On the spike in hazard rates at unemployment benefit expiration: The signalling hypothesis revisited

Bruno Decreuse and Elvira Kazbakova

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Bruno Decreuse†
GREQAM, IDEP, and University of Aix-Marseilles

Elvira Kazbakova‡
GREQAM, University of Aix-Marseilles

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Abstract: We revisit the signalling hypothesis, whereby potential employers use the duration of unemployment as a signal as to the productivity of applicants. We suggest that the quality of such a signal is very low when the unemployed receive unemployment benefits: individuals have good reasons to remain unemployed. Conversely, the signal becomes much more efficient once benefits have elapsed: skilled workers should not stay unemployed in such cases. Therefore, the potential duration of unemployment benefits should drive employers' expectations and their recruitment practices. This mechanism can explain why hazards fall after benefit expiration, and why hazards respond more to the potential duration of benefits than to replacement rates.

Keywords: Worker heterogeneity; Signalling; Hazard rate; Unemployment compensation; Moral hazard

J.E.L. classification: J64; J65; D83

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†Corresponding author. GREQAM, 2, rue de la charité 13236 Marseille cedex 2, France. E-mail: decreuse@univmed.fr

‡GREQAM, 2, rue de la charité 13236 Marseille cedex 2, France. E-mail: elvira@univmed.fr

1 Introduction

This paper is a theoretical contribution to the literature on job search and unemployment insurance. We revisit the signalling hypothesis, whereby potential employers use the duration of unemployment as a signal as to the productivity of applicants. Managers are typically reluctant to interview the longterm unemployed because other managers interviewed and would have hired these workers if they had been productive[1]. We suggest that the quality of such a signal is very low when the unemployed receive unemployment benefits: individuals have good reasons to remain unemployed. Conversely, the signal becomes much more efficient once benefits have elapsed: skilled workers should not stay unemployed in such cases. Therefore, the potential duration of unemployment benefits should drive employers' expectations and their recruitment practices. This mechanism can explain why hazards fall after benefit expiration, and why hazard rates respond more to the potential duration of benefits than to replacement rates.

Why is this important? Our paper is mostly theoretical. However, it can also be used to address two empirical puzzles that standard job search theory hardly explains.

On the one hand, hazard rates increase prior to the exhaustion date, and strongly decline afterwards. Most of the unemployment compensation systems of the OECD countries deliver declining benefits with the unemployment spell. Benefits are proportional to the pre-unemployment wage for short unemployment spells, while they drop to a common standard determined by the public assistance system for longer durations. Since the late 1980s, a number of contributions have shown that the probability of leaving unemployment dramatically rises just prior to benefits lapse (see e.g. Moffitt, 1985, Meyer, 1990, and Katz and Meyer, 1990, for the US, Ham and Rhea, 1987, for Canada, Carling et al, 1996, for Sweden, Joutard and Ruggiero, 1996, and Dormont et al, 2006, for France; Van Ours and Vodopivec, 2006, for Slovenia; see also Card et al, 2007, for a different perspective). However, hazard rates fall after they peak around the exhaustion date. The rise in hazard is predicted by the standard job search theory: reservation wages go down and search efforts go up as the exhaustion date becomes closer (see for instance Mortensen, 1977, 1986, and Van den Berg, 1990). However, the theory also predicts that hazard rates should stay constant afterwards, while they typically fall.

On the other hand, estimates show that the duration of unemployment positively responds to the various components of unemployment compensation generosity. However, it is more responsive to changes in potential duration than to changes in replacement rate. Well-known studies find positive and significant effects on unemployment duration from higher benefits (see for instance Narendranathan et al, 1985, Katz and Meyer, 1990, Van den Berg, 1990, who obtain an elasticity of duration to benefit typically lower than one). However, there is a wide dispersion in estimates, and several studies do not find any effects (see Nickell, 1979, for the UK, Lynch, 1989, for the US, Hujer and Schneider, 1989, for Germany, Groot, 1990, for the Netherlands), or even a negative impact (see Jones, 1996, for Canada). Fewer studies examine the elasticity of average duration to potential duration. However, they conclude that this elasticity is indubitably positive (see Moffitt, 1985, and Katz and Meyer, 1990, for the US, Ham and Rea, 1987, for Canada, Ham et al, 1998, for the Czech and Slovak Republics). This asymmetric response

[1] Oberholzer-Gee (2008) displays direct evidence in favor of such a thesis. He conducted a survey among 766 managers in Switzerland who are responsible for hiring at administrative assistant level. Among the various reasons why they may prefer an employed worker to a 24-month nonemployed, they predominantly answer that they "prefer the candidate with a job because the unemployed applicant is probably not very productive. If she were productive, she would have been hired by another firm."

of unemployment duration to benefit level and benefit duration is nicely illustrated by Katz and Meyer (1990), also quoted in Atkinson and Micklewright (1991). They perform simulations from their estimates and show that a given UI expenditure cut achieved via reducing the length of entitlement has twice the effect on unemployment duration of one coming via a cut in benefit levels. This asymmetry is usually downplayed on the basis of its supposed inconsistency. If the potential duration of benefits plays a role, this must be so because the unemployed lose some income. As the magnitude of the loss is governed by replacement rates, replacement rates should also be part of the story[2].

In this paper, equilibrium hazard rates result from the interplay between workers' job search strategies and employers' hiring strategies. We elaborate on Lockwood (1991), who examines the argument according to which the duration of unemployment conveys a signal on worker's ability, which leads employers to discriminate against the long-term unemployed. We argue that moral hazard effects induced by unemployment compensation alter the value of the signal in a way that is consistent with the two empirical regularities highlighted previously.

A key aspect of our contribution relies on its ability to feature a realistic pattern of hazard rates as an equilibrium outcome of a model with worker heterogeneity, imperfect information, signalling, and moral hazard. Our model follows Lockwood (1991) with the noticeable exceptions that workers are allowed to set their search effort and there is duration-dependent unemployment compensation. There are two types of workers, good and bad, and firms are only willing to hire the good workers. At the beginning of unemployment episode, all workers are fully entitled to unemployment benefits. After a fixed interval of time, benefits fall to a lower level. Workers can set a low search effort or a high one. At the time of interview, firms have a positive probability of detecting a bad worker. This assumption has two implications. First, this ensures that exit rates are lower for the bad workers than for the good ones at given search effort. Second, this drives employers' expectations on workers' type by unemployment duration. The bottom line argument is that workers who stayed unemployed for long have probably been interviewed elsewhere, and some other manager detected something wrong with the worker.

In this environment, agents have to select unemployment duration-contingent strategies that are mutually consistent in equilibrium. Workers set the pace of search effort, while firms set their hiring policy. The model may display different equilibrium configurations. We focus on one of them that is empirically relevant. We name it a baseline equilibrium[3]. A baseline equilibrium features three properties. (i) Bad workers always choose a low search effort. Unlike good workers, bad workers may be rejected by employers on the basis of the signal they send while interviewed. The marginal return to high search effort is so low that they decide not to seek jobs with a high intensity. (ii) Good agents start seeking jobs with a low effort, then set a high effort prior to the potential duration of benefits. The opportunity cost of high search effort goes down with unemployment duration. This leads the good workers to set a high search effort as the exhaustion date becomes closer. (iii) Firms set a larger than the exhaustion date duration above which applicants always get rejected. The proportion of good workers among the applicants rapidly falls with duration around the exhaustion date. Firms adjust their beliefs accordingly and do not hire the long-term unemployed. We provide the set of necessary and sufficient conditions that leads to the existence and uniqueness of such a baseline equilibrium.

[2] For instance, the entitlement effect put forward by Mortensen (1977) can explain why benefit levels may have a negative impact on average duration. However, the potential duration of benefits should also have a negative effect in such a case.

[3] We discuss alternative equilibrium configurations in subsection 4.1.

Then, we analyze how hazard rates respond to changes in the institutional environment. All the different components of unemployment insurance (potential duration and benefit levels before and after the exhaustion date) originate moral hazard effects and as such weaken the signal conveyed by unemployment duration. Therefore, both the duration at which good workers start searching for a job with a high effort, and the cut-off duration above which employers start rejecting applicants, are increasing in unemployment insurance generosity. However, the magnitude of the impact of each component depends on one key parameter: the return to search of a high effort. When this parameter is large, good agents wait for a long time before setting a high search investment. In the non-frictional case where this parameter tends to infinity, individuals set a high effort once they have reached the exhaustion date and immediately get a job offer. Firms rationally expect they will only meet bad agents once the exhaustion date has elapsed and systematically reject all such applicants. More generally, the higher the return to search of a high effort, the closer the cutoff duration to the exhaustion date. Interestingly, benefit levels before and after the exhaustion date only marginally affect this statement. This may explain why hazard rates respond substantially to the potential duration of benefits and not that much to benefit levels.

Very few papers explain the decline in hazard after the exhaustion date. Two types of arguments have been put forward. First, seeking a job may require reducing the leisure time. Mortensen (1977) shows that if leisure and consumption are substitutes, then a fall in benefits raises the opportunity cost of searching and hazard rates go down. Second, benefits can be used to improve job search efficiency. This argument is due to Tannery (1983). In such a case, the opportunity cost of seeking a job is the marginal utility of consumption. When benefits go down, the marginal utility of consumption goes up and search spendings are reduced (see Ben-Horim and Zuckerman, 1988, and Decreuse, 2002).

Our paper is related to contributions that emphasize the role of employers' beliefs and hiring strategies to explain duration dependence in hazard rates. In Blanchard and Diamond (1994), employers can meet several applicants at a time and marginally prefer workers with a short duration. Resulting hazard rates display negative duration dependence. In Coles and Masters (2000), skills depreciate during the unemployment episode. Owing to recruitment costs, employers set a cut-off duration very similar to Lockwood's and ours above which employers reject all applications. Our paper goes a step forward by highlighting the interaction between job search and recruitment strategies. It also argues that the design of unemployment compensation is a key variable affecting the outcome of such an interaction.

The rest of the paper is organized as follows. Section 2 introduces our model and examines individual strategies. Section 3 is devoted to the analysis of equilibrium. Section 4 discusses some empirical implications. Section 5 concludes.

All proofs are gathered in the Appendix.

2 The model

This section introduces the main assumptions of the model, considers the microeconomic choices made by individuals and firms, and characterizes the composition of unemployment by workers' type and duration.

2.1 Environment

We depart from Lockwood (1991) in three ways. First, there are unemployment benefits whose potential duration is finite. Second, workers make search efforts. Third, the matching technology does not depend on the number of vacancies.[4]

We are interested in the steady-state of a continuous time economy populated by a continuum of firms and workers. All agents have the same felicity function v which positively depends on consumption. They discount time at rate $\rho > 0$.

At each instant, a new cohort of individuals of total size $n > 0$ enters unemployment. This cohort is composed of $\pi_0 n$ good individuals, and $(1 - \pi_0)\, n$ bad individuals, $\pi_0 \in (0, 1)$. All these people are initially entitled to unemployment benefits. Workers only differ in their productivity, with $y_g > y_b$. Workers can either be employed, unemployed, or non-participant. To ensure the existence of a steady-state number of unemployed, we assume that agents die/retire at rate n. When unemployed, agents have to seek a job, which means choosing a search effort e. There are two levels of effort: either effort is high and $e = h > 0$, or effort is low and $e = 0$. The cost of effort is ce, $c > 0$. This implies that the cost of a low effort is normalized to zero. The probability of contacting a vacant position in the time interval dt is $m\,(1 + e)\, dt$.

Each firm is endowed with a single job slot, which can either be filled or vacant. Firms endowed with a vacant position must incur the cost $\gamma > 0$. Filled jobs produce either y_g or y_b, depending on the worker's type. There is a single wage $w \in (y_b, y_g)$, which is set exogenously. We discuss alternative assumptions on the wage setting in subsection 4.2. As a result of this wage, firms do not want to hire bad workers who generate negative profits. However, the worker's type is imperfectly observable. Firms receive a private signal on the worker's type at the time of interview. If the worker is good, the signal is good with probability $\phi_g = 1$. If the worker is bad, the signal is good with probability $\phi_b = \phi \in (0, 1)$. It is bad with the complementary probability[5] $1 - \phi$. Once a worker has found a job, he/she leaves the search market forever and enjoys the utility level $W = v\,(w)\,/r$.

The fact that employed workers leave the search market forever deserves further comments. We here follow Lockwood who argues that accounting for job loss would complicate the model in an unexpected way: firms would condition their hiring strategies on cumulative durations of unemployment spells. Although there is some empirical support for this phenomenon, it can be neglected in the first place to shed light on the mechanisms that are specific to this paper.[6]

The flow number of matches depends only on the number and efforts of job-seekers. Let u denote the mass-number of active unemployed, and let $\overline{e}$ denote their average search effort. The total number of matches is $um\,(1 + \overline{e})$. Matching is random, which means that meetings are equiprobably distributed

[4] The latter assumption allows us to neglect the labor demand side of the model, while focusing on the novel aspects that we stress.

[5] This is a reduced form of Lockwood, who assumes that firms can choose whether to test workers prior to hiring them. The parameter ϕ_i is then the probability of passing the test. Good workers are always successful and $\phi_g = 1$, while bad workers may fail and $\phi_b \in (0, 1)$.

[6] One may wonder where workers entitled to unemployment benefits come from in a world without job destruction. We implicitly focus on a particular segment of the job market that is mostly reserved to experienced workers who already spent some time in their first job. The new cohort of workers is composed of those workers who lost or quit their first jobs. Alternative assumptions may be considered to preserve the general meaning of the model. They would come at the cost of losing its simplicity.

between the two sides of the markets, i.e. good and bad workers have the same probability of contacting a vacant job. The contact rate for a vacant position is m/θ, where $\theta = v/[u(1+\overline{e})]$ is the labor market tightness.

The unemployment compensation scheme runs as follows. Unemployed workers receive unemployment benefits $b(s)$ contingent on unemployment duration s. Let T be the potential duration of unemployment benefits. We have:

$$b(s) = \begin{cases} b_{\max} > 0 \text{ if } s \leq T \\ b_{\min} < b_{\max} \text{ else} \end{cases} \tag{1}$$

To ensure that agents do not refuse job offers, $b_{\max} < w$.

2.2 Job-seeker behavior

In this sub-section, we examine the job-seeking behavior of unemployed individuals at given firms' hiring policy.

Let $U_i(s)$ denote the value function of a type$-i$ unemployed whose unemployment duration is s. We have

$$rU_i(s) = \max_{e=h,0} \{v(b(s)) + m(1+e)\phi_i(s)[W - U_i(s)] - ce + U_i'(s)\} \tag{$*$}$$

where $r = \rho + n$ is the effective discount rate, and $\phi_i(s) \in [0,1]$ is the probability that the worker becomes hired once he contacted a vacancy. The reason why this probability is denoted $\phi_i(s)$ will be made clear below. The hazard rate has three components: job availability, summarized by parameter m, worker' search efforts, captured by $(1+e)$, and firms' hiring policy $\phi_i(s)$.

Workers' search behavior depends on the probability of getting the job once interviewed. In the remaining, we solve the optimization problem $(*)$ for a particular hiring policy that will be the equilibrium one. This policy is defined by Assumption A1.

Assumption A1 *Firms' hiring policy is given by*

$$\phi_i(s) = \begin{cases} \phi_i \text{ if } s \leq \Delta, \text{ with } \Delta > T \\ 0 \text{ else} \end{cases}$$

According to this policy, firms reject applicants whenever they receive a bad signal during the interview. This event occurs with probability 0 when the worker is good, and with probability $1-\phi$ when the worker is bad. In addition, employers discriminate against the long-term unemployed: employers do not hire the workers who remain unemployed more time than the cut-off duration Δ. We restrict our attention to the empirically plausible case where $\Delta \geq T$. In what follows, we will refer to this particular configuration as the baseline equilibrium configuration. We will provide additional restrictions later to ensure that such an equilibrium exists.

The optimization problem $(*)$ can be written as follows

$$\begin{aligned} rU_i(s) &= \max_{e=h,0} \{v(b(s)) + m(1+e)\phi_i[W - U_i(s)] - ce + U_i'(s)\} && (2) \\ rU_i(\Delta) &= v(b_{\min}) && (3) \end{aligned}$$

The resulting value function $U_i(s)$ is strictly decreasing for all $s \in [0, \Delta]$.

Let $e_i(s)$ denote the optimal trajectory of effort. This trajectory satisfies $e_i(s) = h$ if and only if

$$\phi_i m \left[W - U_i(s)\right] \geq c \tag{4}$$

This implies that either $e_i(s) = 0$ for all $s \geq 0$, or there exists a unique duration $\sigma_i \in [0, \Delta]$ such that $e_i(s) = h$ iff $s \geq \sigma_i$.

The problem must be solved backward. In the remaining, $a_i = \phi_i m(1 + h)$.

Step 1. If $\phi_i m\left[v(w) - v(b_{\min})\right] < rc$, then $e_i(s) = 0$ for all $s \geq 0$. If not, go to Step 2.

Step 2. Solve the following Cauchy problem for all $s \leq \Delta$

$$\begin{aligned} rx_i^1(s) &= v(b_{\min}) + a_i\left[W - x_i^1(s)\right] - ch + x_i^{1\prime}(s) && (5)\\ rx_i^1(\Delta) &= v(b_{\min}) && (6)\end{aligned}$$

This yields

$$\begin{aligned} x_i^1(s) &= \frac{v(b_{\min}) + a_i W - ch}{r + a_i}\left[1 - e^{-(r+a_i)(\Delta - s)}\right] \\ &\quad + \frac{v(b_{\min})}{r} e^{-(r+a_i)(\Delta - s)}\end{aligned}$$

If $\phi_i m\left[W - x_i^1(T)\right] < c$, then σ_i is such that $\phi_i m\left[W - x_i^1(\sigma_i)\right] = c$. If not, go to Step 3.

Step 3. Solve the following Cauchy problem for all $s \leq T$

$$\begin{aligned} rx_i^2(s) &= v(b_{\max}) + a_i\left[W - x_i^2(s)\right] - ch + x_i^{2\prime}(s) && (7)\\ x_i^2(T) &= x_i^1(T) && (8)\end{aligned}$$

This yields

$$\begin{aligned} x_i^2(s) &= \frac{v(b_{\max}) + a_i W - ch}{r + a_i}\left[1 - e^{-(r+a_i)(T - s)}\right] \\ &\quad + x_i^1(T) e^{-(r+a_i)(T - s)}\end{aligned}$$

If $\phi_i m\left[W - x_i^2(0)\right] < c$, then σ_i is such that $\phi_i m\left[W - x_i^2(\sigma_i)\right] = c$. If not, $e_i(s) = h$ for all $s \leq \Delta$.

Note that $e_b(\Delta) = h$ implies that $e_h(\Delta) = h$. This property allows us to focus on the configuration where $e_g(\Delta) = h$ and $e_b(\Delta) = 0$.

Assumption A2 $\phi m\left[v(w) - v(b_{\min})\right] < rc$

Assumption A3 $m\left[W - x_g^2(0)\right] < c < m\left[W - x_g^1(T)\right]$

The following Proposition summarizes our results.

Proposition 1 Job-seekers' efforts

Let

$$\sigma = T + \ln\left[\frac{v(b_{\max}) + c\frac{r+m}{m} - v(w)}{v(b_{\max}) - v(b_{\min}) + \left(m(1+h)\frac{v(w) - v(b_{\min})}{r} - ch\right)e^{-(r+m(1+h))(\Delta - T)}}\right]^{\frac{1}{r+m(1+h)}} \tag{9}$$

Under Assumptions A1 *to* A3,

(i) Bad workers set $e_b(s) = 0$ *for all* $s \geq 0$;

(ii) Good workers set $e_g(s) = h$ *if* $s \in [\sigma, \Delta]$ *and* $e_g(s) = 0$ *else*

Assumption A2 ensures that bad workers always set the low level of effort. Assumptions A3 guarantees that good workers set the high level of effort before they have reached the potential duration of benefits T, but after some time spent in unemployment – that is $\sigma \in (0, T)$. Note that Assumptions A1 to A3 are necessary and sufficient conditions.

Figure 1 depicts the resulting patterns of hazard rates.

[Insert Figure 1]

The hazard rate of good workers features the typical spike prior to losing benefit entitlement. The spike in hazard lasts until firms discriminate against the long-term unemployed. At the same time, the hazard rate of bad workers is flat throughout the spell of unemployment.

We focus on this particular configuration for simplicity. We discuss alternative equilibrium configurations and some empirical implications in Section 4.

2.3 The composition of unemployment

The previous subsection carefully examines job-seekers' behavior and resulting hazards. In this Subsection, we analyze the implications of such hazard rates on the distribution of unemployment spells, the distribution of type by unemployment duration, and the distribution of contact by type and unemployment duration.

Let $u_i(s,t)$ denote the size of the cohort of type-i unemployed whose unemployment duration is s as of time t. It evolves according to the following partial differential equation:

$$\frac{\partial u_i(s,t)}{\partial s} + \frac{\partial u_i(s,t)}{\partial t} = -\left[\phi_i(s)\, m\,(1+e_i(s)) + n\right] u_i(s,t) \tag{10}$$

$$u_i(0,t) = n\pi_i(0) \tag{11}$$

The total size of the cohort of duration s unemployed is $u(s,t) = u_g(s,t) + u_b(s,t)$. Finally, the number of job-seekers is $U(t) = \int_0^\infty u(s,t)\, ds$. Given that we only focus on a steady-state, $\partial u_i(s,t)/\partial t = 0$ and the dependence vis-à-vis time t will be neglected.

Let $\psi(s)$ denote the pdf of the distribution of unemployment duration, while $\pi_i(s)$ denotes the proportion of type-i unemployed conditional on duration s. By construction

$$\psi(s) = \frac{u(s)}{U} \tag{12}$$

$$\pi_i(s) = \frac{u_i(s)}{u_g(s) + u_b(s)} \tag{13}$$

Finally, let $p_i(s)$ denote the probability of contacting a type-i worker conditional on contacting a worker whose unemployment duration is s. Random matching implies that

$$p_i(s) = \frac{m\left[1+e_i(s)\right] u_i(s)}{m\left[1+e_g(s)\right] u_g(s) + m\left[1+e_b(s)\right] u_b(s)} \tag{14}$$

Proposition 2 Distribution of types and unemployment spells

Under Assumptions A1 *to* A3,

$$\pi_g(s) = \begin{cases} \frac{\pi_0}{\pi_0+(1-\pi_0)e^{m(1-\phi)s}} \text{ if } s<\sigma \\ \frac{\pi_0}{\pi_0+(1-\pi_0)e^{-mh\sigma+m(1+h-\phi)s}} \text{ if } s\in[\sigma,\Delta] \end{cases} \quad (15)$$

$$p_g(s) = \begin{cases} \frac{\pi_0}{\pi_0+(1-\pi_0)e^{m(1-\phi)s}} \text{ if } s<\sigma \\ \frac{(1+h)\pi_0}{(1+h)\pi_0+(1-\pi_0)e^{-mh\sigma+m(1+h-\phi)s}} \text{ if } s\in[\sigma,\Delta] \end{cases} \quad (16)$$

$$\psi(s) = \frac{\sum_{i=g,b} n\pi_i(0)\exp\left\{-\int_0^s [m\phi_i(z)(1+e_i(z))+n]\,dz\right\}}{\int_0^\infty \sum_{i=g,b} n\pi_i(0)\exp\left\{-\int_0^x [m\phi_i(z)(1+e_i(z))+n]\,dz\right\}dx} \ \forall s\in[0,\infty) \quad (17)$$

where σ, $e_g(s)$ and $e_b(s)$ are defined in Proposition 1

Figure 2 depicts the pattern of the proportion of good workers by unemployment duration.

[Insert Figure 2]

As the hazard rate of good workers is always higher than the hazard rate of bad workers, the proportion of good workers falls with the duration of unemployment. Indeed,

$$\pi_g'(s) = -m[1+e_g(s)-\phi]\,\pi_g(s)[1-\pi_g(s)] < 0 \quad (18)$$

Note that the function π_g is continuous on $[0,\triangle]$, while its derivative is not continuous in σ, when good workers start searching harder.

Figure 3 depicts the pattern of the proportion of good agents by contact among individuals who have spent the duration s in unemployment.

[Insert Figure 3]

The function p_g is strictly decreasing on each interval where it is continuous. Indeed,

$$p_g'(s) = -m[1+e_g(s)-\phi]\,p_g(s)[1-p_g(s)] < 0 \quad (19)$$

However, the function p_g jumps upwards in σ, as good workers suddenly become overrepresented among the applicants.

This discontinuity is important because the function p_g shapes employers' beliefs on the composition of applicants by duration. The resulting hiring policy must be consistent with the one that has been postulated in Assumption A1. Typically, the existence of an equilibrium will require that $\lim_{s\uparrow\sigma_-} p_g(s) > p_g(\Delta)$. The probability of contacting a good agent conditional on contacting an unemployed person of duration σ must be larger than the probability of contacting a good agent conditional on contacting an unemployed person of duration Δ. Figure 3 has been drawn assuming that this restriction holds.

Finally, note that the pdf of the unemployment duration distribution has three properties: it is continuous on the support $[0,\Delta]$, it is strictly decreasing in s, reflecting the fact that none can enter the distribution at some positive duration, and its derivative is discontinuous in $s=\sigma$, as good workers start finding jobs at a faster rate.

2.4 Firm behavior

Let $V > \Pi_b$ denote the value of a vacant position, while Π_i is the value of a job filled by a worker of type i. We have[7]

$$\rho V = -\gamma + \frac{m}{\theta}\mathbb{E}_s \max\left\{\mathbb{E}_i\left[\Pi_i - V \mid s\right], 0\right\} \tag{20}$$

$$r\Pi_i = y_i - w \tag{21}$$

Firms endowed with a vacant slot have to predict the type of workers they may meet. They do so on the basis of (i) the unemployment duration s that they observe, (ii) their expectation on the trajectory of individual search efforts, and (iii) the probability $\phi \in (0,1)$ of not detecting a bad worker at the time of interview.

Good workers do not send bad signals. Firms, therefore, can immediately detect a bad worker when they receive a bad signal. As $\Pi_b < V$, firms always reject such workers.

It follows that

$$\begin{aligned}
\mathbb{E}_i\left[\Pi_i - V \mid s\right] = {} & \Pr(\text{signal is good}) \Pr(\text{worker is good} \mid \text{signal is good}) \times (\Pi_g - V) \\
& + \Pr(\text{signal is good}) \Pr(\text{worker is bad} \mid \text{signal is good}) \times (\Pi_b - V) \\
& + \Pr(\text{signal is bad}) \times 0
\end{aligned}$$

Firms' beliefs on workers' types obey the Bayes rule. Therefore,

$$\begin{aligned}
\Pr(\text{worker is good} \mid \text{signal is good}) &= \frac{\Pr(\text{worker is good} \cap \text{signal is good})}{\Pr(\text{signal is good})} \\
&= \frac{p_g(s)}{p_g(s) + p_b(s)\phi}
\end{aligned}$$

and

$$\begin{aligned}
\Pr(\text{worker is bad} \mid \text{signal is good}) &= \frac{\Pr(\text{worker is bad} \cap \text{signal is good})}{\Pr(\text{signal is good})} \\
&= \frac{p_b(s)\phi}{p_g(s) + p_b(s)\phi}
\end{aligned}$$

For a particular firm, we have

$$\rho V = -\gamma + \frac{m}{\theta}\int_0^\infty \psi(s) \max\left\{\left[p_g(s)(\Pi_g - V) + (1 - p_g(s))\phi(\Pi_b - V)\right], 0\right\} ds \tag{22}$$

Expected profits depend on two distributions: the distribution of unemployment durations, and the distribution of workers' types conditional on contact and unemployment duration[8]. Firm's hiring policy $\widetilde{\phi}_i(s)$ follows.

[7] Equation (21) assumes that the job is destroyed whenever the worker retires. This assumption is innocuous, because the value of a vacancy V is driven to 0 in equilibrium.

[8] Note that unemployable workers participate in the search market. This results from the cost structure of search efforts: given zero cost for a low effort, agents are marginally indifferent between searching for a job or not. Assuming an ε-cost would suffice to prevent non-employable workers from searching. In such a case, one would have to consider the distribution of unemployment duration conditional on the fact that $\phi_i(s) > 0$. Working of the model would not be affected.

Proposition 3 Firms' hiring strategy

The policy function $\widetilde{\phi}_i(s)$ *is such that*

$$\widetilde{\phi}_i(s) = \begin{cases} \phi_i \text{ if } p_g(s) \geq -\phi \frac{\Pi_b - V}{\Pi_g - \phi\Pi_b - (1-\phi)V} \\ 0 \text{ else} \end{cases}$$

A duration-s type-i worker that is interviewed is hired with probability ϕ_i if the proportion of good workers among contacted workers is sufficiently large, and is not hired if not.

Finally, there is free entry of new firms on the search market. This drives the value V of vacancy to zero.

3 Equilibrium time-dependence in hazard rates

This section considers the equilibrium of the model. We proceed in three steps. First, we study the existence and uniqueness of equilibrium. Second, we make comparative exercises and study how hazard rates respond to changes in the design of unemployment compensation. Finally, we discuss the impact of the exhaustion date on mean unemployment duration.

3.1 Equilibrium

In a symmetric equilibrium, all firms have the same hiring policy $\phi_i(s)$, all workers of the same type have the same job search behavior $e_i(s)$, agents maximize their gains, and expectations are compatible with equilibrium outcomes. We focus on the particular type of equilibrium that we have highlighted so far. Given that our framework may feature other types of equilibrium, we need to differentiate this equilibrium from other types. In the sequel, we will call it the baseline equilibrium.

Definition Baseline equilibrium

A baseline equilibrium is a vector (σ, Δ, θ) *and a set of four functions* (e_g, e_b, ψ, p_g) *such that*

(i) $e_b(s) = 0$ *for all* $s \geq 0$ *and* $e_g(s) = h$ *iff* $s \in [\sigma, \Delta]$, *where* σ *is defined in Proposition 1*

(ii) ψ *and* p_g *are defined in Proposition 2*

(iii) For $i = g, b$, $f_i(s) = \widetilde{f}_i(s) = \phi_i$ *iff* $s \in [0, \Delta]$, *where* $\widetilde{f}_i(s)$ *is defined in Proposition 3*

(iv) $V = 0$

(v) $0 < \sigma < T < \Delta$

(i) states that the postulated job-seeking behavior is optimal for both types of workers. (ii) recalls that the distribution of unemployment duration and the proportion of good workers by duration are implied by individual strategies. (iii) states that the postulated hiring policy is optimal for firms. (iv) is the free-entry condition. Finally, (v) makes clear that good workers set a high search intensity before benefits have elapsed and potential employers discriminate against the long-term unemployed after the loss of benefit entitlement.

Proposition 4 Existence and uniqueness of a baseline equilibrium

(i) In a baseline equilibrium, we have

$$\sigma = T + \ln \left[\frac{v(b_{\max}) + c\frac{r+m}{m} - v(w)}{v(b_{\max}) - v(b_{\min}) + \left(m(1+h)\frac{v(w)-v(b_{\min})}{r} - ch \right) e^{-(r+m(1+h))(\Delta - T)}} \right]^{\frac{1}{r+m(1+h)}} \tag{JS}$$

$$\Delta = \frac{h}{1+h-\phi}\sigma + \frac{1}{m(1+h-\phi)} \ln \left[\frac{y_g - w}{\phi(w-y_b)} \frac{\pi_0(1+h)}{1-\pi_0} \right] \tag{HS}$$

$$\theta = \frac{m}{\gamma} \int_0^{\Delta} \psi(s) \left[p_g(s)\Pi_g + (1-p_g(s))\phi\Pi_b \right] ds \tag{FE}$$

(ii) There may exist a baseline equilibrium

(iii) If a baseline equilibrium exists, it is unique

(i) The (JS) locus results from good workers' equilibrium job search strategy. The (HS) locus results from firms' equilibrium hiring strategy. This strategy implies that $p_g(\Delta) = -\phi\Pi_b / (\Pi_g - \phi\Pi_b)$, which yields (HS). Finally, tightness results from the free-entry condition. This yields (FE). Given that we assume that job search efforts do not create congestion effects, tightness determination has no feed-back effects on individual choices. Solving the equilibrium can be reduced to finding a couple (σ, Δ) that satisfies (JS) and (HS).

(ii) We provide during the proof of Proposition 4 the set of necessary and sufficient conditions leading to the existence of a baseline equilibrium. These conditions are not particularly appealing. Indeed, we must check that (JS) and (HS) intersect at least once. We must also check that $0 < \sigma^* < T < \Delta^*$. Finally, we must check that $\lim_{s \to \sigma_-} p_g(s) \geq p_g(\Delta)$. This leads to four inequalities that define the parameter space compatible with the existence of a baseline equilibrium. Of course, we show that this parameter space is nonempty.

(iii) The (JS) locus and the (HS) are both strictly increasing. On the one hand, a longer cut-off duration raises the value of search, and good workers delay the moment at which they start searching with high intensity. On the other hand, an increase in σ raises the proportion of bad workers at all durations. In turn, this leads firms to delaying the duration above which they reject all applications. However, the slope of the (JS) curve is always lower than the slope of the (HS) curve. This establishes the uniqueness of equilibrium.

Note that there is a single equilibrium in our model, while there may be multiple equilibria in Lockwood's. This is so because Lockwood assumes that the matching rate m depends on the market tightness θ. This originates a feed-back effect from job creation to the composition of unemployment. Namely, job profitability increases with the mean productivity of the job-seekers. But, tightness raises such a mean productivity. Job profitability may increase with tightness as a result, which explains multiple equilibria. Abstracting from such feed-back effects allows us to focus on the novelty of our paper: the interaction between workers' search and employers' hiring strategies.

Figure 4 depicts the equilibrium.

[Insert Figure 4]

Proposition 4 allows us to interpret the duration-dependence in hazard rates observed in the data as an equilibrium outcome. This results from the interplay between job search and hiring strategies in an environment characterized by matching frictions, worker heterogeneity and asymmetric information on workers' types. Employers have no reasons to discriminate against the unemployed before the exhaustion date because good workers have strong reasons to stay unemployed. Things become different after the exhaustion date because good workers should manage to exit unemployment around the exhaustion date. This explains the fall in hazards after the exhaustion date: employers reject the applications of unemployed who have no reasons to stay unemployed unless they are of the bad type.

3.2 Changes in unemployment compensation scheme

In this Subsection, we examine the impacts of unemployment compensation on equilibrium hazard rates.

Proposition 5 PROPERTIES OF THE BASELINE EQUILIBRIUM

Assume that there exists a baseline equilibrium. Then,

(i) σ^* and Δ^* are strictly increasing in $b_{\min}$, $b_{\max}$ and T

(ii) as h tends to infinity, σ^* and Δ^* tend to T

(i) shows that the various components of unemployment compensation generosity originate moral hazard effects that are detrimental to search efforts. Confronted with a more generous scheme, good workers wait longer to make high search efforts and σ^* increases. However, such moral hazard effects alter the signalling value of unemployment duration. The probability of recruiting a good worker increases at all durations. In turn, employers are less reluctant to hire the long-term unemployed and the cut-off duration Δ^* also increases.

Figure 4 depicts these effects of unemployment insurance. The components of unemployment insurance only affect the (JS) locus that shifts rightward. The equilibrium moves along the (HS) locus that is positively sloped. σ^* and Δ^* increase as a result.

(ii) shows that σ^* and Δ^* tend to T in the non-frictional case where h becomes arbitrarily large. Indeed, good workers await the exhaustion date to set the high search effort. They immediately exit unemployment and no good workers remain among the cohort of unemployed. Employers expect this and do not hire the workers who have overtaken the exhaustion date in unemployment.

According to (i), all the different components of unemployment insurance originate moral hazard effects. But, this says nothing about the magnitude of the different effects. (ii) tells us that the magnitude depends on parameter h. In the non-frictional case where h tends to infinity, the potential duration of benefits governs equilibrium hazard rates, while replacement rates $b_{\max}$ and $b_{\min}$ have no impacts.

This provides a simple explanation to the fact that estimated hazard rates respond less to changes in benefit levels than to changes in potential duration. If good workers can activate a sufficiently efficient technology, they massively exit the unemployment state around the exhaustion date. This drives employers' beliefs who reject all the applicants of a cohort whose unemployment duration is larger than T.

Proposition 5 tells a general lesson. The design of unemployment insurance affects the signalling value of unemployment duration. This should be taken into account by policy makers.

3.3 From benefit expiration to mean unemployment duration

The model can explain why hazard rates mostly respond to the potential duration of benefits. However, the model does not necessarily predict that the average unemployment duration always increases with the exhaustion date. On the one hand, good workers respond to the exhaustion date by delaying high-intensity search. This tends to raise the mean duration. On the other hand, firms are less discriminating against the long-term unemployed, which benefits the bad workers. This tends to lower the mean duration.

Formally, group-i specific mean duration is

$$\overline{s}_i = \int_0^{\infty} e^{-\int_0^s (m\phi_i(z)(1+e_i(z))+n)dz} ds \tag{23}$$

In the limit case where h tends to infinity, we obtain

$$\overline{s}_g = \frac{1-e^{-(m+n)T}}{m+n} \tag{24}$$

$$\overline{s}_b = \frac{1-e^{-(\phi m+n)T}}{\phi m+n} + \frac{e^{-(\phi m+n)T}}{n} \tag{25}$$

Good workers' durations are truncated in the exhaustion date T, date at which they immediately exit the unemployment state. Their mean duration increases with T. Bad workers do not escape unemployment at the exhaustion date. Rather, they start being discriminated against afterwards. As a consequence, $d\overline{s}_b/dT < 0$, and their mean duration decreases with T.

Consider a worker who has just entered unemployment. This worker is good with probability π_0 and bad with probability $1-\pi_0$. Therefore, the expected unemployment duration for such a worker is $\overline{s} = \pi_0 \overline{s}_g + (1-\pi_0)\,\overline{s}_b$. A marginal increase in the exhaustion date leads to

$$\frac{d\overline{s}}{dT} = e^{-(m+n)T}\left[\pi_0 e^{-(1-\phi)mT} - (1-\pi_0)\right] \tag{26}$$

The mean duration of unemployment increases with the exhaustion date whenever $\pi_0 e^{-(1-\phi)mT} > (1-\pi_0)$.

The main reason why the potential duration of benefits has an ambiguous impact on mean unemployment duration is the fact that bad workers do not react to changes in unemployment compensation schemes. Unemployment insurance originates moral hazard effects for the good, and not for the bad. In subsection 4.1, we discuss another equilibrium configuration in which good and bad workers start seeking jobs with a high intensity. In this case, the mean duration increases with the exhaustion date.

4 Discussions

In this section, we discuss several aspects of our model. First, we consider two theoretical issues: the existence of alternative equilibrium configurations, and the case of endogenous wage. Then, we turn to empirical considerations: the model makes predictions on individual hazards that may be confronted to data, and we also make particular assumptions as to employers' information set that merit discussion.

4.1 Alternative equilibrium configurations

In this subsection, we explore the alternative equilibrium configurations that our model may feature. Doing so, we provide further motivations for our focus on a baseline equilibrium.

The panel of Figures 5 shows various combinations of hazard rates for good and bad agents. Each configuration corresponds to a different type of equilibrium. The baseline equilibrium is depicted by Figure 5a to make the comparison easier.

[Insert panel of Figures 5]

Figure 5b shows a Lockwood equilibrium, in which good and bad workers never seek jobs with a high intensity. This type of equilibrium may arise when $m\left[v\left(w\right)-v\left(b_{*\,\min}\right)\right]<rc$, which is not compatible with assumption A3. In such a case, good agents' marginal return to high search effort is lower than marginal cost. A Lockwood equilibrium also arises when h tends to 0. In a Lockwood equilibrium, unemployment insurance design has no effects on job search behavior, and, therefore, no impacts on employers' expectations and recruitment strategies.

Figure 5c depicts a credible set of hazard rates. Both good and bad workers start seeking jobs with high intensity before the exhaustion date. Such a pooling equilibrium may arise when assumption A2 does not hold. The threshold duration is σ_b for bad workers, while it is σ_g for good workers. Figure 5c assumes that $\sigma_g>\sigma_b$, but we may also have $\sigma_g<\sigma_b$. The striking feature is that σ_g generally differs from σ_b. In a pooling equilibrium, unemployment compensation originates moral hazard effects on all the individuals. However, given that good workers benefit from a higher hazard rate at given search effort, the main results featured by Proposition 5 should not be affected. In particular, σ_g^*, σ_b^*, and Δ^* would tend to the exhaustion date T as h would tend to infinity.

Interestingly, this equilibrium configuration may lead to non-ambiguous predictions concerning the impact of the exhaustion date on mean unemployment duration. Consider for instance the case where h tends to infinity. In such a case, the mean durations of good and bad workers are

$$\overline{s}_g = \frac{1-e^{-(m+n)T}}{m+n} \tag{27}$$

$$\overline{s}_b = \frac{1-e^{-(\phi m+n)T}}{\phi m+n} \tag{28}$$

The two durations are now increasing in T. As a result, the average mean duration increases with the exhaustion date.

We have decided not to focus on this equilibrium since it is associated with a technical difficulty that obscures the main message of the paper. When a given group of individuals start seeking jobs with a high intensity, this originates a discontinuity in the function p_g that shapes the proportion of good workers among contacted individuals of a given unemployment duration. This function plays a key role as this drives employers' expectations on workers' types. This discontinuity gives birth to a parametric restriction to make sure that the proportion p_g stays larger than the threshold implied by the zero-profit condition. With two groups of workers, the function p_g is discontinuous in σ_g (where it jumps upward) and σ_b (where it jumps downward). The former case is covered by the definition of a baseline equilibrium. The latter case would be associated to a new parametric restriction.

Figures 5d to 5f show equilibrium configurations that are not empirically credible. In Figure 5d, the function p_g decreases so rapidly that firms discriminate against the unemployed before good agents seek jobs at a high intensity. Then, firms hire the workers again, before turning down all the applicants once the final duration Δ is reached. In Figures 5e, good workers start seeking jobs after the exhaustion date

($\sigma > T$), while firms set a termination date shorter than the exhaustion date ($\Delta < T$) in Figure 5f. These equilibria imply that hazard rates do not peak at the exhaustion date.

4.2 Endogenous wage

The wage does not depend on unemployment duration. This may result from some minimum wage, or collective wage setting. However, one may discuss two alternative assumptions.

First, firms may set the wage as in Lockwood (1991). The monopsony wage would be set so as to guarantee that $W_g(s) = U_g(s)$. Firms have no reasons to choose a wage that yields a utility level larger than the utility reached by a good unemployed person. Similarly, they would not set a lower wage, because good workers would refuse the jobs. The equilibrium wage decreases over the unemployment duration from 0 to the exhaustion date T. However, this would leave no search incentive to the good workers[9]. They would never set the high search effort as a result. Meanwhile, unemployment compensation would not alter workers' search incentives.

Second, there could be wage bargaining over match surplus without possible renegotiation. One of the players' types is unknown to the other player at the time of interview. One way to deal with this issue is to consider a simple process of take-it-or-leave-it offer where the player who initiates the offer is chosen randomly. For simplicity, we consider risk-neutral individuals. Assume that a worker whose unemployment duration is s meets an employer endowed with a vacancy. Suppose that the firm makes the offer with probability $1 - \beta$. The firm won't offer less than $U_g(s)$. If the worker is good, he will refuse the offer. If the worker is bad, he will refuse the contract too. If he accepted, he would reveal that he is bad, leading the firm to reject his application. Similarly, the firm won't offer more than $U_g(s)$. The worker makes the offer with probability β. The firm obtains 0 in that case, while the worker gets the whole pie. Due to type uncertainty, the worker obtains $\overline{Y}(s) = \mathbb{E}_i[Y_i \mid s]$, with $Y_i = y_i/r$. As U_g and $\overline{Y}$ fall over time, the mean wage falls with unemployment duration. The termination date Δ is set so that $\overline{Y}(\Delta) = U_g(\Delta)$. Good workers obtain no rents at the exhaustion date. This implies that they do not seek jobs with a high intensity around the exhaustion date. This may be compatible with the fact that hazard rates peak at the exhaustion date, but this requires additional parametric restrictions. The fixed-wage assumption allows these additional difficulties to be neglected, without losing the main insights.

4.3 From theoretical to empirical hazards

The signalling argument relies on the coexistence of two subpopulations. They correspond to the divide between movers and stayers highlighted by the empirical literature on unobserved heterogeneity. In our paper as in Lockwood, the distinction between movers and stayers is an equilibrium outcome. A key feature of this approach is that ex-ante heterogeneity shapes employers' and workers' beliefs in a way that originates true duration dependence in hazard rates.

Hazard rates have three main properties in our model: (i) they are piecewise continuous, (ii) unobserved good workers benefit from a higher exit rate at all durations than unobserved bad workers, (iii) good workers' and bad workers' hazards respond differently to benefit exhaustion.

[9] There would be no more rents for bad workers. In equilibrium, $W_g(s) = W_b(s) = U_g(s) = U_b(s) = \int_s^{\infty} e^{-r(t-s)} b(t)\,dt$.

(i) is consistent with piecewise constant hazard models introduced by Lancaster (1990). In such models, the hazard is typically written as follows

$$\mu(s \mid x) = \mu_0(s)\, l(x) \tag{29}$$

where x is a vector of observed individual characteristics, and μ_0 is the baseline hazard. The duration axis is divided into M intervals with

$$\mu_0(s) = \begin{cases} \mu_1 \text{ if } s \in [0, \sigma_1] \\ \mu_2 \text{ if } s \in (\sigma_1, \sigma_2] \\ \ldots \\ \mu_M \text{ if } s \in (\sigma_{M-1}, \infty) \end{cases} \tag{30}$$

where μ_k are constant and σ_k are define points in time, $0 < \sigma_1 < \sigma_2 < \ldots < \sigma_{M-1} < \infty$.

In our model, workers have all the same observed characteristics. However, the analysis can be generalized to workers with heterogenous observed characteristics, with the assumption that search markets are segmented by observed characteristics. In the case of good workers, our model predicts three time intervals ($M = 3$), with $\sigma_1 = \sigma$, $\sigma_2 = \Delta$, $\mu_1 = m$, $\mu_2 = m(1 + h)$ and $\mu_3 = 0$.

(ii) Our model hinges on the fact that there are two groups of workers on each market segment, the good and the bad, that cannot be differentiated by employers. These proportions may or may not differ between observable groups, yet the intuition suggests that the initial proportion of good workers should increase with skill level. This means that there is unobserved heterogeneity, and the econometrician should account for it.

(iii) There are several ways to model unobserved heterogeneity to capture the mover-stayer dichotomy. However, they are not all compatible with our model. The simplest way to deal with unobserved heterogeneity is to assume that there is an individual specific component ε in hazard that is independent of both s and x. Formally,

$$\mu(s \mid x, \varepsilon) = \mu_0(s)\, l(x)\, \varepsilon \tag{31}$$

It is then usual to assume a simple functional form for the distribution of the error term, like the Gamma distribution, or, closer to our model, a discrete distribution (see Lancaster, 1979).

However, this hazard function implies that the two subpopulations must experience similar qualitative patterns in baseline hazard rates. By contrast, our model predicts distinctive qualitative patterns across the two groups of workers. In other words, the individual component ε should not be independent of s – and probably of x as well. This requirement may be too strong to be compatible with identification, yet it is an essential feature of our model.

4.4 Further evidence

Dormont et al (2006) provide another type of evidence. They estimate hazard rates by pre-unemployment earnings. They show that the hazard rate of formerly high-paid workers features a spike, while the hazard rate of formerly low-paid workers is fairly smooth around the exhaustion date. Of course, the distinction between low-paid and high-paid workers does not really fit with our model in which there is a single wage. However, this suggests that ability to respond to benefit exhaustion should be positively correlated to skill level.

In addition, our model provides a simple explanation to Dormont et al finding. Suppose that there is a separate search market for each skill, and that formerly high-paid workers compete on the high-skill segment, while formerly low-paid workers compete on the low-skill segment. Suppose also that for one reason or another, estimation techniques used by Dormont et al fail to identify the two groups of workers in each segment. In such a case, they estimate the average hazard among each group of workers. Provided that the proportion of good workers increases with skill level, our model predicts that the mean average rate and the mean peak in such a rate should be higher among the formerly higher-paid workers than among the formerly lower-paid workers.

4.5 Employers' information set

Our model hinges on the assumption that employers can observe both one's unemployment duration *and* one's potential duration of benefits.

Several other papers assume that employers observe the unemployment duration (see e.g. Blanchard and Diamond, 1994, Coles and Masters, 2000). They rely on the fact that cvs implicitly display this information, or that employers should be able to obtain it during the interview. Oberholzer-Gee (2008) offers empirical evidence consistent with this view. He sent applications to jobs advertised in a Swiss newspaper. Applicants only differed with respect to employment status (employed, nonemployed) and unemployment duration (if unemployed). He shows that the longer the spell of nonemployment, the lower is the probability that firms will invite a job applicant for an interview.

The potential duration of benefits may or may not be difficult to observe. There are two main obstacles.

First, eligibility rules may be very complicated. They can vary with former job duration or demographic characteristics. Nevertheless, employers should always be able to evaluate the applicant's situation vis-à-vis the benefit system using the information displayed by the applicant's cv (former job duration and current unemployment duration). In addition, there are certain regularities that depend on the type of advertised job that should be known by employers. For instance, a senior position should mostly attract relatively experienced workers who entered unemployment with full coverage and maximum potential duration[10] (or, similarly, workers benefiting from a longer duration because of their age as in France, Germany or Sweden). Conversely, a very junior position should attract individuals who are mostly non-entitled to unemployment benefits.

Second, the legislation may be volatile. One interesting question concerns the impact of such volatility on employers' beliefs vis-à-vis the long-term unemployed. Intuitively, risk-averse workers facing institutional uncertainty should be more prompt to exit the unemployment state. Unless labor market skills and risk aversion are too negatively correlated, this effect should be stronger for the good workers. Overall, the quality of the signal conveyed by unemployment duration should increase with institutional volatility, strengthening employers' discrimination vis-à-vis the long-term unemployed. The rigorous formalization of this argument is left for future work.

However, employers are likely to know whether one is covered by unemployment benefits or not.

[10] One may argue that signalling problems are less relevant for such workers who have a long work record. This perspective abstracts from human capital transferability problems. For instance, human capital accumulated during the previous job could either be purely specific (in such a case, the worker is bad) or fully transferable to a new job (in such a case the worker is good).

Indeed, it is in the interest of covered workers to say and prove that they are covered. An unemployed worker who is no longer covered can easily be detected in such a case.

5 Conclusion

This paper is a theoretical contribution to the literature on job search and unemployment. We revisit the signalling hypothesis, whereby potential employers use the duration of unemployment as a signal on the productivity of applicants. We suggest that the quality of such a signal is very low when the unemployed get unemployment benefits: individuals have good reasons to stay unemployed. Conversely, the signal becomes much more efficient once benefits have elapsed: skilled workers should not remain unemployed in such cases. Therefore, the potential duration of unemployment benefits should drive employers' expectations and their recruitment practices. This mechanism can explain why hazards fall after benefit expiration, and why hazards respond more to the potential duration of benefits than to replacement rates.

Our paper can be extended in two directions. First, we plan to enrich agents' decision sets to account for non-trivial search and recruitment strategies. Such a model could be calibrated on country data and used to simulate policy changes. Second, beyond its focus on hazard rates, our paper tells that the design of unemployment compensation alters the signalling value of unemployment duration. This should be taken into account by policy makers. Policy implications such as the design of optimal unemployment insurance are on our research agenda.

APPENDIX

A Proof of Proposition 1

Given Assumption A1, the resolution of the maximization problem $(*)$ is given in the main text that precedes Proposition 1.

(i) results from Step 1 of the resolution and Assumption A2.

(ii) results from Step 2, Step 3, and Assumption A3.

By solving, Assumptions A1 to A3 are not only sufficient but also necessary conditions.

B Proof of Proposition 2

Assumption A1 to A4 imply that Proposition 1 holds. Therefore,

$$e_b(s) = 0 \text{ for all } s \geq 0 \tag{32}$$

and

$$e_g(s) = \begin{cases} h \text{ if } s \in [\sigma, \Delta] \\ 0 \text{ else} \end{cases} \tag{33}$$

Solving the Cauchy problem (10)-(11) leads to

$$u_i(s) = n\pi_i(0) \exp\left\{-\int_0^s [f_i(z)\, m\,(1 + e_i(z)) + n]\, dz\right\} \tag{34}$$

One can use these different equations and the definitions of ψ, π_g and p_g given in the main text to show (i) to (iii).

C Proof of Proposition 3

The main text makes it clear that $\widetilde{\phi}_i(s) = \phi_i$ iff

$$p_g(s)(\Pi_g - V) + (1 - p_g(s))\,\phi\,(\Pi_b - V) \geq 0 \tag{35}$$

The result follows.

D Proof of Proposition 4

(i) follows from the definition of a baseline equilibrium. The (JS) locus is implied by Proposition 1. The (HS) locus results from Proposition 2, Proposition 3 and the free-entry assumption $V = 0$. From Proposition 3 and free entry, $f_i(s) = \phi_i$ iff $s \in [0, \Delta]$ implies that $p_g(\Delta) = -\phi\Pi_b/(\Pi_g - \phi\Pi_b)$. From Proposition 2, the function p_g is continuous and strictly decreasing on $(\sigma, \Delta]$. It follows that $\Delta = p_g^{-1}(-\phi\Pi_b/(\Pi_g - \phi\Pi_b))$. The computation leads to (HS). Finally, the (FE) locus results from imposing $V = 0$ in equation (22) that defines the value of a vacancy.

(ii) The definition of baseline equilibrium involves finding a positive vector (σ, Δ, θ) that solves (JS), (HS) and (FE), and satisfies

$$\phi m\,[v(w) - v(b_{\min})] < rc \tag{36}$$

$$0 < \sigma < T < \Delta \tag{37}$$

$$\lim_{s \uparrow \sigma_-} p_g(s) > -\phi \Pi_b / (\Pi_g - \phi \Pi_b) \tag{38}$$

σ and Δ are jointly determined by (JS) and (HS). Then θ follows from (FE). Therefore, the solution can be reduced to finding σ and Δ that solve (JS) and (HS) and satisfy conditions (36) to (38).

We now provide necessary and sufficient conditions for the existence of a baseline equilibrium.

Lemma 1. Let

$$\begin{aligned} A_1 &= v(b_{\max}) + c\frac{r+m}{m} - v(w) \\ A_2 &= v(b_{\max}) - v(b_{\min}) \\ A_3 &= m(1+h)\frac{v(w) - v(b_{\min})}{r} - ch \\ A_4 &= \ln\left[\frac{y_g - w}{\phi(w - y_b)}\frac{\pi_0}{1-\pi_0}(1+h)\right] \end{aligned}$$

Let also

$$\begin{aligned} a_1 &= r + m(1+h) \\ a_2 &= m(1+h-\phi) \end{aligned}$$

Consider the function $P : \mathbb{R} \to \mathbb{R}$ such that

$$P(x) = A_2 e^{-a_1 T} e^{a_1 x} + A_3 \left(e^{a_1 x}\right)^{\frac{1-\phi}{1+h-\phi}} e^{-\frac{a_1}{a_2} A_4} - A_1$$

There exists a baseline equilibrium iff

$$\begin{aligned} \phi m [v(w) - v(b_{\min})] &< rc \qquad \text{(C1)} \\ P(\max\{0, Z_1\}) &< 0 \qquad \text{(C2)} \\ P(\min\{T, Z_2\}) &> 0 \qquad \text{(C3)} \end{aligned}$$

where

$$\begin{aligned} Z_1 &= \frac{1+h-\phi}{h}(T - A_4/a_2) \\ Z_2 &= \frac{1}{m(1-\phi)}\ln\left[\frac{y_g - w}{\phi(w - y_b)}\frac{\pi_0}{1-\pi_0}\right] \end{aligned}$$

Proof. Note first that bad agents never set the high effort iff condition (C1) holds. Using (HS), one can replace Δ in (JS). After simple computations, one obtains

$$P(\sigma) = 0$$

The function P is strictly increasing, with $P(-\infty) = -A_1$ and $P(\infty) = \infty$. It follows that there is a unique σ such that $P(\sigma) = 0$. This σ and associated Δ given by (HS) is an equilibrium candidate. This candidate must satisfy constraints (36) to (38). First, $\sigma > 0$ iff $P(0) < 0$. Second, $\sigma < T$ iff $P(T) > 0$. Third, using (HS), one can see that $\Delta > T$ is equivalent to $\sigma > Z_1$. Therefore, $\Delta > T$ iff $P(Z_1) < 0$. Fourth, $\lim_{s \uparrow \sigma_-} p_g(s) > -\phi \Pi_b / (\Pi_g - \phi \Pi_b)$ is equivalent to $\sigma < p_g^{-1}(-\phi \Pi_b / (\Pi_g - \phi \Pi_b))$. In turn, this is equivalent to $P(Z_2) > 0$. Conditions C2 and C3 result from these four cases.

To conclude the proof, we show that the parameter space defined by conditions C1 to C3 is nonempty. The parameters are set as follows: $r = 5\%$, $\pi_0 = 0.45$, $m = 1.0$, $c = 0.31$; $\phi = 0.3$, $b_{\min} = 0.3$, $b_{\max} = 0.6$, $T = .5$, $w = 1.0$, $y_g = 1.2$, $y_b = 0.8$, $v(x) = 1.0x^{0.5}$. Figure 6 depicts the resulting equilibrium σ^* and Δ^* as h goes from 0 to 100.

[Insert Figure 6]

(iii) Uniqueness is a by-product of the former proof.

E Proof of Proposition 5

(i) The (HS) locus does not depend on the parameters that shape the unemployment compensation scheme. Therefore, we only need to know how $b_{\min}, b_{\max}$ and T affect the (JS) locus. This locus results from

$$m\left[W - x_i^2\left(\sigma, \Delta; b_{\min}, b_{\max}, T\right)\right] = c \tag{39}$$

The function x_i^2 is strictly increasing in $b_{\min}$, $b_{\max}$ (for $\sigma < T$) and T. It is also strictly decreasing in σ. It follows that $\partial\boldsymbol{\sigma}\left(\Delta^*, b_{\min}, b_{\max}, T\right)/\partial b_{\min} > 0$, $\partial\boldsymbol{\sigma}\left(\Delta^*, b_{\min}, b_{\max}, T\right)/\partial b_{\max} > 0$, and $\partial\boldsymbol{\sigma}\left(\Delta^*, b_{\min}, b_{\max}, T\right)/\partial T > 0$. In each case, the (JS) locus shifts rightward in Figure 4. The result follows.

(ii) As h tends to infinity, the (HS) locus tends to the 45-degree line so that $\Delta = \sigma$, while the (JS) locus tends to the vertical line $\sigma = T$. Therefore, σ^* and Δ^* tend to T.

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Not to be published – Numerical simulations

We use Matlab to simulate the model and find equilibrium values for σ^* and Δ^* as h varies from very small ($h = 0.1$) to very large ($h = 100$). The results are reported by Figure 6. The program is based on Appendix D, where we give necessary and sufficient conditions for the existence of a unique baseline equilibrium. The program finds the roots of the function $P(x)$, and checks the set of constraints (C1) to (C3). There are two curves, corresponding to σ^* and Δ^*. These two curves are plain blue whenever all the constraints are satisfied. They are dashed red whenever one of the constraints is not satisfied.

It is difficult to satisfy all the constraints for all possible values of h. In our search for an adequate parameterization, we proceeded as follows. First, set the wage w and the parameters of the benefit system: potential duration T, high replacement rate $b_{\max}$, low replacement rate $b_{\min}$. We normalize w to 1 and broadly replicate the US system with $T = 0.5$, $b_{\max} = 0.6$, and $b_{\min} = 0.3$. Second, set preference parameters: effective discount rate r, utility function v. We set $r = 5\%$, and $v(x) = x^{1/2}$. Then, set the contact rate per unit of search effort m, and the probability that a bad worker sends a good signal ϕ. We set $m = 1.0$, and $\phi = 0.3$. This implies that bad workers need on average three interviews to get hired when good workers need one. Then you can use the residual set of parameters to match the various constraints. The marginal search cost c governs the (JS) locus, while y_g and y_b the output levels, and π_0 the initial proportion of good workers, affect the (HS) locus. We normalize y_g to 1.2 and y_b to 0.8 so that the marginal productivity of good workers is 20% larger than their wage, while the marginal productivity of bad workers is 20% lower than their wage. Finally, we set c to 3.1 and π_0 to 0.45 to match constraints (C1) to (C3). Once there are only two parameters to play with, this may take some time to find the correct combination – remember that we must check all the constraints for all possible values for h, that is from the case where high search efforts are as efficient as low efforts to the Walrasian case where high search efforts instantaneously provide a job.

Here is the program that can be copied and pasted in the Matlab Editor.

```
% September 2008
% Simulations based on Appendix D
% This program computes sigma and delta as functions of h
% The main results are summarized by Figure 1
clc
clear all
close all
% Model parameters
syms z; %duration of employability; "Delta" in the model
syms g; %unemployment duration above which good workers search with high intensity; "sigma" in the model
syms s; %unemployment duration
r=0.05; %effective discount rate
pi1=0.45; %initial proportion of good workers
pi2=1-pi1; %initial proportion of bad workers
c=3.1; %marginal search cost - effort cost is C(e)=c*e
m=1.0; %matching parameter - contact rate is m(1+e)
```

```
n=0.03; %exogenous exit rate
rho=r-n;
f=0.3; %probability that a bad worker sends a good signal; phi in the model
bmin=0.3; %unemployment benefit after potential duration
bmax=0.6; %unemployment benefit before potential duration
w=1.0; %wage
y1=1.2; %output produced by good workers
y2=0.8; %output produced by bad workers
v=@(x) 1.0*(x^0.5); %utility function
W=v(w)/r; %expected utility reached by an employed worker
T=0.5; %potential duration of unemployment benefits
% Simulation
h=0.1; %high search effort
% Stage 0: Checking condition C1
if f*m*(v(w)-v(bmin))>r*c
attentionC1='warning! C1 is not checked!'
h=100
end
i=1; %counter
while h<100
% Stage 1: Computing the various constant
A1=v(bmax)+c*(r+m)/m-v(w);
A2=v(bmax)-v(bmin);
A3=m*(1+h)*(v(w)-v(bmin))/r-c*h;
A4=log(((y1-w)/(f*(w-y2)))*(pi1/pi2)*(1+h));
a1=r+m*(1+h);
a2=m*(1+h-f);
Z1=((1+h-f)/h)*(T-A4/a2);
Z2=1/(m*(1-f))*log(((y1-w)/(f*(w-y2)))*(pi1/pi2));
P=@(x) A2*exp((x-T)*a1)+A3*exp(a1*x*(1-f)/(1+h-f))*exp(-(a1/a2)*A4)-A1;
% Stage 2: Checking conditions C2 and C3
in(i)=0;
in2(i)=0; %C2
in3(i)=0; %C3
C2=max(0,Z1);
if P(C2)>0
attentionC2='warning! C2 is not checked! P(max(0,Z1)) must be lower than 0.';
in(i)=1;
in2(i)=1;
end
C3=min(T,Z2);
if P(C3)<0
```

```
attentionC3='warning! C3 is not checked! P(min(T,Z2)) must be larger than 0.';
in(i)=1;
in3(i)=1;
end
% Stage 3: Computing sigma and delta in equilibrium
options = optimset('MaxFunEvals',10000)
g=fsolve(P,1.5,options); %computing sigma
z=h/(1+h-f)*g+1/m/(1+h-f)*log((y1-w)/f/(w-y2)*pi1*(1+h)/pi2); %computing delta
Hs(i)=h; %vector that contains the various h
Gs(i)=g; %vector that contains the various equilibrium sigma
Zs(i)=z ;%vector that contains the various equilibrium delta
Vg(i)=P(Gs(i)); %ex-post checking that P(sigma)=0
h=h+2;
i=i+1;
end
% Stage 4: Organizing the results
in=logical(in);
in2=logical(in2);
in3=logical(in3);
hs=Hs(in); %h for which conditions C1, and/or C2, and/or C3 are not satisfied
zs=Zs(in); %delta
gs=Gs(in); %sigma
hs_2=Hs(in2); %h for which condition C2 is not satisfied
zs_2=Zs(in2); %delta
gs_2=Gs(in2); %sigma
hs_3=Hs(in3); %h for which condition C3 is not satisfied
zs_3=Zs(in3); %delta
gs_3=Gs(in3); %sigma
% Stage 5: Presenting the results
figure(1)
plot(Hs,Gs)%,'.')
hold on
plot(hs,gs,'w')
plot(hs,gs,'.r')
plot(Hs,Zs)%,'.')
plot(hs,zs,'w')
plot(hs,zs,'.r')
plot([Hs(1) Hs(i-1)], [T T],'g')
title('sigma and Delta as functions of h')
```

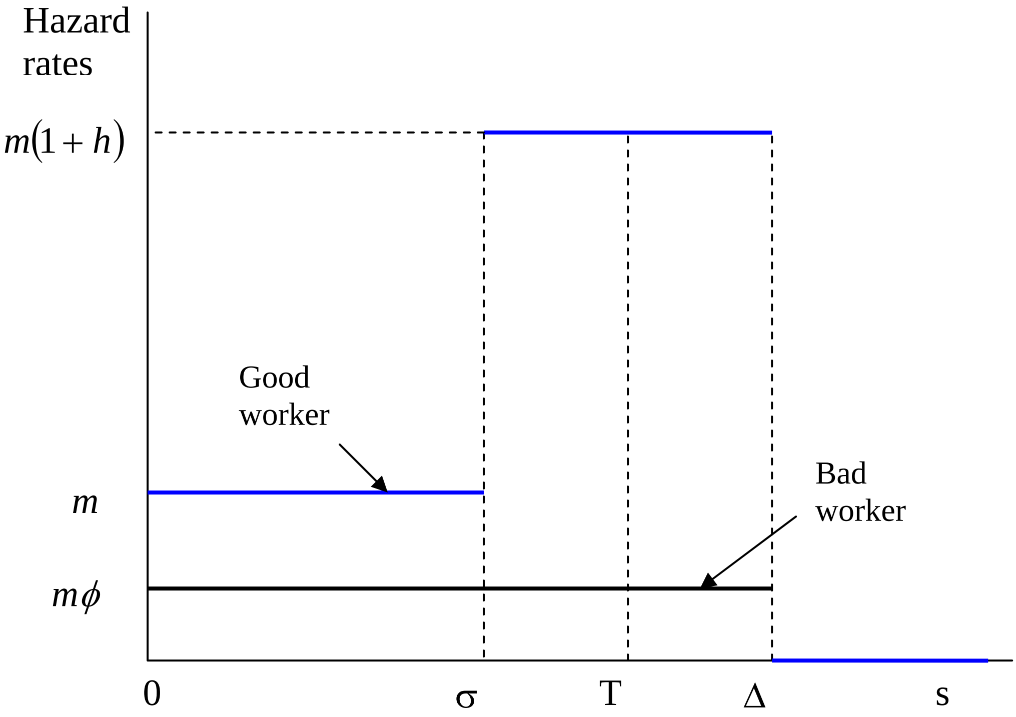

Fig.1 : Hazard rates of good and bad workers – Baseline equilibrium

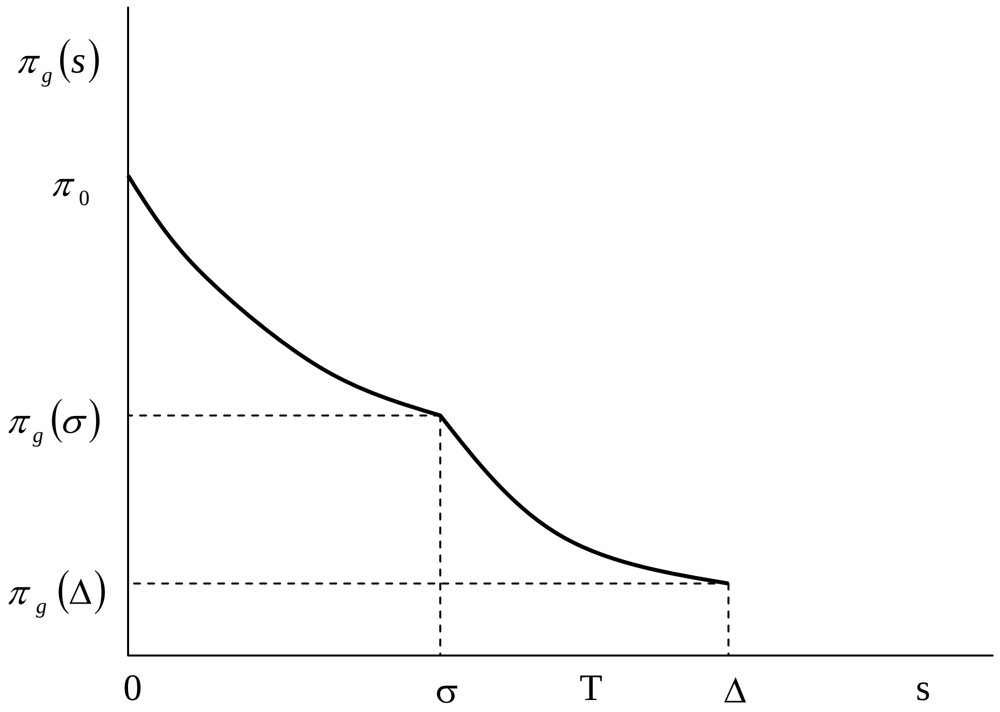

Fig.2: Proportion of good workers by unemployment duration – Baseline equilibrium

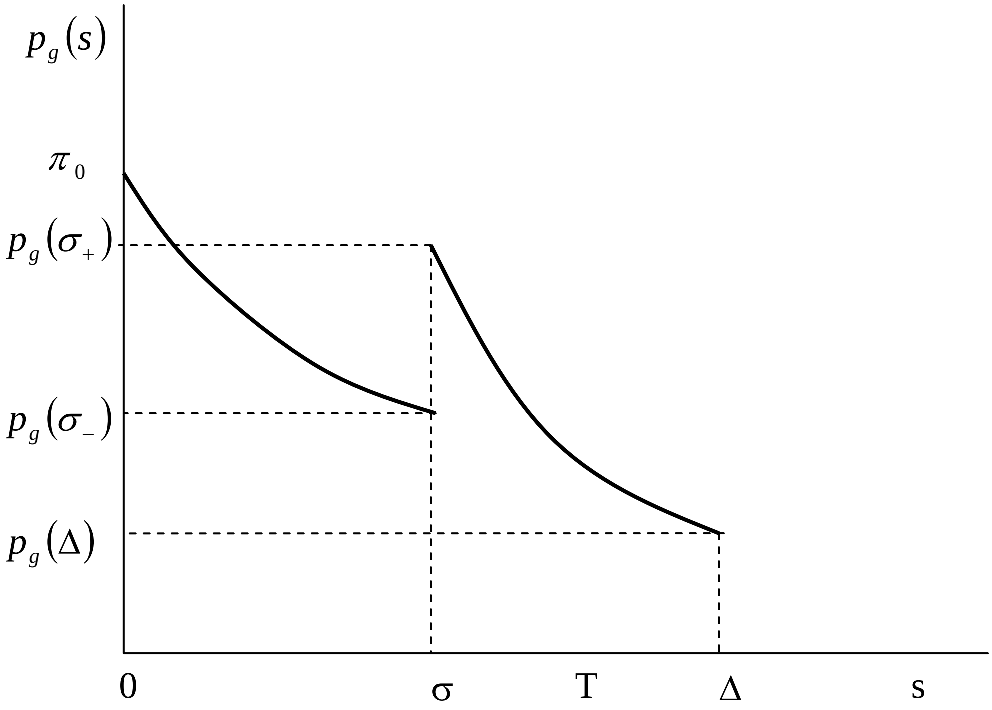

Fig.3: Probability of contacting a good worker by unemployment duration – Baseline equilibrium

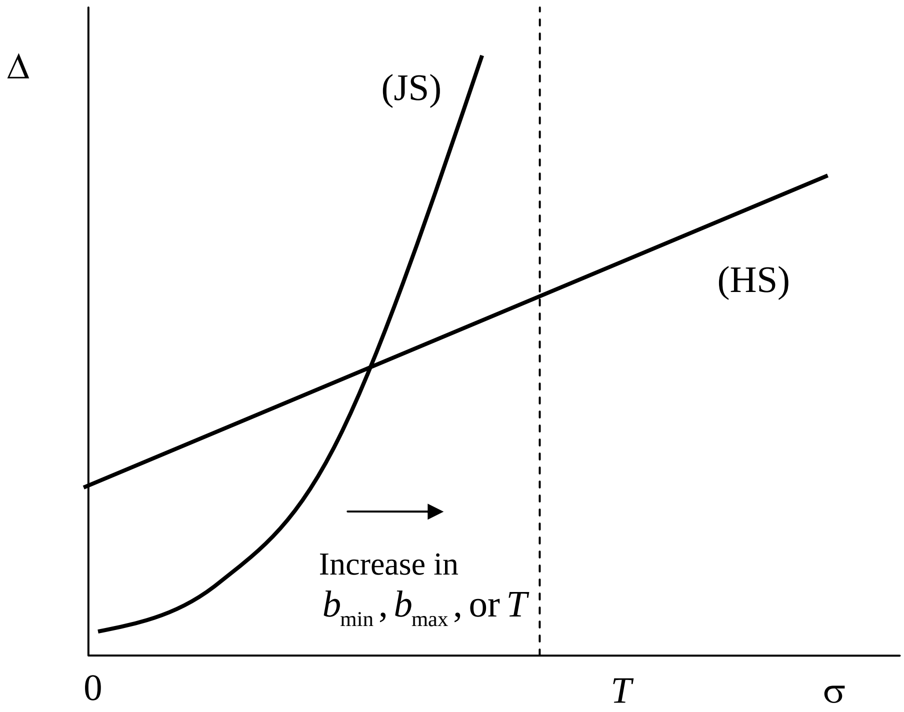

Fig.4: Existence and uniqueness of a baseline equilibrium

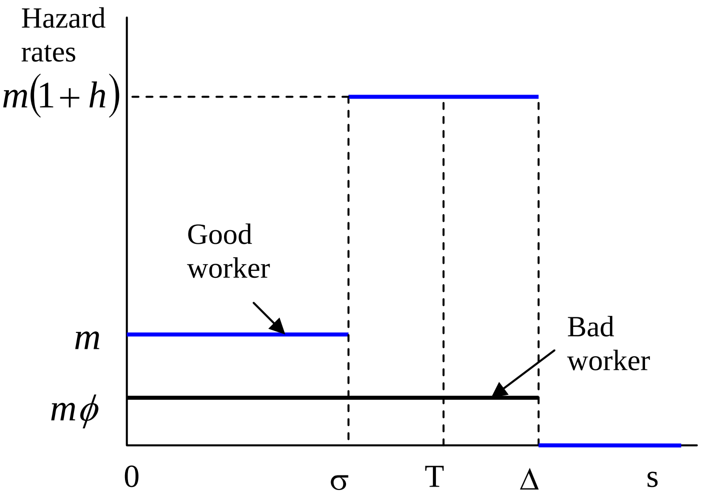

Fig.5a : Baseline equilibrium

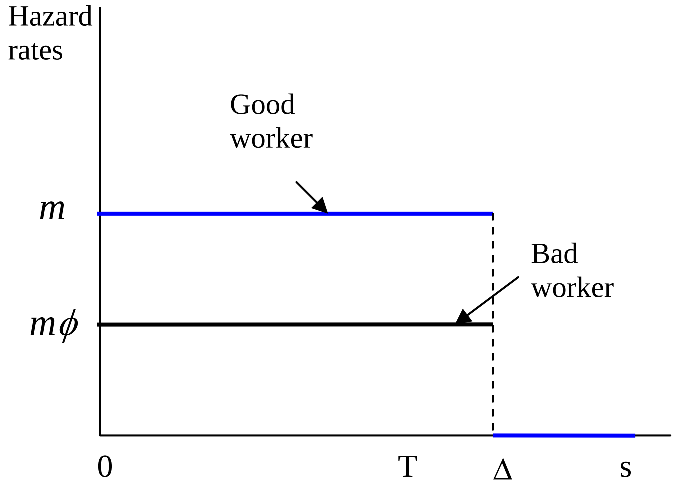

Fig.5b : Lockwood equilibrium

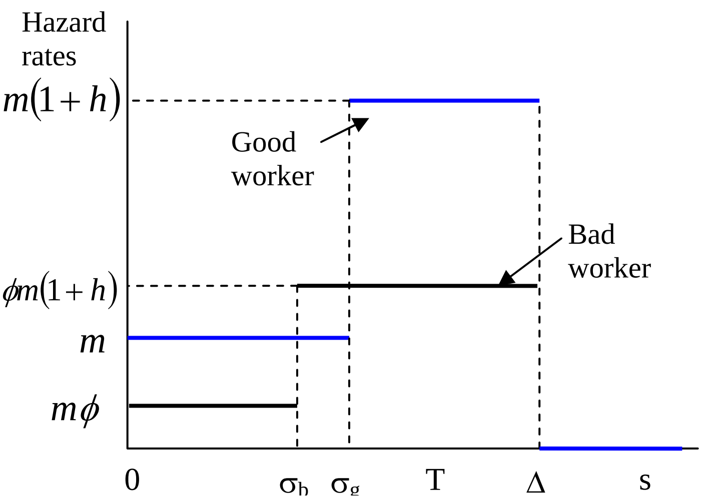

Fig.5c : Pooling equilibrium

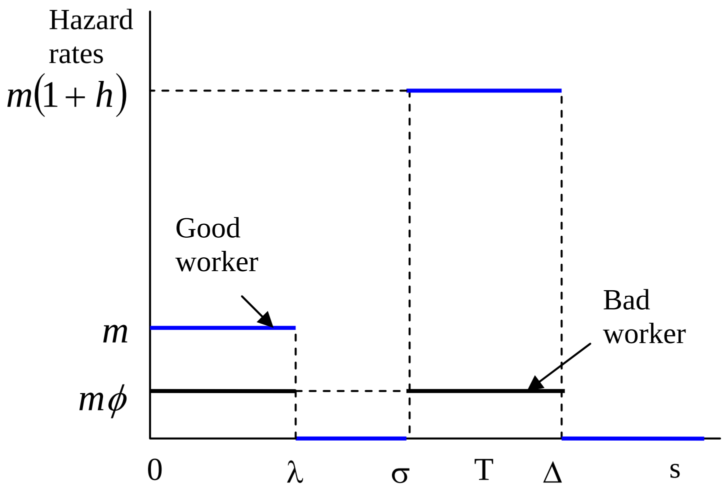

Fig.5d : Hole equilibrium

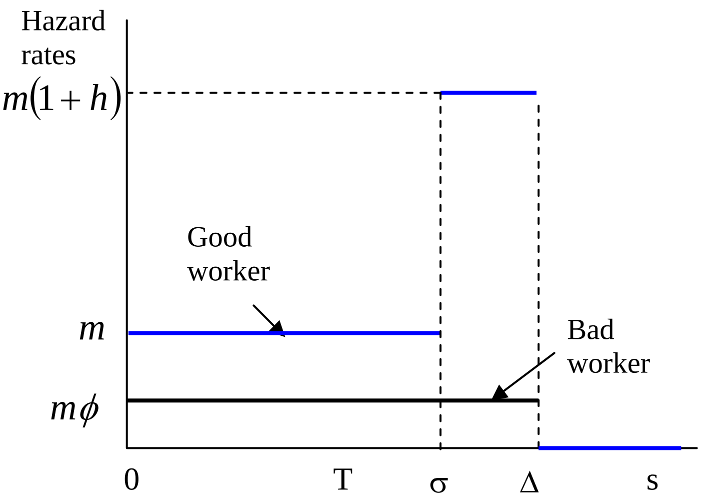

Fig.5e : Late-peak equilibrium

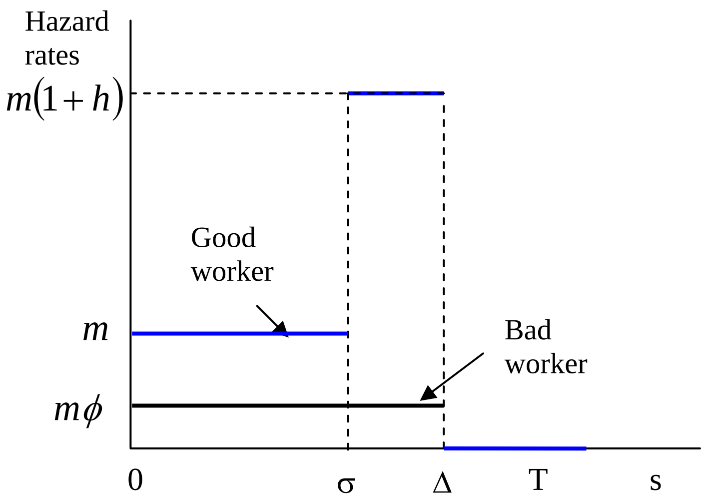

Fig.5f: Early-peak equilibrium

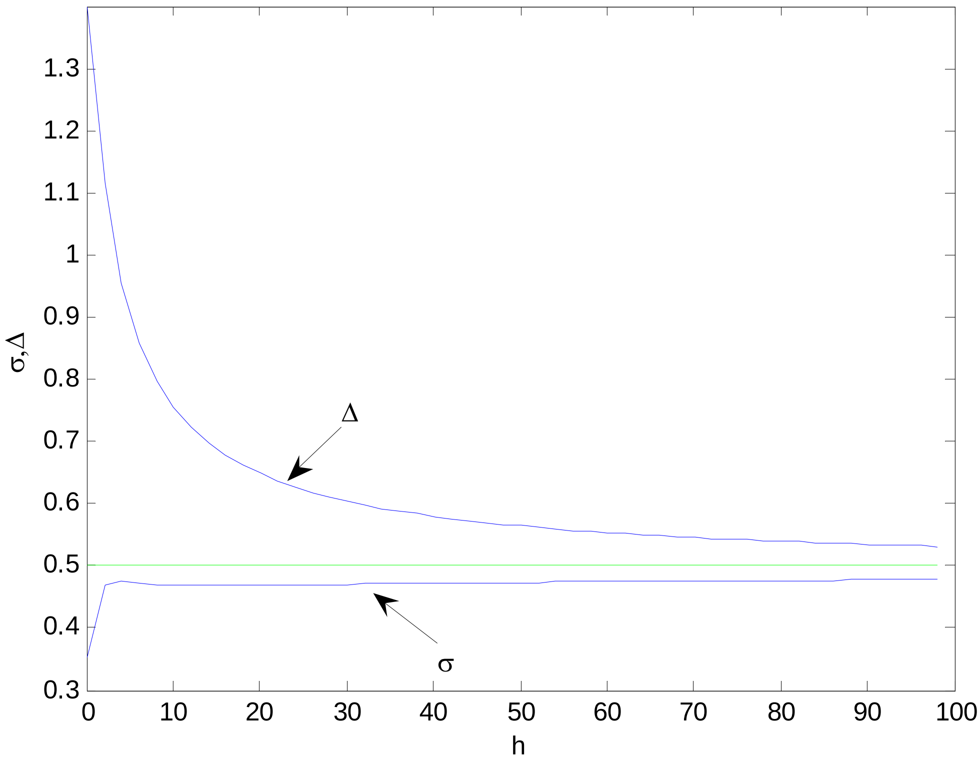

Fig.6 : Equilibrium σ^* and Δ^* as functions of h

Parameters are set as follows:

$r = 5\%$, π_0=.45, m=1.0, c=.31, $\varphi = .3$, $b_{\min}$=.3, $b_{\max}$=.6, T=.5, w=1.0, y_g=1.2, y_b=.8, $v(x)=x^{0.5}$