A Note on Economic Growth and Labor Automation

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January 2022

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1 Abstract

This paper analyzes the relationship between labor automation and economic growth. I build a task-based framework and utility to evaluate how labor automation range can maximize economy in different conditions. I also analyze relationships among labor automation, capital, consumption, and investment. I considered the fact that automation will be expanded due to technological advancements while labor tends to obtain new skills to be more competitive. The best labor automation depends on labor productivity and machine productivity. When labor productivity exceeds machine productivity, labor automation will be less than half of the total tasks. In the opposite case, labor automation will be more than half of the total tasks. I also demonstrated that the investment decreases rapidly when workers become more competitive. When disruptive technologies are introduced, consumption will increase sharply together with labor automation, which is consistent with the first conclusion.

Keywords: Labor automation, economic growth, consumption, investment, technology.

2 Introduction

Studies about economic development date back to the last century. The main economic growth model used in the 1950s was developed by Robert Solow and Trevor Swan (1956) - an exogenous growth model which is not able to explain technology development. To address this issue, an endogenous growth model was proposed by Lucas (1988) and Paul Romer (1986). This model incorporates a new concept of skills, for example, inequity caused by skill difference.

Zeira (1998) proposed a model of economic growth based on labor-capital substitution. Zeira (1998) observed that even though technological progress is beneficial, it is not adopted everywhere. The acceptance level is higher only in countries with high productivity, which in turn significantly amplifies productivity differences between countries. This is an important recent contribution to introduce a task-based framework on labor and capital.

Zeira (1998) explains global differences in output per capita, but it ends there. It remains unclear how this inequity happens. Moreover, it's not accurate enough to explain some phenomena, for instance, significant declines in real wages of low-skill workers, particularly low-skill males. Based on Zeira's (1998) contribution and the study of Autor, Katz, and Kearney (2008) on the canonical model, Acemoglu and Autor (2011) proposed a task-based static framework that high-skill workers tend to benefit from innovations while low skill workers get fewer jobs. In other words, the difference in skill levels causes inequity of wages and job shares decline. The task-based model is, however, incomplete, and the relationship between economic growth and tasks is not mentioned in the studies.

On the other hand, artificial intelligence concepts have advanced rapidly and caused human interventions to be redundant. Frey and Osborne (2013) observe that 47% of US jobs could be automated in one decade or two. Taking these factors into consideration, the task-based framework is then extended based on the previous static model by Acemoglu and Restrepo (2016). Their paper shares the same idea with previous research that because of comparative advantage, automation takes a job from unskilled labor and therefore increases inequity. Acemoglu and Restrepo (2016) also consider dynamic and balanced economic growth and further reveal the relationship between innovation and economic growth. Compared to the previous model, this model turns the direction of research toward automation and the creation of new tasks. More importantly, they claim that there is a balanced growth path in which factor distribution of income and inequity between two skill types stays constant. Acemoglu and Restrepo (2018a) proposed a more comprehensive model including tasks, skills, and economic growth. On this foundation, Acemoglu and Restrepo (2018) show that by increasing the extensive margin of automation by the same amount as the range of tasks, equilibrium wages grow proportionately and the labor share remains constant

Compared with previous studies, Acemoglu and Restrepo (2018) optimized the task-based model for the implication of automation and AI on the demand for labor, wages, and employment. The optimized model not only managed to find a balanced growth but also shows why the creation of new tasks is essential in which labor has a comparative advantage (previous studies did not explain this factor). In addition, the optimized model also presents possible countermeasures to deal with the mismatch between skills and technologies - to simultaneously increase the supply of skills. However, there is something that remains uncovered in these studies, for example, they didn't take the household budget and utility into consideration to analyze productivity growth.

In this paper, using a task-based framework and utility, we analyze relationships among aggregate output, investment, consumption, capital, and automation level. We draw graphs under different situations and explain similarities and differences. We also give a possible reason for differences and present the corresponding strategies.

3 Model

3.1 Firms

A Task-Based Framework, Aggregate output is produced by combining the services of a unit measure of tasks $x \in [O, N]$.

$$\ln Y_t = \int_0^N \ln y_t(x)dx \tag{1}$$

where Y is aggregate output and y(x) is the output of task x. Each task can be produced by human labor,$\ell(x)$, or by machines,$m(x)$. Here we assume in particular that tasks $x \in [0, I]$ are technologically automated, besides, we assume that machines are comparatively productive in tasks $x \in [0, I]$, so they can be only produced by machines, while the rest are not technologically automated, so must be produced with labor:

$$y_t(x) = \begin{cases} \gamma_{Mt}(x)m(x) & if x \in [0, I] \\ \gamma_{Lt}(x)\ell(x) & if x \in (I.N] \end{cases} \tag{2}$$

Here,$\gamma_L(x)$ is the productivity of labor in task x, while $\gamma_m(x)$ is the productivity of machines in automated tasks. We assume that $\ln \gamma_m(x) = x^m \quad and \quad \ln \gamma_L = x^l$. Here,m is the index of productivity of machine in task $x \in [0.I]$, while l is the index of productivity of low-skill and high-skill labor in task $x \in (I.N]$, and we also assume $< 0m, l < 1, m > l$ at first because automation is much more productive than low-skill workers, but later $m < l$(though m and l is both growing) because in tasks that can't be automated high-skill labor have a comparative advantage than machine.

Showed in the Appendix that aggregate output in the equilibrium takes the form:

$$Y = B\left(\frac{K}{I}\right)^{I}\left(\frac{L}{N-I}\right)^{N-I} \tag{4}$$

Where,

$$\ln B = \left(\frac{1}{m+1}\right) I^{m+1} + \left(\frac{1}{l+1}\right) N^{(l+1)}$$

3.2 Individuals

We consider the coefficient of relative risk aversion (CRRA) for the Lifetime utility function of the household. Moreover, we assume that it's a closed economy with no government.

$$\sum_{t=0}^{\infty} \beta^{t}\left[\frac{c_t^{1-\theta}}{1-\theta} - \frac{L_t^{1+\varphi}}{1+\varphi}\right]$$

In addition capital accumulation is defined as followed

$$K_t = (1-\delta)K_{t-1} + i_t$$

Consequently, personal budget constraint formula for each period is:

$$c_t + K_t = w_t L_t + r_t K_{t-1} + (1-\delta)K_{t-1}$$

Here,b_t is national debt,τ_t is tax,and$R$ is interest rate.

Showed in the Appendix,when in an equilibrium,we have:

$$\frac{1}{\beta} + \delta - 1 = B\left(\frac{K}{I}\right)^{I-1}\left(\frac{L}{N-I}\right)^{N-I} \tag{7}$$

Optimization condition of labor:

$$c^{\theta} = Y(N-I)L^{\varphi-1} = e^{\left(\frac{1}{m+1}\right)I^{m+1} + \left(\frac{1}{l+1}\right)N^{(l+1)} - \left(\frac{1}{l+1}\right)I^{l+1}}\left(\frac{K}{I}\right)^{I}\left(\frac{1}{N-I}\right)^{I} L^{(N-I+\varphi-1)} \tag{8}$$

4 Results

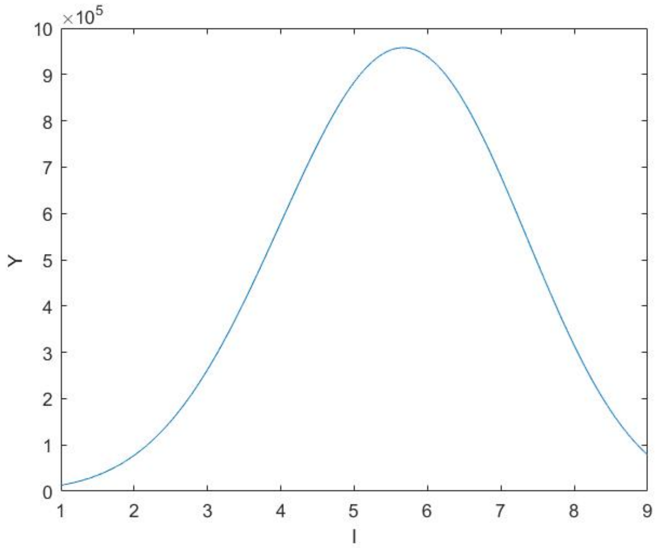
Calibration of Aggregate output and Labor Automation range

Trough calculation is shown in Appendix. We have relation between aggregate output Y and extensive margin of automation, I.

$$Y' = \left[I^m - I^l + \ln\frac{K(N-I)}{LI}\right]\left(\frac{K}{I}\right)^I\left(\frac{L}{N-I}\right)^{N-I} .e^{\left(\frac{1}{m+1}\right)I^{m+1}+\left(\frac{1}{l+1}\right)N^{(l+1)}-\left(\frac{1}{l+1}\right)I^{l+1}}$$

The value of aggregate output depends on each parameter, so let's do some calibration to see how Y varies with I. Assuming $m = 0.3$,$l = 0.2$,$N = 10$,$K = 5$,$L = 5$, in this case, Y can be rewritten as: as:

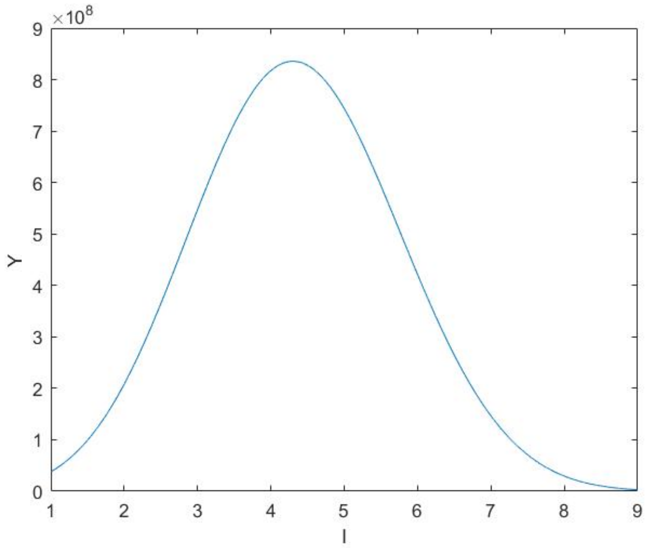
$$Y = \left(\frac{5}{I}\right)^I\left(\frac{5}{10-I}\right)^{10-I} \cdot exp[\left(\frac{1}{1.3}\right)I^{1.3} + \left(\frac{1}{1.2}\right)10^{1.2} - \left(\frac{1}{1.2}\right)I^{1.2}]$$



Graph1

We assume that m grows slowly in a long period, l grows faster because previous low-skill workers become more productive through learning new skills. Assuming $m = 0.4$,$l = 0.5$,$N = 10$,$K = 5$,$L = 5$, in this case, Y can be rewritten as: as:

$$Y = \left(\frac{5}{I}\right)^I\left(\frac{5}{10-I}\right)^{10-I} \cdot exp[\left(\frac{1}{1.4}\right)I^{1.4} + \left(\frac{1}{1.5}\right)10^{1.5} - \left(\frac{1}{1.5}\right)I^{1.5}]$$



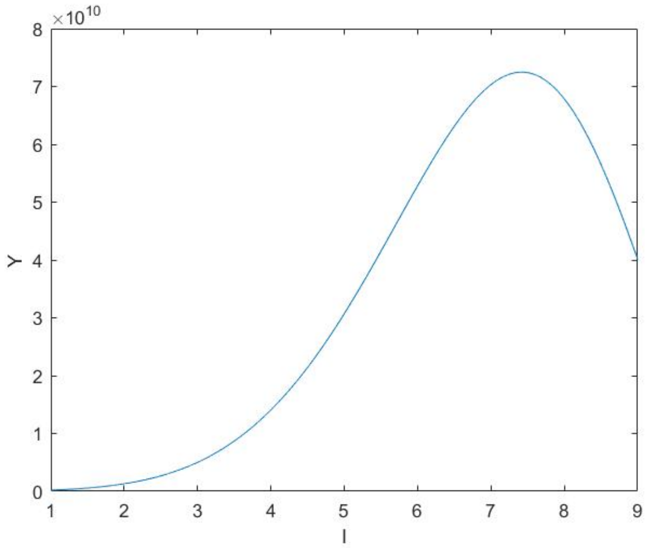
Graph2

Firstly, the shape of the graph has not changed even if we put different parameters, first rises ($Y' > 0$) and comes to the top when $Y' = 0$, then falls ($Y' < 0$), which means there is a perfect value of extensive margin of automation I to maximize aggregate output Y.

Secondly, in Graph 1 and Graph 2, when Y is at peak, the corresponding I value is smaller when l is bigger. The reason is workers have time to adapt to new skills, so they become more competitive over the machine.

However, throughout the history, disruptive technological advancements such as the industrial revolution took place. In this era, AI may result in higher productivity. We assume it happened and made the variable unique set parameter as follows: $m = 0.7$,$l = 0.55$,$N = 10$,$K = 5$,$L = 5$, in this case, Y can be rewritten as:

$$Y = \left(\frac{5}{I}\right)^{I}\left(\frac{5}{10-I}\right)^{10-I} \cdot exp[\left(\frac{1}{1.7}\right)I^{1.7} + \left(\frac{1}{1.55}\right)10^{1.55} - \left(\frac{1}{1.55}\right)I^{1.55}]$$



Graph3

This curve too, first rises and later falls, but it becomes bigger when Y is at peak. This is because workers have no time to adapt to technological progress. They lose the match with the machine temporarily.

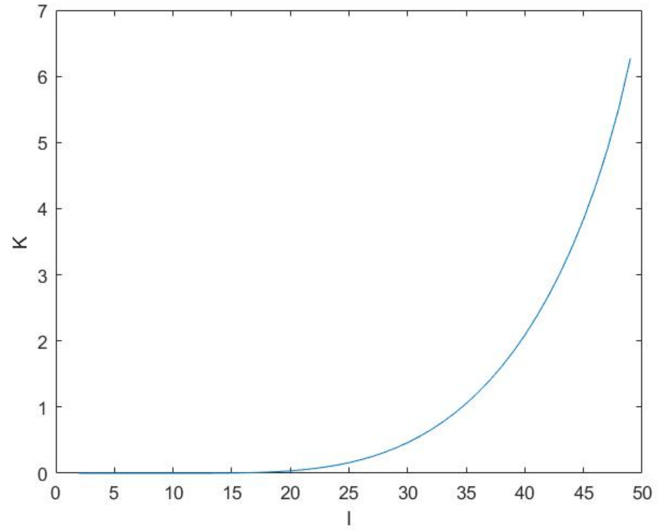
Based on this analysis, we hold the perspective that government should give policy directions for related skill training. The successful case studies show a trend of providing programming training and chances of related employment particularly in China and Japan.

Calibration of Capital and Labor Automation range

On the other hand, we set value of each parameter as:

$\beta = 0.996$,$\delta = 0.04$,$\theta = 1.5$,$\varphi = 2$,$m = 0.2$,$l = 0.3$,$N = 50$,$L = 30$. Expression can be rewritten as:

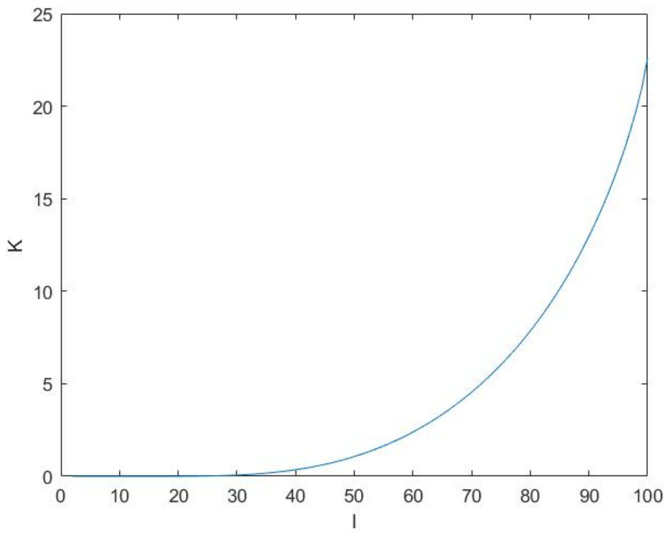
$$\frac{1}{0.996} + 0.04 - 1 = exp[\left(\frac{1}{1.2}\right) I^{1.2} + \left(\frac{1}{1.3}\right) 50^{1.3} - \left(\frac{1}{1.3}\right) I^{1.3}] \cdot \left(\frac{K}{I}\right)^{I-1} \left(\frac{30}{50-I}\right)^{50-I}$$



Graph4

$\beta = 0.996$,$\delta = 0.04$,$\theta = 1.5$,$\varphi = 2$,$m = 0.1$,$l = 0.2$,$N = 60$,$L = 50$expression(7) can be rewritten as :

$$\frac{1}{0.996} + 0.04 - 1 = exp[\left(\frac{1}{1.1}\right) I^{1.1} + \left(\frac{1}{1.2}\right) 60^{1.2} - \left(\frac{1}{1.2}\right) I^{1.2}] \cdot \left(\frac{K}{I}\right)^{I-1} \left(\frac{50}{60-I}\right)^{60-I}$$



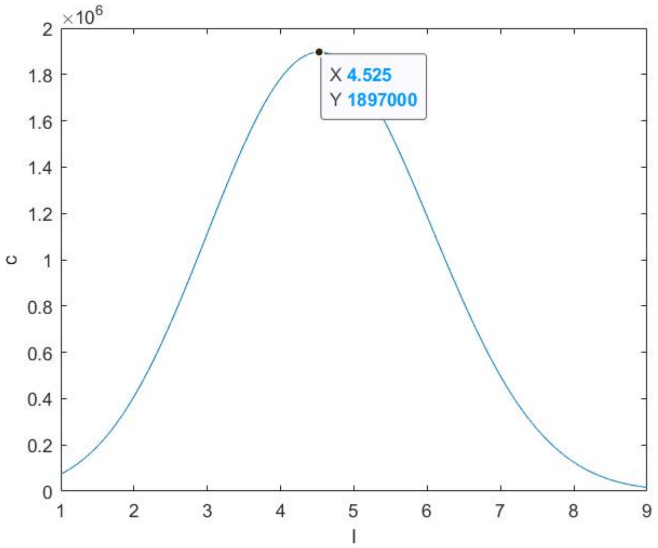
Graph5

Graphs 4 and 5 show that capital K always tends to increase with I, getting bigger no matter how Y varies. It grows more sharply when I get more close to N. This is true because we need more devices when the automation level is getting higher.

Calibration of Consume and Labor Automation range

Set $\theta = 1.5$,$\varphi = 2$,$m = 0.1$,$l = 0.2$,$N = 10$,$L = K = 5$,we have:

$$c^{1.5} = e^{\left(\frac{1}{1.1}\right)I^{1.1} + \left(\frac{1}{1.2}\right)10^{(1.2)} - \left(\frac{1}{1.2}\right)I^{1.2}} \left(\frac{5}{I}\right)^I \left(\frac{1}{10-I}\right)^I L^{(11-I)}$$

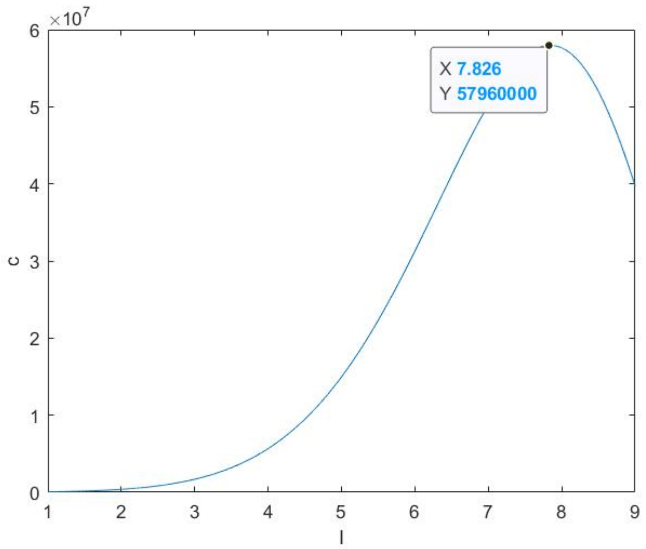


Graph6

Set $\theta = 1.5$,$\varphi = 2$,$m = 0.5$,$l = 0.2$,$N = 10$,$L = K = 5$, under the same value except for m, which grows bigger than l because of advanced technologies such as AI,

$$c^{1.5} = e^{\left(\frac{1}{1.5}\right)I^{1.5} + \left(\frac{1}{1.5}\right)10^{(1.5)} - \left(\frac{1}{1.5}\right)I^{1.5}} \left(\frac{5}{I}\right)^I \left(\frac{1}{10-I}\right)^I 5^{(11-I)}$$

we have graph as follows:

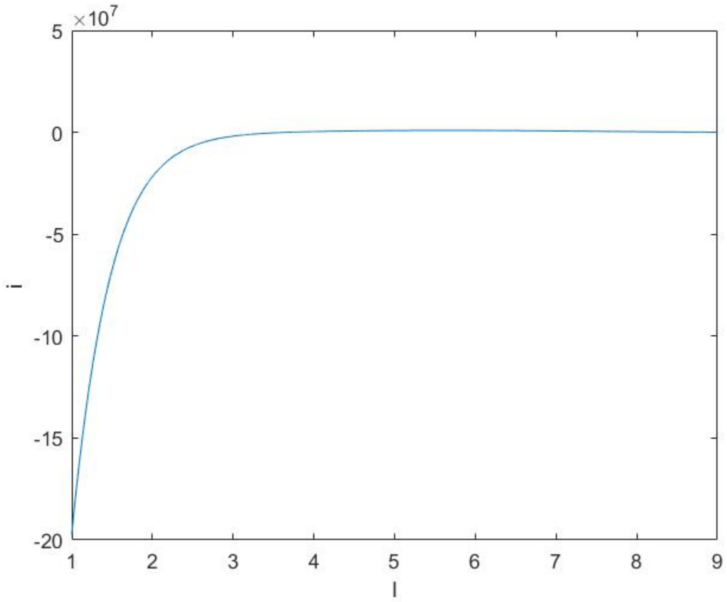


Graph7

We obtain c first, increase and decrease later when I gets bigger. Compared with two graphs, we can conclude that c comes to peak more slowly when m gets bigger.

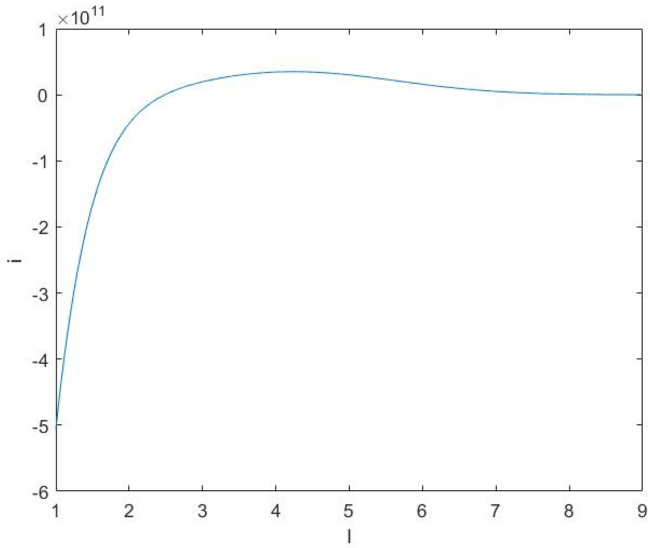
Calibration of Investment and Labor Automation range

$i = Y - c$ holds for any period,set $\theta = 1.5$,$\varphi = 2$,$m = 0.3$,$l = 0.2$,$N = 10$,$L = K = 5$, we have graph as follows:



Graph8

set $\theta = 1.5$,$\varphi = 2$,$m = 0.5$,$l = 0.6$,$N = 10$,$L = K = 5$,



Graph9

From Graph 8 and Graph 9, we can tell that I rises sharply at first because Y then falls slowly. When l is bigger than m, it falls faster but not that fast as c falls. It rises because Y rises with I at first, so the market needs vast investment. It falls slowly mainly because workers learn new skills to become more competitive, so some tasks previously performed by machine are performed by labor, and devices investment reduces.

5 Conclusion

Y increases at first when I increases because at first, low-skill labor is displaced by a cheap machine. The machine has a comparative advantage. So, despite productivity produced by labor and labor share decrease, automation compensates it. However, it decreases later since excessive automation costs more than a high-skill worker can produce (but if there is not enough high-skill labor, we have to consider excessive automation or import labor from other countries).

If workers acquire new skills and become more competitive, not only aggregate output will increase but also the labor ratio increases as tasks performed by machines decrease. Workers need time to be adapted to new technology, and government support can accelerate this process. Before workers get used to mastering new skills, automation is a better choice.

In terms of investment, it will fall faster if workers become more competitive. If there is a revolutionary new technology like AI, consumption will not only get much bigger (because new AI brings numerous productivity) but also automation level corresponding to the maximum production value will increase.

Appendix

To maximize profit,we set price of task x as $p(x)$,and we have:

$$P_t(x) = \begin{cases} \frac{r_t}{\gamma m_t(x)} & if x \in [0.I] \\ \frac{W_t}{\gamma L_t(x)} & if x \in (I, N] \end{cases} \tag{3}$$

In addition,the demand for task x is given by

$$y_t(x) = \frac{Y_t}{p_t(x)}$$

Thus,the demand for machines in task x is

$$k(x) = \begin{cases} \frac{Y}{R} & if x \in [0, I] \\ 0 & if x \in (I, N] \end{cases}$$

and the demand for labor in task x is

$$\ell(x) = \begin{cases} 0 & if x \in [0, I] \\ \frac{Y}{W} & if x \in (I, N] \end{cases}$$

We assume that the supply of capital,K,is equal to total demand for machines

$$K_t = k(x)x = \frac{Y_t}{r_t} I \qquad (5)$$

Similarly,we obtain labor supply:

$$L_t = \ell(x)x = \frac{Y_t}{W_t}(N - I) \qquad (6)$$

Therefore,we obtain rental rate and wage under the condition that the corporate maximize its profit.

$$r_t = \frac{Y_t}{K_t} I \qquad and \qquad W_t = \frac{Y_t}{L_t}(N - I)$$

We next turn to deriving the expression for Y we assume that the price of final good is 1 as numeraire,we have:

$$\int_0^N \ln p(x) dx = 0$$

and,put expression of (x),we have:

$$\int_0^I [\ln r - \ln \gamma_m(x)] \, dx + \int_I^N [\ln \omega - \ln \gamma_L(\lambda)] \, dx = 0$$

Next,we put expressions of W and r in it,we have:

$$\int_0^I [\ln Y - \ln(K/I)) - \ln \gamma_m(x)\Big] dx + \int_I^N [\ln Y - \ln(L/W - I) - \ln \gamma_L(x)]\, dx = 0$$

Consequently:

$$\ln Y = \int_0^I \left[\ln\left(\frac{K}{I}\right) + \ln \gamma_m(x)\right] dx + \int_I^N \left[\ln\left(\frac{L}{N-I}\right) + \ln \gamma_{L(x)}\right] dx$$

$=\int_0^I \ln \gamma_m(x)dx + \int_\tau^N \ln \gamma_L(x)dx$

+I $\ln \frac{K}{I} + (N-I)\ln\left(\frac{L}{N-I}\right)$

As a consequence , we obtain the expression of Y when it is in an equilibrium:

$$Y = B\left(\frac{K}{I}\right)^I \left(\frac{L}{N-I}\right)^{N-I} \qquad (4)$$

Where,

$$B = \exp\left(\int_0^I \ln \gamma_M(x)dx + \int_I^N \ln \gamma_{L(x)dx}\right) = \left(\frac{1}{m+1}\right) I^{m+1} + \left(\frac{1}{l+1}\right) N^{(l+1)}$$

consume and labor

Using Lagrange function ,we obtain:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta}}{1-\theta} - \frac{L_t^{1+\varphi}}{1+\varphi} - \lambda_t \left\{c_t + K_t - w_t L_t - r_t K_{t-1} - (1-\delta)K_{t-1}\right\}\right]$$

By differentiatingc_t,K_t,λ_t,we obtain:

$$c_t : c_t^{-\theta} = \lambda_t$$

$$L_t : L_t^{\varphi} = \lambda_t w_t$$

$$K : \lambda_t = \beta\left[(1 + r_{t+1} - \delta)\,\lambda_{t+1}\right]$$

(Here,we should consider the fact that K_t exists in period t and t+1 because K_{t-1} exists.)

Delete λ_t,we obtain: Euler equations: $c_t^{-\theta} = \beta(1 + r_{t+1} - \delta)c_{t+1}^{-\theta}$

so:

$$c_t^{-\theta} = \beta(1 + r_{t+1} - \delta)c_{t+1}^{-\theta}$$

Optimization condition of labor:

$$L_t : L_t^{\varphi} = c_t^{-\theta} w_t$$

when it comes to a equilibrium: plugging (5) into $1 = \beta(1+r-\delta)$ and (6) into $L : L^{\varphi} = c^{-\theta}w$,we obtain expression (7) and (8)

Aggregate output derivatives

$$\ln Y = \ln B + I \ln \frac{K}{I} + (N-I)\ln\left(\frac{L}{N-I}\right)$$

$=\int_0^I \ln \gamma_m(x)dx + \int_I^N \ln \gamma_L(x)dx + I \ln \frac{K}{I} + (N-I)\ln \frac{L}{N-I}$

$\ln \gamma_m(x) = x^m$ and $\ln \gamma_L = x^l$,therefore,we have

$$\ln B = \left[\left(\frac{1}{m+1}\right) x^{m+1}\right]_0^I + \left[\left(\frac{1}{l+1}\right) - x^{l+1}\right]_I^N$$

$=\left(\frac{1}{m+1}\right) I^{m+1} - \frac{1}{m+1}0^{m+1} + \left(\frac{1}{l+1}\right) N^{l+1} - \left(\frac{1}{1+1}\right) I^{(l+1)}$

$= \left(\frac{1}{m+1}\right) I^{m+1} + \left(\frac{1}{l+1}\right) N^{l+1} - \left(\frac{1}{1+1}\right) I^{(l+1)}$

Plugging equation of $\ln B$ into $\ln Y$,we obtain:

$$\ln Y = \left(\frac{1}{m+1}\right) I^{m+1} + \left(\frac{1}{l+1}\right) N^{(l+1)} - \left(\frac{1}{(l+1)}\right) I^{(l+1)} + I \ln \frac{K}{I} + (N-I)\ln \frac{L}{N-I}$$

DifferentiateI to Y on both sides,we obtain:

$$\frac{Y'}{Y} = I^m - I^l + \ln \frac{K(N-I)}{LI}$$

Hence:

$$Y' = Y\left(I^m - I^l + \ln \frac{K(N-I)}{LI}\right)$$

$=\left[I^m - I^l + \ln \frac{K(N-I)}{LI}\right] \left(\frac{K}{I}\right)^I \left(\frac{L}{N-I}\right)^{N-I} .e^{\left(\frac{1}{m+1}\right)I^{m+1}+\left(\frac{1}{l+1}\right)N^{(l+1)}-\left(\frac{1}{l+1}\right)I^{l+1}}$

References

Acemoglu, Daron, and Pascual Restrepo. "Modeling automation." AEA Papers and Proceedings. Vol. 108. 2018.

Acemoglu, Daron, and Pascual Restrepo. "Artificial intelligence, automation, and work." The economics of artificial intelligence: An agenda. University of Chicago Press, 2018. 197-236.

Acemoglu, Daron, and David Autor. "Skills, tasks and technologies: Implications for employment and earnings." Handbook of labor economics. Vol. 4. Elsevier, 2011. 1043-1171.

Acemoglu, Daron, and Pascual Restrepo. "The race between man and machine: Implications of technology for growth, factor shares, and employment." American Economic Review 108.6 (2018): 1488-1542.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. "Trends in US wage inequality: Revising the revisionists." The Review of economics and statistics 90.2 (2008): 300-323.

Frey, Carl Benedikt, and Michael Osborne. "The future of employment." (2013).

Lucas Jr, Robert E. "On the mechanics of economic development." Journal of monetary economics 22.1 (1988): 3-42.

Romer, Paul M. "Increasing returns and long-run growth." Journal of political economy 94.5 (1986): 1002-1037.

Solow, Robert M. "A contribution to the theory of economic growth." The quarterly journal of economics 70.1 (1956): 65-94.

Zeira, Joseph. "Workers, machines, and economic growth." The Quarterly Journal of Economics 113.4 (1998): 1091-1117.