 \) (otherwise, the model will lose the Schumpeter characteristic), we make the following assumption:

**Assumption 1.** The inequality \( v(1+\lambda_v) > \psi(1+\xi_i) \) needs to hold such that there is a positive entry in the economy.

We are now ready to aggregate firms’ R&D decisions and characterize the balanced growth path of the economy.\(^{20}\) Under Assumption 1, in equilibrium, economic growth is driven by in-house R&D and Schumpeterian creative destruction. Therefore, the growth rate of the technology level is given by

\[
g = \frac{\dot{q}}{q} = z\lambda_v + \phi\lambda_v.
\]

(27)

From (14)-(19), the asset market clearing condition, and the final good market clearing condition, we have

\[
\frac{\dot{q}}{q} = \dot{m} = \dot{a} = \dot{c} = \dot{Y} = \dot{X} = \dot{w} = \dot{C}_M = \dot{C}_E = \dot{C}_F = g.
\]

(28)

Then, by the Euler equation (3), the BGP real interest rate is determined by

\[
r = g + \rho .
\]

(29)

Henceforth, the variable with the superscript “*” attached refers to its equilibrium value. In the BGP equilibrium, the unique marginal value of quality \( v' > 0 \) that satisfies (23) is given by\(^{21}\)

\[^{20}\] Acemoglu and Cao (2015) refer to a BGP as a linear BGP where the value function of an incumbent \( V(q) = vq \).

\[^{21}\] A detailed derivation is provided in Appendix A.
Thus, there is a unique linear BGP, where the value function $V(q) = v^* q$, and $v^*$ is given by (30). Then, the equilibrium innovation rate of incumbents $z^*$ and the equilibrium innovation rate of entrants $e^*$ are simply inferred by inserting $v^*$ into (22) and (26), respectively.

Moreover, (30) shows that $v^*$ is a function of exogenous parameters and the nominal interest rate. Given a constant $i$, we immediately have $V(q) = 0$.\(^{22}\)

Substituting (27) and (29) into (20) yields the BGP creative destruction rate given by\(^{23}\)

$$
\phi^* = \frac{1}{1 + \lambda_o} \left[ \frac{\beta}{v^*} - \frac{1}{\alpha} \lambda_m z^* - \rho \right].
$$

(31)

Thus, equipped with (27), the BGP growth rate of the aggregate variables in the economy is given by

$$
g^* = z^* \lambda_m + \phi^* \lambda_e.
$$

(32)

Eq. (32) clearly shows that economic growth is the sum of the contributions of incumbents and entrants. The incumbents’ contribution to growth, $z^* \lambda_m$, comes from in-house R&D and is equal to the product of the step size of own-product improvement and the innovation rate of incumbents.\(^{24}\) The entrants’ contribution to growth, $\phi^* \lambda_e$, comes from quality improvements by entrants and is equal to the product of the step size of creative destruction and the aggregate creative destruction rate. Henceforth, the changes between $z^* \lambda_m$ and $\phi^* \lambda_e$ stemming from monetary policy are referred to as the “growth composition effect” of monetary policy.

Eq. (32) also reveals an important novelty of our model compared to the existing literature: economic growth is driven by two types of quality improvements (i.e., vertical R&D). If we shut off own-product improvements by incumbents, the economy will degenerate to the baseline Schumpeter model. Moreover, in the second-generation R&D-based growth models characterized by variety expansion and quality improvements, such as Peretto (1998), Howitt (1999), and Segerstrom (2000), the variety expansion by entrants cannot affect the equilibrium growth rate, such that

\(^{22}\) Recall that $V(q)$ is the change in firm value without any changes in the quality level $q$. Thus, when $\dot{v} = 0$, $V(q) = i q + \dot{q} = 0$.

\(^{23}\) See Appendix B for a detailed proof. \(^{24}\) Since the number of intermediate goods industries in the economy is normalized to one (as is the mass of incumbents), $z^*$ then denotes the aggregate R&D rate of incumbents.
there is only one growth engine in the economy. Unlike these studies, in our model, entrants do not invest in the R&D of new products but only engage in quality improvements and, as exhibited in (32), own-product improvements by incumbents and creative destruction by entrants are both growth engines in the economy.

As for the dynamics of the model, (30) implies that the value of quality must jump immediately to its steady-state value $v'$. Consequently, the other relevant variables are also stationary. Therefore, given a constant nominal interest rate $i$, the economy immediately jumps to the BGP along which each variable grows at a constant rate $g'$. We end this section by summarizing these results in Proposition 1.

**Proposition 1.** When there is a positive entry and entrants have to pay a fixed entry cost to enter the market, there exists a unique BGP with the value function of an incumbent that produces an intermediate input with quality $q$ given by $V(q) = v'q$, where $v'$ is the unique marginal value of quality. Given a constant nominal interest rate $i$, the economy immediately jumps to this BGP along which each variable grows at a constant rate $g'$ given by (32).

3. Growth and welfare effects of monetary policy

The previous discussion clearly shows that in the presence of CIA constraints on R&D and entry, the marginal value of being an incumbent $v'$, the aggregate creative destruction rate $\phi'$, and the economic growth rate $g'$ are all functions of the nominal interest rate $i$. Consequently, monetary policy will affect economic growth by changing in-house R&D and creative destruction. In this section, we analyze in detail the effects of monetary policy on own-product improvement, creative destruction, and the long-run economic growth. We begin by summarizing the impact of monetary policy on the equilibrium value of quality $v'$ below.

**Lemma 1.** In the presence of CIA constraints on R&D and entry, the equilibrium marginal value of quality $v'$ increases with the nominal interest rate $i$.

**Proof.** Note (30). ■

The intuition behind Lemma 1 is straightforward. An increase in the nominal interest rate $i$ raises the R&D cost of new firms. In addition, a higher $i$ tightens the CIA constraint on entry and leads the entry to become more expensive, thereby discouraging the entry of new firms. Both these effects make incumbents face lower
competitive pressure and the value of being an incumbent becomes greater. In other words, a higher nominal interest rate \(i\) increases the equilibrium value of quality \(v^*\) by deterring the creative destruction of entrants, and thus the value of incumbent firms rises along the BGP.

Interestingly, the effect of \(i\) on \(v^*\) is not affected by the CIA constraint on in-house R&D. Intuitively, in the presence of a CIA constraint on own-product improvements only (i.e., \(\xi_e = 0\)), a rise in \(i\) increases the R&D costs of incumbents, which tends to reduce the monopolistic profit and hence lead to a negative effect on \(v^*\). However, the decline in the monopolistic profit will discourage the entry of new firms, which in turn generates a positive effect on \(v^*\). Given that \(\xi_e = 0\), the latter positive effect on \(v^*\) is exactly offset by the former negative effect (note that the HJB equation of entrants (23) needs to hold), and hence the CIA constraint on in-house R&D does not affect the impact of \(i\) on the marginal value of quality \(v^*\).

### 3.1. Effects of monetary policy on in-house R&D

In this subsection, we explore the effects of monetary policy on own-product improvements (i.e., the incumbents’ contribution to growth). Given that the step size \(\lambda_m\) is constant, the effect of monetary policy on the incumbents’ contribution to growth \(z^*\lambda_m\) is qualitatively the same as that on the equilibrium innovation rate \(z^*\).

Based on (22), (30), and Lemma 1, we immediately establish the following proposition:

**Proposition 2.** In the presence of CIA constraints on R&D and entry, the equilibrium aggregate in-house R&D of incumbents \(z^*\lambda_m\) increases with the nominal interest rate \(i\).

**Proof.** Substituting (30) into (22) yields the equilibrium innovation rate \(z^*\) given by

\[
 z^* = \left[ \frac{T\lambda_m (1+\xi_e)}{\delta_\zeta_m (1+\lambda_e)(1+\xi_m i)} \right]^{1/\alpha},
\]

where \(T = \left[ \rho \psi \gamma (\gamma - 1) \right]^{(\gamma-1)/\gamma} \delta_\zeta^{\gamma/\gamma} + \psi > 0\). Taking the derivative of \(z^*\) with respect to the nominal interest rate \(i\), we obtain

\[
 \frac{\partial z^*}{\partial i} = \frac{1}{\alpha - 1} \left[ \frac{T\lambda_m}{\delta_\zeta_m (1+\lambda_e)} \right]^{1/\alpha} \frac{\xi_e - \xi_m}{(1+\xi_m i)} \left( 1+\xi_m i \right) \left( 1+\xi_e i \right)^{2-\alpha/\alpha}. 
\]

Given that \(\alpha > 1\) and \(\xi_e > \xi_m\) (see Subsection 2.3.2), we have \(\partial z^*/\partial i > 0\).

---

25 Note that the parameter \(\xi_m\) does not show up in (30).
Specifically, there are two effects of the nominal interest rate on incumbents’ own-product improvements. The first effect is an incentive effect. From Lemma 1, in the presence of a CIA constraint on entrants’ investment (i.e., $\xi > 0$), an increase in the nominal interest rate $i$ leads own-product improvements to be more profitable by increasing the value of quality $v'$. Thus, incumbents have an incentive to set a higher innovation flow rate $z'$. The second effect is a cost effect via the CIA constraint on in-house R&D, because an increase in $i$ will raise the cost of own-product improvements. However, when entrants face more serious liquidity problems relative to incumbents (i.e., $\xi > \xi_m$), the positive incentive effect dominates the negative cost effect. Consequently, a higher nominal interest rate $i$ tends to stimulate own-product improvements and increases the incumbents’ contribution to growth.

3.2. Effects of monetary policy on creative destruction

Turning to entrants, in this subsection, we explore the effects of monetary policy on entrants’ R&D decisions, entry incentives, and the aggregate creative destruction rate. Here we first summarize the effects of the nominal interest rate on the entrants’ R&D efforts (the intensive innovation margin) and the mass of entrants (the extensive innovation margin) into Lemma 2 and Lemma 3, respectively.

**Lemma 2.** In the presence of CIA constraints on R&D and entry, the nominal interest rate $i$ does not affect the equilibrium innovation flow rate of entrants $z'_e$.

**Proof.** Substituting (30) into (26) yields

$$z'_e = \left[ \frac{\rho \psi'}{\delta_e (\gamma - 1)} \right]^{\frac{1}{\gamma}}.$$  \hspace{1cm} (35)

Since the nominal interest rate does not show up in (35), $i$ has no effect on $z'_e$. \hspace{1cm} ■

**Lemma 3.** In the presence of CIA constraints on R&D and entry, the equilibrium mass of entrants $m'_e$ decreases with the nominal interest rate $i$.

**Proof.** Substituting (30) and (33) into (31) and then using (13) and (35) yields the equilibrium mass of entrants given by

$$m'_e = \frac{1}{1 + \lambda_e} \left[ \frac{\rho \psi'}{\delta_e (\gamma - 1)} \right]^{\frac{1}{\gamma}} \frac{\beta (1 + \lambda_e)}{T (1 + \xi_e i) \alpha \left( \frac{T \lambda_m (1 + \xi_e i)}{(1 + \lambda_e)(1 + \xi_m i)} \right)^{\frac{1}{\gamma - 1}} - \rho}. \hspace{1cm} (36)$$

Differentiating (36) with respect to $i$ yields
Given that \( \alpha > 1 \) and \( \xi_e > \xi_m \), we immediately have \( \partial m^*_e / \partial i < 0 \).

Intuitively, when the CIA constraint on the entrants’ investment is present,\(^{26}\) monetary policy has two effects on the R&D efforts of new firms. On the one hand, Lemma 1 indicates that the higher \( i \) increases the value of being an incumbent, which in turn encourages entrants to invest in innovation and set a higher innovation flow rate \( z^* \). On the other hand, in the presence of a CIA constraint on entrants’ investment, a rise in \( i \) increases the entry cost and R&D expenditure of entrants, thereby generating a negative effect on \( z^* \). Under the assumption of the identical extent of the CIA constraints on the entrants’ R&D investment and entry cost, these two opposing effects cancel each other out. Consequently, the nominal interest rate \( i \) does not affect the R&D rate of entrants.

The intuition behind Lemma 3 is also obvious. Specifically, the nominal interest rate \( i \) affects the mass of entrants (market structure) through three channels. Firstly, from Proposition 2, an increase in \( i \) encourages incumbents to set a higher innovation rate \( z' \). Because the two types of innovation (own-product improvement and creative destruction) compete in the economy, the higher \( z' \) enhances the crowding-out effect of in-house R&D on creative destruction. Secondly, an increase in \( i \) raises new firms’ R&D expenditure and entry costs, which reduces the incentives of potential entrants to enter the market and invest in R&D. Both the crowding-out effect and cost effect lead to the exit of R&D firms and reduce the mass of entrants. Finally, the nominal interest rate \( i \) also has an incentive effect on \( m^*_e \). Again, by Lemma 1, a higher \( i \) increases the marginal value of quality \( v' \), which in turn attracts new firms to enter the market and invest in R&D. In our model, the former two negative effects dominate the latter positive effect, thereby leading to a decline in \( m^*_e \) as \( i \) rises.

We are now ready to investigate the effects of monetary policy on the aggregate creative destruction (i.e., the entrants’ contribution to growth). As mentioned above, the entrants’ contribution to growth is given by \( \phi^* \lambda_e \), where \( \phi^* = m^*_e z^*_e \). Similarly,

\(^{26}\) Note that \( z_e \) reported in (26) is a function of \( v \) and the parameter \( \xi_e \) (but not \( \xi_m \)), and Lemma 1 shows that the effect of \( i \) on \( v \) is not affected by \( \xi_m \). Accordingly, the CIA constraint on in-house R&D cannot affect the effect of monetary policy on the entrants’ innovation rate.
given that $\lambda_e$ is a constant value, the effect of monetary policy on the entrants’ contribution to growth is qualitatively the same as that on the aggregate creative destruction rate $\phi'$. Based on Lemma 2 and Lemma 3, we have the following results:

**Proposition 3.** In the presence of CIA constraints on R&D and entry, the equilibrium aggregate creative destruction of entrants $\phi'\lambda_e$ decreases with the nominal interest rate $i$.

**Proof.** Substituting (30) and (33) into (31) yields

$$\phi' = \frac{1}{1 + \lambda_e} \left[ \beta \left(1 + \lambda_e\right) - \frac{\lambda_m}{\alpha} \left( \frac{T \lambda_m (1 + \xi_i)}{\delta_m (1 + \lambda_e)} \right)^{\frac{1}{\alpha-1}} - \rho \right].$$  (38)

Differentiating (38) with respect to $i$ and using (30) and (34), we have

$$\frac{\partial \phi'}{\partial i} = - \frac{1}{(1 + \xi_i)^2 T} \left[ \left( \frac{T \lambda_m}{1 + \lambda_e} \right)^{\frac{\alpha}{\alpha-1}} \frac{(\xi_e - \xi_m)}{\delta_m^{\frac{\alpha}{\alpha-1}}(\alpha - 1)} \left( \frac{1 + \xi_i}{1 + \xi_m} \right)^{\frac{\alpha}{\alpha-1}} + \beta \xi_e \right],$$  (39)

which shows that $\frac{\partial \phi'}{\partial i} < 0$. ■

The standard quality-ladder model usually assumes that there is a unit continuum of R&D firms engaging in creative destruction. Previous studies that examine the growth effect of monetary policy in quality-ladder models, such as Chu and Lai (2013), Chu and Cozzi (2014), Chu et al. (2015), and Huang et al. (2017), find that monetary policy influences creative destruction by affecting the intensive innovation margin $z_e^*$. However, the above analysis clearly shows that monetary policy deters creative destruction mainly by reducing the extensive innovation margin $m_e^*$ (i.e., market structure) in our model. Thus, a novel finding of this paper is that changing the mass of entrants appears to be a more important channel through which monetary policy affects Schumpeterian creative destruction.

### 3.3. Effects of monetary policy on economic growth

In the above subsections, we predict that an increase in the nominal interest rate $i$ tends to affect the composition of innovation and growth, encouraging incumbents’ own-product improvements and deterring entrants’ creative destruction. Thus, the balance of these two offsetting effects determines whether monetary policy is effective in promoting economic growth. Proposition 4 summarizes the effects of the nominal interest rate $i$ on the equilibrium growth rate $g^*$. 

Proposition 4. In the presence of CIA constraints on R&D and entry, (i) for a sufficiently large (small) entry cost $\psi$, the equilibrium economic growth rate $g^*$ increases (decreases) with the nominal interest rate $i$; (ii) for a sufficiently small gap between $\xi_e$ and $\xi_m$, the equilibrium economic growth rate $g^*$ decreases with the nominal interest rate $i$.

Proof. Differentiating (32) with respect to $i$ and using (34) and (38) yields
\[
\frac{\partial g^*}{\partial i} = \frac{1}{(1+\xi_e i)^2} T \left[ \frac{T \lambda_m}{1+\lambda_e} \left( \alpha + (\alpha - 1) \frac{\phi_e}{\phi_m} \right) \alpha (\alpha - 1) \right] \left( \xi_e - \xi_m \right) (1+\xi_e i)^{\phi_e/(\phi_m-\phi_e)} - \beta \lambda_e \xi_e.
\] (40)

Equipped with $T = [\rho \psi^{1/\gamma} / (\gamma - 1)]^{(\gamma - 1)/\gamma} \phi_e$ + $\psi > 0$, the above expression clearly demonstrates the results in Proposition 4.

Intuitively, when the entry cost is sufficiently large, there are fewer entrants investing in R&D in the economy. As a result, the aggregate creative destruction rate is small and the entrants’ contribution to economic growth is limited. Consequently, the negative effect of raising the nominal interest rate $i$ on R&D is weak, which makes the increase in the incumbents’ own-product improvements due to a higher $i$ greater than the associated reduction in the entrants’ creative destruction. Thus, in this case, an increase in the nominal interest rate $i$ stimulates economic growth. Conversely, when the entry cost is sufficiently small, there are many entrants actively investing in R&D, and the entrants’ contribution to growth is sizable. Therefore, the lower entry cost will reinforce the negative effect of raising $i$ on the entrants’ R&D. Therefore, in this case, an increase in the nominal interest rate $i$ will slow down economic growth since the increase in own-product improvements is smaller than the associated reduction in Schumpeterian creative destruction.

Proposition 4 also reveals that the higher nominal interest rate may boost or deter economic growth depending on the size of $\xi_e - \xi_m$. If the gap between $\xi_e$ and $\xi_m$ is sufficiently small, from (34) the increase in the incumbents’ own-product improvements due to a higher nominal interest rate $i$ is limited. Given that $\partial \phi_e^* / \partial i < 0$ in this case, the positive effect of raising $i$ on the incumbents’ contribution to growth is smaller than the associated negative effect on the entrants’ contribution. Consequently, the economic growth rate decreases with the nominal interest rate.

Note that $\phi_e^* \lambda_i = m_i^* \phi_e \lambda_i$. From (35), (36), and (38), we have $\partial \phi_e^* / \partial \psi > 0$, $\partial m_i^* / \partial \psi < 0$, and $\partial \phi_e^* / \partial \psi < 0$, respectively. Accordingly, a higher $\psi$ leads to a decline in the entrants’ contribution to growth by reducing the mass of new R&D firms.
To better understand the growth effects of monetary policy in the benchmark model, we consider two different scenarios, with each being subject to only one type of CIA constraint. In the case where only own-product improvements are subject to a CIA constraint (i.e., $\xi_e = 0$), an increase in $i$ hinders in-house R&D and increases creative destruction.\(^{28}\) (40) indicates that, in this case, the negative effect of $i$ on incumbents dominates the positive effect on entrants, and the equilibrium economic growth rate $g^*$ decreases with $i$. Conversely, in the second case, only the entrants’ investment is subject to a CIA constraint (i.e., $\xi_m = 0$), and an increase in $i$ stimulates own-product improvements and deters creative destruction.\(^{29}\) However, (40) shows that in this case, whether a higher $i$ has a positive or negative effect on growth depends on the magnitude of the relevant parameters. To sum up, compared to the special case where only new firms are subject to a CIA constraint, an increase in the nominal interest rate is more likely to reduce the growth rate in the benchmark model due to the CIA constraint on in-house R&D leading to an additional negative effect on economic growth.

### 3.4. Welfare analysis

In this subsection, we turn to explore the effects of monetary policy on social welfare. Given that the economy is always on the BGP, we impose the balanced growth condition on (1) to derive the steady-state welfare function as

$$U = \frac{1}{\rho} \left( \ln c_0 + \frac{g^*}{\rho} \right),$$

(41)

where $c_0$ is the steady-state level of consumption along the BGP at the instant of time 0. Using the final good market clearing condition $c_i + X_t + C_{me} + C_{te} + C_{pt} = Y_t$ and normalizing the initial quality level $z_0$ to unity, we obtain

$$c_0 = \frac{2\beta - \beta^2}{1 - \beta} - \frac{1}{\alpha} \delta_e z^{\gamma_e} - m_i \left[ \frac{1}{\gamma} \delta_e z^{\gamma_e} + \psi \right].$$

(42)

As noted above, the second and third terms on the right-hand side of (42) are the total expenditures of incumbents and entrants, respectively. Based on the previous discussions and Equations (41) and (42), the welfare effects of increasing the nominal interest rate can be decomposed into three parts. First, an increase in $i$ reduces the welfare level by increasing the R&D expenditure of incumbents. Second, an increase

\(^{28}\) When $\xi_e = 0$, from (34) and (39), we have $\epsilon_c \frac{\partial c}{\partial i} < 0$ and $\epsilon_e \frac{\partial e}{\partial i} > 0$.

\(^{29}\) When $\xi_m = 0$, from (34) and (39), we have $\epsilon_c \frac{\partial c}{\partial i} > 0$ and $\epsilon_e \frac{\partial e}{\partial i} < 0$. 

in $i$ raises the welfare level by decreasing the R&D and entry costs of entrants through a decline in the mass of new R&D firms $m_i'$. Finally, an increase in $i$ affects the welfare level by changing the equilibrium economic growth rate $g'$. Therefore, the impact of a higher $i$ on social welfare is ambiguous, and thus we resort to numerical analysis in the next section.

4. Quantitative analysis and an extension

In this section, we first report the baseline parameterization in Subsection 4.1 and then provide a quantitative analysis on the growth and welfare effects of monetary policy in Subsection 4.2. Finally, in Subsection 4.3, we consider an extension to our benchmark model with an elastic labor supply and a CIA constraint on consumption.

4.1. Calibration

In line with Akcigit and Kerr (2018), we set $\alpha = \gamma = 2$, which implies quadratic R&D cost functions of incumbents and entrants.\(^{30}\) We set the discount rate $\rho$ to a standard value of 0.05, as in Acemoglu and Akcigit (2012). Following Akcigit and Kerr (2018) and Akcigit et al. (2021), we set the step size associated with the incumbents’ in-house R&D to $\lambda_w = 0.05$, whereas the step size of the entrants’ creative destruction is $\lambda_e = 0.08$. This captures the empirical observation that the innovations of new firms have a higher impact on qualities than the own-product improvements of incumbents. As for the extent of the CIA constraints on the entrants’ R&D and entry, we set $\xi_e = 0.5$ as in Huang et al. (2021). To capture the fact that entrants face a larger cash constraint than incumbents, we consider $\xi_m = 0.2$ as our benchmark. As for the production parameter, we consider a value of $\beta = 0.22$ such that the markup ratio of monopolistic intermediate firms $\frac{1}{1-\beta}$ is approximately 1.3, which is within the reasonable range of the markup values of the US economy estimated by the empirical literature; see, for example, Domowitz et al. (1988), Chirinko and Fazzari (1994), and Devereux et al. (1996). Following Akcigit and Kerr (2018), we set the R&D coefficient for incumbents $\delta_w$ to 0.65. Then, in line with Akcigit et al. (2021), we consider the R&D coefficient for entrants $\delta_e$ to be approximately four times that for incumbents, which is set to 2.6. Finally, in line with Jones and Williams (2000) and Zheng et al. (2021), we consider a long-run economic

\(^{30}\) The existing evidence suggests that the elasticity of patents to R&D expenditures is around 0.5, which implies a quadratic curvature (e.g., Hall and Ziedonis, 2001; Blundell et al., 2002). See Akcigit and Kerr (2018) for an extensive discussion on the quadratic cost function.
growth rate of 1.2% such that the cost parameter $\psi = 1.36$. Table 1 summarizes the baseline values of the parameters.

**Table 1: Baseline Parameters**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$\lambda_m$</th>
<th>$\lambda_e$</th>
<th>$\xi_m$</th>
<th>$\xi_e$</th>
<th>$\beta$</th>
<th>$\psi$</th>
<th>$\delta_m$</th>
<th>$\delta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>2</td>
<td>2</td>
<td>0.05</td>
<td>0.05</td>
<td>0.08</td>
<td>0.2</td>
<td>0.5</td>
<td>0.22</td>
<td>1.36</td>
<td>0.65</td>
<td>2.6</td>
</tr>
</tbody>
</table>

4.2. Quantitative analysis

Fig. 1 shows the growth and welfare effects of monetary policy under the baseline parameter values. The top panels of Fig. 1 depict the results reported in Proposition 2 and Proposition 3: the incumbents’ contribution to growth increases with the nominal interest rate, while the entrants’ contribution decreases with the nominal interest rate. Specifically, when $i$ rises from 0 to 0.3, the contribution of incumbents increases by about 0.06%, while the contribution of entrants declines by about 0.12%. As is obvious, the negative effect of raising $i$ on the entrants’ contribution to growth dominates the positive effect on the incumbents’ contribution. Consequently, under our calibrated parameter values, a higher $i$ decreases the equilibrium economic growth rate $g^*$, as illustrated in the bottom left panel of Fig 1. Moreover, the top two panels of Fig. 1 also indicate that an increase in $i$ largely affects the composition of innovation and growth. When $i$ rises from 0 to 0.3, the ratio of the contribution of incumbents to the contribution of entrants increases from about 1.4 to about 2. This result is consistent with previous studies, which find that incumbents contribute more to innovation and growth than entrants (Bartelsman and Doms, 2000; Acemoglu and Cao, 2015; Garcia-Macia et al., 2019).

As for the welfare effect of monetary policy, in the bottom right panel of Fig. 1, we find that social welfare increases with the nominal interest rate $i$. The intuition can be explained as follows. As noted previously, an increase in $i$ reduces the initial consumption $c_0$ by increasing the incumbents’ R&D expenditure while it raises $c_0$ by decreasing the entrants’ R&D and entry expenditure. Given that new firms have lower R&D efficiency (under our parameter values, $\delta_e$ is four times as large as $\delta_m$), the former negative effect is dominated by the latter positive effect and thus the initial level of consumption $c_0$ increases with the nominal interest rate $i$ (i.e., a higher $i$ reallocates social resources from entrants to incumbents and the household).
Furthermore, the positive consumption effect (a higher \( i \) raises welfare by increasing \( c_0 \)) is greater than the negative growth effect (a higher \( i \) reduces welfare by decreasing \( g^* \)), and thus the overall welfare effect of increasing the nominal interest rate is positive in this case.

Interestingly, our model with the EMS is sufficiently flexible to generate a positive relationship between the nominal interest rate and growth.\(^{31}\) We now increase the cost coefficient \( \psi \) to a sufficiently large level of 2.36 as shown in Fig. 2. As mentioned earlier, in this case, fewer entrants actively invest in R&D in the economy. The top two panels of Fig. 2 clearly show that the entrants’ contribution to growth is limited and the economic growth is mainly driven by the own-product improvements of incumbents. Accordingly, an increase in \( i \) leads to a substantial increase in the incumbents’ innovation, which is greater than the associated reduction in the entrants’ creative destruction. Specifically, when \( i \) rises from 0 to 0.3, the incumbents’ contribution to growth increases by about 0.1%, while the entrants’ contribution decreases by about 0.08%. Therefore, in the case where the entry cost is sufficiently large, a rise in \( i \) increases the growth rate \( g^* \), as depicted in the bottom

\(^{31}\) Empirical studies sometimes find a positive relationship between economic growth and the nominal interest rate; see, for example, Kuttner and Mosser (2002), Werner (2005), Wang and Xie (2013), and Lee and Werner (2018).
left panel of Fig. 2. Finally, the bottom right panel of Fig. 2 shows how the welfare level responds to a rise in $i$. Again, in the presence of CIA constraints on R&D and entry, an increase in $i$ reallocates resources from creative destruction to in-house R&D and consumption. Given that the growth effect of monetary policy is positive, a higher $i$ will also raise welfare by increasing $g^*$. Thus, in this case, the overall welfare effect of monetary policy remains positive.

\[ U = \int_0^\infty e^{-\rho t} \left[ \ln c_i + \theta \ln (1 - l_i) \right] dt, \]  
\[ \xi_i c_i + b_i \leq m_i. \]

(44) states that $m_i$ is also used to partly purchase consumption, where $\xi_i \in [0,1]$ measures the extent of the CIA constraint on consumption.

From standard dynamic optimization, the optimal condition for labor supply is
given by
\[ w_t (1 - l_t) = \theta c_t (1 + \xi_t). \]  
(45)

Similar to the benchmark, the aggregate production function becomes
\[ Y_t = \frac{1}{1 - \beta} \bar{q}_t l_t. \]  
(46)

Moreover, from the profit maximization of final good producers, the demand function for labor is
\[ w_t = \frac{\beta}{1 - \beta} \bar{q}_t = \frac{Y_t \beta}{l_t}. \]  
(47)

Substituting (47) into (45) yields
\[ l = \frac{\beta}{\theta c_t Y_t (1 + \xi_t) + \beta}. \]  
(48)

The aggregate expenditure on intermediate goods production becomes
\[ X = (1 - \beta) \bar{q}_t l. \]  
(49)

Meanwhile, the aggregate creative destruction rate can be revised as
\[ \phi_e = \frac{1}{1 + \lambda_e} \left[ \frac{\beta l}{y} - \frac{\lambda}{\alpha} z - \rho \right]. \]  
(50)

By combining (13), (17), (18), (19), (46), (49), and the resource constraint
\[ c + X + C_M + C_E + C_F = Y, \]  
we obtain
\[ \frac{c}{Y} = 2\beta - \beta^2 - \frac{1 - \beta}{l} \left[ \frac{1}{\alpha} \delta_m z^\alpha + \phi_e \left( \frac{1}{\gamma} \delta z^\gamma + \psi \right) \right]. \]  
(51)

Since \( c/Y = \bar{Y}/Y \) along the BGP, \( c/Y \) is a constant value in equilibrium. Therefore, we can solve the three endogenous variables \( \{l, c/Y, \phi_e\} \) using (48), (50) and (51). As for social welfare, the steady-state welfare function is revised as
\[ U = \frac{1}{\rho} \left[ \ln c_0 + \frac{g^*}{\rho} + \theta \ln (1-l^*) \right], \]  
(52)

where
\[ c_0 = \frac{2\beta - \beta^2}{1 - \beta} l^* - \frac{1}{\alpha} \delta_m z^\alpha - m^* \left[ \frac{1}{\gamma} \delta z^\gamma + \psi \right]. \]  
(53)

---

32 It is useful to note that the functions of \( v, \xi, \) and \( z_e \) are the same as in the benchmark. Accordingly, (48) and (50) indicate that when the labor input is elastic and consumption is subject to a CIA constraint, monetary policy will change the labor supply and hence affect both R&D and growth.
In the rest of this subsection, we provide a numerical analysis of the growth and welfare effects of monetary policy in this general case. In particular, we focus on the implications of different degrees of the CIA constraint on consumption. We set the parameter \( \theta \) to 2.7, which implies that the labor supply is approximately equal to the standard value of 1/3. To obtain the same initial growth rate as in the benchmark, we recalibrate \( \beta \) to a value of 0.64.\(^{33}\)

Fig. 3 depicts how the equilibrium labor supply \( l' \) will react in response to an increase in the nominal interest rate \( i \). Three main findings emerge from our numerical analysis. First, in the case of an elastic labor supply without the CIA constraint on consumption (i.e., \( \zeta_x = 0 \)), \( l' \) is decreasing in \( i \). By (48), when \( \zeta_x = 0 \), labor supply is given by \( l' = \beta/\left( \theta(c/Y)^* + \beta \right) \). Intuitively, in the presence of CIA constraints on R&D and entry, increasing the nominal interest rate reallocates resources from innovation to consumption. As a result, the equilibrium consumption-output ratio \( (c/Y)^* \) increases with \( i \), which in turn implies that \( l' \) decreases with \( i \). Second, in the presence of a CIA constraint on consumption (i.e., \( \zeta_x > 0 \)), a higher \( i \) decreases the equilibrium labor supply \( l' \). The intuition is also obvious. When consumption is subject to a CIA constraint, an increase in \( i \) increases the cost of consumption relative to leisure. Therefore, increasing the nominal interest rate \( i \) reduces the labor supply through the consumption-leisure tradeoff.\(^{34}\) Third, the negative effect of a higher \( i \) on the equilibrium labor supply \( l' \) becomes stronger as \( \zeta_x \) increases. The reason for this is that an increase in \( \zeta_x \) further

\[33\] When the production parameter is equal to 0.65, the markup ratio of the intermediate goods firms is about 3. A strand of the literature has quantified the markup ratio of firms, and the estimates range widely. For example, Hall (1990) estimates a markup ratio ranging from 1.5 to 4.

\[34\] Chu and Ji (2016) explore the consumption-leisure tradeoff in a monetary growth model with in-house R&D and variety-expanding R&D by entrants.
increases the cost of consumption, which in turn implies a stronger consumption-leisure tradeoff.

We now investigate the growth effect of monetary policy when consumption is subject to a CIA constraint. Similarly, the top panels of Fig. 4 show that the incumbents’ contribution is increasing in \( i \), while the entrants’ contribution is decreasing in \( i \). It is worth noting that, as illustrated in the top right panel of Fig. 4, the positive effect of raising \( i \) on the incumbents’ contribution is not affected by the extent of the CIA constraint on consumption \( \xi \), and has exactly the same pattern as the top right panel of Fig. 1. The intuition can be explained as follows. The entry of new firms determines the marginal value of quality \( v^* \) and the free entry condition (25) is not influenced by the CIA constraint on consumption. By (22), as long as the unique equilibrium \( v^* \) does not change, incumbents will not change their R&D decision. Furthermore, (50) implies that the decrease in \( i^* \) generates an additional negative effect on the aggregate creative destruction. As a result, in this case, the negative effect of raising \( i \) on the entrants’ contribution becomes stronger than the benchmark. For instance, if \( \xi = 0.3 \), when \( i \) rises from 0 to 0.3, the entrants’ contribution falls by about 0.3%, which is greater than the reduction in the entrants’ contribution presented in the top right panel of Fig. 1. Moreover, when the household

Fig. 4. Growth and welfare effects of monetary policy: CIA on consumption.
faces a greater CIA constraint on consumption, the negative effect of raising \( i \) on the entrants’ contribution to growth becomes more significant, as illustrated in the top right panel of Fig. 4. Therefore, in this case, the growth effect of monetary policy remains negative and becomes more significant than in Fig. 1. In addition, the bottom left panel of Fig. 4 indicates that the negative effect of increasing \( i \) on economic growth becomes stronger as \( \xi_c \) increases.

Finally, the bottom right panel of Fig. 4 shows that under our parameter values, the welfare effect of an increase in the nominal interest rate \( i \) is always positive, and a higher \( \xi_c \) will enhance this positive effect. Intuitively, although a higher \( \xi_c \) leads to a more significant negative effect of increasing \( i \) on creative destruction and growth, it reinforces the positive resource reallocation effect (the higher \( i \) reallocates resources from entrants’ R&D to consumption). Moreover, the decrease in the equilibrium labor supply \( l^* \) implies an increase in leisure, which in turn generates an additional positive effect on welfare. Consequently, in this case, the overall welfare effect of monetary policy remains positive and becomes stronger when consumption is more cash-constrained.

5. Conclusion

This paper explores the effects of monetary policy on market structure, economic growth and social welfare in a monetary Schumpeterian growth model with an EMS and heterogeneous vertical innovations of incumbents and entrants. To incorporate money demand into this Schumpeterian growth model, we impose CIA constraints on R&D and entry. We highlight the crucial role of the EMS in determining the effects of monetary policy and focus on how monetary policy affects the interaction between existing firms’ product improvements and new firms’ creative destruction.

Our analysis shows that an increase in the nominal interest rate leads to a reduction in the aggregate creative destruction by discouraging the entry of new firms. Consequently, a higher nominal interest rate has opposite effects on the two types of quality improvements, promoting incumbents’ own-product improvements and deterring entrants’ creative destruction. The reason is that the decline in creative destruction raises the value of being an incumbent, thereby encouraging incumbents to set higher R&D rates. Accordingly, a higher nominal interest rate changes the composition of economic growth, increasing the incumbents’ contribution to growth while decreasing the entrants’ contribution to growth. In practice, in order to overcome economic depressions and stimulate growth, the monetary authorities in
many developed countries use the nominal interest rate as the monetary policy instrument and cut it to a low level. However, the present paper predicts that the overall growth effect of monetary policy depends on the balance of these two opposing forces.

Appendix A. Proof of Eq. (31).

Differentiating the free entry condition (26) with respect to time $t$ yields

$$V_e(\bar{q}) = \psi(1 + \xi_i)$$

(A.1)

We substitute (13), (26), and (A.1) into (24) and use the linear structure of the incumbent value function to obtain

$$r\psi(1 + \xi_i) - \psi(1 + \xi_i)\bar{q} = \max_{\zeta_e} \left\{ \frac{1}{\gamma} \left[ z_e [v\bar{q}(1 + \lambda_e) - \psi(1 + \xi_i)] - \frac{1}{\gamma} \delta_e \varepsilon_e \bar{q}(1 + \xi_i) \right] \right\}.$$  

(A.2)

Inserting the optimal innovation flow rate of entrants (27) into (A.2) yields

$$(r\bar{q} - \bar{q})\psi(1 + \xi_i) = \left[ \frac{v(1 + \lambda_e) - \psi(1 + \xi_i)}{[\delta_e(1 + \xi_i)]^{(\gamma-1)}} - \delta_e \left[ \frac{v(1 + \lambda_e) - \psi(1 + \xi_i)}{[\delta_e(1 + \xi_i)]^{(\gamma-1)}} \right] \right]^{\gamma/(\gamma-1)} \bar{q}(1 + \xi_i).$$

(A.3)

Dividing both sides of (A.3) by $\bar{q}$ and then using the growth rate of the average quality level (28), we obtain

$$(r - g)\psi(1 + \xi_i) = \left[ \frac{v(1 + \lambda_e) - \psi(1 + \xi_i)}{[\delta_e(1 + \xi_i)]^{(\gamma-1)}} \right]^{\gamma/(\gamma-1)}.$$  

(A.4)

Given the BGP real interest rate (30), (A.4) can be expressed as

$$\rho\psi(1 + \xi_i) = \left[ \frac{v(1 + \lambda_e) - \psi(1 + \xi_i)}{[\delta_e(1 + \xi_i)]^{(\gamma-1)}} \right]^{\gamma/(\gamma-1)}.$$  

(A.5)

To guarantee a positive entry, the inequality $v(1 + \lambda_e) > \psi(1 + \xi_i)$ needs to hold. Then, (A.5) can be rewritten as

$$\left[ \rho\psi \frac{v(1 + \lambda_e)}{[\delta_e(1 + \xi_i)]^{(\gamma-1)}} \frac{\gamma}{\gamma-1} \right]^{\gamma/(\gamma-1)} = v(1 + \lambda_e) - \psi(1 + \xi_i).$$  

(A.6)

Thus, the unique marginal value of quality $v^*$ satisfying (24) is given by

$$v^* = \frac{1}{1 + \lambda_e} \left[ \frac{\rho\psi}{\gamma-1} \right]^{(\gamma-1)/\gamma} \delta_e \left[ \frac{v(1 + \lambda_e)}{[\delta_e(1 + \xi_i)]^{(\gamma-1)}} + \psi \right](1 + \xi_i).$$  

(A.7)

(A.7) shows that $v^*$ is always stationary.

Appendix B. Proof of Eq. (32).
Given a stationary value of quality \( v^* \) and the linear value function \( V(q) = v^* q \), we immediately have \( \dot{V}(q) = 0 \). Then, (21) can be written as
\[
rv^* q = \beta q + z^* \dot{\lambda}_m q - \phi_v v^* q - \frac{1}{\alpha} \delta_m z^\alpha q (1 + \xi_i i). \tag{B.1}
\]
Dividing both sides of (B.1) by \( v^* q \) yields
\[
 r = \frac{\beta}{v^*} + z^* \dot{\lambda}_m - \phi_v - \frac{1}{\alpha} \delta_m z^\alpha (1 + \xi_i i). \tag{B.2}
\]
Substituting the balanced-growth rate (28) and (B.2) into the BGP real interest rate (30), we obtain
\[
\phi_v \lambda_v + \rho = \frac{\beta}{v^*} - \phi_v - \frac{1}{\alpha} \delta_m z^\alpha (1 + \xi_i i). \tag{B.3}
\]
(B.3) can be rewritten as
\[
\phi_v^* = \frac{1}{1 + \lambda_v} \left[ \frac{\beta}{v^*} - \frac{1}{\alpha} \delta_m z^\alpha (1 + \xi_i i) - \rho \right]. \tag{B.4}
\]
Given the innovation rate of incumbents (23), we have
\[
\frac{\delta_m z^\alpha (1 + \xi_i i)}{\alpha v^*} = \frac{1}{\alpha} \lambda_m z^* \tag{B.5}
\]
Substituting (B.6) into (B.5) yields the BGP creative destruction rate given by
\[
\phi_c^* = \frac{1}{1 + \lambda_v} \left[ \frac{\beta}{v^*} - \frac{1}{\alpha} \lambda_m z^* - \rho \right]. \tag{B.6}
\]

References


