Heckscher-Ohlin Theories from Factor Price Equalization to Factor Price Localization vs Empirical Observations from the Leontief Paradox to the Leontief Trade

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ABSTRACT - This paper extends the integrated world equilibrium into effective endowment analyses to obtain the price-trade equilibrium with factor price non-equalization. Trefler (1993)'s effective endowments explored an important logic that a country will export its commodity that is produced by using its effective abundant factor rather than its actual abundant factor intensively. This study shows that the logic favors both the Heckscher-Ohlin trade and the phenomena of the Leontief paradox (this study refers it to Leontief trade). When a country's actual factor abundance is not consistent with its effective factor abundance, Leontief paradox occur. The Leontief trade not only occurs under the presence of factor intensity reversals (FIR) but also occurs with the absence of the FIR. The localized factor prices make sure gains from trade for both countries, no matter it is the Heckscher-Ohlin trade, or it is the Leontief trades. The paper explores factor price definitions of trade patterns, which explain well the skill intensity reversals reported in empirical studies.

Keywords:
Localized factor prices, factor price equalization, factor price non-equalization, General equilibrium of trade, Leontief Paradox, Leontief trade

JEL Classification Code: F10, F15

1. Introduction

The simple motivation is to study how the price-trade equilibrium formed with factor price localization and whether the Leontief paradox can occur as trade consequences when countries have different productivities.

The general trade equilibrium is an essence of international trade theory. The Heckscher-Ohlin model is ideal for presenting the relationship among factor endowments, factor prices, commodity prices, production outputs, and trade flows. International trade is a subject that studies general equilibrium more than any other economic subject.

Samuelson (1948) presented the famous theorem of factor price equalization (FPE). Dixit and Norman (1980, chapter 4) proposed the Integrated World Equilibrium (IWE) to illustrate the factor price equalization, which perfectly fulfilled the factor mobility analysis. They proved that the world prices remain the same when the allocation of factor endowments changes within the factor price equalization (FPE) set in the IWE diagram.

McKenzie (1955) proposed the diversification cone of factor endowments, which is critical to understand factor price equalization (FPE) and trade balance from production constraints. Vanek (1968) proposed the HOV model that presented factor contents of trade. The share of GNP in the HOV model engaged prices with trade and consumption. It resulted in the theoretical and application issue on how to convert the assumption of homothetic taste into consumption balance. Fisher (2011) proposed "goods price diversification cone," which is the counterpart of factor diversification cone. He also offered another insight into the intersection of goods price cones to specify price-trade relationship when countries have different technologies.

The Leontief test (Leontief, 1953) showed that the US, as a capital-abundant country, exported its labor-intensive commodities. It counters the common sense of international economics then. The Leontief paradox impelled the HOV studies aimed to supply alternative approaches to explain it. Leontief (1953) proposed the productivity-equivalent factor (workers) to explain his test results. Trefler (1993) implemented Leontief's idea with factor-argument parameters as effective (equivalent) endowments. The model is also instrumental for theoretical analyses to reach

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factor price non-equalizations[1]. Fisher and Marshall (2008) proposed another excellent approach to involving different technologies using the virtual endowments and the conversion matrix.

Deardorff (1986) presented the diversification cones of the FIRs. He showed the double factor intensity reversals. He suggested a way to turn any model with the FIRs into one without it, and vice versa, by simply redefining goods. Chipman (1969), Trefler (1993), Krugman (2000), Fisher (2011), Leamer (2000), Rassekh and Thompson (1993), and many other studies had argued the need for factor price non-equalization when considering different technologies across countries.

Helpman and Krugman (1985, pp.-24) proposed an insight idea of trade volume that is defined with domestic factor endowments in the IWE diagram. They abstracted a unique principle as "the differences in factor composition are the sole basis of trade." Guo (2015) used their idea to access the price-trade equilibrium in the integrated world economy. This paper extends it to the equilibrium of factor price non-equalization under different productivities. The equilibrium supplies a vehicle to understand trade patterns.

This study shows that there are three trade patterns, under the different production conditions among countries, the Heckscher-Ohlin trade, the FIR Leontief trades, and the mutual Leontief trade. The FIR Leontief trade is caused by the factor intensity reversal, in which one country does Leontief trade, another does the Heckscher-Ohlin trade. The mutual Leontief trade occurs when the actual factor abundances conflict with their effective factor abundances in both countries. It happens without the presence of FIR. The study presents the exact conditions when the mutual Leontief trade occurs. All three trade patterns satisfy the logic of sign predictions that a country exports the services of its effective (or virtual) abundant factor.

The study explored another important feature of the Leontief trade that free trade reward more on same factor in both countries. It just explains the empirical results of skill intensity reversals and the relative wage increasing both in North countries and south countries (see Kurokawa, 2011, Reshef, 2007, Sampson, 2016).

The author organizes this paper into six sections. Section 2 derives the general trade equilibrium of factor price localizations. It shows that the world effective endowments measured by referring local productivities determine localized factor prices. It confirms the comparative advantage theory that localized factor prices ensure gains from trade for both countries. Section 3 illustrates that conceptually there are three trade patterns: the Heckscher-Ohlin trade, the FIR Leontief trade, and the mutual Leontief trade when countries have different productivities. Section 4 presents trade patterns by localized factor prices. discusses the trade effects with the presence of FIRs. Section 5 analyzes the trade patterns by virtual factor endowments. It concludes that the trade patterns under the virtual endowments will be as same as trade patterns under the effective endowments. Section 6 review the empirical studies about the Leontief paradox. It shows that more than half the studies reported Leontief trades. Conceptually, they are acceptable. It implies the co-exist of the Heckscher-Ohlin trade and the Leontief trade. It illustrates that the prediction signs, commonly used by effective endowments and virtual endowments, favor all the three trade patterns this paper. It also shows skill intensity reversals, in empirical studies, are just the localized factor prices by the FIR Leontief trade. The last section is the concluding remarks.

2. The Price-Trade Equilibrium When Countries Have Different Productivities

The Trefler (1993) proposed the first HOV model to incorporate different productivities across countries within the Heckscher-Ohlin framework magnificently. We use it to illustrate the factor price localizations. We use all assumptions in the Trefler model such as free trade, same taste for consumption, constant return of scale, no cost for trade, and productivities different across countries. The illustration is by $2 \times 2 \times 2$ model.

2.1 Review of Trefler Model

The central assumption in the Trefler model is to express productivity differences by factor input requirements as

[1] This paper uses factor price localizations and factor price non-equalizations alternatively for the phenomena that local factors are rewarded differently under the common world commodity prices when countries have different productivities.

$$A^H = \begin{bmatrix} a_{K1}^H & a_{K2}^H \\ a_{L1}^H & a_{L2}^H \end{bmatrix} = \Pi A^F = \begin{bmatrix} \pi_K & 0 \\ 0 & \pi_L \end{bmatrix} A^F \quad (2\text{-}1)$$

where Π is a 2×2 diagonal matrix, its element π_k is the factor productivity-argument parameter, $k = K, L$, K for capital, L for Labor. A^h is the 2×2 technology matrix of country h, its element $a_{ik}^h(w/r)$ is the input requirement of factor k needed to produce one unit of output i, $i=1,2$, $k=L, K$.

Production constraint function and the unit cost function for country H are

$$A^H X^H = V^H \quad (2\text{-}2)$$
$$(A^H)' W^H = P^H \quad (2\text{-}3)$$

For country F, they are

$$\Pi^{-1} A^H X^F = V^F \quad (2\text{-}4)$$
$$(\Pi^{-1} A^H)' W^F = P^F \quad (2\text{-}5)$$

where V^h is the 2×1 vector of factor endowments with elements K as capital and L as labor; X^h is the 2×1 vector of commodity output; W^h is the 2×1 vector of factor prices with elements r as rental and w as wage; P^h is the 2×1 vector of commodity prices with elements p_1^h and p_2^h; $h = H, F$ for countries.

The Trefler model is with a single cone of goods price diversifications[2]. Its factor cost ratio ranks, which show the rays of the cone of goods prices in algebra, are

$$\frac{a_{K1}^H}{a_{K2}^H} = \left(\frac{a_{K1}^F}{a_{K2}^F} = \frac{a_{K1}^H/\pi_K}{a_{K2}^H/\pi_K}\right) > \frac{P_1^*}{P_2^*} > \frac{a_{L1}^H}{a_{L2}^H} = \left(\frac{a_{L1}^F}{a_{L2}^F} = \frac{a_{L1}^H/\pi_L}{a_{L2}^H/\pi_L}\right) \quad (2\text{-}6)$$

where we assume both countries are capital intensive on sector 1. The single cone of goods price diversifications reduces the difficulties of analyses of the price-trade equilibrium. The Trefler model does have two cones of the factor diversifications, which show different productivities across countries. (2-6) also implies the absence of FIRs.

Bernhofen (2011, p104) emphasized the way to calculate factor content as "A country's factor content is defined using the country's domestic technology matrix[3]." This idea is a critical point in analyses of trade equilibrium when countries with different productivities.

The world effective (or equivalent) endowments by referring to country H's productivities are

$$K^{hW} = K^H + \pi_K K^F \quad (2\text{-}7)$$
$$L^{hW} = L^H + \pi_L L^F \quad (2\text{-}8)$$

The world effective endowments by country F's productivities are

$$K^{fW} = K^F + K^H/\pi_K \quad (2\text{-}9)$$
$$L^{fW} = L^F + L^H/\pi_L \quad (2\text{-}10)$$

We use the lowercase character h to indict the country referred to its productivities.

2.3 Factor Price Localizations at Equilibrium

Trefler (1993) described that the factor price equalization hypothesis and the HOV theorem hold in his equivalent-productivities system[4]. When the effective system is built (or mapped) by the referring to country H's productivities, the equalized factor price is the localized factor prices in country H. Similarly, when the effective system is built (or mapped) by referring to country F's productivities, the equalized factor price in the system is country F's prices.

Let express an effective productivity system formally by referring country H's productivities. Equations (2-2) and (2-3) for country H are still the same. Rewrite (2-4) and (2-5) as

$$A^H X^F = V^{hF} \quad (2\text{-}11)$$
$$(A^H)' W^H = P^F \quad (2\text{-}12)$$

where

$$V^{hF} = \Pi V^F \quad (2\text{-}13)$$
$$W^H = \Pi^{-1} W^F \quad (2\text{-}14)$$

[2] See Fisher (2011) for the cone of goods price diversification.

[3] The sign predictions both by effective endowments and by virtual endowments say this also.

[4] Fisher (2011) also mentioned that factor price equalization and Hechscher-Ohlin theorem hold in the virtual endowment system.

Equations (2-2), (2-3), (2-11), and (2-12) compose the effective system, or mapped system by matrix A^H. Mathematically it is an Heckscher-Ohlin model exactly. It is just model that Feenstra (2004, pp.) described the equalized factor price in the effective endowments as "Let A now denote the amounts of effective factors needed per unit of output in each industry. We continue to assume that factor price equalization holds in terms of effective factor prices, so with identical technologies, the matrix A is the same across countries". All the theorems and equilibrium result in the Heckscher-Ohlin model can apply to it. Guo (2015) proposed the price-trade equilibrium under the assumption of same technologies[5]. It can be applied to the effective system above directly as

$$s^h = \frac{1}{2}\left(\frac{K^h}{K^{hW}} + \frac{L^h}{L^{hW}}\right) \qquad h = (H, F) \tag{2-15}$$

$$W^{*H} = \begin{bmatrix} L^{HW} \\ K^{HW} \end{bmatrix} \tag{2-16}$$

$$P^* = (A^H)'\, W^{*H} \tag{2-17}$$

$$W^{*F} = \Pi W^{*H} \tag{2-18}$$

$$F_K^h = s^h K^{hW} - K^h = -\frac{1}{2}\frac{K^h L^{hW} - K^{hW} L^h}{L^{hW}} \qquad h = (H, F) \tag{2-19}$$

$$F_L^h = s^h L^{hW} - L^h = \frac{1}{2}\frac{K^h L^{hW} - K^{hW} L^h}{K^{hW}} \qquad h = (H, F) \tag{2-20}$$

$$T_1^h = s^h\, x_1^W - x_1^h \qquad (h = H, F) \tag{2-21}$$

$$T_2^h = s^h x_2^W - x_2^h \qquad (h = H, F) \tag{2-22}$$

We assume $w^{*H} = K^{hW}$ to drop one market clear condition by Walras equilibrium law. w^{*H} serves as "benchmark" price to be referred to by the other three factors' prices and two world commodity prices. Equation (2-18) is by the assumption of the Trefler model.

The numerators of (2-19) and (2-20) show that when

$$\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}} \tag{2-23}$$

then $F_K^H < 0$ and $F_L^H > 0$. It just says that a country exports the services of its effective abundant factor.

Equations (2-19) through (2-22) imply that a country being effective-capital abundance will export its capital-intensive goods and import its labor-intensive goods.

The key relationships for localized factor prices are

$$\frac{w^{*H}}{r^{*H}} = \frac{K^{hW}}{L^{hW}} \tag{2-29}$$

$$\frac{w^{*F}}{r^{*F}} = \frac{K^{fW}}{L^{fW}} \tag{2-30}$$

Appendix B derives factor price localization by trade volume defined by domestic factors and specified by world factor endowments measured by referring to domestic technology matrix. The approach is proposed by Helpmand and Krugman (1985) for factor price equalization.

3. Trade Patterns Specified by Effective Endowments

3.1 the logic of trade direction when countries have different productivities

The Heckscher-Ohlin theorem guides the trade direction under the same technologies. The HOV studies did accumulate some understanding of trade patterns when countries have different productivities also.

Trade direction of factor content (Trefler 1993)
a country exports the services of a factor that is effective abundant factor, compared to another factor.

This logic is widespread accepted, when countries have different productivities, in the HOV studies. The sign predictions for effective endowments and virtual endowments both use the logic (see section 6 for the sign predictions cited).

[5] Appendix A is the price-trade equilibrium proposed by Guo (2019), for the Heckscher-Ohlin model.

Trade direction of commodities

A country will export a commodity that is produced by using its effective abundant factor rather than is actual abundant factor intensively.

It is a natural extension of the trade direction of factor services. No literature addresses it directly. Section 5.2 presents the proof for virtual factor endowments, which is valid for effective endowments also[6]. The general trade equilibrium in the last section can proved it also.

When the actual factor abundance in a country is conflict with its effective factor abundance, the Leontief trade occurs. It is the whole logical basics of this paper.

3.2 Factor Intensity and Factor Intensity Reversals

The Heckscher-Ohlin theories defines the factor intensities between two industries as[7]

$$\frac{a_{K1}}{a_{L1}} > \frac{a_{K1}}{a_{L2}} \tag{3-1}$$

The most theorems of Heckscher-Ohlin model require the absence of factor intensity reversals (FIRs), which is defined as the elasticities and substitutions in a production function[8]. When two countries have different production functions by different parameters, it has more chances to present the factor intensity reversals. The system functions (2-2) through (2-5) are snapshot of the production productions, in linear system, by giving a group of factor endowments. The system functions should be with the capacity to express factor intensity reversals.

For the Trefler model by (2-2) through (2-5), the productivity (or technologies) differences can be presented both by cones of factor diversifications and by cones of goods price diversifications in the HOV studies (these two types of cones can reflect factor intensities and factor intensity reversals from production functions.)

The Heckscher-Ohlin model only analyzes factor intensity between industries, since two countries' technologies are same. When countries have different productivities (or technologies), the following cones of factor diversifications may occur,

$$\frac{a_{K1}^H}{a_{L1}^H} > \frac{a_{K1}^H}{a_{L2}^H} \tag{3-2}$$

$$\frac{a_{K1}^F}{a_{L1}^F} < \frac{a_{K1}^F}{a_{L2}^F} \tag{3-3}$$

It implies that country H is capital intensity to product commodity 1, and country F is capital intensive to product commodity 2. This is a factor intensity reversal across countries. It is an essential term related to trade direction when countries have different productivities.

The goods price diversification cone is an idea that make sure factor prices are positive when the vector of commodity price falls within it. The intersection of goods price diversification cones will make sure that both countries' factor prices will be positive when the vector of world commodity price falls within it. The intersection of goods price diversification cones can be used to illustrate factor intensity well also. Appendix C presents the goods price diversification cone and Intersection cone of two goods price diversification cones, geometrically.

The following cost requirement ratio ranks are typical for normal factor intensity across countries,

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{K1}^F}{a_{K2}^F} > \frac{a_{L1}^H}{a_{L2}^H} > \frac{a_{L1}^F}{a_{L2}^F} \tag{3-4}$$

They are also the rays of the two goods price cones of the two countries. The intersection cone is

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6 There are two approaches to prove the trade direction of commodity output by the direction of factor content of trade. One is by Leamer (1984,p.9-10). Another is by Helpman and Krugman (1985, p17). Both can be extended to analyze the commodity trade direction under effective endowments or virtual endowments.

7 It uses the approach by Leamer (1984, p.9-10).

8 Constant-elasticity-of-substitution (CES) production can specify the factor intensity reversals. When both goods have Cobb-Douglas production function, the factor intensity reversal are impossible (see Bhagwati, Panagariya, and Srinivasan (1998, p.61). However, when assume technology differences and using it in production function even by Cobb-Douglas functions, the factor intensity reversals will be more significant.

$$\frac{a_{K1}^F}{a_{K2}^F} > \frac{a_{L1}^H}{a_{L2}^H} \tag{3-5}$$

Goods prices by free trade must fall in this cone[9]. Equation (3-4) shows that both countries are factor intensity in industry 1. It is the case of the absence of factor intensity reversal. The equations (3-2) and (3-3) can be characterized by

$$|A^H||A^F| > 0 \tag{3-6}$$

where $|A^h|$ is the determinant of technology matrix A^h of country h, $h = H, F$. $|A^h| > 0$ means that country h is capital-intensive in industry 1. $|A^h| < 0$ means that country h is labor-intensive in industry 1.

Look another cost requirement ratio ranks

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{L1}^H}{a_{L2}^H} > \frac{a_{K1}^F}{a_{K2}^F} \tag{3-7}$$

It implies (3-2) and (3-3). We call it factor intensity reversal across countries (Briefly, we still call it FIR as it is described in production function analyses). The intersection cone of two goods price diversification cones is

$$\frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{L1}^H}{a_{L2}^H} \tag{3-8}$$

The equations (3-7) and (3-8) can be characterized by

$$|A^H||A^F| < 0 \tag{3-9}$$

We do not introduce any new definition for factor intensity reversals but identify it in presentation in the system equations to help to see its trade consequence.

3.3 The Heckscher-Ohlin trade

This study finds that there are three trade patterns conceptually when countries have different productivities.

The first one is the Heckscher-Ohlin trade, which is well-known when countries have same technologies. It says that a country abundant in the endowment of a factor will export the commodity that use this factor intensively. It is also a trade pattern when countries have different productivities. The following conditions make the Heckscher-Ohlin trade occur, assuming the absence of FIR,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-11}$$

$$\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}} \tag{3-12}$$

$$\frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{3-13}$$

Equation (3-11) is about the actual factor abundance[10]. Equations (3-12) and (3-13) uses effective endowments. If both countries are capital intensive in product 1 as (3-4), county H will export product 1; and country F will export product 2. The feature of this trade pattern is that both countries' actual factor abundances are consistent with their effective factor abundance.

3.3 Mutual Leontief Trade

Assume the absence of factor intensity reversal. The following conditions will lead to the mutual Leontief trade,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-14}$$

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{3-15}$$

$$\frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}} \tag{3-16}$$

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[9] The relative commodity price must fall within the intersection cone as $\frac{a_{L1}^F}{a_{L2}^F} > \frac{p_1^*}{p_1^2} > \frac{a_{L1}^H}{a_{L2}^H}$, see Fisher (2011).

[10] Feenstra and Taylor (2012, p102) first used term "actual factor endowment" to different from "effective factor endowment".

both countries' actual factor abundances are conflict with their effective factor abundances. If both countries are capital intensive in producing 1 as described as (3-4), County H will export product 2, and country F will export product 1.

We illustrate how it happens. Assuming that country H be actual factor abundant as (3-14), if the following is true,

$$\frac{K^H L^F}{L^H K^F} < \frac{\pi_K}{\pi_L} \tag{3-17}$$

equations (3-15) and (3-16) will occur. Equation (3-17) can be rewritten as the following separately

$$\frac{K^H}{L^H} < \frac{\pi_K K^F}{\pi_L L^F} = \frac{K^{hF}}{L^{hF}} \tag{3-18}$$

$$\frac{K^F}{L^F} > \frac{K^H/\pi_K}{L^H/\pi_L} = \frac{K^{fH}}{L^{fH}} \tag{3-19}$$

Equation (3-18) implies[11] (3-15). And equation (3-19) implies (3-16). As we illustrated in 3.1 that *A country will export a commodity that is produced by using its effective abundant factor intensively,* county H will export product 2, and country F will export product 1.

The mutual Leontief trade may occur within the Trefler model with the absence of the FIR. It is a new trade pattern that we get to notice by this study. The scope of the presence of the Leontief trade is much larger than what we expected before.

Appendix D is a numerical example for the mutual Leontief trade.

3.4 The FIR Leontief Trade - Factor Conversion Trade

The factor intensity reversals source the FIR Leontief trade. We specify the Trefler model a little bit differently by assuming that technological matrices of the two countries be

$$A^H = \psi A^F = \begin{bmatrix} 0 & \theta_K \\ \theta_L & 0 \end{bmatrix} A^F \tag{3-20}$$

where ψ is a 2×2 anti-diagonal matrix, its element θ_k is the productivity-across-factor-argument parameter, $k = K, L$. Denote

$$A^H = \begin{bmatrix} a_{K1}^H & a_{K2}^H \\ a_{L1}^H & a_{L2}^H \end{bmatrix} \tag{3-21}$$

The technology matrix in country F is

$$A^F = \psi^{-1} A^H = \begin{bmatrix} \frac{1}{\theta_L} a_{L1}^H & \frac{1}{\theta_L} a_{L2}^H \\ \frac{1}{\theta_K} a_{K1}^H & \frac{1}{\theta_K} a_{K2}^H \end{bmatrix} \tag{3-22}$$

Those two matrices compose a model with the FIRs as

$$A^H X^H = V^H \tag{3-23}$$

$$(A^H)' W^H = P^H \tag{3-24}$$

$$\psi^{-1} A^H X^F = V^F \tag{3-25}$$

$$(\psi^{-1} A^H)' W^F = P^F \tag{3-26}$$

Rewrite (3-25) as

$$A^H X^F = \psi V^F = \begin{bmatrix} \theta_L L^F \\ \theta_K K^F \end{bmatrix} \tag{3-27}$$

It shows that the equivalent or effective endowments[12] measured by country H' productivities to produce commodity X^F in country F. The world effective endowments by referring to country H's productivities are

$$K^{hW} = K^H + \theta_L L^F \ , \qquad L^{hW} = L^H + \theta_K K^F \tag{3-28}$$

Similarly, the world effective endowments by referring to country F's productivities are

$$K^{fW} = K^F + L^F/\theta_L \ , \qquad L^{fW} = L^F + K^F/\theta_K \tag{3-29}$$

11 $\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} < \frac{K^{hF}}{L^{hF}}$ is always true.

12 It can be expressed also as $\begin{bmatrix} K^{hF} \\ L^{hF} \end{bmatrix} = \left(A^H {A^F}^{-1}\right) \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \frac{\theta_L L^F}{\theta_K K^F}$.

The cost requirement ratios, which write down the rays of goods price diversification cones in algebra, are

$$\frac{a_{K1}^H}{a_{K2}^H} = (\frac{a_{L1}^F}{a_{L2}^F} = \frac{a_{K1}^H/\theta_L}{a_{K2}^H/\theta_L}) \ , \quad \frac{a_{L1}^H}{a_{L2}^H} = (\frac{a_{K1}^F}{a_{K2}^F} = \frac{a_{L1}^H/\theta_K}{a_{L2}^H/\theta_K}) \tag{3-30}$$

It is also the case of the single cone of goods prices diversification, in which the two cones intersected fully, but in reversal. If country H is capital-intensive to produce commodity 1,

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{L1}^H}{a_{L2}^H} \tag{3-31}$$

by (3-30), country F will be capital-intensive to produce commodity 2,

$$\frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{K1}^F}{a_{K2}^F} \tag{3-32}$$

The model presents the FIR. The following conditions judge the FIR Leontief trade,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-33}$$

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{3-34}$$

$$\frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{3-35}$$

By inequality (3-31), country H is labor effective abundant. It will export commodity 2 since it is labor intensity to product commodity 2 by (3-31). And country F will export commodity 1, by (3-35) that it is effective labor abundant and by and (3-32) that it is labor intensive to produce community 2. The commodity trades equilibrate in the normal way[13] as

$$T^H = -T^F \tag{3-36}$$

Both countries are effective labor abundant by (3-34) and (3-35). Both countries will export labor services and import capital services. We call it the reversals of factor content of trade. Both countries export commodities that are produced by using the same factor intensively, but they export different commodities since each country is factor intensive at different industry.

Let see how (3-34) and (3-35) occurs. Equation (3-34) implies[14]

$$\frac{K^H}{L^H} < \frac{K^{hF}}{L^{hF}} = \frac{\theta_L L^F}{\theta_K K^F} \tag{3-37}$$

It means that country H is effective labor abundant. Equation (3-37) can be rewritten as

$$\frac{K^F}{L^F} < \frac{L^H/\theta_K}{K^H/\theta_L} = \frac{K^{fH}}{L^{fH}} \tag{3-38}$$

It implies that country F also is effective labor abundant.

Equations (3-34) and (3-35) show that effective labor is abundant in both countries. Under comparative advantage law, a country exports its product with relative advantage to produce. Net exported factor will be rewarded with higher price than its price in autarky. Both countries' labor will be rewarded better. It is another new characteristic of the FIR Leontief trade. Section 4 shows that like the Heckscher-Ohlin trade, the Leontief trade can make sure of gains from trade for both countries.

The Trefler FIR model is a Trefler model mathematically. The result of general trade equilibrium (2-15) through (2-22) can be applied to the Trefler FIR model.

With factor content of trade reversals, both countries will consume more on their effective scarce factor, embodied in the trade flows[15]. It is a new type of comparative advantages to use global resource more efficiently.

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[13] Some studies explained their conclusion as that both countries export two commodities and imports same two commodities. This explanation is not valid. It built another paradox.

[14] The following always holds, $\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}} > \frac{K^{hF}}{L^{hF}} = \frac{\theta_L L^F}{\theta_K K^F}$, if $\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}}$.

[15] Free trade transforms the global effective abundant factor into global effective scarce factor, embodied in the commodity trade flows. The FIR Leontief trade phenomenon is a little bit like the "black hole" in astronomy (Black hole is defined as that a region of space having a gravitational field so intense that no matter or radiation can escape). Free trade traps or absorbers the global effective abundant factor, which cannot "escape" from the market. At the same time, free trade is also like the "white hole" (the white hole is a hypothetical region of spacetime, which cannot be entered from the outside, although matter and light can escape from it. In this sense, it is the reverse of a black hole, which can only be entered from the outside and from which matter and light cannot escape). Free trade releases or transforms the global effective scarce factor to both countries.

For a particular case when country F produces commodity 1 by using technologies of industry 2 in country H and produces commodity 2 by using the technologies of industry 1 in country H as

$$\psi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{3-39}$$

the localized factor prices will be[16]

$$\frac{w^{*H}}{r^{*H}} = \frac{r^{*F}}{w^{*F}} \tag{3-40}$$

It reflects the switch order of goods in the analyses by production function for the Heckscher-Ohlin model, which deals with FIR.

Deardorff (2006, page 102) defined the factor intensity reversal as "A property of technology (page 268) for two industries such as that their ordering of relative factor intensities is different at different factor price. For example, one industry may be relatively capital intensive compared to the other at high relative wages and labor intensive at low relative wage". This study exams the same issue reversely. We use factor intensity reversals illustrate factor prices reversal. The results and conclusion are same about localized factor prices.

Appendix E is numerical example of the FIR Leontief trade. Appendix F shows how the FIR Leontief trade occurs under higher dimension.

The Leontief trades are easy to be identified under the Heckscher-Ohlin framework with different productivities. For the two-country economy, if only one country's actual factor abundance conflicts with its effective factor abundance, it is the FIR Leontief trade. If both country's actual factor abundance conflicts with their effective factor abundance, it is the mutual Leontief trade. No way to make sure that actual factor abundance is always are consistent with effective factor abundance, therefore, no way to make sure there is only the Heckscher-Ohlin trade.

4. Factor Price Definitions of the Trade Patterns

There are two alternative ways to defining factor abundance[17]. Country H is said being capital abundant by either of the followings,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \qquad \text{"}phsical\ defination\text{"} \tag{4-1}$$

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} \qquad \text{"}price\ defination\text{"} \tag{4-2}$$

where w^{ah} is autarky wage in country h; and r^{ah} is autarky rental in country h, $h = H, F$. The price definition is not as popular as the physical definition since both autarky prices and free trade prices are not available before. Guo (2019) proved the logic of autarky prices as

$$\frac{w^{ah}}{r^{ah}} = \frac{K^h}{L^h} \qquad (h = H, F) \tag{4-3}$$

It is a useful condition to defined trade patterns by factor prices.

The localized wage-rental ratio for a country is

$$\frac{w^{*h}}{r^{*h}} = \frac{K^{hW}}{L^{hW}} \qquad (h = H, F) \tag{4-4}$$

The Heckscher-Ohlin trade is specified by the physical factor abundances, in the last section, as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}\ , \qquad \frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}}\ , \qquad \frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{4-5}$$

Substituting (4-3) and (4-4) into the inequalities above yields,

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}\ , \qquad \frac{w^{aH}}{r^{aH}} > \frac{w^{*H}}{r^{*H}}\ , \qquad \frac{w^{aF}}{r^{aF}} < \frac{w^{*F}}{r^{*F}} \tag{4-6}$$

It is the factor price definition of the Heckscher-Ohlin trade when countries have different productivities. We assume two countries engaged on free trade immediately form a Leontief trade pattern in (4-6). It may be not realistic in real world. We just want show relationships among variables within the system.

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[16] The equilibrium result in section 2 can present it exactly.

[17] see Bhagwati, Panagariya, and Srinivasan (1998, p.63).

It illustrates that the trade will benefit capital services in country H and labor in country F. Free trades benefit the effective-abundant factors, which are actual abundant factors also for the Heckscher-Ohlin trade.

From (4-6), two possible ranks of wage-rental ratios are

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{aF}}{r^{aF}} \tag{4-7}$$

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{aF}}{r^{aF}} \tag{4-8}$$

They show

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} \tag{4-9}$$

$$\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} \tag{4-10}$$

We call (4-9) that country H is labor reward intensity, and that country F is capital reward intensity, under world commodity prices. (4-10) reverses (4-9). The Heckscher-Ohlin trade can generate both, depending on if $\frac{K^{hW}}{L^{hW}} > \frac{K^{fW}}{L^{fW}}$ or not.

It implies that the wage-rental ratio $\frac{w^{*H}}{r^{*H}}$ in country H is still higher than the ratio $\frac{w^{*F}}{r^{*F}}$ in country F. It is not expected by Heckscher-Ohlin theory. It is kind of the factor reward intensity reversal[18] from the view of the Heckscher-Ohlin theory.

From the localized factor prices, it is reasonable. Equation (4-9) shows that the relative capital price in country H, $\frac{r^{*H}}{w^{*H}}$, does improved respective to autarky price. It is a trade consequence; it may occur if $\frac{K^{hW}}{L^{hW}} > \frac{K^{fW}}{L^{fW}}$.

Country H imports labor service. However, it does it by two possible factor reward intensities.

The mutual Leontief trade by physical factor abundance is expressed as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}, \qquad \frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}}, \qquad \frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}} \tag{4-10}$$

Substituting (4-3) and (4-4) into them yields,

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}, \qquad \frac{w^{aH}}{r^{aH}} < \frac{w^{*H}}{r^{*H}}, \qquad \frac{w^{aF}}{r^{aF}} > \frac{w^{*F}}{r^{*F}} \tag{4-11}$$

It shows that the trade will benefit labor in country H and capital in country F. They are actual scarce factors of each country. However, they are effective abundant factors of each country. Rewrite (4-11) as

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} > \frac{w^{*F}}{r^{*F}} \tag{4-12}$$

It shows that labor in country H is relative expensive in autarky; it is even more relatively expensive after free trade. From the view of the Heckscher-Ohlin theories, it is the factor reward intensity reversals also. Equation (4-12) implies that the wage-rental ratio in country H, $\frac{w^{*H}}{r^{*H}}$, is even higher than its ratio in autarky, $\frac{w^{aH}}{r^{aH}}$. It is with only factor intensity as

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} \tag{4-13}$$

From it, we judge that (4-9) is expected by the Heckscher-Ohlin trade. (4-10) is as same as (4-13), which is for the Leontief trade.

The FIR Leontief trade is specified as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}, \qquad \frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}}, \qquad \frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{4-14}$$

Substituting (4-3) and (4-4) into them yields

[18] Equation (3-36) is a typical factor reward intensity reversal. Deardorff (1986) discussed this topic systematically. Empirical studies used this term differently. Some of them checked their result by the Stolper-Samuelson theorem, if it can't be explained, they call it factor reward intensity reversal. Some of them checked by which factor should be benefited by the Heckscher-Ohlin theorem, if it is not consistent, they call it factor reward intensity reversal. This study does the same thing for the term usage.

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}, \qquad \frac{w^{aH}}{r^{aH}} < \frac{w^{*H}}{r^{*H}}, \qquad \frac{w^{aF}}{r^{aF}} < \frac{w^{*F}}{r^{*F}} \tag{4-15}$$

It depicts that the trade will benefit labors in both countries, which are effective-abundant factor worldwide. And capital is the effective-scarce factor worldwide.

From (4-14), two possible wage-rental ratio ranks are

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{aF}}{r^{aF}} \tag{4-16}$$

$$\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} \tag{4-17}$$

Those two inequality chains show (4-9) and (4-10) also. It implies both the Heckscher-Ohlin trade and the FIR Leontief trade can generate different factor reward intensities.

A unique feature for the FIR Leontief trade is that trade benefits the effective abundant factor, which are same factor for both countries. For example, both countries' wage-rental ratios are higher than their autarky wage-rental ratio as (4-16) and (4-17) . Trade compensates consumptions more on effective scarce factor for both countries. Trade converts the part of effective abundant factor into effective scarce factor in consumption equilibrium. Free trade makes the usage of resources more efficiently. The technology differences, indicated by $|A^H||A^F| < 0$, endow factor intensity reversals that source factor abundance reversals, the reversals factor content of trade, and factor price reversals. It just explains the phenomime of skill intensity reversal reported in many empirical studies[19]. It shows a new trade mechanism of consumption compensation.

The structures of localized factor prices are as complex as trade patterns do. Some of patterns looks strange in the first glace. However, all the price patterns above make sure of gains from trades. The factor price definitions of trade patterns show a new way to view practices of international trade.

4.2 The Dual between Trade Pattern and Localized Factor Prices

We now summarize the discussions as a theorem.

Theorem – *A country's Localized factor Prices are determined by the world effective endowments measured by referring to its domestic productivities. The localized factor prices specify three trade patterns, the Heckscher-Ohlin trade, the FIR Leontief trade, and the mutual Leontief trade, ensuring that both countries gain from free trade.*

Proof

Equation (2-16) through (2-18) presents the structure of localized factor prices. This section shows the factor price definitions of trade patterns. It implies that the trade patterns are trades consequences conceptually. Appendix F illustrates the gains from trade by localized factor prices.

End Proof

Regardless of whether the model is the same technical model or not, gains from trade are the basic requirements of an equilibrium solution for models under the Heckscher-Ohlin frameworks[20].

4.3 Stolper-Samuelson Trade Effect Reversals and Rybczynski Trade Effect Reversals

Some empirical studies explained their results by the Stolper-Samuelson trade effects or the Rybczynski trade effects. When the presence of FIR in the models, both the Stolper-Samuelson trade effect reversals and the

[19] Section 6 will discuss it in detail.

[20] The localized factor prices also satisfied with Helpman (1984) restrictions between factor price differences and factor content of trade

$$(w^j - w^i)'F^{ij} > 0$$
$$(w^j - w^i)'\left(F^{ij} - F^{ji}\right) > 0$$

where w^j is the vector of payment in country j and F^{ij} is the vector of factor content of trade exported from country *j* to country *i*, *i=1,2*, and *j=1,2*. This can be displayed numerically for the three trade patterns.

Rybczynski trade effect reversals will occur. Therefore, we cannot use Heckscher-Ohlin theories directly explaining trade phenomena when countries have different productivities.

The Stolper-Samuelson theorem[21] states that a rise in the price of a commodity will increase the real reward of the factor used intensively in the sector and decrease the real reward of the other factor. It depends on factor intensities, which are same under the assumption of same technologies.

With the presence of FIR, the signs of $|A^h|$ two countries are different. If one country is capital intensive in sector 1; another country will be capital intensive in sector 2. A rise in price of a commodity will cause different factor price increase in each country. Therefore, the trade effect reversal happens.

Both the Stolper-Samuelson trade effects and the Rybczynski trade effects depend on the sign of $|A^h|$.
If we assume that commodity remain unchanged when the commodity price rising as Stolper-Samuelson theorem did, we can specify the Stolper-Samuelson trade effects for countries with different productivities as that under specific economic assumptions (constant returns to scale, perfect competition, equality of the number of factors to the number of products, world output and factor endowments stay no changes), a rise in the price of a commodity will increase the price of the domestic factor that is used intensively in the sector and decrease the price of other domestic factor[22]. This is a general statement. It will be true no matter the presence of FIR or the absence of FIR.

Similarly, we describe the Rybczynski trade effect as that an increase in the factor will increase by a greater proportion of the commodity output of the domestic sector which uses the factor intensively and decrease the domestic output of the other sector[23]. It implies that if an increase in a factor in country H causes the commodity increase in a sector, the increase of that factor in country F will cause the commodity increase in another sector, when the FIR presents.

5. Analyses of Trade Patterns by the Virtual Endowments

The idea of the virtual endowments presented the full technologies difference across countries in the Heckscher-Ohlin framework (see Fisher and Marshall, 2008). It is more complex in model structure and equilibrium analyses, although its mathematical expression is still concise. Fisher (2011) proposed the interception of goods diversification cones, which explored the most challenging part of the model's general trade equilibrium with virtual endowments[24].

Fisher (2011) also mentioned that under the virtual endowment assumptions, the classical Heckscher-Ohlin theory holds when technologies and factor prices are identical to those of the reference country. The behaviors and the trade patterns of the virtual endowment model are identical to the Trefler model's behaviors.

The $2 \times 2 \times 2$ model with virtual endowments can be expressed as

$$A^h X^h = V^h \qquad (h = H, F) \tag{5-1}$$

$$(A^h)' W^h = P^h \qquad (h = H, F) \tag{5-2}$$

where $A^H \neq A^F$ in general. We refer the model to the Fisher-Marshall model[25] or the Heckscher-Ohlin-Ricardo model[26]. The world virtual endowments referring to the home country's technology can be expressed with the conversion matrix as

$$V^{hW} = V^H + (A^H)^{-1} A^F V^F \tag{5-3}$$

The world virtual endowments referring to the foreign country's technology can be expressed with the conversion matrix as

$$V^{fW} = V^F + (A^F)^{-1} A^H V^H \tag{5-4}$$

where $V^{hW}$is the vector of is the factor services needed to produce world commodity x^wusing a reference to the technology matrix of country h as A^h , $h = h, f$. We use the same notations as those used in the effective endowments.

[21] See Wong (1995, p.34)

[22] This result can be derived to use the derivation (Suranovic, 2010, chapter 115-2) to both countries separately.

[23] This result can be derived to use the derivation (Suranovic, 2010, chapter 115-3) to both countries separately.

[24] Appendix C presents a geometrical expression of the interception of goods diversification cones.

[25] In their original notation, they consider the indirect primary factors by intermediate input in their empirical analysis, such as $A = B(I - \tilde{A})$, where B is input-output matrix, $\tilde{A}$ is directe factor requirement matrix.

[26] Davis (1995) studied this kind of model, referred it as Heckscher-Ohlin-Ricardo model.

Unlike the Trefler model, the model of virtual endowments is with two price diversification cones (see Appendix C). The intersection cone of two goods price cones will bring some difficulty to analyze the trade consequence. The solution of price-trade equilibrium by referring country H's technology is slightly different from the solution by country F's technology. It needs more studies about it for a theoretical solution.

5.1 Derivation of the FIR Leontief trade

This paper only introduces one trade pattern, namely FIR Leontief trade as an illustration.

Leamer (1984, pp.8-9) supplied a unique way to prove the Heckscher-Ohlin theorem analytically. We now extend Leamer's analysis to virtual factor endowments.

Assume the model is with presence of FIRs, in which[27] $|A^H| > 0$ and $|A^F| < 0$. Assume also that country H is actual-capital abundance; country H is virtual-labor abundance.

In (3-34) and (3-35), we proved that with presence of FIRs, if country H is effective labor abundant, country F will be effective abundant also. We will show that it is true also for virtual endowments.

The vector of the output exports in the home country is the difference between production output X^H and consumption[28] C^H:

$$T^H = C^H - H^H = A^{H-1}(s^H V^{hW} - V^H) \tag{5-9}$$

which is $(A^H)^{-1}$ times the vector of excess virtual factor supplies,

$$F^H = s^H V^{hW} - V^H = \begin{bmatrix} s^H K^{hW} - K^H \\ s^H L^{hW} - L^H \end{bmatrix} = \begin{bmatrix} K^{hW}(s^H - K^H/K^{hW}) \\ L^{hW}(s^H - L^H/L^{hW}) \end{bmatrix} \tag{5-10}$$

The home country will export the services of labor and import the services of capital, by the assumptions. Therefore, the vector of factor content of trade in the home country is with signs

$$F^H = \begin{bmatrix} + \\ - \end{bmatrix} \tag{5-11}$$

The signs of trade flow of the home country from equation (5-11) will be

$$T^H = (A^H)^{-1} F^H = \begin{bmatrix} + & - \\ - & + \end{bmatrix} \begin{bmatrix} - \\ + \end{bmatrix} = \begin{bmatrix} - \\ + \end{bmatrix} \tag{5-12}$$

This is due to the home country is capital intensive in output 1 by $|A^H| > 0$.

The vector of the output exports in the foreign country is

$$T^F = C^F - H^F = (A^F)^{-1}(s^F V^{fW} - V^F) \tag{5-12}$$

which is $(A^F)^{-1}$ times the vector of excess virtual factor supplies:

$$F^F = s^F V^{fW} - V^F = \begin{bmatrix} s^F K^{fW} - K^F \\ s^F L^{fW} - L^F \end{bmatrix} = \begin{bmatrix} K^{fW}(s^F - K^H/K^{fW}) \\ L^{fW}(s^F - L^H/L^{fW}) \end{bmatrix} \tag{5-13}$$

The signs of trade flow of the foreign country from equation (5-12) will be

$$T^F = -T^H = \begin{bmatrix} + \\ - \end{bmatrix} \tag{5-14}$$

We assume it is a normal commodity trade as $T^F = -T^H$.

The matrix of country A^F satisfies

$$(A^F)^{-1} = \begin{bmatrix} - & + \\ + & - \end{bmatrix} \tag{5-15}$$

This is due to the foreign country is capital intensive in output 2 by $|A^F| < 0$.

If the factor content of trade in the foreign country is with signs

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[27] Leamer used the inversion matrix of technology matrix as

$$A^{-1} = \begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix}^{-1} = \begin{bmatrix} a_{L2} & -a_{K2} \\ -a_{L1} & a_{K1} \end{bmatrix} / |A|$$

where $|A| = a_{L1} a_{L2} \left({}^{a_{K1}}/_{a_{L1}} - {}^{a_{K2}}/_{a_{L2}} \right) > 0$

[28] For this paper, export is expressed by negative value.

$$F^F = \begin{bmatrix} - \\ + \end{bmatrix} \tag{5-16}$$

The signs of trade flow of the foreign country in equation (5-14) will hold as

$$T^F = (A^F)^{-1}F^H = \begin{bmatrix} - & + \\ + & - \end{bmatrix}\begin{bmatrix} - \\ + \end{bmatrix}=\begin{bmatrix} + \\ - \end{bmatrix} \tag{5-17}$$

(5-16) implies that country F is effective labor abundant also. Country F will export the services of labor and import the services of capital also. Therefore, it is the FIR Leontief trade.

5.2 *Geometric Presentations of Trade Patterns*

We use the IWE diagram with the virtual endowments to illustrate both the Leontief trades' geometric presentation[29] .

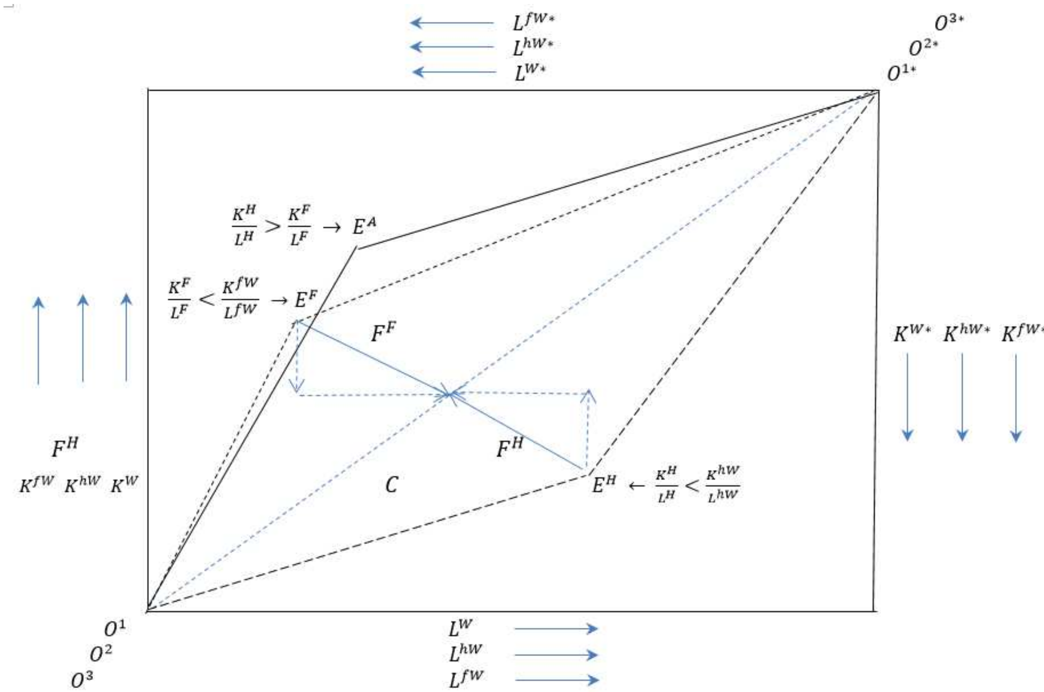

Figure 4 Multiscale IWE Diagram for the FIR Leontief Trade

Figure 4 draws an IWE diagram for the FIR Leontief trade. It is a multiscale diagram that merges the three charts. The densities of each diagram's scales are different. The right-upper corner is with three origins for the foreign country. The lower-left corner is with three origins for the home country. Dimension O^1O^{1*}, presenting chart 1, is for world actual factor endowments. Dimension O^2O^{2*} is for virtual factor endowments measured by referring to the home country's technology. Dimension O^3O^{3*} is for virtual factor endowments measured by referring to the foreign country's technology.

For a given allocation of actual factor endowments of two countries at E^A, there are two respective allocations of virtual factor endowments E^H and E^{F*}. Allocation E^A is the vector from the home origin O^1. It is above the diagonal line. It specifies that country H is actual capital abundance as

$$\frac{K^H}{L^H} > \frac{K^W}{L^W} \tag{5-24}$$

E^H is the vector from home origin. It writes down the allocation of virtual factor endowments of two countries, measured by country H's technology. It is below the diagonal line. It signifies that country H is virtual labor abundance as

[29] Davis and Weinstein (2000) talked about the new perspective of Integrated World Equilibrium (IWE). They mentioned, "A breakdown of FPE and a multiple-cone view of the world will importantly inform additional work on the Heckscher-Ohlin-Vanek model."

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{5-25}$$

E^F is from foreign origin, it shows the allocation of the virtual factor endowments of two countries, measured by referring to the foreign country's technology. It is below the diagonal line from the view of foreign origin. It implies that the foreign country is virtual labor abundance as

$$\frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{5-26}$$

Vectors F^F and F^H show that both countries export labor services and import capital services.

Figure 5 draws an IWE diagram for the mutual Leontief trade.

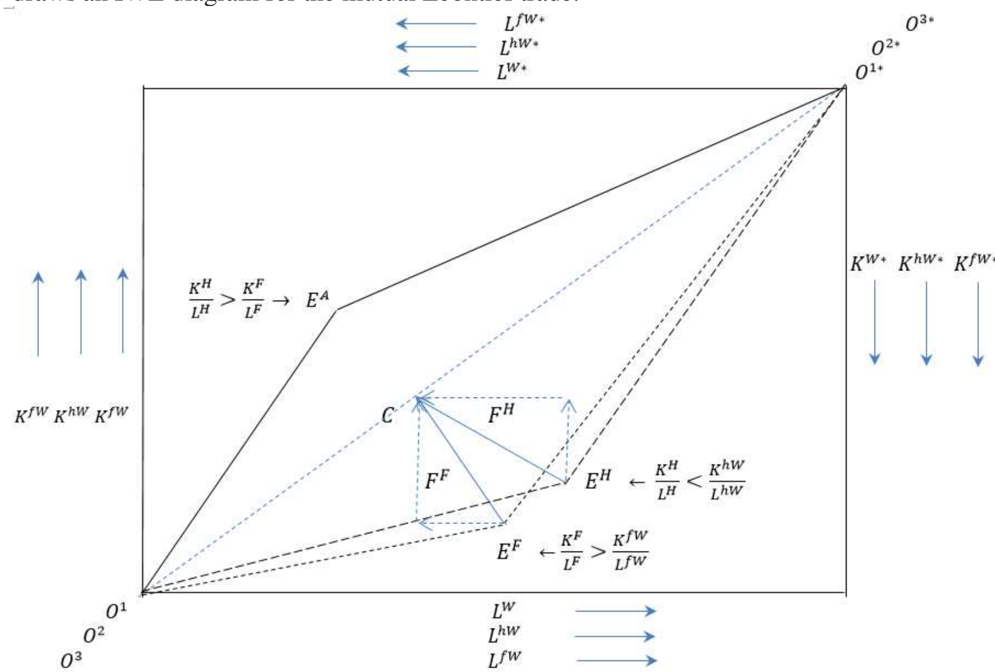

Figure 5. Multiscale IWE Diagram for the Mutual Leontief Trade

Allocation E^A is the vector from the home origin O^1. It says that country H is actual capital abundance as

$$\frac{K^H}{L^H} > \frac{K^W}{L^W} \tag{5-27}$$

Point E^H is below the diagonal line. It signifies that country H is virtual labor abundance as

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{5-28}$$

Point E^F is below the diagonal line from the view of foreign origin. It signifies that the foreign country is virtual capital abundance as

$$\frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}} \tag{5-29}$$

There are two vectors of factor content of trade, $F^H$and F^F, in Figure 5. Vector F^H says that country H, as an actual capital abundant country, exports labor services, and imports capital services. Similarly, vector F^F writes down that the foreign country, as an actual labor-abundant country, exports capital services, and imports labor services.

Similarly, we can draw an IWE diagram for the Heckscher-Ohlin trade.

Helpman and Krugman (1985, pp.17) proved the trade pattern by IWE diagram. We can use their approach to illustrate trade direction of commodity outputs by Figure 4 and Figure 5.

6. Discussions of empirical studies related

a. Empirical Studies Showed Co-existences of the Heckscher-Ohlin trade and the Leontief Trade

Kwok and Yu (2005) investigated the 52 countries' data using differentiated factor intensity techniques and concluded that the Leontief paradox "is found to be either disappeared or eased."

More than a hundred econometric pieces of literature about the Leontief paradox were published between the 1960s and the 1990s. Half of them concluded that the paradox persists; and half was consistent with the Heckscher-Ohlin theory. The half to half results confused economists then. Nevertheless, all the tests are still meaningful from the view of factor price localization of this paper.

The empirical studies in this period mostly used sign prediction based on the same technology assumption,

$$(V_k^i - s^i V_k^W)F_k^i > 0 \tag{6-1}$$

The reality is that countries are with different productivities. If (6-1) is failed in a study with its data, it implies actual factor abundance is conflict with effective factor abundance. Therefore, the failure implies the Leontief trade. Half of the tests at this period reported the Leontief trade[30], which are denied by lacking an adequate conceptual foundation. This paper does supply the conceptual description of Leontief trade. Based on the half of test results at this period, we may see the co-existence of the Heckscher-Ohlin trade and Leontief trades.

6.2 The sign predictions by the effective endowments and virtual endowments favor both Leontief trade and Heckscher-Ohlin Trade

In empirical studies, the sign prediction for the effective endowments can be written as

$$(V_k^i - s^i \textstyle\sum_j \pi_k^i V_k^j)F_k^i > 0 \tag{6-2}$$

where V_k^j is the element of vector V^j which is defined as $V^j = \Pi_j^{-1} A^0 y^j$. And V_k^j is the factor service needed to product country j's commodity y^j using a reference to productivity in country i as A^0. F_k^i is the factor services exported by country i.

The sign prediction for virtual endowments is

$$(V_k^{vi} - s^i \textstyle\sum_j V_k^{vj})F_k^{vi} > 0 \tag{6-3}$$

where V_k^{vj} is the element of vector V^{vj} which is defined as $V^{vj} = A^0 y^j$. V_k^{vj} is the factor service needed to product country j's commodity y^j using a reference to technology matrix in country i as A^0. F_k^{vi} is the factor service exported by country i.

Both the Heckscher-Ohlin trade and the Leontief trades under the logic (6-2) and (6-3). They are derived from the logic of these signs. Therefore, the signs above favor both the Heckscher-Ohlin trade and the Leontief trades. It is not sufficient to use those test results to clear the issue of the Leontief paradox simply.

6.3 Skill Intensity Reversal (Factor Reward Intensity Reversal) as the Leontief Trades

Some studies in this century show evidence of the Leontief trade by factor intensity reversals. Kurokawa (2011) showed "clear-cut evidence for the existence of the skill intensity reversal" in his empirical study of the USA-Mexico economy. Sampson (2016) interpreted his assignment reversals of skilled workforce between North and South by factor intensity reversal. Takahashi (2004) studied the postwar Japanese economy. He interpreted Japan's economic growth as a capital-intensity reversal.

Reshef (2007) claimed "One of the most prevalent economic phenomena in the last two decades of the 20th century has been the increase in skill premia in many countries around the globe skilled workers have been receiving a higher share of income and higher wages relative to their less-skilled fellow workers. The magnitude of this increase varies considerably across countries but is economically large almost everywhere". He cited other five studies which presented same results[31]. Kozo and Yoshinori (2017) found the existence of factor intensity reversals in their study as well. They wrote, "Using newly developed region-level data; however, we argue that the abandonment of factor

[30] We cite fewer of test with Leontief trade ant the period. Keesing (1966) inspected the factor contents of trade in some OECD countries and reported that US exports have higher skill input than their imports[30]. Heller (1976) studied the Japanese economy and documented the changes in trade factor contents. Roskamp (1963) noted that in 1954 West German experts were more labor-intensive than imports. Baldwin (1971) showed that U.S. imports were 27% more capital-intensive than U.S. exports in the 1962 trade data, using a measure like Leontief's.

[31] Acemoglu (2003), Behrman et al. (2003), Gorg, and Strobl (2002), and Hoekman and Winters (2005). All of them is about the phenomenon that skill intensive reversal has indeed been global, as both developed, and less-developed countries have experienced it.

intensity reversals in the empirical analysis has been premature. Specifically, we find that the degree of the factor intensity reversals is higher than that found in previous studies on average".

Sampson (2016) specially mentioned in his study, "Therefore, assignment reversals offer a new explanation for why trade liberalization has led to increased wage inequality not only in the relative skill abundant North but also in the relative skill scarce South." Equations (4-15) through (4-17) presents the reward increasing of same factor in both countries. It is another typical character of the Leontief FIR trade.

Conclusion

This paper derives the price-trade equilibrium with factor-price localizations when countries have different productivities. The Leontief trade and the factor price localizations posts a way to describe the complexity of international trade in real world. The localized factor prices are not neutral to the Leontief paradox. The differences between autarky prices and localized factor prices can specify the Leontief trades. Both the Heckscher-Ohlin trade and the Leontief trades are derived from effective endowments and virtual endowments. The Leontief trades satisfy the Heckscher-Ohlin framework's core idea that effective or virtual factor abundance determined trade directions. Like the Heckscher-Ohlin trade, the Leontief trades are under the law of comparative advantage, it makes sure gains from free trade.

More than half of the empirical studies reported the evidence of existences of the Leontief paradox. These results cannot be ignored conceptually from view of factor price localizations, which suggests the co-exist[32] of the Heckscher-Ohlin trade and Leontief trades.

The study shows that the mutual Leontief trade can occur without the presence of FIR.

Technically, the paper shows that the sign predictions, commonly used in empirical studies, by effective endowments or by the virtual endowments, favor both the Heckscher-Ohlin trade and the Leontief trades. Therefore, the accuracy improvements in sign predictions in those studies do not mean there is no room for the Leontief trades[33].

The Leontief trade and localized factor prices explain the phenomenon of factor intensity reversal and skill intensity reversal well.

The Stolper-Samuelson trade effects and the Rybczynski trade effects are related and determined by domestic factor intensity or domestic technology input matrix. The study shows that the price-trade mechanism is more complex than we understand by original Heckscher-Ohlin analyses.

Appendix A - The price-trade equilibrium when countries have same technologies

Dixit and Norman (1985, chapter 4) found the mobility property of factor price equalization that world prices will remain unchanged when world factor endowments redistributed within the FPE set in the IWE chart. It implies $\frac{w^*}{r^*}$ is constant. The trade balance of factor content can be expressed as

$$\frac{w^*}{r^*} = -\frac{(s^h - \frac{K^h}{K^W})}{(s^h - \frac{L^h}{L^W})} \frac{K^W}{L^W} \tag{A-1}$$

Guo (2015) introduced a constant

[32] The Heckscher-Ohlin trade is defaulted as only trade pattern for some empirical studies by effective endowments and virtual endowments. It is clearly missing something from the signs used to predict trade direction. The hechsher-Ohlin theory and factor price equalization works within the effective endowment system and virtual endowment system, which are the mapped mathematical system. In the actual world, it is with factor price localizations and the co-existence of the Heckscher-Ohlin trade and the Leontief trade.

[33] On the contrary, the Leontief trades are presented because the accuracy improvements of sign predictions are majorly by including the Leontief trades.

$$\varphi = -\frac{(s^h - \frac{K^h}{K^W})}{(s^h - \frac{L^h}{L^W})} \tag{A-2}$$

It implies

$$\frac{w^*}{r^*} = \varphi \frac{K^W}{L^W} \tag{A-3}$$

It is the IWE equilibrium by Dixit and Norman. Using Helpman and Krugman's trade volume defined with domestic factor endowments, Guo (2015) proved that $\varphi = 1$, by three different approaches. One of them is by using the trade volume Helpman and Krugman (1985, pp.23) defined by domestic factor endowments. The integrated world equilibrium is then expressed as

$$w^* = K^W \tag{A-4}$$

$$r^* = L^W \tag{A-5}$$

$$p_1^* = a_{k1}L^W \ + a_{L1}K^W \tag{A-6}$$

$$p_2^* = a_{k2}L^W + a_{L2}K^W \tag{A-7}$$

$$s^h = \frac{1}{2}\left(\frac{K^h}{K^W} + \frac{L^h}{L^W}\right) \qquad h = (H, F) \tag{A-8}$$

Appendix B – The Integrated World Equilibrium by Effective Endowments

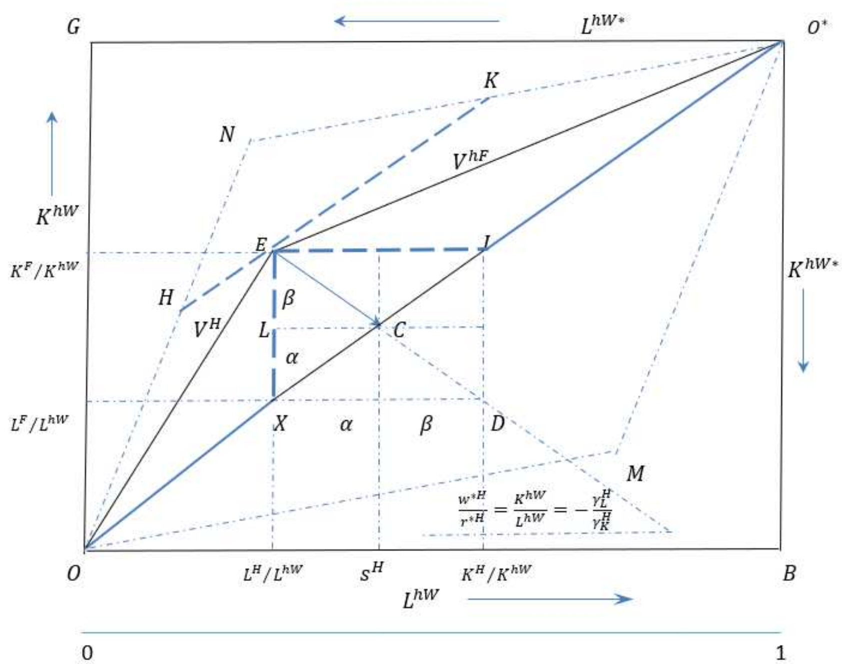

Figure 1 Trade Box and Equal Trade Volume Line by Effective Factor Endowments

Figure 1 is the IWE diagram extended to present effective endowments. The dimensions of the diagram represent the effective endowments measured by referring to the productivities of the home country to produce world commodities. The home country's origin is the lower-left corner, and the foreign country is from the right-upper corner. ON and OM are the rays of the cone of factor diversifications in the home country. Any point within the parallelogram formed by ONO^*M is an available allocation of effective endowments of two countries. V^{hF} is the vector of effective endowments of country F measured by the productivities of country H.

Corresponding the rays of goods price diversification cone in (2-6), there is a relationship for the range of the share of GNP as

$$\frac{K^H}{K^{hW}} > s^H > \frac{L^H}{L^{hW}} \tag{B-1}$$

where

$$\frac{K^H}{K^{hW}} = s^H_{max}(p) = s\left(\begin{bmatrix} a^H_{K1} \\ a^H_{K2} \end{bmatrix}\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x^W_1 + a_{K2}x^W_2} \tag{B-2}$$

$$\frac{L^H}{L^{hW}} = s^H_{min}(p) = s\left(\begin{bmatrix} a^H_{L1} \\ a^H_{L2} \end{bmatrix}\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x^W_1 + a^H_{L2}x^W_2} \tag{B-3}$$

Figure 1 presents (B-1) as trade box $EJXD$.

Suppose that E is the allocation describing the distribution of the world effective endowments. Country H is effective capital abundant at this allocation (we will use this assumption for all analyses of this study). Point C is the trade equilibrium point. It shows the sizes of the consumption of the two countries.

We propose that each country's trade volume is the function of the local (or domestic) factor endowments and localized factor prices by using Helpman and Krugman's idea about trade volume.

Helpman and Krugman (1985, p.23) defined trade volume by domestic factors constrained with world factor endowments. They illustrated that there are some variables (γ_L , γ_K) for all equal trade volumes lines, which satisfy the following relationships:

$$VT = \gamma_L L^H + \gamma_K K^H \tag{B-4}$$

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \tag{B-5}$$

The equal trade volume curves in the FPE set are straight lines, which are parallel to the diagonal line OO^* in the IWE diagram. The two equations ensure that a higher difference in factor composition leads to a higher trade volume and that trade volume is zero if factor endowments distribute at the diagonal line OO^* in the IWE diagram. They showed that one of γ_L, γ_K is negative. If country H is capital abundant, its two variables are $\gamma_K > 0$ and $\gamma_L < 0$.

We slightly change (B-4) and (B-5) by using the world effective endowments as

$$VT^h = \gamma^h_L L^h + \gamma^h_L K^h \qquad (h = H, F) \tag{B-6}$$

$$-\frac{\gamma^h_L}{\gamma^h_K} = \frac{K^{hW}}{L^{hW}} \qquad (h = H, F) \tag{B-7}$$

In Figure 1, $\overline{HK}$ is an equal trade volume line by effective endowments.

Vector V^H , the factor endowments in country H, can be written as

$$V^H = \begin{pmatrix} K^H \\ L^H \end{pmatrix} = \overrightarrow{OG} + \overrightarrow{EG} \tag{B-8}$$

$\overline{OG}$ stands for the part of the factor endowments that is under the proportion (composition) of world factor consumptions as

$$\overrightarrow{OG} = \begin{pmatrix} \lambda_L K^{hW} \\ \lambda_L L^{hW} \end{pmatrix} \tag{B-9}$$

$\overline{EG}$ is the excessive capital services, which is out of the proportion of world factor equivalent consumptions. We express it as

$$\overrightarrow{EG} = \begin{pmatrix} (\alpha + \beta)K^{hW} \\ 0 \end{pmatrix} \tag{B-10}$$

Rewrite it as

$$V^H = \begin{pmatrix} K^H \\ L^H \end{pmatrix} = \begin{pmatrix} \lambda_L K^{hW} \\ \lambda_L L^{hW} \end{pmatrix} + \begin{pmatrix} (\alpha + \beta)K^{hW} \\ 0 \end{pmatrix} \tag{B-11}$$

The trade volume (B-6) can be rewritten as a dot product of V^H and the pair of the variables $(\gamma^H_K \quad \gamma^H_L)$

$$VT^H = (\gamma^H_K \quad \gamma^H_L)\begin{pmatrix} K^H \\ L^H \end{pmatrix} \tag{B-12}$$

where the two variables are marked with superscript h to for country. Substituting (B-11) into the above yields

$$VT^H = (\gamma^H_K \quad \gamma^H_L) \cdot \left(\overrightarrow{OG} + \overrightarrow{EG}\right) = (\gamma^H_K \quad \gamma^H_L)\begin{pmatrix} \lambda_L K^{hW} \\ \lambda_L L^{hW} \end{pmatrix} + (\gamma^H_K \quad \gamma^H_L)\begin{pmatrix} (\alpha + \beta)K^{hW} \\ 0 \end{pmatrix} \tag{B-13}$$

The first term on the right side above is zero by (B-7),

$$(\gamma^H_K \quad \gamma^H_L)\begin{pmatrix} \lambda_L K^{hW} \\ \lambda_L L^{hW} \end{pmatrix} = 0 \tag{B-14}$$

Simplify (B-13) as

$$VT^H = (\alpha + \beta)K^{hW}\gamma^H_K \tag{B-15}$$

The vertical line EG, in quantity as $(\alpha+\beta)K^W$, is the differences of factor composition described by Helpman and Krugman. It is just the vertical boarder of trade box. Its value by free trade is trade volume.
The trade volume by EG is

$$VT^H = (\alpha+\beta)K^{hW}r^{*H} \tag{B-16}$$

It implies

$$\gamma_K^H = r^{*H} \tag{B-17}$$

The trade volume of factor content of trade, in country H, can be expressed also as

$$VT = 2F_K^H r^{*H} = 2\beta K^{hW} r^{*H} \tag{B-18}$$

Substituting (B-18) into (B-16) yields

$$\beta = \alpha \tag{B-19}$$

It implies

$$\frac{w^{*H}}{r^{*H}} = \frac{K^{hW}}{L^{hW}} \tag{B-20}$$

Similarly, we can obtain

$$\frac{w^{*F}}{r^{*F}} = \frac{K^{fW}}{L^{fW}} \tag{B-21}$$

The solution can be extended to the case of multiple countries as the Heckscher-Ohlin model does (See Guo, 2019)

Appendix C - The goods price diversification cone and Intersection of two goods price diversification cones

Fisher (2011) proposed the terms of "the goods price diversification cone" and "the intersection of two price diversification cones". The goods price diversification cone is the counterpart of the diversification cone of factor endowments. It is an especially important concept for price-trade equilibriums. The intersection of two price diversification cones illustrates what makes sure that the rewards of two sets of localized factor prices are positive when countries have different technologies.

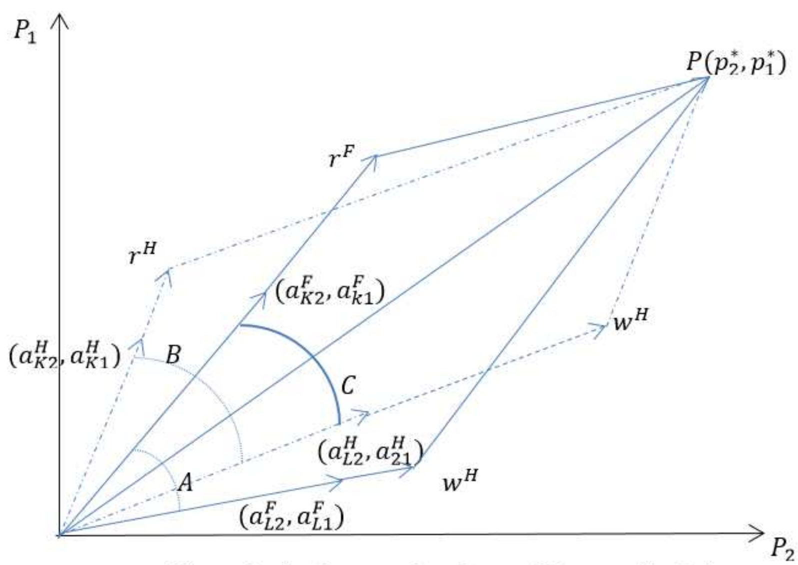

Figure 1 the Intersection Cone of Commodity Price

We draw the cost requirement vectors (a_{K2}^H, a_{K1}^H), (a_{L2}^H, a_{L1}^H), (a_{K2}^F, a_{K1}^F), and (a_{L2}^F, a_{L1}^F) in the ranks (2-5) in Figure 1. After multiplying each of them by their payments of respective factors, these vectors create two cones of output prices, labeled as cone A and cone B. There is an overlapped part of two cones of goods prices. The overlap of two cones is the intersection of two goods price cones, labeled as cone C, which is the space spanned by vectors (a_{K2}^F, a_{K1}^F)

and (a_{L2}^H, a_{L1}^H). It is clear from Figure 1 that the rewards for four factors of the two countries will be positive, if and only if the world output price vector (p_1^*, p_2^*) lies in the intersection of two goods price diversification cones.

Appendix C – Numerical Example of the Mutual Leontief Trade

The technological matrix for the home country, in this example, is

$$A^H = \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The technological matrix for the foreign country is

$$A^F = \begin{bmatrix} 1/0.9 & 0 \\ 0 & 1/0.6 \end{bmatrix} \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The factor intensities of the two countries are

$$a_{K1}^H / a_{L1}^H = 3.0 > a_{K2}^H / a_{L2}^H = 0.75$$
$$a_{K1}^F / a_{L1}^F = 3.0 > a_{K2}^F / a_{L2}^F = 0.75$$

The home country is capital intensive in industry 1, and the foreign country is capital intensive in industry 1 too. This system is with the absence of FIRs. We take the factor endowments for the two countries as

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 3000 \\ 2850 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 3500 \\ 4300 \end{bmatrix}$$

The outputs of the two countries are

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 700 \\ 900 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 826.60 \\ 670.0 \end{bmatrix}$$

The home country is actual labor abundant as

$$\frac{K^H}{L^H} = \frac{3000}{2850} = 1.05 > \frac{K^F}{L^F} = \frac{3500}{4300} = 0.81$$

The home country is effective labor abundant as

$$\frac{K^H}{L^H} = \frac{3000}{2850} = 1.05 < \frac{0.9 * K^F}{0.6 * L^F} = \frac{3155}{2580} = 1.22$$

Therefore, the home country exports labor services and exports commodity 2 since commodity 1 uses labor intensively.

The foreign country is effective capital abundant also as

$$\frac{K^F}{L^F} = \frac{3500}{4300} = 0.81 > \frac{K^H/0.9}{L^H/0.6} = 0.70$$

The foreign country will export capital services also. It will export commodity 1 that is produced in using capital intensively.

The share of GNP of the home country is $s^H = 0.5734$. The trade flows and the factor contents of trades by the share of GNP are:

$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} 73.0 \\ -105.0 \end{bmatrix}, \qquad \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = \begin{bmatrix} -73.0 \\ 105.0 \end{bmatrix}$$
$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} -287.5 \\ +246.4 \end{bmatrix}, \qquad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} 449.1 \\ -273.8 \end{bmatrix}$$

At the equilibrium, the world prices and the localized factor prices are

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 4.0714 \\ 2.8751 \end{bmatrix}, \quad \begin{bmatrix} r^{*H} \\ w^{*H} \end{bmatrix} = \begin{bmatrix} 0.8571 \\ 1.0 \end{bmatrix}, \quad \begin{bmatrix} r^{*F} \\ w^{*F} \end{bmatrix} = \begin{bmatrix} 0.5142 \\ 0.9 \end{bmatrix}$$

Here, we assume $w^{*H} = 1$.

The autarky prices for the two countries are

$$\begin{bmatrix} r^{aH} \\ w^{aH} \end{bmatrix} = \begin{bmatrix} 0.7307 \\ 1.0 \end{bmatrix}$$

$$\begin{bmatrix} r^{aF} \\ w^{aF} \end{bmatrix} = \begin{bmatrix} 0.7083 \\ 1.0 \end{bmatrix}$$

Here, we assume $w^{aH} = 1$, and $w^{aF} = 1$.

The gains from trade are

$$g^H = W^{aH} \cdot F^H = 36.66$$
$$g^H = W^{aH} \cdot F^F = 65.60$$

The critical point, which decides the trade directions in this example is

$$\frac{K^H L^F}{L^H K^F} = 1.29 < \frac{\pi_K}{\pi_L} = \frac{0.9}{0.6} = 1.5$$

If the technological matrix for the foreign country is

$$A^F = \begin{bmatrix} 1/0.9 & 0 \\ 0 & 1/0.8 \end{bmatrix} \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The critical point will be

$$\frac{K^H L^F}{L^H K^F} = 1.29 > \frac{\pi_K}{\pi_L} = \frac{0.9}{0.8} = 1.11$$

It will be the Heckscher-Ohlin trade. At that moment, the actual factor abundance still is

$$\frac{K^H}{L^H} = \frac{3000}{2850} = 1.05 > \quad \frac{K^F}{L^F} = \frac{3500}{4300} = 0.81$$

The home country is effective capital abundant as

$$\frac{K^H}{L^H} = \frac{3900}{2850} = 1.36 > \quad \frac{0.9 * K^F}{0.8 * L^F} = 0.91$$

The foreign country is effective labor abundant as

$$\frac{K^F}{L^F} = \frac{3500}{4300} = 0.81 < \quad \frac{K^H/0.9}{L^H/0.8} = 0.93$$

The trade pattern now changed. It shows that trade pattens are extremely sensitive to the ratio $\frac{\pi_K}{\pi_L}$. For multiple countries analyses, it needs check to see what trade pattern for each country.

Appendix D – Numerical Example of the FIR Leontief Trade

The technological matrix for the home country, in this example, is

$$A^H = \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The matrix for the foreign country is

$$A^F = \begin{bmatrix} 0.0 & 1/0.9 \\ 1/0.8 & 1.0 \end{bmatrix} \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The factor intensities of the two countries are

$$a^H_{K1}/a^H_{L1} = 2.0 > \ a^H_{K2}/a^H_{L2} = 0.5$$
$$a^F_{K1}/a^F_{L1} = \ 0.562 < a^F_{K2}/a^F_{L2} = 2.25$$

The home country is capital intensive in product 1, and the foreign country is capital intensive in industry 2. The system is with the presence of the FIRs. We take the factor endowments for the two countries as

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 4200 \\ 3000 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 3187.5 \\ 2666.6 \end{bmatrix}$$

The outputs of the two countries are

$$\begin{bmatrix} x^H_1 \\ x^H_2 \end{bmatrix} = \begin{bmatrix} 1200.0 \\ 600.0 \end{bmatrix}, \qquad \begin{bmatrix} x^F_1 \\ x^F_2 \end{bmatrix} = \begin{bmatrix} 500.0 \\ 900.0 \end{bmatrix}$$

The home country is actual capital abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \quad \frac{K^F}{L^F} = \frac{3187.5}{2666.6} = 1.19$$

The home country is effective capital abundant also as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \quad \frac{K^{hF}}{L^{hF}} = \frac{2400}{2550} = 0.94$$

Therefore, the home country exports capital services and exports commodity 1 since commodity 1 uses the capital intensively.

The foreign country is effective capital abundant also as

$$\frac{K^F}{L^F} = \frac{318.75}{2666.6} = 1.19 > \quad \frac{K^{fH}}{L^{fH}} = \frac{3750}{4666} = 0.80$$

Therefore, the foreign country exports capital services too. It will export commodity 2 since commodity 2 used the capital intensively. The home country is with the Heckscher-Ohlin trade and the foreign country is with the Leontief trade.

The share of GNP is, $s^H = 0.5884$. The trade flows and the factor contents of trades by the share of GNP are:

$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} -199.6 \\ 282.6 \end{bmatrix}, \qquad \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = \begin{bmatrix} 199.6 \\ -282.6 \end{bmatrix}$$

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} -316.2 \\ 265.9 \end{bmatrix}, \qquad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} -332.8 \\ 351.3 \end{bmatrix}$$

We see that both countries export capital services and import labor services. The trade converts the globally effective abundant factor into the globally scarce factor. However, the trade flows are normal, country H exports product 1 and country F exports product 2.

At the equilibrium, the world prices and the localized factor prices are

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 4.0227 \\ 2.8409 \end{bmatrix}, \quad \begin{bmatrix} r^{*H} \\ w^{*H} \end{bmatrix} = \begin{bmatrix} 0.8409 \\ 1.0000 \end{bmatrix}, \quad \begin{bmatrix} r^{*F} \\ w^{*F} \end{bmatrix} = \begin{bmatrix} 0.8000 \\ 0.7568 \end{bmatrix}$$

Here, we assume $w^{*H} = 1$.

The autarky prices for the two countries are

$$\begin{bmatrix} r^{aH} \\ w^{aH} \end{bmatrix} = \begin{bmatrix} 0.7142 \\ 1.0 \end{bmatrix}$$

$$\begin{bmatrix} r^{aF} \\ w^{aF} \end{bmatrix} = \begin{bmatrix} 0.8366 \\ 1.0 \end{bmatrix}$$

Here, we assume $w^{aH} = 1$, and $w^{aF} = 1$.

The gains from trade are

$$g^H = W^{aH} \cdot F^H = 40.04$$

$$g^H = W^{aH} \cdot F^F = 73.27$$

Appendix E – The FIR Leontief Trade for Many Factors and Many Commodities

The FIR Leontief trade also occurs in the models with many commodities, many factors, and many countries. A straightforward way to specify a FIRs model in high dimensions is by switching a pair of rows in its technology matrix. Row-switching matrix S_{ij}, like the following, switches all matrix elements on row *i* with their counterparts on row *j*.

$$S_{ij} = \begin{bmatrix} 1 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 1 & & & & & & \\ & & & 0 & 0 & 1 & & & \\ & & & 0 & \ddots & 0 & & & \\ & & & 1 & 0 & 0 & & & \\ & & & & & & 1 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 1 \end{bmatrix}$$

The corresponding elementary matrix is obtained by swapping row i and row j of the identity matrix. Since the determinant of the identity matrix is unity, $\det[S_{ij}] = -1$. It follows that for any square matrix A (of the correct size), we have $\det[S_{ij}A] = -\det[A]$. Using a row-switching operation, we can implement a FIRs model. It is also available for non-square (not even) technology matrix. The conversion trade not only occurs for the even model (factor number equals to output number) but also for the non-even model. To specify a non-even FIR model, just use a square Row-switching matrix S_{ij}.

We present a numerical example to display a conversion trade for $4 \times 4 \times 2$ model. The technological matrix for country H is

$$A^H = \begin{bmatrix} 3.0 & 1.2 & 1.3 & 0.9 \\ 1.1 & 2.0 & 0.9 & 1.4 \\ 0.7 & 1.5 & 2.1 & 1.0 \\ 1.6 & 1.7 & 0.8 & 1.5 \end{bmatrix}$$

The technology matrix for country F is

$$A^F = \psi \quad A^H$$

where

$$\psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A^F = \begin{bmatrix} 3.0 & 1.2 & 1.3 & 0.9 \\ 1.1 & 2.0 & 0.9 & 1.4 \\ 1.6 & 1.7 & 0.8 & 1.5 \\ 0.7 & 1.5 & 2.1 & 1.0 \end{bmatrix}$$

The third row and fourth row A^F are switched from A^H.

The factor endowments of the two countries are

$$V^H = \begin{bmatrix} 4253 \\ 4189 \\ 3631 \\ 4098 \end{bmatrix}, \qquad V^F = \begin{bmatrix} 3690 \\ 4975 \\ 3865 \\ 4080 \end{bmatrix}$$

The world effective abundant by the home productivities are

$$V^{hW} = \begin{bmatrix} 8333 \\ 8054 \\ 8606 \\ 7788 \end{bmatrix}$$

The world effective abundant by foreign productivities are

$$V^{fW} = \begin{bmatrix} 8333 \\ 8054 \\ 7788 \\ 8606 \end{bmatrix}$$

We see that the values of V_3^{fW} and V_4^{fW} are reversals of V_3^{hW} and V_4^{hW}. Both countries are effective abundant at factor 4 related to factor 3

$$\frac{v_3^H}{v_4^H} = \frac{3631}{4098} = 0.886 < \frac{v_3^{hW}}{v_4^{hW}} = \frac{8606}{7788} = 1.105$$

$$\frac{v_3^F}{v_4^F} = \frac{3864}{4080} = 0.947 < \frac{v_3^{fW}}{v_4^{fW}} = \frac{7788}{8606} = 0.949$$

That will cause the factor content reversals between factor 3 and factor 4.

Appendix F – Gains from trade by localized factor prices

We express the gains from trade for country H as

$$W^{aH} \cdot F^{H} > 0 \qquad \text{(F-1)}$$

where $W^{aH}$is autarky factor prices in country H. It can be expressed as[34]

$$W^{aH} = \begin{bmatrix} K^{H} \\ L^{H} \end{bmatrix} \qquad \text{(F-2)}$$

The factor content of trade of country H by (2-19) and (2-20) is

$$F^{H} = \begin{bmatrix} -\frac{1}{2}\frac{K^{H}L^{hW}-L^{hW}L^{H}}{L^{hW}} \\ \frac{1}{2}\frac{K^{H}L^{hW}-K^{hW}L^{H}}{K^{hW}} \end{bmatrix} \qquad \text{(F-3)}$$

Substituting (F-2) through (F-3) into (F-1) yields

$$\begin{bmatrix} L^{H} & K^{H} \end{bmatrix} \begin{bmatrix} -\frac{1}{2}\frac{K^{H}L^{hW}-L^{hW}L^{H}}{L^{hW}} \\ \frac{1}{2}\frac{K^{H}L^{hW}-K^{hW}L^{H}}{K^{hW}} \end{bmatrix} > 0 \qquad \text{(F-4)}$$

Reduced it to

$$\frac{(K^{H}L^{hW}-K^{hW}L^{H})^{2}}{2K^{hW}L^{hW}} > 0 \qquad \text{(F-5)}$$

Similarly, we can obtain the gain from trade for country F.

Appendix G Simulations of Trade Patterns Numerically

Fisher (2011) suggested using the middle of intersection cone of two goods price cones as the price solution. We can use it as do numerical simulations of trade patterns. Giving the price by this way, using different combinations of assumed production matrix and factor endowments, it can show trade patterns. The advantage of this approach is that it is quite simple and easy to work with higher-demission matrix.

This paper suggests adopting the following share of GNP to do simulation,

$$s^{H} = \frac{1}{4}\left(\frac{K^{H}}{K^{fW}} + \frac{L^{H}}{L^{fW}} + \frac{K^{fH}}{K^{fW}} + \frac{L^{fH}}{L^{fW}}\right) \qquad \text{(G-1)}$$

The advantage of this approach is that it may be the theoretical solution. When simplifying it to the matrices as the Trefler model, it is equation (2-15). And when simplifying it as the same technologies, it is equation (A-8).

Reference

Acemoglu D (2003) Patterns of Skill Premia, The Review of Economics Studies 70, 199-230.

Behrman J B and Szekely M (2003) Economic Policy and Wage Differentials in Latin America, Center for Global Development Working Paper 29.

Bertrand J T (1972*)* An Extension of the N- Factor Case of Factor Proportions Theory*,* Kyklos*,* July 1972*,* 25*,* 592-96. https://doi.org/10.1111/j.1467-6435.1968.tb00141.x

Baldwin R E. (1979) Determinants of the Commodity Structure of U.S. Trade. The American Economic Review. 61 (1): 126–146. https://doi.org/10.2307/1924829

Bhagwati J & Panagariya A & Srinivasan T N (1998) Lectures on International Trade, 2nd Edition, MIT Press Books, The MIT Press.

[34] see Guo (2019)

Chipman JS (1969) Factor price equalization and the Stolper–Samuelson theorem. International Economic Review, 10(3), 399−406, https://doi.org/10.2307/2525651

Davis, DR. (1995) Intra-Industry Trade: A Heckscher-Ohlin-Ricardo Approach, Journal of International Economics, 39 (3-4) 201-226. https://doi.org/10.1016/0022-1996(95)01383-3

Davis DR & Weinstein DE (2001) An account of global factor trade, American Economic Review, 91(5), 1423−1453. https://DOI: 10.1257/aer.91.5.1423

Deardorff AV (1982) General Validity of the Heckscher-Ohlin Theorem, The American Economic Review, Vol. 72, No. 4 (Sep. 1982), 683-694. https://doi.org/10.1142/9789814340373_0011

Deardorff AV (1986) FIRLESS FIRWOES: How Preferences can Interfere with the Theorems of International Trade, Journal of International Economics, 20(1-2), 131-142. https://doi.org/10.1016/0022-1996(86)90065-6

Deardorff AV (1994) The possibility of factor price equalization, revisited. Journal of International Economics, 36(1-2), 167–175. https://doi.org/10.1016/0022-1996(94)90063-9

Dixit, A.K. and Norman V. (1980), *Theory of International Trade,* James Nisbet, Welwyn, and Cambridge University Press.

Feenstra CR (2004) Advanced International Trade Theory and Evidence, Princeton University Press.

Feenstra CR, Taylor AM (2012) International Economics, Worth, 2nd edition, 2012.

Fisher E O'N, Marshall KG (2008) Factor content of trade when countries have different technology, http://www.calpoly.edu.efisher/FCT.pdf.

Fisher E. O'N (2011) Heckscher–Ohlin theory when countries have different technologies. International Review of Economics & Finance, 20(2), 202–210. https://doi.org/10.1016/j.iref.2010.11.009

Fisher E O'N, Marshall, KG. (2011) The Structure of the American Economy. Review of International Economics, 19(1), 15–31. https://doi.org/10.1111/j.1467-9396.2010.00928.x

Gorg H, Strobl E (2002) Relative Wages, Openness and Skill-Biased Technological Change, The Institute for the Study of Labor (IZA), Discussion Paper No. 596.

Guo B (2005) Endogenous Factor-Commodity Price Structure by Factor Endowments, International Advances in Economic Research, November 2005, Volume 11, Issue 4, p 484

Guo B (2015) General Trade Equilibrium of Integrated World Economy, MPRA Paper No. 111118, https://mpra.ub.uni-muenchen.de/111118/ . Manuscript Unpublished.

Guo B (2019) Trade Effects Based on Trade Equilibrium, Theoretical and Applied Economics, Asociatia Generala a Economistilor din Romania - AGER, vol. 0(1(618), S), pages 159-168.

Heller PS (1976) Factor Endowment Change and Comparative Advantage: The Case of Japan, 1956-1969. The Review of Economics and Statistics, 58(3), 283. https://doi.org/10.2307/1924950

Helpman E (1984) The Factor Content of Foreign Trade. The Economic Journal, 94(373), 84. https://doi.org/10.2307/2232217

Helpman E, Krugman PR (1985) *Market Structure and Foreign Trade,* Cambridge, MIT Press.

Hoekman B, Winters LA (2005) "Trade and Employment: Stylized Facts and Research Findings", World Bank Policy Research Working Paper No. 3676 (August 2005).

Jones RW (1956) Factor Proportions and the Heckscher-Ohlin Theorem. The Review of Economic Studies, 24(1), 1. https://doi.org/10.2307/2296232

Jones RW (1965) The Structure of Simple General Equilibrium Models. Journal of Political Economy, 73(6), 557–572. https://doi.org/10.1086/259084

Keesing DB (1967) Labor Skill and Comparative Advantages, American Economic Review, February 1967, 75,38-49.

Kozo K, Yoshinori K (2017) Factor Intensity Reversals Redux, RIETI Discussion Paper Series 17-E-021, available at https://www.rieti.go.jp/jp/publications/dp/17e021.pdf,
Manuscript Unpublished.

Krugman PR (2000) Technology, trade and factor prices. Journal of International Economics, 50(1), 51–71. https://doi.org/10.1016/s0022-1996(99)00016-1

Kurokawa Y (2011) Is a skill intensity reversal a mere theoretical curiosum? Evidence from the US and Mexico. Economics Letters, 112(2), 151–154. https://doi.org/10.1016/j.econlet.2011.04.005

Kwok YK, Yu, ESH (2005) Leontief Paradox and Role of Factor Intensity Measurement, available at https://www.iioa.org/conferences/15th/pdf/kwok_yu.pdf

Lerner AP (1952) Factor Prices and International Trade. Economica, 19(73), 1. https://doi.org/10.2307/2549912

Leamer EE (1980) The Leontief Paradox, Reconsidered. Journal of Political Economy, 88(3), 495–503. https://doi.org/10.1086/260882

Leamer EE (1984) Source of International Comparative Advantage, The MIT Press, Cambridge, Massachusetts, London, England.

Leamer EE, Levinsohn J (1995) International trade theory: evidence, in Gene M Grossman and Kenneth S. Rogoff(eds), Handbook of International Economics, vol. III. New York: Elsevier Science.

Leontief W (1953) Domestic Production and Foreign Trade; The American Capital Position Re-Examined. Proceedings of the American Philosophical Society. 97 (4): 332–349. JSTOR 3149288.

McKenzie LW (1955) Equality of Factor Prices in World Trade, Econometrica, 23(3), 239. https://doi.org/10.2307/1910382

McKenzie LW (1987) General Equilibrium, The New Palgrave, A dictionary of economics, 1987, v. 2, pp 498-512.

Minhas BS (1962) The Homohypallagic Production Function, Factor-Intensity Reversals, and the Heckscher-Ohlin Theorem. Journal of Political Economy, 70(2), 138–156. https://doi.org/10.1086/258610

Reshef A (2007) Heckscher-Ohlin and the Global Increase of Skill Premia: Factor Intensity Reversals to the Rescue. Working Paper, New York University.

Rassekh F, Thompson H (1993) Factor Price Equalization: Theory and Evidence. Journal of Economic Integration, 8(1), 1–32. https://doi.org/10.11130/jei.1993.8.1.1

Roskamp KW (1963) Factor Proportion and Foreign Trade: The case of West Germany, Weltwirtschaftliches Archiv, 1963, 319-326.

Sampson T (2016) Assignment Reversals: Trade, Skill Allocation and Wage Inequality. Journal of Economic Theory, 163: 365-409.

Samuelson PA (1938) Welfare Economics and International Trade. American Economic Review, 28, 261-66. Reprinted in ed. JE Stiglitz (1966), 775-780.

Samuelson PA (1948) International Trade and the Equalization of Factor Prices. The Economic Journal, 58(230), 163. https://doi.org/10.2307/2225933

Samuelson PA (1949) International factor-price equalization once again. Economic Journal, 59, 181–97. Reprinted in ed. JE Stiglitz (1966), 869-885.

Samuelson PA (1953) Prices of Factors and Good in General Equilibrium. The Review of Economic Studies, 21(1), 1. https://doi.org/10.2307/2296256

Samuelson PA (1971) Ohlin was right. Swedish Journal of Economics, 73, 365–84.

Samuelson PA (1992) Factor-Price Equalization by Trade in Joint and Non-Joint Production. Review of International Economics, 1(1), 1–9. https://doi.org/10.1111/j.1467-9396.1992.tb00002.x

Suranovic SM (2010) International Trade Theory and Policy, online text book, https://internationalecon.com/Trade/Tch115/T115-2.php

Takahashi H (2004) The capital-intensity reversal in the postwar Japanese economy: Why did Japan grow so fast during 1955-1975?, Online at http://mpra.ub.uni-muenchen.de/29876/ Manuscript Unpublished.

Trefler D (1993) International Factor Price Differences: Leontief was Right! Journal of Political Economy, 101(6), 961–987. https://doi.org/10.1086/261911

Vanek J (1968) THE FACTOR PROPORTIONS THEORY: THE N? FACTOR CASE. Kyklos, 21(4), 749–756. https://doi.org/10.1111/j.1467-6435.1968.tb00141.x

Woodland A (2013) General Equilibrium Trade Theory, Chapter 3, in Bernhofen D, Falvey R, Greenaway D, and Kreickemeier U (Eds.), *Palgrave Handbook of International Trade,* Palgrave Macmillan.

Wong K-Y (1995) International Trade in Goods and Factor Mobility, The MIT Press, London, England.

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