Wage-rise contract and mixed Cournot duopoly competition with profit-maximizing and socially concerned firms

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Abstract
This paper investigates a Cournot game model with a nonlinear demand function where a profit-maximizing firm competes against a socially concerned firm. The timing of the game is as follows. In stage one, each firm non-cooperatively decides whether to offer a wage-rise contract policy as a strategic commitment device. In stage two, after observing the rival’s decision in stage one, each firm non-cooperatively chooses its actual output. The paper presents the equilibrium solutions of the model.

Keywords: Cournot model; Corporate social responsibility; Profit-maximizing firm; Socially concerned firm; Wage-rise contract
JEL classification: C72; D21; L20

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1. Introduction

There have been some theoretical studies that incorporate socially concerned firms well known for their corporate social responsibility activities (see, for example, Fanti and Buccella, 2018; Garcia, Leal and Lee, 2019; Goering, 2007; Han, 2019; Kopel and Brand, 2012; Kopel, Lamantia and Szidarovszky, 2014; Kopel, 2015; Lamberti and Tampieri, 2012; Planer-Friedrich and Sahm, 2018; Xu, 2014). Each socially concerned firm maximizes its own profit plus a fraction of consumer surplus. For example, Lambertini and Tampieri (2012) examine a Cournot oligopoly market with pollution where a socially concerned firm competes with profit-maximizing firms. Lambertini and Tampieri show that the socially concerned firm obtains higher profits compared to profit-maximizing firms and its presence improves economic welfare. Kopel, Lamantia and Szidarovszky (2014) consider a Cournot oligopoly model in which profit-maximizing firms and socially concerned firms coexist and compete with each other. It is shown that socially concerned firms can have larger market shares and profits than profit-maximizing firms. Fanti and Buccella (2018) consider a two-stage game model in which two firms produce homogeneous network goods. In the first stage, each firm chooses its level of corporate social responsibility, and in the second stage, standard Cournot competition takes place. It is shown that the equilibrium in which both firms have social concerns is more profitable than simple profit-seeking for sufficiently intense network externalities. Furthermore, Planer-Friedrich and Sahm (2018) consider a three-stage duopoly model in which two profit-maximizing firms compete with each other. In the first stage, each firm simultaneously and independently determines its additional objective function, choosing either all consumers or their own customers only. In the second stage, each firm simultaneously and independently decides upon the weight of the additional objective function chosen in the first stage. In the third stage, each firm simultaneously and independently decides upon its output level. Planer-Friedrich and Sahm show that each firm prefers to care for all consumers; that is, it acts like a socially concerned firm.
In the present paper, I consider a Cournot game model with a nonlinear demand function where a profit-maximizing firm competes against a socially concerned firm. The timing of the game is as follows. In the first stage, each firm non-cooperatively decides whether to offer a wage-rise contract policy (WRCP) as a strategic commitment device. In the second stage, after observing the rival’s decision in the first stage, each firm non-cooperatively chooses its actual output. The purpose of this study is to present the equilibrium solutions of the model.

The remainder of the paper is organized as follows. Section 2 introduces the basic setting of the model. Section 3 provides supplementary explanations of the model. Section 4 discusses the equilibrium outcomes of the model. Finally, the paper is concluded in Section 5.

2. Model

I consider a mixed market composed of a profit-maximizing firm (firm 1) and a socially concerned firm (firm 2). Entry decisions are not considered. When \( i \) and \( j \) are used to refer to firms in an expression, they should be understood to represent (firm) 1 and (firm) 2 with \( i \neq j \). The inverse demand or price function is given by \( p = a - Q^2 \), where \( a \in (Q^2, \infty) \) represents a constant parameter, and \( Q = \sum_{i=1}^{2} q_i \) represents the industry output.

The market is modelled by means of the following two-stage game. In stage one, each firm non-cooperatively decides whether or not to adopt WRCP. If firm \( i \) adopts WRCP, then it chooses an output level \( q_i^* \in [0, \infty) \) and a wage premium rate \( t_i \in (0, \infty) \). Furthermore, firm \( i \) agrees to pay each employee a wage premium uniformly if it actually produces more than \( q_i^* \). In stage two, after observing the rival’s decision in stage one, each firm non-cooperatively chooses its actual output \( q_i \in (0, \infty) \).

\(^1\) For details see Ohnishi (2003, 2007).
Therefore, firm $i$’s profit function is given by
\[
\pi_i = \begin{cases} 
(a - Q^2)q_i - m_i q_i^2 & \text{for } q_i \leq q_i^*, \\
(a - Q^2)q_i - m_i q_i^2 - (q_i - q_i^*)t_i & \text{for } q_i \geq q_i^*, 
\end{cases}
\] (1)

where $m_i \in (0, \infty)$ represents firm $i$’s marginal cost of production. Firm 1 seeks to maximize (1).

Furthermore, firm 2’s objective function is given by
\[
V_2 = \pi_2 + \theta \cdot CS,
\] (2)
where $CS$ represents consumer surplus and $\theta \in [0, 1]$ is the share of the consumer surplus. Therefore, (1) can be rewritten by
\[
V_2 = \begin{cases} 
(a - Q^2)q_2 - m_2 q_2^2 + \theta \left[ \int_0^Q (a - X^2) dX - (a - Q^2)Q \right] & \text{for } q_2 \leq q_2^*, \\
(a - Q^2)q_2 - m_2 q_2^2 - (q_2^2 - q_2^*)t_2 + \theta \left[ \int_0^Q (a - X^2) dX - (a - Q^2)Q \right] & \text{for } q_2 \geq q_2^*. 
\end{cases}
\] (3)

In this paper, I use the well-known solution concept called subgame perfection.

3. Supplementary explanations

I first derive firm 1’s best reaction function from (1). Firm 1’s reaction function for $q_i < q_i^*$ is defined by
\[
R_i(q_2) = \arg \max_{q_i \geq 0} \left[ (a - Q^2)q_i - m_i q_i^2 \right],
\] (4)
and firm 1’s reaction function for $q_i > q_i^*$ is defined by
\[
R_i'(q_2) = \arg \max_{q_i \geq 0} \left[ (a - Q^2)q_i - (m_i + t_i) q_i^2 + t_i q_i^2 \right].
\] (5)

Therefore, if firm 1 selects $q_i^*$ and offers WRCP, then its best reply is given by
Firm 1 chooses $q_1$ in order to maximize $\pi_1$, given $q_2$. Hence, the first-order condition for (4) is
\[ a - 3q_1^2 - 4q_1q_2 - q_2^2 - 2m_1q_1 = 0, \tag{7} \]
and the second-order condition is
\[ -3q_1 - 2q_2 - m_1 < 0. \tag{8} \]
On the other hand, the first-order condition for (5) is
\[ a - 3q_1^2 - 4q_1q_2 - q_2^2 - 2m_1q_1 - 2t_1q_1 = 0, \tag{9} \]
and the second-order condition is
\[ -3q_1 - 2q_2 - m_1 - t_1 < 0. \tag{10} \]
Hence, I obtain
\[ R'_1(q_2) = \frac{-2q_1 - q_2}{3q_1 + 2q_2 + m_1} \tag{11} \]
and
\[ R''_1(q_2) = \frac{-2q_1 - q_2}{3q_1 + 2q_2 + m_1 + t_1}. \tag{12} \]
The following lemma is now immediate.

**Lemma 1:** Both $R_1(q_2)$ and $R'_1(q_2)$ are downward-sloping.

Second, I derive firm 2’s best reaction function from (3). Firm 2’s reaction function for $q_2 < q^*_2$ is defined by
\[ R_2(q_1) = \arg \max_{q_2 \in \mathbb{R}} \left\{ (a - Q^2)q_2 - m_2q_2^2 + \theta \left[ \int_0^Q (a - X^2) dX - (a - Q^2)Q \right] \right\}, \tag{13} \]
and firm 2’s reaction function for \( q_2 > q_2^* \) is defined by

\[
R_2^*(q_i) = \arg \max_{q_2 \geq 0} \left\{ (a - Q^2) q_2 - (m_2 + t_2) q_2^2 + t_2 q_2^2 + \theta \left[ \int_0^1 (a - X^2) dX - (a - Q^2) Q \right] \right\}. \tag{14}
\]

Hence, if firm 2 selects \( q_2^* \) and adopts WRCP, then its best reply is given by

\[
R_2^W (q_i) = \begin{cases} 
R_2 (q_i) & \text{for } q_2 < q_2^*, \\
q_2^* & \text{for } q_2 = q_2^*, \\
R_2^* (q_i) & \text{for } q_2 > q_2^*.
\end{cases} \tag{15}
\]

Firm 2 chooses \( q_2 \) in order to maximize \( V_2 \), given \( q_i \). Therefore, the first-order condition for (13) is

\[
a + 2\theta (q_1 + q_2)^2 - q_1^2 - 4q_1 q_2 - 3q_2^2 - 2m_2 q_2 = 0, \tag{16}
\]

and the second-order condition is

\[
2q_1 (\theta - 1) + q_2 (2\theta - 3) - m_2 < 0. \tag{17}
\]

On the other hand, the first-order condition for (14) is

\[
a + 2\theta (q_1 + q_2)^2 - q_1^2 - 4q_1 q_2 - 3q_2^2 - 2m_2 q_2 - 2t_2 q_2 = 0, \tag{18}
\]

and the second-order condition is

\[
2q_1 (\theta - 1) + q_2 (2\theta - 3) - m_2 - t_2 < 0. \tag{19}
\]

Hence, I have

\[
R'_2 (q_i) = -\frac{q_1 (2\theta - 1) + 2q_2 (\theta - 1)}{2q_1 (\theta - 1) + q_2 (2\theta - 3) - m_2} \tag{20}
\]

and

\[
R''_2 (q_i) = -\frac{q_1 (2\theta - 1) + 2q_2 (\theta - 1)}{2q_1 (\theta - 1) + q_2 (2\theta - 3) - m_2 - t_2}. \tag{21}
\]

The following lemma is presented.

**Lemma 2:** (i) If \( \theta < (2q_1 + q_2)/(2q_1 + q_2) \), then \( R_2 (q_i) \) and \( R'_2 (q_i) \) slope downwards.
(ii) If \( \theta > \left( 2q_1 + q_2 \right)/2 \left( q_1 + q_2 \right) \), then \( R_2(q_1) \) and \( R'_2(q_1) \) slope upwards.

Third, I prove the following two lemmas, which provide characterizations of WRCP as a strategic commitment device.

**Lemma 3:** If firm \( i \) adopts WRCP, then at equilibrium its actual quantity coincides with \( q_i^* \).

Proof: I consider the possibility that \( q_i \) is larger than \( q_i^* \). Firm 1’s profit is

\[
(a - Q_1^2)q_i - m_i q_i^2 - (q_i^2 - q_i^{*2}) \eta_i.
\]

Here, firm 1 must pay its employees wage premiums \( (q_i - q_i^*) \eta_i \). That is, firm 1 can improve its profit by raising \( q_i^* \), and the equilibrium point does not change in \( q_i \geq q_i^* \). Hence, \( q_i > q_i^* \) does not result in an equilibrium.

Next, I consider the possibility that \( q_i \) is smaller than \( q_i^* \). In this case, firm 1’s marginal cost of production does not change. It is impossible for firm 1 to change its output in equilibrium because such a strategy is not credible. That is, WRCP does not function as a strategic commitment.

The proof of firm 2 is similar to that of firm 1, and thus it is omitted here. Q.E.D.

**Lemma 4:** Firm \( i \)'s payoff maximizing output is lower when it offers WRCP than when it does not.

Proof: I prove the case for firm 2. From (3), it is seen that WRCP will never decrease firm 2’s marginal cost of production. The first-order condition for \( q_2 > q_2^* \) is
\[ a + 2\theta (q_1 + q_2)^2 - q_1^2 - 4q_1q_2 - 3q_2^2 - 2m_2q_2 - 2r_2q_2 = 0, \]

where \( t \) is positive. To satisfy the first-order condition,

\[ a + 2\theta (q_1 + q_2)^2 - q_1^2 - 4q_1q_2 - 3q_2^2 - 2m_2q_2 \]

must be positive.

Since the proof of firm 1 is very similar to that of firm 2, it is omitted here. Q.E.D.

### 4. Equilibrium outcomes

In this section, I consider the following two cases.

**Case 1:** \( \theta < \frac{(2q_1 + q_2)}{2(q_1 + q_2)} \)

**Case 2:** \( \theta > \frac{(2q_1 + q_2)}{2(q_1 + q_2)} \)

In Case 2, firm 1’s output is a strategic substitute, while firm 2 treats its output as a strategic complement.

#### 4.1. Case 1

This case is illustrated in Figure 1, where \( R_i \) is firm \( i \)’s reaction curve with no WRCP, \( \pi_i^A \) is firm 1’s iso-profit curve extending through \( A \), and \( V_2^A \) is firm 2’s iso-payoff curve extending through \( A \). For explanation, the figure is drawn simply. Since Case 1 is strategic substitutes in which goods are perfect substitutes; that is, \( R_i \) slopes down. Point \( A \) is the equilibrium solution with no WRCP as a strategic commitment device.

If firm 1 offers WRCP, its marginal cost of production increases and thus it decreases its output (Lemma 4). In Figure 1, if firm 1 chooses \( q_1^* \) and offers WRCP, then its reaction curve shifts down for \( q_1 > q_1^* \) and becomes the kinked bold lines. Therefore, firm 1’s unilateral solution can occur at a point like \( D \). From Figure 1, it is seen that firm 1’s profit is lower at
than at A while firm 2’s objective function value is higher at D than at A. Here, if firm 2 chooses $q_2^*$ and offers WRCP, then its reaction curve shifts to the left for $q_2 > q_2^*$ and becomes the kinked bold broken lines. Therefore, the bilateral WRCP solution can become B. Firm 2’s objective function value is lower at B than at D, while firm 1’s profit is higher at B than at D. Firm 2’s unilateral solution can occur at C. Firm 2’s objective function value is lower at C than at A, while firm 1’s profit is higher at C than at A. In addition, firm 1’s profit at B is lower than at C, while firm 2’s objective function value is higher at B than at C.

Each firm’s WRCP adoption decreases its optimal output. Given the output level of firm i, decreasing firm j’s output decreases the total market output. Furthermore, since firm i’s optimal strategy increases its output because of strategic substitutes, firm j’s payoff decreases. Since firm j’s WRCP adoption decreases its output and payoff, it never offers WRCP. Hence, the equilibrium occurs at A in Figure 1.

The equilibrium of this case can be stated in the following proposition.

**Proposition 1:** In $\theta < (2q_1 + q_2)/2(q_1 + q_2)$, there exists an equilibrium point in which neither firm offers WRCP as a strategic commitment.

Proof: Lemmas 1 and 2 (i) state that $R_i(q_j)$ and $R'_i(q_j)$ are downward-sloping, namely strategic substitutes. Lemma 4 states that firm i’s optimal output is lower when it offers WRCP than when it does not. Decreasing firm i’s output increases firm j’s amount of demand because of substitute goods, and the optimal strategy increases firm j’s output because of strategic substitutes. Increasing firm j’s output decreases firm i’s amount of demand and payoff. Firm j’s reaction function gives its optimal output for each output of firm i. In firm j’s reaction function, decreasing firm i’s output never decreases firm j’s output. Even if firm i’s output decreases,
since firm \( j \)'s output never decreases, firm \( i \)'s payoff decreases because of substitute goods. Thus, Proposition 1 is proved. Q.E.D

Proposition 1 says that neither firm offers WRCP in the case of \( \theta < \frac{2q_i + q_j}{2(q_i + q_j)} \).

Since firm \( i \)'s WRCP adoption increases its marginal cost of production, it decreases its optimal output; that is, firm \( i \) that adopts WRCP becomes a less aggressive competitor. If the adoption of WRCP makes firm \( i \) behave less aggressively, then firm \( j \) has an incentive to be more aggressive, and therefore firm \( i \) loses share in the market. Thus, neither firm offers WRCP.

4.2. Case 2

Case 2 is illustrated in Figure 2. In this case, firm 2’s output is a strategic complement. Point \( E \) is the equilibrium solution with no WRCP offered. Since firm \( i \)'s WRCP adoption increases its marginal cost of production, it decreases its optimal output (Lemma 4). On \( R_1 \), firm 1’s profit is highest at \( S \). If firm 1 chooses \( q_1^s \) and offers WRCP, then its reaction curve shifts down for \( q_1 > q_1^s \) and becomes the kinked bold lines. Hence, firm 1’s unilateral solution can occur at \( S \). From Figure 2, it is seen that firm 2’s objective function value is lower at \( S \) than at \( E \). On the other hand, if firm 2 chooses \( q_2^s \) and offers WRCP, then its reaction curve shifts to the right for \( q_2 > q_2^s \) and becomes the kinked bold broken lines. Therefore, firm 2’s unilateral solution can occur at \( F \). From Figure 2, it is seen that firm 1’s profit is higher at \( F \) than at \( S \). Hence, firm 1 has no incentive to choose \( q_2^s \) and offer WRCP.

The following result differs notably from that of Case 1.

**Proposition 2:** In \( \theta > \frac{2q_i + q_j}{2(q_i + q_j)} \), there exists an equilibrium point in which only
firm 2 offers WRCP as a strategic commitment.

Proof: I first consider firm 1’s Stackelberg leader output when neither firm offers WRCP. If firm 1 is the Stackelberg leader, then it maximizes \( \pi_i(q_1, R_2(q_1)) \) with respect to \( q_1 \).

Therefore, firm 1’s Stackelberg leader output satisfies the first-order condition:

\[
\frac{\partial \pi_1}{\partial q_1} + \frac{\partial \pi_1}{\partial q_2} \frac{\partial R_2}{\partial q_1} = 0.
\]

Here \( \frac{\partial \pi_1}{\partial q_2} \) is negative, and \( \frac{\partial R_2}{\partial q_1} \) is positive (Lemma 2 (ii)). To satisfy the first-order condition, \( \frac{\partial \pi_1}{\partial q_1} \) must be positive. Hence, firm 1’s Stackelberg leader output is smaller than its Cournot output. At equilibrium, \( q_1 \) coincides with \( q_1^* \) (Lemma 3). Furthermore, firm 1’s profit-maximizing output is lower when it adopts WRCP than when it does not (Lemma 4). A decrease in firm 1’s output is decided by the value of \( t_1 \). Let \( t_1 \) be a variable that can take any value more than zero. In \( R'_1 \), firm 1’s profit is highest at its Stackelberg leader point.

Therefore, if firm 2 does not offer WRCP, then firm 1 selects its Stackelberg solution and maximizes its profit. However, firm 2’s objective function value at firm 1’s Stackelberg solution is lower than at the Cournot-Nash solution with no WRCP. Hence, firm 2 does not want firm 1 to offer WRCP.

Next, I consider firm 2’s Stackelberg leader output when neither firm offers WRCP. If firm 2 is the Stackelberg leader, then it maximizes \( V_2(R_1(q_2, q_2)) \) with respect to \( q_2 \).

Therefore, firm 2’s Stackelberg leader output satisfies the first-order condition:

\[
\frac{\partial V_2}{\partial q_2} + \frac{\partial V_2}{\partial q_1} \frac{\partial R_1}{\partial q_2} = 0.
\]

Here, \( \frac{\partial V_2}{\partial q_1} \) is positive, and \( \frac{\partial R_1}{\partial q_2} \) is negative (Lemma 1). To satisfy the first-order condition, \( \frac{\partial V_2}{\partial q_1} \) must be positive. Hence, firm 2’s Stackelberg leader output is lower than
its Cournot output. At equilibrium, $q_2$ coincides with $q_2^*$ (Lemma 3). Furthermore, Lemma 4 shows that firm 2’s payoff-maximizing output is lower when it offers WRCP than when it does not. A decrease in firm 2’s output is decided by the value of $t_2$. Let $t_2$ be a variable that can take any value more than zero. In $R_1$, firm 2’s profit is highest at its Stackelberg leader point. Therefore, firm 2 chooses $q_2^{*S}$ corresponding to its Stackelberg solution and offers WRCP. Hence, firm 1’s profit will increase. Our equilibrium concept is subgame perfection and all information in the model is common knowledge. Thus, firm 1 has no incentive to offer WRCP, and only firm 2 adopts WRCP. Q.E.D.

5. Conclusion

I have examined two cases of a game model with a concave demand function where a profit-maximizing firm competes with a socially concerned firm. Each firm is allowed to offer WRCP as a strategic commitment. I have presented the respective equilibrium solutions of the two cases. I have found that the equilibrium of Case 2 may be profitable for both firms.

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Figure 1: $\theta < \left( \frac{2q_1 + q_2}{2(q_1 + q_2)} \right)$
Figure 2: \[ \theta > \frac{(2q_1 + q_2)}{2(q_1 + q_2)} \]