Shirking and Capital Accumulation under Oligopolistic Competition

Zhou, Haiwen and Zhou, Ruhai

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Haiwen Zhou & Ruhai Zhou

Abstract

In this infinite horizon model, unemployment results from the existence of efficiency wages. Consumers choose saving optimally and there is capital accumulation. Firms producing intermediate goods engage in oligopolistic competition and choose technologies to maximize profits. A more advanced technology has a higher fixed cost but a lower marginal cost of production. In the steady state, it is shown that an increase in population size or a decrease in the discount rate leads intermediate good producers to choose more advanced technologies and the wage rate increases. Interestingly, the equilibrium unemployment rate decreases with the size of the population.

Keywords: Unemployment, increasing returns to scale, capital accumulation, choice of technology, oligopolistic competition

JEL Classification Numbers: E24, J64, L13, O14

1. Introduction

Modern production is associated with extensive use of machines. This usage of machines has various implications. First, the usage of machines can lead to oligopolistic firms dominating industries. Machines are fixed costs and the existence of significant fixed costs in production leads to increasing returns to scale, and firms with larger levels of output have cost advantages over those with smaller levels of output. With increasing returns in production, distribution, and management, there is a tendency for an industry to become monopolized by a firm. However, antitrust policies may prevent monopoly from happening. Thus, oligopoly became an important type of market structure for developed countries since the Second Industrial Revolution (Chandler, 1990). Standard textbooks on microeconomics recognize the importance of oligopolistic competition as a type of market structure. One example is Pindyck and Rubinfeld (2005, p. 441) who write “oligopoly is a prevalent form of market structure. Examples of oligopolistic industries include automobiles, steel, aluminum, petrochemicals, electrical equipment, and computers.”

Second, firms can choose the number of machines. A more automatic production process uses more machines and uses a smaller number of workers. A choice of the degree of automation is a choice of the level of technology, and technology choices by firms are commonly observed. One example of technology choice is the adoption of containers, which was one of the most important innovations in the transportation sector in the twentieth century. Levinson (2006) illustrates the adoption of containers as a choice of technology in the transportation sector. Adoption of containers began in the 1950s. At that time, the loading and unloading of cargos were handled by longshoremen and were labor-intensive. Marginal costs were high when wage rates were high. Compared with the technology of using longshoremen to load and unload cargos, marginal costs of loading and unloading through containerization decreased sharply. However, the adoption of containers led to sharp rises in fixed costs of production because specially designed cranes, containerships, and container ports had to be built. Third, when more machines are used, workers may be replaced, and unemployment may result. Unemployment is an economically and politically important question. Weitzman (1982) argues that increasing returns should be a foundation in explaining the existence of unemployment.

In this paper, we study unemployment in an infinite horizon model in which oligopolistic firms choose technologies to maximize profits and workers choose saving optimally. Unemployment results from the existence of efficiency wages (Shapiro and Stiglitz, 1984; Kimball, 1994; Phelps, 1994). The final good is produced by combining a continuum of intermediate goods. Firms producing the same intermediate good engage in oligopolistic competition and also choose technologies to maximize profits. A more advanced technology has a higher fixed cost but a lower marginal cost of production. The existence of fixed costs leads to increasing returns to scale in production.

In the steady state, we show that an intermediate good producer's equilibrium level of technology increases with population size. That is, when the size of the market is larger, intermediate good producers adopt technologies substituting labor for capital even with the existence of unemployment. We show that an increase in the size of the population reduces rather than increases the unemployment rate. This result shows that incorporating increasing returns to scale in production into a model of unemployment leads to interesting implications. With constant returns to scale in production, an increase in population size increases labor supply and can lead to a higher unemployment rate. If there are increasing returns in production and there are full employment, it is intuitive that real wage rate increases with population size. With the existence of unemployment in this model, the real wage rate still increases with population size. The reason behind this is that a higher wage rate leads to a lower unemployment rate through the non-shirking constraint of a worker.

This model is related to the literature on unemployment based on increasing returns. Weitzman (1982) argues that constant returns to scale in production are associated with perfect competition. With constant returns to scale, there is no possibility for the existence of unemployment. He believes that the existence of increasing returns should be a foundation of unemployment theory. In this model, increasing returns are also important. However, there are some significant differences between Weitzman (1982) and this one. First, in his model, firms locate around a circle and a firm only competes with two neighboring firms. In this model, firms producing intermediate goods engage in Cournot competition and compete with all firms producing the same intermediate input. Second, the modeling of unemployment is different. In his model, lack of demand leads to unemployment. In this model, potential shirking leads to equilibrium unemployment as a discipline device. Third, in his model, labor is the only factor of production. In this model, both labor and capital are factors of production and there is capital accumulation.

This paper is related to models of unemployment based on the existence of efficiency wages. The seminal work of Shapiro and Stiglitz (1984) demonstrates the existence of unemployment in a general equilibrium model in which firms engage in perfect competition. However, out-of-the-steady-state dynamics and capital accumulation are not addressed in Shapiro and Stiglitz. Whereas Kimball (1994) has addressed out-of-the-steady-state dynamics of an efficiency wage model and Phelps (1994) has incorporated saving behavior of workers, in their stimulating paper Brecher, Chen, and Choudhri (2010) have considered a dynamic model of shirking and unemployment incorporating capital accumulation and public debt.[1] While this paper is closely related to Brecher, Chen, and Choudhri (2010), there are two substantial differences between their model and this one. First, technology choice is not addressed in their model while it is the focus of this paper. Second, in their model there are constant returns in production and firms engage in perfect competition. In this model there are increasing returns in production and firms producing intermediate goods engage in oligopolistic competition. With increasing returns in production, we show that unemployment rate can decrease with the size of the population. For models with increasing returns, Zhou (2018) studies a two-sector general equilibrium model in which firms engage in oligopolistic competition and unemployment is a result of the existence of

[1] Brecher, Chen, and Choudhri's general model can lead to different policy implications on wage and interest taxes (subsidies).

efficiency wages. Impact of international trade is then addressed. Wen and Zhou (2020) have addressed the impact of financial and trade integration in a model of technology choice with the presence of efficiency wages in a general equilibrium model. In Zhou (2020), the interactions among a firm's choices of technology, output, and monitoring intensity are studied in a general equilibrium model in which firms engage in oligopolistic competition. One significant difference between this paper and those models is that those models do not allow for capital accumulation.

The plan of the paper is as follows. Section 2 specifies the model. Utility maximization of a representative household, profit maximization of a final good producer, profit maximization of a representative firm producing an intermediate good, and market-clearing conditions are established. Section 3 addresses stability of the steady state. Section 4 conducts comparative statics to explore properties of the steady state. Section 5 discusses some potential generalizations and extensions of the model and concludes.

2. The model

Time is continuous. If there is no confusion, variables will not be indexed with time. We assume that the economy has a continuum of identical households and each household has one individual.[2] There is one final good and its price is P. To eliminate a firm's potential market power in the labor market, we assume that there is a continuum of intermediate goods indexed by a real number $\varpi \in [0,1]$ and the price of an intermediate good is $p(\varpi)$.[3] All intermediate goods are assumed to have the same costs of production and enter the production of the final good in a symmetric way. The size of the population is L, which is exogenously given and does not change over time. The amount of capital is K, which is endogenously determined.

2.1. Household utility maximization

[2] In Brecher, Chen, and Choudhri (2010), each household has a continuum of individuals. A household will make decisions for individuals belonging to this household. The purpose of their specification is to abstract from complications due to wealth differences among individuals because the shirking decision of a worker is affected by the amount of wealth of a worker. To maximize a utility function that is an average of the utilities of individual members, each household chooses the same consumption level and shirking behavior for all members of this household.

[3] A firm's market power in the labor market can make the analysis of a firm's optimization problem complicated. With firms from the sector with a continuum of intermediate goods demanding labor in the labor market, an intermediate good producer is one of the infinite number of firms demanding labor and will take the wage rate as given.

Let $Z(t)$ denote employment status at time t: $Z(t) = 1$ if an individual is employed and $Z(t) = 0$ if unemployed. If an individual shirks, $S = 1$; if an individual does not shirk, then $S = 0$.

Each individual is endowed with one unit of labor. The subjective discount rate of a consumer is ρ. The cost of effort for an individual is δ, a positive constant. The amount consumed of an individual of the final good at time t is $C(t)$. For the constant $\theta > 0$, her utility function is specified as

$$U = \int_0^1 e^{-\rho t}\left[\frac{C(t)^{1-\theta}}{1-\theta} - \delta Z(1-S)\right] dt. \tag{1}$$

For an employed worker, the exogenous job separation rate at each moment is b. The probability that a worker's shirking is detected is q, an exogenously given positive constant. A shirker caught is fired immediately. The probability for an unemployed individual to find a job at each moment is a.

The interest rate is r. The wealth of a household is X. The constraints faced by a household are

$$\dot{X} = rX + wZ - PC, \tag{2}$$

$$\dot{Z} = a(1-Z) - (b+qS)Z, \tag{3}$$

$$S(1-S) = 0, \tag{4}$$

$$X(0) = X_0, Z(0) = Z_0. \tag{5}$$

Equation (2) is the evolution of assets for a household. Equation (3) is the evolution of the percentage of individuals unemployed. Equation (4) says that an individual is either employed or unemployed. Equation (5) shows the initial conditions for assets and employment rate.

The Lagrangean function for a household's optimization problem is

$$\Gamma \equiv \frac{C^{1-\theta}}{1-\theta} - \delta Z(1-S) + \mu(rX + wZ - PC)$$
$$+\eta[a(1-Z) - (b+qS)Z] + \lambda S(1-S). \tag{6}$$

In the above equation, μ and η are costate variables, and λ is a Lagrange multiplier. Specifically, μ is the shadow value of wealth and η is the shadow value of employment. In addition to equations (2)-(5), the following conditions are necessary for a household's optimization:

$$\frac{\partial\Gamma}{\partial C} = C^{-\theta} - \mu P = 0, \tag{7}$$

$$\frac{\partial\Gamma}{\partial S} = (\delta - \eta q)Z + \lambda(1-2S) = 0, \tag{8}$$

$$\dot{\mu} = \rho\mu - r\mu, \tag{9}$$

$$\dot{\eta} = \rho\eta + \delta(1 - S) - \mu w + \eta(a + b + qS), \tag{10}$$

$$\lim_{t\to\infty} e^{-\rho}\ \mu X = 0, \tag{11}$$

$$\lim_{t\to\infty} e^{-\rho t}\eta Z = 0. \tag{12}$$

Equations (11) and (12) are the transversality conditions. The second order condition with respect to S is $\frac{\partial^2\Gamma}{\partial S^2} = -2\lambda \leq 0$. For this condition to be satisfied, it is necessary that $\lambda \geq 0$. With $S = 0$ in equation (8), it is clear that $(\delta - \eta q)Z \leq 0$, or $\delta - \eta q \leq 0$. That is, $\eta q \geq \delta$ is a necessary condition to prevent a worker from shirking. The interpretation of this condition is as follows. An individual compares the cost and benefit in deciding whether to shirk. The benefit from shirking is the saving of effort δ. Since η is the shadow value of employment and q is the additional probability of losing job from shirking, the cost of shirking for a worker is ηq. An individual will not shirk if the cost is not lower than the benefit: $\eta q \geq \delta$.

When an individual is indifferent between shirking and not shirking, following Shapiro and Stiglitz (1984), we assume that an individual will choose not to shirk: $S = 0$. Competition drives the market wage to the point that keeps households indifferent between shirking and not shirking. That is, the condition $\eta q \geq \delta$ holds with equality in equilibrium:

$$\eta = \frac{\delta}{q}. \tag{13}$$

The value of η in equation (13) is constant over time: $\dot{\eta} = 0$. Plugging the values of η and $\dot{\eta}$ into equation (10), the wage rate can be expressed as

$$w = \frac{(\rho+a+b+q)\delta}{\mu q}. \tag{14}$$

2.2. Profit maximization of a final good producer

The final good is produced by combining a continuum of intermediate goods (He and Yu, 2015; Chu and Ji, 2016; Ji and Seater, 2020) without incurring additional costs. Firms producing the final good engage in perfect competition. The final good can be used either for consumption or investment. Let ε denote a constant not smaller than one. If the amount of intermediate good ϖ used for producing the final good is $q_t(\varpi)$, output of the final good Q_t is given by

$$Q_t = \left[\int_0^1 q_t^{\frac{\varepsilon-1}{\varepsilon}}(\varpi)d\varpi\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{15}$$

A firm producing the final good treats the prices of intermediate goods and the final good as given and chooses quantities of intermediate goods to maximize profit $P\left[\int_0^1 q_t^{\frac{\varepsilon-1}{\varepsilon}}(\varpi)d\varpi\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p(\varpi)q(\varpi)d\varpi$. From a final good producer's profit maximization, the absolute value of the elasticity of demand for an intermediate input is constant and equals ε. Also, the relationship between the price of the final good and prices of intermediate goods is given by

$$P = [\int_0^1 p(\varpi)^{1-\varepsilon}d\varpi]^{\frac{1}{1-\varepsilon}}. \quad (16)$$

2.3. Profit maximization of an intermediate good producer

The number of identical firms producing intermediate good ϖ is $m(\varpi)$. Firms producing the same intermediate good engage in Cournot competition. To produce an intermediate good, it is assumed that there is a continuum of technologies indexed by a positive real number n (Zhou, 2004, 2009, 2011; Gong and Zhou, 2014).[4] A higher value of n indicates a more advanced technology. For technology n, the level of fixed costs in terms of the amount of capital used is $f(n)$ and the level of marginal cost in terms of the amount of labor used is $\beta(n)$. To capture capital-labor substitution in production,[5] we assume that fixed costs increase while marginal cost decreases with the level of technology: $f'(n) > 0$ and $\beta'(n) < 0$.[6] The level of output of an intermediate good producer is x and its price is p. An intermediate good producer's profit is the difference between total revenue px and costs of production $f(n)r + \beta(n)xw$, or $px - f(n)r - \beta(n)xw$. An intermediate good producer takes the interest rate, the wage rate, and other firms' outputs and technologies as given and chooses its own output and technology optimally to maximize profit.

[4] Zhou (2004) studies the mutual dependence between the division of labor and the extent of the market in a general equilibrium model incorporating the choice of technologies based on the tradeoff between fixed and marginal costs of production. There are two important differences between this model and Zhou (2004). First, Zhou (2004) is a full-employment model while this one allows for the existence of unemployment. Second, Zhou (2004) is a one-period model with labor as the only factor of production while this one is a dynamic model with capital accumulation.

[5] Prendergast (1990) discusses technology choices in three industries: nuts and bolts, iron founding, and machine tools. In those industries, higher levels of output lead firms to choose technologies with higher fixed costs but lower marginal costs of production.

[6] For the second order condition for a firm's optimal choice of technology to be satisfied, we also assume that $f''(n) \geq 0$ and $\beta''(n) \geq 0$. This means that fixed costs increase at a nondecreasing rate with the level of technology and marginal cost decreases at a nonincreasing rate with the level of technology.

As pointed out by a reviewer, in the consumer (worker) side, shirking is a decision variable in this model, and it may be detected in probability q. But this is not considered on the firm side. Implicitly shirking detection is done by firms and the associated costs are not considered explicitly. Alternatively, shirking on the firm side can be modelled explicitly by a cost like Zhou (2020) who considers a firm's choice of monitoring intensity. A higher level of detection probability leads to a higher monitoring cost. For simplicity, choice of monitoring intensity is not considered in this model. The setup in this model can be interpreted as a fixed level of detection probability and the associated monitoring cost is combined with other fixed costs captured by $f(n)$.

An intermediate good producer's optimal choice of output yields $p + x\frac{\partial p}{\partial x} = \beta w$. Remembering that the absolute value of a final good producer's elasticity of demand of an intermediate good is constant at ε, thus an intermediate good producer's optimal choice of output yields

$$p\left(1 - \frac{1}{\varepsilon m}\right) = \beta w. \tag{17}$$

An intermediate good producer's optimal choice of technology yields

$$-f'(n)r - \beta'(n)xw = 0. \tag{18}$$

Equation (18) shows that an intermediate good producer will choose a more advanced technology if its level of output is higher.

The number of firms producing an intermediate good is a real number rather than restricted to be an integer. Firms will enter the intermediate good sector until the level of profit is zero.[7] Zero profit for an intermediate good producer requires that

$$px - fr - \beta xw = 0. \tag{19}$$

2.4. Market-clearing conditions

For the market for capital, each of the m intermediate good producers demands f units of capital. Integrating over all intermediate goods, total demand for capital is $\int_0^1 m(\varpi)f(\varpi)d\varpi$. Total supply of capital in a period is K. The clearance of the market for capital requires

$$\int_0^1 m(\varpi)f(\varpi)d\varpi = K. \tag{20}$$

[7] See Liu and Wang (2010) for an example of models with Cournot competition and free entry.

For the market for labor, total demand for labor is $\int_0^1 \frac{m(\varpi)\beta(\varpi)x(\varpi)}{Z} d\varpi$ and total supply for labor is L. Equilibrium in the labor market requires that

$$\int_0^1 \frac{m(\varpi)\beta(\varpi)x(\varpi)}{Z} d\varpi = L. \tag{21}$$

In equilibrium, all intermediate goods have the same number of firms producing it using the same level of technology, have the same level of output, and charge the same price. Since all intermediate goods are symmetric in terms of production and consumption and total measure of intermediate goods is one, for simplicity we drop the integration operation for intermediate goods. In equilibrium, assets of households equal capital stock: $X = K$. For the rest of the paper, a representative intermediate good is used as the numeraire: $p \equiv 1$. With this normalization, the price of the final good equals 1: $P = 1$.

3. Stability of the steady state

Pugging the value of m from equation (20) into equation (17) to derive the value of w, plugging this value of w and the value of x from equation (19) into equation (18) yields

$$-\beta(n)f'(n) - [K - f(n)]\beta'(n) = 0. \tag{22}$$

The above equation defines the level of technology as a function of the capital stock implicitly: $\beta = \beta[n(K)]$ and $f = f[n(K)]$. Marginal and fixed costs are functions of technology, which is endogenously determined and is a function of capital stock. Thus, marginal and fixed costs can be viewed as functions of capital stock. Applying implicit function theorem on (22) yields $\beta'(K) < 0$ and $f'(K) > 0$. That is, a higher amount of capital stock will induce intermediate good producers to choose more advanced technologies and marginal cost of production decreases.

From the set of equilibrium conditions, we can derive the following set of three equation defining the evolution of three variables K, Z, and μ as functions of exogenous parameters:[8]

$$\dot{K} = L\left(\frac{Z}{\beta(K)} - \mu^{-\frac{1}{\theta}}\right), \tag{23a}$$

$$\dot{z} = \left[\frac{q\mu}{\delta\beta(K)}\left(1 - \frac{f(K)}{\varepsilon K}\right) - \rho - q\right](1 - Z) - b, \tag{23b}$$

[8] The derivation of (23a) - (23c) is as follows. First, equation (23a) comes from plugging the value of C from equation (7) and the value of w from equation (17) into equation (2). Second, equation (23b) comes from plugging the value of a from equation (14) into equation (3). Finally, equation (23c) comes from plugging the value of r from equation (19) into equation (9).

$$\dot{\mu} = \mu\left(\rho - \frac{ZLf(K)}{\varepsilon\beta(K)K^2}\right). \tag{23c}$$

Equation (23a) can be understood as follows: for terms in the parenthesis, since Z is the employment rate and β is marginal cost of production, $\frac{Z}{\beta(K)}$ is output per capita, and $\mu^{-\frac{1}{\theta}}$ is consumption per capita. The difference between output and consumption leads to capital accumulation.

Let a bar over a variable denotes its steady-state value. In a steady state, variables do not change over time. The steady-state value of K, Z, and μ is defined by setting equations (23a)-(23c) to zero:

$$L\left(\frac{Z}{\beta(K)} - \mu^{-\frac{1}{\theta}}\right) = 0,$$

$$\left[\frac{q\mu}{\delta\beta(K)}\left(1 - \frac{f(K)}{\varepsilon K}\right) - \rho - q\right](1 - Z) - b = 0,$$

$$\mu\left(\rho - \frac{ZLf(K)}{\varepsilon\beta(K)K^2}\right) = 0.$$

Log linearization of equations (23a) - (23c) around the steady state yields

$$\begin{pmatrix} \dot{K} \\ \dot{Z} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} K - \bar{K} \\ Z - \bar{Z} \\ \mu - \bar{\mu} \end{pmatrix}. \tag{24}$$

For (24), $a_{11} = -\frac{ZL\beta'}{\beta^2}$, $a_{12} = \frac{L}{\beta}$, $a_{13} = \frac{1}{\theta}\mu^{-\frac{1+\theta}{\theta}}L$, $a_{21} = \frac{q\mu f(1-Z)}{\delta\beta\varepsilon K^2}$, $a_{22} = -\frac{b}{1-Z}$, $a_{23} = \left[\frac{q}{\delta\beta(K)}\left(1 - \frac{f(K)}{\varepsilon K}\right)\right](1 - Z)$, $a_{31} = -\frac{\mu ZL}{\varepsilon}\left(\frac{f'}{\beta K^2} - \frac{f\beta'}{\beta^2 K^2} - \frac{2f}{\beta K^3}\right)$, $a_{32} = -\frac{\mu Lf(K)}{\varepsilon\beta(K)K^2}$, $a_{33} = 0$. Let g_1, g_2, and g_3 denote the three characteristic roots of (24). Since the sum of the eigenvalues of a square matrix equals the trace of this matrix, we have

$$g_1 + g_2 + g_3 = a_{11} + a_{22} + a_{33} = -\frac{ZL\beta'}{\beta^2} - \frac{b}{1-Z}. \tag{25}$$

With $\beta' < 0$ and thus $-\frac{ZL\beta'}{\beta^2} > 0$, $\frac{b}{1-Z} > 0$, the overall sign of (25) is ambiguous. One interpretation of (25) is as follows. We can rewrite $-\frac{ZL\beta'}{\beta^2} - \frac{b}{1-Z}$ as $\frac{Z}{1-Z}\left(-\frac{(1-z)L\beta'}{\beta^2} - \frac{b}{Z}\right)$, and the sign of this expression is the same as the sign of $-\frac{(1-z)L\beta'}{\beta^2} - \frac{b}{Z}$. For the first term $\frac{(1-Z)L}{\beta}\frac{(-\beta')}{\beta}$, $\frac{(1-Z)L}{\beta}$ is the level of output produced if unemployed individuals find jobs and $\frac{(-\beta')}{\beta}$ is the rate of decline in marginal cost. Thus, the first term $-\frac{(1-z)L\beta'}{\beta^2}$ can be interpreted as the force leading to output increase. The second term $\frac{b}{Z}$ is the exogenous job separation rate adjusted by the percentage of individuals employed and it can be interpreted as the force leading to output reduction. If the force leading to output increase is large enough and dominates the force leading to output reduction, capital stock is going to explode, and the system may not be stable. Otherwise, the system is stable. In the special case if there is no choice of technology, $a_{11} = 0$. In this case, $g_1 + g_2 + g_3 < 0$ and the system is either stable or a saddle path. In (25), $-\frac{ZL\beta'}{\beta^2} - \frac{b}{1-Z}$ can be shown to be equal to $\frac{-bLf}{Lf-\varepsilon\rho \quad ^2} - \frac{\varepsilon\rho K^2\beta'}{f\beta}$, and the advantage from doing this is that this expression is a function of K only. With $\beta' < 0$ and thus $-\frac{\varepsilon\rho K^2\beta'}{f\beta} > 0$, $\frac{-bLf}{Lf-\varepsilon\rho\beta K^2} < 0$, the overall sign of (25) is ambiguous. In the following, we impose the following restriction which can be checked once fixed cost and marginal cost functions are specified:[9]

$$a_{11} + a_{22} + a_{33} = \frac{-bLf}{Lf-\varepsilon\rho\beta K^2} - \frac{\varepsilon\rho \quad ^2\beta'}{f\beta} < 0. \tag{26}$$

The product of the eigenvalues of a square matrix equals the determinant of this matrix. In general, the sign of $g_1 g_2 g_3$ is indeterminate.[10] Given that (26) is valid, there are two possibilities. First, if $g_1 g_2 g_3 > 0$, there are two negative eigenvalues and one positive eigenvalue. In this case, the steady state has a two-dimensional stable manifold and is a saddle path.[11] Second, if $g_1 g_2 g_3 < 0$, there are two subcases. In the first subcase, there are three negative eigenvalues, and the steady state is stable. In the second subcase, there are two positive eigenvalues and one negative eigenvalue, and the steady state has a one-dimensional stable manifold and is a saddle path.

4. Steady state

From equations (2), (3), and (9), the following equations should be valid in the steady state:

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[9] One example to check the stability of (24) is as follows. If we specify that $f(n) = \tau n$ and $\beta(K) = \frac{s}{n}$, where τ and s are real positive numbers, then from equation (22), $n = \frac{K}{2\tau}$. Thus $\frac{-bLf}{Lf-\varepsilon\rho \quad ^2} - \frac{\varepsilon\rho \quad ^2\beta'}{f\beta}$ becomes $\frac{-L(b-2\varepsilon\rho)-8s\tau\varepsilon^2\rho^2}{L-4\varepsilon\rho s\tau}$ and a sufficient condition for the trace to be negative is that $b - 2\rho > 0$ if $\varepsilon = 1$. This inequality is always satisfied if we use $b = 0.045$ and $\rho = 0.01$ as in Brecher, Chen, and Choudhri (2010, p. 1399).

[10] In the special case if there is no choice of technology, it can be shown that $g_1 g_2 g_3 \propto -\frac{\rho\mu q(1-Z)}{\theta\delta ZL} + \frac{2}{K}[b + (\rho + q)(1 - Z)] + \frac{2bZ}{\theta K(1-Z)}$. For this expression, the first term $-\frac{\rho\mu q(1-Z)}{\theta\delta ZL}$ is negative and other terms are positive. Even in this special case, the sign of $g_1 g_2 g_3$ is undetermined.

[11] One example of this case is as follows. For $q = 0.1$, $\rho = 0.01$, $b = 0.05$, $\delta = 0.05$, $\varepsilon = 4$, $L = 1.5$, $f(n) = n^{2/5}$, and $\beta(n) = \frac{4}{n^{2/3}}$, the three eigenvalues are -0.500463, -0.0200298, and 0.183116. The steady values are given by K =19.2648, Z =0.714864, and μ =0.882396.

$$rX + wZ - C = 0, \tag{27}$$

$$a(1 - Z) - bZ = 0, \tag{28}$$

$$\rho - r = 0, \tag{29}$$

In a steady state, equations (7), (13)-(14), (17)-(21), and (27)-(29) form a system of eleven equations defining eleven variables C, w, Z, η, μ, K, r, m, p, x, and n as functions of exogenous parameters. From this system of equations defining the steady state, we can derive the following system of three equations defining three variables Z, n, and w in the steady state as functions of exogenous variables:[12]

$$\Omega_1 \equiv w - \delta\left(1 + \frac{\rho}{q} + \frac{b}{q(1-Z)}\right)\frac{Z^\theta}{\beta^\theta} = 0, \tag{30a}$$

$$\Omega_2 \equiv -\beta' f w - (1 - \beta w) f' = 0, \tag{30b}$$

$$\Omega_3 \equiv ZL - \frac{\rho f \beta}{\varepsilon(1-\beta w)^2} = 0. \tag{30c}$$

Partial differentiation of equations (30a) - (30c) with respect to Z, n, w, q, L, ρ, and ε yields

$$\begin{pmatrix} \frac{\partial \Omega_1}{\partial Z} & \frac{\partial \Omega_1}{\partial n} & \frac{\partial \Omega_1}{\partial w} \\ 0 & \frac{\partial \Omega_2}{\partial n} & \frac{\partial \Omega_2}{\partial w} \\ \frac{\partial \Omega_3}{\partial Z} & \frac{\partial \Omega_3}{\partial n} & \frac{\partial \Omega_3}{\partial w} \end{pmatrix} \begin{pmatrix} dZ \\ dn \\ dw \end{pmatrix} = -\begin{pmatrix} \frac{\partial \Omega_1}{\partial q} \\ 0 \\ 0 \end{pmatrix} dq - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Omega_3}{\partial L} \end{pmatrix} dL - \begin{pmatrix} \frac{\partial \Omega_1}{\partial \rho} \\ 0 \\ \frac{\partial \Omega_3}{\partial \rho} \end{pmatrix} d\rho - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Omega_3}{\partial \varepsilon} \end{pmatrix} d\varepsilon. \tag{31}$$

According to the correspondence principle (Samuelson, 1983, chap. 9), stability requires that the determinant of the coefficient matrix of endogenous variables of (25) to be negative: $\Delta < 0$. This condition can be checked once the fixed cost and marginal cost functions are specified such as in footnote 11. With Δ nonsingular, a unique steady state exists.

Labor market efficiency can be affected by government policies, such as the existence and the level of unemployment benefits. A higher value of q can be viewed as a more efficient labor market. The following proposition studies the impact of a change in the level of labor market efficiency on endogenous variables.

[12] The derivation of (30a) - (30c) is as follows. First, equation (30a) comes from equation (14). Second, equation (30b) comes from plugging the value of x from equation (19) into equation (18). Third, equation (30c) comes from equation (21).

Proposition 1: In the steady state, an increase in labor market efficiency leads to an increase in the wage rate and intermediate good producers choose more advanced technologies. An increase in the level of labor market efficiency leads to a lower equilibrium unemployment rate.

Proof: Applying Cramer's rule on (31) yields

$$\frac{dw}{dq} = \frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial u}/\Delta > 0,$$

$$\frac{dn}{dq} = -\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial w}\frac{\partial\Omega_3}{\partial u}/\Delta > 0,$$

$$\frac{dZ}{dq} = \frac{\partial\Omega_1}{\partial q}\left(\frac{\partial\Omega_2}{\partial w}\frac{\partial\Omega_3}{\partial n} - \frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial w}\right)/\Delta.$$

Since the sign of $\frac{\partial\Omega_2}{\partial w}\frac{\partial\Omega_3}{\partial n} - \frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial w}$ is ambiguous, the sign of $\frac{dZ}{dq}$ is ambiguous. However, from equation (30a), since either an increase in q or w will lead to a higher Z, it is clear that Z increases. ■

The intuition behind Proposition 1 is as follows. With a more efficient labor market, unemployment rate decreases. Through the non-shirking condition for a worker, the equilibrium wage rate increases. A higher wage rate does not change the marginal cost while increases the marginal benefit of choosing a more advanced technology. Thus, an intermediate good producer adopts a more advanced technology.

Like the proof of Proposition 1, it can be shown that an increase in the exogenous job separation rate will reduce the equilibrium wage rate and leads intermediate good producers to choose less advanced technologies.

Population size is related to market size, which affects technology choice. The following proposition studies the impact of a change in population size on endogenous variables.

Proposition 2: In the steady state, a higher population size leads intermediate good producers to choose more advanced technologies, the wage rate increases, and the unemployment rate decreases.

Proof: Applying Cramer's rule on (31) yields

$$\frac{dn}{dL} = \frac{\partial\Omega_1}{\partial u}\frac{\partial\Omega_2}{\partial w}\frac{\partial\Omega_3}{\partial L}/\Delta > 0,$$

$$\frac{dw}{dL} = -\frac{\partial\Omega_1}{\partial u}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial L}/\Delta > 0,$$

$$\frac{dZ}{dL} = \frac{\partial \Omega_3}{\partial L}\left(\frac{\partial \Omega_1}{\partial w}\frac{\partial \Omega_2}{\partial n} - \frac{\partial \Omega_1}{\partial n}\frac{\partial \Omega_2}{\partial w}\right)/\Delta.$$

Since the sign of $\frac{\partial \Omega_1}{\partial w}\frac{\partial \Omega_2}{\partial n} - \frac{\partial \Omega_1}{\partial n}\frac{\partial \Omega_2}{\partial w}$ is ambiguous, the sign of $\frac{dZ}{dL}$ is ambiguous. However, from equation (30a), since either an increase in n or w will lead to a higher Z, it is clear that Z increases. ■

The intuition behind Proposition 2 is as follows. With the existence of fixed costs of production, there are increasing returns to scale in production. A higher level of output makes the adoption of more advanced technologies more profitable because the higher fixed costs can be spread over a higher level of output. A more advanced technology leads to a lower average cost of production. Since intermediate good producers and firms producing the final good earn profits of zero, a lower average cost shows up as a higher equilibrium wage rate because the price of an intermediate good is normalized to one. While each unit of output uses a lower number of workers, the level of output is higher, and this generates a higher demand for labor. Since the impact from output increase dominates the impact from a lower marginal cost, the equilibrium unemployment rate is lower with a higher population!

Casual observation shows there is no monotonic relationship between population size and unemployment rate for countries. This result that unemployment rate may decrease with population size highlights the implication of incorporating increasing returns to scale in production into a model of unemployment. Without unemployment, it is intuitive that real wage rate increases with population size if there are increasing returns in production. With unemployment in this model, the real wage rate still increases with population size. Through the non-shirking constraint, a higher wage rate leads to a lower unemployment rate.

Psychological studies such as Duckworth (2016) have shown that other things equal more patient individuals are more successful. The degree of patience of an individual can be captured by the discount rate. The following proposition studies the impact of a change in the discount rate on the equilibrium level of technology and the wage rate.

Proposition 3: In the steady state, a higher discount rate leads intermediate good producers to choose less advanced technologies and the wage rate decreases. Also, the employment rate decreases.

Proof: Applying Cramer's rule on (31) yields

$$\frac{dn}{d\rho} = \frac{\partial \Omega_2}{\partial w}\left(\frac{\partial \Omega_1}{\partial u}\frac{\partial \Omega_3}{\partial \rho} - \frac{\partial \Omega_1}{\partial \rho}\frac{\partial \Omega_3}{\partial u}\right)/\Delta < 0,$$

$$\frac{dw}{d\rho} = \frac{\partial \Omega_2}{\partial n}\left(\frac{\partial \Omega_1}{\partial \rho}\frac{\partial \Omega_3}{\partial u} - \frac{\partial \Omega_1}{\partial u}\frac{\partial \Omega_3}{\partial \rho}\right)/\Delta < 0.$$

From equation (30a), since either a decrease in n or w will lead to a higher Z, it is clear that Z decreases. ■

The intuition behind Proposition 3 is as follows. A higher discount rate means a higher interest rate, which increases the marginal cost of adopting a more advanced technology. Thus, intermediate good producers choose a less advanced technology in equilibrium. With a less advanced technology, the equilibrium wage rate is lower.

The elasticity of substitution among intermediate goods can be used to capture the degree of competition among intermediate goods. A higher elasticity of substitution means a higher degree of competition. The following proposition studies the impact of a change in the elasticity of substitution among intermediate goods on endogenous variables.

Proposition 4: In the steady state, a higher elasticity of substitution among intermediate goods leads to the adoption of more advanced technologies by intermediate good producers and the wage rate is higher. Also, the employment rate increases.

Proof: Applying Cramer's rule on (31) yields

$$\frac{dn}{d\varepsilon} = \frac{\partial \Omega_1}{\partial Z}\frac{\partial \Omega_2}{\partial w}\frac{\partial \Omega_3}{\partial \varepsilon}/\Delta > 0,$$

$$\frac{dw}{d\varepsilon} = -\frac{\partial \Omega_1}{\partial Z}\frac{\partial \Omega_2}{\partial n}\frac{\partial \Omega_3}{\partial \varepsilon}/\Delta > 0.$$

From equation (30a), since either an increase in n or w will lead to a higher Z, it is clear that Z increases. ■

The intuition behind Proposition 4 is as follows. A higher elasticity among intermediate goods means that a price reduction by an intermediate good producer will lead to a higher level of output. A higher level of output induces an intermediate good producer to adopt more advanced technologies.

4. Conclusion

In this paper, we have studied an infinite horizon model of unemployment in which intermediate good producers engage in oligopolistic competition and choose technologies to maximize profits. Unemployment is the result of the existence of efficiency wages and saving of workers is allowed. We have shown that an increase in population size, a decrease in the discount rate, or a higher elasticity of substitution among intermediate goods leads firms producing intermediate goods to choose more advanced technologies and a higher wage rate results in the steady state. Interestingly, an increase in the size of the population leads to a lower unemployment rate!

There are some possible generalizations and extensions of the model. First, this paper studies a closed economy. To address the impact of opening up to international trade on unemployment, it may be interesting to extend the model to an open economy. Second, it is assumed that there exists a menu of technologies. In reality, new technologies may need to be discovered through costly research and development efforts. Incorporation of endogenous development of technologies can be a valuable avenue for future research. Finally, in this model, government is absent. With the existence of market imperfections in this model, it will be interesting to introduce the government into the model and to study the impact of various government policies such as employment subsidies.

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