Service-led industrialization in developing economies: Some implications of technology gap dynamics

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Service-led industrialization in developing economies: Some implications of technology gap dynamics*

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Abstract

Can expansion of modern-services such as telecommunications, banking & finance and business services boost industrialization in developing countries? We explore this question in a two-sector Kaleckian model where an autonomously growing service sector generates market for a demand-constrained domestic industry but the latter faces competition from technologically-superior imports. We show that it is possible to have a steady state in this model, where domestic industry grows at the same rate as the service sector with positive industrial employment growth. Convergence to this steady state, however, requires domestic industry to increase its rate of technical change in response to increasing import competition. We find that improvements in the conditions for technological progress in the domestic industrial sector, say because of policy interventions that helps in upgrading technology, can increase relative size of domestic industry. On the other hand, an increase in the pace of technological progress abroad or an increase in the elasticity of imports of industrial product with respect to technology gap between the domestic industry and its foreign competitor reduces the same.

JEL Codes: O14, O19, O41, F63, F68

Keywords: Modern services and industrialization, imports, technology gap, developing countries, two-sector Kaleckian model

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1. Introduction

One of the stylized facts of economic development is that the industry or manufacturing sector starts to become secondary to services, especially in terms of employment share, at high per capita income levels. However, lately, employment and output shares of industry in many developing economies, particularly in Latin America and Africa, are declining at lot lower levels of per capita income than what can be expected from the development experience of currently advanced economies. This earlier than expected decline in employment and/or output share of industry in developing economies has been christened as 'premature de-industrialization' in the literature.[1]'Premature de-industrialization' implies developing economies are going to find expansion of the industrial sector an increasingly difficult task. This is a concern because traditionally the industrial sector has been identified as the 'engine of growth' and the key for development because of its technological dynamism as well as it ability to absorb resources trapped in low productivity sectors. (Kaldor 1989)

Amidst this concern for 'premature de-industrialization', some services like banking & finance, telecommunications and business services may have emerged as alternatives for developing economies today. (Eichengreen and Gupta (2013) and Rodrik (2016)) The advantage of these services is that the two main arguments which have traditionally been made against regarding services as leading sectors of growth - that they are technologically stagnant activities, *à la* Baumol (1967), and their non-tradable nature - do not seem to apply, mainly due to considerable application of modern information and communication technologies (ICTs). Eichengreen and Gupta (2013) considering a cross section of 91 countries over the period 1950 to 2005 find that because of services that are receptive to ICTs, the share of service sector in national output is expanding at much lower levels of per capita income after 1990 than before. An outstanding example of expansion of these services in a developing economy can be found in the case of India, where these services have been amongst the fastest growing sectors for some time and have according to Eichengreen and Gupta (2011) increased their combined share in GDP from an average of 3.5 percent in 1950-70 to an average of 18.4 percent in 2000-08. As opposed to these services, the share in GDP of the industrial sector in India has been more or less stagnant since the 1990s.

In this paper, instead of trying to ascertain if these services can provide an alternative to industrialization, we inquire whether an autonomous expansion in these services can affect expansion of manufacturing or industry in developing economies. The question is interesting for two main reasons. First, an autonomous expansion of

[1]See for example Palma (2005), Dasgupta and Singh (2006), Amirapu and Subramanian (2015), and Rodrik (2016).

services like telecommunications, banking & finance and business services in a developing economy should generate potential markets for the industrial sector. Growth of telecommunication services, for example, creates growth in demand for telecommunication equipment manufacturing. Similarly, expansion of software and other business services and also banking and financial services should generate demand for computer hardware, electrical and electronic products in the economy. Second, the domestic industrial sector of the developing economy will face import competition in these markets.[2] In a typical developing economy one expects the industrial sector to be constrained by technology inferior to the frontier one. The extent to which domestic industry then succeeds in capturing these markets should depend, among other things, on its technology gap vis-a-vis the producers of imports. Thus industrialization or de-industrialization in a developing economy may depend on expansion of these services as well as the relative rates of technological progress in the domestic and foreign frontier industries.

This paper explores the implications of these ideas in a partial equilibrium growth model for a developing economy. In this model, an autonomously growing service sector generates final demand for a manufactured product. For this demand, a domestic industrial sector, characterized by the presence of excess capacity, competes with technologically superior imports from an advanced economy. We show that a long-run steady state can exists in this model where the domestic industrial sector grows at the same rate as the service sector. Stability of the steady state crucially depends on the manner in which import competition affects technological progress in the domestic industrial sector. In case increasing import competition decreases labour productivity growth in the domestic industrial sector then the steady state is unstable. On the other hand, if increasing import competition has the opposite effect because rising competition from imports may force the domestic sector to either innovate or learn from imports then the steady state can be locally stable. Comparative static exercises at the locally stable steady state suggests that improvements in the conditions for technological progress in the domestic industrial sector, say because of policy interventions that helps the domestic industrial sector in upgrading technology, can contribute towards industrialization in developing economies. Similarly, an increase in the pace of technological progress abroad and an increase in the elasticity of imports of industrial product with respect to technology gap between the domestic industry and its foreign competitor can be factors behind de-industrialization.

[2]Telecommunication equipment manufacturing industry of India is an interesting example in this context. The Indian telecommunication equipment industry used to cater to a captive domestic market provided by public sector telecommunication service enterprises. In the post-liberalization period, despite rapid growth in the telecommunication service segment which continues to the present day, the domestic telecommunication equipment manufacturing sector has not been able to compete with imports. See Mani (2008). See also Chaudhuri (2013), for a similar account of both telecommunication equipment manufacturing and computer hardware industries of India.

One prediction of this model is that there need not be any convergence in labour productivity growth of the domestic industrial sector and its foreign competitor, as we show that the technology gap between the two grows at a constant positive rate in the steady state. Rodrik (2013), however, provides empirical evidence in favour of a strong tendency for unconditional convergence in labour productivity growth in formal organised manufacturing industries. We also present an alternative model in which labour productivity growth in the domestic industrial sector is assumed to be an increasing function of its technology gap with its foreign competitor. In this case, there can be a locally stable long-run steady state with not only balanced growth between the service sector and the domestic industrial sector but also a constant technology gap between the latter and its foreign competitor. Exogenous improvements in the labour productivity growth of the domestic industrial sector as well as that of its foreign competitor have essentially the same implications for industrialization or de-industrialization in this case too.

Regarding the structure of the paper, the next section describes the short-run output determination in the model economy. In this section we describe the two domestic sectors and also specify a manufacturing imports function. Section 3 describes capital accumulation in the two domestic sectors. Section 4 describes the evolution of the technology gap due to technological progress in industrial sectors of advanced/frontier and developing economies. Here we specify a technological progress function for the domestic industrial sector, according to which labour productivity growth depends on import competition. Section 5 presents the long run dynamics of the model. In section 6 we present some comparative statics results relating to the long run steady state and uses these in section 7 to look at probable causes of industrialization or de-industrialization implied by the model. Section 8 contains an alternative version of the model which allows for convergence in labour productivity growth of the domestic industrial sector and its foreign competitor. There are two major departures in this version from the main model. One, we use a slightly more general specification for the import function. And two, technological progress in the domestic industrial sector is determined by technology gap instead of import competition. And finally, section 9 contains concludes the paper.

2. Short Run of the Model

Consider a developing economy which has a fairly developed service sector with a given stock of capital say K_s. By a fairly developed service sector we mean services like business services including software services, banking and other financial services and telecommunication services form a significant part of the sector. These services use modern information and communication technologies (ICTs) and are tradable.[3]

[3]See for example Eichengreen and Gupta (2013).

For simplicity we assume that the service sector produces only one service, whose output we denote as X_s. We assume that X_s is always equal to its full capacity level. This implies

$$X_s = \bar{u}_s K_s \tag{1}$$

where $\bar{u}_s$ is a positive constant which represents the full capacity output-capital ratio. (1) requires that there is always enough demand for the service sector to produce up to its full capacity level at a fixed price level which allows a feasible constant price mark-up on unit cost. This is possible for example, if there is unlimited export demand for the service and the sector is competitive at an exogenously determined world price. Without any loss of generality, let $\bar{u}_s = 1$.

The service sector provides a market for a certain manufactured product. For example, telecommunication services can be expected to give rise to a demand for telecommunication equipment required for switching and transmission and also terminal equipment like phone sets. Similarly growing software services and business services require computer hardware and other electronics and electrical products and the same can be said about financial services. In the model we assume that the production of the service induces a proportional final demand for a particular manufactured product, which we refer as *the* manufactured product. Specifically let $d_i = \lambda X_s$ where d_i is the real demand for the manufactured product and λ is a positive constant. (1) and $\bar{u}_s = 1$ imply

$$d_i = \lambda K_s \tag{2}$$

There also exists a domestic industrial sector which produces the manufactured product. Like the service sector, has a given stock of capital say K_i. Capital stocks of these two sectors - K_i and K_s - consists of a single capital good which we assume that both sectors can always procure at a fixed price $P_K = 1$. The domestic industrial sector to compete with imports for the market created by the service sector. As a result, it might operate below its full capacity output level.

We assume that foreign producers of the manufacturing product, whom we collectively refer to as the foreign industrial sector, have more advanced production technology than the domestic industrial sector. This can be captured by specifying that $0 < x_i < x_i^f$ where x_i and x_i^f are the labour productivities in the domestic and the foreign industrial sectors respectively. x_i and x_i^f are determined by the respective production technologies of the two sectors, which we assume to be fixed in the short run. The ratio $\mu = x_i^f / x_i$ captures the extent of technology gap between the domestic and the foreign industrial sectors, with greater μ implying a greater technology gap.[4]

[4] An alternate specification for the technology gap can be $x_i^f - x_i$, but we prefer $\frac{x_i^f}{x_i}$ because it allows use of logarithmic differentiation to compute growth rates.

$0 < x_i < x_i^f$ implies technology gap $\mu > 1$. We assume that total imports of the manufacturing product in real terms, M, is a strictly increasing function of μ,

$$M = M(\mu) \tag{3}$$

where M is a function $M : (1, \infty) \mapsto \mathbb{R}_{++}$ such that the derivative $M'(\mu) > 0$. We assume that the elasticity of imports of the manufactured product with respect to the technology gap between the domestic and the foreign industrial sectors, $\sigma = \frac{\mu M'(\mu)}{M(\mu)} > 0$ is a constant. This specification is inspired from the balance of payments constrained growth (BPCG) models where imports depend on the ratio of foreign price in domestic currency to domestic price and the real income in the domestic economy, with negative and positive constant elasticities with respect to the former and the latter.[5] In set-ups with cost-plus pricing, a ceteris paribus rise in μ has a negative effect on the ratio of foreign price to domestic price. However, in specifying (3), we ignore effects of the real income, the exchange rate, the possible wage differential between the domestic and the foreign industrial sectors on import of the manufactured product to focus solely on implication of the technology gap in determining competitiveness of the domestic industrial sector.[6]

The idea underlying (3) is that a greater technology gap between the domestic and the foreign industrial sectors means more of the imported manufactured product is preferred to its domestically produced counterpart. Thus, we need to be careful about our assumption regarding pricing in the domestic industrial sector. In case the domestic industry is a price taker then technological superiority of the foreign industry implies imports capture the entire market for the manufacturing product. On the other hand, the standard assumption of a fixed mark-up pricing rule is also not tenable in an open economy framework. Blecker (1989) argues that in an open economy it is difficult for firms to pass on any cost disadvantage to consumers by proportionately raising price without adversely affecting their competitiveness. And by the same token, it may be beneficial for firms to increase mark-ups in the event of any cost advantage as long as they stay competitive. Therefore Blecker (1989) suggests a flexible mark-up pricing rule which takes into account the competitiveness of domestic firms. In this paper, therefore, we envision a scenario in which the domestic industry compromises its price mark-up in order to account for the technology gap and remain competitive.[7] We can capture this in the following manner. Let the price of the

[5] See, for example, Thirlwall (2011) and Blecker (2021).

[6] In section 8, we work with a slightly more general import function. See (23), in which the total imports of the manufactured product is also a function of the demand for the manufactured product in addition to μ.

[7] A possible recourse that firms might employ in order to counter the technology gap and remain competitive is cost cutting. However scope for cost cutting in the absence of technological progress is limited.

domestic manufactured product be,

$$P_i = (1 + z_i)\frac{W}{x_i} \tag{4}$$

$(W/x_i) > 0$ in (4) is the wage cost incurred per unit output in domestic industry which for simplicity is assumed to be a constant through out the paper.[8] On the other hand the price mark-up in domestic industry z_i is a decreasing function of $m = \frac{M}{K_i}$, i.e.,

$$z_i = z(m) \tag{5}$$

where z is a C^1 function $z : \mathbb{R}_{++} \mapsto \mathbb{R}_{++}$ such that $\lim_{m \to 0} z(m) = \bar{z} > 0$ is some target mark-up and $z'(m) < 0$. We can think of $m = \frac{M}{K_i}$ as the extent of import competition that the domestic industrial sector faces. From (3), a larger technology gap μ means greater imports of the manufactured product and therefore the extent of import competition facing the domestic industrial sector is also higher. Thus, (5) implies that the greater is μ the lower is z_i.

The extent of technology gap between domestic and foreign industrial sectors μ is fixed in the short run as labour productivities of both domestic and foreign industry are given. Therefore, it follows from (3) - (5) that both total imports of the manufacturing product $M(\mu)$ and price of the domestic manufactured product P_i are also fixed in the short run. Then, from (2), output of the domestic industrial sector in the short run is $X_i = \lambda K_s - M(\mu)$ or

$$\frac{X_i}{K_i} = \lambda k_s - m \tag{6}$$

where $\frac{X_i}{K_i}$ is the degree of capacity utilization in the domestic industrial sector and $k_s = \frac{K_s}{K_i}$ is the relative capital stock of the service sector with respect to the domestic industrial sector. The analysis is economically meaningful only if capacity utilization in the domestic industrial sector is positive, i.e. $\lambda k_s > m$ in (6). We show later in the paper that the economy can settle in a steady state where $\lambda k_s > m$.

3. Capital Formation in Service and Domestic Industry

This section discusses the nature of capital formation in the two sectors of the domestic economy by specifying their investment rates. For the domestic industrial sector,

[8] In other words, we are assuming that the nominal wage, W, in the domestic industrial sector grows at the same rate as the labour productivity of the sector when the latter is not a constant- as is the case when we consider the long-run dynamics of the model. This assumption helps us in solving for the long-run steady state in the model.

we assume a standard linear Kaleckian investment function. Specifically, let

$$\frac{I_i}{K_i} = \alpha + \beta\frac{X_i}{K_i} + \tilde{\gamma}r_i \tag{7}$$

where α, β and $\tilde{\gamma}$ are positive constants. I_i, $\frac{I_i}{K_i}$, $\frac{X_i}{K_i}$ and r_i are investment, rate of investment, rate of capacity utilization and rate of profit in the domestic industrial sector respectively. We know from (6) that $\frac{X_i}{K_i} = \lambda k_s - m$. The rate of profit of the domestic industrial sector is

$$r_i = \frac{P_i X_i}{P_K K_i}\left(1 - \frac{W}{P_i x_i}\right)$$

Using (4)-(5) and $P_K = 1$ in the previous equation, we obtain the following expression for r_i.

$$r_i = z(m)\frac{X_i}{K_i}\frac{W}{x_i} \tag{8}$$

Substituting for $\frac{X_i}{K_i}$ from (6) and for r_i from (8) in (7), we can obtain the rate of investment in domestic industry as the following function of k_s and m.

$$\frac{I_i}{K_i} = \alpha + (\beta + \gamma z(m))(\lambda k_s - m) \tag{9}$$

where $\gamma = \tilde{\gamma}\frac{W}{x_i}$ is a positive constant. On the other hand, we assume that the service sector grows because of exogenous reasons, like expected growth in services exports. In the previous section we discussed that the service sector always produces at its full capacity level at a fixed price. Fixed price of service and capital along with full capacity utilization means both degree of capacity utilization and rate of profit in the service sector are constants as long as we abstract away from changes in income distribution in the sector. Therefore, with constant rates of capacity utilization and profit we can assume that the investment rate of the service sector is a constant, i.e.

$$\frac{I_s}{K_s} = \delta \tag{10}$$

where δ is a positive constant. I_s and $\frac{I_s}{K_s}$ are investment and investment rate of the service sector respectively. We assume that there is no depreciation of capital in either of the sectors.

Parameters α and δ in (9) and (10) respectively are autonomous investment rates of the two sectors. Kalecki (1971) included an autonomous term in the investment function to capture influences of "past economic, social and technological developments".[9] We can think of α and δ as effects of various exogenous factors, sector specific or otherwise, like favourable changes in policy environment, increase in ex-

[9] Kalecki (1971, p. 173)

pected export growth, exogenous technological progress nd emergence of new markets on investment rates of domestic industry and service sector respectively. In this paper we are trying to explore implications of autonomous expansion of modern services for industry in developing economies, particularly in the context of 'premature deindustrialization'. Therefore, it makes sense to argue that rate of investment in the service sector is high whereas autonomous rate of investment in the domestic industrial sector is low. Specifically, we assume that $\delta > \alpha$.

4. Dynamics of Manufacturing Technology Gap

In the long run μ is no longer constant but changes due to technological progress in both domestic and foreign industrial sectors. For the purpose of this paper we take technological progress in the foreign industrial sector as independent of the concerned developing economy. Therefore we assume that labour productivity in the foreign industrial sector grows at an exogenous rate $\hat{x}_i^f > 0$. On the other hand, for the domestic industrial sector, technological progress is captured through a technological progress function.

$$\hat{x}_i = f(m) \tag{11}$$

where $\hat{x}_i$ is the growth rate of labour productivity in the domestic industrial sector and f is a C^1 function $f : \mathbb{R}_+ \mapsto \mathbb{R}_{++}$. Since the ratio m reflects the extent of import competition that the domestic industrial sector faces, $f(0) > 0$ means there is some technological progress in the domestic industrial sector even in the absence of import competition.

As regards to $f'(m)$, the derivative of $f(m)$, we consider two different ways in which increased import competition can affect technological progress in the domestic industrial sector. The first follows from Kaldor and Mirrlees (1962), where it is argued that growth in labour productivity is the result of introduction of new machinery which depends on gross investment. Given the capital stock of the service sector, (6) implies that the greater are imports relative to the capital stock of the domestic industrial sector, the lower is the capacity utilization in domestic industry. Thus from (9), smaller is the new addition to its capital stock. Another possible reason why an increase in m can possibly have a negative effect on $\hat{x}_i$ is that it may impede 'learning by doing'[10] by slowing the investment rate. If these effects are strong then we can assume $f'(m) < 0$ for all $m \in \mathbb{R}_+$.

The second way in which import competition can affect technological progress in the domestic industrial sector is if an increase in imports forces the domestic industrial sector to adopt new technology in order to remain competitive, assuming that there are limits to which it can compromise on its price mark-up. In spirit, this argument

[10] Arrow (1962)

is similar to Bhaduri (2006), where labour saving technological progress is a result of intra-capitalist competition over market share and is further reinforced by rise in real wages. Here we are not concerned with the income distribution aspect. However increased competition from imports can have a role similar to that of domestic intra-capitalist competition.[11] A related treatment of technological progress can be found in Grossman and Helpman (1991), where the knowledge stock of the economy is specified as an increasing function of trade volume to capture possible international knowledge spillovers from interactions between domestic and foreign agents via trade. Technological dynamism through increased competition is one of the major promised virtues of liberalization. If increased import competition forces the domestic industry to either make new innovations or learn from imports then we can assume $f'(m) > 0$ for all $m \in \mathbb{R}_+$. In the next section we would consider implications of both these cases on the long run dynamics of the model.

Logarithmic differentiation of μ, yields the long run change in the technology gap as the difference between growth rates of labour productivities in domestic and industrial sectors. Specifically, growth rate of μ is

$$\hat{\mu} = \hat{x}_i^f - \hat{x}_i = \hat{x}_i^f - f(m) \tag{12}$$

5. Long-Run Dynamics of the Model

The long-run dynamics of the model results from changes in k_s and m. The relative capital stock ratio of the service sector with respect to industrial sector k_s may change because of different growth rates of investment in the two sectors. Since there is no depreciation of capital in either of the sectors, the growth rate of k_s is the rate of investment of the service sector less that of the domestic industrial sector for all $k_s \in \mathbb{R}_{++}$. Therefore from (9) and (10) we have the following expression for rate of change in k_s, for all $k_s \in \mathbb{R}_{++}$.

$$\dot{k_s} = k_s\{\delta - \alpha - (\beta + \gamma z(m))(\lambda k_s - m)\} \tag{13}$$

where $\dot{k_s}$ is the rate of change in k_s. Similarly, the extent of import competition faced by the domestic industrial sector, m, changes in the long run because changes in imports of the manufactured product caused by the dynamics of technology gap discussed in the previous section and because of investment in the domestic industrial sector. Logarithmic differentiation of (3) yields the growth rate of imports as $\hat{M} = \sigma\hat{\mu}$. Now we can derive an expression for rate of change in m in a fashion similar to

[11] Also see Guha (2021), who argues that the intra-capitalist competition part of Bhaduri's argument is more relevant for technical change in demand-constrained economies than inter-class competition because of existence of excess resources including labor.

that used to derive (13). For all $m \in \mathbb{R}_{++}$, the rate of growth of m, by definition of m, is rate of growth of imports of the manufactured product less the growth rate of capital stock of the domestic industrial sector. Using (9) and (12), we can derive the following expression for rate of change in m, for $m \in \mathbb{R}_{++}$.

$$\dot{m} = m\{\sigma(\hat{x}_i^f - f(m)) - \alpha - (\beta + \gamma z(m))(\lambda k_s - m)\} \tag{14}$$

where $\dot{m}$ is the rate of change in m. (13) and (14) constitute a system of two differential equations in two variables, k_s and m, which describes the long-run dynamics of the model. We discussed in section 2 that for our analysis to be economically meaningful we need $m < \lambda k_s$ because otherwise output in the domestic industrial sector is not positive. In the rest of this section we look for conditions which ensure existence and local stability of a steady state where $0 < m < \lambda k_s$ is satisfied.

5.1 Existence of Steady State

$(k_s, m) \in \mathbb{R}^2_{++}$ is a steady state if and only if $\dot{k}_s = 0$ in (13), $\dot{m} = 0$ in (14) and $\lambda k_s > m$. Now from (13) and (14), $\dot{k}_s = 0$ and $\dot{m} = 0$ for any $(k_s, m) \in \mathbb{R}^2_{++}$ if and only if $\delta = \alpha + (\beta + \gamma z(m))(\lambda k_s - m)$ and $\sigma(\hat{x}_i^f - f(m)) = \alpha + (\beta + \gamma z(m))(\lambda k_s - m)$. We use these two equations to find out necessary conditions for existence of steady state in Proposition 1 and Corollaries 1.1 and 1.2.

Proposition 1. *Define* $\psi(m) = \sigma(\hat{x}_i^f - f(m)) - \delta$. $\psi(m) = 0$ *at steady state.*

Proof. Let $(k_s^*, m^*) \in \mathbb{R}^2_{++}$ be a steady state. Then from (13), $\delta = \alpha + (\beta + \gamma z(m^*))(\lambda k_s^* - m^*)$ and from (14), $\sigma(\hat{x}_i^f - f(m^*)) = \alpha + (\beta + \gamma z(m^*))(\lambda k_s^* - m^*)$. Therefore it follows that $\sigma(\hat{x}_i^f - f(m^*)) = \delta$ or $\psi(m^*) = 0$. □

Proposition 1 implies imports of the manufactured product must grow at the same rate as the service sector in steady state, i.e δ. Since growth rate of imports of the manufactured product is determined by the growth rate of the technology gap, the latter must grow at a positive rate at steady state for imports to grow at the rate δ. This implies growth rate of labour productivity of the foreign industrial sector $\hat{x}_i^f > \frac{\delta}{\sigma}$ because growth rate of labour productivity of the domestic industrial sector is positive by definition. Corollary 1.1 states this formally.

Corollary 1.1. *If a steady state exists then* $\sigma\hat{x}_i^f > \delta$.

Proof. Suppose $(k_s^*, m^*) \in \mathbb{R}^2_{++}$ be a steady state and $\sigma\hat{x}_i^f \leq \delta$. From Proposition 1, $\psi(m^*) = \sigma(\hat{x}_i^f - f(m^*)) - \delta = 0$. It follows $f(m^*) = \hat{x}_i^f - \delta/\sigma$. This leads to a contradiction as $\sigma\hat{x}_i^f \leq \delta$ implies $f(m^*) \leq 0$ while by definition $f(m^*) > 0$. □

Assuming $\delta < \sigma \hat{x}_i^f$, Corollary 1.2 states another necessary condition for existence of steady state. In case increases in import competition reduce the pace of technological progress in the domestic sector, i.e. $f'(m) < 0$, then steady states do not exist if the pace of technological progress in the domestic industrial sector in the absence of import competition is too small, i.e. $f(0) \leq \hat{x}_i^f - \frac{\delta}{\sigma}$. On the other hand, in case increases in import competition increase the pace of technological progress in the domestic sector, i.e. $f'(m) > 0$, then once again steady states do not exist if the pace of technological progress in the domestic industrial sector in the absence of import competition is too large, i.e. $f(0) \geq \hat{x}_i^f - \frac{\delta}{\sigma}$.

Corollary 1.2. *If a steady state exists then* $[(\forall m \in \mathbb{R}_+)(f'(m) < 0) \longrightarrow (f(0) > \hat{x}_i^f - \frac{\delta}{\sigma})]$ *and* $[(\forall m \in \mathbb{R}_+)(f'(m) > 0) \longrightarrow (f(0) < \hat{x}_i^f - \frac{\delta}{\sigma})]$.

Proof. Let $(k_s^*, m^*) \in \mathbb{R}_{++}^2$ be a steady state. From Proposition 1, $\psi(m^*) = 0$. We have two cases to rule out in order to prove the claim. First consider the case, $(\forall m \in \mathbb{R}_+^2)(f'(m) < 0)$ and $f(0) \leq \hat{x}_i^f - \frac{\delta}{\sigma}$. $f(0) \leq \hat{x}_i^f - \frac{\delta}{\sigma}$ implies $\psi(0) \geq 0$. And $(\forall m \in \mathbb{R}_+^2)(f'(m) < 0)$ implies derivative of $\psi(m)$, $\psi'(m) = -\sigma f'(m) > 0$ for all $m \in \mathbb{R}_+^2$ as $\sigma > 0$. Then it follows from $\psi(0) \geq 0$ and $m^* > 0$ that $\psi(m^*) > 0$, which is a contradiction. Similarly, in the other case, $(\forall m \in \mathbb{R}_+)(f'(m) > 0)$ and $f(0) \geq \hat{x}_i^f - \frac{\delta}{\sigma}$ imply $\psi(m^*) < 0$, which again is a contradiction. □

As regards sufficient conditions for existence and uniqueness of steady state, it can be shown that our assumption $\delta > \alpha$, along with a set of boundary restriction on the technological progress function $f(m)$ for each of the two cases - $(\forall m \in \mathbb{R}_+)(f'(m) < 0)$ and $(\forall m \in \mathbb{R}_+)(f'(m) > 0)$ - suffices. Proposition 2 states these restrictions.

Proposition 2. *Let* $\delta > \alpha$. *If* $(\forall m \in \mathbb{R}_+)(f'(m) < 0)$ *then a sufficient condition for existence of a unique steady state is* $\lim_{m \to \infty} f(m) < \hat{x}_i^f - \frac{\delta}{\sigma} < f(0)$. *Similarly if* $(\forall m \in \mathbb{R}_+)(f'(m) > 0)$ *then a sufficient condition for existence of a unique steady state is* $f(0) < \hat{x}_i^f - \frac{\delta}{\sigma} < \lim_{m \to \infty} f(m)$.

Proof. It is given that $\delta > \alpha$. First, consider $(\forall m \in \mathbb{R}_+)(f'(m) < 0)$. Suppose $\lim_{m \to \infty} f(m) < \hat{x}_i^f - \frac{\delta}{\sigma} < f(0)$. Define $\psi(m) = \sigma(\hat{x}_i^f - f(m)) - \delta$. $\hat{x}_i^f - \frac{\delta}{\sigma} < f(0)$ implies $\psi(0) < 0$ and $\lim_{m \to \infty} f(m) < \hat{x}_i^f - \frac{\delta}{\sigma}$ implies $\lim_{m \to \infty} \psi(m) > 0$. $\lim_{m \to \infty} \psi(m) > 0$ means for sufficiently large values of m, $\psi(m) > 0$. Let $\bar{m} > 0$ be such that $\psi(\bar{m}) > 0$. Since $\psi(0) < 0$ and $\psi(\bar{m}) > 0$, it follows from the *Intermediate Value Theorem* that there exists $m \in [0, \bar{m}]$ such that $\psi(m) = 0$. Moreover since $\psi'(m) = -\sigma f'(m)$, $(\forall m \in \mathbb{R}_+)(f'(m) < 0)$ implies there can be only one value of m for which $\psi(m) = 0$. Similarly when $(\forall m \in \mathbb{R}_+)(f'(m) > 0)$, $f(0) < \hat{x}_i^f - \frac{\delta}{\sigma} < \lim_{m \to \infty} f(m)$ implies there exist a unique $m \in (0, \infty)$ such that $\psi(m) = 0$. Let

$m^* \in \mathbb{R}_{++}$ be such that $\psi(m^*) = 0$ and let $k_s^* = \frac{\delta-\alpha}{\lambda(\beta+\gamma z(m^*))} + \frac{m^*}{\lambda}$. Since $\delta > \alpha$, it follows that $k_s^* > 0$ and $m^* < \lambda k_s^*$. Thus (k_s^*, m^*) is the unique steady state. $\square$

Let $(k_s^*, m^*) \in \mathbb{R}^2_{++}$ be a steady state. We know from Proposition 1 that $\psi^*(m) = 0$ or using the definition of $\psi(m)$,

$$f(m^*) = \hat{x}_i^f - \frac{\delta}{\sigma} \tag{15}$$

Also from the proof of Proposition 2, we know that

$$k_s^* = \frac{\delta - \alpha}{\lambda(\beta + \gamma z(m^*))} + \frac{m^*}{\lambda} \tag{16}$$

Suppose $\sigma\hat{x}_i^f > \delta$ and $\lim_{m\to\infty} f(m) < \hat{x}_i^f - \frac{\delta}{\sigma} < f(0)$ if $f'(m) < 0$ for all $m \in \mathbb{R}_+$ or $f(0) < \hat{x}_i^f - \frac{\delta}{\sigma} < \lim_{m\to\infty} f(m)$ if $f'(m) > 0$ for all $m \in \mathbb{R}_+$. Then, from Propositions 1 and 2 these conditions along with our assumption $\delta > \alpha$ ensure that a steady state (k_s^*, m^*) exists and is unique for both cases of technological progress in the domestic industrial sector. At the steady state both the capital stock of the domestic industrial sector and imports of the manufactured product grow at the same rate at which the capital stock of the service sector grows, i.e. δ. From (1) and (6), outputs of both service and domestic industrial sectors also grow at the rate δ. Rate of growth of labour productivity of the domestic industrial sector is given by (15), which implies that the technology gap between foreign and domestic industrial sectors μ grows at a constant rate $\frac{\delta}{\sigma}$ in steady state which results in a constant rate of growth of imports of the manufactured product. The growth rate of employment in the domestic industrial sector not only depends upon the growth of the service sector but also on the elasticity of the imports function as well as the pace of technological progress in the foreign industrial sector. Using (16), it can be easily checked that the steady state growth rate of employment in the domestic industrial sector is equal to $\delta(1 + \frac{1}{\sigma}) - \hat{x}_i^f$. Thus, employment in the domestic industrial sector grows in the steady state if and only if $\delta(1 + \frac{1}{\sigma}) > \hat{x}_i^f$.

5.2 Stability of Steady State

In this section we discuss stability of the steady state (k_s^*, m^*). It will become evident that the manner in which import competition affects technological progress in domestic industrial sector, i.e. the sign of $f'(m)$, is crucial for local stability of the steady state. We can ascertain local stability properties of the steady state (k_s^*, m^*) by linearizing the system of differential equations (13) and (14) around (k_s^*, m^*) as long as the corresponding linearized system is hyperbolic. The latter is true if both trace and determinant of the Jacobian matrix evaluated at (k_s^*, m^*) are non-zero. Moreover, the steady state is stable when the determinant of the said matrix is positive and its trace negative. On the other hand negative determinant or positive trace of the said matrix implies that the steady state is unstable.

The Jacobian matrix of the system of differential equations (13) and (14) evaluated at (k_s^*, m^*) is

$$\begin{bmatrix} -k_s^*(\beta+\gamma z(m^*))\lambda & k_s^*\{-\gamma z'(m^*)(\lambda k_s^* - m^*) + (\beta+\gamma z(m^*))\} \\ -m^*(\beta+\gamma z(m^*))\lambda & m^*\{-\sigma f'(m^*) - \gamma z'(m^*)(\lambda k_s^* - m^*) + (\beta+\gamma z(m^*))\} \end{bmatrix}$$

with determinant,

$$D = \lambda k_s^* m^*(\beta+\gamma z(m^*))\sigma f'(m^*) \tag{17}$$

and trace,

$$T = -k_s^*(\beta+\gamma z(m^*))\lambda + m^*\{-\sigma f'(m^*) - \gamma z'(m^*)(\lambda k_s^* - m^*) + (\beta+\gamma z(m^*))\}$$

or using (16),

$$T = -(\delta-\alpha) - m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} \tag{18}$$

The steady state is unstable if an increase in import competition decreases the rate of technological progress in the domestic industrial sector. Suppose an increase in the extent of import competition decreases the rate of labour productivity growth of the domestic industrial sector. Then $f'(m^*) < 0$. Now suppose the economy is at a non-steady state position $(k_s^*, \bar{m})$ where $\bar{m} - m^* = \epsilon > 0$. If ϵ is sufficiently small then $f'(m^*) < 0$ implies $f(\bar{m}) < f(m^*)$, i.e. growth rate of labour productivity in the domestic industrial sector is lower at $(k_s^*, \bar{m})$ than at the steady state. This means imports of the manufactured commodity must grow at a faster rate at $(k_s^*, \bar{m})$ than at the steady state because, from (12), $f(\bar{m}) < f(m^*)$ means a faster growth in the technology gap between foreign and domestic industrial sectors compared to that in the steady state. Also, from (5) and (6), $\epsilon > 0$ means a lower mark-up and a lower capacity utilization in the domestic industrial sector at $(k_s^*, \bar{m})$ than at the steady state. Then, from (10), it follows that the growth rate of capital stock of the domestic industrial sector must be lower at $(k_s^*, \bar{m})$ than at the steady state. Since $\dot{m} = 0$ at the steady state, a faster growth rate of imports of the manufactured commodity and a slower growth rate of capital stock of the domestic industrial sector compared to their steady state growth rates imply $\dot{m} > 0$. Therefore there is a tendency for the economy to move further away from the steady state from points such as $(k_s^*, \bar{m})$. Formally $f'(m^*) < 0$ means $D < 0$, which is a sufficient condition for the steady state to be a saddle point. Proposition 3 states this.

Proposition 3. *If $f'(m^*) < 0$ then the steady state is a saddle point.*

Proof. Suppose $f'(m^*) < 0$. Since λ, k_s^*, m^*, β, γ, $z(m^*)$ and σ are all positive, $f'(m^*) < 0$ implies $D < 0$ in (17). $\square$

On the other hand, if an increase in import competition increases the rate of technological progress in the domestic industrial sector then it is possible that the steady state is locally stable. In this case, the rate of growth of labour productivity of the domestic industrial sector increases with an increase in the extent of import competition, which means $f'(m^*) > 0$. Now, suppose, once again, that the economy is at $(k_s^*, \bar{m})$. This time, if ϵ is sufficiently small, $f'(m^*) > 0$ implies $f(\bar{m}) > f(m^*)$. This means, unlike in the previous case, imports of the manufactured commodity grows at a lesser rate at $(k_s^*, \bar{m})$ than at the steady state. This tends to decrease m. However, $\bar{m} > m^*$ also means that the growth rate of capital stock of the domestic industrial sector at $(k_s^*, \bar{m})$ is lower than at the steady state. This then tends to increase m at $(k_s^*, \bar{m})$. It is the net effect of a lower growth rate of imports of the manufactured commodity and a lower growth rate of capital stock of the domestic industrial sector at $(k_s^*, \bar{m})$ compared to their steady state values that determines the sign of $\dot{m}$ at $(k_s^*, \bar{m})$. If the positive effect of on m of a lower growth rate of capital stock of the domestic industrial sector dominates the negative effect on m of a slower growth rate of imports then $\dot{m} > 0$ at $(k_s^*, \bar{m})$, which makes the steady state unstable. Formally while $f'(m^*) > 0$ ensures $D > 0$, for local stability of the steady state sign of T is also important. From (18), the sign of T not only depends on $f'(m^*)$ but also on $z'(m^*)$ which reflects the effect of increase in imports competition on the price mark-up in the domestic industrial sector near the steady state. If $z'(m^*)$ takes a high absolute value then T can be positive and in that case the steady state is locally unstable. Proposition 4 states a sufficient condition for the steady state to be unstable even when $f'(m^*) > 0$.

Proposition 4. *If $f'(m^*) > 0$ and $m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} < -(\delta - \alpha)$ then the steady state is unstable.*

Proof. Suppose $f'(m^*) > 0$ and $m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} < -(\delta - \alpha)$. Since λ, k_s^*, m^*, β, γ, $z(m^*)$ and σ are all positive, $f'(m^*) > 0$ implies $D > 0$ in (17). Also $m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} < -(\delta - \alpha)$ implies $-(\delta - \alpha) - m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} > 0$. Then, from (18), $T > 0$. $\square$

However, if the absolute value of $z'(m^*)$ is sufficiently low so that $T < 0$ then the steady state is locally asymptotically stable when the growth rate of labour productivity in the domestic industrial sector is an increasing function of the extent of import competition. In this case, for example, the tendency for m to decrease because of a lower growth rate of imports of the manufactured commodity at a non-steady state point such as $(k_s^*, \bar{m})$ compared to that at the steady state more than counteracts the tendency for m to increase because of lower growth rate of capital stock of the domestic industrial sector at such points. As a consequence, at points such as $(k_s^*, \bar{m})$, $\dot{m} < 0$, which tends to push the economy towards the steady state. Proposition 5 states a sufficient condition for the steady state to be locally asymptotically stable when $f'(m^*) > 0$.

Proposition 5. *If $f'(m^*) > 0$ and $m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} > -(\delta - \alpha)$ then the steady state is locally asymptotically stable.*

Proof. Suppose $f'(m^*) > 0$ and $m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} > -(\delta - \alpha)$. Since λ, k_s^*, m^*, β, γ, $z(m^*)$ and σ are all positive, $f'(m^*) > 0$ implies $D > 0$ in (17). Also $m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} > -(\delta - \alpha)$ implies $-(\delta - \alpha) - m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} < 0$. Then, from (18), $T < 0$. □

A convenient result which follows from the above proposition is that the steady state is locally asymptotically stable when $f'(m^*) > 0$ if, at the steady state, the absolute value of the elasticity of the price mark-up with respect to the extent of import competition is less than one.

Corollary 5.1. *Let $\sigma_z^* = \frac{m^* z'(m^*)}{z(m^*)}$. If $f'(m^*) > 0$ and $\sigma_z^* > -1$ then the steady state is locally asymptotically stable.*

Proof. Suppose $f'(m^*) > 0$ and $\sigma_z^* > -1$. Now from (18), $T = -(\delta - \alpha) - m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} = -\{\beta + \gamma z(m^*)(1 + \sigma_z^*)\}\frac{(\delta-\alpha)}{\beta+\gamma z(m^*)} - m^*\sigma f(m^*)$. Since $\delta > \alpha$, it follows that $f'(m^*) > 0$ and $\sigma_z^* > -1$ imply $T < 0$. The claim then follows from Proposition 5. □

6. Comparative Statics

In this section we analyze effects on the steady state of various shocks. Comparative static exercises are meaningful only if a steady state exists and is at least locally stable. Since, from Proposition 3, we know that the steady state is not stable when rate of growth of labour productivity in the domestic industrial sector is a decreasing function of extent of import competition, in this section we are only going to consider the other case where $f'(m) > 0$ for all $m \in \mathbb{R}_+$. We assume that $\sigma\hat{x}_i^f > \delta$ and $f(0) < \hat{x}_i^f - \frac{\delta}{\sigma} < \lim_{m\to\infty} f(m)$. As a result, it follows from Corollary 1.1 and Proposition 2 that an unique steady state exists. Let $(k_s^*, m^*) \in \mathbb{R}_{++}^2$ be the steady state. It therefore satisfies (15) and (16). Also, let $\sigma_z^* = \frac{m^* z'(m^*)}{z(m^*)} > -1$. Since

$f'(m) > 0$ for all $m \in \mathbb{R}_+$ implies $f'(m^*) > 0$, it follows from Corollary 5.1 that (k_s^*, m^*) is locally asymptotically stable steady state. [12]

Let us first consider the effect of a *ceteris paribus* increase in the autonomous rate of investment in the service sector, δ, on the steady state. From (15), a *ceteris paribus* increase in δ means a lower rate of technological progress in the the domestic industrial sector at the new steady state. Since $f'(m^*) > 0$, a lower rate of technological progress must imply that the domestic industrial sector faces less import competition at the new steady state. That is, for (15) to hold m^* must decrease with an increase in δ. The effect on the steady state relative capital stock ratio of the service sector, however, is ambiguous. On one hand, an increase in δ increases the growth rate of capital stock of the service sector which tends to increase k_s^*. On the other hand, the rate of growth of capital stock of the domestic industrial sector also increases as a result of the decrease in m^* associated with a *ceteris paribus* increase in δ. This increases both capacity utilization $\lambda k_s^* - m^*$ and the price mark-up, since $z'(m^*) < 0$, in the domestic industrial sector. This increase in the growth rate of capital stock of the domestic industrial sector tends to decrease k_s^*. We show in Proposition 6 that if the increase in the steady state growth rate of imports because of an infinitesimal decrease in m^*, $\sigma f'(m^*)$, is greater than the increase in the growth rate of capital stock of the domestic industrial sector, $\beta + \gamma z(m^*) - (\lambda k_s^* - m^*)\gamma z'(m^*)$, then k_s^* increases with a *ceteris paribus* increase in δ.

Proposition 6. $[f'(m^*) > 0 \longrightarrow \frac{\partial m^*}{\partial \delta} < 0 \wedge [\frac{\partial k_s^*}{\partial \delta} > 0 \longleftrightarrow \sigma f'(m^*) > \beta + \gamma z(m^*) - (\lambda k_s^* - m^*)\gamma z'(m^*)]]$

Proof. Suppose $f'(m^*) > 0$. Partially differentiating (15) with respect to δ yields $\frac{\partial m^*}{\partial \delta} = \frac{-1}{\sigma f'(m^*)}$. Since $\sigma > 0$ and $f'(m^*) > 0$ it follows that $\frac{\partial m^*}{\partial \delta} < 0$. Next partially differentiating (16) with respect to δ yields

$$\frac{\partial k_s^*}{\partial \delta} = \frac{1}{\lambda(\beta + \gamma z(m^*))}[1 - \frac{(\delta - \alpha)\gamma z'(m^*)}{(\beta + \gamma z(m^*))}\frac{\partial m^*}{\partial \delta}] + \frac{1}{\lambda}\frac{\partial m^*}{\partial \delta}$$

Substituting for $\frac{\partial m^*}{\partial \delta}$ in the above expression and then using (16) we obtain,

$$\frac{\partial k_s^*}{\partial \delta} = \frac{1}{\lambda(\beta + \gamma z(m^*))}[1 + (\lambda k_s^* - m^*)\frac{\gamma z'(m^*)}{\sigma f'(m^*)}] - \frac{1}{\lambda \sigma f'(m^*)}$$

Now since $\lambda > 0$, $\frac{1}{\lambda(\beta+\gamma z(m^*))}[1 + (\lambda k_s^* - m^*)\frac{\gamma z'(m^*)}{\sigma f'(m^*)}] - \frac{1}{\lambda \sigma f'(m^*)} > 0$ if and only if $\frac{1}{(\beta+\gamma z(m^*))}[1 + (\lambda k_s^* - m^*)\frac{\gamma z'(m^*)}{\sigma f'(m^*)}] - \frac{1}{\sigma f'(m^*)} > 0$ or, rearranging the terms, $\sigma f'(m^*) > \beta + \gamma z(m^*) - (\lambda k_s^* - m^*)\gamma z'(m^*)$. Thus $\frac{\partial k_s^*}{\partial \delta} > 0$ if and only if $\sigma f'(m^*) > \beta + \gamma z(m^*) -$

[12] For the results of this section, it is inconsequential whether one assumes $\sigma_z^* > -1$ or $m^*\{\frac{(\delta-\alpha)\gamma z'(m^*)}{\beta+\gamma z(m^*)} + \sigma f'(m^*)\} > -(\delta - \alpha)$.

$(\lambda k_s^* - m^*)\gamma z'(m^*)$. □

A *ceteris paribus* increase in any of the three parameters in the investment function of the domestic industrial sector -α, β and γ- has no effect on m^* as they do not feature in (15). However a *ceteris paribus* increase in either α or β or γ decreases k_s^* because the growth rate of capital stock of domestic industrial sector increases while there is no effect on the growth rate of the capital stock of the service sector. Similarly an increase in λ has no effect on m^* but decreases k_s^*. Proposition 7 shows this formally.

Proposition 7. $\frac{\partial m^*}{\partial \Upsilon} = 0 \wedge \frac{\partial k_s^*}{\partial \Upsilon} < 0$ *where* $\Upsilon \in \{\alpha, \beta, \gamma, \lambda\}$

Proof. Partially differentiating (15) with respect to Υ yields $\frac{\partial m^*}{\partial \Upsilon} = 0$ where $\Upsilon \in \{\alpha, \beta, \gamma, \lambda\}$. Partially differentiating (16) with respect to α yields $\frac{\partial k_s^*}{\partial \alpha} = \frac{-1}{\lambda(\beta+\gamma z(m^*))} < 0$ as λ, β and $z(m^*)$ are all positive. Partially differentiating (16) with respect to β yields $\frac{\partial k_s^*}{\partial \beta} = \frac{-(\delta-\alpha)}{\lambda(\beta+\gamma z(m^*))^2} < 0$ as λ and δ are positive and $\delta > \alpha$. Partially differentiating k_s^* with respect to γ yields $\frac{\partial k_s^*}{\partial \gamma} = \frac{-(\delta-\alpha)z(m^*)}{\lambda(\beta+\gamma z(m^*))} < 0$ as δ, λ and $z(m^*)$ are positive $\delta > \alpha$. Finally partially differentiating k_s^* with respect to λ yields $\frac{\partial k_s^*}{\partial \lambda} = \frac{-1}{\lambda^2}[\frac{(\delta-\alpha)}{(\beta+\gamma z(m^*))} + m^*] < 0$ as δ, β, γ, m^* and $z(m^*)$ are positive and $\delta > \alpha$. □

Let us next consider the effect of a *ceteris paribus* increase in the rate of growth of labour productivity of the foreign industrial sector, $\hat{x}_i^f$, on the steady state. From (15), we can see that the effect of an increase in $\hat{x}_i^f$ on the steady state growth rate of labour productivity of the domestic industrial sector is opposite to that of an increase in δ. That is a larger value of $\hat{x}_i^f$ is associated with a larger rate of growth of labour productivity of the domestic industrial sector. Since $f'(m^*) > 0$, the extent of import competition that the domestic industrial sector faces must be greater at the new steady state for (15) to hold. Thus a greater $\hat{x}_i^f$ implies a larger value of m^*. And, from (16), it follows that at the new steady state the capital stock ratio of the service sector relative to that of the domestic industrial sector must be greater. This is because a larger value of m^* means that the growth rate of capital stock of the domestic industrial sector is less than δ at the old value of k_s^*, as both capacity utilization and price mark-up in the sector are now lower. Proposition 8 summarizes this discussion on the effect of a *ceteris paribus* increase in $\hat{x}_i^f$ on the steady state.

Proposition 8. $[f'(m^*) > 0 \longrightarrow \frac{\partial k_s^*}{\partial \hat{x}_i^f} > 0 \wedge \frac{\partial m^*}{\partial \hat{x}_i^f} > 0]$.

Proof. Suppose $f'(m^*) > 0$. Partially differentiating (15) with respect to $\hat{x}_i^f$ yields $\frac{\partial m^*}{\partial \hat{x}_i^f} = \frac{1}{f'(m^*)}$. Since $f'(m^*) > 0$ it follows $\frac{\partial m^*}{\partial \hat{x}_i^f} > 0$. Next, partially differentiating

(16) with respect to $\hat{x}_i^f$ yields

$$\frac{\partial k_s^*}{\partial \hat{x}_i^f} = \frac{1}{\lambda}[\frac{-z'(m^*)(\delta - \alpha)}{(\beta + \gamma z(m^*))^2}\frac{\partial m^*}{\partial \hat{x}_i^f} + \frac{\partial m^*}{\partial \hat{x}_i^f}]$$

Since $\lambda > 0$, $z'(m^*) < 0$ and $\alpha < \delta$, $\frac{\partial m^*}{\partial \hat{x}_i^f} > 0$ implies $\frac{\partial k_s^*}{\partial \hat{x}_i^f} > 0$. □

The effect of a *ceteris paribus* increase in σ - the elasticity of the import function for the manufactured commodity with respect to the technology gap between foreign and domestic industrial sectors - on the steady state is the same as that of $\hat{x}_i^f$. Proposition 9 shows this.

Proposition 9. $[f'(m^*) > 0 \longrightarrow \frac{\partial k_s^*}{\partial \sigma} > 0 \wedge \frac{\partial m^*}{\partial \sigma} > 0]$.

Proof. $f'(m^*) > 0$. Partially differentiating (15) with respect to σ yields $\frac{\partial m^*}{\partial \sigma} = \frac{\partial m^*}{\partial \sigma} = \frac{\delta}{\sigma^2 f'(m^*)}$. Since $\delta > 0$ and $f'(m^*) > 0$ it follows $\frac{\partial m^*}{\partial \sigma} > 0$. Next partially differentiating (16) with respect to σ yields

$$\frac{\partial k_s^*}{\partial \sigma} = \frac{1}{\lambda}[\frac{-z'(m^*)(\delta - \alpha)}{(\beta + \gamma z(m^*))^2}\frac{\partial m^*}{\partial \sigma} + \frac{\partial m^*}{\partial \sigma}]$$

Since $\lambda > 0$, $z'(m^*) < 0$ and $\alpha < \delta$, $\frac{\partial m^*}{\partial \sigma} > 0$ implies $\frac{\partial k_s^*}{\partial \sigma} > 0$. □

Finally, let us consider the effect of an upward shift in the technological progress function of the domestic industrial sector, $f(m)$. If the technological progress function of the domestic industrial sector shifts up then the rate of growth of labour productivity of the domestic industrial sector corresponding to every feasible value of m increases. Since the steady state growth rate of labour productivity of the domestic industrial sector does not change with a *ceteris paribus* upward shift in the technological progress function of the sector and $f'(m^*) > 0$, for the steady state condition (15) to hold m^* must decrease. Since a decrease in m^* means a greater steady state growth rate of capital stock of the industrial sector at the old value of k_s^* (as already explained while discussing the effect of an increase in δ), k_s^* must decrease. We prove this in Proposition 10.

Proposition 10. *Let f_1 and f_2 be such that for all $m \in \mathbb{R}_+$, $f_1(m) < f_2(m)$ and $f_1'(m) > 0$. Let $(k_s^1, m^1) \in \mathbb{R}^2_{++}$ be the steady state corresponding to f_1 and $(k_s^2, m^2) \in \mathbb{R}^2_{++}$ be the steady state corresponding to f_2. Then $k_s^1 > k_s^2$ and $m^1 > m^2$.*

Proof. Since (k_s^1, m^1) and (k_s^2, m^2) are steady states, (15) and (16) hold for both (k_s^1, m^1) and (k_s^2, m^2). From (15), $f_1(m^1) = \hat{x}_i^f - \frac{\delta}{\sigma}$ and $f_2(m^2) = \hat{x}_i^f - \frac{\delta}{\sigma}$. Therefore it follows that $f_1(m^1) = f_2(m^2)$. Let us first suppose $m^1 = m^2$. Since $f_1(m) < f_2(m)$ for all $m \in \mathbb{R}_+$, $m^1 = m^2$ implies $f_1(m^1) < f_2(m^2)$. This is a contradiction. Next, suppose $m^1 < m^2$. Now, $f_1(m^2) < f_2(m^2)$, as $f_1(m) < f_2(m)$ for all $m \in \mathbb{R}_+$. And since $f_1'(m) > 0$ for all $m \in \mathbb{R}_+$, $m^1 < m^2$ implies $f_1(m^1) < f_1(m^2)$. $f_1(m^1) < f_1(m^2)$ and $f_1(m^2) < f_2(m^2)$ imply $f_1(m^1) < f_2(m^2)$, which again is a contradiction. Therefore it must be the case that $m^1 > m^2$. Also, from (16), $k_s^1 = \frac{\delta-\alpha}{\lambda(\beta+\gamma z(m^1))} + \frac{m^1}{\lambda}$ and $k_s^2 = \frac{\delta-\alpha}{\lambda(\beta+\gamma z(m^2))} + \frac{m^2}{\lambda}$. Since $z'(m) < 0$ for all $\mathbb{R}_+$, $m^1 > m^2$ implies $k_s^1 > k_s^2$. □

7. Industrialization and De-industrialization

In the development economics literature, a decline in the output share of the manufacturing sector is often taken as an indicator of de-industrialization, premature or otherwise.[13] In the context of the partial equilibrium nature of our model, instead of the output share, we can examine the ratio of the value of output of the domestic industrial sector to the total value of outputs of domestic industrial sector and the service sector at a locally stable steady state. This ratio, in general, can be defined as

$$\xi_i = \frac{P_i X_i}{P_i X_i + P_s X_s} = \frac{X_i}{X_i + pX_s} \tag{19}$$

where P_s is the constant price of the service and $p = \frac{P_s}{P_i}$ is the relative price of the service in terms of the manufactured product. From (4.4) and (4.5), we can express p as a function of m such that $p(m) = \frac{P_s x_i}{(1+z(m))W}$ with derivative $p'(m) = \frac{-P_s x_i z'(m)}{(1+z(m))^2 W} > 0$ for all $m \in \mathbb{R}_{++}$. Substituting $p(m)$ for p and using (4.1) along with the assumption $\bar{u}_s = 1$ and (4.6), we can express ξ_i as a function of k_s and m such that,

$$\xi_i = \frac{\lambda k_s - m}{\lambda k_s - m + p(m)k_s}$$

Therefore value of ξ_i at a steady state (k_s^*, m^*) is a constant,

$$\xi_i^* = \frac{(\lambda k_s^* - m^*)}{(\lambda k_s^* - m^*) + p(m^*)k_s^*} \tag{20}$$

or, using (16),

$$\xi_i^* = \frac{\lambda(\delta - \alpha)}{(\lambda + p(m^*))(\delta - \alpha) + m^* p(m^*)(\beta + \gamma z(m^*))} \tag{21}$$

Even though it is tempting to interpret increase (decrease) in ξ_i^* as industrialization (de-industrialization), it is important to emphasize that industrialization and de-industrialization are macro phenomena whereas the domestic industrial sector of our model will generally constitute only a segment of the industrial sector in any economy. Nonetheless, an increase (decrease) in ξ_i^*, implies a transition process or a process of structural change in which there is more (less) than proportionate growth in the

[13] For example see Rodrik (2016).

output of the domestic industrial sector compared to that of the service sector in the model. Therefore, it more apt to interpret any factor that increases (decreases) ξ_i^* as a possible contributing factor to industrialization (de-industrialization) in the economy.

Let us now consider reasons for changes in ξ_i^* in the model. It can be easily verified from (21) that an increase in either α, β or γ decrease ξ_i^* whereas an increase in λ increases ξ_i^*. Other than these parameters, ξ_i^* also depends upon m^* and δ. However changes in either m^* or δ do not have unambiguous effects on ξ_i^*. For example, if m^* increases then $p(m^*)$ and $m^*p(m^*)$ increase, which tend to decrease ξ_i^*, whereas $z(m^*)$ decreases, which tends to increase ξ_i^*. Intuitively, an increase in m^* has two effects on ξ_i^*- one positive and the other negative. The positive effect is due to the fall in the mark-up of the domestic industrial sector which, tends to decrease the growth rate of capital stock of the sector, and as a result of which k_s^* tends to increase as the steady state growth rate of capital stock of the service sector does not change. A higher value of k_s^*, *ceteris paribus*, means a higher steady state rate of capacity utilization $(\lambda k_s^* - m^*)$ and therefore greater levels of output along the steady state for the sector.[14] The negative effect, on the other hand, is due to the fact that an increase in m^* also tends to reduce value of output of the domestic industrial sector along the steady state. This is because, one, the price of the domestic industrial sector's product relative to that of the service falls, as $p'(m^*) > 0$. And two, a higher value of m^*, *cetris paribus*, means a lower value of $(\lambda k_s^* - m^*)$ and therefore lower levels output along the steady state. Proposition 11 shows if the absolute value of the elasticity of the price mark-up with respect to the extent of import competition is less than one at the steady state, i.e $\sigma_z^* > -1$, then the negative effect dominates the positive effect and therefore, an increase in m^* decreases ξ_i^*.

Proposition 11. $\sigma_z^* = \frac{m^*z'(m^*)}{z(m^*)} > -1 \longrightarrow \frac{\partial \xi_i^*}{\partial m^*} < 0$

Proof. Suppose $\sigma_z^* = \frac{m^*z'(m^*)}{z(m^*)} > -1$. Differentiating (21) with respect to m^* yields

$$\frac{\partial \xi_i^*}{\partial m^*} = -\frac{\lambda(\delta-\alpha)^2 p'(m^*)}{\{(\lambda+p(m^*))(\delta-\alpha)+m^*p(m^*)(\beta+\gamma z(m^*))\}^2} - \frac{\lambda(\delta-\alpha)\{(p(m^*)+m^*p'(m^*))(\beta+\gamma z(m^*))+m^*p(m^*)\gamma z'(m^*)\}}{\{(\lambda+p(m^*))(\delta-\alpha)+m^*p(m^*)(\beta+\gamma z(m^*))\}^2}$$

[14] In case of an increase in α, β or γ the opposite is true as growth rate of capital stock of the industry sector increases. On the other hand, in case of an increase in λ, there is no effect on the steady state degree of capacity utilization in the domestic industrial sector as it can be easily verified, using (16) and Proposition 7, that $\frac{\partial(\lambda k_s^* - m^*)}{\partial \lambda} = 0$. However, the term $p(m^*)k_s^*$ in the denominator of (20) decreases since, from Proposition 7, $\frac{\partial k_s^*}{\partial \lambda} < 0$ and therefore ξ_i^* increases. A lower value of $p(m^*)k_s^*$ means lower service sector output along the steady state, expressed in terms of the output of the domestic industrial sector.

Since λ, $p'(m^*)$ and $\delta - \alpha$ are all positive, $\frac{d\xi_i^*}{dm^*} < 0$ if $(p(m^*)+m^*p'(m^*))(\beta+\gamma z(m^*))+m^*p(m^*)\gamma z'(m^*) > 0$. Now, $(p(m^*) + m^*p'(m^*))(\beta + \gamma z(m^*)) + m^*p(m^*)\gamma z'(m^*) = m^*p'(m^*)(\beta + \gamma z(m^*)) + \beta p(m^*) + \frac{\gamma p(m^*)(1+\sigma_z^*)}{z(m^*)}$, which is positive if $\sigma_z^* > -1$ as β, γ, m^*, $p(m^*)$, $p'(m^*)$ and $z(m^*)$ are all positive. □

We know from Corollary 5.1 that the steady state (k_s^*, m^*) is locally asymptotically stable if $f'(m^*) > 0$ and $\sigma_z^* > -1$. Also, from the comparative statics exercises of the previous section, we know that $f'(m^*) > 0$ implies an increase in $\hat{x}_i^f$ or σ increases m^* whereas an upward shift in the function $f(m)$ decreases m^*. Therefore if $f'(m^*) > 0$ and $\sigma_z^* > -1$ then it follows from Proposition 11 that an increase in the pace of growth of technological progress abroad, $\hat{x}_i^f$, or in the elasticity of imports, σ, decreases ξ_i^*, whereas any exogenous improvement in the conditions of technological progress in the domestic industrial sector increases ξ_i^*.

The effect of an increase in the autonomous investment rate of the service sector on ξ_i^* is, however, slightly more complicated. Note that partially differentiating (21) with respect to δ yields

$$\frac{\partial \xi_i^*}{\partial \delta} = \frac{\lambda}{A^2}[A - (\delta - \alpha)p(m^*) - (\delta - \alpha)B\frac{\partial m^*}{\partial \delta}] \tag{22}$$

where $A = (\lambda+p(m^*))(\delta-\alpha)+m^*p(m^*)(\beta+\gamma z(m^*))$ and $B = (p(m^*)+m^*p'(m^*))(\beta+\gamma z(m^*)) + m^*p(m^*)\gamma z'(m^*)$. Now $\lambda > 0$, $A > 0$ and $(\delta - \alpha)p(m^*) > 0$. On the other hand, the sign of B is ambiguous as $z'(m^*) < 0$ and, from Proposition 6, $\frac{\partial m_i^*}{\partial \delta} < 0$ if $f'(m^*) > 0$. Therefore, it follows from (22) that the sign of $\frac{\partial \xi_i^*}{\partial \delta}$ is ambiguous. However, it is possible that at values of m^* close to zero, $\frac{\partial \xi_i^*}{\partial \delta} > 0$. To see this, first note that, from the proof Proposition 11, $B > 0$ if $\sigma_z^* > -1$. Also, $p'(m^*) > 0$ and $\lim_{m^* \to 0} p(m^*) = \frac{P_s x_i}{(1+\bar{z})W}$. Then, it follows from (22) that if $\frac{P_s x_i}{(1+\bar{z})W}$ is small enough then $f'(m^*) > 0$ and $\sigma_z^* > -1$ implies that $\frac{\partial \xi_i^*}{\partial \delta} > 0$ at value of m^* is sufficiently close to zero.

8. Convergence in Manufacturing Productivity Growth

Rodrik (2013) shows a strong tendency for unconditional convergence in manufacturing productivity growth. This is at odds with the analysis presented so far in the paper as the technology gap between domestic and foreign industrial sectors μ grows at a constant rate in the steady state. The convergence result of Rodrik (2013) however points to an average tendency. For example, according to Amirapu and Subramanian (2015), the average labour productivity growth of Indian manufacturing sectors is 14 percent lower than the world figure whereas for China it is 17 percent higher.[15] Nevertheless, in this section, we examine implications of convergence in a simple variant of the model presented in the previous sections.

In this section, we want the growth rate of labour productivity of the domestic industrial sector to become equal to that of the foreign industrial sector in a steady state. For this purpose, first of all, we need to replace the imports function (3) with

$$M = \theta(\mu)\lambda K_s \tag{23}$$

where $\theta(\cdot)$ is a function $\theta : (1,\infty) \mapsto (0,1)$ with derivative $\theta'(\mu) > 0$ for all $\mu \in (1,\infty)$. Thus we now assume that total imports of the manufactured commodity is a fraction of its demand, i.e. λK_s and that fraction is an increasing function of technology gap μ. The disadvantage of our earlier imports function (3) is the following. Since we are looking for convergence in labour productivity growth rates of domestic and foreign industrial sectors in a steady state, (3) implies that the extent of import competition that domestic industry faces m is zero at any steady state where μ is a constant. On the other hand, with (23) we can have $m > 0$ at such steady states. Now, (23) implies short run output of domestic industrial sector is

$$X_i = (1 - \theta(\mu))\lambda K_s \tag{24}$$

For simplicity, we assume that flexible mark-up pricing has no effect on the investment rate of domestic industry, i.e. $\tilde{\gamma} = 0$ in (7). Substituting for X_i from (24) to (7) along with the assumption $\tilde{\gamma} = 0$ yields the following expression for rate of capital stock of domestic industrial sector.

$$g_i = \alpha + \beta(1 - \theta(\mu))\lambda k_s \tag{25}$$

Next, a strong tendency for convergence of productivity growth for manufacturing means economies with the most inferior technology compared to the frontier experience the fastest growth rate of productivity in manufacturing industries. In our terms we can capture this by postulating the growth rate of labour productivity of domestic industry to be an increasing function of the technology gap μ. Thus we replace (11) by

$$\hat{x}_i = \chi(\mu) \tag{26}$$

where χ is a function $\chi : (1,\infty) \mapsto \mathbb{R}_{++}$ with derivative $\chi'(\mu) > 0$ for all $\mu \in (1,\infty)$.

[15] Moreover according to Amirapu and Subramanian (2015) there is almost no convergence when it comes to Indian industries in the core specification of Rodrik (2013). Amirapu and Subaramanian acknowledge Rodrik in a footnote for this information.

So the growth rate of μ now is

$$\hat{\mu} = \hat{x}_i^f - \chi(\mu) \tag{27}$$

From (10), (25) and (27), the long-run dynamics is now captured by the following system of two differential equations in two variables $k_s \in \mathbb{R}_{++}$ and $\mu \in (1, \infty)$.

$$\begin{aligned} \dot{k}_s &= k_s\{\delta - \alpha - \beta(1 - \theta(\mu))\lambda k_s\} \\ \dot{\mu} &= \mu(\hat{x}_i^f - \chi(\mu)) \end{aligned} \tag{28}$$

A meaningful steady state of (28) requires $(\exists k_s \in (0, \infty))(\exists \mu \in (1, \infty))(\delta - \alpha - \beta(1 + \theta(\mu))\lambda k_s = 0 \wedge \hat{x}_i^f - \chi(\mu) = 0)$. Proposition 12 shows, like Proposition 2 in case of the previous model, that $\delta > \alpha$ and a couple of boundary conditions on the function χ - specifically, $\lim_{\mu \to 1} \chi(\mu) < \hat{x}_i^f$ and $\lim_{\mu \to \infty} \chi(\mu) > \hat{x}_i^f$ - ensure existence and uniqueness of steady state of (28).

Proposition 12. *Let $\delta > \alpha$. Then there exists a unique steady state of (28) if $\lim_{\mu \to 1} \chi(\mu) < \hat{x}_i^f < \lim_{\mu \to \infty} \chi(\mu)$.*

Proof. It is given that $\delta > \alpha$. Suppose $\lim_{\mu \to 1} \chi(\mu) < \hat{x}_i^f < \lim_{\mu \to \infty} \chi(\mu)$. Define $\varphi(\mu) = \hat{x}_i^f - \chi(\mu)$. $\varphi(\mu)$ is a continuous function on $(1, \infty)$ as $\chi(\mu)$ is a continuous function on $(1, \infty)$. Next, $\lim_{\mu \to 1} \chi(\mu) < \hat{x}_i^f < \lim_{\mu \to \infty} \chi(\mu)$ implies $\lim_{\mu \to 1} \varphi(\mu) < 0 < \lim_{\mu \to \infty} \varphi(\mu)$. Now, $\lim_{\mu \to 1} \varphi(\mu) < 0$ means for values of $\mu \in (1, \infty)$ sufficiently close to one, $\varphi(\mu) > 0$. And $\lim_{\mu \to \infty} \varphi(\mu) > 0$ means for sufficiently large values of $\mu \in (1, \infty)$, $\varphi(\mu) < 0$. Let $\mu_1 \in (1, \infty)$ and $\mu_2 \in (1, \infty)$ be such that $\varphi(\mu_1) > 0$ and $\varphi(\mu_2) < 0$. Then, it follows from the *Intermediate Value Theorem* that there exists $\mu \in [\mu_1, \mu_2]$ such that $\varphi(\mu) = 0$. Moreover, since $\varphi'(\mu) = -\chi'(\mu) < 0$ for all $\mu \in (1, \infty)$ as $\chi'(\mu) > 0$ for all $\mu \in (1, \infty)$, there can be only one value of μ for which $\varphi(m) = 0$. Let $\tilde{\mu} \in (1, \infty)$ be such that $\varphi(\tilde{\mu}) = 0$ and let $\tilde{k}_s = \frac{\delta - \alpha}{\beta(1 - \theta(\tilde{\mu}))\lambda}$. $\delta > \alpha$ implies $k_s^* \in \mathbb{R}_{++}$ as β and λ are positive and $\theta(\tilde{\mu}) \in (0, 1)$. Thus, $(\tilde{k}_s, \tilde{\mu})$ is the unique steady state of (28). □

In rest of the section we will assume that conditions given by Proposition 12 for existence and uniqueness of steady state of (28) hold. Let $(\tilde{k}_s, \tilde{\mu})$ be the steady state of (28). Then, from the proof of Proposition 12, we have

$$\hat{x}_i^f = \chi(\tilde{\mu}) \tag{29}$$

$$\tilde{k}_s = \frac{\delta - \alpha}{\beta(1 - \theta(\tilde{\mu}))\lambda} \tag{30}$$

Like the previous model, in this model too, both output and capital stock of the domestic industrial sector in steady state grow at the same rate at which capital stock of the service sector grows, i.e. δ. The steady state growth rate of employment in this case is $\delta - \hat{x}_f^f$, which is positive if and only if $\delta > \hat{x}_f^f$. The extent of import competition faced by the domestic industrial sector is also a constant in this steady state, given by $\tilde{m} = \theta(\tilde{\mu})\lambda\tilde{k}_s$. The main difference between steady states in the two models, however, is that (29) implies technology gap between the domestic and foreign industry sector, μ, is constant in steady state of this model whereas (15) implies μ grows at a constant positive rate $\frac{\delta}{\sigma}$ in steady state of the previous model. Moreover, the tendency for convergence ensures that the steady state is locally asymptotically stable, as shown below in Proposition 13.

Proposition 13. $(\tilde{k}_s, \tilde{\mu})$ *is locally asymptotically stable steady state of* (28).

Proof. The Jacobian matrix of (28) evaluated at the steady state $(\tilde{k}_s, \tilde{\mu})$ is

$$\begin{bmatrix} -\tilde{k}_s\beta(1-\theta(\tilde{\mu}))\lambda & \tilde{k}_s^2\beta\theta'(\tilde{\mu})\lambda \\ 0 & -\tilde{\mu}\chi'(\tilde{\mu}) \end{bmatrix}$$

with trace $-\tilde{k}_s\beta(1-\theta(\tilde{\mu}))\lambda - \tilde{\mu}\chi'(\tilde{\mu})$ and determinant $\tilde{k}_s\beta(1-\theta(\tilde{\mu}))\lambda\tilde{\mu}\chi'(\tilde{\mu})$. Since $\chi'(\mu) > 0$, it follows that trace is negative and determinant is positive as $\tilde{k}_s$, $\tilde{m}$, β and λ are all positive and $\theta(\tilde{\mu}) < 1$. □

At the steady state, the technology gap between the foreign and the domestic industrial sectors $\tilde{\mu}$ is completely determined by the rate growth of labour productivity of the foreign industrial sector $\hat{x}_i^f$ and the technological progress function of the domestic industrial sector (26). On the other hand, the steady state relative capital stock of the service sector $\tilde{k}_s$ depends not only on $\tilde{\mu}$ but also on parameters of the investment functions of the two sectors - α, β and δ - and demand for the manufactured product per unit service output λ. Propositions 14 shows that the effect of an increase in α, β and λ on the steady state is same in this model as in the previous model, given by Proposition 7.

Proposition 14. $\frac{\partial\tilde{\mu}}{\partial\Upsilon_1} = 0 \wedge \frac{\partial\tilde{k}_s}{\partial\Upsilon_1} < 0$ *where* $\Upsilon_1 \in \{\alpha, \beta, \lambda\}$

Proof. Partially differentiating (29) with respect to Υ_1 yields $\frac{\partial\tilde{\mu}}{\partial\Upsilon_1} = 0$ where $\Upsilon_1 \in \{\alpha, \beta, \lambda\}$. Partially differentiating (30) with respect to α yields $\frac{\partial\tilde{k}_s}{\partial\alpha} = -\frac{1}{\beta(1-\theta(\tilde{\mu}))\lambda} < 0$ as β and λ are positive and $\theta(\mu) \in (0, 1)$. Next, partially differentiating (30) with respect to β yields $\frac{\partial\tilde{k}_s}{\partial\alpha} = -\frac{(\delta-\alpha)}{\{\beta(1-\theta(\tilde{\mu}))\lambda\}^2} < 0$ as δ, β and λ are positive, $\theta(\mu) \in (0, 1)$ and $\delta > \alpha$. Finally, partially differentiating (30) with respect to λ yields $\frac{\partial\tilde{k}_s}{\partial\alpha} = -\frac{(\delta-\alpha)}{\{\beta(1-\theta(\tilde{\mu}))\lambda\}^2} < 0$ as δ, β and λ are positive, $\theta(\mu) \in (0, 1)$ and $\delta > \alpha$. □

Effect of an increase in δ on the steady state is however different in this model compared to that in the previous one given by Proposition 6. Proposition 15 shows that an increase in δ has no effect on $\tilde{\mu}$ but increases $\tilde{k}_s$. This means that the steady state extent of import competition $\tilde{m} = \theta(\mu)\lambda\tilde{k}_s$ in this case increases as a result of an increase in δ whereas in the previous model at the locally stable steady, m^*, from Proposition 6, decreased.

Proposition 15. $\frac{\partial\tilde{\mu}}{\partial\delta} = 0 \wedge \frac{\partial\tilde{k}_s}{\partial\delta} > 0$

Proof. Partially differentiating (29) with respect to δ yields $\frac{\partial\tilde{\mu}}{\partial\delta} = 0$. And partially differentiating (30) with respect to δ yields $\frac{\partial\tilde{k}_s}{\partial\delta} = \frac{1}{\beta(1-\theta(\tilde{\mu}))\lambda} > 0$ as β and λ are positive and $\theta(\mu) \in (0, 1)$. □

On the other hand, an increase in the pace of technological progress in the foreign industrial sector, x_i^f, or an upward shift in the technological progress function of the domestic industrial sector has the same effect on the steady sate of this model as in the locally stable steady state of the previous model, given by Propositions 8 and 10. Proposition 16 shows an increase in $\hat{x}_i^f$ increases both $\tilde{\mu}$ and $\tilde{k}_s$ whereas Proposition 17 shows if the technological progress function of the domestic industrial sector, given by (26), shifts up then both $\tilde{\mu}$ and $\tilde{k}_s$ decrease. Therefore the steady state extent of import competition $\tilde{m} = \theta(\tilde{\mu})\lambda\tilde{k}_s$ increases because of an increase in x_i^f whereas decreases when the function $\chi(\mu)$ shifts up.

Proposition 16. $\frac{\partial\tilde{\mu}}{\partial\hat{x}_i^f} > 0 \wedge \frac{\partial\tilde{k}_s}{\partial\hat{x}_i^f} > 0$

Proof. Partially differentiating (29) with respect to $\hat{x}_i^f$ yields $\frac{\partial\tilde{\mu}}{\partial\hat{x}_i^f} = \frac{1}{\chi'(\tilde{\mu})} > 0$ as $\chi'(\tilde{\mu}) > 0$. And partially differentiating (30) with respect to $\hat{x}_i^f$ yields $\frac{\partial\tilde{k}_s}{\partial\hat{x}_i^f} = \frac{(\delta-\alpha)\beta\theta'(\tilde{\mu})\lambda}{\{\beta(1-\theta(\tilde{\mu}))\lambda\}^2} > 0$ as δ, β and λ are positive, $\delta > \alpha$ and $\theta'(\mu) > 0$. □

Proposition 17. *Let χ_1 and χ_2 be such that for all $\mu \in (1, \infty)$, $\chi_1(\mu) < \chi_2(\mu)$. Let $(k_{s1}, \mu_1) \in \mathbb{R}_{++} \times (1, \infty)$be the steady state of (28) corresponding to χ_1 and let $(k_{s2}, \mu_2) \in \mathbb{R}_{++} \times (1, \infty)$be the steady state of (28) corresponding to χ_2. Then, $k_{s1} > k_{s2}$ and $\mu_1 > \mu_2$.*

Proof. Since (k_{s1}, μ_1) and (k_{s2}, μ_2) are both steady states of (28), steady state conditions (29) and (30) hold at (k_{s1}, μ_1) and (k_{s2}, μ_2) for χ_1 and χ_2 respectively. Thus, from (29), $\chi_1(\mu_1) = \chi_2(\mu_2)$. Let us first suppose $\mu_1 = \mu_2$. Since $\chi_1(\mu) < \chi_2(\mu)$ for all $\mu \in (1, \infty)$, $\mu_1 = \mu_2$ implies $\chi_1(\mu_1) < \chi_2(\mu_2)$. This, however, is a contradiction. Next, suppose $\mu_1 < \mu_2$. Now, $\mu_1 < \mu_2$ implies $\chi_1(\mu_1) < \chi_1(\mu_2)$ as $\chi_1'(\mu) > 0$ for all $\mu \in (1, \infty)$. And since $\chi_1(\mu) < \chi_2(\mu)$ for all $\mu \in (1, \infty)$, $\chi_1(\mu_2) < \chi_2(\mu_2)$. Thus it follows $\chi_1(\mu_1) < \chi_2(\mu_2)$, which again is a contradiction. Then, it must be the case that $\mu_1 > \mu_2$. Also, from (30), $k_{s1} = \frac{\delta-\alpha}{\beta(1-\theta(\mu_1))\lambda}$ and $k_{s2} = \frac{\delta-\alpha}{\beta(1-\theta(\mu_2))\lambda}$. Now, $\mu_1 > \mu_2$

implies $\theta(\mu_1) > \theta(\mu_2)$ as $\theta'(\mu) > 0$ for all $\mu \in (1, \infty)$. Therefore, it follows from $\theta(\mu_1) > \theta(\mu_2)$ that $k_{s1} > k_{s2}$. □

In the previous model, we discussed factors that can contribute to industrialization or de-industrialization using the value of the ratio ξ at locally stable steady state. In this model too, we can derive a similar expression for ξ_i at the steady state $(\tilde{k}_s, \tilde{\mu})$. Substituting in (19) for X_i form (24) and X_s from (1) along with the assumption $\bar{u}_s = 1$, we obtain

$$\xi_i = \frac{(1-\theta(\mu))\lambda}{p(m) + (1-\theta(\mu))\lambda}$$

where, from (23), $m = \theta(\mu)\lambda k_s$ and $p(m) = \frac{P_s}{P_i}$ is same as in section 7. Thus, value of ξ_i at $(\tilde{k}_s, \tilde{\mu})$ is

$$\tilde{\xi}_i = \frac{(1-\theta(\tilde{\mu}))\lambda}{p(\tilde{m}) + (1-\theta(\tilde{\mu}))\lambda} \tag{31}$$

where $\tilde{m} = \theta(\tilde{\mu})\lambda\tilde{k}_s$. Proposition 18 shows that an increase in either $\tilde{\mu}$ or $\tilde{k}_s$ unambiguously decreases $\tilde{\xi}_i$.

Proposition 18. $\frac{\partial \tilde{\xi}_i}{\partial \tilde{k}_s} < 0 \wedge \frac{\partial \tilde{\xi}_i}{\partial \tilde{\mu}} < 0$

Proof. Partially differentiating (31) with respect to $\tilde{k}_s$ yields

$$\frac{\partial \tilde{\xi}_i}{\partial \tilde{k}_s} = -\frac{(1-\theta(\tilde{\mu}))\lambda^2 p'(\tilde{m})\theta(\tilde{\mu})}{\{p(\tilde{m}) + (1-\theta(\tilde{\mu}))\lambda\}^2}$$

Since $p'(\tilde{m})$ is positive and $0 < \theta(\tilde{\mu}) < 1$, it follows that $\frac{\partial \tilde{\xi}_i}{\partial \tilde{k}_s} < 0$. Similarly, partially differentiating (31) with respect to $\tilde{\mu}$ yields,

$$\frac{\partial \tilde{\xi}_i}{\partial \tilde{\mu}} = -\frac{\{p(\tilde{m})\theta'(\tilde{\mu})\lambda + (1-\theta(\tilde{\mu}))\lambda p'(\tilde{m})\frac{\partial \tilde{m}}{\partial \tilde{\mu}}\}}{\{p(\tilde{m}) + (1-\theta(\tilde{\mu}))\lambda\}^2}$$

Since λ, $p(\tilde{m})$, $p'(\tilde{m})$, $\theta'(\tilde{\mu})$ are all positive and $\theta(\tilde{\mu}) \in (0,1)$, $\frac{\partial \tilde{\xi}_i}{\partial \tilde{\mu}} < 0$ if $\frac{\partial \tilde{m}}{\partial \tilde{\mu}} > 0$. Now, differentiating $\tilde{m} = \theta(\tilde{\mu})\lambda\tilde{k}_s$ with respect to $\tilde{\mu}$ yields, $\frac{\partial \tilde{m}}{\partial \tilde{\mu}} = \theta'(\tilde{\mu})\lambda\tilde{k}_s + \theta(\tilde{\mu})\lambda\frac{\partial \tilde{k}_s}{\partial \tilde{\mu}} = \theta'(\tilde{\mu})\lambda\tilde{k}_s\{1 + \frac{\theta(\tilde{\mu})}{\beta(1-\theta(\tilde{\mu}))\lambda}\}$, where the last equality is obtained using (30). Since β, λ, $\theta'(\tilde{\mu})$ and $\tilde{k}_s$ are all positive and $\theta(\tilde{\mu}) \in (0,1)$, it follows that $\frac{\partial \tilde{m}}{\partial \tilde{\mu}} > 0$ and, as a result, $\frac{\partial \tilde{\xi}_i}{\partial \tilde{\mu}} < 0$. □

Propositions 16, 17 and 18 imply that an increases in $\hat{x}_i^f$ unambiguously decrease $\tilde{\xi}_i$ whereas an upward shift of the $\chi(p)$ unambiguously increase it. Thus, even in presence of a strong tendency towards convergence in manufacturing productivity growth, exogenous improvements in the conditions for technological progress in the domestic industrial sector can contribute towards industrialization whereas an increase in the pace of technological progress in the frontier can contribute towards de-industrialization. [16]

9. Conclusion

The advent of 'premature de-industrialization' means developing economies are going to find it increasingly difficult to effect sustained growth and development relying on industry alone. Many economists see hope instead in services like finance, telecommunications and software, which because of advances in information and communication technologies have become tradable as well as technologically dynamic.[17] In this paper we argue that expansion of these services in a developing economy, by generating demand, can contribute towards growth of its industrial sector, or at least a segment of it. Nonetheless, in the context of trade liberalization, the domestic manufactures in developing economies may be constrained by competition from technologically superior foreign competitors. For the majority of this paper, we try to bring these ideas together in a partial equilibrium growth model where a typical Kaleckian domestic industrial sector competes with technologically superior foreign manufacturers for final demand generated by an autonomously growing service sector for a particular manufacturing product. In order to focus on the implications of technology gap, we use a reduced form version of the import function generally used in the BPCG literature by assuming imports of the manufactured product depend only on the technology gap between the domestic industrial sector and its foreign competitor(s). We show that in this model, it is possible to have a long-run steady state in which output and capital stock of the domestic industrial sector grow at the same rate as the capital stock of the service sector. Moreover, if the autonomous growth rate of the service sector is sufficiently large then there can also be positive employment growth in the domestic industrial sector at this steady state.[18]

However, convergence to this steady state crucially depends on the nature of technological progress in the domestic industrial sector and the resulting dynamics of its technology gap *vis-a-vis* foreign manufacturers. For example, if labour productivity growth in the domestic industrial sector is negatively related to the extent of its import competition then we show that the steady state is unstable. On the other hand, if increasing import competition forces the domestic industrial sector to innovate,

[16]Increase in parameters of the two for investment functions in this model have very different effect on $\tilde{\xi}_i$ compared to that on ξ_i^*. For example, an increase in δ unambiguously decreases $\tilde{\xi}_i$ whereas, in the previous section, an increase in δ can increase ξ_i^*. Similarly, while an increase in either α or β decreases ξ_i^* in the previous section, $\tilde{\xi}_i$ increases. However, parameters of investment functions affect $\tilde{\xi}_i$ only through the steady state relative price of service $p(\tilde{m})$ in this model.

[17]See for example Rodrik (2016), Dasgupta and Singh (2006), and Eichengreen and Gupta (2013)

[18]This is also true if pace of technological progress enjoyed by the foreign competitors of the domestic industrial sector is sufficiently low and/or imports of the manufacturing product are less responsive to technology gap between domestic and foreign producers.

thus increasing the growth rate of its labour productivity, then the steady state can be locally stable. A key for convergence in this case, as shown by Corollary 5.1, is the responsiveness of the price mark-up in the domestic industrial sector to changes in the extent of import competition as captured by the ratio of imports of the manufactured product to its capital stock, m. When conditions for local stability of this steady state as given by either Proposition 5 or Corollary 5.1, then Propositions 8 and 9 show that, a higher rate of technological progress of abroad or a higher elasticity of the imports function means that the domestic industrial sector faces greater import competition in the steady state, m^*. On the other hand, Propositions 6 and 10 show that, faster rates of growth in the service sector and exogenous improvements in the pace of technological progress in the domestic industrial sector have the exactly opposite effect on m^*.

In section 7, we argue that increase (decrease) in the ratio of the value of output of the domestic industrial sector to the total value of outputs of domestic industrial sector and the service sector at a locally stable steady state, ξ_i^*, implies a process of structural change in which output of the domestic industrial sector grows at a proportionately higher (lower) rate than that of the service sector. We show that ξ_i^* depends on m^* as well as the demand for the manufactured product per unit service output, λ, the growth rate of the service sector, δ, and parameters of the investment of the domestic industrial sector. Proposition 11 shows that when conditions for local stability, as given by Corollary 5.1 holds, then an increase in m^* decreases ξ_i^*. This result along with Proposition 10 then suggests that exogenous improvements in the pace of technological progress in manufacturing sectors that depend on services for demand can contribute towards industrialization in developing economies. Similarly, Propositions 8 and 9, together with Proposition 11, suggest that an increase in the growth rate of labour productivity of foreign competitors of the domestic industrial sector and in the elasticity of the imports function can contribute towards de-industrialization. In this section we also show that ξ_i^* can increase as a result of increase in λ whereas an increase in either α, β or γ decreases ξ_i^*. The effect of an increase in δ on ξ_i^* is, however, ambiguous.

Another property of the steady state of this model is that the technology gap between the domestic and the foreign industrial sectors, μ, grows at a constant rate. This persistent lack of convergence in labour productivity growth of these two sectors, however, may be perceived as a limitation of the model, given the empirical evidence provided by Rodrik (2013) in favour of a strong tendency towards unconditional convergence in manufacturing productivity growth. In section 8, we consider a simple alternative the model in which we assume, in order to account for a tendency towards convergence, that labour productivity growth in the domestic industrial sector is a strictly increasing function of the technology gap, μ, and a slightly modified version of the import function. In this version of the model, too, we show that a locally stable long-run steady state can exist in which output and capital stock of the domestic industrial sector grows at the same rate as the capital stock of the service sector along with a constant technology gap $\tilde{\mu}$. Propositions 16 and 17 along with Proposition 18 suggest that even in presence of a tendency towards convergence in manufacturing productivity growth, exogenous improvements in the growth rate of labour productivity in the domestic industrial sector can contribute towards industrialization whereas an increases in growth rate of labour productivity of its foreign competitor can contribute towards de-industrialization.

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