Endogenous childcare costs in RD based model

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abstract

In an AI society, ICT is being introduced in all sectors. This trend is expected to significantly impact an aging society with a declining birthrate, which is expected to accelerate in the future. The development of medical care and improvements in diet will promote longer life expectancy, while the spread of online services will promote more efficient labor, and the development of home appliances will greatly reduce the burden of housework and childcare. In this paper, we analyze how the increase in longevity and disposable time of households in an AI society will affect the decline in fertility based on an R&D model.

Keywords: Endogenous Childcare cost - Endogenous lifetime - Two-sector growth model

JEL classification: E00 - O11 - P23

1. Introduction

As is well known, fertility rates in developed countries have been declining rapidly over the past half-century, especially in South Korea, where the total fertility rate in 2020 will be 0.92, and in Japan, where it will be 1.33. It is due to unmarried and late marriages, which may be attributed to the relative opportunity cost of having children due to the diversity of values and declining incomes associated with lower economic growth rates. The government's policy to reduce the birthrate provides free nursery schools, compulsory education, and childcare allowances in kind or cash. However, the effects of these policies are far from clear. On the other hand, life expectancy increases rapidly due to the expansion of medical technology and the quality and variety of food products. 2020 life expectancy in Japan will be 87.74 years for women and 81.64 years for men, increasing for the ninth consecutive year. These are the factors that are thought to be creating a declining birthrate and aging population. The main challenges of an aging society with a declining birthrate include a decline in economic growth due to a shrinking working population, insufficient financial resources for social security, and reduced public capital investment due to declining tax revenues. In recent years, investment in AI (artificial intelligence), IT (Information Technology), and ICT (Information and Communication Technology) has increased rapidly in developed countries. In the United States, in particular, ICT investment has nearly doubled in the past 30 years. ICT and IT are terms that mean almost the same thing, but they are used differently in specific terms, and the key is what they focus on. IT is a term that refers to computer-related technology itself, including hardware, software, and infrastructure.

On the other hand, ICT emphasizes the transmission of information and refers to how technology is used in healthcare, education, and other areas or the methodology for doing so. The total fertility rate was 1.84 in 1980 and 1.71 in 2019. Although the total fertility rate is declining, the rate of decrease is smaller than in Japan. IT investment in the U.S. and Japan in 2017 shows that Japan has about one-third the IT investment and about half the GDP per capita of the U.S. This gap has continued to widen since 2000. From the above data, it can be read that raising GDP through IT investment increases disposable income, which in turn increases fertility and survival rates. Specifically, increased disposable income can purchase new home appliances developed through innovation, reducing the time spent on household chores or opportunity costs. Alternatively, parents can reduce the amount of time they spend on child-rearing and education by hiring babysitters when they can afford to or through the spread of AI-based education systems.

On the other hand, life expectancy is expected to increase due to the development of medical technology through IT investment, improved food quality, variety, and improved care functions through AI. Hirazawa and Yakita (2017) show a positive correlation exists between income and survival, and they endogenized its viability. In this study, we analyze how the opportunity cost of childcare is endogenized in the endogenization of their survival rate and how innovation through research and development, such as IT investment, affects demographics through the endogenization of the two. We keep it simple and build on the endogenous length model based on Romer's (1990) R&D, incorporating the opportunity cost of childcare and endogenization of survival. Romer (1990), in his dynamic equation in the quantity of innovation in the R&D sector, argued that the incremental amount of innovation depends frankly on developers and the quantity of existing innovation; furthermore, the population growth rate is assumed to be zero and that the rate of economic growth is always constant. His model relies heavily on the scale effect. Jones (1995) considers diminishing returns on the number of developers and the amount of existing innovation and shows that the equilibrium growth path is stable and converges to it through time. We define innovation dynamics equations to depend not on the number of researchers per se but the ratio of researchers to the total workforce. The percentage of researchers is also increasing in the U.S. and France, where IT investment is increasing rapidly. Trade-offs occur as innovation (economic growth rate) increases in our model. It is an increase in survival and a decrease in the cost of childcare. As income increases due to innovation, survival increases, and households increase their savings for old age. It increases capital, which increases the output of final goods and thus boosts GDP, but the decrease in disposable income during the working years leads to a decrease in the number of children as a luxury good. In other words, the effect on the population growth rate is negative.

On the other hand, the decrease in childcare costs associated with increased innovation lowers the opportunity cost of childcare and increases income during the working period. Its effect on population growth is positive. Therefore, this study analyzes the rate of decline of the child care cost for the population growth rate to be positive. In the model, we use a two-sector model based on the R&D model, and we use Diamond's (1965) two-period overlapping generations model. The first period is the work period, and households holding one unit of time allocate it to work or childcare. Childhood is not considered here because it is assumed to be dependent on parents and accounted for identically in the household. The opportunity cost of childcare in the first generation is endogenized and is assumed to decrease as income increases. The second generation is assumed to be old and does not engage in labor. The survival rate in the second period is assumed to be endogenous and increases with income.

2. Model

2.1 Individuals

This study uses a life-cycle model, which defines survival as constant and determines savings and consumption based on budget constraints at the point in time when the economic agent is in existence; the OLG model uses a two-generation overlap model with two periods of survival and three or more periods of Multiple generation overlap models exist. In this model, the schooling period is excluded, and the model starts from the point when people enter society and begin working. In other words, it is a long-term analysis with a single period of 30 to 40 years. In addition, household heterogeneity is excluded here because a representative household is assumed. The OLG model was first developed by Samuelson (1958), and his model is characterized by the fact that it does not consider capital accumulation since there is no production sector, and a fixed number of non-durable goods are provided each period. The optimal solution of the model is for each generation in each period to consume all the given consumption goods. The intergenerational externality he showed us is that the generation in the second-period issues credits to obtain consumption goods from the generation in the first period. This externality assumes that a generation exists after an infinite period and that the population growth rate is positive, making it a Ponzi Game. One of Diamond's model characteristics is the introduction of capital accumulation: subjects in the two periods consume, but their budget is income from labor supply in the first generation and capital income in the second generation. (The sum of principal and interest income.) The first-period generation allocates income from labor to consumption or savings for the second period. One common feature of both models is the existence of an externality of intergenerational income transfers, and the possibility exists that the competitive equilibrium is not Pareto optimal because of “over-accumulation”. The policy is that the optimal solution is sought through government intervention and intergenerational redistribution in the form of social security. However, as the capital accumulation progresses and harvest diminishes, the equilibrium value becomes less than the "Golden rule" (the value of capital that would maximize aggregate consumption in macroeconomic terms) when the population growth rate is greater than the interest rate. It is since each generation focuses only on its utility and does not view the economy from the perspective of the next generation or the long term.

We set up a two-period OLG model consisting of adult and old. In the first period, which includes childhood, each individual has one unit of time allocated to work or childcare. In this analysis, there is no distinction between males and females, and each individual determines how many children he or she will have. This analysis does not distinguish between men and women but determines how many children each individual will have. Child-rearing needs time and goods. Income earned from labor is distributed to consumption in both periods. From the above, an individual's utility depends on the consumption in each period and the number of children in the first period. The utility function is shown as follows:

$$u_t = \log c_t + \beta \log d_t + \gamma \log n_t \,, \tag{1}$$

where $\beta > 0,\ \gamma > 0$ indicates the preference rate for the second period's consumption and the number of children. When each individual in the first period raises units of children, they must incur δn_t units of final goods and ρn_t units of time. Furthermore, w_t indicates the wage rate and the disposable working income will be $(1-\rho n_t)w_t$, and consumption and savings are represented by c_t and s_t. We will assume here $\rho(w_t)$ that the childcare cost depends on the wage rate. Moreover, childcare costs diminish as income increases, $\rho'(w_t) < 0,\ \rho''(w_t) > 0$. For example, private nursery schools, babysitters, and improvements in the quality of home appliances, housing, cars, and other products that come with AI will increase disposable time. Thus, the budget constraint for each individual in the first period is shown as follows:

$$c_{1,t} + s_t + \delta n_t = [1 - \rho(w_t) n_t] w_t \,. \tag{2}$$

Now, let us rewrite the equation, focusing on the number of children where δn_t represents the cost to have children because it is the same as luxury goods. Furthermore, raising children needs the opportunity cost of parent and this time indicates ρ to raise children. Parents' expense of cost per child is marked as $\rho w_t + \delta$. We consider a perfectly competitive insurance market shown by Yaari (1965) and Blanchard (1985). This market will deposit savings at the last period. Insurance companies invest their savings in a capital market, and the return on that investment will be paid to the surviving individuals in a second period. Details will be discussed later; according to Hirazawa and Yakita (2017), the survival rate is considered dependent on the wage rate, $\lambda_t(w_t)$. Therefore, the second-period budget constraint is shown as follows:

$$c_{2,t+1} = \frac{1 + r_{t+1}}{\lambda(w_t)} s_t \,. \tag{3}$$

By substituting the budget constraints in both periods for utility function, we can rewrite it as follows:

$$\max_{s_t, n_t} \log\{[1-\rho(w_t)n_t]w_t - \delta n_t - s_t\} + \beta\lambda(w_t)\log\left[\frac{1+r_{t+1}}{\lambda(w_t)}s_t\right] + \gamma \log n_t \,. \tag{4}$$

Solving the optimization problem concerning savings and the number of children yields the following optimal solution.

$$s_t = \frac{\beta\lambda(w_t)w_t}{1+\beta\lambda(w_t)+\gamma}, \tag{5}$$

$$n_t = \frac{\gamma w_t}{[1+\beta\lambda(w_t)+\gamma][\rho(w_t)w_t+\delta]}. \tag{6}$$

1.2 Productions

2.2.1 Final goods sector

This model has three sectors; a final-goods sector produces the consumption/capita good using labor. Furthermore, an intermediate-goods sector has monopoly firms and an R&D sector indicated by Romer (1990) and Jones (1995). First, the final goods market is perfectly competitive, and the production function is shown as follows:

$$Y_t = L_{Y,t}^{1-\alpha}\int_0^{A_t} x_{j,t}^{\alpha}\, dj\,, \qquad 0<\alpha<1, \tag{7}$$

The above production function can be rewritten as follows:

$$Y_t = L_{Y,t}^{1-\alpha}A_t\left(\frac{K_t}{A_t}\right)^{\alpha} = \left(A_t L_{Y,t}\right)^{1-\alpha}K_t^{\alpha}, \qquad 0<\alpha<1, \tag{8}$$

Taking the logarithm of both sides of the above equation and differentiating to time yields the next equation:

$$\log Y_t = (1-\alpha)\log A_t + (1-\alpha)\log L_{Y,t} + \alpha\log K_t \tag{9}$$

$$\frac{\dot{Y}_t}{Y_t} = (1-\alpha)\frac{\dot{A}_t}{A_t} + (1-\alpha)\frac{\dot{L}_t}{L_t} + \alpha\frac{\dot{K}_t}{K_t} \tag{10}$$

where the growth rate of total factor productivity is denoted by $(1-\alpha)\frac{\dot{A}_t}{A_t}$. As discussed later, TFP is determined by the ratio of researchers to all workers. It is, therefore, very different from the Solow model, which depends on the rate of population growth and the rate of technological progress.

$$w_t = (1-\alpha)L_{Y,t}^{-\alpha}\int_0^{A_t} x_{j,t}^{\alpha}\, dj = (1-\alpha)\frac{Y_t}{L_{Y,t}}\,, \tag{11}$$

$$q_{j,t} = \alpha L_{Y,t}^{1-\alpha}x_{j,t}^{\alpha-1}\,, \tag{12}$$

where w_t represents the wage rate and $q_{j,t}$ is the price of the j th intermediate good. The demand function of its intermediate good is shown as follows:

$$x_{j,t} = \left(\frac{\alpha}{w_t}\right)^{\frac{1}{1-\alpha}} L_{Y,t} \ . \tag{13}$$

2.2.2 Intermediate goods sector

We consider intermediate good firms with different interests, and this market is monopolistic competition. Each company issues stock to raise funds and employs a labor unit, producing various consumer goods. When a new blueprint is created, the firm in an R&D sector accepts a patent from the government that allows it to produce the new type of intermediate good exclusively. The patent is assumed to be valid indefinitely in this model for simplicity. The patent is then sold to the firm in a middle sector. The firm's profit function in an intermediate goods sector is as follows:

$$max \qquad \pi_{j,t} = q_{j,t}(x_{j,t})x_{j,t} - w_t x_{j,t} \ , \tag{14}$$

$$s,t \qquad x_{j,t} = \left(\frac{\alpha}{q_{j,t}}\right)^{\frac{1}{1-\alpha}} L_{Y,t} \ , \tag{15}$$

where $q_{j,t}(x_{j,t})$ indicates the demand function of an intermediate goods. The first order condition is as follows:

$$\frac{dq_{j,t}(x_{j,t})}{dx_{j,t}} x_{j,t} + q_{j,t} - w_t = 0 \ , \tag{16}$$

Then solve the above equation for $q_{j,t}$.

$$q_{j,t} = \frac{1}{1 + \left[\frac{dq_{j,t}(x_{j,t})}{dx_{j,t}} \frac{x_{j,t}}{q_{j,t}}\right]} w_t \ , \tag{17}$$

where the braces indicate an elasticity of demand for a price. Then, the above equation can be rewritten by the following equation:

$$\frac{dq_t}{dx_t}\frac{x_t}{q_t} = (\alpha - 1) \ , \tag{18}$$

$$q_{j,t} = \frac{1}{1 + (\alpha - 1)} w_t, \leftrightarrow \quad q_{j,t} = q_t = \frac{1}{\alpha} w_t \ , \tag{19}$$

Next, we substitute the above equation into the demand function for the intermediate good to obtain the following equation:

$$x_{j,t} = x_t = \left(\frac{\alpha^2}{w_t}\right)^{\frac{1}{1-\alpha}} L_{Y,t} \ , \tag{20}$$

Furthermore, the profit function can be rewritten as follows:

$$\pi_{j,t} = \pi_t = \frac{1-\alpha}{\alpha} w_t x_t \, . \tag{21}$$

The value of the blueprint in period t should be equal to the present value of a profit v_t that the firms in an intermediate sector can earn by purchasing it.

$$v_t = \sum_{\tau=t+1}^{\infty} \frac{\pi_\tau}{\prod_{u=t+1}^{\tau}(1+r_u)} . \tag{22}$$

By using the formula for the sum of infinite geometric series, the above equation can be rewritten as follows:

$$v_t = \frac{\pi_{t+1}}{(1+r_{t+1})} + \frac{\pi_{t+2}}{(1+r_{t+1})(1+r_{t+2})} + \cdots \frac{v_{t+1}}{(1+r_{t+1})} + \frac{v_{t+2}}{(1+r_{t+1})(1+r_{t+2})} + \cdots \infty, \tag{23}$$

$$v_t = \frac{\frac{\pi_{t+1}}{(1+r_{t+1})}}{1-\frac{1}{(1+r_{t+1})}} + \frac{\frac{(v_{t+1}-v_t)}{(1+r_{t+1})}}{1-\frac{1}{(1+r_{t+1})}} = \frac{\frac{\pi_{t+1}}{(1+r_{t+1})}}{\frac{(1+r_{t+1})-1}{(1+r_{t+1})}} + \frac{\frac{(v_{t+1}-v_t)}{(1+r_{t+1})}}{\frac{(1+r_{t+1})-1}{(1+r_{t+1})}} = \frac{\pi_{t+1}}{r_{t+1}} + \frac{(v_{t+1}-v_t)}{r_{t+1}}, \tag{24}$$

Therefore, the no-arbitrage equation for the new blueprint, which indicates the current value of blueprint equals income gain π_{t+1} and capital gain $v_{t+1} - v_t$ in the future, is shown as follows:

$$r_{t+1} v_t = \pi_{t+1} + v_{t+1} - v_t \, , \tag{25}$$

where the left-hand side shows the return of the invested funds which were deposited in a financial institution for only one period, and the right-hand side shows the sum of the profit earned by purchasing the patent of blueprint, using it in production for one period, and the yield gained by selling it immediately afterward. In equilibrium, these coincide.

2.2.3 R&D sector

The R&D-based models in the endogenous growth are indicated by Romer (1990), Grossman and Helpman (1991a, 1991b, 1991c), and Aghion and Howitt (1992). Romer (1990) distinguishes between skilled and unskilled workers and states that only skilled workers will be engaged in the R&D sector. Furthermore, In Romer (1990), there is a scale effect, i.e., if the population growth rate is positive, the economic growth rate is also positive, whereas, in Jones (1995), the growth rate is independent of the number of laborers but depend on the labor growth rate. The developed knowledge is a new type of capital good, and A_t denotes its quantity which offers an increase of input in an R&D sector boosts productivity proportionally. The difference equation in the blueprint is shown as follows:

$$A_{t+1} - A_t = \delta\theta_t \frac{L_{At}}{L_t} = \delta\theta_t\mu_t\ , \tag{26}$$

where $L_{A,t}$ represents the number of laborers in the R&D sector. $\mu_t > 0$ is the labor ratio in the R&D sector to the total number of laborers. In other words, we consider that the R&D sector, which is the foundation of economic growth, should incorporate the researcher's ratio to other sectors, not the number of researchers involved in the sector itself. Moreover, we consider the productivity of firms in the R&D sector depends on existing knowledge produced previous R&D production indicated by Romer (1990), Grossman and Helpman (1991), and Jones (1995). Individuals take advantage of the existing stock of knowledge to invest in new designs. The productivity of firms in an R&D sector is indicated as follows:

$$\theta_t = A_t^{\phi}, \tag{27}$$

where A_t represents the stock of technological knowledge. Jones (1995) showed that $\phi < 1$ means fishing out in which the rate of innovation decreases with knowledge. Furthermore, $\phi > 0$ indicates the positive external returns (In other words, standing on the shoulders of giants). $\phi = 0$ means the constant returns to scale (there is no externality) in which the arrival rate of an additional idea is independent of the stock. Romer (1990) defines it as $\phi = 1$, but the external effect is too large to be realistic. We consider the firms in an R&D sector that create a new variety of intermediate goods. This sector's market is perfectly competitive, and firms in intermediate goods demand these blueprints, which the firms in an R&D sector create. When a new blueprint is discovered, firms receive from the government the exclusive right to produce fresh intermediate good and sell it to firms in an intermediate goods sector: it is said as a patent. The number of blueprints developed equals the number of intermediate goods because they are purchased by firms that produce intermediate goods. The firm's profit function in the R&D sector is presented as follows:

$$\pi_t^A = \upsilon_t(A_{t+1} - A_t) - w_t L_{A,t} = \upsilon_t\delta A_t^{\phi}\mu_t - w_t L_{A,t} = \left(\frac{\upsilon_t\delta A_t^{\emptyset}}{L_t} - w_t\right)L_{A,t} = \frac{\upsilon_t\delta L_{A,t}A_t^{\emptyset}}{L_t} - w_t L_{A,t}, \tag{28}$$

The profit-maximizing condition is indicated as follows:

$$\upsilon_t = \frac{w_t L_t}{\delta A_t^{\emptyset}} \tag{29}$$

3. Market

In the labor market, an individual's available supply of time is $1 - \rho(w_t)n_t$, and labor is employed in the final goods sector, medium goods sector, and R&D sector. The following equation represents the condition in the labor market equilibrium:

$$L_{Y,t} + A_t x_t + L_{A,t} = [1 - \rho(w_t)n_t]w_t N_t\ . \tag{30}$$

Aggregate savings will be invested or purchased in an R&D sector and asset market. The equilibrium condition is shown as follows:

$$(A_{t+1} - A_t)\upsilon_t + A_t\upsilon_t = A_{t+1}\upsilon_t = s_t N_t\ , \tag{31}$$

where N_t means the number of populations at t period. The equilibrium condition in a final goods sector is shown as follows:

$$Y_t = c_{1,t}N_t + c_{2,t}\lambda_{t-1}N_{t-1} + \delta n_t N_t \,, \tag{32}$$

The final goods are consumed by individuals in the t, t+1 period and used to raise children. We can rewrite the above equation by using the utility-maximizing conditions as follows:

$$Y_t = \left(\frac{\alpha^2}{w_t}\right) A_t L_{Y,t} \,, \tag{33}$$

Furthermore, we substitute the above equation for profit-maximizing condition as follows:

$$w_t = (1-\alpha)^{1-\alpha} A_t^{1-\alpha} \alpha^{2\alpha(1-\alpha)}, \tag{34}$$

$$w_t = \hat{\alpha} A_t^{1-\alpha}, \tag{35}$$

Where $\hat{\alpha} \equiv \alpha^{2\alpha}(1-\alpha)^{1-\alpha}$ and the growth rate of a blueprint in an R&D sector can be rewrite by next equation:

$$g_{A,t} \equiv \frac{(A_{t+1} - A_t)}{A_t}, \tag{36}$$

$$1 + g_{A,t} = \frac{A_{t+1}}{A_t} = \frac{s_t N_t}{v_t} = \frac{\delta\beta\lambda(w_t)}{[1+\beta\lambda(w_t)+\gamma]L_t} N_t A_t^{\emptyset-1} \,, \tag{37}$$

where per capita labor supply, as shown in the household sector, subtracts the cost of childcare from a unit of time owned, and the overall labor supply is then multiplied by the number of adult populations. Taking these into account, we obtain the following equation.

$$1 + g_{A,t} = \frac{\delta\beta\lambda(w_t)}{[1+\beta\lambda(w_t)+\gamma][1-\rho(w_t)n_t]} A_t^{\emptyset-1} \,, \tag{38}$$

The growth of the population can be indicated as follows:

$$1 + g_L = 1 + \frac{N_{t+1} - N_t}{N_t} = n_t, \tag{39}$$

Therefore, the dynamic equation of the population is as follows:

$$N_{t+1} = n_t N_t. \tag{40}$$

4. Relationship between fertility and per capita wage income

Following Hirazawa and Yakita (2017), we define $\lambda(w_t)$ as follows:

$$\lambda(w_t) = \frac{v}{1 + \chi e^{-\psi w_t}}, \tag{41}$$

where $\nu \in (0,1]$ and $\chi,\ \psi > 0$. We will differentiate the survival rate λ concerning wage rate.

$$\lambda'(w_t) = \frac{\nu\psi\chi e^{-\psi w_t}}{(1 + \chi e^{-\psi w_t})^2} > 0\,, \tag{42}$$

$$\lambda''(w_t) = \frac{\nu\psi^2\chi e^{-\psi w_t}}{(1 + \chi e^{-\psi w_t})^3}\left(\chi e^{-\psi w_t} - 1\right), \tag{43}$$

where if $w_\lambda \equiv \frac{1}{\psi}\log\chi$, $(e^{\psi w_\lambda} \equiv \chi)$, the equation can be rewritten as follows:

$$\lambda''(w_t) = \nu\psi^2\left(\chi e^{-\psi w_\lambda} - 1\right) = 0\,, \tag{44}$$

where $e \cong 2.718$ is Napier's constant, the bottom of the natural logarithm, and the range of values of $e^{-\psi w_t}$ when $\psi w_t > 0$ is $0 < e^{-\psi w_t} < 1$. Therefore, if $0 < \chi \leq 1$, the sign of λ " becomes negative. Then, the relationship between w and λ is concave for any value of w_t. Similarly, if $\chi > 1$ and $w_\lambda > \frac{1}{\psi}\log\chi$, then it indicates $\lambda''(w_t) < 0$. If $\chi > 1$ and $w_\lambda < \frac{1}{\psi}\log\chi$, then it shows $\lambda''(w_t) > 0$. In other words, w_λ indicates the threshold between concave and convex functions. The value of λ when the wage is approximated to zero and infinity is as follows:

$$\lim_{w_t \to 0} \lambda(w_t) \equiv \underline{\lambda} = \frac{\nu}{1 + \chi}\,, \tag{45}$$

$$\lim_{w_t \to +\infty} \lambda(w_t) \equiv \overline{\lambda} = \nu\,. \tag{46}$$

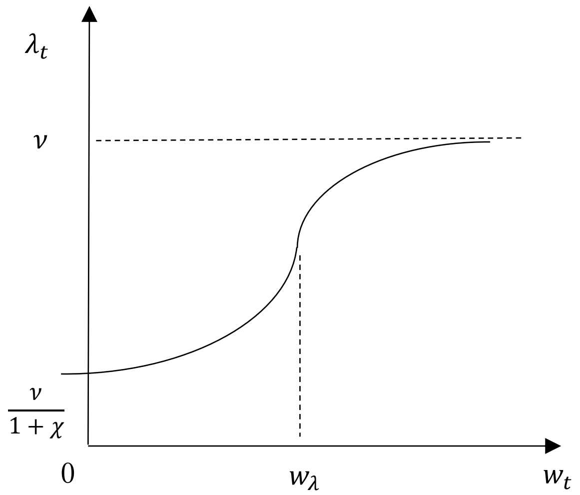

Fig1. The case of $\chi > 1$.

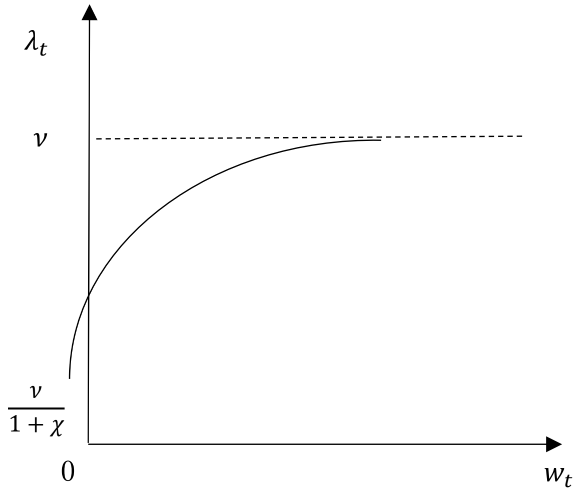

Fig2. The case of $0 < \chi \leq 1$

Next, we analyze the parenting time function, and the function is shown as follows:

$$\rho(w_t) = \chi e^{-\varphi w_t} + \epsilon w_t \tag{47}$$

where $\nu \in (0,1]$ and $\chi,\ \psi > 0$. We will differentiate the survival rate λ concerning wage rate.

$$\rho'(w_t) = -\varphi\, \chi e^{-\varphi w_t} + \epsilon < 0, \tag{48}$$

$$\rho''(w_t) = \varphi^{\,2} \chi e^{-\varphi w_t} > 0, \tag{49}$$

where if $w_\rho \equiv -\frac{1}{\varphi}\log \chi$, $(e^{-\varphi w_\rho} \equiv \chi)$, the equation can be rewritten as follows:

$$\rho''(w_t) = \frac{\chi e^{\varphi w_\rho} - 1}{(1 + \chi e^{-\varphi w_\rho})^3} = 0, \tag{50}$$

where $e \cong 2.718$ is Napier's constant, the bottom of the natural logarithm, and the range of values of $e^{-\psi w_t}$ when $\psi w_t > 0$ is $0 < e^{-\psi w_t} < 1$. Therefore, if $0 < \chi \leq 1$, the sign of ρ " becomes negative. Then, the relationship between w and λ is convex for any value of w_t. Similarly, if $\chi > 1$ and $w_\rho > \frac{1}{\psi}\log \chi$, then it indicates $\rho''(w_t) < 0$. If $\chi > 1$ and $w_\rho < \frac{1}{\psi}\log \chi$, then it shows $\rho''(w_t) > 0$. In other words, w_ρ indicates the threshold between concave and convex functions. The value of ρ when the wage is approximated to zero and infinity is as follows:

$$\lim_{w_t \to 0} \rho(w_t) \equiv \underline{\rho} = \frac{\nu}{2}\ , \tag{51}$$

$$\lim_{w_t \to +\infty} \rho(w_t) \equiv \overline{\rho} = \infty\ . \tag{52}$$

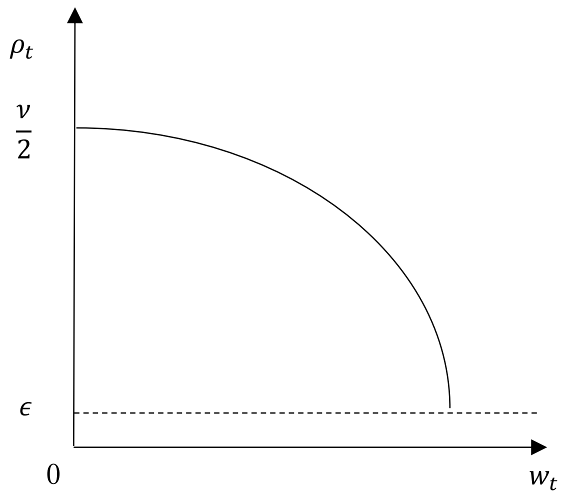

Fig.3.

5. Balanced Growth Path (Long-run growth)

This chapter analyzes the long-term suburbs: we derive the number of children in A when income goes to infinity. (53)

$$\lim_{w_t \to \infty} n_t = n^* = \infty \,.$$

To investigate whether $A_{t+1} > A_t$ or $A_{t+1} = A_t$ at each point of the (A_t, N_t) plane, we set $g_{A,t} = 0$ in Eq. (38) as follows:

$$\mathrm{F}(A_t) \equiv n_t = 1 - \frac{\delta\beta\lambda(w_t)A_t^{\emptyset-1}}{[1+\beta\lambda(w_t)+\gamma]\rho(w_t)}, \tag{54}$$

Above this nonlinear curve, ideas continue to emerge ($A_{t+1} > A_t$). Below the curve, ideas remain constant ($A_{t+1} = A_t$). We differentiate $\mathrm{F}(A_t)$ concerning A_t using $w_t = \hat{\alpha}A_t^{1-\alpha}$as follows:

$$\mathrm{F}'(A_t) \equiv \frac{d\mathrm{F}(A_t)}{dA_t} = \frac{\beta\delta\rho(w_t)[1+\beta\lambda(w_t)+\gamma]\left[(1-\emptyset)\lambda(w_t)A_t^{\emptyset-2} - \lambda'(w_t)\hat{\alpha}(1-\alpha)A_t^{\emptyset-\alpha-1}\right]}{\{[1+\beta\lambda(w_t)+\gamma]\rho(w_t)\}^2}$$

$$+\frac{A_t^{\emptyset-\alpha-1}\beta\delta\lambda(w_t)(1-\alpha)\hat{\alpha}\{[1+\beta\lambda(w_t)+\gamma]\rho'(w_t) + \rho(w_t)\beta\lambda'(w_t)\}}{\{[1+\beta\lambda(w_t)+\gamma]\rho(w_t)\}^2} \tag{55}$$

In the first item of the above equation, the conditional expression for the sign to be positive is shown.

$$(1-\emptyset)\lambda(w_t)A_t^{\emptyset-2} > \lambda'(w_t)\hat{\alpha}(1-\alpha)A_t^{\emptyset-\alpha-1} \tag{56}$$

$$\lambda'(w_t) < \frac{(1-\emptyset)\lambda(w_t)}{\hat{\alpha}(1-\alpha)A_t^{1-\alpha}} \tag{57}$$

The condition for the sign of the second item to be positive is then shown as follows:

$$-[1+\beta\lambda(w_t)+\gamma]\rho'^{(w_t)} < \rho(w_t)\beta\lambda'(w_t), \tag{58}$$

$$\rho'(w_t) > -\frac{\beta\rho(w_t)\lambda'(w_t)}{[1+\beta\lambda(w_t)+\gamma]}, \tag{59}$$

$$\frac{(1-\emptyset)\lambda(w_t)}{\hat{\alpha}(1-\alpha)A_t^{1-\alpha}} > \frac{\lambda'(w_t)}{\rho'(w_t)} > -\frac{[1+\beta\lambda(w_t)+\gamma]}{\beta\rho(w_t)}, \tag{60}$$

In the above equation, the sign of the ratio of survival rate to marginal childcare costs in the middle is negative, so the sign on the leftmost side is satisfied and is rewritten as follows:

$$\frac{\lambda'(w_t)}{\rho'(w_t)} > -\frac{[1+\beta\lambda(w_t)+\gamma]}{\beta\rho(w_t)}, \tag{61}$$

In the above equation, it is intuitively clear that this inequality holds if the marginal childcare cost reduction is very large. To analyze the economy in the long run, we derive the values of Eq. (42) and Eq. (48) infinitely far in time.

$$\lim_{w_t\to\infty}\lambda'(w_t) = \lim_{w_t\to\infty}\frac{\nu\psi\chi e^{-\psi w_t}}{(1+\chi e^{-\psi w_t})^2} = 0, \tag{62}$$

$$\lim_{w_t\to\infty}\rho'(w_t) = \lim_{w_t\to\infty} -\varphi\,\chi e^{-\varphi w_t} + \epsilon = \epsilon > 0, \tag{63}$$

where consider A again using the above equation.

$$0 = \frac{\lambda'(w_t)}{\rho'(w_t)} > -\frac{[1+\beta\lambda(w_t)+\gamma]}{\beta\rho(w_t)}, \tag{64}$$

where clearly, this inequality is satisfied. In other words, more R&D (innovation) indicates an increasing birth rate, resulting in positive population growth. It indicates that the fertility rate will increase if the rate of increase in efficiency of housework and childcare, which is associated with technological advances in AI and IT, increases more than the survival rate in the long run.

6. Concluding remarks

This study used a two-sector model based on an R&D model to analyze how economic growth associated with innovation (AI-Imation and IT-Imation) affects demographics. Primarily, we endogenized survival and child care opportunity costs to model the impact of increased innovation on them. A trade-off relationship existed between these two impacts on population growth rates, and conditions for population growth (expansion of fertility) were derived. In conclusion, we showed that population growth is possible if the increase in innovation converges to a constant value in the long term.

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