Subsidies, Entry, and Economic Growth in a Schumpeterian Model with Incumbents and Entrants

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Abstract

This paper explores the effects of various subsidies in a Schumpeterian model with incumbents and entrants. We find that subsidizing the production of intermediate goods or subsidizing incumbents' in-house R&D serves to promote economic growth. However, the growth effect of the subsidy on entrants' R&D is ambiguous. Moreover, we show that various types of subsidies have different effects on the entry of new firms and market structure. Finally, we calibrate the model and find that the subsidy on intermediate goods production is more effective than R&D subsidies in terms of promoting growth and raising welfare.

JEL classification: O31, O38, O40
Keywords: innovation, subsidies, economic growth, market structure, incumbents and entrants

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1. Introduction

It is a common belief that the social return on R&D investment generally exceeds the private return, thereby leading to an underinvestment in R&D relative to the social optimal level (Jones and Williams, 1998, 2000; Grossmann et al., 2013).[1] To correct the distortion resulting from this well-known R&D underinvestment, R&D subsidy policies are considered to be an effective instrument in many countries. Thus, analyzing the effects of subsidies has been of interest to researchers and policy-makers for a long time, and various types of subsidies have been examined in the literature.[2]

Moreover, some existing evidence suggests that both the improvement of existing products and creative destruction are beneficial to economic growth. For example, Garcia-Macia et al. (2019) provide empirical evidence to support the view that both the product improvements of incumbents and the creative destruction of entrants are growth engines that cannot be ignored, and that own-product improvements appear to be even more important than creative destruction. Bartelsman and Doms (2000) also suggest that the entry of new firms only accounts for about 25% of productivity improvements. Given these empirical findings and the importance of subsidies, several questions then naturally arise: Should we subsidize the R&D investment of incumbents or the R&D investment of new entrants? Do distinct types of R&D subsidies all serve to promote economic growth, and how do they affect the composition of the innovation that drives economic growth?

In this study, we develop a Schumpeterian growth model in which both the in-house R&D of incumbents and the creative destruction of entrants act as growth engines, and use it to shed some light on the above important questions. The novelty of our model is that we consider simultaneous innovations by existing and new firms, and highlight the crucial role of an endogenous market structure (EMS) on the effects of subsidy policies.[3] To be more specific, the model we deal with can be described as follows. In each industry, there is a monopolistic quality leader (i.e., the incumbent), and this firm can invest in in-house R&D to improve its own-product. At the same time, potential entrants engage in creative destruction in order to replace the incumbent. In addition, we introduce a fixed entry cost into the model to generate the endogenous entry of new firms.[4] Thus the market structure, measured by the number of firms, is endogenously

[1] See Hall et al. (2010) for a survey of the literature that measures the private and social returns to R&D.
[2] See Becker (2015) for a survey of the literature that explores the effects of subsidies on R&D investment and economic growth.
[3] See Erto (2009) for a more extensive discussion on EMS, and see Chu et al. (2021) and Huang et al. (2021) for recent studies that consider EMS.
[4] See also Baldwin and Robert-Nicoud (2008), Gustafsson and Segerstrom (2010), and Chu et al. (2017) who

determined by the competition between incumbents and entrants. These novel features of our model lead to predictions that differ from those of previous studies and offer new insights.

As mentioned above, in the economy, intermediate goods are produced by monopolistic enterprises. Due to the well-known monopoly-pricing distortion, in addition to R&D subsidies, we will also consider including a production subsidy. More specifically, this paper revisits the policy implications of three types of subsidies: a subsidy on the production of intermediate goods, a subsidy on in-house R&D, and a subsidy on creative destruction.[5] To the best of our knowledge, this is the first study that considers both non-R&D and R&D subsidies in a Schumpeterian model, in which in-house R&D and creative destruction jointly promote economic growth.

Within this growth-theoretic framework, we find that the three types of subsidies have quite different effects on economic growth. Firstly, the subsidy on the intermediate-goods production increases the number of entrants, stimulates creative destruction, and promotes innovation and growth. Secondly, subsidizing in-house R&D is also effective in promoting economic growth, but it increases the incumbents' contribution to economic growth and reduces the entrants' contribution. Intuitively, although the subsidy on in-house R&D investment can promote incumbents' innovations, it also reduces Schumpeterian creative destruction due to the crowding-out effect. However, the rise in the incumbents' contribution to growth induced by subsidizing in-house R&D dominates the reduction in the entrants' contribution, so that the net growth effect of the in-house R&D subsidy is positive. Thirdly, the effect of subsidizing the entrants' R&D investment on economic growth is ambiguous, and depends on the size of the fixed entry cost. Intuitively, although the subsidy on creative destruction increases the entrants' R&D, it also leads incumbents to be replaced more frequently, thereby reducing the value of the incumbents' R&D investment. In the case where the entry cost is sufficiently small, there are more new firms actively investing in R&D. Under such a situation, subsidizing the entrants' R&D leads to a substantial increase in the entrants' innovation, which is greater than the reduction in in-house R&D. Hence subsidizing the entrants' R&D is effective in promoting economic growth. Conversely, when the entry cost is sufficiently large, the increase in the entrants' contribution to growth resulting from

incorporate a fixed entry cost into the R&D-based growth model, but they do not consider a framework featuring both in-house R&D and creative destruction.

[5] Chu et al. (2016) also explore the implications of R&D subsidies on growth in a Schumpeterian model with EMS. However, in their model, entrants introduce new varieties of products to the market and compete with incumbents for market share. Therefore, they argue that an increase in the number of firms will reduce the economic growth rate, which is different from our findings.

subsidizing the entrants' R&D is smaller than the decrease in the incumbents' contribution. Consequently, in this case, subsidizing the entrants' R&D tends to slow down economic growth.

We also calibrate the model to quantitatively examine the impact of subsidies on the market structure and to determine which subsidy policy instrument is more effective in boosting growth and raising welfare. Under our calibrated parameter values, the mass of entrants increases with the subsidy rate for intermediate production, but decreases with the two R&D subsidy rates. Our results show that the subsidy on in-house R&D or on intermediate goods production has a more significant impact on the composition of innovation (i.e., the relative proportions of own-product improvements and creative destruction) than the R&D subsidy on entrants. Moreover, our quantitative analysis suggests that the subsidy on intermediate goods production is more growth-promoting and welfare-enhancing than the R&D subsidies on either incumbents or new entrants. As for the two R&D subsidies, in terms of promoting economic growth and improving social welfare, we find that the R&D subsidy on incumbents is more effective than the R&D subsidy on entrants, especially when the entry cost is relatively high.

1.1. Related literature

This study is related to the literature on *innovation-driven* growth. Romer (1990) is the seminal study that develops a variety-expanding growth model in which growth is driven by the creation of new products. Then Segerstrom et al. (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) develop the Schumpeterian quality-ladder growth models in which innovation and economic growth are driven by the quality improvements of existing products.[6] Jones (1995) argues that these studies feature a counterfactual scale effect of the population size on growth and develops a semi-endogenous model, in which the growth rate is scale-invariant.[7] Peretto (1998), Howitt (1999), and Segerstrom (2000) combine both variety expansion (horizontal R&D) and quality improvement (vertical R&D) in their models and also remove the scale effect.[8] The current paper differs from these seminal studies by incorporating both the own-product improvements of incumbents and Schumpeterian creative destruction of new entrants into the model and by investigating the important interaction between these two types of vertical R&D.

This study is more closely associated with several recent growth models in which

[6] See Aghion, Akcigit, and Howitt (2014) for a survey of Schumpeterian growth theory.
[7] See also Segerstrom (1998), who considers a semi-endogenous growth model.
[8] Their models are known as the second-generation R&D-based growth models.

economic growth is driven by the innovations of both incumbents and entrants. Acemoglu and Cao (2015) provide a tractable framework for the analysis of economic growth that is driven by both incremental R&D for own-product improvements and radical R&D for creative destruction. A recent study by Acemoglu et al. (2018) extends the basic Schumpeterian model by assuming that incumbents and entrants invest in R&D and allow for heterogeneous entrants with different innovative capacities. Moreover, Akcigit and Kerr (2018) build up an elegant growth model, in which incumbents invest in internal innovations to improve their existing products and engage in external innovations to acquire new product lines, while new firms also invest in R&D in order to become intermediate producers of a successful innovation. Along the lines of this strand of the literature, this paper turns the focus to explore the effects of non-R&D and R&D subsidies in this vintage of the Schumpeterian growth model. Our reduced-form modeling of innovations by incumbents and entrants allows us to provide a tractable analysis of the effects of various subsidies on the interaction between in-house R&D and Schumpeterian creative destruction.

Our study is also related to the literature regarding the effects of R&D subsidies in the R&D-based growth models; see, for example, Segerstrom (1998), Zeng and Zhang (2007), Sener (2008), Impullitti (2010), Chu and Lai (2014), Chu and Cozzi (2018), Yang (2018), and Chan et al. (2022). These studies mostly consider either variety-expanding models or quality-ladder models. Only a few studies, such as Segerstrom (2000), Chu et al. (2016), Chu and Wang (2020), and Akcigit et al. (2021), explore the effects of R&D subsidies in the Schumpeterian growth model with two types of innovation. However, none of these studies consider two vertical R&D subsidies (i.e., subsidies for in-house R&D and creative destruction). A recent study by Iwaisako and Ohki (2019) finds that subsidizing either leader's or follower's R&D has a positive effect on both leader's and follower's innovation, thereby stimulating economic growth. However, they do not compare the effects of the two types of R&D subsidies and investigate how they affect the composition of economic growth. Consequently, a novel contribution of our study is to provide a complete comparison of R&D subsidies on own-product improvements and creative destruction to fill this gap. More importantly, we explore how these two types of R&D subsidies affect the composition of innovation and growth as well as the endogenous market structure.

Finally, this study also contributes to the literature that explores the mixed use of R&D subsidies and non-R&D subsidies. Grossmann et al. (2013) consider a time-varying subsidy on R&D and a constant subsidy on intermediate-goods production in a semi-endogenous growth model. Zeng and Zhang (2007) and Hu et al. (2019) examine the effects of a subsidy on final output, a subsidy on the purchase of intermediate goods, and a subsidy on R&D based on two leading approaches, the variety-expanding and quality-ladder approaches, respectively. Furthermore, Li and Zhang (2014) consider subsidies for R&D and the purchase of intermediate goods in the Matsuyama (1999) model of growth through cycles. The present paper complements these studies by incorporating subsidies for the production of intermediate goods, in-house R&D, and creative destruction into a Schumpeterian model, in which in-house R&D and creative destruction jointly promote economic growth.[9]

The rest of the paper proceeds as follows. Section 2 presents the Schumpeterian model. In Section 3, we analyze the growth effects of various subsidies. Section 4 provides a quantitative analysis. The final section concludes.

2. A Schumpeterian model with incumbents and entrants

To analyze the effects of subsidies on economic growth and growth composition, we extend the basic Schumpeterian growth model by adding three features: (1) we allow incumbents to invest in in-house R&D to improve the quality of their products; (2) we incorporate entry costs to generate the endogenous entry of new firms; and (3) we consider subsidies for intermediate goods production, incumbents' in-house R&D investment, and entrants' R&D investment. Our model is essentially based on Akcigit and Kerr (2018), in which both the innovations of incumbent firms and new entrants promote economic growth. Our analysis provides a complete closed-form solution for the balanced growth path of the economy. Given that the quality-ladder model has been well studied, the standard features of the model will be briefly described below, whereas new features will be described in more detail.

2.1. Household

In the economy, the population size is normalized to unity, and there is a representative household that has the following lifetime utility function:

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt \,, \tag{1}$$

where $\rho > 0$ is the subjective discount rate and C_t denotes consumption of the final good at time t. The household inelastically supplies one unit of labor to earn wage income and is subject to the following asset-accumulation equation:

[9] Alternatively, we can also incorporate subsidies for final output and the purchase of intermediate goods into the model. In fact, in our model, the subsidies for either final output production or the purchase of intermediate goods are equivalent. Zeng and Zhang (2007) also find an equivalent relation between subsidies for final output and subsidies for the purchase of intermediate goods.

$$\dot{A}_t = r_t A_t + w_t + T_t - C_t , \tag{2}$$

where A_t is the real value of financial assets (in the form of equity shares in intermediate goods firms) owned by the household, r_t is the real interest rate, w_t is the real wage rate, and T_t is lump-sum taxes imposed by the government.[10] From standard dynamic optimization, the familiar Euler equation is given by

$$\frac{\dot{C}_t}{C_t} = r_t - \rho . \tag{3}$$

2.2. Final good

The unique final good is produced by competitive firms that employ labor and a continuum of intermediate goods indexed by $i \in [0,1]$ as inputs. The final good serves as the numéraire throughout the paper. In line with Acemoglu and Cao (2015) and Akcigit and Kerr (2018), the production function is given by

$$Y_t = \frac{1}{1-\beta}\left(\int_0^1 q_{it}^{\beta} x_{it}^{1-\beta} di\right) L_t^{\beta} , \tag{4}$$

where $\beta \in (0,1)$. x_{it} is the quantity of the intermediate good of type i used in the production, and the productivity of x_{it} depends on its own quality q_{it}. Because population is constant at $L_t = 1$ and labor is supplied inelastically, we omit the parameter L_t in the rest of the paper.

Based on the profit maximization of final good producers, the equilibrium wage rate is[11]

$$w_t = \beta Y_t , \tag{5}$$

and the conditional demand function for x_{it} is

$$x_{it} = p_{it}^{-\frac{1}{\beta}} q_{it} , \tag{6}$$

where p_{it} is the price of intermediate good i.

2.3. Intermediate goods and R&D

There is a unit continuum of industries $i \in [0,1]$ that produce differentiated intermediate goods. In each industry i, there is an industry leader that holds a patent on the highest-quality version of intermediate good i and that becomes the incumbent

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[10] To avoid distortions in resource allocation, following Barro and Sala-i-Martin (2004), the subsidies on production or R&D would have to be financed by a lump-sum tax rather than a wage tax.

[11] The profit maximization of the final good firms yields the conditional demand function for the labor input: $L_t = \beta Y_t / w_t$. Given that labor supply is inelastic and normalized to unity (i.e., $L_t = 1$), the equilibrium wage rate is then given by $w_t = \beta Y_t$.

firm of the industry. Following Acemoglu and Cao (2015) and Akcigit and Kerr (2018), we assume that the final good producers will only use the highest-quality version of each intermediate good. Therefore, each type of intermediate good is produced by a single monopolistic firm, and the incumbent can always charge the monopoly price.[12] There are two sources of innovation in the economy: in-house R&D and creative destruction. In-house R&D is carried out by incumbent firms in an attempt to improve the quality of their existing products. Meanwhile, creative destruction takes place when new firms enter the market and become new intermediate goods producers after making a successful innovation.

2.3.1. Incumbents

To simplify the expressions below, we assume that the marginal cost of producing one unit of intermediate good $i \in [0,1]$ is constant at the level of $1-\beta$ units of the final good. The incumbent firm's profit at time t is $\pi_{it} = [p_{it} - (1-s_x)(1-\beta)]x_{it}$, where s_x is the subsidy rate for the production of intermediate goods. The profit-maximization of each incumbent gives its optimal price and quantity, which are respectively given by

$$p_{it} = 1 - s_x, \tag{7}$$

$$x_{it} = (1-s_x)^{-\frac{1}{\beta}} q_{it}. \tag{8}$$

Using (7) and (8), the amount of monopolistic operating profit in industry i is

$$\pi_{it} = \beta (1-s_x)^{1-\frac{1}{\beta}} q_{it}. \tag{9}$$

As indicated in (9), the monopolistic profit of an incumbent is proportional to the quality of its product, and thus each incumbent has an incentive to engage in in-house R&D in quality improvement. To achieve an instantaneous Poisson arrival rate of successful innovation in quality improvement $z_{it} \geq 0$, we follow Akcigit and Kerr (2018) and assume that the flow R&D cost of an incumbent firm is[13]

$$C_m(z_{it}, q_{it}) = \frac{1}{2}\delta_m z_{it}^2 q_{it}. \tag{10}$$

The parameter $\delta_m > 0$ denotes the productivity of in-house R&D. In (10), the flow innovation cost $C_m(z_{it}, q_{it})$ is proportional to the quality of the intermediate good i. This specification implies that improving a higher-quality intermediate good is more

[12] See Akcigit and Kerr (2018) who provide a two-stage price-bidding game to rationalize this assumption.
[13] Existing evidence suggests that the elasticity of patents to R&D expenditures is around 0.5, which implies a quadratic curvature (e.g., Hall and Ziedonis, 2001; Blundell et al., 2002). See Akcigit and Kerr (2018) for an extensive discussion on the quadratic cost function.

expensive. In addition, the government intervenes in in-house R&D through subsidies at the rate of $s_m \in (0,1)$. When an innovation by incumbent firm i is successful, the quality level of its product improves by a step size $\lambda_m > 0$ such that $q_{i(t+\Delta t)} = (1+\lambda_m)q_{it}$.

2.3.2. Entrants

At any time, there is a mass of entrants who seek to take over the market position of incumbent firms. Entrants do not produce any input and only engage in R&D investment to innovate over the existing intermediate goods. If an entrant's innovation succeeds, that entrant will become the new quality leader of an industry $i \in [0,1]$.[14] Similar to the R&D cost function of an incumbent firm, to achieve an instantaneous Poisson arrival rate of successful innovation $z_{et} \geq 0$, each entrant firm bears the following flow R&D cost:

$$C_e(z_{et}, \bar{q}_t) = \frac{1}{2}\delta_e z_{et}^2 \bar{q}_t. \tag{11}$$

The parameter $\delta_e > 0$ denotes the productivity of creative destruction by entrants. Differing from that of incumbents, in (11) the flow R&D cost of entrants is specified to be proportional to the average quality level in the economy, denoted by $\bar{q}_t = \int_0^1 q_{it} di$.[15] This specification implies that entrants who engage in R&D to replace incumbents need to incur higher flow costs in a technologically more advanced economy. Moreover, the government intervenes in entrants' R&D through subsidies at the rate of $s_e \in (0,1)$. In addition to the variable R&D expenditure, each entrant also requires an entry cost $\psi \bar{q}_t$ to set up a new firm, where ψ is a cost parameter.[16] As we will show later, this fixed cost ensures that our model has a unique balanced growth path (BGP). Because potential new entrants seek to obtain leadership over intermediate inputs that they do not currently own, their innovations have wide breadth and applications. Therefore, to model this feature, we follow Akcigit and Kerr (2018) and assume that each entrant achieves a breakthrough in any intermediate industry $i \in [0,1]$ with equal probability. Thus, in terms of expectations, an entrant's successful innovation leads to an increase in the average quality level from $\bar{q}_t$ to $(1+\lambda_e)\bar{q}_t$, where $\lambda_e > 0$ represents the step size of creative destruction.

The mass of entrants is endogenous and denoted by m_t. Thus, the creative

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[14] In other words, entrants threaten the ability of incumbents to remain in the market via Schumpeterian creative destruction.

[15] Since there is a unit continuum of intermediate industries, the average quality level equals the aggregate technology level.

[16] The entry cost is proportional to the average quality level in the economy. This assumption captures the idea that it is more expensive to enter the market in a technologically more advanced economy.

destruction rate ϕ_{et} is also endogenous and determined by aggregating the innovation flow rates across the mass of entrants.[17] We consider that all entrants are homogeneous in the economy, and then the innovation flow rate is equal across entrant firms. Therefore, the rate of creative destruction at time t is given by

$$\phi_{et} = m_t z_{et} . \tag{12}$$

Eq. (12) indicates that the mass of new entrants m_t and the instantaneous Poisson arrival rate z_{et} of new firms determine the frequency of successful innovations coming from creative destruction ϕ_{et}.

2.4. Aggregations

Substituting (8) into (4) yields the following aggregate production function:

$$Y_t = \frac{1}{1-\beta}(1-s_x)^{1-\frac{1}{\beta}}\bar{q}_t . \tag{13}$$

Using (8), we can infer that the aggregate expenditure on intermediate goods production $X_t = (1-\beta)\int_0^1 x_{it} di$ is given by

$$X_t = (1-\beta)(1-s_x)^{-\frac{1}{\beta}}\bar{q}_t . \tag{14}$$

Substituting (13) into (5), the real wage rate is

$$w_t = \frac{\beta}{1-\beta}(1-s_x)^{1-\frac{1}{\beta}}\bar{q}_t . \tag{15}$$

Moreover, based on (10)-(12), the total expenditure on in-house R&D C_{Mt}, the total variable R&D spending by entrants C_{Et}, and the total entry cost C_{Ft} are respectively given by

$$C_{Mt} = \frac{1}{2}\delta_m z_{it}^2 \bar{q}_t , \tag{16}$$

$$C_{Et} = \frac{1}{2} m_t \delta_e z_{et}^2 \bar{q}_t , \tag{17}$$

$$C_{Ft} = m_t \psi \bar{q}_t . \tag{18}$$

2.5. Government

The government intervenes in the production of intermediate goods, the in-house

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[17] In fact, ϕ_{et} is the expected rate of creative destruction. Throughout this study, ϕ_{et} is simply called the rate of creative destruction since this will not cause any confusion.

R&D of incumbents, and the creative destruction of entrants by imposing subsidy rates s_x, s_m, and s_e, respectively. Moreover, the government collects tax revenues in the amount T_t from the household. As a result, the balanced-budget condition is

$$T_t = s_x X_t + s_m C_{Mt} + s_e C_{Et}, \tag{19}$$

and T_t changes endogenously to balance the government budget.

2.6. Equilibrium definitions

The competitive equilibrium in this economy consists of a time path of allocations $\{C_t, A_t, Y_t, x_{it}, X_t, C_m(z_{it}, q_{it}), C_e(z_{et}, \bar{q}_t), C_{Mt}, C_{Et}, C_{Ft}\}_{t=0}^{\infty}$, a time path of prices $\{w_t, r_t, p_{it}, V(q_{it})\}_{t=0}^{\infty}$,[18] and a time path of the mass of entrants, innovation flow rates and creative destruction rates $\{m_t, z_{it}, z_{et}, \phi_{et}\}_{t=0}^{\infty}$. Also, at each instance of time,

(i) the household maximizes utility taking $\{w_t, r_t\}$ as given;

(ii) competitive final good firms maximize profits taking $\{w_t, p_{it}, q_{it}\}$ as given;

(iii) incumbent firms produce $\{x_{it}\}$ and choose $\{z_{it}, p_{it}\}$ to maximize expected profits taking $\{r_t, \phi_{et}\}$ and the subsidy rates s_x and s_m as given;

(iv) entrants make entry decisions and choose z_{et} to maximize expected net return taking $\{r_t\}$ and the subsidy rates s_x, s_m, and s_e as given;

(v) the final good market clears such that $C_t + X_t + C_{Mt} + C_{Et} + C_{Ft} = Y_t$;

(vi) the asset market clears such that the value of monopolistic firms adds up to the value of the household's asset: $\int_0^1 V(q_{it})di = A_t$;

(vii) the government balances its budget reported in (19).

2.7. Optimal innovation decisions

Before solving the BGP of the economy, in this subsection, we first characterize incumbents' and entrants' R&D decisions. To obtain the optimal innovation flow rates, we consider the profit-maximization problem of incumbents and entrants, respectively.

2.7.1. Incumbents' maximization problem

We focus on the symmetric equilibrium. Thereafter, the subscript i is suppressed since this will simplify the notation and not cause any confusion. Let $V(q)$ denote the value function of an incumbent firm. The value function $V(q)$ is a function of intermediate good quality q, and the incumbent firm maximizes its expected return by choosing its optimal R&D flow rate. The incumbent's value function satisfies the

[18] $V(q)$ is the value of an incumbent firm with intermediate good quality q, which we will discuss in detail in the next subsections.

following standard Hamilton-Jacobi-Bellman (henceforth HJB) equation:

$$rV(q)-\dot{V}(q)=\max_{z}\{\pi-C_m(z,q)(1-s_m)+z[V((1+\lambda_m)q)-V(q)]-\phi_e V(q)\}. \quad (20)$$

As mentioned in (12), $\phi_e(=m_e z_e)$ is the rate of creative destruction at which the incumbent loses the market position and exits the economy.[19] The term $\dot{V}(q)$ on the left-hand side of (20) represents the change in the incumbent firm value without any successful innovations (i.e., the quality level q in the industry does not change). The right-hand side of (20) is the sum of four terms. The first term, π, is the monopolistic profit of production reported in (9), while the second term, $C_m(z,q)(1-s_m)$, is the net flow cost paid by the incumbent for improving the quality of its product. The last two terms represent changes in the value of the incumbent due to innovation either by the incumbent itself or by an entrant. The term $z[V((1+\lambda_m)q)-V(q)]$ is the probability-weighted change in the incumbent firm value due to quality improvement by itself (at the arrival rate z, and the step size of the quality improvement is λ_m), while $\phi_e V(q)$ is the expected value loss due to creative destruction by an entrant (the incumbent is replaced at the arrival rate ϕ_e).

The first-order condition of the maximization problem in (20) yields the optimal innovation flow rate of incumbents:

$$z=\frac{V((1+\lambda_m)q)-V(q)}{\delta_m q(1-s_m)}. \quad (21)$$

The numerator of (21) is the change in an incumbent's value due to own-product improvement $V((1+\lambda_m)q)-V(q)$ (i.e., the gain from in-house R&D). Moreover, the denominator represents the adjusted R&D cost coefficient $\delta_m(1-s_m)$ (the government subsidizes in-house R&D at the rate s_m), weighted by the quality level q.

2.7.2. Entrants' maximization problem

We denote $V_e(\bar{q})$ as the expected value of an entrant from entering the market before successful innovation, which is a function of the average quality level $\bar{q}$. In other words, $V_e(\bar{q})$ is the ex-ante value of an innovation. This value satisfies the following HJB equation:

$$rV_e(\bar{q})-\dot{V}_e(\bar{q})=\max_{z_e}\{z_e E_{i\in[0,1]}[V((1+\lambda_e)q_i)-V_e(\bar{q})]-C_e(z_e,\bar{q})(1-s_e)\}. \quad (22)$$

The term $\dot{V}_e(\bar{q})$ represents the derivative of the ex-ante value with respect to time t.

[19] Since there is a unit continuum of intermediate input industries, the rate of creative destruction in each industry equals the aggregate creative destruction rate.

The first term on the right-hand side of (22) is the probability-weighted expected change in value when the entrant achieves a breakthrough and takes over the market position of the current incumbent. The second term $C_e(z_e,\bar{q})(1-s_e)$ is the variable cost borne by the entrant in terms of the final good.

From (22) we can solve the following optimal innovation flow rate of entrants:

$$z_e = \frac{E_{i\in[0,1]}[V((1+\lambda_e)q_i)-V_e(\bar{q})]}{\delta_e\bar{q}(1-s_e)}. \tag{23}$$

Similarly, the numerator of (23) $E_{i\in[0,1]}[V((1+\lambda_e)q_i)-V_e(\bar{q})]$ is the expected change in an entrant firm's value when the entrant becomes a new incumbent upon a successful innovation. The denominator represents the adjusted R&D cost coefficient $\delta_e(1-s_e)$, weighted by the average quality level $\bar{q}$.

2.8. BGP properties and growth decomposition

To solve for the value function $V(q)$, in line with Acemoglu and Cao (2015), we focus on the linear BGP, along which the value function of a monopolist incumbent firm with quality q is linear in q.[20] Specifically, we conjecture that the incumbent value function exhibits a linear form $V(q)=vq$, where $v>0$ is the marginal (and average) value of quality. Given this linear form, it follows from (21) that the optimal innovation flow rate of incumbents is given by

$$z = \frac{v\lambda_m}{\delta_m(1-s_m)}. \tag{24}$$

When there is a positive entry, the free entry condition requires that the ex-ante value of an innovation equal the fixed entry cost. Therefore, in equilibrium, the free entry condition is

$$V_e(\bar{q}) = \psi\bar{q}. \tag{25}$$

Substituting (25) into (23) and also specifying the linear form of the entrant value function, the optimal innovation flow rate of entrants can be rewritten as

$$z_e = \frac{v(1+\lambda_e)-\psi}{\delta_e(1-s_e)}. \tag{26}$$

Moreover, in order to ensure an equilibrium featuring positive entry and aggregate creative destruction, we make the following assumption:

Assumption 1. The inequality $v(1+\lambda_e)>\psi$ needs to hold such that there is a positive

[20] For a definition and more detailed discussion of the linear BGP, see Acemoglu and Cao (2015) and Akcigit and Kerr (2018).

entry in the economy.

Under Assumption 1, in the equilibrium, incumbents and entrants will engage in in-house R&D and creative destruction in the economy, respectively. Then, the growth rate of the technology level is given by

$$g = \frac{\dot{\bar{q}}}{\bar{q}} = z\lambda_m + \phi_e \lambda_e . \tag{27}$$

Moreover, from (13)-(18), the asset market clearing condition, and the final good market clearing condition, we have

$$\frac{\dot{\bar{q}}}{\bar{q}} = \frac{\dot{A}}{A} = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{w}}{w} = \frac{\dot{C}_M}{C_M} = \frac{\dot{C}_E}{C_E} = \frac{\dot{C}_F}{C_F} = g . \tag{28}$$

Then, given the Euler equation (3), the real interest rate along the BGP is

$$r = g + \rho . \tag{29}$$

Henceforth, the variable with the superscript "*" refers to its equilibrium value. In the BGP equilibrium, the unique v^* that satisfies (22) is given by[21]

$$v^* = \frac{\sqrt{2\rho\psi\delta_e(1-s_e)} + \psi}{1+\lambda_e} . \tag{30}$$

Eq. (30) reveals that $v^* > 0$ when entrants have to pay a fixed entry cost (i.e., $\psi > 0$) to enter the market. Therefore, there is a unique linear BGP, where the incumbent value function $V(q) = v^* q$, and v^* is given by (30). Given v^*, the equilibrium R&D arrival rates of incumbents and entrants are simply given by (24) and (26), respectively. From (30), we immediately have that $\dot{V}(q) = 0$.

Substituting (27) and (29) into (20), the BGP creative destruction rate is given by[22]

$$\phi_e^* = \frac{1}{1+\lambda_e}\left[\frac{\beta(1-s_x)^{1-\frac{1}{\beta}}}{v^*} - \frac{1}{2}\lambda_m z^* - \rho\right]. \tag{31}$$

Thus, the BGP growth rate of the aggregate variables in the economy can be expressed as

$$\underbrace{g^*}_{\substack{Growth \\ in\ Output}} = \underbrace{z^*\lambda_m}_{\substack{Contribution \\ of\ Incumbents}} + \underbrace{\phi_e^*\lambda_e}_{\substack{Contribution \\ of\ Entrants}} . \tag{32}$$

Eq. (32) indicates that output growth is decomposed into the contribution of two sources. The first source, $z^*\lambda_m$, arises from the quality improvements of incumbents, which are

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21 See Appendix A for a detailed derivation.

22 See Appendix B for a detailed proof.

equal to the incumbents' innovation arrival rate multiplied by the step size of their quality improvements. The second source, $\phi_e^* \lambda_e$, comes from the creative destruction of entrants, which is equal to the aggregate creative destruction rate multiplied by the step size of creative destruction. The expression reported in (32) indicates that the distinct types of subsidy policies will affect the BGP growth rate by way of changes in the composition of the incumbents' contribution to economic growth $z^* \lambda_m$ and the entrants' contribution to economic growth $\phi_e^* \lambda_e$. Accordingly, the changes between $z^* \lambda_m$ and $\phi_e^* \lambda_e$ stemming from distinct types of subsidies are henceforth dubbed as the "growth composition effect" of subsidies.

Eq. (32) reveals an important novelty compared to the existing literature. In their second-generation R&D-based growth models, Peretto (1998), Howitt (1999) and Segerstrom (2000) combine both variety expansion (horizontal R&D) and quality improvement (vertical R&D) in their models, and find that the equilibrium growth rate is not affected by the variety expansion of entrants. In a departure from these studies, this paper allows both incumbents and new entrants to undertake quality improvements, and, as exhibited in (32), both types of vertical R&D (own-product improvements by incumbents and Schumpeterian creative destruction by new entrants) are powerful in governing the long-run growth rate.

As for the dynamics of the model, (30) implies that the marginal value of quality must jump immediately to its steady-state value v^*. Given a stationary v^*, the other variables in the economy are also stationary. Therefore, in association with a given subsidy rate, the economy immediately jumps to the unique BGP along which each variable grows at a constant rate g^*. Based on the above discussions, we can establish the following proposition:

Proposition 1. *Supposing that Assumption 1 holds and entrants have to bear a fixed entry cost to enter the market, there exists a unique BGP along which the incumbent value function with the intermediate input quality* q *is given by* $V(q) = v^* q$*, where* v^* *is the unique marginal value of quality. In association with a given subsidy rate, the economy immediately jumps to this BGP along which each aggregate variable grows at a constant rate* g^**, where* g^* *is reported in (32).*

3. Growth and welfare effects of various subsidies

In this section, we analyze the impact of subsidies on economic growth, growth composition, and social welfare. Lemma 1 summarizes the impact of subsidies on the equilibrium marginal value of quality v^*.

Lemma 1. *(a) The equilibrium marginal value of quality* v^* *decreases with the subsidy rate for creative destruction* s_e*; (b) the equilibrium marginal value of quality* v^* *is not affected by the subsidy rate for the production of intermediate goods* s_x *and the subsidy rate for in-house R&D* s_m.

Proof. See (30). ■

Intuitively, an increase in the subsidy on entrants' R&D investment promotes entrants' innovation, which makes incumbents more likely to be replaced and exit the market, thus reducing the marginal value of incumbents v^*. Interestingly, subsidies on the production of intermediate goods or incumbents' R&D investment will not affect v^*. The intuition can be explained as follows. A rise in s_x lowers the after-subsidy input costs of the intermediate goods production and a rise in s_m reduces the after-subsidy R&D costs, both of which tend to increase the value of incumbents and hence lead to a positive effect on v^*. However, a higher incumbent firm value will attract the entry of new firms, which in turn generates a negative effect on the marginal value of quality v^*. The former positive effect on v^* is exactly offset by the latter negative effect, and hence a rise in either s_x or s_m is powerless in affecting the value of v^*.[23]

3.1. Effects of subsidies on in-house R&D

In this subsection, we analyze the effects of subsidies on incumbents' in-house R&D. Based on Lemma 1 and (24), we immediately establish the following proposition:[24]

Proposition 2. *(a) The equilibrium in-house R&D is not affected by the subsidy rate for the production of intermediate goods* s_x*; (b) the equilibrium in-house R&D increases with the subsidy rate for in-house R&D* s_m*; (c) the equilibrium in-house R&D decreases with the subsidy rate for creative destruction* s_e.

Proof. Substituting (30) into (24) yields the equilibrium innovation rate of incumbents:

$$z^* = \frac{v^* \lambda_m}{\delta_m (1-s_m)} = \frac{\left[\sqrt{2\rho\psi\delta_e(1-s_e)}+\psi\right]\lambda_m}{\delta_m(1+\lambda_e)(1-s_m)}. \tag{33}$$

From (33), we have $\partial z^*/\partial s_x = 0$, $\partial z^*/\partial s_m > 0$, and $\partial z^*/\partial s_e < 0$. ■

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23 The free entry of new firms will make the free entry condition (25) hold again. Given that $\psi \bar{q}$ does not change, $V_e(\bar{q})$ must remain unchanged. Therefore, the unique v^* that satisfies (22) returns to its original level.

24 One point related to Proposition 2 should be mentioned here. Given that the incumbents' contribution to growth is $z^* \lambda_m$ as reported in (32) and the step size of the incumbents' quality improvements λ_m is a constant value, the effect of subsidies on the equilibrium innovation flow rate z^* of incumbents is qualitatively the same as that on the incumbents' contribution to growth.

The first equality in (33) shows that the incumbents' innovation flow rate depends on v^* and is independent of s_x. Moreover, from Lemma 1, we know that s_x cannot affect v^*. Thus, subsidizing the production of intermediate goods cannot affect the innovation decision of incumbents. As for the subsidy rate s_m, although a rise in the subsidy rate for the production of intermediate goods s_m does not affect v^*, it reduces the cost of in-house R&D. Thus, an increase in s_m stimulates the innovation of incumbents. In addition, Lemma 1 shows that a rise in the subsidy rate for entrants' creative destruction s_e has a negative effect on v^*, which in turn reduces the incentive for incumbents to invest in R&D to improve the quality of their products.

3.2. Effects of subsidies on creative destruction

In this subsection, we explore the effects of subsidies on the entrants' innovation decision and entry incentives. As mentioned earlier, the mass of entrants and entrants' R&D efforts jointly determine the aggregate creative destruction. Here we first summarize the effects of subsidies on the intensive and extensive innovation margins into Lemma 2 and Lemma 3, respectively.[25]

Lemma 2. *(a) The equilibrium innovation flow rate z_e^* of entrants increases with the subsidy rate for creative destruction s_e; (b) the equilibrium innovation flow rate of entrants z_e^* is not affected by the subsidy rate for the production of intermediate goods s_x and the subsidy rate for in-house R&D s_m.*

Proof. Substituting (30) into (26) yields the equilibrium innovation flow rate of entrants:

$$z_e^* = \sqrt{\frac{2\rho\psi}{\delta_e(1-s_e)}}. \tag{34}$$

From (34), we obtain $\partial z_e^* / \partial s_x = \partial z_e^* / \partial s_m = 0$ and $\partial z_e^* / \partial s_e > 0$. ■

Lemma 3. *(a) The equilibrium mass of entrants m^* increases with the subsidy rate s_x for the production of intermediate goods; (b) the equilibrium mass of entrants m^* decreases with the subsidy rate s_m for in-house R&D; (c) the equilibrium mass of entrants m^* may increase or decrease with the subsidy rate s_e for creative destruction, depending on the relative size of the parameters.*

Proof. Substituting (30) and (33) into (31) and then combining with (12) and (34) yields

$$m^* = \frac{\beta(1-s_x)^{1-\frac{1}{\beta}}}{2\rho\psi + \sqrt{2\rho\psi^3 / \delta_e(1-s_e)}} - \frac{\lambda_m^2\left[\delta_e(1-s_e) + \sqrt{\psi\delta_e(1-s_e)/(2\rho)}\right]}{2\delta_m(1+\lambda_e)^2(1-s_m)} - \sqrt{\frac{\rho\delta_e(1-s_e)}{2(1+\lambda_e)^2\psi}}. \tag{35}$$

[25] In the literature, the innovation flow rate of entrants is called the intensive innovation margin, and the number of entrants is called the extensive innovation margin.

From (35) we immediately obtain $\partial m^{*}/\partial s_{x} > 0$ and $\partial m^{*}/\partial s_{m} < 0$. Differentiating (35) with respect to s_{e} yields

$$\frac{\partial m^{*}}{\partial s_{e}} = -\frac{\beta(1-s_{x})^{1-\frac{1}{\beta}}\sqrt{\rho\psi^{3}/2\delta_{e}}(1-s_{e})^{-\frac{3}{2}}}{\left(2\rho\psi+\sqrt{2\rho\psi^{3}/\delta_{e}(1-s_{e})}\right)^{2}} + \frac{\lambda_{m}^{2}\left[\delta_{e}+\sqrt{\psi\delta_{e}/\left(2^{3}\rho(1-s_{e})\right)}\right]}{2\delta_{m}(1+\lambda_{e})^{2}(1-s_{m})}$$

$$+\sqrt{\frac{\rho\delta_{e}}{2^{3}(1+\lambda_{e})^{2}\psi(1-s_{e})}}. \tag{36}$$

Eq. (36) clearly indicates that the sign of $\partial m^{*}/\partial s_{e}$ is ambiguous, and depends on the relative size of the parameters. ■

The intuition behind Lemma 2 is obvious. From (26) and Lemma 1, we can easily observe that the subsidy rates s_{x} and s_{m} do not affect the equilibrium innovation flow rate of entrants z_{e}^{*}. As for the subsidy rate s_{e}, it generates two offsetting effects on the entrants' innovation decision. On the one hand, an increase in s_{e} reduces the R&D investment cost of entrants, and hence promotes Schumpeterian creative destruction. On the other hand, as indicated in Lemma 1, the higher s_{e} reduces the value of being an incumbent, which in turn discourages new firms from entering the market. In our model, the former positive effect of the subsidy rate s_{e} on z_{e}^{*} dominates the latter negative effect, thereby leading to the result that z_{e}^{*} increases with the subsidy rate for entrants' R&D s_{e}.

In addition, the intuition for Lemma 3 can be explained as follows. A higher s_{x} increases the production profits of incumbents, which in turn attracts the entry of new firms. As for s_{m}, due to the crowding-out effect of in-house R&D on entrants' innovation, a rise in s_{m} tends to stimulate the own-product improvements of incumbents. This in turn crowds out entrants' R&D, and then leads to a decline in the mass of entrants. However, a rise in s_{e} leads to two offsetting effects on the market structure (i.e., the number of entrants). On the one hand, a higher s_{e} reduces the R&D costs of entrants, which in turn attracts new firms to the market. On the other hand, based on Lemma 1, an increase in s_{e} reduces the marginal value of being an incumbent, thereby reducing the incentive of new firms to enter the market and invest in R&D to replace the incumbent. Consequently, the impact of the subsidy rate s_{e} on the market structure is ambiguous.

According to (12), in equilibrium the aggregate creative destruction rate is given by $\phi_{e}^{*} = m^{*}z_{e}^{*}$. Based on Lemma 2 and Lemma 3, and given that the step size λ_{e} is constant, we are now in a position to explore the effects of subsidies on the equilibrium aggregate creative destruction (i.e., the entrants' contribution to R&D and growth).[26] Proposition 3 summarizes the main results:

Proposition 3. (a) *The equilibrium* aggregate *creative destruction increases with the subsidy rate* s_x *for the production of intermediate goods; (b) the equilibrium* aggregate *creative destruction decreases with the subsidy rate* s_m *for in-house R&D; (c) the equilibrium* aggregate *creative destruction increases with the subsidy rate* s_e *for creative destruction.*

Proof. Substituting (30) and (33) into (31) yields

$$\phi_e^* = \frac{\beta(1-s_x)^{1-\frac{1}{\beta}}}{\sqrt{2\rho\psi\delta_e(1-s_e)}+\psi} - \frac{\lambda_m^2\left[\sqrt{2\rho\psi\delta_e(1-s_e)}+\psi\right]}{2\delta_m(1+\lambda_e)^2(1-s_m)} - \frac{\rho}{1+\lambda_e}. \tag{37}$$

Differentiating (37) with respect to s_x, s_m, and s_e, respectively, we immediately have $\partial\phi_e^*/\partial s_x > 0$, $\partial\phi_e^*/\partial s_m < 0$, and $\partial\phi_e^*/\partial s_e > 0$. ■

Lemma 2 shows that neither s_x nor s_m will affect the equilibrium innovation flow rate z_e^* of entrants, while Lemma 3 shows that they are powerful in affecting the number of new entrants m^* that engage in R&D. A higher s_x tends to increase the entry of new firms, and hence promotes the aggregate creative destruction. On the contrary, a rise in s_m reduces the mass of new entrants, thereby causing a decline in the aggregate creative destruction. Interestingly, while subsidizing entrants' R&D has a definite positive effect on z_e^* and an ambiguous effect on m^*, a higher subsidy rate s_e definitively promotes the aggregate creative destruction. In other words, even if an increase in the subsidy rate s_e decreases the entrants' extensive innovation margin, this negative effect will be dominated by the associated positive effect on the entrants' intensive innovation margin.

3.3. Effects of subsidies on the growth rate

In the previous two subsections, we have analyzed in detail how subsidies affect the incumbents' and entrants' contribution to economic growth, respectively. We are now ready to investigate the effects of subsidies on the equilibrium growth rate. From (24) and (30)-(32), we immediately have the following proposition:

Proposition 4. *(a) The equilibrium economic growth rate* g^* *increases with the subsidy rate for the production of intermediate goods* s_x*; (b) the equilibrium economic*

[26] Similarly, given that the entrants' contribution to growth is $\phi_e^*\lambda_e$ and the step size of creative destruction λ_e is a constant value, the effect of subsidies on the equilibrium aggregate creative destruction rate ϕ_e^* is qualitatively the same as that on the entrants' contribution to growth.

growth rate g^* increases with the subsidy rate for in-house R&D s_m; (c) for a sufficiently small (large) entry cost ψ, the equilibrium economic growth rate g^* increases (decreases) with the subsidy rate for creative destruction s_e.

Proof. See Appendix C. ■

The economic intuition behind Proposition 4 can be explained as follows. Eq. (32) indicates that the determination of the growth rate is composed of two sources: the first source is the incumbents' quality improvements $z^*\lambda_m$ and the second source is the entrants' creative destruction $\phi_e^*\lambda_e$. As a result, we can then deal with how the distinct types of subsidy policies govern the economic growth rate by way of how they affect the two types of innovations that jointly promote economic growth, namely, $z^*\lambda_m$ which comes from incumbents and $\phi_e^*\lambda_e$ which comes from entrants.

Proposition 2 and Proposition 3 show that the higher s_x does not affect the in-house R&D of incumbents $z^*\lambda_m$ (with λ_m being constant), but it does increase the Schumpeterian creative destruction of entrants $\phi_e^*\lambda_e$ (with λ_e being constant). Therefore, subsidizing intermediate goods production is effective in stimulating economic growth. In addition, based on Proposition 2 and Proposition 3, subsidizing in-house R&D stimulates the own-product improvements of incumbents and deters the Schumpeterian creative destruction of entrants. The impact of s_m on g^* is thus determined by these two conflicting forces. In our model, the positive effect dominates the negative effect, thereby causing g^* to be positively related to s_m.[27]

As indicated in Proposition 2 and Proposition 3, in contrast to the subsidy rate for in-house R&D s_m, a rise in the subsidy rate for creative destruction s_e tends to lower the incumbents' innovation $z^*\lambda_m$ and improve the entrants' creative destruction $\phi_e^*\lambda_e$. Therefore, a rise in s_e generates an ambiguous impact on economic growth that crucially depends on the relative extent between the former negative effect and the latter positive effect. Intuitively, when the entry cost ψ is sufficiently small, the mass of new entrants in the economy is sizable, and so is the aggregate creative destruction (note that $\phi_e^*\lambda_e = m^* z_e^*\lambda_e$). Consequently, the lower entry cost ψ will reinforce the latter positive effect and lead to a substantial increase in creative destruction that more than offsets the associated reduction in the incumbents' own-production improvements. Therefore, in this case, an increase in s_e is more likely to result in a rise in the equilibrium growth rate g^*. Conversely, in the case where the entry cost is sufficiently large, fewer entrants will invest in R&D, and the aggregate creative destruction will thus be limited. As is

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[27] See Appendix C.

evident, in this case, an increase in s_e leads to decline in g^*, since the increase in the creative destruction of entrants triggered by the higher s_e is smaller than the associated reduction in the in-house R&D of incumbents.

3.4. Welfare analysis

This subsection analyzes the welfare effects of distinct types of R&D subsidies. By imposing balanced growth, the social welfare function (i.e., the indirect lifetime utility of the representative household) can be expressed as

$$U=\frac{1}{\rho}\left(\ln C_0+\frac{g^*}{\rho}\right), \tag{38}$$

where C_0 is the steady-state level of consumption along the BGP at the initial instant of time 0. Using the final good market clearing condition $Y_t=C_t+X_t+C_{Mt}+C_{Et}+C_{Ft}$ and normalizing the initial aggregate quality level $\bar{q}_0$ to unity, we obtain

$$C_0=\frac{1}{1-\beta}(1-s_x)^{1-\frac{1}{\beta}}-(1-\beta)(1-s_x)^{-\frac{1}{\beta}}-\frac{1}{2}\delta_m z^{*2}-\left(\frac{1}{2}\delta_e z_e^{*2}+\psi\right)m^*. \tag{39}$$

The four terms on the right-hand side of (39) are total output, production expenditures for intermediate goods, R&D expenditures of incumbents, and total expenditures of entrants, respectively. As is evident, following a rise in distinct types of R&D subsidies, the welfare level will change in response by way of changes in C_0 and the balanced growth rate g^*. Differentiating social welfare U by s_x, s_m and s_e, respectively, we obtain

$$\frac{\partial U}{\partial s_x}=\frac{1}{\rho}\left[\frac{1}{C_0}\left(\frac{1}{\beta}(1-s_x)^{-\frac{1}{\beta}}+\frac{1-\beta}{\beta}(1-s_x)^{-\frac{1}{\beta}-1}-\left(\frac{1}{2}\delta_e z_e^{*2}+\psi\right)\underbrace{\frac{\partial m^*}{\partial s_x}}_{+}\right)+\frac{1}{\rho}\underbrace{\frac{\partial g^*}{\partial s_x}}_{+}\right], \tag{40}$$

$$\frac{\partial U}{\partial s_m}=\frac{1}{\rho}\left[-\frac{1}{C_0}\left(\delta_m z^*\underbrace{\frac{\partial z^*}{\partial s_m}}_{+}+\left(\frac{1}{2}\delta_e z_e^{*2}+\psi\right)\underbrace{\frac{\partial m^*}{\partial s_m}}_{-}\right)+\frac{1}{\rho}\underbrace{\frac{\partial g^*}{\partial s_m}}_{+}\right], \tag{41}$$

$$\frac{\partial U}{\partial s_e}=\frac{1}{\rho}\left[-\frac{1}{C_0}\left(\delta_m z^*\underbrace{\frac{\partial z^*}{\partial s_e}}_{-}+m^*\delta_e z_e^*\underbrace{\frac{\partial z_e^*}{\partial s_e}}_{+}+\left(\frac{1}{2}\delta_e z_e^{*2}+\psi\right)\underbrace{\frac{\partial m^*}{\partial s_e}}_{+,-}\right)+\frac{1}{\rho}\underbrace{\frac{\partial g^*}{\partial s_e}}_{+,-}\right]. \tag{42}$$

(40)-(42) clearly show that it is very difficult for us to provide a clear analytical result to solve how the welfare level is affected in response to a change in s_x, s_m, and s_e. Accordingly, we must resort to a numerical analysis.

4. Quantitative analysis

This section provides a quantitative assessment by resorting to a numerical analysis. To be more specific, we examine numerically the relative contributions of incumbents and entrants to growth, the effects of subsidies on market structure, the welfare effects of subsidies, and which type of subsidy is more effective in promoting economic growth and raising social welfare.

4.1. Calibration

The benchmark parameters we set are adopted from commonly-used values in the existing literature or calibrated to match the empirical data. In line with Acemoglu et al. (2018) and Akcigit and Kerr (2018), the discount rate ρ is set to 0.02. Following Akcigit and Kerr (2018) and Akcigit et al. (2021), the step size of entrants' innovations is set to $\lambda_e = 0.08$. As for the production parameter, we calibrate the parameter β by setting the markup ratio of monopolistic intermediate firms in the absence of a production subsidy as $1/(1-\beta) = 1.5$,[28] which is within the reasonable range of the markup values for the US economy (Domowitz et al., 1988; Chirinko and Fazzari, 1994; Devereux et al., 1996; Jones and Williams, 2000). As in Akcigit and Kerr (2018), the R&D cost coefficient of incumbents is set to $\delta_m = 0.65$. Moreover, following Akcigit et al. (2021), the cost coefficient of entrants is set to be about four times δ_m, i.e., $\delta_e = 2.6$. Finally, the parameters $\{\lambda_m, \psi\}$ are set so as to match the following two moments. First, following Zeng and Zhang (2007), the equilibrium economic growth rate is equal to $g^* = 3\%$.[29] Second, in line with the Acemoglu and Cao (2015) estimation, the entrants' contribution to economic growth is half that of the incumbents, i.e., $\phi_e^* \lambda_e = z^* \lambda_m / 2$. Table 1 summarizes our baseline parameterization.

Table 1: Baseline Parameters

Parameters	ρ	λ_m	λ_e	β	ψ	δ_m	δ_e
values	0.02	0.0793	0.08	0.34	1.8	0.65	2.6

4.2. Numerical results

Fig. 1 depicts the effects of R&D subsidies under our benchmark parameter values. The top left panel in Fig. 1 presents the results reported in Lemma 1: v^* decreases with s_e but is not affected by s_x and s_m. Moreover, the top right panel in Fig. 1 indicates

[28] Note the monopolistic price of intermediate goods in (7).

[29] According to the Conference Board Total Economy Database, the average growth rate of GDP in the US for the last 30 years is roughly 3%.

that the mass of new firms is significantly positively related to the subsidy rate s_x but moderately negatively related to the subsidy rate s_m. In particular, while Lemma 3 shows that the impact of subsidizing entrants' R&D on the mass of entrants is ambiguous, as exhibited in the top right panel in Fig. 1, a higher s_e tends to decrease the equilibrium mass of entrants m^* under our benchmark parameter values.

Next, the middle two panels in Fig. 1 confirm the results reported in Propositions 2 and 3. Specifically, a rise in s_x does not affect the incumbents' contribution to growth but has a significant positive effect on the entrants' contribution. The higher s_m greatly increases the incumbents' contribution to growth but has a weaker negative effect on the entrants' contribution. In addition, a rise in s_e leads to a negative effect on the incumbents' contribution to growth and a positive effect on the entrants' contribution, but both effects are relatively insignificant.

Finally, the bottom left panel in Fig. 1 displays how the balanced growth rate will react in response to a rise in each of the three distinct subsidy rates, respectively.[30] Three main findings emerge from this panel. First, the economic growth rate increases with the subsidy rate s_x and the subsidy rate s_m, but decreases with the subsidy rate s_e. Second, the growth effect of the production subsidy rate s_x is more significant than that of the two R&D subsidy rates s_m and s_e. This finding regarding the relative growth effect between non-R&D subsidies and R&D subsidies differs from Zeng and Zhang (2007), but is consistent with Hu et al. (2019). Third, subsidizing the incumbents' R&D is more effective than subsidizing the entrants' R&D in terms of promoting economic growth. In their previous studies on R&D subsidies, both Zeng and Zhang (2007) and Hu et al. (2019) consider only one type of R&D subsidy (a subsidy on product expansion or creative destruction). This paper explicitly deals with simultaneous innovations from both incumbents and entrants, and is thus able to explore the relative superiority between subsidizing the R&D of incumbents (i.e., own-product improvements) and subsidizing the R&D of new firms (i.e., creative destruction) in terms of stimulating economic growth.

The bottom right panel in Fig. 1 shows how the welfare level will respond following a rise in each of the three distinct subsidy rates, respectively. Obviously, the social welfare is significantly positively related to the subsidy rate for intermediate goods production s_x and moderately positively related to the subsidy rate for the

[30] Under our calibrated parameter values, we do not find a nonlinear (i.e., inverted U-shape) relationship between subsidies and growth. In fact, the large body of recent literature finds that subsidies on average are effective in stimulating R&D and growth, while only a few studies provide evidence of an inverted U-shaped effect of subsidies. For a more detailed discussion, see Becker (2015).

incumbents' in-house R&D s_m, but moderately negatively related to the subsidy rate for the entrants' R&D s_e. This clearly indicates that, in terms of raising the social welfare level, subsidizing intermediate goods production is more effective than the two R&D subsidies. Moreover, by comparing the relative welfare effect between the two R&D subsidy rates s_m and s_e, it is quite clear that subsidizing the incumbents' R&D is superior to subsidizing the entrants' R&D in terms of raising social welfare.

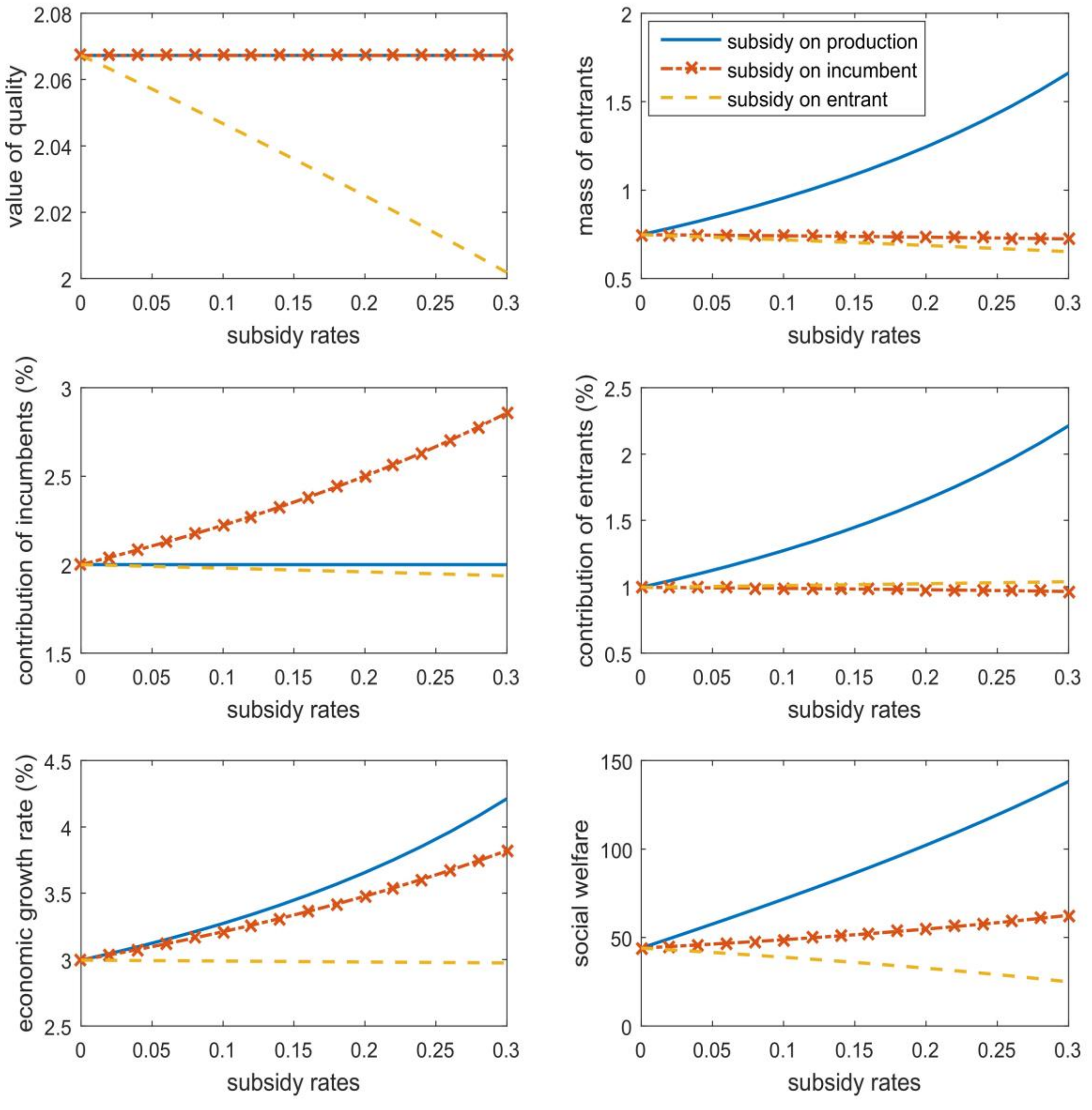

Fig. 1. Growth and welfare effects of subsidies (ψ = 1.8).

To shed light on the importance of the entry cost ψ, in Fig. 2 we perform a numerical analysis to examine whether our findings in Fig. 1 are robust when ψ is lowered from its benchmark value of 1.8 to a relatively low value of 0.8. Several important findings emerge from Fig. 2. First, in this case, the impact of the three subsidies on the EMS is similar to that under the benchmark values. However, by comparing Fig. 1 and Fig. 2, we find that a lower entry cost ψ will sharply increase the number of new firms engaged in R&D in the economy. As mentioned earlier, a higher m^* leads to incumbents being more likely to be replaced and exit the market. Therefore, compared with the case in Fig. 1, the marginal value of the incumbents v^* decreases as exhibited in the top left panel in Fig. 2.

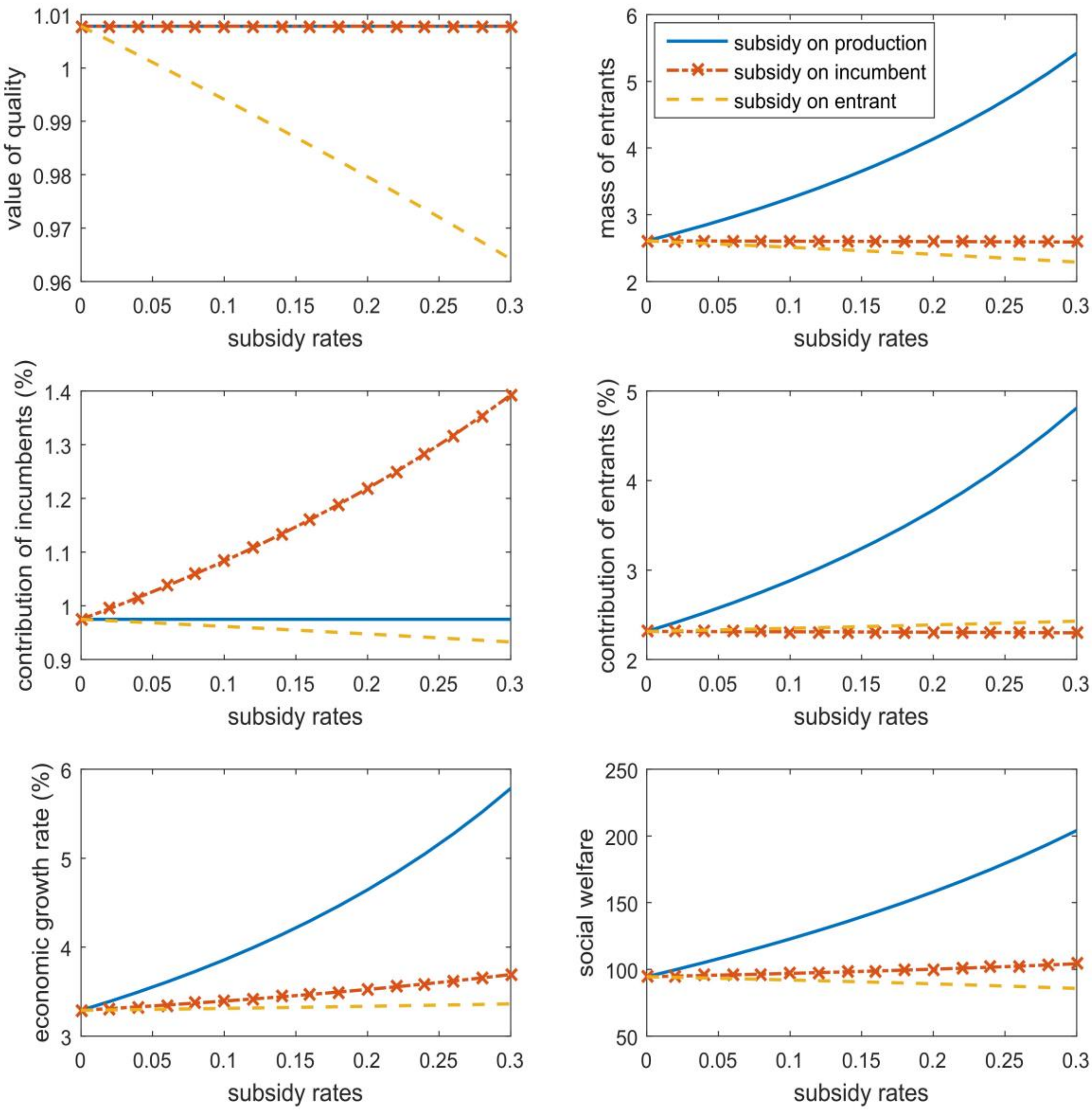

Fig. 2. Growth and welfare effects of subsidies ($\psi = 0.8$).

Second, by comparing the middle two panels in Fig. 2 with those in Fig. 1, it is observed that the incumbents' contribution to growth decreases while the entrants' contribution rises. For instance, throughout the range $s_x \in [0,0.3]$, the incumbents' contribution to growth roughly declines from 2%-2.9% to 1% -1.4%, and the new entrants' contribution to growth roughly increases from 1%-2.2% to 2.3%-4.8%. The rationale behind this finding can be explained intuitively. On the one hand, the previous analysis clearly shows that, when the entry cost is relatively low, more new entrants are motivated to enter the market, thereby causing a rise in the entrants' contribution to growth. On the other hand, more new entrants make it easier for incumbents to be replaced, which lowers the incentive for incumbents to invest in R&D and therefore reduces the incumbents' contribution to growth.

Third, it is clear from the bottom panels in Fig. 2 that, in terms of stimulating economic growth and social welfare, subsidizing intermediate goods production is more effective than the two R&D subsidies. As is obvious, a decline in the entry cost ψ does not affect the relative superiority for improving economic growth and social welfare among s_x, s_m, and s_e. Consequently, our quantitative analysis shows that over a wide range of the entry cost, subsidizing entrants' R&D seems to be less effective in boosting economic growth and improving social welfare.

5. Conclusion

In this study, we revisit the implications of three types of subsidies for economic growth in a Schumpeterian growth model with incumbents and entrants. The salient feature is that incumbents invest in R&D to improve the quality of their products and entrants invest in R&D in an attempt to replace the incumbents, and both the innovations of incumbents and entrants serve as growth engines of the economy. Within this framework, we investigate how three distinct types of subsidies affect entry incentives, the innovation decisions of incumbents and entrants, and the composition of R&D that drives economic growth.

The key prediction of our analysis is that the subsidy on intermediate goods production or the incumbents' R&D investment is effective in promoting economic growth, but whether the subsidy on the entrants' R&D investment can promote growth is ambiguous, and depends on the extent of the entry costs. Furthermore, these three types of subsidies all importantly change the composition of innovation that drives economic growth. More specifically, subsidizing the intermediate goods production stimulates the entrants' contribution to growth but does not affect the incumbents' contribution. Subsidizing in-house R&D increases the incumbents' contribution to growth and reduces the entrants' contribution. Conversely, the subsidy on entrants' R&D reduces the incumbents' contribution to growth and raises the entrants' contribution.

Our quantitative analysis shows that the subsidy on intermediate goods production is more growth-enhancing and welfare-improving than the R&D subsidies. Moreover, in terms of promoting growth and raising welfare, the R&D subsidy to incumbents seems more effective than the R&D subsidy to entrants, especially when the entry cost is high. Equipped with these results, policy-makers need to take into consideration all types of subsidies and the relative contribution of incumbents and entrants to economic growth when they intervene in R&D activities by using subsidy policy instruments.

Appendix A. Proof of (30).

Differentiating the free entry condition (25) with respect to time t yields

$$\dot{V}_e(\bar{q}) = \psi\dot{\bar{q}}. \quad \text{(A.1)}$$

Substituting (25) and (A.1) into (22) and using the linear form of the incumbent value function, we obtain

$$r\psi\bar{q} - \psi\dot{\bar{q}} = \max_{z_e}\left\{z_e[v\bar{q}(1+\lambda_e) - \psi\bar{q}] - \frac{1}{2}\delta_e z_e^2\bar{q}(1-s_e)\right\}. \quad \text{(A.2)}$$

Inserting the optimal innovation flow rate of entrants (26) into (A.2) gives rise to

$$r\psi\bar{q} - \psi\dot{\bar{q}} = \frac{[v(1+\lambda_e) - \psi]^2}{2\delta_e(1-s_e)}\bar{q}. \quad \text{(A.3)}$$

Dividing both sides of (A.3) by $\bar{q}$ and using the equilibrium economic growth rate reported in (27) yield

$$(r-g)\psi = \frac{[v(1+\lambda_e) - \psi]^2}{2\delta_e(1-s_e)}. \quad \text{(A.4)}$$

Given the equilibrium real interest rate in (29), (A.4) can be expressed as

$$\rho\psi = \frac{[v(1+\lambda_e) - \psi]^2}{2\delta_e(1-s_e)}. \quad \text{(A.5)}$$

To guarantee a positive entry of new firms, we impose the restriction $v(1+\varphi) > \psi$. Then, from (A.5) we have

$$\sqrt{2\rho\psi\delta_e(1-s_e)} = v(1+\lambda_e) - \psi. \quad \text{(A.6)}$$

Thus, the unique value of quality v^* satisfying (A.6) is given by

$$v^* = \frac{\sqrt{2\rho\psi\delta_e(1-s_e)} + \psi}{1+\lambda_e}. \quad \text{(A.7)}$$

Eq. (A.7) shows that v^* is always stationary.

Appendix B. Proof of (31).

Given a stationary value of quality v^* and the linear incumbent value $V(q)=v^*q$, we have $\dot{V}(q)=0$. Eq. (20) then becomes

$$rv^*q=\beta(1-s_x)^{1-\frac{1}{\beta}}q+z^*v^*\lambda_m q-\phi_e v^*q-\frac{1}{2}\delta_m z^{*2}q(1-s_m). \tag{B.1}$$

Dividing both sides of (B.1) by v^*q yields

$$r=\frac{\beta(1-s_x)^{1-\frac{1}{\beta}}}{v^*}+z^*\lambda_m-\phi_e-\frac{\delta_m z^{*2}(1-s_m)}{2v^*}. \tag{B.2}$$

Substituting (B.2) and the balanced-growth rate (27) into the BGP real interest rate (29), we obtain

$$z^*\lambda_m+\phi_e\lambda_e+\rho=\frac{\beta(1-s_x)^{1-\frac{1}{\beta}}}{v^*}+z^*\lambda_m-\phi_e-\frac{\delta_m z^{*2}(1-s_m)}{2v^*}. \tag{B.3}$$

Eq. (B.3) can be rewritten as

$$\phi_e=\frac{1}{1+\lambda_e}\left[\frac{\beta(1-s_x)^{1-\frac{1}{\beta}}}{v^*}-\frac{\delta_m z^{*2}(1-s_m)}{2v^*}-\rho\right]. \tag{B.4}$$

Using (24), we have

$$\frac{\delta_m z^{*2}(1-s_m)}{2v^*}=\frac{1}{2}\lambda_m z^*. \tag{B.5}$$

Inserting (B.5) into (B.4) yields the following BGP creative destruction rate:

$$\phi_e^*=\frac{1}{1+\lambda_e}\left[\frac{\beta(1-s_x)^{1-\frac{1}{\beta}}}{v^*}-\frac{1}{2}\lambda_m z^*-\rho\right]. \tag{B.6}$$

Appendix C. Proof of Proposition 4.

Taking the derivative of $g^*=z^*\lambda_m+\phi_e^*\lambda_e$ with respect to s_x and using (31), we obtain

$$\frac{\partial g^*}{\partial s_x}=\frac{\lambda_e(1-\beta)}{v^*(1+\lambda_e)}\left(\frac{1}{1-s_x}\right)^{\frac{1}{\beta}}. \tag{C.1}$$

Substituting (30) into (C.1) gives rise to

$$\frac{\partial g^*}{\partial s_x}=\frac{\lambda_e(1-\beta)}{\sqrt{2\rho\psi\delta_e(1-s_e)}+\psi}\left(\frac{1}{1-s_x}\right)^{\frac{1}{\beta}}. \tag{C.2}$$

Taking the derivative of $g^*=z^*\lambda_m+\phi_e^*\lambda_e$ with respect to s_e yields

$$\frac{\partial g^*}{\partial s_e}=\lambda_m\frac{\partial z^*}{\partial s_e}+\lambda_e\frac{\partial \phi_e^*}{\partial s_e}. \tag{C.3}$$

Equipped with the BGP creative destruction rate (31), (C.3) can be rewritten as

$$\frac{\partial g^*}{\partial s_e}=\frac{\lambda_m(2+\lambda_e)}{2(1+\lambda_e)}\frac{\partial z^*}{\partial s_e}-\frac{\beta(1-s_x)^{1-\frac{1}{\beta}}\lambda_e}{1+\lambda_e}\frac{1}{v^{*2}}\frac{\partial v^*}{\partial s_e}. \tag{C.4}$$

The equilibrium innovation flow rate of incumbent firms z^* can be rewritten as

$$z^*=\frac{\left[\sqrt{2\rho\psi\delta_e(1-s_e)}+\psi\right]\lambda_m}{\delta_m(1+\lambda_e)(1-s_m)}. \tag{C.5}$$

Differentiating (C.5) with respect to s_e yields

$$\frac{\partial z^*}{\partial s_e}=-\frac{\lambda_m\sqrt{\rho\psi\delta_e}}{\delta_m(1+\lambda_e)(1-s_m)\sqrt{2(1-s_e)}}. \tag{C.6}$$

In addition, by differentiating (30) with respect to s_e, we obtain

$$\frac{\partial v^*}{\partial s_e}=-\frac{\sqrt{\rho\psi\delta_e}}{(1+\lambda_e)\sqrt{2(1-s_e)}}. \tag{C.7}$$

Substituting (C.6), (C.7), and (30) into (C.4) yields

$$\frac{\partial g^*}{\partial s_e}=\left[\frac{\beta(1-s_x)^{1-\frac{1}{\beta}}\lambda_e\sqrt{\rho\psi\delta_e}}{\left(\sqrt{2\rho\psi\delta_e(1-s_e)}+\psi\right)^2}-\frac{(2+\lambda_e)\sqrt{\rho\psi\delta_e}\lambda_m^2}{2\delta_m(1-s_m)(1+\lambda_e)^2}\right]\frac{1}{\sqrt{2(1-s_e)}}. \tag{C.8}$$

Using similar procedures to the above, we can infer the following expression:

$$\frac{\partial g^*}{\partial s_m}=\frac{\left[\sqrt{2\rho\psi\delta_e(1-s_e)}+\psi\right](2+\lambda_e)\lambda_m^2}{2\delta_m(1+\lambda_e)^2(1-s_m)^2}. \tag{C.9}$$

From (C.2), (C.8), and (C.9), we immediately have Proposition 4.

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