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Auction design for the allocation of carbon emission allowances: uniform or discriminatory price?

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Auction design for the allocation of carbon emission allowances: uniform or discriminatory price?

Abstract: Only four states used auction in Phase Ⅰ (2005-2007) of the European Union Emission Trading System, of which four used a uniform-price sealed auction format. Here we discuss whether the auction should adopt a uniform-price or discriminatory-price format using an agent-based carbon allowances auction model established for the purpose. The main conclusions are as follows: (1) when carbon allowances are relatively scarce, the government should use a discriminatory-price auction; when carbon allowances are relatively abundant, the government should use a uniform-price auction. (2) Uncertainty of the generating cost reduces the ability of an auction to know bidders' private values, which will reduce the government's revenue and reduce auction efficiency. (3) Compared with the discriminatory-price auction, the uniform-price auction can prevent large bidders from obtaining excessive profits. (4) The uniform-price auction is relatively insensitive to market structure. However, a monopoly market is more likely to develop under the discriminatory-price auction format. The results of the model have some policy implications for designing carbon market mechanisms in the future.

Keywords: Agent-based model; Carbon allowances auction; Discriminatory-price auction; Uniform-price auction

1. Introduction

The Kyoto Protocol has set emission reduction targets of 5.2% of 1990 levels for six greenhouse gases for thirty-eight OECD countries and transition economies for the commitment period of 2008-1012. To help member states achieve the reduction targets at low cost, the Kyoto Protocol introduced three flexible mechanisms: (1) Emissions Trading (ET); (2) Joint Implementation (JI); and (3) Clean Development Mechanism (CDM).

In ET, how initial carbon allowances can be distributed among enterprises in a fair way to achieve the greenhouse gas emissions reduction targets is an important issue [1]. The initial allocation of carbon allowances is the basis for designing the carbon emissions trading system. Methods studied by researchers include grandfathering (giving companies permits based on historical output or emissions), auction and the combination of these two modes [2].

Carbon emission rights are essentially a kind of public goods. The free grandfathering allocation modes are neither efficient nor fair. On the other hand, the carbon auction provides a so-called "double dividend effect" [3]. Under an auction mechanism, carbon allowances are allocated to the agents who need them most, allowing exploitation of their economic value. The auction revenue can be used to finance environmental protection programs, reducing the tax burden otherwise needed for such measures. In designing the carbon auction, which factors are critical and which auction format is better are the issues of interest to us and which we address here.

This paper is structured as follows: in Section 2, we begin with an analysis of the critical factors for carbon auction, taking the European Union Emission Trading System (EU ETS) as an example; in Section 3, we establish an agent-based carbon allowance auction model (CAAM); Section 4 presents the questions we want answers to and sets the parameters for the model taking China as an example; we analyze the model's empirical results and their implications in Section 5; finally, some policy recommendations and suggestions for future research are presented in Section 6.

2. Review of existing EU ETS auctions and related literature

The EU ETS of the carbon market currently adopts a combination of grandfathering and auction. Member states are allowed to auction up to 5% of their overall carbon allowances in the first phase (2005-2007) and up to 10% in the second phase (2008-2012). The remainder is distributed for free by governments. Only three states chose to auction carbon allowances in the first phase (Hungary, Ireland and Lithuania).

[Insert Table 1]

Standard auctions are of two types: static (sealed) auction and dynamic (clock) auction. The static auction can be divided into three types according to the different clearing prices,: uniform-price auction, discriminatory-price auction and second-price auction (see Table 1).
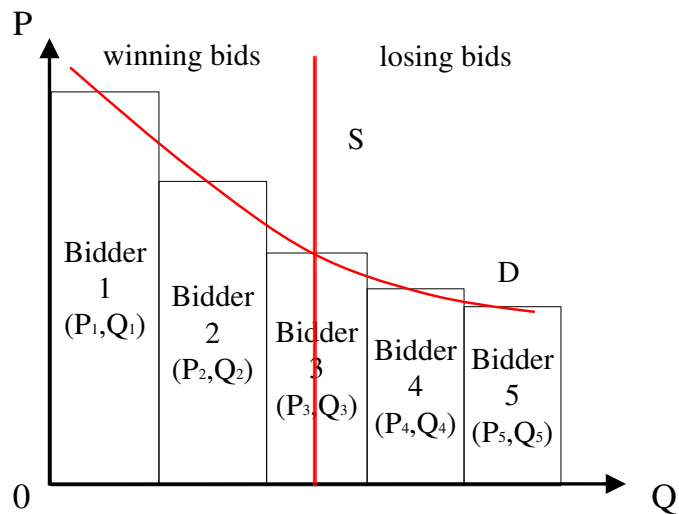
Fig. 1. Supply-demand curves of an auction

Fig. 1 is a classic diagram of supply and demand in an auction. The supply is constant and the price inelastic. We assume that there are five bidders (1-5), whose bid prices and bid quantities are P_1-P_5 and Q_1-Q_5, respectively. Bids of 1 and 2 and part of 3's bids are winning bids. Under the uniform-price auction, the market clearing price is P_3; and under the discriminatory-price auction, the clearing prices are bidders' own bid prices. Under the second-price auction, the clearing prices are the opportunity costs of the carbon allowances. In the dynamic auction, a price is given by the auctioneer in each round. The bidders adjust their bid quantities according to the price. The auctioneer raises or lowers the price in each round until the sum of bid quantities equals the quantity of auction goods.

Two types of auctions frequently considered in EU ETS auctions, are: (1) static sealed auction and (2) dynamic ascending auction. The main difference between these two types is the auction rounds. The static auction has only one round, while the dynamic auction has multiple rounds. All the states that chose the auction in the first

phase of EU ETS used the static sealed auction because of its relative simplicity over dynamic auction. Hungary, Ireland and Lithuania used the uniform-price auction format. Their market participants included not only factories needing carbon allowances to produce goods, but also financial firms. Denmark did not use an auction format directly, but used an agent to sell the carbon allowances, believing this would achieve larger revenue because of the agent's experiences [4].

[Insert Table 2]

[Insert Table 3]

Tables 2 and 3 list the reserved and auctioned carbon allowances of the four states in the first phase. Because the actual auctioned carbon allowances include those of closed plants and those reserved for new entrants re-entering the market, the actual auctioned allowances were larger.

Research on auctions applied to environmental public goods has mainly focused on the sulfur dioxide market Cason and Plott [5] did a study on the sulfur dioxide market. They found that compared with the uniform-price auction, the discriminatory-price auction in the Environmental Protection Agency (EPA) resulted in a lower market clearing price and less revenue if sulfur dioxide allowances could only be got from auction. However, the allowances auctioned were only a small part of total sulfur dioxide allowances on the actual market. Therefore, the potential negative impact in their conclusions may be exaggerated. Joskow et al. [6] argue that Cason and Plott's supposition is wrong regarding the EPA's actual auction of sulfur dioxide allowances because auction was not the only way to get allowances. Joskow

et al. [6] believe that the EPA's auction achieved the basic objective of stimulating transactions in the private market, although did not identify which auction format demonstrated better performance. Wang and Zender [7] found that uniform-price auction and discriminatory-price auction could not be strictly distinguished, at least in theory: in some cases there is price equilibrium in a discriminatory-price auction with lower expected revenue than that provided by a uniform-price auction, and vice versa in other cases.

More studies have focused on the carbon market recently. Cramton and Kerr [8] believe that an auction of carbon permits is the best way to achieve domestic carbon caps, and an auction is preferred to grandfathering, because it reduces tax distortions and provides greater incentives for innovation. Holt et al. [9] argue that carbon auction should use a uniform-price auction format because of advantages of simplicity, relatively transparency, price discovery, and so on. Klemperer [10] believes that there is no auction mechanism which can be applied to all cases. For example, while ascending auction is usually considered more transparent, its implementation is also more expensive and complicated. Mandell [11] studied the frequency of carbon auctions. He believes that high frequency auctions will result in high transaction costs and could easily lead to collusion, but could also be more conducive to cash flow management. Cook et al. [12] studied whether the British Government should sell or auction their remaining carbon quotas. They considered four scenarios and believed that the quota should be sold at market prices when there is strong liquidity in EU ETS, but that a uniform-price or ascending clock auction format may be better when

there is weak liquidity. Matthes and Neuhoff [13] conclude seven key decisions on auction design options, including the auction format, the auction frequency, the clearing price and so on. Compared with the single-round auction, although the multiple-round auction provides a price-discovery function, it also results in higher transaction costs. Because of the existence of the carbon market, the price-discovery function of the multiple-round auction is not very important. Burtraw et al. [14] tested three auctions based on an experimental economics method. They found that compared with the clock auction, the uniform-price auction and the discriminatory-price auction bring about larger revenue, whether or not there are exchanges between bidders. Koesrindartoto [15] studied whether treasury auction should be a uniform-price or a discriminatory-price format based on a multi-agent model, finding that compared with the secondary market price, the results were more sensitive to market structure, the supply-demand relations and bidders' learning behavior.

Therefore, we study a single-round auction in this paper, specifically focusing on whether carbon auctions should use a uniform-price or a discriminatory-price format.

3. Methodology: Agent-based Carbon Allowances Auction Model (CAAM)

This section presents an agent-based simplified CAAM. We assume that the auctioneer is the government and the bidders include two types of agents: gas-fired power plants and coal-fired power plants. Gas-fired power plants need a permits to generate one unit of electricity, while coal-fired power plants need b permits to

generate one unit of electricity. The private value (v_i) of an allowance to $agent_i$ is defined in equation (1):

$$v_i = \begin{cases} \dfrac{e - c_i}{a}, where\ agent\ I\ is\ a\ gas\text{-}fired\ power\ plant \\ \dfrac{e - c_i}{b}, where\ agent\ I\ is\ a\ coal\text{-}fired\ power\ plant \end{cases} \tag{1}$$

In the equation above, e is electricity price; c_i is unit generation cost for $agent_i$; and v_i is the maximum price the agent would pay for an allowance unit. Each agent gives its initial bid price which is randomly generated between zero and the private value known only to the agent and not to the other bidders. The market clearing price (cp) is then formed. For agent i, if the clearing price is greater than the private value, agent i gives up; otherwise, agent i adjusts its bid price (bid_i) according to the following three strategies:

(1) Risk-seeking: $bid_i = cp + \dfrac{3}{4} \times (v_i - cp)$

(2) Risk-neutral: $bid_i = cp + \dfrac{1}{2} \times (v_i - cp)$

(3) Risk-averse: $bid_i = cp + \dfrac{1}{4} \times (v_i - cp)$

The main learning algorithm considered for bidders in this paper is the reinforcement learning algorithm developed by Erev and Roth [16]. What makes this reinforcement learning algorithm different from others is that it aims to mimic how real humans learn. The Roth-Erev reinforcement learning algorithm includes two principles as follows:

● Successful choices in the past means similar actions are more likely adopted in the future. - Experimentation effect.

- Compared with past actions, recent actions have a greater impact on the decision-making. - Recency effect

At first, every agent assigns an equal possibility to each of its strategies, as shown in equation (2):

$$p_{ik}(1) = \frac{1}{K} \tag{2}$$

In equation (2), $p_{ik}(T)$ is the possibility of agent i choosing strategy k in the Tth round. K is the number of strategies (Here $K=3$). If we assume that $\pi(i,k,T)$ is the profit of agent i choosing strategy k at the end of the Tth round, the propensity value of agent i choosing strategy k, q_{ik}, can be calculated in equation (3):

$$q_{ik}(T+1) = (1-r)q_{ik}(T) + E(i,k,k',T,K,e) \tag{3}$$

$$E(i,k,k',T,e) = \begin{cases} \pi(i,k',T)(1-e), & k=k' \\ \pi(i,k',T)\dfrac{e}{K-1}, & k \neq k' \end{cases} \tag{4}$$

In equation (4), e and r denote the experiment parameter and the recency parameter, respectively. In the $T+1$th round, the possibility of agent i choosing strategy k, $p_{ik}(T+1)$ can be calculated as equation (5):

$$p_{ik}(T+1) = \frac{q_{ik}(T+1)}{\displaystyle\sum_{m=1}^{K} q_{im}(T+1)} \tag{5}$$

4. Empirical analysis

In this paper, we mainly want to examine the revenues of the government and bidders in different scenarios as well as the fairness issue, which can be used to weigh the pros and cons of the two auction mechanisms, uniform-price and

discriminatory-price auctions.

4.1 Problem description

(1) Comparison of two auction mechanisms under different supply-demand relations

From the analysis of the four states that used auctions in the first phase of EU ETS (2005-2007), we can see that the timing for the success of the auction is important. The carbon price fell very quickly in 2007. The main reason was a change in the supply-demand relation. Therefore, we consider a variety of supply-demand relations, represented by RCAP ($relative\ capacity$, ($\dfrac{demand}{sup\ ply}$)), as shown in Table 4.

[Insert Table 4]

(2) Comparison of two auction mechanisms under generation cost uncertainty

As the model here is a general model, we consider the impact of uncertainty in generation cost on auction results. Two scenarios are considered:

Scenario 1: There is high uncertainty in generation cost.

The generation cost of gas is randomly set between e_1 and e_2; the generation cost of coal is randomly set between e_3 and e_4.

Scenario 2: There is low uncertainty in generation cost.

The generation cost of gas is randomly set between f_1 and f_2; the generation cost of coal is randomly set between f_3 and f_4.

The relative size of the parameters characterizes the difference between the generation costs of gas and coal.

(3) Comparison of two auction mechanisms with different learning behaviors

According to Roth and Erev's experiment, setting *r* and *e* to 0.1 and 0.2, respectively, is most consistent with the real human decision-making process. By setting *e=(K-1)/K* (no learning), *e=0.2* (best-fit learning) and *e=0* (no-bias learning), we can simulate the auction under different learning behaviors.

(4) Comparison of the two auction mechanisms with different market structures

We mainly set up three different market structures as follows:

● Many small bidders - *competitive market*

● A mixture of two types of bidders - *monopolistic competition market*

● A few large bidders – *monopoly market*

The classification of bidders' types is based on their generation cost.

4.2 Parameter setting

According to the results of Cong and Wei [17], in China, the coal-fired power plants need to emit 1.3 tons of carbon dioxide for 1KKWH electricity; while the gas-fired power plants only need to emit 0.7 ton of carbon dioxide for 1KKWH electricity. The unit generation cost of coal is 51.24 $/KKWH, while the unit generation cost of gas is 117 $/KKWH. Considering the data above, we set the parameters as shown in Table 5:

[Insert Table 5]

For example, in the scenario of low uncertainty, the generation cost of gas is randomly set in [7, 8], and the generation cost of coal in [3, 4]. This roughly reflects the ratio of the unit generation cost of coal to the unit generation cost of gas in China.

5. Result analysis and discussion

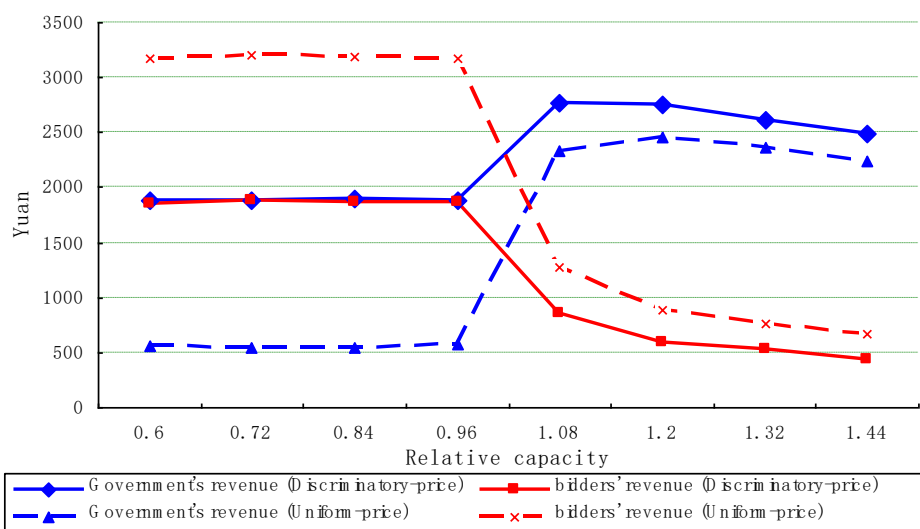## 5.1 Comparison of two auction mechanisms under different supply-demand relations

Fig. 2. Revenue of government and bidders under different market supply-demand relations

Fig. 2 shows that, under a uniform-price auction, when the carbon allowance supply is greater than demand (0.6<$RCAP$<0.96), the bidders' revenue is greater than the government's; and when the supply is less than demand (1.08<$RCAP$<1.44), the government's revenue is greater than the bidders'. Under a discriminatory-price auction, when the carbon allowance supply is greater than demand (0.6<$RCAP$<0.96), the bidders' revenue and the government's revenue are approximately equal; when the demand is greater than supply (1.08 <$RCAP$ <1.44), the government's revenue is greater than the bidders'.

When the carbon allowance is a scarce commodity, the government's revenue is greater than the bidders' revenue in the two auction mechanisms. This is because

when the market is a sellers' market, the government has an advantage. And in this case, if the government uses discriminatory-price auction, it can get more revenue, which allows maximizing of the economic value of the allowances. When the carbon allowance supply is large, the market is a buyers' market and the bidders have an advantage. In this case, discriminatory-price auction will result in less revenue for the government compared with uniform-price auction.

Therefore, in order to maximize the economic value of carbon allowances, when the allowances are relatively scarce, the government should use a discriminatory-price auction; and when the allowances are relatively abundant, the government should use a uniform-price auction.

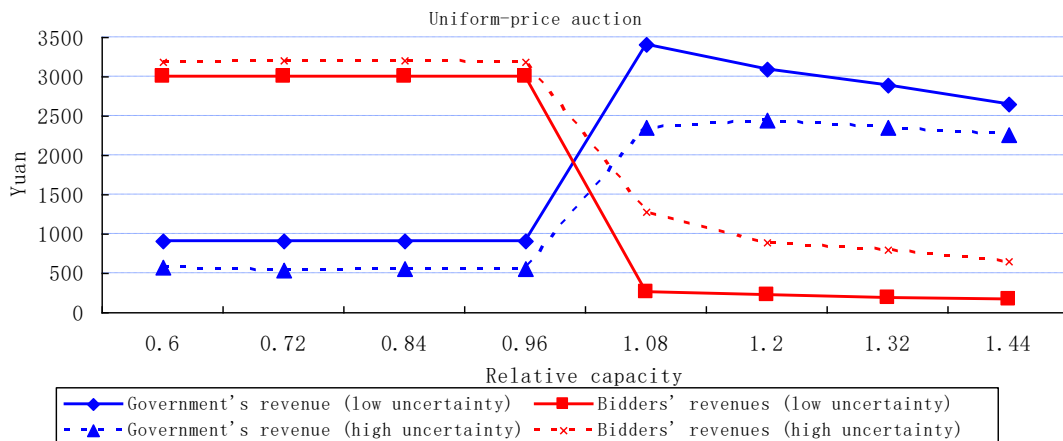## 5.2 Comparison of two auction mechanisms under generation cost uncertainty

Fig. 3. Impact of uncertainty in generation cost under uniform-price auction

Fig. 3 shows that under a uniform-price auction the government's revenue with low uncertainty is larger than that with high uncertainty; and the bidders' revenue with

low uncertainty is less than that with high uncertainty. Therefore, when there is large uncertainty in generation cost, the uniform-price auction is more beneficial to bidders than to the government.
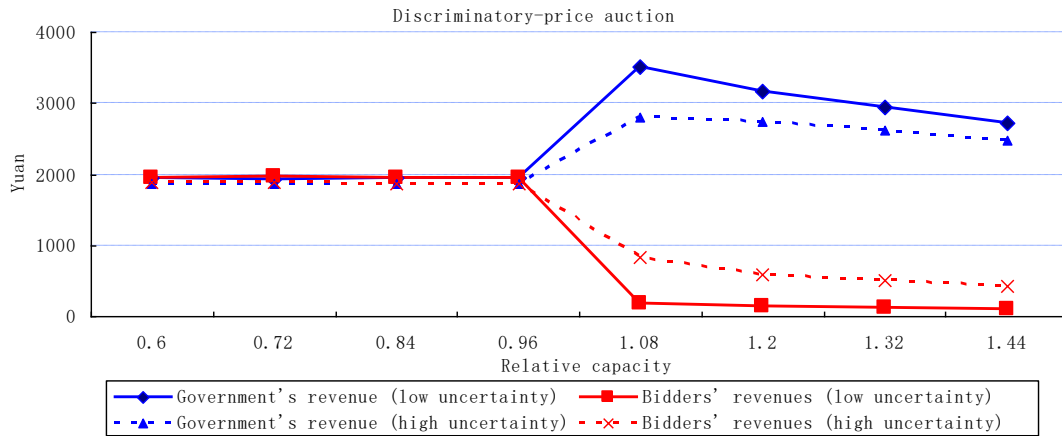

Fig. 4. Impact of uncertainty in generation cost under discriminatory-price auction

Fig. 4 shows that under a discriminatory-price auction, bidders' revenue with low uncertainty is less than that with high uncertainty; and the government's revenue with low uncertainty is larger than that with high uncertainty.

Therefore, in both the auction formats, uniform-price and discriminatory price, the uncertainties in generation cost are more beneficial to bidders than to the government.

Uncertainty in generation cost reduces the ability of an auction to know bidders' private values, which lets bidders hide their true bid more easily, reduces the government's revenue and lowers auction efficiency. At the same time, we can see from Fig. 4, that when the carbon allowance supply is greater than demand, uncertainty in generation cost has little impact on the revenues of the government and bidders under a discriminatory-price auction. When the carbon allowance supply is

less than demand, the government's revenue is greater under a discriminatory-price auction compared with a uniform-price auction.

Therefore, when there is high uncertainty in generation cost, no matter what the market supply-demand relation is, the government should use a discriminatory-price auction format.

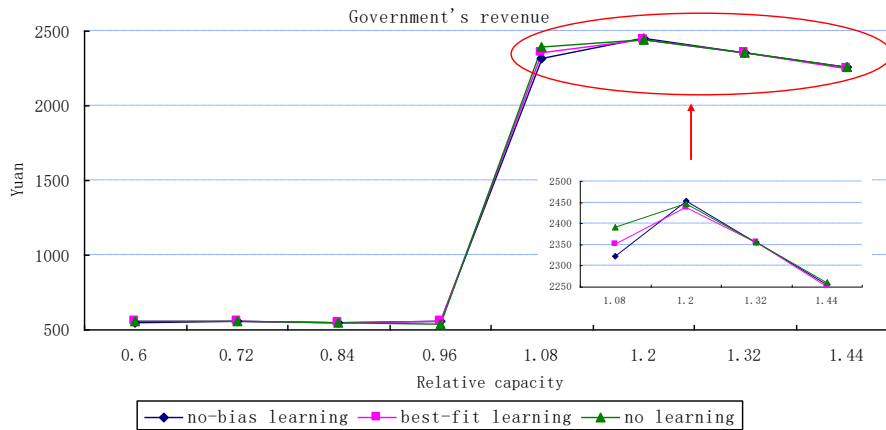## 5.3 Comparison of two auction mechanisms with different learning behaviors

Fig. 5. Government's revenue when bidders have learning ability under uniform-price auction
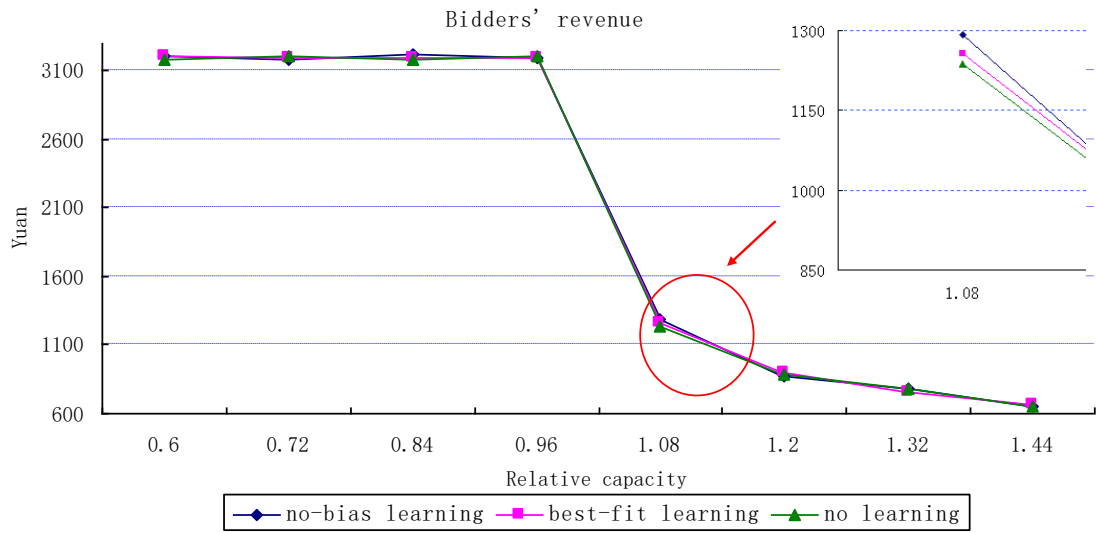
Fig. 6. Bidders' revenue when bidders have learning ability under uniform-price

auction

Figs. 5 and 6 show that under a uniform-price auction the bidders' learning

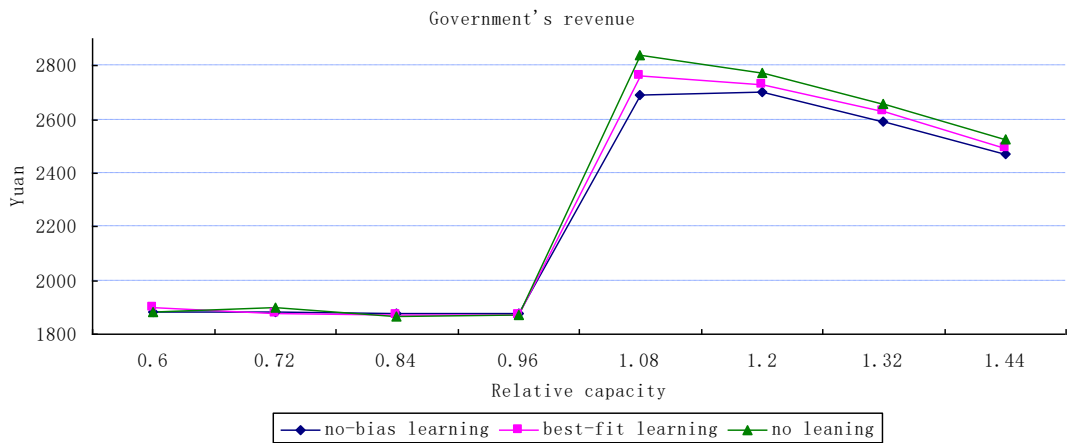behavior does not have much impact on the revenues of the government or bidders.

Fig. 7. Government's revenue when bidders have learning ability under
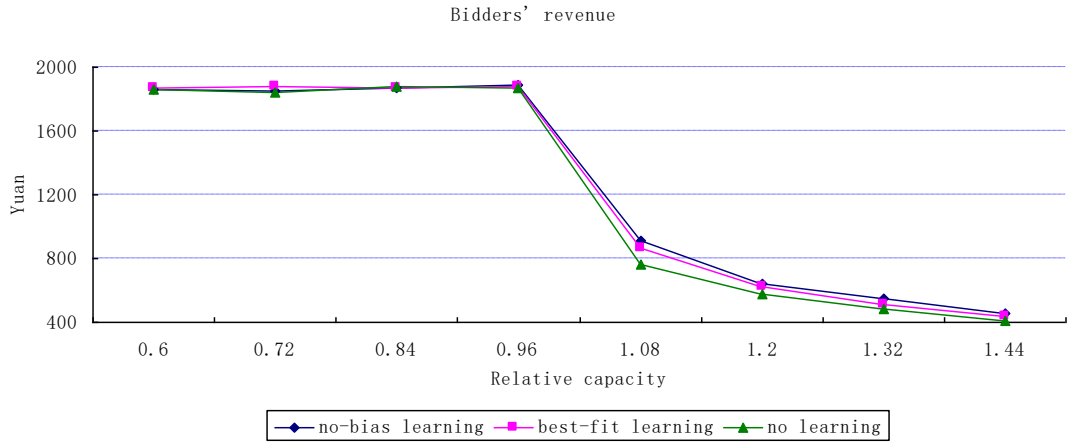
discriminatory-price auction

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Fig. 8. Bidders' revenue when bidders have learning ability under

discriminatory-price auction

Figs. 7 and 8 show that in case of discriminatory-price auction, when carbon allowances are abundant, the bidders' learning behavior has little impact. When carbon allowances are scarce, the bidders' learning behavior has large impact. When the bidders have a no-bias learning behavior, their revenue is the largest and the government's revenue is the least.

Therefore, compared with discriminatory-price auction, uniform-price auction can effectively prevent bidders from learning, making the excess return obtained by bidders due to learning behavior the least. Under a discriminatory-price auction, bidders can obtain excess return by learning. Therefore, discriminatory-price auction is relatively unfair for small bidders without learning ability compared with uniform-price auction.

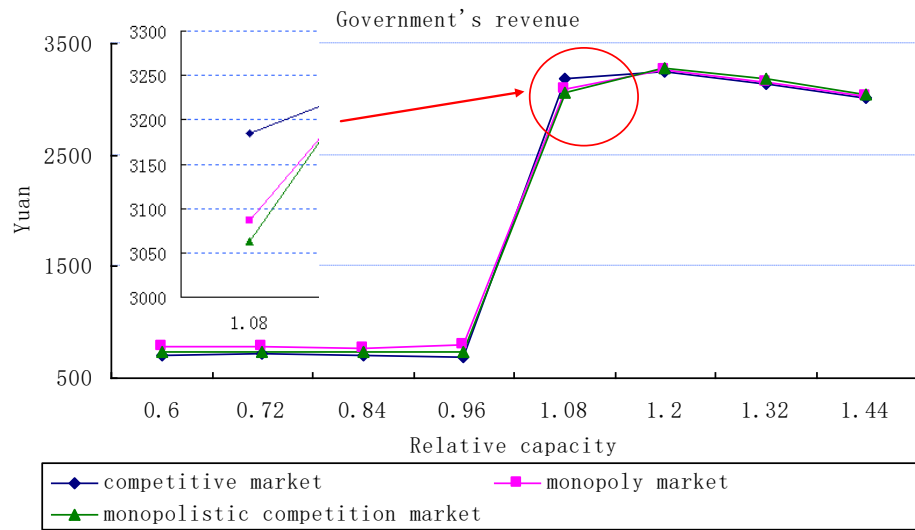## 5.4 Comparison of two auction mechanisms with different market structures

Fig. 9. Impact of different market structures on government's revenue under a

uniform-price auction
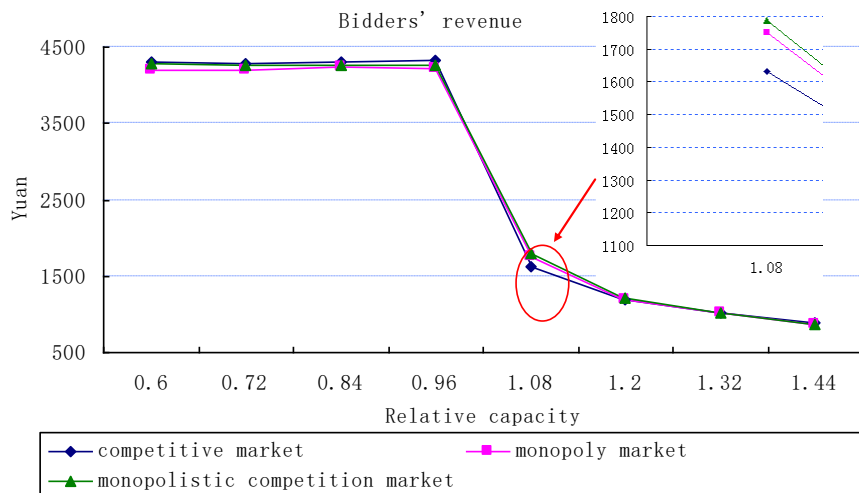

Fig.10. Impact of different market structures on bidders' revenue under uniform-price

auction

Figs. 9 and 10 show that under a uniform-price auction, when the carbon allowance supply is greater than demand, the government's revenue is greater in a

monopoly market compared with other market structures; and when supply and demand are roughly equal, the government's revenue is the largest in a competitive market and bidders' revenue is the largest in a monopoly market. However, we should also note that there is little impact of market structure under a uniform-price auction.
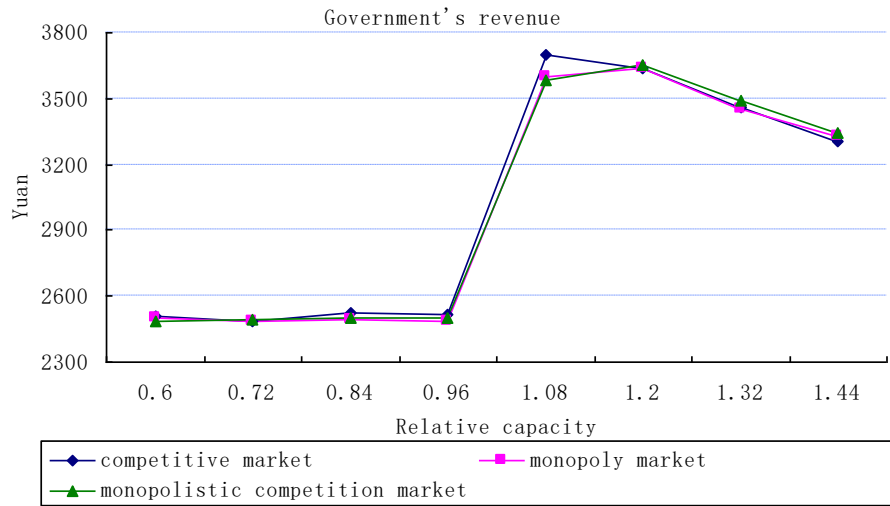

Fig. 11. Impact of different market structures on government revenue under
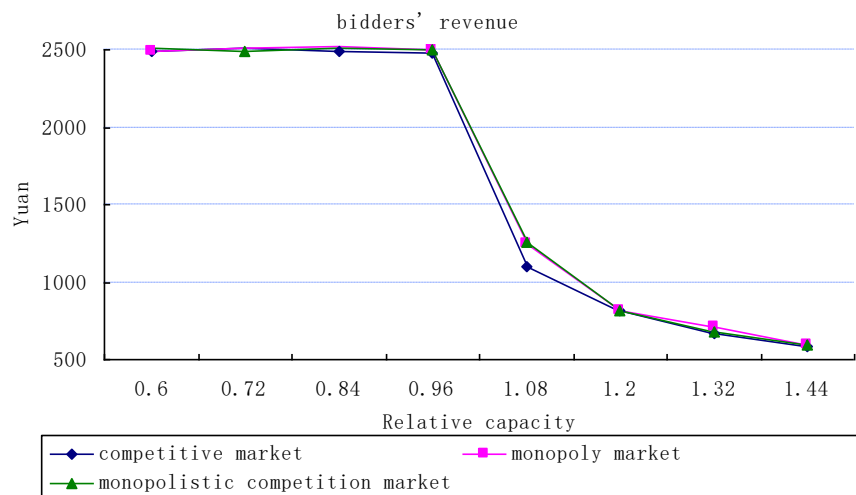
discriminatory-price auction

Fig. 12. Impact of different market structures on bidders' revenue under

discriminatory-price auction

Figs. 11 and 12 show that under a discriminatory-price auction, when the carbon

allowance supply and demand are roughly equal, the government's revenue is greater and the bidders' revenue is less in a competitive market, compared with the other two market structures. This shows that when the advantages of the government and bidders are roughly equal, the greater the number of small bidders, the better for the auction: the government obtains more revenue from the competition.

Therefore, a uniform-price auction is relatively insensitive to market structure. However, a monopoly market is more likely to develop under a discriminatory-price auction. Thus, a uniform-price auction is fairer for small bidders.

6. Main conclusions and further research

In this paper, based on the review of the EU ETS current carbon allowance auction mechanism, we analyzed whether uniform-price auction or discriminatory-price auction should be used for carbon allowance auction. We established an agent-based CAAM and performed simulation analysis with reference to China's actual situation of carbon allowances. The main conclusions are as follows:

(1) When the market is a sellers' market, the government has an advantage and would get more revenue under a discriminatory-price auction. When the market is a buyers' market, bidders have an advantage and the government would get more revenue under a uniform-price auction. Therefore, to maximize the economic value of carbon allowances, when the allowances are relatively scarce, the government should use a discriminatory-price auction format; and when the allowances are in large supply, the government should use uniform-price auction format.

(2) Uncertainty in generation cost reduces the ability of an auction to know the

bidders' private values, which would reduce the government's revenue and reduce auction efficiency. Under a discriminatory-price auction, when the carbon allowance supply is larger than demand, uncertainty has little impact on the revenues of the government and bidders. When the carbon allowance supply is less than demand, the government would obtain greater revenue under a discriminatory-price auction compared with a uniform-price auction. Therefore, when there is high uncertainty in generation cost, no matter what the market supply-demand relation is, the government should use a discriminatory-price auction format.

(3) Compared with discriminatory-price auction, uniform-price auction can prevent bidders from learning. This is because bidders obtain little excess return from learning behavior compared with discriminatory-price auction. Therefore, discriminatory-price auction is relatively unfair for small bidders without learning ability.

(4) Under a uniform-price auction, the impact of market structure is relatively small. When the advantages of the government and bidders are roughly equal, the greater the number of small bidders, the better for the auction: the government obtains more revenue from the competition. Therefore, a uniform-price auction is relatively insensitive to market structure. On the other hand, under a monopoly market is more likely to develop under a discriminatory-price auction. Thus, a uniform-price auction is fairer for small bidders.

In summary, a discriminatory-price auction is more suitable in terms of maximizing revenue for the government, but a uniform-price auction is better in terms

of fairness to bidders, especially small bidders.

The results here are based on a simplified auction market model. In reality, there may be collusion between bidders, which is the focus of our future research.

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Table 1 Standard Auction Formats

Static (sealed) auction			Dynamic (clock) auction	
Uniform-price auction (Most EU allowance auctions)	Discriminatory-price auction	Second-price sealed auction (Vickrey auction)	Ascending auction (English auction)	Descending auction (Dutch auction)

Table 2 Carbon allowances reserved for auction in the EU ETS first phase

EU Allowances (EUA)	Ireland	Hungary	Lithuania
Reserved carbon allowances	502,201	1,420,000	552,000
Proportion of reserved carbon allowances to total carbon allowances	0.75%	2.5%	1.5%

Table 3 Actual auctioned carbon allowances in the EU ETS first phase

EU Allowances (EUA)	Ireland	Hungary	Lithuania
2005	0	0	0
2006	250,000 +963,000	1,197,000	0
2007	0	1,177,500	552,000
Actual auctioned carbon allowances in the EU ETS first phase (2005-2007)	1,213,000	2,374,500	552,000
Proportion of actually auctioned carbon allowances to total carbon allowances in the EU ETS first phase	1.81%	4.18%	1.5%

Table 4 Different market supply-demand relations (*RCAP*)

RCAP	0.6	0.72	0.84	0.96	1.08	1.2	1.32	1.44

Table 5 Parameters of the carbon allowances auction model

Parameter	Value	Parameter	Value
a	1	e_4	6
b	2	f_1	7
e	12	f_2	8
e_1	5	f_3	3
e_2	10	f_4	4
e_3	2		