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Modern services led growth and development in a structuralist dual economy: long-run implications of skilled labor constraint*

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Abstract

Motivated by the South Asian experience, this paper highlights the importance of the rate of expansion of skilled labor force for service-led growth and development in economies characterized by large populations and low average education and skill endowments using a dual economy model. The model economy consists of a skilled-labor intensive service sector and a skilled-labor indifferent industrial sector - both Kaleckian, in the sense that they maintain excess capacity and operate under conditions of imperfect competition. Labor market is fragmented. There is unlimited supply of unskilled labour but skilled labor is scarce and grows only at a finite rate. Growth of skilled labor supply is only fractionally explained by growth in real wage of skilled labor while the rest depends on education policy of the government. Since government policies take time to adjust to the needs of the private sector, we argue that effect of education policy on skilled labor supply growth can be treated as autonomous. The main result of this paper shows that the model economy can converge to a steady state characterized by balanced sectoral growth at a rate equal to the autonomous part of skilled labor growth. Also, increase in returns to skilled labor can drive up the output share of modern services as the two are positively related in the steady state. The model also shows that the supply side can determine growth in structuralist models despite persistence of unemployed resources.

JEL Codes: O14, O15, O41, I25

Keywords: developing economies, dual economy model, modern services, skilled labor, education policy

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1. Introduction

A remarkable aspect of recent growth experience in South Asia is rapid growth of services that are receptive to modern information and communication technologies (ICTs) such as telecommunications, banking & finance, software & IT services and various other business services. These services are often collectively referred to as modern services in the literature. For some time now, modern services have been among the fastest growing sectors in India and have increased their combined share in GDP from an average of 3.5 percent in 1950-70 to an average of 18.4 percent in 2000-08. (Eichengreen and Gupta 2011) In other South Asian economies, like Bangladesh and Sri Lanka, too these services have grown at a faster rate than other services between 2000 and 2006. (Ghani 2010) The rise in importance of such services in South Asian economies, particularly in the Indian economy, has prompted calls in the literature to take services seriously as a leading sector for growth and development. (Bosworth and Maertens 2010; Ghani 2010; Eichengreen and Gupta 2011; Eichengreen and Gupta 2013)

A characteristic feature of modern services is their skill intensiveness. For example, in case of India, Nayyar (2013), using various NSSO Employment and Unemployment Surveys between 1983 and 2004-05, finds that 53.3%, 49% and 25.1% of the sectoral labor force of business services, banking & finance and telecommunications respectively were graduates and above. Nayyar (2013) complements these statistics by showing that the percentage of labor force in these sectors constituted by professional, technical, executive and managerial workers is also high. Similar statistics are also provided by Amirapu and Subramanian (2015). Bosworth and Maertens (2010) find evidence of higher average years of schooling of employees in service sector compared to both agriculture and industry in almost all South Asian economies. This paper highlights the importance of the rate of expansion of skilled labor force for service-led growth and development in economies characterized by large populations and low average education and skill endowments using a two-sector structural growth model consisting of industry and a service sector.

While there is an extremely rich dual economy literature that focuses on transition of a primitive agricultural economy into a modern industrial economy, services are generally invoked in the context of 'de-industrialization' in advanced economies. Baumol (1967) argues that services are technological stagnant activities compared to industry, which increases employment share of the service sector in advanced economies causing stagnation. Dutt (1992) shows that Baumol's argument is crucially based on

the assumption of full employment of resources. Moreover, advances in ICTs and emergence of modern services has narrowed the distinction between industry and service sector in terms technological progressiveness. In our model, we distinguish between the two sectors by assuming that the service sector employs only skilled labor whereas industry is indifferent about employing skilled and unskilled labor. Further, in order to emphasise on scarcity of skilled labor in developing economies, we assume, one, unskilled labor supply is always perfectly elastic and, two, increase in real wage of skilled labor only fractionally explains increase in skilled labor supply in the long run.

Such an assumption regarding skilled labor growth is a simplification it can, nevertheless, be justified. Growth of the real wage of skilled labor can increase the rate of growth of skilled labor supply as increasing returns to skilled labor, which is expected when skill intensive sectors expand, can induce both demand for and private provisioning of skill training.¹ However both private and public higher educational institutions impart skill training in developing economies. Education policies of the government, particularly those regarding accessibility, affordability and funding of public higher educational institutions, have an effect on skilled labor supply growth. Since government policies take time to adjust to the needs of the private sector, not that they are necessarily designed to do so, we can treat their effect on the growth rate of skilled labor supply as autonomous. Our main result indicates that government's education policy can be crucial if developing economies are to bank on modern services. This is because we show that the model economy can converge to a long-run steady state where both sectors grow at the same rate determined by the autonomous part of skilled labor growth. This result is achieved despite the fact that we allow for excess capacity in both the sectors and there is no scarcity of unskilled labor. Another important implication of our analysis is that increase in returns to skilled labor can drive up the output share of modern services as we show that the two are positively related in the steady state.

The paper is organised in the following manner. Next section describes the main assumptions of the model. Section 3 presents the short run of the model, where capital stocks of both sectors and all prices are given and output levels adjust to equate demand and supply. Section 4 discusses the long-run dynamics of the model caused by growth in capital stocks of the two sectors as well as real wage and supply

¹For example, Dhalman (2010) notes the growth in private provision of higher education in response to rapid service sector growth in India, at times initiated by the software industry itself.

of skilled labor. While in the main model, only consumption demand for service is considered, section 5 contains an extension which also considers service input demand from industry. And finally, section 6 concludes the paper.

2. Model

Consider a closed developing economy with two sectors - industry and service - and a segmented labor force - skilled and unskilled labor. Throughout the paper, notations with subscript i correspond to industry and those with subscript s correspond to the service sector. The industrial sector produces a good that is required both for consumption and as a capital good. Service sector produces a skill intensive service, which, to begin with, we assume is only required for consumption.² Production in both the sectors requires capital and labor as inputs. Capital is assumed not to depreciate. Industry is indifferent between skilled and unskilled labor and requires labor as a variable input (with constant labor coefficient) for which it pays a nominal wage w_i . On the other hand, for convenience, we assume that the service sector has no requirement for unskilled labor instead requires skilled labor as technical, managerial and administrative staff in a fixed proportion, $\eta > 0$, to its capital stock at a nominal wage w_s . Employment levels in the two sectors are $E_i = \frac{X_i}{x_i}$ and $E_s = \eta K_s$ respectively, where X_j , K_j and x_j denote output, capital stock and labor productivity of sector $j \in \{i, s\}$.

As is standard in the post-Keynesian/Kaleckian growth literature, we assume that firms in both sectors operate under conditions of imperfect competition and always maintain excess capacity to meet demand contingencies. Price levels in both the sectors are arrived at using the cost-plus pricing method.³ In case of industry sector we assume that the price, P_i , is fixed by applying a constant mark-up factor, $z_i > 1$, on its unit direct cost, which is unit labor cost. Therefore, $P_i = z_i(\frac{w_i}{x_i})$ where $x_i > 0$ is assumed to be a constant. In case of the service sector, since unit direct cost is zero, we use another variant of cost-plus pricing called full-cost pricing. Accordingly we assume that service price, P_s is fixed by applying a constant mark-up, $z_s > 1$, on its overhead labor cost at full capacity output. Thus $P_s = z_s(\frac{w_s}{x_s^*})$ where $x_s^* > 0$ is the productivity of overhead labor in the service sector at full capacity output, assumed constant. We denote the relative price of the service in terms of the industrial good, $\frac{P_s}{P_i}$, as p and the wage in sector $j \in \{i, s\}$ in terms of the industrial good, $\frac{w_j}{P_i}$, as

²We relax this assumption in Section 5 to include overhead service demand from the industrial sector.

³See, for example, Lavoie (1992, pp. 129–133).

ω_j . For the sake of brevity, p , ω_s and ω_i are henceforth referred to as the relative price of the service, the real wage of skilled labor and the real wage of unskilled labor respectively.

Rates of accumulation/investment of both sectors are assumed to be linear increasing functions of their respective rates of profit. In case of the industry sector the rate of profit $r_i = \frac{P_i X_i - w_i E_i}{P_i K_i} = \frac{(z_i - 1) X_i}{z_i K_i}$, where $\frac{X_i}{K_i}$ is the degree of capacity utilisation of industry. Let the rate of investment of industry, since z_i is a constant, be,

$$\frac{I_i}{K_i} = \alpha + \beta \frac{X_i}{K_i} \quad (1)$$

where $\alpha \geq 0$ and $\beta > 0$ are constants. Similarly the rate of profit in the service sector is $r_s = \frac{P_s X_s - w_s E_s}{P_s K_s} = p \frac{X_s}{K_s} - \omega_s \eta$, where $\frac{X_s}{K_s}$ is the degree of capacity utilisation of the service sector. Let rate of investment of the service sector be,

$$\frac{I_s}{K_s} = \delta + \gamma \left(p \frac{X_s}{K_s} - \omega_s \eta \right) \quad (2)$$

where $\delta > 0$ and $\gamma > 0$ are constants. I_j is investment demand of sector $j \in \{i, s\}$. The terms α and δ represent autonomous investment rates of the industrial and the service sectors respectively. The autonomous component of investment function represents “a slowly changing magnitude depending...on past economic, social and technological developments”.⁴ It captures effect of various factors like policy environment, emergence of new markets, evolution of tastes and preferences, etc along with exogenous innovations on private investment. We have assumed δ to be positive whereas α to be non-negative, $\alpha = 0$ may be true if industry is extremely stagnant.

In order to keep the savings behaviour simple, we assume that all profits are saved while all wages are spent on consumption. This implies that consumption expenditure in the economy is $C_i + C_s = w_i \frac{X_i}{x_i} + w_s \eta K_s$ where C_j denotes consumption expenditure incurred on the product of sector $j \in \{i, s\}$. As regards to the allocation of consumption expenditure, we assume $C_s = \theta C_i$ where $\theta > 0$ is a constant.⁵ Thus consumption demands for the two sectors can be written as

$$\frac{C_i}{P_i} = \frac{1}{(1 + \theta)} \left\{ \omega_i \frac{X_i}{x_i} + \omega_s \eta K_s \right\} \quad (3)$$

⁴Kalecki (1971, p. 173)

⁵A Cobb-Douglas utility function over the industrial good and the service is sufficient for this.

and

$$\frac{C_s}{P_s} = \frac{\theta}{p(1+\theta)} \left\{ \omega_i \frac{X_i}{x_i} + \omega_s \eta K_s \right\} \quad (4)$$

As far as supply of labor is concerned, we assume that there is unlimited supply of unskilled labor at some exogenously determined nominal wage in industry, w_i . This means P_i and ω_i are also constants. Supply of skilled labor is scarce. For the purpose of the short run, we assume the real wage of skilled labor ω_s is always such that the service sector is able to meet its requirement. In the long-run, however, the growth rate of skilled labor in the economy is

$$\hat{S} = S_0 + \varphi \hat{\omega}_s \quad (5)$$

where $\hat{\omega}_s$ is growth rate ω_s and S_0 and φ are positive constants. From (5), if real wage of skilled labor is constant, i.e. $\hat{\omega}_s = 0$, then skilled labor supply grows at the constant rate S_0 , which we refer to as the autonomous growth rate of skilled labor supply. We can think of S_0 as some sort of normal rate at which higher educational institutes autonomously generate additional skilled labor when there is no change in the return to skilled labor ω_s . As argued in the previous section, S_0 depends on the education policy of the government.

3. Short Run of the Model

In the short run, we treat K_i and K_s as arbitrary constants whereas take ω_s to be given at a level such that employment requirement in the service sector, ηK_s is met. With ω_s fixed, it also follows that the relative price of service p is a constant in the short run. Instead, we assume that both the sectors produce in the short run to meet demand at their respective constant price levels according to $\dot{X}_j = \psi_j [d_j - X_j]$ where \dot{X}_j is the rate of change in X_j , $\psi_j > 0$ is a constant and d_j is demand for sector $j \in \{i, s\}$. Now $d_i = \frac{C_i}{P_i} + I_i + I_s$ and $d_s = \frac{C_s}{P_s}$. Using (1), (2), (3) and (3) to substitute for d_i and d_s allows us to represent the short-run output dynamics as the following system of linear differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i \left[- \left\{ 1 - \frac{\omega_i}{(1+\theta)x_i} - \beta \right\} X_i + \gamma p X_s + \alpha K_i + \delta K_s \right. \\ &\quad \left. + \left(\frac{1}{1+\theta} - \gamma \right) \omega_s \eta K_s \right] \\ \dot{X}_s &= \psi_s \left[\frac{\theta \omega_i X_i}{p(1+\theta)x_i} + \frac{\theta \omega_s \eta K_s}{p(1+\theta)} - X_s \right] \end{aligned} \quad (6)$$

In the short-run equilibrium demand $\dot{X}_i = \dot{X}_s = 0$. From (6), in short-run equilibrium we have:

$$X_i = \frac{1}{\Delta} [\alpha K_i + \{\delta + \frac{(1-\gamma)}{(1+\theta)} \omega_s \eta\} K_s] \quad (7)$$

$$X_s = \frac{\theta}{p(1+\theta)} \left[\frac{\alpha \omega_i K_i}{x_i \Delta} + \frac{\delta \omega_i K_s}{x_i \Delta} + \left\{ \frac{\omega_i (1-\gamma)}{x_i (1+\theta) \Delta} + 1 \right\} \omega_s \eta K_s \right] \quad (8)$$

where $\Delta = 1 - \frac{(1+\gamma\theta)\omega_i}{(1+\theta)x_i} - \beta$. Proposition 1 states a sufficient condition for existence and stability of a unique short-run equilibrium represented by (7) and (8).⁶

$\alpha K_i + \{\delta + \frac{(1-\gamma)}{(1+\theta)} \omega_s \eta\} K_s$ is the total autonomous demand for the industrial sector's output, which is a result of autonomous investment in the two sectors (αK_i and δK_s) and the fixed wage bill of the service sector ($\omega_s \eta K_s$). On the other hand, $\frac{1}{\Delta}$ is the expenditure multiplier for the industry sector. This is because, each unit produced in the industrial sector generates an additional $\frac{\omega_i}{(1+\theta)x_i}$ units of consumption and an additional β units of investment demand for the industrial good. Further $\frac{\omega_i}{(1+\theta)x_i}$ units of consumption demand for the industrial good is associated with $\frac{\theta \omega_i}{p(1+\theta)x_i}$ units of demand for the service which in turn increases service sector's investment demand by $\frac{\gamma \theta \omega_i}{(1+\theta)x_i}$ units. So $\Delta = 1 - \frac{(1+\gamma\theta)\omega_i}{(1+\theta)x_i} - \beta$ is the leakage from each unit of industrial output. Proposition 1 shows that $\Delta > 0$ and $\gamma < 1$ ensure existence of a unique and asymptotically stable short run equilibrium.

Proposition 1. *If $\Delta > 0$ and $\gamma < 1$ then there exists a unique and asymptotically stable short-run equilibrium.*

Proof. Note that $\Delta > 0$ and $\gamma < 1$ implies $\frac{1}{\Delta} [\alpha K_i + \{\delta + \frac{(1-\gamma)}{(1+\theta)} \omega_s \eta\} K_s] > 0$ as all the remaining terms are positive. Thus it follows from (6)-(8) that there exists a unique $(X_i, X_s) \in \mathbb{R}_{++}^2$ such that $\dot{X}_i = \dot{X}_s = 0$. For stability, note that the Jacobian for (6) is

$$\begin{bmatrix} -\psi_i \left\{ 1 - \frac{\omega_i}{(1+\theta)x_i} - \beta \right\} & \psi_i \gamma p \\ \frac{\psi_s \theta \omega_i}{p(1+\theta)x_i} & -\psi_s \end{bmatrix}$$

with trace $-\psi_i \left\{ 1 - \frac{\omega_i}{(1+\theta)x_i} - \beta \right\} - \psi_s = -\psi_i \left\{ \Delta + \frac{\gamma \theta \omega_i}{(1+\theta)x_i} \right\} - \psi_s$ and determinant $\psi_i \psi_s \Delta$. Since $\psi_i > 0$ and $\psi_s > 0$, $\Delta > 0$ implies that trace is negative and determinant is positive. \square

Following Proposition 1, we maintain $\Delta > 0$ and $\gamma < 1$ throughout the rest of the

⁶There is an implicit assumption here that full capacity output-capital ratios of the two sectors are large enough to allow for (7) and (8).

paper. In the next section we consider the long-run dynamics of the model, where K_i , K_s and ω_s are not constants. However, before proceeding, let us consider how the short-run equilibrium outputs of the two sectors are affected by changes in K_i , K_s and ω_s . It can be easily inferred from (7) and (8) that a higher stock of capital in either of the two sectors, *ceteris paribus*, implies higher equilibrium output in both the sectors. The reason is that higher K_i or K_s means higher autonomous demand for the industrial output, which increases production in the sector. Higher production in the industrial sector is accompanied by higher employment and an increase in the wage bill of the sector. This in turn increases consumption demand for both the industrial good and the service. On the other hand, an increase in ω_s increases the short-run equilibrium output of the industrial sector but decreases that of the service sector. Differentiating (7) with respect to ω_s yields the partial derivative $\frac{\partial X_i}{\partial \omega_s} = \frac{(1-\gamma)\eta K_s}{\Delta(1+\theta)}$. Similarly differentiating (8) with respect to ω_s and then using (8) we obtain, $\frac{\partial X_s}{\partial \omega_s} = -\frac{X_s}{p} \frac{\partial p}{\partial \omega_s} + \frac{\theta \eta K_s}{p(1+\theta)} \left\{ \frac{(1-\gamma)\omega_i}{x_i \Delta(1+\theta)} + 1 \right\}$. Since $p = \frac{z_s \omega_s}{x_s^*}$, $\frac{\partial X_s}{\partial \omega_s} = -\frac{\theta \omega_i (\alpha K_i + \delta K_s)}{\omega_s p (1+\theta) x_i \Delta}$. Thus $\gamma < 1$ and $\Delta > 0$ imply $\frac{\partial X_i}{\partial \omega_s} > 0$ and $\frac{\partial X_s}{\partial \omega_s} < 0$. Intuitively, when $\gamma < 1$, an increase in ω_s increases net demand for the industrial good from the service sector. In case of the service sector, although demand for the service increases because of the increase in ω_s , both directly by raising its wage bill and indirectly because of the increase in the wage bill of the industrial sector, this increase is not sufficient enough to counteract the fall in demand for the service due to the increase in its relative price p .

4. Long Run of the Model

Investments undertaken by both the sectors result into capital accumulation in the long-run. Also supply of skilled labor in the economy grows in the long run in response to the growth in demand for skilled labor. In order to focus on the resulting dynamics we assume that the economy is always in a short-run equilibrium given by (7) and (8). Since there is no depreciation of capital stock, substituting X_i from (7) in (1) yields the growth rate of capital stock of industry, g_i as follows:

$$g_i = \alpha_0 + \beta_0 k_s + \beta_1 \omega_s k_s \quad (9)$$

where $\alpha_0 = \alpha(1 + \frac{\beta}{\Delta}) \geq 0$, $\beta_0 = \frac{\beta \delta}{\Delta} > 0$, $\beta_1 = \frac{\beta(1-\gamma)\eta}{\Delta(1+\theta)} > 0$ and $k_s = \frac{K_s}{K_i}$ is the relative capital stock of the service sector. Similarly, substituting X_s from (8) in (2) yields

the growth rate of capital stock of the service sector, g_s , as follows:

$$g_s = \delta_0 + \frac{\alpha_1}{k_s} + \gamma_0 \omega_s \quad (10)$$

where $\delta_0 = \delta(1 + \frac{\gamma\theta\omega_i}{(1+\theta)x_i\Delta}) > 0$, $\alpha_1 = \frac{\alpha\gamma\theta\omega_i}{(1+\theta)x_i\Delta} \geq 0$ and $\gamma_0 = \frac{\gamma\eta}{(1+\theta)} \{ \frac{(1-\gamma)\theta\omega_i}{(1+\theta)x_i\Delta} - 1 \}$.

Growth rates of capital stock of the two sectors are functions of the relative capital stock of the service sector, k_s , and the real wage rate of skilled labour, ω_s . The partial derivative of g_i with respect to k_s is $\beta_0 + \beta_1\omega_s > 0$ and of g_s with respect to k_s is $-\frac{\alpha_1}{k_s^2} < 0$. From the discussion in the previous section, we know that a *ceteris paribus* increase in capital stock of the service sector increases short equilibrium output levels of both the sectors. For the industrial sector this means an increase in the degree of capacity utilization $\frac{X_i}{K_i}$, which in turn increases the growth rate of its capital stock. For the service sector, however, the increase in short run equilibrium output is not enough to counter the decrease in its degree of capacity utilization because of the increase in its capital stock. From (8), $\frac{\partial(X_s/K_s)}{\partial K_s} = -\frac{\theta\alpha\omega_i K_i}{K_s^2 p(1+\theta)x_i\Delta} < 0$. As a result, a *ceteris paribus* increase in K_s decreases the growth rate of capital stock in the sector. Similarly, a *ceteris paribus* increase in the real wage of skilled labour increases the growth rate of capital stock of industrial sector. The partial derivative of g_i with respect to ω_s is $\beta_1 k_s > 0$. This is because the increase in ω_s increases short-run equilibrium output of the industrial sector as shown in the previous section.

Sign of the partial derivative of g_s with respect to ω_s , γ_0 , is however ambiguous. γ_0 is equal to γ times the partial derivative of the service sector's rate of profit, r_s , with respect to ω_s . The partial derivative of r_s with respect to ω_s is comprised of a direct effect and an indirect effect on r_s due to any infinitesimal change in ω_s . The direct effect itself consist of two opposing effects. First, a unit increase in ω_s increases the demand for consumption of the service, which increases its output per unit its capital stock by $\frac{\theta\eta}{p(1+\theta)}$. As a result r_s increases by $\frac{\theta\eta}{(1+\theta)}$. Second, there is also a decrease in r_s by η because of increase in the overhead wage bill per unit capital stock of the service sector. The total direct effect of a unit increase in ω_s on r_s is negative, i.e. $-\frac{\eta}{(1+\theta)}$. On the other hand, a unit increase in ω_s also increases the service sector's demand for industrial output by $\frac{(1-\gamma)\eta K_s}{(1+\theta)}$, which gives rise to additional industrial output of $\frac{(1-\gamma)\eta K_s}{(1+\theta)\Delta}$. The latter induces additional wage income in the industry sector, a constant part of which generates additional consumption demand for services and increases capacity utilization of the service sector by $\frac{\theta}{p(1+\theta)} \times \frac{(1-\gamma)\eta}{(1+\theta)\Delta} \times \frac{\omega_i}{x_i} = \frac{\theta(1-\gamma)\eta\omega_i}{p(1+\theta)^2 x_i \Delta}$. As a result, r_s increases by $\frac{\theta(1-\gamma)\eta\omega_i}{(1+\theta)^2 x_i \Delta}$. This is the positive indirect effect of a unit

increase of ω_s on r_s . The partial derivative of r_s with respect to ω_s is the sum of these two effects, which is $\frac{\theta(1-\gamma)\eta\omega_i}{(1+\theta)^2x_i\Delta} - \frac{\eta}{(1+\theta)}$. The partial derivative of g_s with respect to ω_s , then, is simply $\gamma\{\frac{\theta(1-\gamma)\eta\omega_i}{(1+\theta)^2x_i\Delta} - \frac{\eta}{(1+\theta)}\} = \frac{\gamma\eta}{(1+\theta)}\{\frac{(1-\gamma)\theta\omega_i}{(1+\theta)x_i\Delta} - 1\} = \gamma_0$. Therefore $\gamma_0 > 0$ when the indirect positive effect of a change in ω_s on r_s dominates its direct negative effect. Similarly $\gamma_0 < 0$ when the direct negative effect of a change in ω_s on r_s dominates its indirect positive effect.

The rate of change in k_s is $\dot{k}_s = k_s(g_s - g_i)$ for all $k_s \in \mathbb{R}_{++}$. Substituting for g_i and g_s from (9) and (10) respectively, we can express \dot{k}_s as a function of k_s and ω_s such that,

$$\dot{k}_s = k_s(\delta_0 + \frac{\alpha_1}{k_s} + \gamma_0\omega_s - \alpha_0 - \beta_0k_s - \beta_1\omega_s k_s) \quad (11)$$

for all $k_s \in \mathbb{R}_{++}$. In the short run, we assumed that ω_s is such that $E_s = \eta K_s$. This means in the long run, ω_s must adjust so that growth in skilled labor supply keeps up with capital accumulation in the service sector. Thus, using from (5) and (10), the rate of change in ω_s is

$$\dot{\omega}_s = \phi \omega_s(\delta_0 + \frac{\alpha_1}{k_s} + \gamma_0\omega_s - S_0) \quad (12)$$

for all $\omega_s \in \mathbb{R}_{++}$ where $\phi = \frac{1}{\varphi}$ is a positive constant. (11) and (12) form a system of two differential equations in two variables, $(k_s, \omega_s) \in \mathbb{R}_{++}^2$, which describes the long-run dynamics of the model.

4.1 Existence of Steady State

For a steady state, we need a $(k_s^*, \omega_s^*) \in \mathbb{R}_{++}^2$ such that $\dot{k}_s = \dot{\omega}_s = 0$. Assuming $\gamma_0(S_0 - \alpha_0) + \alpha_1\beta_1 \neq 0$, Proposition 2 states a set of necessary and sufficient conditions which ensures existence of a unique steady state.

Proposition 2. $[\gamma_0(S_0 - \alpha_0) + \alpha_1\beta_1 \neq 0] \longrightarrow [(\exists(k_s, \omega_s) \in \mathbb{R}_{++}^2)(\dot{k}_s = \dot{\omega}_s = 0 \wedge (\forall(k'_s, \omega'_s) \in \mathbb{R}_{++}^2)(\delta_0 + \frac{\alpha_1}{k'_s} + \gamma_0\omega'_s - \alpha_0 - \beta_0k'_s - \beta_1\omega'_s k'_s = 0 \wedge \delta_0 + \frac{\alpha_1}{k'_s} + \gamma_0\omega'_s - S_0 = 0 \longrightarrow k'_s = k_s \wedge \omega'_s = \omega_s)) \longleftrightarrow (S_0 - \alpha_0 > 0) \wedge ((S_0 - \delta_0)(S_0 - \alpha_0) > \alpha_1\beta_0 \wedge \gamma_0(S_0 - \alpha_0) + \alpha_1\beta_1 > 0) \vee ((S_0 - \delta_0)(S_0 - \alpha_0) < \alpha_1\beta_0 \wedge \gamma_0(S_0 - \alpha_0) + \alpha_1\beta_1 < 0)]]$

Proof. Suppose $\gamma_0(S_0 - \alpha_0) + \alpha_1\beta_1 \neq 0$. Let $\dot{k}_s = \dot{\omega}_s = 0$ at (k_s^*, ω_s^*) . Then from (11)

and (12),

$$\delta_0 + \frac{\alpha_1}{k_s^*} + \gamma_0 \omega_s^* = \alpha_0 + \beta_0 k_s^* + \beta_1 \omega_s^* k_s^* \quad (13)$$

$$\delta_0 + \frac{\alpha_1}{k_s^*} + \gamma_0 \omega_s^* = S_0 \quad (14)$$

From (13) and (14), $\alpha_0 + \beta_0 k_s^* + \beta_1 \omega_s^* k_s^* = S_0$ or,

$$k_s^* = \frac{S_0 - \alpha_0}{\beta_0 + \beta_1 \omega_s^*} \quad (15)$$

Using (15) we can eliminate k_s^* from (14) and uniquely obtain,

$$\omega_s^* = \frac{(S_0 - \delta_0)(S_0 - \alpha_0) - \alpha_1 \beta_0}{\gamma_0(S_0 - \alpha_0) + \alpha_1 \beta_1} \quad (16)$$

ω_s^* is defined since $\gamma_0(S_0 - \alpha_0) + \alpha_1 \beta_1 \neq 0$. Thus $\omega_s^* > 0$ if and only if $[(S_0 - \delta_0)(S_0 - \alpha_0) > \alpha_1 \beta_0 \wedge \gamma_0(S_0 - \alpha_0) + \alpha_1 \beta_1 > 0] \vee [(S_0 - \delta_0)(S_0 - \alpha_0) < \alpha_1 \beta_0 \wedge \gamma_0(S_0 - \alpha_0) + \alpha_1 \beta_1 < 0]$. And with $\omega_s^* > 0$, $k_s^* > 0$ if and only if $S_0 > \alpha_0$. \square

4.2 Stability of Steady State

Assuming that $(k_s^*, \omega_s^*) \in \mathbb{R}_{++}^2$ is a steady state, Proposition 3 states a sufficient condition for its local stability.

Proposition 3. *If the wage share in industry $\frac{\omega_i}{x_i} < 1 - \frac{\beta S_0}{S_0 - \alpha}$ then the steady state (k_s^*, ω_s^*) is locally asymptotically stable.*

Proof. Suppose $\frac{\omega_i}{x_i} < 1 - \frac{\beta S_0}{S_0 - \alpha}$. The Jacobian matrix of the system of differential equations (11) and (12) at (k_s^*, ω_s^*) is

$$J(k_s^*, \omega_s^*) = \begin{bmatrix} -k_s^* \left\{ \frac{\alpha_1}{(k_s^*)^2} + \beta_0 + \beta_1 \omega_s^* \right\} & k_s^* (\gamma_0 - \beta_1 k_s^*) \\ -\frac{\phi \omega_s^* \alpha_1}{(k_s^*)^2} & \phi \omega_s^* \gamma_0 \end{bmatrix}$$

with trace $Tr(J(k_s^*, \omega_s^*)) = -\frac{\alpha_1}{k_s^*} - k_s^* (\beta_0 + \beta_1 \omega_s^*) + \phi \omega_s^* \gamma_0$ and determinant $|J(k_s^*, \omega_s^*)| = -\phi \omega_s^* \{ \gamma_0 (S_0 - \alpha_0) + \alpha_1 \beta_1 \}$. From definitions of α_0 , α_1 , β_1 , γ_0 and Δ it follows that $\gamma_0 (S_0 - \alpha_0) + \alpha_1 \beta_1 = \frac{\gamma \eta (S_0 - \alpha)}{(1 + \theta) \Delta} (\frac{\omega_i}{x_i} - 1 - \frac{\beta S_0}{S_0 - \alpha})$. Since $S_0 > \alpha$ and γ , η , θ and Δ are all positive, $\gamma_0 (S_0 - \alpha_0) + \alpha_1 \beta_1 < 0$ if and only if $\frac{\omega_i}{x_i} < 1 - \frac{\beta S_0}{S_0 - \alpha}$. Therefore $\frac{\omega_i}{x_i} < 1 - \frac{\beta S_0}{S_0 - \alpha}$ implies $|J(k_s^*, \omega_s^*)| > 0$ as ϕ and ω_s^* are positive. Also, since $S_0 > \alpha_0$ and α_1 and β_1 are positive, $\gamma_0 (S_0 - \alpha_0) + \alpha_1 \beta_1 < 0$ only if $\gamma_0 < 0$. From $\gamma_0 < 0$, it follows that $Tr(J(k_s^*, \omega_s^*)) < 0$ as k_s^* , ω_s^* , α_1 , β_0 , β_1 and ϕ are all positive. \square

When wage share in industry is less than $1 - \frac{\beta S_0}{S_0 - \alpha}$ the total derivative of g_s with respect to ω_s is negative at the steady state. To see this, note from (10) that the total derivative $\frac{dg_s}{d\omega_s} = -\frac{\alpha_1}{k_s^2} \frac{dk_s}{d\omega_s} + \gamma_0$. Since at the steady state $k_s^* = \frac{S_0 - \alpha_0}{\beta_0 + \beta_1 \omega_s^*}$, $\frac{dg_s}{d\omega_s}$ at (k_s^*, ω_s^*) is given by $\frac{dg_s^*}{d\omega_s^*} = \frac{\alpha_1 \beta_1}{S_0 - \alpha_0} + \gamma_0$, which is negative if and only if $\gamma_0(S_0 - \alpha_0) + \alpha_1 \beta_1 < 0$. And from the proof of Proposition 3 we know that $\gamma_0(S_0 - \alpha_0) + \alpha_1 \beta_1 < 0$ if and only if $\frac{\omega_i}{x_i} < 1 - \frac{\beta S_0}{S_0 - \alpha}$. Thus, another way of interpreting Proposition 3 is that the steady state (k_s^*, ω_s^*) is locally asymptotically stable if $\frac{dg_s^*}{d\omega_s^*} < 0$. Now $\frac{dg_s^*}{d\omega_s^*} < 0$ only if the partial derivative of g_s with respect to ω_s , γ_0 , is sufficiently negative, i.e. $\gamma_0 < -\frac{\alpha_1 \beta_1}{S_0 - \alpha_0}$. This condition is likelier to hold, among other factors, lower the value of α i.e., the rate of autonomous investment in the industrial sector. In fact, we prove in Appendix A that if $\alpha = 0$ then $\gamma_0 < 0$ implies global asymptotic stability of the steady state.

4.3 Properties of Steady State

Propositions 2 and 3 imply that a unique and locally asymptotically stable steady state exists in the model if $S_0 > \alpha_0$ and $(S_0 - \delta_0)(S_0 - \alpha_0) < \alpha_1 \beta_0 \wedge \gamma_0(S_0 - \alpha_0) + \alpha_1 \beta_1 < 0$. At this steady state, the relative capital stock of the service sector and real wage of skilled labor are given by k_s^* in (15) and ω_s^* in (16) respectively. This steady state has the property of balanced sectoral growth as output, capital stock and employment in both the sectors grow at the same constant rate S_0 .⁷ Policy interventions, such as increase in public investment in higher educational institutions that impart skills required by modern services, to the extent that they increase S_0 , increase the steady-state growth rate of the model.

As regards to the structure of the economy along the steady state, while k_s^* can be taken as an indicator, a more conventional indicator is output share. By definition, output share of the service sector is $\frac{pX_s}{X_i + pX_s} = 1 - \frac{X_i}{X_i + pX_s} = 1 - \frac{X_i}{1 + pX_s/X_i}$. Using (7) and (8), we can express the steady state output share of the service sector, say ξ_s^* , as a function of k_s^* and ω_s^* such that

$$\xi_s^* = 1 - \frac{1}{1 + \frac{\theta}{(1+\theta)} \left\{ \frac{\omega_i}{x_i} + \frac{\Delta}{\frac{1}{\eta \omega_s^*} \left(\frac{\alpha}{k_s^*} + \delta \right) + \frac{(1-\gamma)}{(1+\theta)}} \right\}} \quad (17)$$

Proposition 4 shows that ξ_s^* is positively related to both k_s^* and ω_s^* .

Proposition 4. $\frac{\partial \xi_s^*}{\partial k_s^*} > 0$ and $\frac{\partial \xi_s^*}{\partial \omega_s^*} > 0$

Proof. See Appendix B. □

⁷A positive steady-state rate of profit in the service sector further requires $S_0 > \delta$.

A higher level of k_s^* , for any given level of K_i , means larger output in both sectors. So increase in k_s^* tends to increase ξ_s^* by increasing output of the service sector. On the other hand, as discussed in section 3, an increase in ω_s^* increases output in industry whereas decreases output in the service sector and at the same time increases steady state relative price of the service $p^* = \frac{z_s \omega_s^*}{x_s^*}$ which ensures that $\frac{\partial \xi_s^*}{\partial \omega_s^*} > 0$. Identifying effects of changes in underlying parameters that determine k_s^* and ω_s^* on ξ_s^* is, however, slightly difficult, not least because k_s^* and ω_s^* are themselves inversely related. Nonetheless, we can make some predictions for values of α close to zero. For example, $\frac{d\xi_s^*}{dS_0} = \frac{\partial \xi_s^*}{\partial k_s^*} \frac{\partial k_s^*}{\partial S_0} + \frac{\partial \xi_s^*}{\partial \omega_s^*} \frac{\partial \omega_s^*}{\partial S_0}$. If $\alpha = 0$ then $\frac{\partial \omega_s^*}{\partial S_0} < 0$ since $\gamma_0 < 0$ and also, from the proof of Proposition 4, $\frac{\partial \xi_s^*}{\partial k_s^*} = 0$. Proposition 4 then implies $\frac{d\xi_s^*}{dS_0} < 0$ for $\alpha = 0$. Further, under the conditions for local stability and existence of steady state as derived in Propositions 2 and 3, it is easy to check that $\frac{\partial \xi_s^*}{\partial k_s^*}$, $\frac{\partial k_s^*}{\partial S_0}$, $\frac{\partial \xi_s^*}{\partial \omega_s^*}$ and $\frac{\partial \omega_s^*}{\partial S_0}$ are all differentiable and, therefore, continuous functions of α . This means that there must be a range values of α near zero where $\frac{d\xi_s^*}{dS_0} < 0$. Thus, in case autonomous rate of investment in industry (α) is extremely low, a low value of the parameter S_0 can cause to a high steady state output share of the service sector because of a high steady state real wage of skilled labor. In a similar vein, it is possible to show that a high rate of autonomous investment in the service sector (δ) can cause a high steady state output share of the service sector if α is sufficiently small.

5. An Extension

A major source of demand for the kinds of services we are considering in this paper is service input demand from industry. For example, use of modern telecommunication services can bring down transaction costs and ensure smooth functioning of production operations. Demand for various business services including software services comes from industry in order to streamline and increase efficiency of production and delivery systems. Bhagwati (1984) argues that growth in industry causes ‘splintering’, i.e. many of its in-house activities get out-sourced to the service sector. Let us suppose that input demand for modern services from industry is overhead type and industry’s input bill for the service can be expressed as $N_s = \lambda P_i K_i$ where λ is a positive constant.⁸ Demand for services d_s changes to $\frac{C_s}{P_s} + \frac{N_s}{P_s}$. As a consequence, instead of (9) and (10), we now have $g_i = \bar{\alpha}_0 + \beta_0 k_s + \beta_1 \omega_s k_s$ and $g_s = \bar{\delta}_0 + \frac{\bar{\alpha}_1}{k_s} + \gamma_0 \omega_s$ where $\bar{\alpha}_0 = \alpha_0 + \frac{\beta \gamma \lambda}{\Delta} > 0$ and $\bar{\alpha}_1 = \alpha_1 + \gamma \lambda \left\{ \frac{\gamma \theta \omega_i}{(1+\theta)x_i \Delta} + 1 \right\} > 0$. Since g_i and g_s have the same forms as in section 4, the long-run dynamics is also similar.

⁸This is related to ‘splintering’ as input demand for service increases with expansion in industry. Dutt (1992) uses a similar specification.

6. Conclusion

Expansion of skilled labor force is crucial if developing economies such as the South Asian ones are to benefit from skill-intensive modern services in the long run. This paper argues that government's education policy in developing economies, given the significance of public institutes in higher education, is a major determinant of skilled labor supply growth. Our main result (Proposition 2) shows that it can also determine the long-run steady state growth rate of the economy even in the absence of capacity constraints and unlimited supply of unskilled labor if, broadly speaking, skill-intensive sectors have inter-linkages with other sectors. A sufficient condition for local stability of steady state in our model is that the rate of accumulation in the service sector, g_s , is negatively related to returns to skilled labor, ω_s . Since growth is demand-led in the model, a sufficiently low wage share in industry ensures this (Proposition 3). The constant rate at which both sectors grow in the steady state is equal to S_0 , the rate at which supply of skilled labor grows in the absence of any change in ω_s . We interpret S_0 as the normal rate at which higher educational institutions generate additional skilled labor supply in the economy. If changes in the education policy of the government can increase S_0 then the steady state growth rate of the model unambiguously increases. Our model also implies that an increase in output share of modern services may be a reflection of increase in skilled labor wage (Proposition 4). Further both a decrease in S_0 and an increase δ can increase steady state wage of skilled labor, particularly if industry is stagnated (in the sense of near-zero values of α). Finally, we conclude by drawing attention to two obvious limitations of the model. First, the closed economy framework can not account for one of the most exciting aspects of modern services - their tradability. However, given our emphasis on skilled labor constraint, including exports as an additional source of service demand in the model is unlikely to make significant difference. Second, and more importantly, our model does not take into consideration of the macroeconomic effects of investment expenditure required to generate additional supply of skilled labor. This is a problem because in developing economies such as the South Asian ones, resource crunch is a severe obstacle in front of policy makers trying to boost the average skill endowment of the population. A proper exposition of this aspect of the problem requires extension of the model to include a third sector which imparts skill training in the economy backed by both fiscal measures and private investment.⁹

⁹One way to think within a two-sector framework is to assume that service is used for both consumption and acquiring skills by workers. Additional skilled labor is then financed by substitution away from consumption on part of the workers.

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Appendix A

Proposition 5 identifies a sufficient condition for global stability of (k_s^*, ω_s^*) , where k_s^* and ω_s^* are from (15) and (16) respectively, using a modification of Olech’s Theorem due to Ito (1978).

Proposition 5. *If $\alpha = 0$ and $\gamma_0 < 0$ then the steady state (k_s^*, ω_s^*) of the system of*

differential equations (11) and (12) is asymptotically stable in \mathbb{R}_{++}^2 .

Proof. Suppose $\alpha = 0$ and $\gamma_0 < 0$. Since $\alpha = 0$ implies $\alpha_0 = \alpha_1 = 0$, the system of differential equations (11) and (12) reduces to $\dot{k}_s = G(k_s, \omega_s)$ and $\dot{\omega}_s = H(k_s, \omega_s)$ where $G(k_s, \omega_s) = k_s(\delta_0 + \gamma_0\omega_s - \beta_0k_s - \beta_1\omega_s k_s)$ and $H(k_s, \omega_s) = \phi\omega_s(\delta_0 + \gamma_0\omega_s - S_0)$ and from (16) and (15) the steady state is $k_s^* = \frac{S_0}{\beta_0 + \beta_1\omega_s^*} > 0$ and $\omega_s^* = \frac{S_0 - \delta_0}{\gamma_0} > 0$. Note $G(k_s^*, \omega_s^*) = G(k_s^*, \omega_s^*) = 0$. $G(k_s, \omega_s)$ and $H(k_s, \omega_s)$ are differentiable functions in \mathbb{R}_{++}^2 with partial derivatives $\frac{\partial G}{\partial k_s} = \frac{G}{k_s} - k_s(\beta_0 + \beta_1\omega_s)$, $\frac{\partial G}{\partial \omega_s} = k_s(\gamma_0 - \beta_1k_s)$, $\frac{\partial H}{\partial k_s} = 0$ and $\frac{\partial H}{\partial \omega_s} = \frac{H}{\omega_s} + \phi\omega_s\gamma_0$, which are all continuous functions in \mathbb{R}_{++}^2 . Hence $G(k_s, \omega)$ and $H(k_s, \omega)$ are C^1 functions in \mathbb{R}_{++}^2 . Next, since ϕ , β_0 and β_1 are positive, $\gamma_0 < 0$ implies $\frac{\partial G}{\partial k_s} - \frac{G}{k_s} + \frac{\partial H}{\partial \omega_s} - \frac{H}{\omega_s} = -k_s(\beta_0 + \beta_1\omega_s) + \phi\omega_s\gamma_0 < 0$ for all $(k_s, \omega_s) \in \mathbb{R}_{++}^2$. Similarly $\gamma_0 < 0$ implies $(\frac{\partial G}{\partial k_s} - \frac{G}{k_s})(\frac{\partial H}{\partial \omega_s} - \frac{H}{\omega_s}) - \frac{\partial G}{\partial \omega_s} \frac{\partial H}{\partial k_s} = -k_s(\beta_0 + \beta_1\omega_s)\phi\omega_s\gamma_0 > 0$ for all $(k_s, \omega_s) \in \mathbb{R}_{++}^2$. Finally $(\frac{\partial G}{\partial k_s} - \frac{G}{k_s})(\frac{\partial H}{\partial k_s} - \frac{H}{k_s}) = -k_s(\beta_0 + \beta_1\omega_s)\phi\omega_s\gamma_0 \neq 0$ for all $(k_s, \omega_s) \in \mathbb{R}_{++}^2$. The claim then follows from Theorem 2 of Ito (1978). \square

Appendix B

Partial differentiation of (17) with respect to k_s yields

$$\begin{aligned} \frac{\partial \xi_s^*}{\partial k_s^*} &= -1 \times -\frac{1}{\left[1 + \frac{\theta}{(1+\theta)} \left\{ \frac{\omega_i}{x_i} + \frac{\Delta}{\frac{1}{\eta\omega_s^*} \left(\frac{\alpha}{k_s^*} + \delta \right) + \frac{(1-\gamma)}{(1+\theta)}} \right\} \right]^2} \\ &\quad \times -\frac{\theta\Delta}{(1+\theta) \left\{ \frac{1}{\eta\omega_s^*} \left(\frac{\alpha}{k_s^*} + \delta \right) + \frac{(1-\gamma)}{(1+\theta)} \right\}^2} \times -\frac{\alpha}{\eta\omega_s^* k_s^{*2}} \end{aligned}$$

Using (7) and (17), we can simplify the above equation to,

$$\frac{\partial \xi_s^*}{\partial k_s^*} = \frac{(1 - \xi_s^*)^2 \theta \alpha \eta K_i^2 \omega_s^*}{(1 + \theta) \Delta X_i^2} \quad (18)$$

Similarly, partial differentiation of (17) with respect to ω_s yields

$$\begin{aligned} \frac{\partial \xi_s^*}{\partial \omega_s} &= -1 \times -\frac{1}{\left[1 + \frac{\theta}{(1+\theta)} \left\{ \frac{\omega_i}{x_i} + \frac{\Delta}{\frac{1}{\eta\omega_s^*} \left(\frac{\alpha}{k_s^*} + \delta \right) + \frac{(1-\gamma)}{(1+\theta)}} \right\} \right]^2} \\ &\quad \times -\frac{\theta\Delta}{(1+\theta) \left\{ \frac{1}{\eta\omega_s^*} \left(\frac{\alpha}{k_s^*} + \delta \right) + \frac{(1-\gamma)}{(1+\theta)} \right\}^2} \times -\frac{1}{\eta\omega_s^{*2}} \times \left(\frac{\alpha}{k_s^*} + \delta \right) \end{aligned}$$

Again using (7) and (17), we can simplify the above equation to,

$$\frac{\partial \xi_s^*}{\partial \omega_s^*} = \frac{(1 - \xi_s^*)^2 \theta \eta K_s^2}{(1 + \theta) \Delta X_i^2} \left(\frac{\alpha}{k_s^*} + \delta \right) \quad (19)$$

The assumption $\Delta > 0$ implies $\frac{\partial \xi_s}{\partial k_s} > 0$ and $\frac{\partial \xi_s}{\partial \omega_s} > 0$ as by definition $\xi_s < 1$ and all terms in (18) and (19) are positive.