The heterogeneity of Okun’s law: A metaregression analysis

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The heterogeneity of Okun’s law: A metaregression analysis

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Abstract

Okun’s law is an extremely influential parameter in empirical research and policy analysis, based on the sizable number of estimates from this perspective. Nevertheless, it is also subject to considerable heterogeneity. We first show graphical and statistical evidence on the existence of a high level of heterogeneity among Okun’s law estimates in existing research, then analyze potential sources of heterogeneity. Using 1,213 estimates of Okun’s law for various countries, regions, and time periods, separate metaregressions are estimated; one using estimates with the unemployment rate as the dependent variable, and the other with output as the dependent variable. Our findings indicate that the specification of the underlying model of the relationship has an effect on the magnitude of Okun’s parameter. Differential labor market characteristics may also explain part of the observed heterogeneity. Finally, the results are also found to be influenced by methodological issues, such as the type of data (time series or panel data), the frequency of the data (annual or quarterly), the spatial coverage of the estimates (country, region, or group of countries), whether more variables are included in estimations, and whether a dynamic or static, symmetric or asymmetric model is estimated. This paper contributes to highlight the heterogeneity affecting the estimates of Okun’s law and that needs to be taken into account. In order to know the "true" relationship between unemployment and economic growth, researchers should bear in mind that there are a number of methodological choices that have consequences for the results.

Keywords: Okun’s Law; heterogeneity; metaregression

JEL Codes: C55, E23, E24, J60

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1. Introduction

When we attempt to answer the question of how much economic activity must grow to reduce unemployment, there does not appear to be a single definitive answer, as it depends on several variable and research design factors. The specific characteristics of each economy, such as the production structure, labor legislation, and other aspects of the labor market (informality, self-employment, social security coverage, and other relevant considerations) are some of the factors that affect the unemployment–output relationship. Are the observed estimates of this relationship affected by studies’ design? Could the choice of model or other methodological decisions explain some of the observed differences?

The unemployment–output relationship, known as Okun’s law, has been estimated for several countries or regions in different time periods, establishing a long list of published studies regarding the law. Our literature review reveals a high degree of heterogeneity in the parameter that measures this relationship (Okun’s coefficient). For example, the Okun’s coefficients estimated by Perman and Tavera (2005) for several European countries are between −0.8 and −0.05 (Spain and Luxembourg, respectively) for the period 1970–2002, which implies that the unemployment rate in Spain falls by 0.8 percentage points (pp) as output grows by 1%, while in Luxembourg the fall is only 0.05 pp. This indicates that unemployment in Spain is highly sensitive to output, whereas the response of unemployment in Luxembourg is very low.

Heterogeneity is also evident between estimates for the same country or region. For example, estimates for the United Kingdom vary between −0.68 and −0.05 (Perman and Tavera, 2005 and Palombi et al., 2015, respectively), which could be explained by the time period of estimations, if the relationship was unstable, or if there was a structural break, but it suggests that methodological approach used for estimation also matters.

From a policy perspective, it is imperative to know the “true” effect of the unemployment–output relationship for appropriate policy design and decision-making. However, with the considerable heterogeneity among estimates of Okun’s law, the existence of a single “true” value effect at all times and places is irrelevant. Instead, it is more appropriate to explore this heterogeneity. Thus, we assert that an analysis of the different dimensions
of heterogeneity of the Okun’s law, providing empirical evidence regarding the variables that may be influencing the results, will offer a significant contribution to the literature.

To do so, we explore whether the observed heterogeneity of Okun’s law is related to different sources, exploring 1) the underlying theoretical model of the relationship, 2) the features of each labor market that make the relationship between unemployment and output more or less sensitive, and 3) the methodological approach used to estimate the law.

Our investigation of the heterogeneity of Okun’s law is conducted through a meta-analysis, including the estimation of metaregressions incorporating 1,213 estimates of Okun’s coefficients collected from previously published studies. After presenting evidence of the considerable heterogeneity between estimates of the law and revealing the lack of a common, representative coefficient, we introduce the results of metaregressions, assuming that the studies’ effect sizes differ and that the collected studies represent a random sample of a larger number of studies.

We confirm that labor market features can explain part of the heterogeneity, but the specification of the underlying model of the relationship also has an effect on the magnitude of Okun’s parameter. The results are also influenced by methodological decisions, such as the type of data used (time series or panel data), the frequency of the data (annual or quarterly), geographic coverage (country, region, or group of countries), the number of variables included, and the use of dynamic or static, symmetric or asymmetric model estimation approaches.

The remainder of this paper is structured into six sections. Section 2 provides a brief overview of Okun’s law. Section 3 presents the different sources of heterogeneity. Section 4 details methodological approach, including a description of the criteria adopted to create the metadataset, and the metaregression techniques applied, followed by a description of the variables used in our regressions and descriptive statistics. Section 5 presents the results and section 6 concludes.
2. Okun’s law

The unemployment–output relationship has been an aspect of the economic research agenda since Okun (1962) applied the initial estimation to the United States to examine how much output the economy could produce under conditions of full employment. Full employment is a key goal of economic policy, and from a Keynesian economic perspective, Okun considered it relevant to ascertain how far the real economy was from achieving it, to aid the formulation of appropriate fiscal and monetary policies to stimulate aggregate demand, and consequently, employment.

Okun’s research presented an empirical analysis of quarterly US data for the period 1947:2–1960:4, demonstrating an inverse and statistically significant relationship between unemployment and output in the US, and concluding that for every percentage point of output growth above normal or potential growth, the unemployment rate of the US would fall by about 0.3 pp.

Knowledge regarding the validity and the magnitude of Okun’s law is essential for economic policy development, as these insights uncover details on the responsiveness of unemployment to economic growth, or the cost of maintaining idle labor resources. The usefulness of this parameter is reflected by the enormous number of studies devoted to its estimation. The economic literature on this subject has grown over time, verifying its validity for other countries and time periods, applying one or several of the original models, analyzing the relationship of output to unemployment or of unemployment to output, incorporating adjustments to the original versions or attempting to explain the differences.

3. Source of heterogeneity

The existing research findings have a high degree of heterogeneity, and there is considerable variance among studies that cannot be attributed to measured sampling error alone (Higgins and Thompson, 2002). We identify three likely sources of heterogeneity in applications of Okun’s law. 1) The underlying theoretical model of the relationship, 2) the features of each labor market that make the relationship between unemployment and output more or less sensitive, and 3) methodological diversity.
3.1. Theoretical model of the relationship

Okun (1962) used three different models to estimate the relationship between unemployment and economic growth, finding a strong statistical relationship between the two variables. While the researcher estimated the relationship from models using unemployment as the dependent variable, he also analyzed the relationship in the opposite direction. This led to Okun’s relationship being estimated, in some cases, with the unemployment rate, and in others, with the output as the dependent variable in subsequent studies. Consequently, two critical questions emerge. 1) Does the relationship go from output to unemployment or from unemployment to output, and are the results from the two models comparable? 2) Is the estimated coefficient sensitive to the model specification?

Regarding the first question, on econometric grounds, Barreto and Howland (1993) criticize the use of the inverse value of the estimated coefficient to indicate effects in the opposite direction. They argue that the coefficient has only one reading corresponding to the estimated model, and independent of the “true” causal relationship, the researcher must choose between models, depending on the variable to be predicted from the past values of both variables. Given that these two measures of Okun’s law are not comparable, heterogeneity analyses of the law must be conducted separately; one for the results of models with unemployment as the dependent variable (U_model), and another for those with output as dependent variable (Y_model).

Regarding the second question, Okun estimated the relationship using three different models:

**First-difference model (OKUN_I)**

This model considers the relationship between the change in the unemployment rate and the change in output:

\[ u_t - u_{t-1} = \beta_0 + \beta_1 g_{yt} \]  \hspace{1cm} (1)

where \( u_t \) is unemployment rate in time t, and \( g_{yt} \) is real GDP growth in time t. The \( \beta_1 \) coefficient is the parameter of interest showing the fall in unemployment when GDP grows 1% above its “normal” growth rate.
The heterogeneity of Okun’s law: A metaregression analysis

Gap-model (OKUN_.II)
The original version of the gap model is as follows:

\[ u_t = \theta_0 + \theta_1 \left( \frac{Y_t^p - Y_t}{Y_t^p} \right) \]  

where \( Y_t \) and \( Y_t^p \) are the current and potential real GDP, respectively, and \( \theta_0 \) is the natural rate of unemployment. \( Y_t^p \) is unobservable, and for the US, Okun assumed it was the level of GDP that matches with a natural 4% rate of unemployment. As this rate is also unobservable, later research applying this specification of the law used the following equation:

\[ (u_t - u_t^*) = \gamma_0 + \gamma_1 (Y_t - y_t^*) \]  

where the variable \( y_t \) represents the logarithm of real GDP and the asterisk indicates the potential level of GDP, while \( u_t^* \) is the natural unemployment rate resulting from frictional and structural unemployment. As the asterisked variables are unobservable, both variables are estimated using different methods to decompose the series into trend and cycle components. \( \gamma_1 \) is the coefficient of interest that indicates how far the unemployment rate deviates from its natural level when output deviates from its potential level by 1%.

Fitted trend and elasticity model (OKUN_.III)
The first-difference model and the gap-model have been the most commonly used methods of researchers studying Okun’s law. This is unquestionably related to the evolution of the field of econometrics since Okun’s original work. Based on current knowledge, it is problematic to estimate model (4) without proper variable cointegration analysis, or to include a trend variable in the model that could be absorbing much of the variability. However, there are also a few estimates of the law using fitted trend and elasticity models:

\[ \ln E_t = \delta_0 + \delta_1 \ln Y_t - \delta_2 t \]  

where \( E_t \) is the employment rate (employed/labor force) \( (E_t=1-u_t) \). \( \delta_1 \) corresponds to employment–output elasticity, and \( \delta_2 \) is the trend coefficient that arises from multiplying the growth rate of potential GDP with the employment–output elasticity.

Belmonte and Polo (2004) demonstrated that models (1), (2), and (4) proposed by Okun are similar under certain assumptions; therefore, it is not
surprising the Okun's estimates made yielded similar results ($\beta_1 = -0.3; \theta_1 = 0.36$, and $\delta_1$ ranging from 0.35 to 0.4). Some of these assumptions are that the natural rate of unemployment, potential GDP, and potential employment are constant. Indeed, Okun's gap-model assumed that the natural rate of unemployment for the US was 4% in that period, and the parameter of interest was estimated under that assumption.

The existence of a unique and invariant natural unemployment rate has been questioned in the economic literature. In addition, the natural unemployment rate is unobservable and difficult to estimate; thus, studies estimating this version of the law use filters to decompose time series into trends and cycles. Various filters are used, including Hodrick and Prescott (HP-filter, Hodrick and Prescott, 1997)), Bakter and King (Bandpass-BP-filters, Bakter and King, 1999), Beveridge and Nelson (BN-filter, Beveridge and Nelson, 1981), linear trend (LTREND), and quadratic trend (QTREND), or from modeling such as the Kalman-filter (Kalman, 1960) or Harvey’s method (Harvey, 1985; 1989). The question is, are the estimation results sensitive to the model or filter used? Studies that present estimates using more than one model or more than one filter remain inconclusive, and in cases wherein differences are evident, the sign of the bias is unclear.

### 3.2. Features of individual labor markets

As noted previously, an increasing amount of research has found that some characteristics that differentiate labor markets explain part of the heterogeneity of Okun's law.

Some authors find employment protection legislation (EPL) to prevent the rapid adjustment of employment to changes in GDP, as it generates hiring and/or firing costs for firms, with effects on the unemployment-output relationship (Balakrishnan, et al., 2010; Blanchard, 1997; Sögner and Stiassny, 2002). With high costs, firms choose not to lay off staff in recessions, resulting in so-called labor hoarding, and the unemployment rate reacting weakly to changes in GDP. Given that the EPL differs across countries, this could be expected to explain at least part of the differences between researchers' Okun coefficients. Despite this logical assumption, other authors find that the variable used to measure the degree
of EPL fails to explain the estimated differences in Okun’s law across countries (Ball et al., 2019; Porras-Arena and Martín-Román, 2021).

Other features of labor markets include variables that researchers have found to be explanatory factors for differences in Okun’s coefficients between countries or regions. Such variables include labor productivity, productive specialization of the economy (Villaverde and Maza, 2009; Herwartz and Niebuhr, 2011), and labor market characteristics that affect the quality of employment, such as work in the informal sector, the proportion of self-employment when it functions as “refuge employment,” and occupations without social security (Porras-Arena and Martín-Román, 2019; 2021).

3.3. Methodological issues

In addition to determining the direction of the relationship between unemployment and output and the Okun’s model to estimate, researchers must decide on other methodological issues that may also be sources of heterogeneity among the results. For example, are there omitted variables in the relationship? Prachowny (1993) argued that the estimates made by Okun (1962) and later by Gordon (1984) produce higher values than the “real” outcomes due to the omission of relevant variables, estimating a model that also included other variables, such as installed capacity, labor supply, and hours worked, and obtaining a significantly lower coefficient of the relationship (in absolute value) than that of Okun and Gordon. Based on this finding, other authors have also included these or other variables in the model (Freeman, 2001; Katos et al., 2004; Folawewo and Adeboje, 2017; Liu et al., 2018), and it is to be expected that the inclusion of additional variables in the estimated relationship would reduce the absolute value of Okun’s coefficient, explaining part of the observed heterogeneity.

Researchers must also choose the type of data to use, time series or panel data? The literature review reveals that most studies use time series, but there are also several studies that use panel data. Estimations with panel data always include more observations, which affects the precision of the estimated parameters. In contrast, the econometric methodology for approaching such estimations differs according to the type of data used, which can also be a source of heterogeneity. In this case, there is no a priori
idea of the sign of the effect of using one type of data or another on heterogeneity.

Is the relationship linear? Some authors estimate a nonlinear relationship between the variables, showing differential effects depending on the business cycle phase, i.e., the effect of output on unemployment would differ in recessions than in expansions (Cevik et al., 2013; Palombi et al., 2015; Valadkhani, 2015; among others). Such studies have not developed theoretical arguments to support the possible asymmetric relationship; thus, there is no specific expected result. Instead, they have focused on testing the nonlinearity of the relationship by highlighting the error of not including it in estimations (Liquitaya and Lizarazu, 2003; Harris and Silverstone, 2001; Virén, 2001; Pérez et al., 2003; Marinkov and Geldenhuys, 2007).

Is the relationship static or dynamic? Okun’s original formulations assume a static relationship between unemployment and output, but several authors have argued that this is too restrictive and does not allow for the capture of possible correlations with past values (Knotek, 2007). In this sense, various studies present dynamic estimates of the law, arguing that the inclusion of variable lags also solves problems of serial correlation in the error terms (Mossa, 1997; Canarella, and Miller, 2017; among others). In these cases, the effect of GDP on unemployment (or of unemployment on GDP) is not measured by only the coefficient of the current explanatory variable, but also by the total effect, which also considers the effects of lagged variables.

Does the periodicity of the data used for the estimates have an effect on the results? Does it make a difference whether annual, semi-annual, or quarterly data are used? The Okun’s coefficient of a model with annual data is, in general, larger than the coefficient of the current relationship between the variables of a model with quarterly data. The time it takes for variables to adjust to shocks is one of the factors behind this phenomenon. This is also related to the above, as, in many cases, dynamic models are also estimated using quarterly data (Ball et al., 2017). In these cases, only the total effect, which considers the effects of lagged variables, will be comparable with the coefficient estimated with annual data.

Econometrics has also made considerable advances since the time of Okun’s (1962) estimations, which is reflected in the heterogeneity of
econometric approaches used in subsequent Okun’s law estimations (ordinary least squares [OLS], generalized least squares [GLS], seemingly unrelated regressions [SUR], fully modified ordinary least squares [FMOLS], dynamic ordinary least square [DOLS], maximum likelihood [ML]).

Is the relationship stable over time? Some empirical evidence suggests that Okun’s law is unstable over time, and that in many cases, the relationship is stronger in more recent periods (Moosa, 1997; Sögner and Stiaassny, 2002; Perman and Tavera, 2005; Knotek, 2007; Balakrishnan et al., 2010; Porras-Arena and Martín-Román, 2019). Consequently, some of the observed heterogeneity between Okun’s coefficients may be due to estimates’ corresponding to different time periods.

4. Meta-analysis

Per Glass (1976), meta-analysis refers to an analysis of analyses; a statistical analysis of the results of individual studies that address the same question for the purpose of comparing the results to elicit one unified conclusion to that question. Nevertheless, as noted by Deeks et al. (2021), if there is considerable variation between the studies included, it may be misleading to quote an average value for the effect, and the conclusions will be less clear. Instead, a meta-analysis is more appropriate for exploring the factors behind the variability.

As a background to our study, Perman et al. (2015) conducted a meta-analysis of Okun’s law. The aim of the research was to determine whether an evident common representative empirical coefficient of Okun’s law emerged from previous work. They used 269 observed estimates of the law to measure the size of the “true” effect, applying a specific methodological meta-analysis approach. In a second stage, Perman et al. (2015) also estimated a multivariate metaregression, but with the objective of eliminating factors that might be affecting the estimate of the common effect. Our meta-analysis endeavors to explore the factors that may explain the heterogeneity, as with the work of Lichter et al. (2015) and Aiello and Bonanno (2019) regarding other economic problems. We present previous empirical evidence of a high degree of heterogeneity among the estimates that render the estimation of a common effect irrelevant.
We follow most of the meta-analysis guidelines proposed by Stanley et al. (2013) and Havránek et al. (2020).

4.1. Data

A critical first stage of the work is searching for, reading, and selecting the relevant literature that will be part of the meta-analysis, and constructing the database to be used in metaregression analyses after coding the information collected from the chosen articles.

To this end, the criteria used to select the data that will be included in the database must be defined in advance. In our case, we reference Perman et al. (2015), using the same criteria.

- Source: Empirical studies on Okun’s law published after 1980 in journals included in EconLit database of the American Economic Association.3
- Article selection criteria:
  a) The words “Okun’s law” must be present in the title or abstract.
  b) The selected articles must clearly specify at least one estimate of Okun’s law and apply a measure of the precision of the estimate (standard deviation or t-statistic).
  c) Articles should also clearly detail the methodology used for estimation.

Applying the aforementioned selection criteria, a total of 163 articles were identified, and we selected 64 studies (see Appendix 1). Articles were excluded due to several factors, including the aim of the study not referencing the law and not presenting related estimations of the coefficient of interest to our research, or, although focusing on the law, they were theoretical works, did not clearly present the results obtained in a way that was comparable with the others, did not present standard deviation or t-statistics as a measure of the precision of the estimate(s), or did not clearly present the methodology used.

Using the selected studies, we constructed a database with 1,213 estimates of the Okun’s law, corresponding 683 observations to estimated

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3 As Perman et al. (2015) have pointed out, econometric methods have evolved, especially since the 1980s, and therefore they consider it reasonable to select papers published from that year onwards, to make them comparable.
models of unemployment rate as the dependent variable and 530 observations of output as dependent variable.

4.2. Metaregressions

When heterogeneity is high among collected data, no single “true” effect size can be determined, and each observation may vary widely from the mean of the effects, which is the result of conventional simple meta-analysis estimates, rendering the estimated mean irrelevant. When this occurs, metaregression offers an alternative to simple meta-analysis that aims to relate effect size to one or more characteristics of the studies involved (Thompson and Higgins, 2002). Metaregression is a linear regression of study effect sizes on study-level variables (moderators) to analyze whether heterogeneity between studies can be explained by one or more moderators.

Some relevant differences between metaregression and simple regression include: 1) larger studies having more influence than smaller studies, since in a metaregression each study is weighted by its respective precision measure, and 2) metaregression’s consideration of residual heterogeneity among the observed effects that have not been modeled by the explanatory variables.

Metaregression models include fixed-effects (FE) and random-effects (RE) models. These models differ, in that the FE model assumes that all variability between studies can be explained through the moderators included, while the RE model accounts for residual heterogeneity that is unexplained by the moderators. The FE model only considers within-study variation, and the null hypothesis states that the common true effect is not associated with the variable of interest, whereas the null hypothesis of the RE model, considering both within-study and between-study variance, states that the mean of the true effect is not associated with that variable (Spineli and Pandis, 2020). The choice of model depends on assumptions regarding the characteristics of the studies included in the meta-analysis. A fixed effects metaregression will be adequate if there are sufficient grounds to claim that the “true” effects estimated by the studies are identical; however, this assumption is highly unlikely to be met. In addition to methodological considerations that may have affected the results of estimates, as noted, characteristics of the phenomena under study are often
additional factors of heterogeneity; therefore, it is most appropriate to estimate a metaregression using a random effects model (Berkey et al. 1995).

We have \( k \) studies with underlying true effects \( \theta_j \), and a between-study heterogeneity \( \tau^2 \) \((\text{var}(\theta_j) = \tau^2)\). From each study, we have \( \hat{\theta}_j \) (estimate of \( \theta_j \)) so that \( \text{E}(\hat{\theta}_j|\theta_j) = \theta_j \) and \( \text{var}(\hat{\theta}_j|\theta_j) = \sigma_j^2 \). The RE model for estimation is as follows:

\[
\hat{\theta}_j = x_j \beta + \epsilon_j = x_j \beta + u_j + \epsilon_j
\]

(5)

weighted by \( w_j^* = \frac{1}{\sigma_j^2 + \tau^2} \), where \( \epsilon_j^* \sim N(0, \sigma_j^2 + \tau^2) \).

\( \hat{\theta}_j \) is the effect size observed, \( \sigma_j^2 \) its variance, \( x_j \) a \((1 \times p)\) vector of moderators, and \( \beta \) a \( p \times 1 \) vector of unknown coefficients. The error term \( \epsilon_j^* \) includes a random effects term, \( u_j \sim N(0, \tau^2) \), to account for the remainder of heterogeneity not explained by moderators.

There are several methods to estimate \( \tau^2 \), and we present results using two of them, restricted maximum likelihood (REML) and DerSimonian and Laird (DL). The REML method produces an unbiased, nonnegative estimate of \( \tau^2 \) and is the default estimation method in the Stata software because it performs well in most scenarios. The DL is a popular estimation method because it does not make any assumptions regarding the distribution of random effects and does not require iteration (Stata Meta-analysis Reference Manual -Release 17).

The study-level variables or moderators included in vector \( x_j \) capture the sources of heterogeneity, such as the model used, type of data (time series or panel), level of data (country or region), frequency of data (quarterly or annual), and other relevant considerations, including country dummy variables capturing each labor market’s differential features and time variables. The final variables included are the mean year of estimation period of each study, to identify potential shifts in the Okun’s relationship over recent decades, and a dummy variable of studies’ year of publication, to capture methodological advances.

\(^4\) The FE model assumes \( \tau^2 = 0 \).
The robustness testing of the results is performed by estimating the model by weighted least squares (WLS), using the inverse of the variance of the observations as weights (Stanley and Doucouliagos 2015).

We used the Stata software for estimations, as it offers a suite of commands to perform meta-analyses. Even if a primary regression has the same basic approaches as metaregression, the computations have notable differences. It is essential to use a specifically designed software to perform a metaregression (Borenstein *et al.* 2017).

### 4.3. Descriptive statistics

Figure 1 presents the distribution of Okun’s coefficients in our database. The graph on the left shows the distribution of coefficients estimated with unemployment rate as the dependent variable. The simple mean is $-0.30$, with a standard deviation of 0.22 (Table 1). Notably, the vast majority of estimates lie within zero and $-0.5$ (87.6%), which is unsteady, as there are many observations that exceed the absolute value of 0.5. The right figure presents the coefficients estimated with output as the dependent variable. In this case, the data is more dispersed; the mean is $-1.76$, the standard deviation is 2.29, and the majority of the observation (73.4%) lies within zero and $-2$.

Table 1 describes all the variables used in the metaregressions, indicating the relative weight of each in the database constructed. Most of the coefficients in both databases were estimated using the OLS method. The fitted trend and elasticity model (OKUN_III) is rarely used to estimate Okun’s law. For the rest of the variables, most of the estimates in both databases use static, symmetric, time series, bivariate, annual, and country-level models. Among those estimating the gap-model (OKUN_II), most use the HP-filter to decompose the series into trend and cycle components.

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5 Lichter *et al.* (2015) did the reverse, estimating by weighted least squares and then performing robustness analysis with a metaregression with random effects.
Table 1. Description of variables and descriptive statistics

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description of the variable</th>
<th>U_model (1)</th>
<th>Y_model (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>Dummy, 1 if the study uses OLS, 0 otherwise.</td>
<td>60.9</td>
<td>85.8</td>
</tr>
<tr>
<td>OTHER_OLS</td>
<td>Dummy, 1 if the study uses other than OLS, 0 otherwise.</td>
<td>31.1</td>
<td>14.2</td>
</tr>
<tr>
<td>OKUN_I</td>
<td>Dummy, 1 if the study uses first difference-model, 0 otherwise.</td>
<td>63.4</td>
<td>20.0</td>
</tr>
<tr>
<td>OKUN_II</td>
<td>Dummy, 1 if the study uses gap-model, 0 otherwise.</td>
<td>36.2</td>
<td>80.0</td>
</tr>
<tr>
<td>OKUN_III</td>
<td>Dummy, 1 if the study uses fitted trend and elasticity model, 0 otherwise.</td>
<td>0.4</td>
<td>0.0</td>
</tr>
<tr>
<td>STAT_MOD</td>
<td>Dummy, 1 if the model is static, 0 otherwise.</td>
<td>67.7</td>
<td>91.1</td>
</tr>
<tr>
<td>DYN_MOD</td>
<td>Dummy, 1 if the model is dynamic, 0 otherwise.</td>
<td>30.0</td>
<td>4.0</td>
</tr>
<tr>
<td>COINT_MOD</td>
<td>Dummy, 1 if the study uses cointegration model, 0 otherwise.</td>
<td>2.3</td>
<td>4.9</td>
</tr>
<tr>
<td>SYM_MOD</td>
<td>Dummy, 1 if the model is symmetric, 0 otherwise.</td>
<td>88.0</td>
<td>74.3</td>
</tr>
</tbody>
</table>

Figure 1. Distribution of Okun’s coefficient.
Notes: The figure on the left corresponds to the Okun’s coefficients estimated with models using unemployment rate (U_dep) as the dependent variable and the one on the right with models using output (Y_dep) as dependent variable. For illustrative purposes, estimates exceeding the absolute value of 10 were excluded from the plot on the right.
<table>
<thead>
<tr>
<th>Dummy Variable</th>
<th>Description</th>
<th>Mean (1)</th>
<th>SE (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ASYM_MOD (3)</strong></td>
<td>Dummy, 1 if the model is asymmetric, 0 otherwise.</td>
<td>12.0</td>
<td>25.7</td>
</tr>
<tr>
<td><strong>TIME_SERIES</strong></td>
<td>Dummy, 1 if the study uses time series data, 0 otherwise.</td>
<td>81.7</td>
<td>83.0</td>
</tr>
<tr>
<td><strong>PANEL_DATA</strong></td>
<td>Dummy, 1 if the study uses panel data, 0 otherwise.</td>
<td>18.3</td>
<td>17.0</td>
</tr>
<tr>
<td><strong>TWO_VAR</strong></td>
<td>Dummy, 1 if the study uses only two variables, unemployment and output, 0 otherwise.</td>
<td>78.5</td>
<td>89.8</td>
</tr>
<tr>
<td><strong>MORE_VAR</strong></td>
<td>Dummy, 1 if the study uses more than two variables, 0 otherwise.</td>
<td>21.5</td>
<td>10.2</td>
</tr>
<tr>
<td><strong>YEAR</strong></td>
<td>Dummy, 1 if the study uses annual data, 0 otherwise.</td>
<td>53.0</td>
<td>97.5</td>
</tr>
<tr>
<td><strong>NO_YEAR</strong></td>
<td>Dummy, 1 if the study uses quarterly or biannual data, 0 otherwise.</td>
<td>47.0</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>CTY_LEVEL</strong></td>
<td>Dummy, 1 if the study is at country level, 0 otherwise.</td>
<td>67.4</td>
<td>48.7</td>
</tr>
<tr>
<td><strong>REG_LEVEL</strong></td>
<td>Dummy, 1 if the study is at region level estimate, 0 otherwise.</td>
<td>9.4</td>
<td>45.5</td>
</tr>
<tr>
<td><strong>CTY_GR_LEVEL</strong></td>
<td>Dummy, 1 if the study is at country group level, 0 otherwise.</td>
<td>17.2</td>
<td>5.8</td>
</tr>
<tr>
<td><strong>OTHER_LEVEL</strong></td>
<td>Dummy, 1 if the study is at the population group level (e.g. age, sex), 0 otherwise.</td>
<td>6.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>FILT_HP</strong></td>
<td>Dummy, 1 if the gap-model uses HP filter, 0 otherwise.</td>
<td>75.3</td>
<td>40.3</td>
</tr>
<tr>
<td><strong>FILT_BN</strong></td>
<td>Dummy, 1 if the gap-model uses Beveridge Nelson filter, 0 otherwise.</td>
<td>2.4</td>
<td>11.3</td>
</tr>
<tr>
<td><strong>FILT_BP</strong></td>
<td>Dummy, 1 if the gap-model uses Band-Pass filter, 0 otherwise.</td>
<td>4.1</td>
<td>14.9</td>
</tr>
<tr>
<td><strong>FILT_LT</strong></td>
<td>Dummy, 1 if the gap-model uses Linear Trend filter, 0 otherwise.</td>
<td>1.2</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>FILT_Q</strong></td>
<td>Dummy, 1 if the gap-model uses HP filter, 0 otherwise.</td>
<td>1.2</td>
<td>10.2</td>
</tr>
<tr>
<td><strong>FILT_OTHER</strong></td>
<td>Dummy, 1 if the gap-model uses other type of filter or model, 0 otherwise.</td>
<td>15.8</td>
<td>23.3</td>
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</table>

<table>
<thead>
<tr>
<th>Description</th>
<th>Mean (2)</th>
<th>SE (2)</th>
</tr>
</thead>
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<tr>
<td><strong>OKUN</strong></td>
<td>Observed Okun’s coefficients</td>
<td>-0.3</td>
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<tr>
<td><strong>M_YEAR_OBS</strong></td>
<td>Mean year of estimation period</td>
<td>1995</td>
</tr>
<tr>
<td><strong>M_YEAR_PUB</strong></td>
<td>Mean year of publication</td>
<td>2013</td>
</tr>
</tbody>
</table>

Number of observations: 683

(1) Database with Okun’s coefficient estimated using unemployment as dependent variable.
(2) Database with Okun’s coefficient estimated using GDP as dependent variable.
(3) In meta-regression we distinguish between coefficient estimates for recessionary periods from estimates for expansionary periods (ASYM_MOD_REC and ASYM_MOD_EXP).
5. Results

In this section we first present empirical evidence of the high heterogeneity in the data and then detail our estimation results.

5.1. Assessing heterogeneity

We first demonstrate evidence of high heterogeneity among Okun’s estimates graphically, followed by some statistics that further confirm this extreme heterogeneity.

![Figure 2. Okun’s coefficients by country.](image)

Notes: The figure on the left corresponds to the Okun’s coefficients estimated with models using unemployment rate (U_dep) as the dependent variable and the one on the right with models using output (Y_dep) as the dependent variable. For illustrative purposes, estimates exceeding the absolute value of 10 were excluded from the plot on the right. GC refers to group of countries.

Figure 2 presents the distribution of Okun’s coefficient by country or group of countries. The white dots indicate the mean value of the coefficient per country, revealing that the mean values differ significantly, with extreme heterogeneity of the Okun coefficients between countries. Indeed, in the U_dep database, the maximum mean value (in absolute value) is 0.81 (South Africa), and the minimum 0.006 (Belarus), and 10.15 (Japan) and 0.75 (Spain), respectively, in the Y_dep database. A high dispersion of coefficients within each country is also observed, particularly countries such as South Africa, the US, Spain, Denmark, and the Czech Republic, among...
The heterogeneity of Okun’s law: A metaregression analysis

others, in the U_dep database, and Japan, Austria, Switzerland, France, the US, and Greece in the Y_dep database.

The Galbraith plot is also used to detect heterogeneity among studies. On the y-axis are the standardized effect sizes, and on the x-axis, are the corresponding precision measures (inverse standard error). It offers an alternative to forest plots (the most used plot in meta-analyses) for summarizing results when there are many studies (Stata Meta-analysis Reference Manual -Release 17). Heterogeneity is assessed by observing the variation of the studies around the slope of the regression line that capture the overall effect size. For this purpose, the plot also draws a confidence interval (CI). High heterogeneity between studies will be evident if a sizable number of points occur outside the CI. We expect around 95% of the studies to lie within the shaded area (indicating 95% CI) in the absence of high heterogeneity. Studies with low precision are near the origin, and the precision of studies increases toward the right on the x-axis. In our case, there is a wide dispersion of points in both databases and a lot of them are outside the shaded area, indicating high heterogeneity between estimates of the Okun’s law (Figure 3).

Figure 3. Galbraith plots.
Notes: The figure on the left corresponds to the Okun’s coefficients estimated with models using unemployment rate (U_dep) as the dependent variable, and the one on the right models using output (Y_dep) as the dependent variable. For illustrative purposes, estimates with 1/se_j>200 in the U_dep database were excluded from the plot.
We also performed box plots to detect outliers (Figure 4). As demonstrated, the values of Okun’s coefficient lower than $-0.8$ are outliers in $U_{\text{dep}}$ and lower than $-4$ in $Y_{\text{dep}}$. Outliers often hinder and distort analyses, and therefore, we will present the results with and without outliers, to visualize whether their inclusion modifies the conclusions.

A commonly used statistical test to indicate the extent of heterogeneity is Cochran’s $\chi^2$ test or the Q-test (also known as a homogeneity test). The Q-test sums the squared deviations of each study’s estimate ($\hat{\theta}_j$) from the estimated overall effect ($\hat{\theta}$) (the weight of each study mirrors that of the meta-analysis). The statistic then compares with a $\chi^2$ distribution ($k-1$ degrees of freedom, where $k$ is the number of studies), obtaining a p-value.

The null hypothesis is, $H_0: \theta_1 = \theta_2 = \ldots = \theta_K = \theta$ and the Q-test statistic is calculated as follows:

$$Q = \sum_{j=1}^{K} w_j (\hat{\theta}_j - \hat{\theta})^2 = \sum_{j=1}^{K} w_j \hat{\theta}_j^2 - \frac{(\sum_{j=1}^{K} w_j \hat{\theta}_j)^2}{\sum_{j=1}^{K} w_j}$$

where $w_j = 1/\sigma_j^2$ and $\sigma_j^2$ is the variance of each study.

Nevertheless, this test does not provide relevant information regarding heterogeneity in all cases because it has poor power in a few cases.
The heterogeneity of Okun’s law: A metaregression analysis

studies circumstances, and excessive power to detect inconsequential heterogeneity when there are many studies (Higgins and Thompson, 2002; Higgins et al., 2003).

For this reason, Higgins and Thompson (2002) proposed two additional measures of heterogeneity, $H^2$ and $I^2$. For a random effect model the measures are:

\[ H^2 = \frac{\hat{\tau}^2 + s^2}{s^2} \]

\[ I^2 = \frac{\hat{\tau}^2}{\hat{\tau}^2 + s^2} \times 100 \]

where

\[ s^2 = \frac{\sum_{j=1}^{K} w_j - \sum_{j=1}^{K} w_j^2 / \sum_{j=1}^{K} w_j}{K-1} \]

is the within-study variance, and $\hat{\tau}^2$ is an estimator of the between-study variance. A value of $H^2$ close to unity indicates homogeneity between studies, meaning that $\hat{\tau}^2$ is practically equal to zero and all the variance corresponds to the within-study variance. $I^2$ indicates the proportion of variation between the studies due to heterogeneity relative to the pure sampling variation, indicating what proportion of the observed variability would remain if each study in the meta-analysis had a large sample size and with the consequence of minimal sampling error. An $I^2$ percentage above 75% suggests considerable heterogeneity (Higgins et al., 2003). Among other advantages, the authors asserted that the $I^2$ statistic does not inherently depend on the number of studies in the meta-analysis as does the Q-test.

In Table 2, we present the statistics indicating the level of heterogeneity of the information contained in both databases (U_dep and Y_dep) for the databases with and without outliers, using the two previously introduced methods for estimating $\tau^2$, REML, and DL. First, the results for the Q-test reject the homogeneity of the estimates of Okun’s law in both databases; however, as already noted, this test may not be reliable for databases with few or many studies (as in our case). Therefore, we add the results of $H^2$ and $I^2$ statistics, confirming the results of the Q tests. In both databases, $H^2$ is far from unity and $I^2$ indicates that most of the variance corresponds to between-study variability and to within-study variability to a much smaller extent.
The heterogeneity of Okun’s law: A metaregression analysis

Table 2. Heterogeneity statistics

<table>
<thead>
<tr>
<th>Database</th>
<th>Method of estimation</th>
<th>Test of homogeneity</th>
<th>r²</th>
<th>p-value</th>
<th>Q</th>
<th>r²</th>
<th>H²</th>
<th>I² (%)</th>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ &lt;</td>
<td>-0.8</td>
<td>REML</td>
<td>664</td>
<td>44836.41</td>
<td>0.000</td>
<td>0.0229</td>
<td>743.76</td>
<td>99.87</td>
</tr>
<tr>
<td></td>
<td>DL</td>
<td>0.0021</td>
<td>67.52</td>
<td>98.52</td>
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<td></td>
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<tr>
<td>θ without</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>restrictions</td>
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<td>46553.17</td>
<td>0.000</td>
<td>0.0265</td>
<td>838.53</td>
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</tr>
<tr>
<td></td>
<td>DL</td>
<td>0.0021</td>
<td>68.26</td>
<td>98.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y_dep (2)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>θ &lt;</td>
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<td>0.3542</td>
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<td>91.43</td>
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<td></td>
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<td>92.59</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>restrictions</td>
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<td>7240.29</td>
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<td>0.4118</td>
<td>12.89</td>
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<td>13.69</td>
<td>92.69</td>
<td></td>
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<td></td>
<td></td>
</tr>
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</table>

(1) Database with Okun’s coefficient estimated using unemployment as dependent variable.
(2) Database with Okun’s coefficient estimated using GDP as dependent variable.
(3) Database without outliers and without restrictions.

With the graphical and statistical confirmation of the presence of high heterogeneity between the studies in both databases, it only remains to explore this heterogeneity by means of a metaregression, using explanatory variables of the study characteristics that may influence the estimated effect sizes.

It is notable that most meta-analyses examine the problem of publication bias in the data, since the chosen database comes from estimated effects published in peer-reviewed journals. This occurs when the acceptance of articles for publication is conditional on the effects falling within a certain range of values and/or high levels of significance of the estimated parameters. Publication bias is a crucial consideration when a meta-analysis is conducted with the aim of determining the “true effect” of a particular phenomenon, but this is not relevant in our case, where the focus is on identifying the variables to explain the observed heterogeneity.
5.2. Metaregression results

As shown in Table 3, several metaregressions were estimated to test the significance of the variables under different criteria. First, each database (U_dep and Y_dep) was estimated separately. Then, metaregressions with random effects were estimated using two different methods of estimating the between-study variance (REML and DL) and also using WLS. In addition, two estimations were performed; one with the complete database and another without outliers (values greater than 0.8 and 4 in absolute value are considered outliers in the U_dep and Y_dep databases). It is notable that the coefficients of each variable estimated were very similar.

As noted previously, we identify three sources of heterogeneity among the estimates of Okun’s law, which are confirmed by the results of metaregressions: 1) the underlying theoretical model of the relationship, 2) the features of each labor market that make the relationship between unemployment and output more or less sensitive, and 3) the methodological diversity of estimations. Our results regarding a group of variables coincide with some results obtained by Perman et al. (2015), but for other variables, we obtain contradictory results in terms of sign or level of significance, and we also included some variables that they did not include.

Regarding the underlying theoretical model, the metaregressions were initially performed separately between the estimations using the unemployment rate as the dependent variable (U_dep), and those using output (Y_dep) because the results are not comparable. Second, as observed in the metaregression results using U_dep (Table 3), the choice between OKUN_I, OKUN_II, or OKUN_III models yields significantly different results (the omitted variable was OKUN_I). This implies that not all cases estimating the Okun relationship with a first-difference model obtained the same result as with the gap-model or the fitted-trend and elasticity model. As noted, Okun obtained similar results with the three models for the US under the fulfillment of some assumptions, such as the

---

6 Perman et al. (2015) estimated a metaregression with the entire database, and then separately; however, as the authors note in discussing some of the results, they retain the inverse of the estimated Okun’s law for models with output as the endogenous variable to make them comparable to the Okun’s law obtained when unemployment is endogenous. We contend that even keeping the inverse of the coefficient of one of the databases does not make these parameters comparable (Barreto and Howland, 1993).
natural unemployment rate of 4%; however, this assumption is not valid for any time period or location. Perman et al. (2015) obtained similar results. In their estimations, the variable LEVEL, which indicated that the variables were in levels as opposed to first differences, was found to be significant.

In addition, the estimates of Okun’s law using the gap model have applied some kind of filter to decompose the series and obtain the gaps with respect to natural or trend values. As shown in Table 4, the choice of the filter to decompose the series between trend and cycle can also generate significant systematic differences between the results of the Okun’s law estimates. Indeed, the metaregression on the U_dep database indicates that the estimates using Beveridge and Nelson (FILT_BN) and linear trend filters (FILT_LTREND) differ from those that used the Hodrick and Prescott filter (FILT_HP) (the omitted variable). It is notable that for Perman et al. (2015) the filter used was not a significant variable. In our estimate using the Y_dep database, the variables indicating the filters used were not significant with respect to the HP filter, which is the omitted variable, but when only the variables indicating the filter used are included in the model, are FILT_BN and FILT_OTHER filters significant, which could suggest that when more variables are included in the model, they present some kind of collinearity, which usually manifests itself in problems regarding the significance of the variables.

To capture the second source of heterogeneity, i.e., the features of each labor market that make the relationship between unemployment and output more or less sensitive, we introduced dummy variables by country or group of countries in metaregressions (Tables 3 and 4). The omitted variable was the US, and most of the country dummy variables were significant with a negative sign (with some exceptions). Lichter et al. (2015) also used country dummy variables to capture cross-national differences in a metaregression analysis regarding the own-wage elasticity of labor demand, whereas Perman et al. (2015) did not include them. Instead, the authors only distinguished the degree of economic development, using developed or developing countries. We contend that this distinction is not adequate, as there are important distinctions in terms of labor market institutions or labor market features between countries at the same degree of economic development. Our results suggest that the relationship between unemployment and economic activity is weaker in most countries other than
in the US (the Okun’s coefficients are in absolute value in metaregressions). Some of the differential characteristics of country labor markets mentioned in section 2.3 may be factors that make the country dummy variables significant in the metaregressions, (e.g., EPL, the proportion of self-employment, informal employment, the sectoral distribution of employment, and other relevant considerations).

The third source of heterogeneity relates to the methodological diversity of the estimates. As shown in Table 3, many variables indicating methodological choices were significant in metaregressions in both databases, including the use of estimation methods other than ordinary least squares (OTHER_OLS), the use of static rather than dynamic models (STAT_MOD), the estimation of Okun’s coefficient indicating that an asymmetric relationship in recessions is different than symmetric (ASYM_MOD_RES), the use of time series rather than panel data (TIME_SERIES), the inclusion of more than two variables in the model (MORE_VAR), the use of annual rather than quarterly or semi-annual data (YEARLY), the estimation of the relationship for a country rather than regions within a country (CTY_LEVEL), and average year of the estimation period (M_YEAR_OBS).

There is no clear explanations regarding why the use of estimation methods other than OLS could generate systematically different estimates of Okun’s law; however, it could be that this variable picks up some other unobservable common effect.\(^7\)

In the case of studies using unemployment as a dependent variable, the Okun’s coefficient estimated from a static model (STAT_MOD) would be lower in absolute value than that resulting from a dynamic relationship. This is because the dynamic model captures both the contemporaneous effect between variables and the total effect. Perman et al. (2015) obtained similar results in the same way, as such models will capture the total cumulated or long-run effect of the exogenous variable on the endogenous variable. In contrast, the variable STAT_MOD in the regression with output as the dependent variable (Y_dep) was significant with an opposite sign. This may be related to the minimal number of dynamic estimates in

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\(^7\) This is not a variable included by Perman et al. (2015).
this database, most of which are from a single study, which may not be representative of this problem.

In other cases, the variables have effects with opposite signs between the two databases. For example, the variable ASYM_MOD_RES, which takes a positive sign in U_dep and negative in Y_dep. This implies that the estimated effects during economic crises and recessions will be larger than those for the whole sample, when the relationship estimated is from output on unemployment, and lower when the inverse relationship is estimated. This result is one of the contributions of this research, as Perman et al. (2015) did not differentiate between estimates with symmetric and asymmetric modeling.

Using a more recent database (M_YEAR_OBS) may influence larger estimations of Okun’s coefficients (in absolute values) in U_dep, based on the evidence that the effect of output on unemployment has been growing over time, in general. This result contradicts that obtained by Perman et al. (2015), who found more recent databases to lead to smaller Okun coefficients (in absolute values). In the other database (Y_dep), with the inverse relationship, the effects of unemployment on output have been diminishing more recently; confirming this, the M_YEAR_OBS variable is significant with a negative sign. Therefore, we can assert that some of the observed heterogeneity between Okun’s coefficients may be due to estimates that correspond to different time periods.

In contradiction to Prachowny (1993), given that the MORE_VAR variable was found to be significant and with a positive sign in U_dep, it follows that including additional variables in the modeling of the Okun relationship may lead to larger estimated coefficients (in absolute values) than those including only unemployment and output. This result is congruent with Perman et al. (2015). Using the Y_dep database, the MORE_VAR variable was found to have a low level of significance (0.1).
Table 3. Meta-regression of Okun’s law (the Okun’s coefficients in regressions are in absolute value)

<table>
<thead>
<tr>
<th>Variables (3)</th>
<th>Random Effect meta-regression (RE)</th>
<th>WLS (6)</th>
<th>Random Effect meta-regression (RE)</th>
<th>WLS (6)</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>RMLE (4)</td>
<td>DL (5)</td>
<td>RMLE (4)</td>
<td>DL (5)</td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
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<td>0.013</td>
<td>0.028 ***</td>
<td>0.027 ***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>OKUN_II</td>
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<td>0.082 ***</td>
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<td>0.094 ***</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>OKUN_III</td>
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<td>0.200 ***</td>
<td>0.189 ***</td>
<td>0.380 ***</td>
</tr>
<tr>
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<td>(0.094)</td>
<td>(0.065)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>STAT_MOD</td>
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<td>-0.128 ***</td>
<td>-0.103 ***</td>
<td>-0.107 ***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>COUTN_MOD</td>
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<td>0.029 **</td>
<td>0.042 ***</td>
</tr>
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<td></td>
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<td>(0.033)</td>
<td>(0.012)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>ASYM_MOD_RES</td>
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<td>0.047 *</td>
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<tr>
<td></td>
<td>(0.026)</td>
<td>(0.027)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
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<td>-0.003</td>
<td>-0.002</td>
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<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>TIME_SERIES</td>
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</tr>
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<td>(0.026)</td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>MORE_VAR</td>
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<td>0.052 ***</td>
<td>0.014 **</td>
<td>0.017 ***</td>
</tr>
<tr>
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<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>YEARLY</td>
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<td>0.134 ***</td>
<td>0.123 ***</td>
<td>0.126 ***</td>
</tr>
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<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.006)</td>
<td>(0.006)</td>
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<tr>
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<td>0.252 ***</td>
<td>0.274 ***</td>
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<td></td>
<td>(0.033)</td>
<td>(0.033)</td>
<td>(0.013)</td>
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<td>0.286 ***</td>
<td>0.225 ***</td>
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<td></td>
<td>(0.034)</td>
<td>(0.032)</td>
<td>(0.014)</td>
<td>(0.014)</td>
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<td>0.320 ***</td>
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<td></td>
<td>(0.037)</td>
<td>(0.035)</td>
<td>(0.016)</td>
<td>(0.015)</td>
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<td>M_YEAR_OBS</td>
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<td>0.003 ***</td>
<td>0.003 ***</td>
<td>0.003 ***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>_cons</td>
<td>-5.269 ***</td>
<td>-6.169 ***</td>
<td>-5.327 ***</td>
<td>-5.928 ***</td>
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<td></td>
<td>(1.368)</td>
<td>(1.355)</td>
<td>(0.627)</td>
<td>(0.619)</td>
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<tr>
<td>Country dummy variables</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
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</tbody>
</table>

Notes: Standard errors in parentheses. *, **, and *** correspond to significance level of 0.1, 0.05, and 0.01, respectively. The omitted variables are: US, OLS, OKUN_I, DYN_MOD, SYM_MOD, PANEL_DATA, TWO_VAR, NO_YEARLY, and REG_LEVEL. (1) Database with Okun’s coefficient estimated using unemployment as dependent variable. (2) Database with Okun’s coefficient estimated using GDP as dependent variable. (3) For description of the variables see Table 1. (4) RMLE: restricted maximum likelihood, the default method of estimation of tax2. (5) DL: DerSimonian-Laird method of estimation of tax2. (6) Weighted least squares (WLS) using the inverse of the squared standard error of the parameter estimate as weight.

(a) Estimated model without outliers: Okun’s coefficient lower than -0.8 are outliers in U_dep and lower than -4 in Y_dep. (b) Estimated model with the complete database.
Table 4. Meta-regression of Okun’s law gap-model \( (\text{OKUN}_{\text{II}}=1) \). U_{dep} database (1) (the Okun’s coefficients are in absolute value)

<table>
<thead>
<tr>
<th>Variables (2)</th>
<th>Random Effect meta-regression (RE)</th>
<th></th>
<th></th>
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</thead>
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<tr>
<td></td>
<td>RMLE (3)</td>
<td>DL (4)</td>
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</tr>
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<td>OTHER_OLS</td>
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<td>0.108 ***</td>
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<td>(0.022)</td>
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<td>-0.143 ***</td>
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<td>(0.044)</td>
<td>(0.041)</td>
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<td>ASYM_MOD_RES</td>
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<td>0.078 *</td>
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<td>(0.044)</td>
<td>(0.042)</td>
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<td>(0.041)</td>
<td>(0.039)</td>
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<tr>
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<td>0.273 ***</td>
<td>0.272 ***</td>
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</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.063)</td>
<td></td>
</tr>
<tr>
<td>YEARLY</td>
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<td>0.160 ***</td>
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<td>(0.042)</td>
<td>(0.040)</td>
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<tr>
<td>CTY_LEVEL</td>
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<td>0.069 **</td>
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<td></td>
<td>(0.030)</td>
<td>(0.029)</td>
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<td>M_YEAR_OBS</td>
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<td>(0.002)</td>
<td>(0.002)</td>
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<tr>
<td>FILT_BP</td>
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<td>0.021</td>
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</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.060)</td>
<td></td>
</tr>
<tr>
<td>FILT_BN</td>
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<td>0.645 ***</td>
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</tr>
<tr>
<td></td>
<td>(0.210)</td>
<td>(0.209)</td>
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<td>-0.038</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.067)</td>
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<tr>
<td>FILT_LTREND</td>
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<td>0.457 ***</td>
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<tr>
<td></td>
<td>(0.149)</td>
<td>(0.147)</td>
<td></td>
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<tr>
<td>FILT_OTRO</td>
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<td>0.002</td>
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<tr>
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<td>(0.028)</td>
<td>(0.026)</td>
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<tr>
<td>_cons</td>
<td>-18.202 ***</td>
<td>-18.079 ***</td>
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<td></td>
<td>(3.678)</td>
<td>(3.493)</td>
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</tr>
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<td>Country dummy variables</td>
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<tr>
<td>Number of obs</td>
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</tr>
<tr>
<td>R-squared (%)</td>
<td>69.92</td>
<td>70.44</td>
<td></td>
</tr>
<tr>
<td>Wald chi2</td>
<td>472.09</td>
<td>519.77</td>
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<tr>
<td>Prob &gt; chi2</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses. *, **, and *** correspond to significance level of 0.1, 0.05, and 0.01, respectively. The omitted variables are: US, OLS, DYN_MOD, SYM_MOD, PANEL_DATA, NO_YEARLY, REG_LEVEL, and FILT_HP. (1) Database with Okun’s coefficient estimated using unemployment as dependent variable. (2) For description of the variables see Table 1. (3) RMLE=restricted maximum likelihood, the default method of estimation of tau2. (4) DerSimonian–Laird method of estimation of tau2.
The periodicity of the data used for the estimates also has effects on the results. The variable YEARLY is significant in both databases, with positive sign, meaning that the Okun’s coefficient of a model with annual data is larger than the coefficient of the current relationship between the variables of a model with quarterly data. As noted, the time it takes for variables to adjust to shocks is one of the factors behind this phenomenon.

Like us, Perman et al. (2015) distinguished between databases with time series or panel data used in Okun’s law estimations, but found no significant differences. In our case, the TIME_SERIES variable was significant in both databases, indicating that estimating Okun’s law with time series variables yields systematically different results than those obtained with panel data.

Finally, the spatial level of the Okun’s law estimation also affects the results and is confirmed as another source of heterogeneity. Indeed, while most of the estimates correspond to countries, others refer to regions within countries, groups of countries, or groups of people within countries (by gender). The omitted variable was REG_LEVEL, and as demonstrated in Table 3, a different level than a regional level positively impacts the law. This is because there is greater diversity and heterogeneity at the regional level, and the relationship is influenced by the unique labor market characteristics. In some regions, the relationship is stronger, and in others, it is lower or even not verified (Porrás-Arena and Martín-Román, 2019). These differences disappear in the aggregate when the relationship is estimated at national levels, or among groups of countries or groups of people at national level.

6. Conclusions

Since the Okun’s law allows a determination of the responsiveness of unemployment to output, or cost in terms of production of keeping labor resources idle, it is an extremely relevant knowledge for economic policy. The importance of this parameter is reflected by the enormous number of studies estimating Okun’s coefficient.
In this article, we have shown graphical and statistical evidence of the existence of a high level of heterogeneity among the estimates of Okun’s law. This observed heterogeneity is not only between countries or regions, but also within countries or regions; therefore, the usual meta-analysis procedure of estimating to find the “true common effect” is no longer logical and it is far more relevant to analyze the factors that may explain this heterogeneity.

Estimating metaregressions, we analyze the influence of three possible sources of heterogeneity: 1) the theoretical specification of the underlying model of the relationship, 2) labor market characteristics, and 3) methodological approaches.

Regarding the specification of the model, we first find that since the relationship has been estimated from output to unemployment and also in the opposite direction, the analysis of heterogeneity must be conducted separately since these parameters are not comparable (not even the inverse of one of the coefficients is comparable with the other). Second, while Okun estimated the relationship in the US, using three different models (in first differences, in gaps, and trend-adjusted and elasticity), and obtained similar results, this does not hold for all countries or regions. This implies that researchers should consider this finding when estimating the relationship, and the recommendation is to estimate the relationship with more than one specification, analyzing whether there are significant differences. Third, to estimate the model in gaps, it is necessary to apply some kind of filter on time-series prior to the estimation, and according to the results obtained in the metaregressions, the choice of filter can also be a source of heterogeneity. Again, the recommendation is to use more than one filter and compare the results.

As noted, although the literature has made progress in investigating the variables that refer to labor market characteristics that may explain the differences observed between the estimates of the Okun’s law for countries or regions within the same country, some of these variables include the weight of self-employment, informal employment, sectoral distribution of employment, and EPL, among others. As a means of capturing these differences we included country dummy variables in the metaregressions, which were significant, in most cases. This confirms the existence of
unobservable and idiosyncratic variables in each labor market in our database that also explain part of the observed heterogeneity.

Finally, methodological approach also matters in explaining the differences between estimates of Okun’s law, which has direct consequences for the choices the researcher must make in approaching the study of the law. We find the estimation period to be important; therefore, research estimating Okun’s coefficient should include some kind of stability analysis of the law. The type of data used for the estimations, such as time-series or panel-data, or annual, quarterly, or semi-annual frequency, are also critical aspects of researcher consideration, bearing in mind that the choice of data may generate some level of bias in the estimations. The level of coverage for which the Okun’s relationship is estimated has an effect on the results. Estimates at the country level generally indicate stronger relationships between unemployment and output than those at the level of regions within the same country. Consequently, if the objective is to obtain the Okun’s relationship of a territory in depth, it is advisable to estimate it for the economy as a whole as well as for each region reflecting such diversity. Finally, it is also important to recognize that the dynamic or static specification of the model also has consequences on the results, as well as the specification of a symmetric or asymmetric relationship. It is therefore recommended to begin from a more general specification, such as the dynamic one, assessing the significance of variable lags as a way of capturing the “true” dynamics of the relationship, and not limiting the estimation to a static relationship, which is more restrictive. In contrast, the relationship between unemployment and output may be stronger in recessionary periods than when it is estimated without including this consideration; therefore, the linearity of the relationship should also be examined, not assuming a priori that it always holds.
The heterogeneity of Okun’s law: A metaregression analysis

References


The heterogeneity of Okun’s law: A metaregression analysis


Appendix 1

List of articles used to create the metadatabase:


