A simple General Constrained Dynamics (GCD) model for demand, supply and price shocks

Glötzl, Erhard

JKU

15 March 2022

A simple General Constrained Dynamics (GCD) model
for demand, supply and price shocks

Erhard Glötzl
Linz, Austria
erhard.gloetzl@gmail.com

February 2022

Key words: macroeconomic models, demand shock, supply shock, price shock, constraint dynamics, GCD, DSGE, out-of-equilibrium dynamics, Lagrangian mechanics, stock flow consistent, SFC

JEL: A12, B13, B41, B59, C02, C30, C54, C60, E10, E70

Abstract

In economics balance identities as e.g. C+K'-Y(L,K) = 0 must always apply. Therefore, they are called constraints. This means that variables C,K,L cannot change independently of each other. In General Equilibrium Theory (GE), the solution for equilibrium is obtained as optimisation under the above or similar constraints. The standard method for modelling dynamics in macroeconomics are Dynamic Stochastic General Equilibrium (DSGE) models. Dynamics in DSGE models result from the maximisation of an intertemporal utility function that results in the Euler-Lagrange equations. The Euler-Lagrange equations are differential equations that determine the dynamics of the system. In Glötzl, Glötzl, und Richters (2019) we have introduced an alternative method to model dynamics, which is constitutes a natural extension of GE theory. It is based on the standard method for modelling dynamics under constraints in physics. We therefore call models of this type "General Constrained Dynamic (GCD)" models. GCD models can be seen as an alternative to DSGE models to model the dynamics of economic processes. DSGE models are used in particular to analyse economic shocks. For this reason, the aim of this article is to show how GCD models are formulated and how they can be used to model economic shocks such as demand, supply, and price shocks. Since the goal of this paper is to lay out the fundamental principles to the formulation of such GCD models, very simple macroeconomic models are used for illustrative purposes. All calculations can easily be carried out with the open-source program GCDconfigurator, which also allows for the integration of shocks.

Contents

Abstract 2
Contents 3
1. Introduction 4
2. GCD models for non-intertemporal utility functions 7
3. Modeling of supply, demand and price shocks 10
   3.1. 2 different types of shocks 10
   3.2. Examples of demand shocks 10
   3.3. Examples of supply shocks 11
   3.4. Price shock 12
   3.5. Policy shocks 13
4. Topics to be discussed 13
5. Model A1, (1 household, 1 firm, 1 good, without interest) 15
   5.1. Overview of the setup 15
   5.2. Description of the A1 model in detail 16
6. Calculations with model A1 on various shocks 21
7. Calculations with model B1 for central bank polices in case of inflation and deflation shock 29
   7.1. Overview of the setup of model B1 29
   7.2. Inflation and deflation shock as variable shock for the price 31
   7.3. Model inflation and deflation shock as model shock 34
8. Summary 35
Acknowledgements 35
References 36

1. Introduction

The standard method in macroeconomics for modelling dynamics is DSGE (Dynamic Stochastic General Equilibrium). Dynamics in DSGE models are generated by maximising an intertemporal utility function, which leads to the Euler-Lagrange equations. The Euler-Lagrange equations are differential equations which the dynamics must satisfy.

Recently there has been a renewed interest in alternative approaches in macroeconomics. In Zaman (2020) four different methodological principles are presented which lie outside the framework of the conventional approach. One of these concepts is called GCD (General Constrained Dynamics) and is based on the standard method of physics for modelling a dynamic under constraints. It can be seen as a natural extension of the GE theory for modelling dynamics in economics and can be thought of as an alternative to DSGE. The method was first introduced in Glötzl (2015) under the name Newtonian Constrained Dynamics, a name that was later changed to General Constrained Dynamics. The principles of GCD, an encompassing review of the literature and an application of GCD to the microeconomic Edgeworth box model are presented in Glötzl, Glötzl, und Richters (2019). In Richters und Glötzl (2020) it is shown that SFC models (stock flow consistent models (Godley und Lavoie 2012)) can be understood as special forms of GCD models. In Richters (2021) a more complex macroeconomic model is used to show that GCD models converge to the classical equilibrium solution under some assumptions. In Glötzl (2022c) we show how macroeconomic GCD models can be built in a systematic way and how they can be used for macroeconomic analysis. In this respect, we want to point out that all calculations for all GCD models with non-intertemporal utility functions can be performed easily and conveniently with the open source program GCDconfigurator, which is published in GitHub (Glötzl und Binter 2022) which can be downloaded under

https://github.com/lbinter/gcd

All Mathematica program codes used for calculations of the various GCD models can be downloaded under

https://www.dropbox.com/sh/npis47xjqkecggv/AAAMzCVhmhDYIIhoB5MfATFya?dl=

DSGE models are typically used to analyse economic shocks. The target of this article is to show how any kind of economic shock, e.g. demand, supply or price shocks, can be modelled in the GCD framework. In this paper we limit ourselves to GCD models with non-intertemporal utility functions.

In contrast to DSGE models, all previously published GCD models were based on non-intertemporal utility functions. Since intertemporal utility functions are essential in many applications and only intertemporal utility functions are used in DSGE models, (Glötzl 2022a) describes the principles of formulating intertemporal GCD models (IGCD) and shows that these IGCD models can be seen as a generalisation of and alternative to DSGE models. Also in such GCD models with intertemporal utility functions shocks can be integrated in the same way as shown in the following for GCD models with non-intertemporal utility functions.

Non-intertemporal GCD models and intertemporal GCD models can be considered as an essential contribution to solve problem 8 of the 18 major problems of dynamics listed by Steve Smale in 1991 (Smale 1991; 1997; 1998; Smale Institute 2003)

In chapter 2 we give a brief introduction the GCD method for non-intertemporal utility functions.

In chapter 3 we describe the different types of economic shocks and how they can be modelled.

In chapter 4 we discuss the possible model applications related to economic shocks.

In chapter 5 we describe the simple macroeconomic model A1, into which we then integrate supply, demand and price shocks in chapter 6.

In chapter 6 we show the results of the numerical calculations of the impact of various shocks introduced into the A1 model.

In chapter 7 we describe the model B1, which includes a commercial bank and a central bank to describe money creation. We present numerical calculations for the impact of inflation and deflation shocks.

In chapter 8 we give a summary.

2. GCD models for non-intertemporal utility functions

In general, a dynamic economic model is described by agents and variables that describe any stock or flow of goods, resources, financial liabilities or other variables or parameters such as prices or interest rates. The behaviour of these variables is described by behavioural equations. The behaviour of these variables can be restricted by economic constraints, which are described by additional equations. In particular, all balance sheet identities are subject to such constraints. In general, the introduction of additional constraints to the behavioural equations can lead to the system of equations becoming overdetermined and thus unsolvable. The GCD method is a "closure" method to make a system of equations solvable by introducing additional Lagrange multipliers. It can also be understood as a method to transfer the concept of Lagrange multipliers from optimization problems under constraints to dynamic systems under constraints. This is done in analogy to what is done in classical mechanics.

The GCD method is described in detail in Glötzl, Glötzl, und Richters (2019). We will therefore limit ourselves to the explanation of a simple example with 2 agents (each with 1 non-intertemporal utility function) and 1 constraint.

We explain the principle for 2 agents A,B and 2 variables x_1,x_2.
The utility functions of A,B are $U^A(x_1,x_2), U^B(x_1,x_2)$. The interest of A is to change x_1,x_2 so that the increase of his utility function is maximal. This is given, if the change of x_1,x_2 is done in the direction of the gradient of $U^A(x_1,x_2)$, i.e.

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \textit{ proportional to } \begin{pmatrix} \dfrac{\partial U^A}{\partial x_1} \\ \dfrac{\partial U^A}{\partial x_2} \end{pmatrix}$$

The interest of A in a change of the variables does not lead alone to an actual change, because the household must have also the power and/or possibility of actually implementing its change desire. For example, a household cannot or can only partially enforce its additional consumption desire, e.g., to go to the cinema or go on vacation, because it is possibly quarantined or the borders are closed. This limitation of the possibility to enforce his consumption change requests is described by a (possibly time-dependent and endogenously determined) "power factor" μ_C^H. In general, the change request for each of the variables is described by "power factors" $\mu_{x_1}^H, \mu_{x_2}^H$. Considering the power factors, the following applies to the change of x_1, x_2 (due to the interest of A and the power of A to enforce this interest)

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \textit{ proportional to } \begin{pmatrix} \mu_{x_1}^A \dfrac{\partial U^A}{\partial x_1} \\ \mu_{x_2}^A \dfrac{\partial U^A}{\partial x_2} \end{pmatrix}$$

Just as A has an interest, to change x_1, x_2, also B has an interest to change these two variables. The actual change is therefore the result of the two individual efforts to change, weighted with the power factors. We therefore call this behaviour **"individual utility optimization"**.

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} \mu_{x_1}^A \dfrac{\partial U^A}{\partial x_1} \\ \mu_{x_2}^A \dfrac{\partial U^A}{\partial x_2} \end{pmatrix} + \begin{pmatrix} \mu_{x_1}^B \dfrac{\partial U^B}{\partial x_1} \\ \mu_{x_2}^B \dfrac{\partial U^B}{\partial x_2} \end{pmatrix} \qquad \text{<2.1>}$$

This equation of motion <2.1> describe the temporal development of (x_1, x_2) under the condition that there are no constraints that restrict the temporal development. It is therefore referred to as the **ex-ante equation of motion.**

If a constraint

$$Z(x_1, x_2) = 0$$

exists, there arises an additional constraint force f^Z to the ex-ante force which ensures that the constraint is fulfilled at all times. In physics, this constraint force is perpendicular to the constraint at all times due to the so-called d'Alembert principle, i.e.

$$f^Z(x_1, x_2) = \begin{pmatrix} f_1^Z(x_1, x_2) \\ f_1^Z(x_1, x_2) \end{pmatrix} = \lambda \begin{pmatrix} \dfrac{\partial Z(x_1, x_2)}{\partial x_1} \\ \dfrac{\partial Z(x_1, x_2)}{\partial x_2} \end{pmatrix} \qquad \text{<2.2>}$$

The time-dependent factor $\lambda = \lambda(t)$ is called Lagrange multiplier, as in the case of optimisation under constraints.

In economic models, the constraint force does not necessarily have to be perpendicular to the constraint at any point in time due to a special economic principle as in physics, but in most cases it is plausible to model constraint forces in a similar way to physics, namely perpendicular to the constraint.

From <2.1> and <2.2> we get the equation of motion, which is called **ex post equation of motion**:

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} \mu_{x_1}^A \dfrac{\partial U^A(x_1,x_2)}{\partial x_1} \\ \mu_{x_2}^A \dfrac{\partial U^A(x_1,x_2)}{\partial x_2} \end{pmatrix} + \begin{pmatrix} \mu_{x_1}^B \dfrac{\partial U^B(x_1,x_2)}{\partial x_1} \\ \mu_{x_2}^B \dfrac{\partial U^B(x_1,x_2)}{\partial x_2} \end{pmatrix} + \lambda \begin{pmatrix} \dfrac{\partial Z(x_1,x_2)}{\partial x_1} \\ \dfrac{\partial Z(x_1,x_2)}{\partial x_2} \end{pmatrix} \qquad \text{<2.3>}$$

For J agents with the designations $j = 1,2,...,J$
I Variables with the designations x_i $i = 1,2,...,I$ $x = (x_1, x_2, ..., x_I)$
K Constraints with the designations Z_k $k = 1,2,...,K$
the general GCD model equations result analogously

$$x_i' = \sum_{j=1}^{J} \mu_{x_i}^j \frac{\partial U^j(x)}{\partial x_i} + \sum_{k=1}^{K} \lambda_k \frac{\partial Z^k(x)}{\partial x_i} \qquad i = 1,2,...,I \qquad \text{<2.4>}$$

Remark: If constraint conditions depend on time derivatives of variables
If a constraint depends not only on $x = (x_1, x_2, ... x_I)$ but also on $x' = (x_1', x_2', ... x_I')$ or higher derivatives $x'' = (x_1'', x_2'', ... x_I'')$, ... ,i.e.
$0 = Z(x, x', x'', ...)$
the constraint forces are always to be derived from the highest time derivative of the variables (Flannery 2011), i.e.

$$\frac{\partial Z(x,x')}{\partial x_i'} \textit{ instead of } \frac{\partial Z(x,x')}{\partial x_i} \qquad \textit{resp.} \quad \frac{\partial Z(x,x',x'')}{\partial x_i'} \textit{ instead of } \frac{\partial Z(x,x',x'')}{\partial x_i}$$

3. Modeling of supply, demand and price shocks

3.1. 2 different types of shocks

Basically, a shock can lead to 2 fundamentally different types of shocks:

(1) Variable shock

All model parameters remain unchanged, but at the time of the shock t_s one or more model variables $V \in \{C, L, K, M^H, M^F, S, p, w\}$ abruptly change by the factor f_V from V to $f_V V$

$V(t_s) \to f_V V(t_s)$

Interpretation: The basic behaviour of all agents remains the same, but an external event suddenly changes the value of a variable (e.g. the price of energy). The system restarts, as it were, with this new value as the starting value.

(2) Model shock

All variables remain unchanged, but one or more model parameters or power factors $\pi \in \{\alpha, \beta, \gamma, \mu_C^H, \mu_L^H, \mu_L^F, \mu_K^F, \mu_{M^H}^H, \mu_S^F, \mu_p^F, \mu_w^F\}$ are no longer constant but change over time, i.e. $\pi \to \pi(t)$. For the sake of simplicity, we describe the temporal behaviour of such a parameter $\pi(t)$ by multiplication with a sawtooth curve:

$\pi(t) = \pi\,\sigma(t)$

where the sawtooth curve is defined by

t_s *time of the shock*

f *shock factor*

d *duration of the linearly decreasing shock effects*

$$\sigma(t) = \begin{cases} 1 & for\ t < t_s \\ f & for\ t = t_s \\ linear\ from\ f\ to\ 1 & for\ t_s < t < t_s + d \\ 1 & for\ t_s + d_i^j \le t \end{cases}$$

3.2. Examples of demand shocks

For example, a demand shock N can have three different causes:

(1) **Variable shock**: A demand shock can be triggered by the fact that at the time of the demand shock t_N consumption C is reduced by a factor f_C^N from C to $f_C^N C$:

$$C \to f_C^N C$$

At the same time, the constraint Z_3 must always be fulfilled, even during the shock. This is always guaranteed by the numerical solution method for differential-algebraic equations of Mathematica NDSolve. In addition, of course, one can make any other assumptions, such as that production Y and investment K' remain the same and that everything that is consumed less $(1-f_N^C)C(t_N)$ is stored, i.e.

$$S'(t_A) \to S'(t_A) + (1-g_C^N)C(t_A)$$

The dynamic system then continues to develop with these new initial values.

(2) **Model shock:** The model parameter γ describes the consumption preference of the household. A demand shock can be triggered by the changes of γ over time according to a sawtooth curve:

$$\gamma \to \gamma(t) = \sigma(t)\gamma$$

(3) **Model shock**: The power factor μ_C^H describes the power of the household to actually enforce its consumption interests (e.g. due to quarantine measures). A demand shock can be triggered by a change of μ_C^H in time according to a sawtooth curve:

$$\mu_C^H \to \mu_C^H(t) = \sigma(t)\,\mu_C^H$$

3.3. Examples of supply shocks

For example, a supply shock A at the time can have the following causes:

(1) **Model and variable shock**: A supply shock could be triggered by the fact that the production function

$$Y(L,K) = \beta L^{\alpha} K^{1-\alpha}$$

changes over time according to a sawtooth curve with a shock factor f_β^A. This initially corresponds to a model shock, because this is described by the fact that the parameter β changes according to a sawtooth curve with a shock factor f_β^A:

$$\beta \rightarrow \beta(t) = \sigma(t)\beta$$

At the same time, production Y suddenly changes by the shock factor f_β^A at the time of the shock t_A:

$$Y(t_A) \rightarrow f_\beta^A Y(t_A)$$

Because of the constraint

$$Z_3 = 0 = Y(L,K) - C - K' - S'$$

at the time t_A, therefore, C, K', S' must also change so that the constraint is fulfilled. This leads to sudden changes in at least one of the variables or in all of them. This is always guaranteed by NDSolve. For example, one can also make additional more precise assumptions about the behaviour of the other variables, e.g. one could assume that at the time t_A also C, K', S' change by the shock factor f_β^A, i.e.

$$C(t_A) \rightarrow f_\beta^A C(t_A), \qquad K'(t_A) \rightarrow f_\beta^A K'(t_A), \quad S'(t_A) \rightarrow f_\beta^A S'(t_A)$$

and that the dynamic system can then adapt to these new starting conditions and the time-varying parameter

$$\beta(t) = \sigma_\beta^A(t)\beta$$

(2) **Model shock:** The model parameter α describes the labour intensity of production. A supply shock can be triggered by the changes of α over time according to a sawtooth curve

$$\alpha \rightarrow \alpha(t) = \sigma(t)\alpha$$

(3) **Model shock:** The power factor μ_K^F describes the power of the firm to actually enforce its investment interests (e.g. because of administrative regulations). A supply shock can be triggered by the fact that the power factor μ_K^F changes over time according to a sawtooth curve:

$$\mu_K^F \rightarrow \mu_K^F(t) = \sigma(t)\mu_K^F$$

3.4. Price shock

For example, a price shock P can be modelled by changing the price p at the time t_P by the factor f_p^P

$$p(t_P) \rightarrow f_p^P\, p(t_P)$$

This corresponds to a variable shock.

3.5. Policy shocks

In addition, a wide variety of fiscal and monetary policy measures that apply from certain time points can of course also be interpreted as economic policy shocks and modelled in the same way, e.g:

- government measures:
  - Tax reform
  - Increase or decrease in public debt
  - etc.
- Changes in central bank policy:
  - From money supply control to interest rate control
  - Change in the inflation target
  - Purchase programmes
  - etc.

4. Topics to be discussed

In the economy, a shock can occur for a number of reasons, e.g.
- sudden changes in raw material prices
- sudden changes in consumer behaviour due to quarantine regulations
- sudden production restrictions due to a disruption in the supply chain
- etc.

From an economic point of view, there are 2 fundamental topics related to shocks:
(1) Forecasting: How will the economic variables change?
(2) Evaluating countermeasures: What measures can be taken to overcome the shock as quickly as possible or with as little effort as possible?
Possible measures are, for example:

- Various forms of financial assistance from the government to firms
- Various forms of financial benefits to consumers
- Different ways of financing additional government expenditure
- short-time working models
- Central bank monetary policy measures
- Organisational measures, e.g. relieving companies of administrative regulations, extending opening hours in the retail sector, etc. Such

organisational measures are expressed in the models by changes in power factors or other parameters.

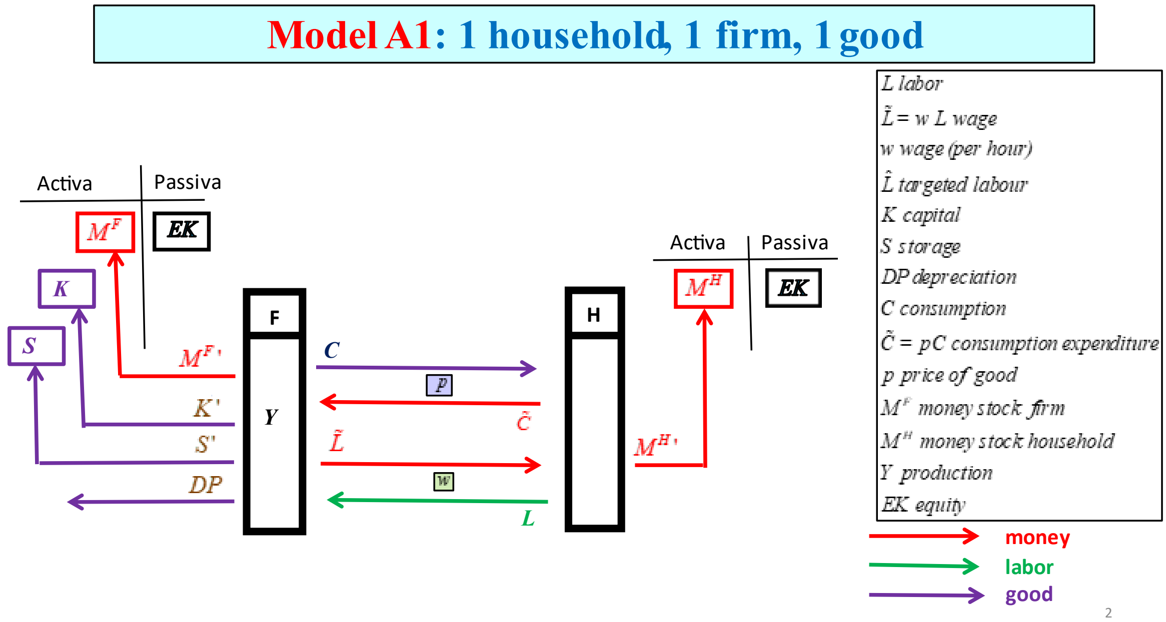
The target of section C. is to show that GCD models are basically suitable for answering these 2 questions and that this can be done very easily and conveniently with the help of the open-source program GCDconfigurator. The additions necessary to incorporate shocks into a model programmed with GCDconfigurator are very easy to program.

In order to apply GCD models to real economic situations, they would of course have to be extended accordingly and adapted to the real conditions.

Using model A1 as an example, we show how special supply, demand and price shocks can be modelled and what effects they have on the further course of the economy.
Using model B1, we show how central bank measures have different effects on a price shock depending on whether the central bank pursues a monetary policy or an interest rate policy.
In order to make the effects clearly visible, the model calculations are carried out for very strong shocks of a magnitude that is unlikely to occur in reality.

5. Model A1, (1 household, 1 firm, 1 good, without interest)

5.1. Overview of the setup



Model A1: basic equations

algebraically defined variables

$Y(L,K) = \quad \beta L^{\alpha} K^{1-\alpha}$ "production function"

$DP(K) = \quad \widehat{dp}\, K$ "depreciation"

utility functions

$U^H(C,L,MH) = \quad C^{\gamma} - \left(\hat{L} - L\right)^2 - (\hat{M}^H - M^H)^2$ "utility function household"

$U^F(Y,L,S) = \quad pY - w\,L - (\hat{S} - S)^2$ "utility function firm"

constraints

$Z^H = 0 = wL - pC - M^{H}\,'$ *for money of household H*

$Z^F = 0 = pC - wL - M^{F}\,'$ *for money of firm F*

$Z_1 = 0 = Y(L,K) - C - K' - S' - DP$ *for good 1 of firm F*

3

With the aid of the GCDconfigurator programme, the differential-algebraic equation system of the A1 model is calculated as follows:

Model A1 : diff.-alg. equation system

$$\mathrm{uF}[t] == -(\mathrm{sdach} - s[t])^2 - l[t] \times w[t] + p[t] \times y[t]$$
$$\mathrm{uH}[t] == \mathrm{cH}[t]^{\gamma} - (\mathrm{ldach} - l[t])^2 - (\mathrm{mHdach} - \mathrm{mH}[t])^2$$

$$\mathrm{dp}[t] == \mathrm{dpdach}\, k[t]$$
$$\mathrm{inv}[t] == k'[t]$$
$$y[t] == \beta\, k[t]^{1-\alpha}\, l[t]^{\alpha}$$

$$\mathrm{cH}'[t] == \gamma\, \mu\mathrm{HcH}\, \mathrm{cH}[t]^{-1+\gamma} + p[t]\, \lambda_1[t] - p[t]\, \lambda_2[t] - \lambda_3[t]$$
$$k'[t] == (1-\alpha)\, \beta\, \mu\mathrm{Fk}\, k[t]^{-\alpha}\, l[t]^{\alpha}\, p[t] - \lambda_3[t]$$
$$l'[t] == 2\, \mu\mathrm{Hl}\, (\mathrm{ldach} - l[t]) + \mu\mathrm{Fl}\, \left(\alpha\, \beta\, k[t]^{1-\alpha}\, l[t]^{-1+\alpha}\, p[t] - w[t]\right) - w[t]\, \lambda_1[t] + w[t]\, \lambda_2[t] + \alpha\, \beta\, k[t]^{1-\alpha}\, l[t]^{-1+\alpha}\, \lambda_3[t]$$
$$\mathrm{mF}'[t] == -\lambda_1[t]$$
$$\mathrm{mH}'[t] == 2\, \mu\mathrm{HmH}\, (\mathrm{mHdach} - \mathrm{mH}[t]) - \lambda_2[t]$$
$$p'[t] == \beta\, \mu\mathrm{Fp}\, k[t]^{1-\alpha}\, l[t]^{\alpha} + \mathrm{cH}[t]\, \lambda_1[t] - \mathrm{cH}[t]\, \lambda_2[t]$$
$$s'[t] == 2\, \mu\mathrm{Fs}\, (\mathrm{sdach} - s[t]) - \lambda_3[t]$$
$$w'[t] == -\mu\mathrm{Fw}\, l[t] - l[t]\, \lambda_1[t] + l[t]\, \lambda_2[t]$$

$$0 == \mathrm{cH}[t] \times p[t] - l[t] \times w[t] - \mathrm{mF}'[t]$$
$$0 == -\mathrm{cH}[t] \times p[t] + l[t] \times w[t] - \mathrm{mH}'[t]$$
$$0 == -\mathrm{cH}[t] - \mathrm{dpdach}\, k[t] + \beta\, k[t]^{1-\alpha}\, l[t]^{\alpha} - k'[t] - s'[t]$$

$$\mathrm{cH}[0] == \mathrm{k0}^{1-\alpha}\, \mathrm{l0}^{\alpha}\, \beta$$
$$k[0] == \mathrm{k0}$$
$$l[0] == \mathrm{l0}$$
$$\mathrm{mF}[0] == \mathrm{mF0}$$
$$\mathrm{mH}[0] == \mathrm{mH0}$$
$$p[0] == \mathrm{p0}$$
$$s[0] == \mathrm{s0}$$
$$w[0] == \mathrm{w0}$$

5.2. Description of the A1 model in detail

The one good serves as both a consumption good and an investment good. We assume that vertical constraint forces occur.
Since the target is first to show the principle, we choose the production function and the utility functions as simple as possible.
We choose a simple Cobb-Douglas production function as the production function, and the goods excreted per year (depreciation) are proportional to the capital stock. This results in the 2 necessary algebraically defined variables. They are necessary because they occur in the utility functions or constraints.

$$Y(L,K) = \beta L^{\alpha} K^{1-\alpha} \qquad \beta > 0,\ 0 < \alpha < 1$$
$$DP(K) = dp\, K \qquad 0 \le dp \le 1 \qquad \text{<5.1>}$$

In addition, one can be interested, for example, in net investment, for which one defines as a further algebraically defined variable

$$inv(K) = K' \qquad \text{<5.2>}$$

Households want to consume with decreasing marginal utility. Consumption of consumer goods C leads to a utility for households in the amount of C^{γ} with $0 < \gamma < 1$. They strive for a desired working time $\hat{L}$. Deviations from the desired working time $\hat{L}$ lead to a reduction of utility by $(L - \hat{L})^2$. In addition, households aim to keep cash in the amount of $\hat{M}^H$. Deviations from the desired cash position $\hat{M}^H$ lead to a reduction in utility by $(\hat{M}^H - M^H)^2$. This leads to the **utility function for the household**

$$U^H = C^{\gamma} - (\hat{L} - L)^2 - (\hat{M}^H - M^H)^2 \qquad 0 < \gamma < 1 \qquad \text{<5.3>}$$

For the company, in the simplest case, the utility initially consists of the goods produced, which are valued at the selling price, i.e. pY. The produced goods are used for:

C Sales = Consumption
S' change in inventory
K' changes in productive capital stock

In principle, it would be possible to weight the utility of these uses differently. For the sake of simplicity, we will refrain from doing so. Therefore, this utility is reduced by the cost of labor and the cost of storage, which we evaluate through the deviations from the planned inventory. For simplicity, we assume that holding money in cash has no influence on the utility. This leads to the **utility function for the firm**

$$U^F = pY(L,K) - wL - (\hat{S} - S)^2 = p\beta L^{\alpha} K^{1-\alpha} - wL - (\hat{S} - S)^2 \qquad \text{<5.4>}$$

From the model graph, it can be seen that the following **constraints** must be satisfied:

$$Z_1 = 0 = wL - pC - M^{H}{}' \qquad \textit{for money of household } H$$
$$Z_2 = 0 = pC - wL - M^{F}{}' \qquad \textit{for money of firm } F \qquad \text{<5.5>}$$
$$Z_3 = 0 = Y(L,K) - C - K' - S' \qquad \textit{for good } 1 \textit{ of firm } F$$

According to the methodology of GCD models, the interest or desire of households to change consumption is the greater the more the utility changes when consumption changes, i.e., the interest is proportional to $\frac{\partial U^H}{\partial C}$. However, the interest in changing consumption does not in itself lead to an actual change in consumption, because the household must also have the power or opportunity to actually implement its desire to change consumption. For example, a household cannot or can only partially enforce its additional consumption wish, e.g., to go to the cinema or on holiday, because it is in quarantine or the borders are closed. This restriction of the possibility to enforce his or her consumption change wishes is described by a (possibly time-dependent) "power factor" μ_C^H. Analogously, the firm could have an interest $\frac{\partial U^F}{\partial C}$ and power μ_C^F to influence consumption. In the specific case $\frac{\partial U^F}{\partial C} = 0$. This results in the following behavioural equation for the **ex-ante planned change in consumption**

$$C' = \mu_C^H \frac{\partial U^H}{\partial C} + \mu_C^F \frac{\partial U^F}{\partial C} = \mu_C^H \gamma C^{\gamma-1}$$

<5.6>

The same considerations apply to labour L as to consumption. Even the household's wish to increase or reduce working time does not in itself lead to an actual change in working time, because the household must also have the power or possibility to actually implement its wish to change. For example, a household might not be able to enforce its wish to increase working time, or only partially, because it is on short-time working or unemployed, or it might not be able to enforce its wish to reduce working time because it is contractually obliged to work overtime. This restriction of the possibility to enforce his wishes for a change in working time is also described by a (possibly time-dependent) power factor, which we denote with μ_L^H. The same applies to the firm's ability to influence working time.

Therefore, the behavioural equation for the **ex-ante planned change in working time** is as follows

$$L' = \mu_L^H \frac{\partial U^H}{\partial L} + \mu_L^F \frac{\partial U^F}{\partial L} = 2\mu_L^H (\hat{L} - L) + \mu_L^F (p\beta\alpha L^{\alpha-1} K^{1-\alpha} - w)$$

The ex-ante behavioural equations for the other variables result analogously.

However, the plans of the 2 agents household and firm to change consumption C, labour L and the other variables cannot be enforced independently of each other, because the constraints

$$\begin{aligned}
&Z_1 = 0 = wL - pC - M^{H}{}' && \textit{für Geld von Haushalt H} \\
&Z_2 = 0 = pC - wL - M^{F}{}' && \textit{für Geld von Firma F} \qquad \langle 5.7\rangle \\
&Z_3 = 0 = Y(L,K) - C - K' - S' - DP && \textit{für Gut 1 von Firma F}
\end{aligned}$$

lead to constraint forces, which we assume are vertical constraint forces. The constraint force for the change in consumption therefore results in

$$\lambda_1 \frac{\partial Z_1}{\partial C} + \lambda_2 \frac{\partial Z_2}{\partial C} + \lambda_3 \frac{\partial Z_3}{\partial C} = -\lambda_1 p + \lambda_2 p - \lambda_3$$

The behavioural equation for the actual **ex-post change in consumption** is therefore

$$C' = \mu_C^H \frac{\partial U^H}{\partial C} + \lambda_1 \frac{\partial Z_1}{\partial C} + \lambda_2 \frac{\partial Z_2}{\partial C} + \lambda_3 \frac{\partial Z_3}{\partial C} = \mu_C^H \gamma C^{\gamma-1} - \lambda_1 p + \lambda_2 p - \lambda_3 \qquad \text{<5.8>}$$

Analogously, the actual **ex-post change in labour** is as follows

$$L' = \mu_L^H \frac{\partial U^H}{\partial L} + \mu_L^F \frac{\partial U^F}{\partial L} + \lambda_1 \frac{\partial Z_1}{\partial L} + \lambda_2 \frac{\partial Z_2}{\partial L} + \lambda_3 \frac{\partial Z_3}{\partial L} =$$
$$= 2\mu_L^H (\hat{L} - L) + \mu_L^F (p\beta\alpha L^{\alpha-1} K^{1-\alpha} - w) + \lambda_1 w - \lambda_2 w + \lambda_3 \alpha\beta L^{\alpha-1} K^{1-\alpha}$$

This also applies analogously to the company's investments. In the case of the company, too, the actual implementation of ex-ante planned investment increases can be prevented by real restrictions, e.g. by interruptions in supply chains. In the same way, a desired reduction in investment may not be possible to the desired extent because the project is a large-scale project of many years' duration. These restrictions can in turn be described by a (possibly time-dependent) power factor μ_K^B. This results in the following behavioural equation for the actual ex-post change in capital

$$K' = \mu_K^F \frac{\partial U^F}{\partial K} + \lambda_1 \frac{\partial Z_1}{\partial K} + \lambda_2 \frac{\partial Z_2}{\partial K} + \lambda_3 \frac{\partial Z_3}{\partial K'} = \mu_K^F p\beta(1-\alpha) L^{\alpha} K^{-\alpha} - \lambda_3 \qquad \text{<5.9>}$$

Note that we have to use $\frac{\partial Z_3}{\partial K'}$ instead of $\frac{\partial Z_3}{\partial K}$ because the constraint forces are always derived from the highest time derivative of the variables (see Remark in chapter 2 (Flannery 2011).

The equations of behaviour for M^H, M^F, S, p, w are derived analogously. In sum, this results in the model equations *differentiell behavioural equations*

$$C' = \mu_C^H \frac{\partial U^H}{\partial C} + \mu_C^F \frac{\partial U^F}{\partial C} + \lambda_1 \frac{\partial Z_1}{\partial C} + \lambda_2 \frac{\partial Z_2}{\partial C} + \lambda_3 \frac{\partial Z_3}{\partial C} =$$

$$= \mu_C^H \gamma C^{\gamma-1} - \lambda_1 p + \lambda_2 p - \lambda_3$$

$$L' = \mu_L^H \frac{\partial U^H}{\partial L} + \mu_L^F \frac{\partial U^F}{\partial L} + \lambda_1 \frac{\partial Z_1}{\partial L} + \lambda_2 \frac{\partial Z_2}{\partial L} + \lambda_3 \frac{\partial Z_3}{\partial L} =$$

$$= \mu_L^H (\hat{L} - L) + \lambda_1 w - \lambda_2 w + \lambda_3 \alpha\beta L^{\alpha-1} K^{1-\alpha}$$

$$K' = \mu_K^H \frac{\partial U^H}{\partial K} + \mu_K^F \frac{\partial U^F}{\partial K} + \lambda_1 \frac{\partial Z_1}{\partial K} + \lambda_2 \frac{\partial Z_2}{\partial K} + \lambda_3 \frac{\partial Z_3}{\partial K'} =$$

$$= \mu_K^F p\beta(1-\alpha) L^\alpha K^{-\alpha} - \lambda_3$$

$$M^{H}{}' = \mu_{M^H}^H \frac{\partial U^H}{\partial M^H} + \mu_{M^H}^F \frac{\partial U^F}{\partial M^H} + \lambda_1 \frac{\partial Z_1}{\partial M^{H}{}'} + \lambda_2 \frac{\partial Z_2}{\partial M^H} + \lambda_3 \frac{\partial Z_3}{\partial M^H} =$$

$$= 2\mu_{M^H}^H \left(\hat{M}^H - M^H\right) - \lambda^H$$

$$M^{F}{}' = \mu_{M^F}^H \frac{\partial U^H}{\partial M^F} + \mu_{M^F}^F \frac{\partial U^F}{\partial M^F} + \lambda^H \frac{\partial Z^H}{\partial M^{F}{}'} + \lambda^B \frac{\partial Z^B}{\partial M^F} + \lambda_1 \frac{\partial Z_1}{\partial M^F} =$$

$$= -\lambda_2$$

$$S' = \mu_S^H \frac{\partial U^H}{\partial S} + \mu_S^F \frac{\partial U^F}{\partial S} + \lambda_1 \frac{\partial Z_1}{\partial S} + \lambda_2 \frac{\partial Z_2}{\partial S} + \lambda_3 \frac{\partial Z_3}{\partial S'} =$$

$$= \mu_S^F \left(\hat{S} - S\right) - \lambda_3$$

$$p' = \mu_p^H \frac{\partial U^H}{\partial p} + \mu_p^F \frac{\partial U^F}{\partial p} + \lambda_1 \frac{\partial Z_1}{\partial p} + \lambda_2 \frac{\partial Z_2}{\partial p} + \lambda_3 \frac{\partial Z_3}{\partial p} =$$

$$= \mu_p^F \beta K^{1-\alpha} L^\alpha - \lambda_1 c + \lambda_2 c$$

$$w' = \mu_w^H \frac{\partial U^H}{\partial w} + \mu_w^F \frac{\partial U^F}{\partial w} + \lambda_1 \frac{\partial Z_1}{\partial w} + \lambda_2 \frac{\partial Z_2}{\partial w} + \lambda_3 \frac{\partial Z_3}{\partial w} =$$

$$= -\mu_w^F L + \lambda_1 L - \lambda_2 L$$

Or written in a clearer way

differentiell behavioural equations

$$C' = \mu_C^H \gamma C^{\gamma-1} - \lambda_1 p + \lambda_2 p - \lambda_3$$

$$L' = 2\mu_L^H (\hat{L} - L) + \mu_L^F \left(\alpha\beta K^{1-\alpha} L^{-1+\alpha} p - w\right) + \lambda_1 w - \lambda_2 w + \lambda_3 \alpha\beta K^{1-\alpha} L^{-1+\alpha}$$

$$K' = \mu_K^F \beta\left(1-\alpha\right) L^\alpha K^{-\alpha} p - \lambda_3$$

$$M^{H}{}' = 2\mu_{M^H}^H \left(\hat{M}^H - M^H\right) - \lambda_1$$

$$M^{F}{}' = -\lambda_2$$

$$S' = \mu_S^F \left(\hat{S} - S\right) - \lambda_3$$

$$p' = \mu_p^F \beta K^{1-\alpha} L^\alpha - \lambda_1 c + \lambda_2 c$$

$$w' = -\mu_w^F L + \lambda_1 L - \lambda_2 L$$

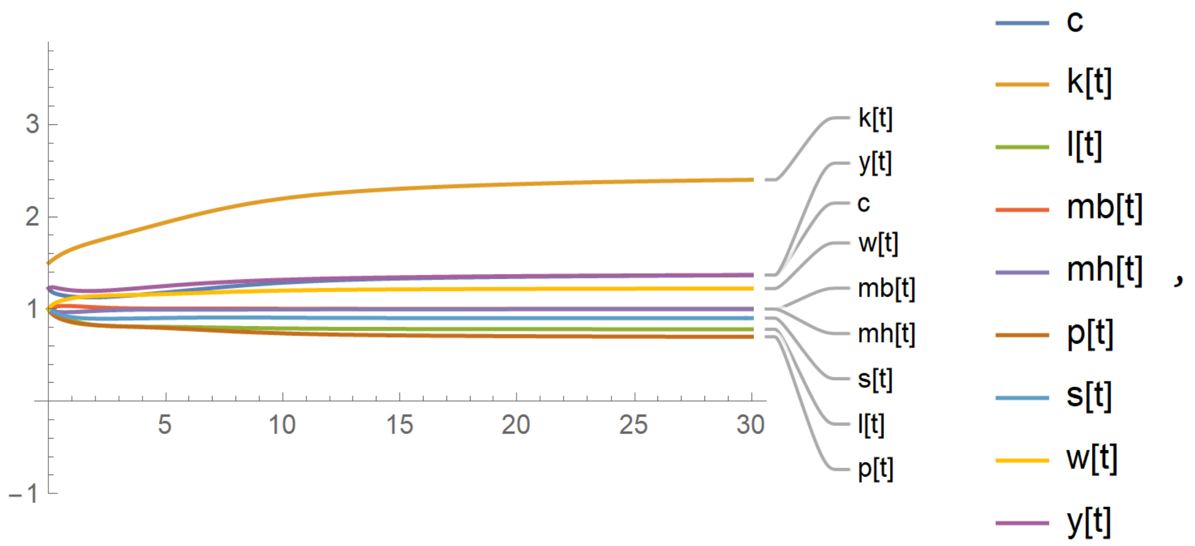
6. Calculations with model A1 on various shocks

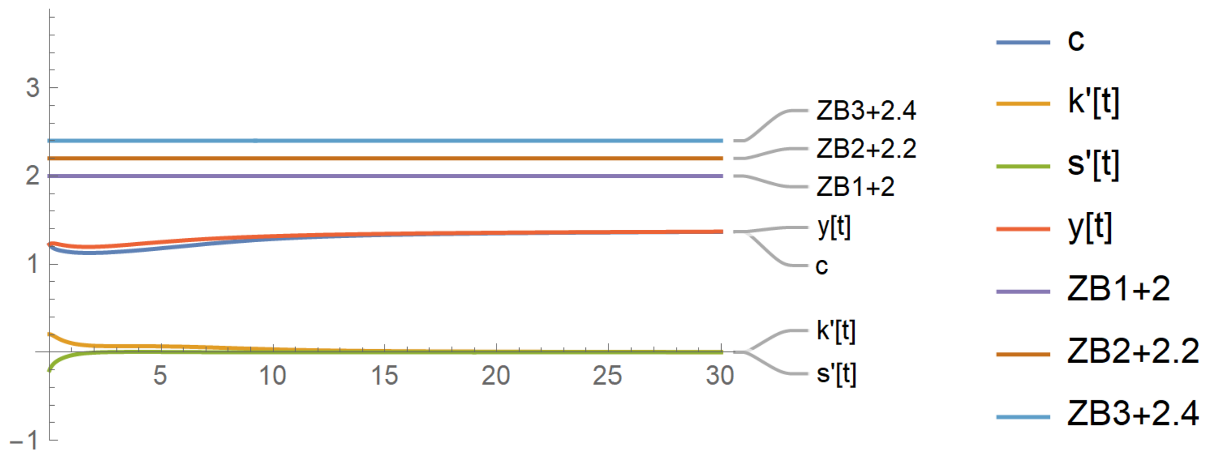
We model the following shocks

```
(*Price shock as variable shock,
 Variable shock for p[t] at time tnp, p[t] jumps to fpp x p[t] at time tnp*)
WhenEvent[t == tp, {p[t] → fpp p[t]}],
bei Ereignis

(*Demand shock as variable shock,
 Variable shock for C[t] at time tnc, C[t] jumps to fnc x c[t] at time tnc*)
WhenEvent[t == tnc, {c[t] → fnc c[t]}],
bei Ereignis


(* Demand shock as model shock (shock of power of household),
Model shock for μHc[t] at time tn, decays over the time period of dn,
σnμHc[t] a sawtooth curve,
σnμHc[0]=1, jumps at time tn to fnμHc, goes back linearly to 1 in the time period of dn,
μHc[t]=μHc x σnμHc[t] a sawtooth curve,
μHc[0]=μHc, jumps at time tn to μHc x fnμHc, goes back linearly to μHc in the time period of dn *)

σnμHc[t] == σnμHc0[t] + σnμHc1[t] (t - tn),
σnμHc0[0] == 1, σnμHc1[0] == 0,
WhenEvent[t == tn, {σnμHc0[t] → fnμHc, σnμHc1[t] → (1 - fnμHc) / dn}],
bei Ereignis
WhenEvent[t == tn + dn, {σnμHc0[t] → 1, σnμHc1[t] → 0}],
bei Ereignis
 (* Supply shock as model shock (shock of power of firm),
 Model shock for μFk[t] at time ta, decays over the time period of da,
 σaμFk[t] a sawtooth curve,
 σaμFk[0]=1, jumps at time tn to faμFk, goes back linearly to 1 in the time period of da,
 μFk[t]=μFk x σaμFk[t] a sawtooth curve,
 μFk[0]=μFk, jumps at time ta to μFk x faμFk, goes back linearly to μFk in the time period of da *)


 σaμFk[t] == σaμFk0[t] + σaμFk1[t] (t - ta),
 σaμFk0[0] == 1, σaμFk1[0] == 0,
 WhenEvent[t == ta, {σaμFk0[t] → faμFk, σaμFk1[t] → (1 - faμFk) / da}],
 bei Ereignis
 WhenEvent[t == ta + da, {σaμFk0[t] → 1, σaμFk1[t] → 0}],
 bei Ereignis

 (*Supply shock as model shock (technology shock),
 Model shock for β[t] at time taβ, decays over the time period of daβ ,
 σaβ[t] a sawtooth curve,
 σaβ[0]=1, jumps at time taβ to faβ, goes back linearly to 1 in the time period of daβ,
 β[t]=β x σaβ[t] a sawtooth curve,
 β[0]=β, springt zum Zeitpunkt taβ auf β x faβ, goes back linearly to β in the time period of daβ *)
```

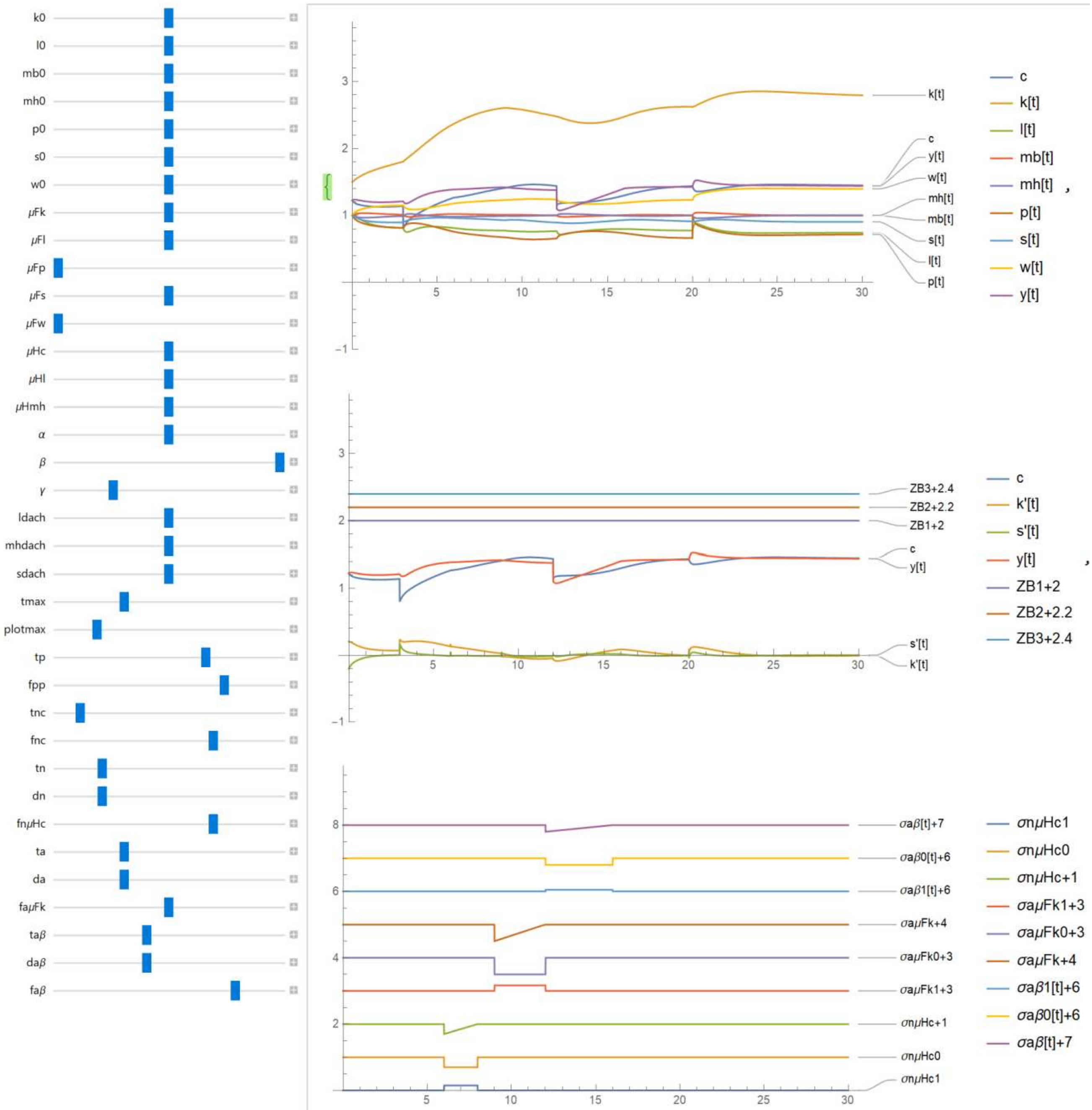
https://www.dropbox.com/s/cdewtdh2zrwfrfp/Modell%20A1%20SCHOCK%20Version%2024.nb?dl=0

With no shocks

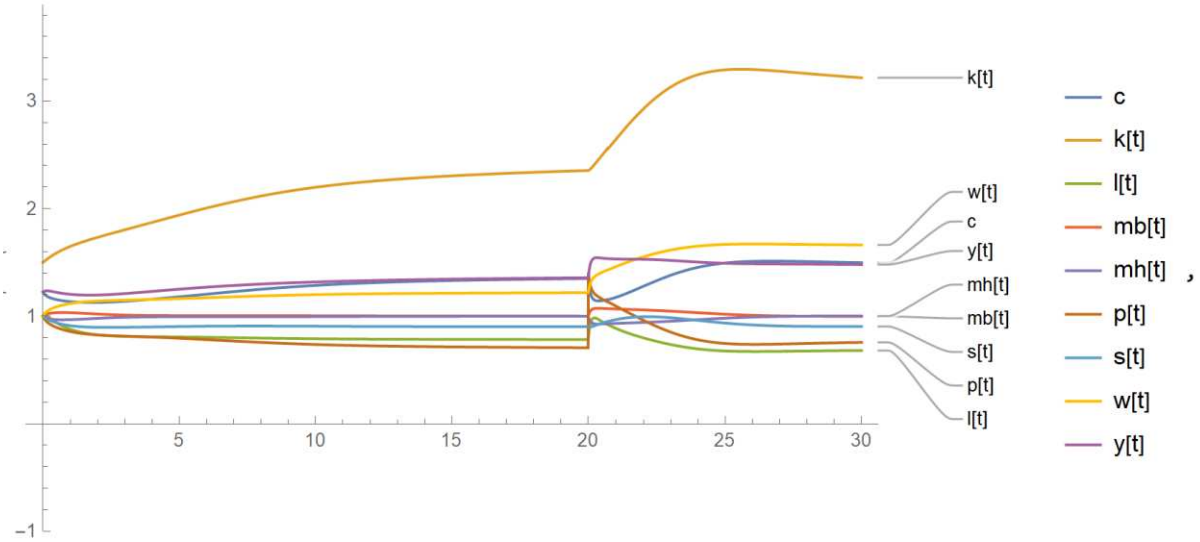


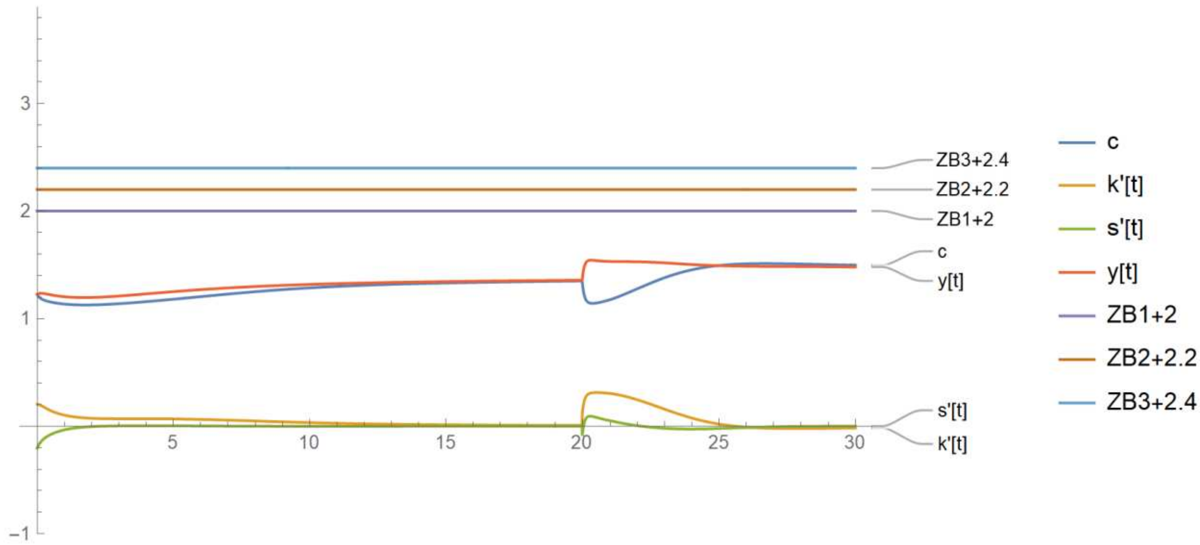


With multiple shocks



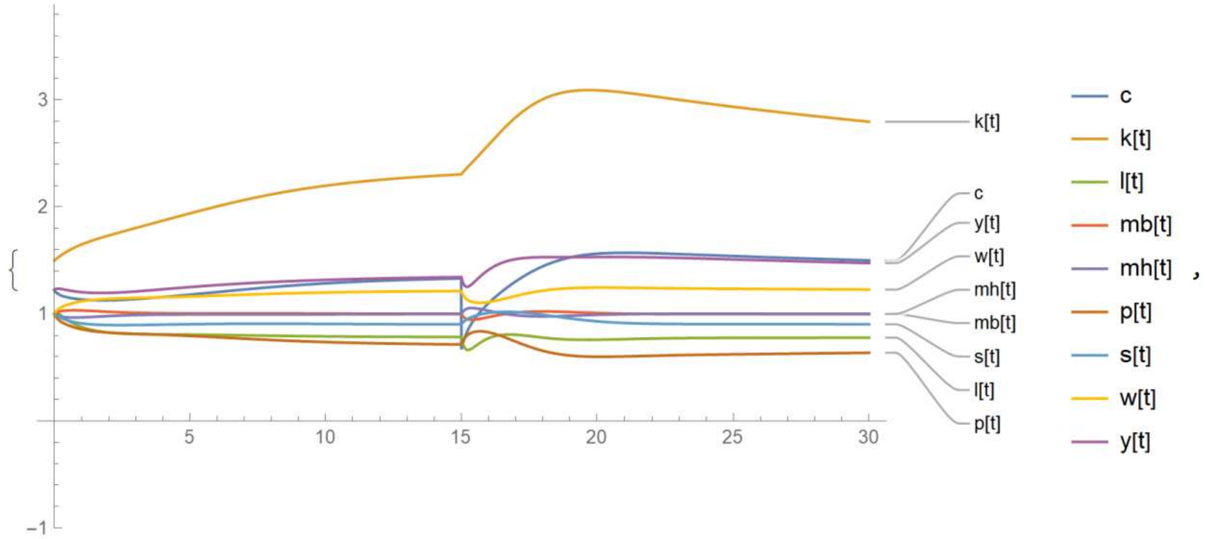
Price shock at $t = 20, \qquad p \to 2\,p$

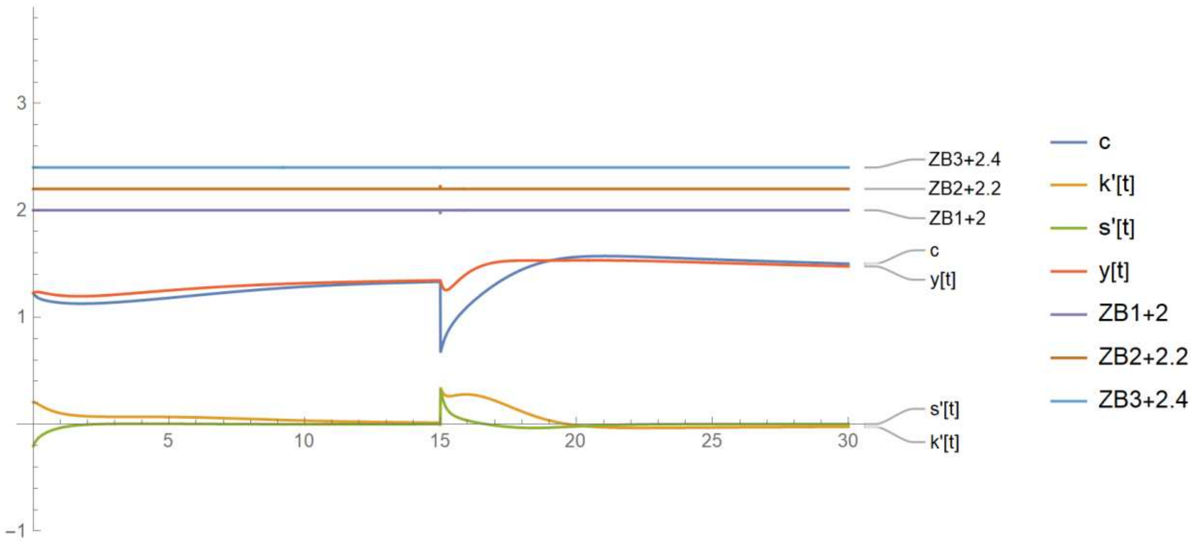




Demand shock due to variable shock (consumption shock)

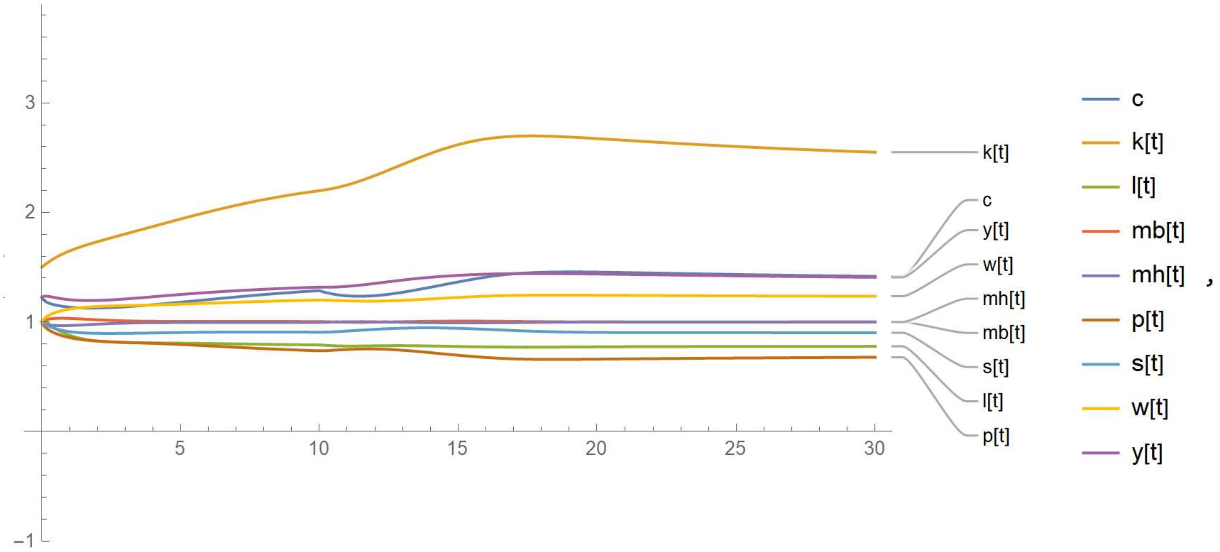
at $t = 15$ $\qquad C \to 0.5\,C$

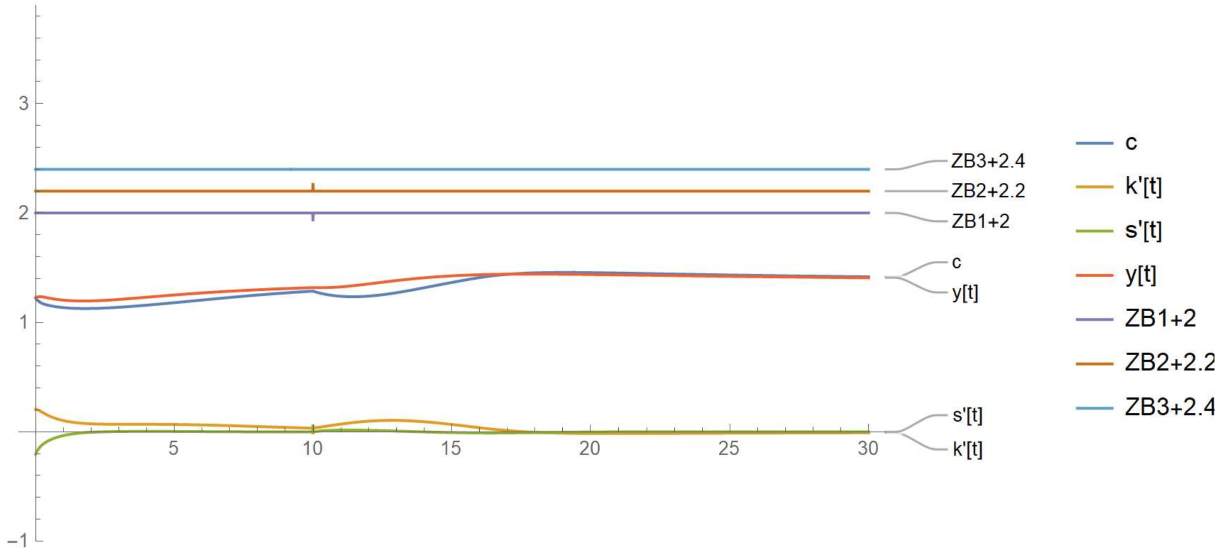




Demand shock due to model shock (shock to power of households)

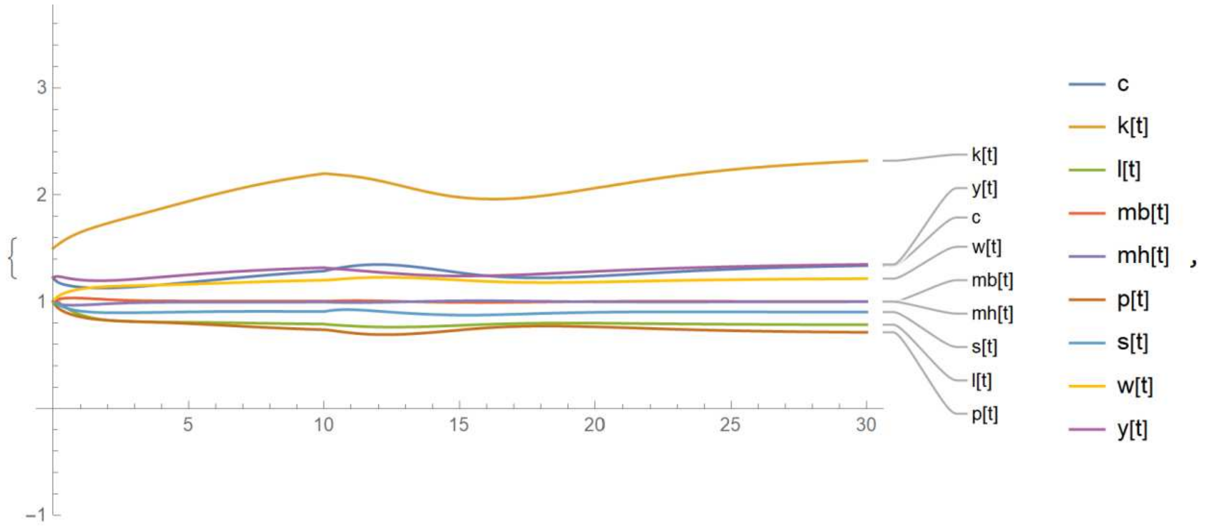
at $t=10$ $\mu_C^H \to 0.5\mu_C^H$ thereafter, the power of the households increases again linearly to μ_C^H within the time period $dn=5$

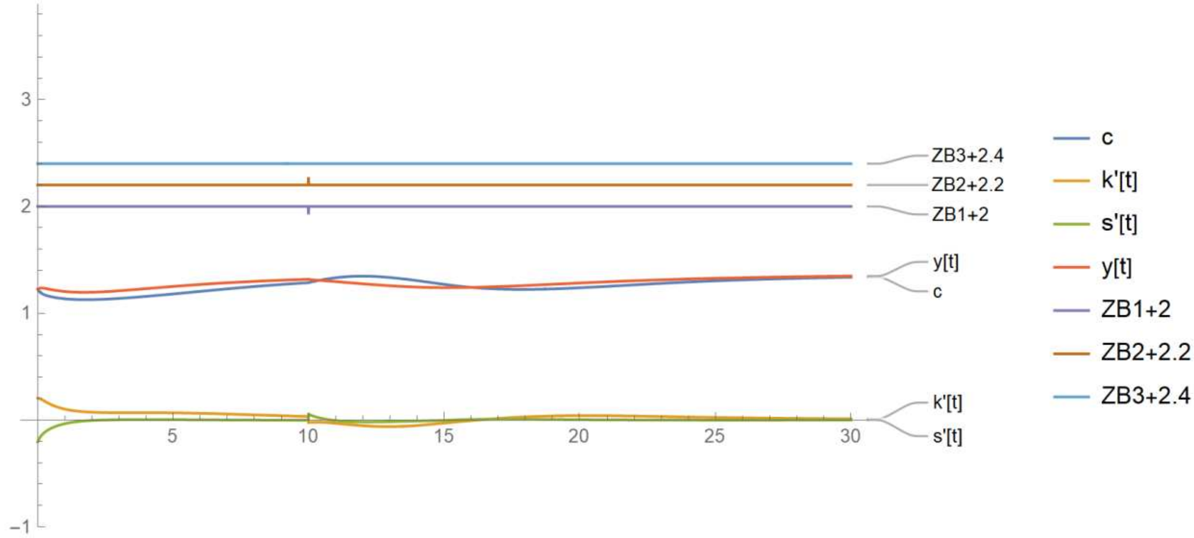


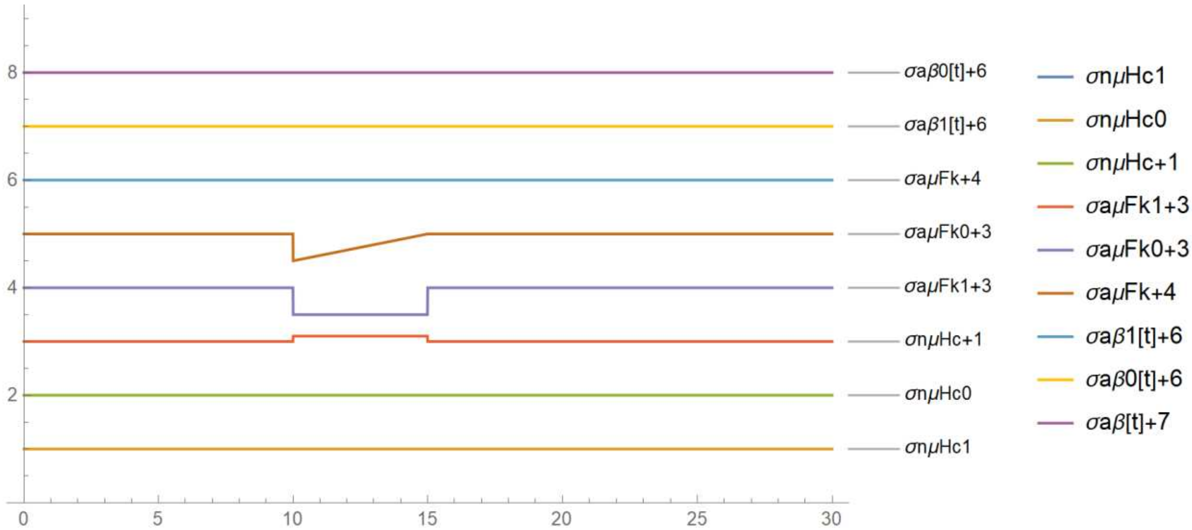


Supply shock due to model shock (shock to power of the firm)

at $t = 10 \quad \mu_K^F \to 0.5\mu_K^F$ thereafter, the power of the firm increases again linearly to μ_K^F within the time period $da = 5$

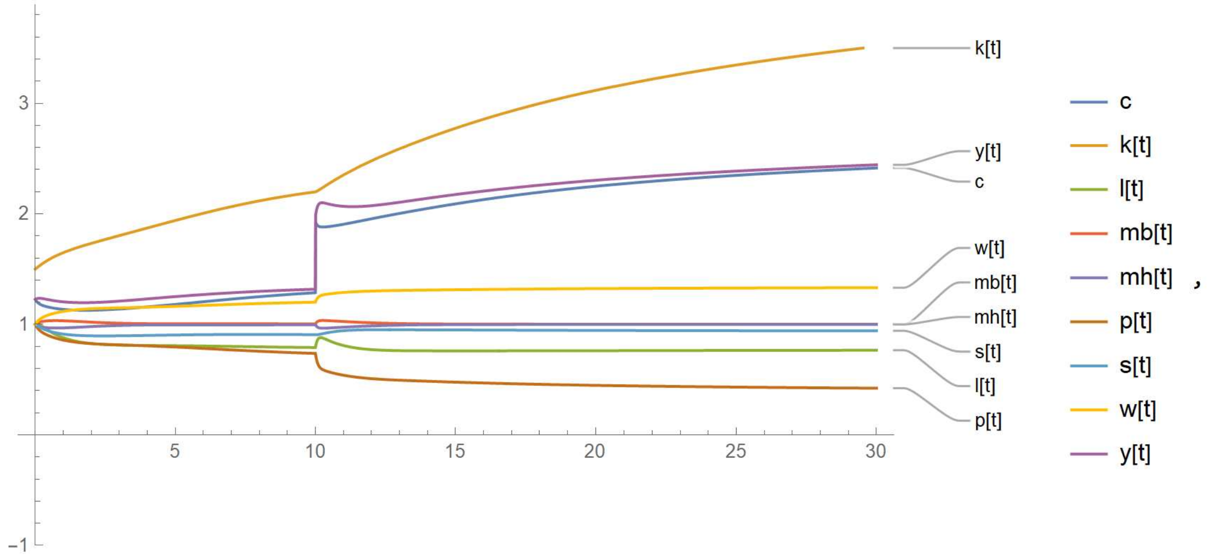


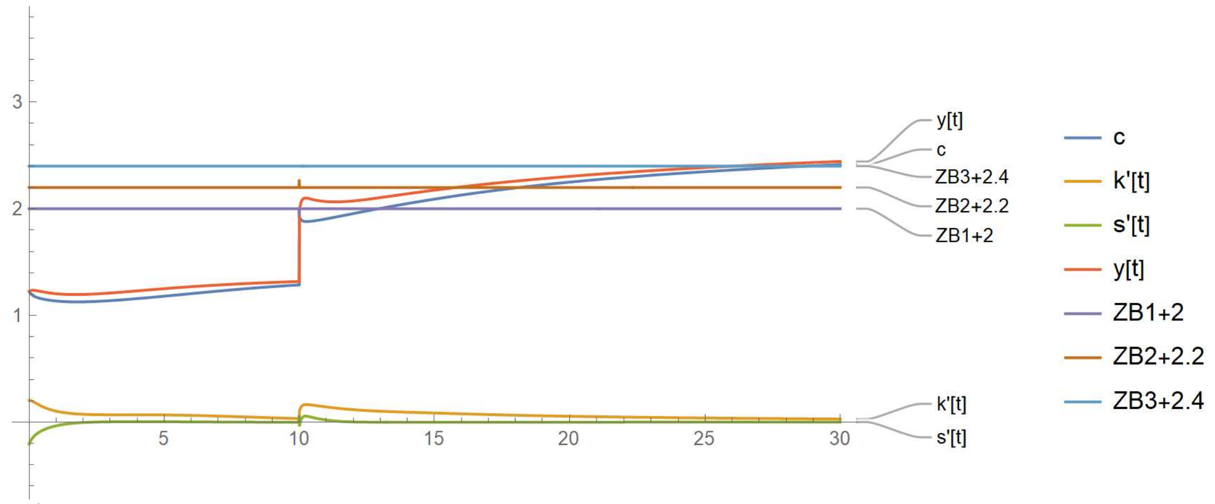


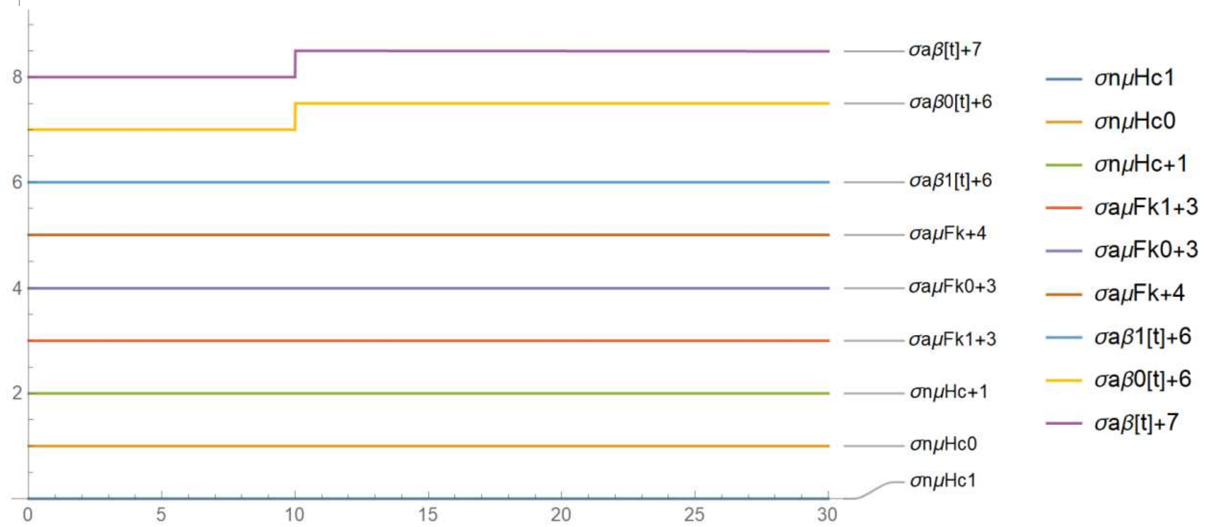


Supply shock due to model shock (technology jump)

at $t = 10 \quad \beta \to 1.5\beta$ i.e. a technological jump occurs, the resulting increase in productivity is maintained permanently







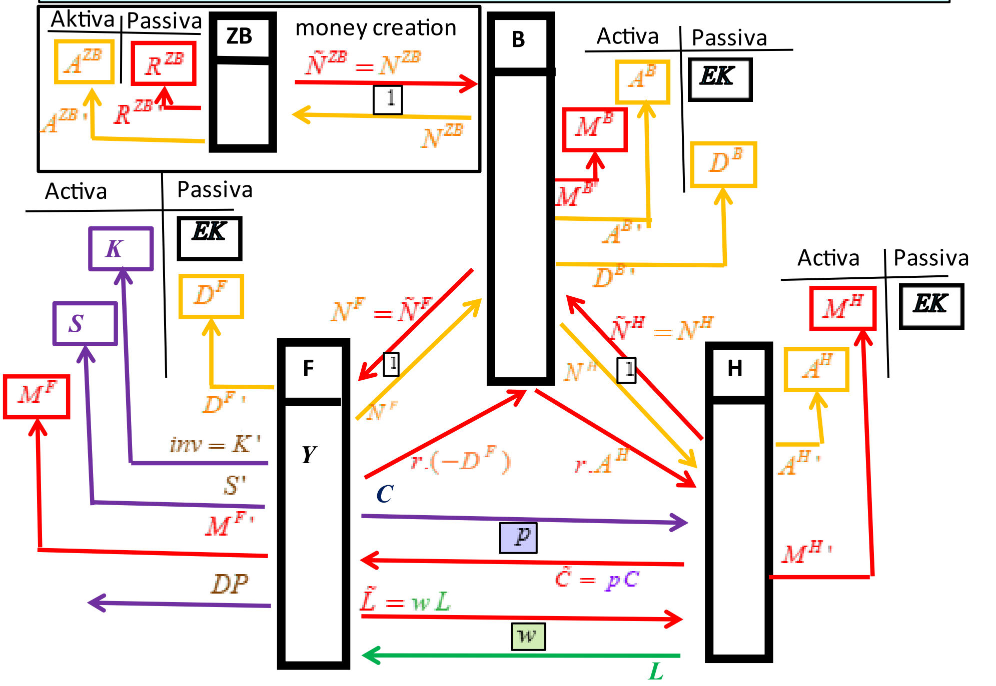
7. Calculations with model B1 for central bank polices in case of inflation and deflation shock

7.1. Overview of the setup of model B1

The target of model B1 is to model the money creation process by the central bank in a simplified way.
In model B1, the central bank is seen as an endogenous money creator and the bank is seen as an endogenous credit creator. The central bank's target is to keep inflation $\frac{p'}{p}$ at the target inflation $\hat{p} = 0.02$ i.e. 2% by means of interest rate policy ($\delta = 1$) and monetary-supply policy($\delta = 0$).
In this model B1, the central bank's interest rate policy is still modeled in a very simplified way. We assume that the policy rate is constant 0 (banks do not pay interest to the central bank) and that the central bank can, however, influence the interest rate directly. That the policy rate is constant 0 is possible and does not cause the bank to borrow arbitrarily from the central bank, since the bank is assumed to have a constant 0 utility function. This means that the bank has no particular interest in lending to firms or receiving savings deposits from households. Thus, the bank lends endogenously and accepts savings deposits endogenously.

In (Glötzl 2022b) we present more realistic models, in which shocks can be integrated in the same way as in model B1: In model B2, we model the behaviour of the central bank according to the Taylor rule. All these simplifying restrictions regarding money creation hold also for models C1, C2, because in models C1, C2 we are concerned with modeling the government. It is only in the much more comprehensive model D2 that the restrictions on the modeling of money creation and the modeling of the government are largely abandoned.

Model B1: 1 household, 1 firm, 1 good, 1 bank. 1 central bank



L labor
$\tilde{L} = w\,L$ wage
w wage (per hour)
$\hat{L}$ targeted labour
K capital
S storage
DP depreciation
$inv = K'$ net - investment
C consumption
$\tilde{C} = pC$ consumption expenditure
p price of good
M^F money stock firm
M^H money stock household
$\hat{M}^H$ targeted money stock household
M^B money stock bank
$\tilde{N}^{ZB}$ money creation central bank
$\tilde{N}^F$ credit money from bank to firm
$\tilde{N}^H$ savings money from household to bank
D^F, D^B debts
A^B, A^H account
$r.(-D^F), r.A^H$ interest payments
EK equity

Pay attention when establishing the constraints:
(1) Claims A have a positive sign, liabilities D have a negative sign
(2) Banks' equity capital is 0. They do not make profits.

Model B1 : basic equations

algebraically defined variables

$Y(L,K) = \quad \beta L^{\alpha} K^{1-\alpha}$ "production function"

$DP(K) = \quad \widehat{dp}\, K$ "depreciation"

utility functions

$U^H(C,L,M^H) = \quad C^{\gamma} - \left(\hat{L} - L\right)^2 - (\hat{M}^H - M^H)^2 + r\,A^H$ "utility function household"

$U^F(Y,L,S) = \quad pY - w\,L - (\hat{S} - S)^2 - r(-D^F)$ "utility function firm"

$U^B = 0$ "utility functionbank"

$U^{ZB}(r,p,N^B) = \quad \left(-\delta r + (1-\delta)N^{ZB}\right)(\hat{p} - \frac{p'}{p})$ "utility function central bank"

constraints

$Z_1 = 0 = w\,L - pC + r\,A^H - N^H - M^H{}'$ for money flow of household H

$Z_2 = 0 = -w\,L + pC - r(-D^F) + N^F - M^F{}'$ for money flow of firm F

$Z_3 = 0 = N^{ZB} - N^F + r(-D^F) - r\,A^H + N^H - M^B{}'$ for money flow of bank B

$Z_4 = 0 = -N^{ZB} - R^{ZB}{}'$ for money flow of central bank ZB

$Z_5 = 0 = Y(L,K) - C - S' - DP - K'$ for flow of good 1 of firm F

$Z_6 = 0 = N^H - A^H{}'$ for accounts / debts flow of household H

$Z_7 = 0 = -N^F - D^F{}'$ for accounts / debts flow of firm F

$Z_8 = 0 = -N^{ZB} + N^F - N^H - D^B{}' - A^B{}'$ for accounts / debts flow of of bank B

$Z_9 = 0 = N^{ZB} - A^{ZB}{}'$ for accounts / debts flow of central bank ZB

Model B1 : diff.-alg. equation system

```mathematica
uB[t] == 0
uF[t] == dF[t] × r[t] - (sdach - s[t])^2 - l[t] × w[t] + p[t] × y[t]
uH[t] == cH[t]^γ - (ldach - l[t])^2 - (mHdach - mH[t])^2 + aH[t] × r[t]
uZB[t] == (pdach - ps[t]/p[t]) ((1 - δ) nZB[t] - δ r[t])

dp[t] == dpdach k[t]
inflation[t] == ps[t]/p[t]
inv[t] == k'[t]
y[t] == β k[t]^(1-α) l[t]^α

aB'[t] == -λ2[t]
aH'[t] == μHaH r[t] - λ4[t] - r[t] λ6[t] + r[t] λ8[t]
aZB'[t] == -λ5[t]
cH'[t] == γ μHcH cH[t]^(-1+γ) + p[t] λ7[t] - p[t] λ8[t] - λ9[t]
dB'[t] == -λ2[t]
dF'[t] == μFdF r[t] - λ1[t] - r[t] λ6[t] + r[t] λ7[t]
k'[t] == (1 - α) β μFk k[t]^(-α) l[t]^α p[t] - λ9[t]
l'[t] == 2 μHl (ldach - l[t]) + μFl (α β k[t]^(1-α) l[t]^(-1+α) p[t] - w[t]) - w[t] λ7[t] +
   w[t] λ8[t] + α β k[t]^(1-α) l[t]^(-1+α) λ9[t]
mB'[t] == -λ6[t]
mF'[t] == -λ7[t]
mH'[t] == 2 μHmH (mHdach - mH[t]) - λ8[t]
nF'[t] == -λ1[t] + λ2[t] - λ6[t] + λ7[t]
nH'[t] == -λ2[t] + λ4[t] + λ6[t] - λ8[t]
nZB'[t] == (1 - δ) μZBnZB (pdach - ps[t]/p[t]) - λ2[t] - λ3[t] + λ5[t] + λ6[t]
p'[t] == β μFp k[t]^(1-α) l[t]^α + μZBp ps[t] ((1-δ) nZB[t] - δ r[t])/p[t]^2 + cH[t] λ7[t] - cH[t] λ8[t] - λ10[t]
ps'[t] == -μZBps ((1-δ) nZB[t] - δ r[t])/p[t] + λ10[t]
r'[t] == μHr aH[t] + μFr dF[t] - δ μZBr (pdach - ps[t]/p[t]) + (-aH[t] - dF[t]) λ6[t] +
   dF[t] λ7[t] + aH[t] λ8[t]
rZB'[t] == -λ3[t]
s'[t] == 2 μFs (sdach - s[t]) - λ9[t]
w'[t] == -μFw l[t] - l[t] λ7[t] + l[t] λ8[t]

0 == -nF[t] - dF'[t]
0 == nF[t] - nH[t] - nZB[t] - aB'[t] - dB'[t]
0 == -nZB[t] - rZB'[t]
0 == nH[t] - aH'[t]
0 == nZB[t] - aZB'[t]
0 == -nF[t] + nH[t] + nZB[t] - aH[t] × r[t] - dF[t] × r[t] - mB'[t]
0 == nF[t] + cH[t] × p[t] + dF[t] × r[t] - l[t] × w[t] - mF'[t]
0 == -nH[t] - cH[t] × p[t] + aH[t] × r[t] + l[t] × w[t] - mH'[t]
0 == -cH[t] - dpdach k[t] + β k[t]^(1-α) l[t]^α - k'[t] - s'[t]
0 == ps[t] - p'[t]

aB[0] == aB0
aH[0] == aH0
aZB[0] == aZB0
cH[0] == 1/2 k0^(1-α) l0^α β
dB[0] == dB0
dF[0] == dF0
k[0] == k0
l[0] == l0
mB[0] == mB0
mF[0] == mF0
mH[0] == mH0
nF[0] == nF0
nH[0] == nH0
nZB[0] == nZB0
p[0] == 2 k0^(-1+α) l0^(-α) (-nH0 + aH0 r0 + l0 w0)/β
ps[0] == ps0
r[0] == r0
rZB[0] == rZB0
s[0] == s0
w[0] == w0
```

13

7.2. Inflation and deflation shock as variable shock for the price

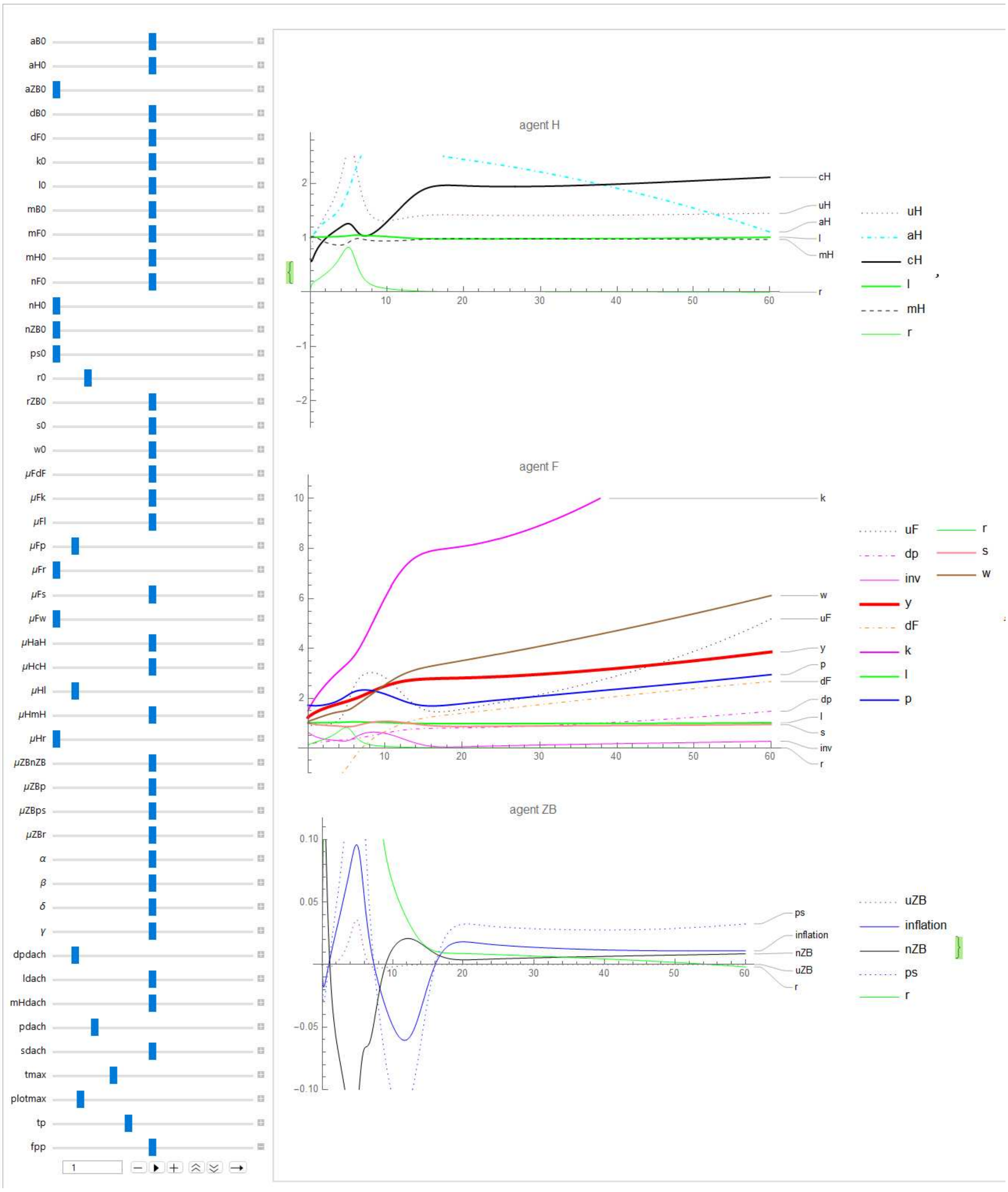
The simplest way to model an inflation respectively deflation shock is to model it as a variable shock for the price as shown in chapter 5.
For example, we use:
inflation shock: $p \rightarrow 1.5\,p$
deflation shock: $p \rightarrow 0.5\,p$
at time $t = 20$, because by this time the system has already settled in.

https://www.dropbox.com/s/th868lbth9ahz9i/Modell%20B1%20SCHOCK%20Version%203.ndsolve.nb?dl=0

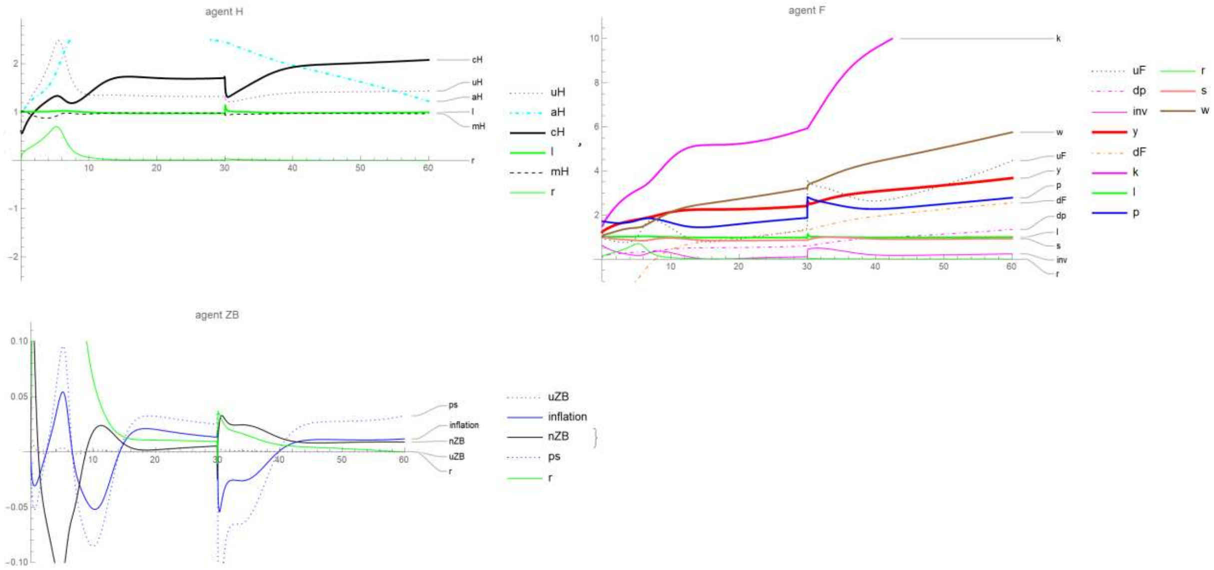
Certainly, one could also interpret these calculations in economic terms. But without prior adjustment of the models to real conditions, a real interpretation is not really serious. Therefore, we will not comment further on the calculated graphs.
As emphasized several times, the target of this book is to present the methodology of the GCD models in principle and to give an idea of what can be done with them and in what form. For application to concrete economic questions, the GCD

models still need to be adapted to real conditions. This is one of the tasks that still has to be done in the future.

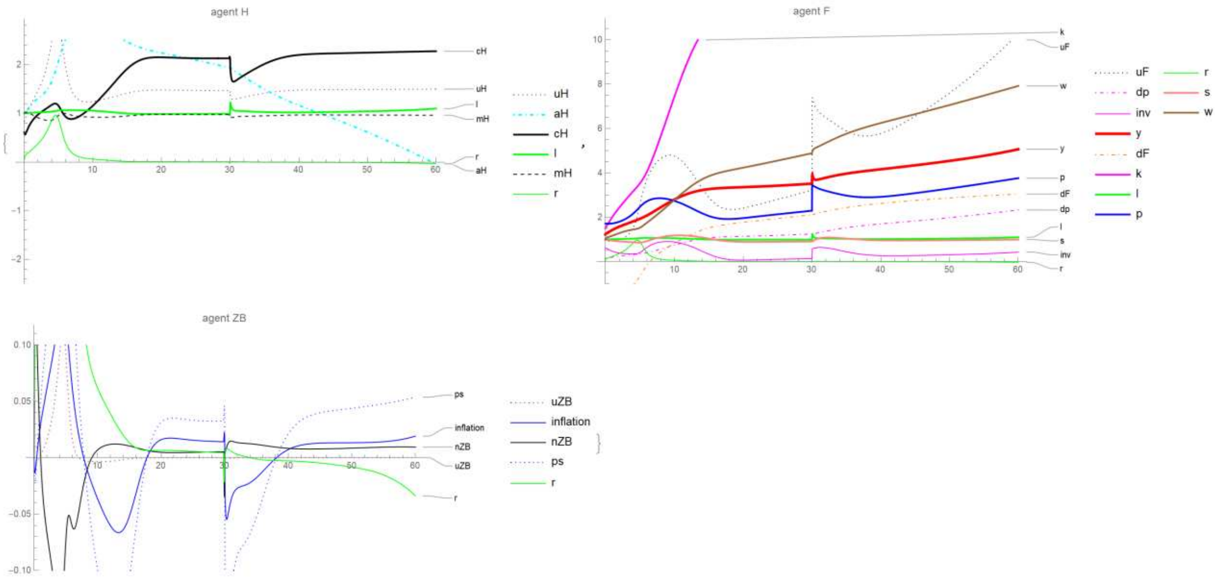
Model B1 without shock



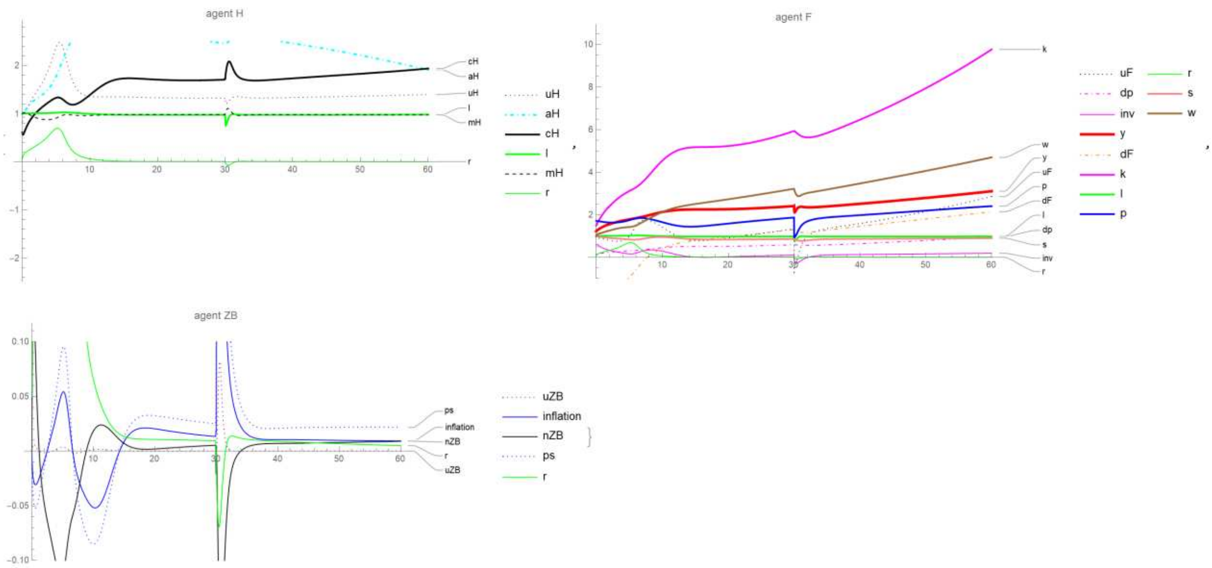
Inflation shock $p \to 1.5\,p$, pure money supply policy of central bank $\delta = 0$



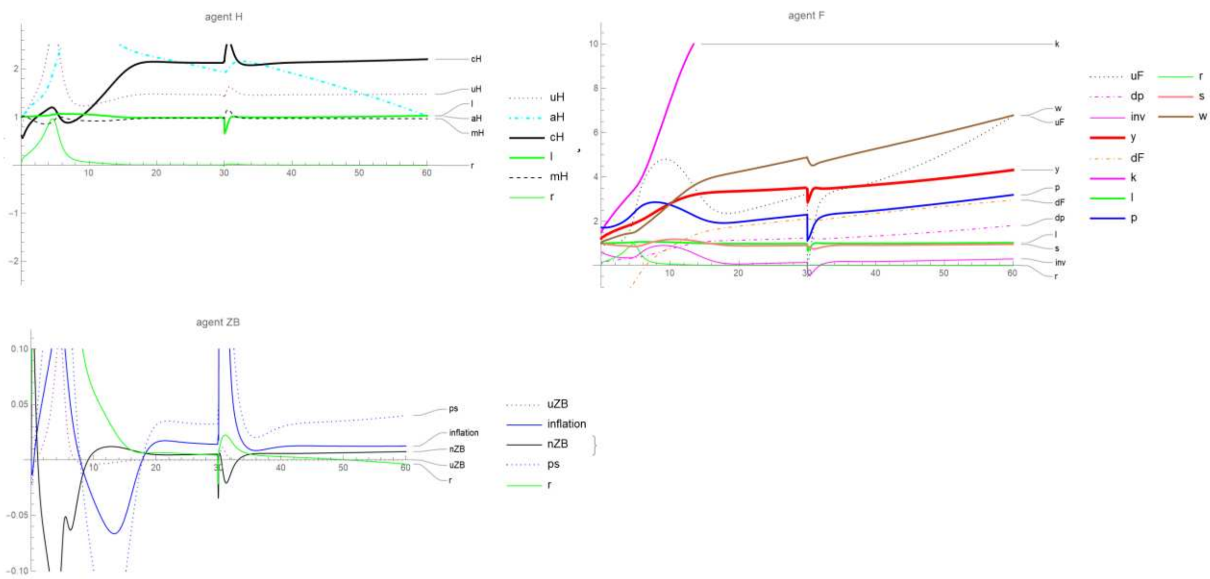
Inflation shock $p \to 1.5\,p$, pure interest policy of central bank $\delta = 1$



deflation shock $p \to 0.5\,p$ **, pure money supply policy of central bank** $\delta = 0$



deflation shock $p \to 0.5\,p$ **, pure interest policy of central bank** $\delta = 1$



7.3. Model inflation and deflation shock as model shock

Another possibility to model an inflation or deflation shock would be:
Introduce an agent A who has some power μ_{ps}^{A} to influence the price change $ps = p'$. (Consider e.g. OPEC as agent, which has the intention and the power to influence the trend in oil prices). If A intends to increase the price p at time t_0 for 1 year this leads to an inflation or deflation shock which can be modeled in the following way:

$$U^A = -(ps - ps)^2$$

$$\mu_{ps}^A(t) = 0 \qquad for\ t \in [0, t_0]\ and\ \ t > t_0 + 1$$

$$\mu_{ps}^A(t) = 1 \qquad for\ t \in [t_0, t_0 + 1]$$

We give this as an example, but do not calculate this model in detail.
Obviously, there are a lot of other possibilities to model price respectively inflation or deflation shocks.

8. Summary

GCD models are a natural extension of GE theory. They are based on the standard method for modelling dynamics under constraints in physics and can be thought of as an alternative to DSGE models. DSGE models are typically used to analyse economic shocks. For this reason, the aim of this article was to show how any kind of economic shock, e.g. demand, supply or price shocks, can be modelled within the framework of GCD models.

Acknowledgements

My thanks go above all to my son Florentin, who has advised me on all economic matters over the years, and especially to Oliver Richters, who has been a faithful companion in all matters of economics and physics for many years and without whom many of the publications would not have been possible.

References

Flannery, Martin Raymond. 2011. „D'Alembert–Lagrange analytical dynamics for nonholonomic systems". *Journal of Mathematical Physics* 52 (3): 032705. https://doi.org/10.1063/1.3559128.

Glötzl, Erhard. 2015. „Why and How to overcome General Equilibrium Theory". MPRA Paper 66265. https://mpra.ub.uni-muenchen.de/66265/1/MPRA_paper_66265.pdf.

———. 2022a. „General Constrained Dynamic (GCD) models with intertemporal utility functions". https://www.dropbox.com/s/269ki6287fc8atk/A%20general%20constrained%20dynamic%20model%20with%20intertemporal%20utility%20function%20FINAL%20Version%202%20maten.docx?dl=0.

———. 2022b. „Macroeconomic General Constrained Dynamic models (GCD models)". https://www.dropbox.com/s/2h8o3rg0r4b008b/Macroeconomic%20GCD%20Modelle%20A%2CB%2CC%20FINAL%20Version%202.docx?dl=0.

Glötzl, Erhard, und Lucas Binter. 2022. „GCDconfigurator, GitHub". https://github.com/lbinter/gcd.

Glötzl, Erhard, Florentin Glötzl, und Oliver Richters. 2019. „From Constrained Optimization to Constrained Dynamics: Extending Analogies between Economics and Mechanics". *Journal of Economic Interaction and Coordination* 14 (3): 623–42. https://doi.org/10.1007/s11403-019-00252-7.

Godley, Wynne, und Marc Lavoie. 2012. *Monetary economics: an integrated approach to credit, money, income, production and wealth*. 2. Aufl. Basingstoke and New York: Palgrave Macmillan.

Richters, Oliver. 2021. „Modeling the Out-of-Equilibrium Dynamics of Bounded Rationality and Economic Constraints". *Journal of Economic Behavior & Organization* 188 (August): 846–66. https://doi.org/10.1016/j.jebo.2021.06.005.

Richters, Oliver, und Erhard Glötzl. 2020. „Modeling Economic Forces, Power Relations, and Stock-Flow Consistency: A General Constrained Dynamics Approach". *Journal of Post Keynesian Economics* 43 (2): 281–97. https://doi.org/10.1080/01603477.2020.1713008.

Smale Institute. 2003. „Smale Institute Mathematics & Computation". 2003. http://www.smaleinstitute.com/problem.html.

Smale, Steve. 1991. „Dynamics Retrospective: Great Problems, Attempts That Failed". *Physica D: Nonlinear Phenomena* 51 (1–3): 267–73. https://doi.org/10.1016/0167-2789(91)90238-5.

———. 1997. „Mathematical Problems for the Next Century“. Fields Institute, Toronto, Juni. http://www.cityu.edu.hk/ma/doc/people/smales/pap104.pdf.

———. 1998. „Mathematical problems for the next century“, nachgedruckt in V. Arnold, M. Atiyah, P. Lax, B. Mazur: Mathematics: Frontiers and Perspectives 2000, American Mathematical Society 2000, 2: 7–15.

Zaman, Asad. 2020. „New Directions in Macroeconomics“. *International Econometric Review (IER)* 12 (1): 1–23. https://doi.org/10.33818/ier.747603.