Wage-rise contract and mixed Cournot duopoly competition with profit-maximizing and socially concerned firms

Ohnishi, Kazuhiro

24 March 2022

Wage-rise contract and mixed Cournot duopoly competition with profit-maximizing and socially concerned firms

Kazuhiro Ohnishi*

Institute for Economic Sciences, Japan

Abstract

This paper investigates a Cournot game model with a nonlinear demand function where a profit-maximizing firm competes against a socially concerned firm. The timing of the game is as follows. In stage one, each firm non-cooperatively decides whether to offer a wage-rise contract policy as a strategic commitment device. In stage two, after observing the rival's decision in stage one, each firm non-cooperatively chooses its actual output. The paper presents the equilibrium solutions of the model.

Keywords: Cournot model; Corporate social responsibility; Profit-maximizing firm; Socially concerned firm; Wage-rise contract

JEL classification: C72; D21; L20

* Email: ohnishi@e.people.or.jp

1. Introduction

There have been some theoretical studies that incorporate socially concerned firms well known for their corporate social responsibility activities (see, for example, Fanti and Buccella, 2018; García, Leal and Lee, 2019; Goering, 2007; Han, 2019; Kopel and Brand, 2012; Kopel, Lamantia and Szidarovszky, 2014; Kopel, 2015; Lambertini and Tampieri, 2012; Planer-Friedrich and Sahm, 2018; Xu, 2014). Each socially concerned firm maximizes its own profit plus a fraction of consumer surplus. For example, Lambertini and Tampieri (2012) examine a Cournot oligopoly market with pollution where a socially concerned firm competes with profit-maximizing firms. Lambertini and Tampieri show that the socially concerned firm obtains higher profits compared to profit-maximizing firms and its presence improves economic welfare. Kopel, Lamantia and Szidarovszky (2014) consider a Cournot oligopoly model in which profit-maximizing firms and socially concerned firms coexist and compete with each other. It is shown that socially concerned firms can have larger market shares and profits than profit-maximizing firms. Fanti and Buccella (2018) consider a two-stage game model in which two firms produce homogeneous network goods. In the first stage, each firm chooses its level of corporate social responsibility, and in the second stage, standard Cournot competition takes place. It is shown that the equilibrium in which both firms have social concerns is more profitable than simple profit-seeking for sufficiently intense network externalities. Furthermore, Planer-Friedrich and Sahm (2018) consider a three-stage duopoly model in which two profit-maximizing firms compete with each other. In the first stage, each firm simultaneously and independently determines its additional objective function, choosing either all consumers or their own customers only. In the second stage, each firm simultaneously and independently decides upon the weight of the additional objective function chosen in the first stage. In the third stage, each firm simultaneously and independently decides upon its output level. Planer-Friedrich and Sahm show that each firm prefers to care for all consumers; that is, it acts like a socially concerned firm.

In the present paper, I consider a Cournot game model with a nonlinear demand function where a profit-maximizing firm competes against a socially concerned firm. The timing of the game is as follows. In the first stage, each firm non-cooperatively decides whether to offer a wage-rise contract policy (WRCP) as a strategic commitment device.[1] In the second stage, after observing the rival's decision in the first stage, each firm non-cooperatively chooses its actual output. The purpose of this study is to present the equilibrium solutions of the model.

The remainder of the paper is organized as follows. Section 2 introduces the basic setting of the model. Section 3 provides supplementary explanations of the model. Section 4 discusses the equilibrium outcomes of the model. Finally, the paper is concluded in Section 5.

2. Model

I consider a mixed market composed of a profit-maximizing firm (firm 1) and a socially concerned firm (firm 2). Entry decisions are not considered. When i and j are used to refer to firms in an expression, they should be understood to represent (firm) 1 and (firm) 2 with $i \neq j$. The inverse demand or price function is given by $p = a - Q^2$, where $a \in \left(Q^2, \infty\right)$ represents a constant parameter, and $Q = \sum_{i=1}^{2} q_i$ represents the industry output.

The market is modelled by means of the following two-stage game. In stage one, each firm non-cooperatively decides whether or not to adopt WRCP. If firm i adopts WRCP, then it chooses an output level $q_i^* \in [0, \infty)$ and a wage premium rate $t_i \in (0, \infty)$. Furthermore, firm i agrees to pay each employee a wage premium uniformly if it actually produces more than q_i^*. In stage two, after observing the rival's decision in stage one, each firm non-cooperatively chooses its actual output $q_i \in (0, \infty)$.

[1] For details see Ohnishi (2003, 2007).

Therefore, firm i's profit function is given by

$$\pi_i = \begin{cases} \left(a - Q^2\right) q_i - m_i q_i^2 & \text{for } \ q_i \le q_i^*, \\ \left(a - Q^2\right) q_i - m_i q_i^2 - \left(q_i - q_i^*\right) t_i & \text{for } \ q_i \ge q_i^*, \end{cases} \tag{1}$$

where $m_i \in (0,\infty)$ represents firm i's marginal cost of production. Firm 1 seeks to maximize (1).

Furthermore, firm 2's objective function is given by

$$V_2 = \pi_2 + \theta \cdot CS, \tag{2}$$

where CS represents consumer surplus and $\theta \in [0,1]$ is the share of the consumer surplus. Therefore, (1) can be rewritten by

$$V_2 = \begin{cases} \left(a - Q^2\right) q_2 - m_2 q_2^2 + \theta\left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q\right] & \text{for } \ q_2 \le q_2^*, \\ \left(a - Q^2\right) q_2 - m_2 q_2^2 - \left(q_2^2 - q_2^{*2}\right) t_2 + \theta\left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q\right] & \text{for } \ q_2 \ge q_2^*. \end{cases} \tag{3}$$

In this paper, I use the well-known solution concept called subgame perfection.

3. Supplementary explanations

I first derive firm 1's best reaction function from (1). Firm 1's reaction function for $q_1 < q_1^*$ is defined by

$$R_1(q_2) = \arg\max_{q_1 \ge 0} \left[\left(a - Q^2\right) q_1 - m_1 q_1^2\right], \tag{4}$$

and firm 1's reaction function for $q_1 > q_1^*$ is defined by

$$R_1^t(q_2) = \arg\max_{q_1 \ge 0} \left[\left(a - Q^2\right) q_1 - \left(m_1 + t_1\right) q_1^2 + t_1 q_1^{*2}\right]. \tag{5}$$

Therefore, if firm 1 selects q_1^* and offers WRCP, then its best reply is given by

$$R_1^W(q_2) = \begin{cases} R_1(q_2) & \text{for} \quad q_1 < q_1^*, \\ q_1^* & \text{for} \quad q_1 = q_1^*, \\ R_1^t(q_2) & \text{for} \quad q_1 > q_1^*. \end{cases} \tag{6}$$

Firm 1 chooses q_1 in order to maximize π_1, given q_2. Hence, the first-order condition for (4) is

$$a - 3q_1^2 - 4q_1q_2 - q_2^2 - 2m_1q_1 = 0, \tag{7}$$

and the second-order condition is

$$-3q_1 - 2q_2 - m_1 < 0. \tag{8}$$

On the other hand, the first-order condition for (5) is

$$a - 3q_1^2 - 4q_1q_2 - q_2^2 - 2m_1q_1 - 2t_1q_1 = 0, \tag{9}$$

and the second-order condition is

$$-3q_1 - 2q_2 - m_1 - t_1 < 0. \tag{10}$$

Hence, I obtain

$$R_1'(q_2) = \frac{-2q_1 - q_2}{3q_1 + 2q_2 + m_1} \tag{11}$$

and

$$R_1^{t\prime}(q_2) = \frac{-2q_1 - q_2}{3q_1 + 2q_2 + m_1 + t_1}. \tag{12}$$

The following lemma is now immediate.

Lemma 1: Both $R_1(q_2)$ and $R_1^t(q_2)$ are downward-sloping.

Second, I derive firm 2's best reaction function from (3). Firm 2's reaction function for $q_2 < q_2^*$ is defined by

$$R_2(q_1) = \arg\max_{q_2 \ge 0} \left\{ \left(a - Q^2\right) q_2 - m_2 q_2^2 + \theta \left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q \right] \right\}, \tag{13}$$

and firm 2's reaction function for $q_2 > q_2^*$ is defined by

$$R_2^t(q_1) = \arg\max_{q_2 \geq 0} \left\{ \left(a - Q^2\right) q_2 - \left(m_2 + t_2\right) q_2^2 + t_2 q_2^{*2} + \theta \left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q \right] \right\}. \quad (14)$$

Hence, if firm 2 selects q_2^* and adopts WRCP, then its best reply is given by

$$R_2^W(q_1) = \begin{cases} R_2(q_1) & \text{for} \quad q_2 < q_2^*, \\ q_2^* & \text{for} \quad q_2 = q_2^*, \\ R_2^t(q_1) & \text{for} \quad q_2 > q_2^*. \end{cases} \quad (15)$$

Firm 2 chooses q_2 in order to maximize V_2, given q_1. Therefore, the first-order condition for (13) is

$$a + 2\theta\left(q_1 + q_2\right)^2 - q_1^2 - 4q_1q_2 - 3q_2^2 - 2m_2q_2 = 0, \quad (16)$$

and the second-order condition is

$$2q_1\left(\theta - 1\right) + q_2\left(2\theta - 3\right) - m_2 < 0. \quad (17)$$

On the other hand, the first-order condition for (14) is

$$a + 2\theta\left(q_1 + q_2\right)^2 - q_1^2 - 4q_1q_2 - 3q_2^2 - 2m_2q_2 - 2t_2q_2 = 0, \quad (18)$$

and the second-order condition is

$$2q_1\left(\theta - 1\right) + q_2\left(2\theta - 3\right) - m_2 - t_2 < 0. \quad (19)$$

Hence, I have

$$R_2'(q_1) = -\frac{q_1\left(2\theta - 1\right) + 2q_2\left(\theta - 1\right)}{2q_1\left(\theta - 1\right) + q_2\left(2\theta - 3\right) - m_2} \quad (20)$$

and

$$R_2^{t\prime}(q_1) = -\frac{q_1\left(2\theta - 1\right) + 2q_2\left(\theta - 1\right)}{2q_1\left(\theta - 1\right) + q_2\left(2\theta - 3\right) - m_2 - t_2}. \quad (21)$$

The following lemma is presented.

Lemma 2: (i) If $\theta < \left(2q_1 + q_2\right)/2\left(q_1 + q_2\right)$, then $R_2(q_1)$ and $R_2^t(q_1)$ slope downwards.

(ii) If $\theta > (2q_1 + q_2)/2(q_1 + q_2)$, then $R_2(q_1)$ and $R_2^t(q_1)$ slope upwards.

Third, I prove the following two lemmas, which provide characterizations of WRCP as a strategic commitment device.

Lemma 3: If firm i adopts WRCP, then at equilibrium its actual quantity coincides with q_i^*.

Proof: I consider the possibility that q_1 is larger than q_1^*. Firm 1's profit is

$$\left(a - Q^2\right)q_1 - m_1 q_1^2 - \left(q_1^2 - q_1^{*2}\right)t_1 .$$

Here, firm 1 must pay its employees wage premiums $(q_1 - q_1^*)t_1$. That is, firm 1 can improve its profit by raising q_1^*, and the equilibrium point does not change in $q_1 \geq q_1^*$. Hence, $q_1 > q_1^*$ does not result in an equilibrium.

Next, I consider the possibility that q_1 is smaller than q_1^*. In this case, firm 1's marginal cost of production does not change. It is impossible for firm 1 to change its output in equilibrium because such a strategy is not credible. That is, WRCP does not function as a strategic commitment.

The proof of firm 2 is similar to that of firm 1, and thus it is omitted here. Q.E.D.

Lemma 4: Firm *i*'s payoff maximizing output is lower when it offers WRCP than when it does not.

Proof: I prove the case for firm 2. From (3), it is seen that WRCP will never decrease firm 2's marginal cost of production. The first-order condition for $q_2 > q_2^*$ is

$$a+2\theta\left(q_1+q_2\right)^2-q_1^2-4q_1q_2-3q_2^2-2m_2q_2-2t_2q_2=0,$$

where t_1 is positive. To satisfy the first-order condition, $a+2\theta\left(q_1+q_2\right)^2-q_1^2-4q_1q_2-3q_2^2-2m_2q_2$ must be positive.

Since the proof of firm 1 is very similar to that of firm 2, it is omitted here. Q.E.D.

4. Equilibrium outcomes

In this section, I consider the following two cases.

Case 1: $\theta<\left(2q_1+q_2\right)/2\left(q_1+q_2\right)$

Case 2: $\theta>\left(2q_1+q_2\right)/2\left(q_1+q_2\right)$

In Case 2, firm 1's output is a strategic substitute, while firm 2 treats its output as a strategic complement.

4.1. Case 1

This case is illustrated in Figure 1, where R_i is firm *i*'s reaction curve with no WRCP, π_1^A is firm 1's iso-profit curve extending through A, and V_2^A is firm 2's iso-payoff curve extending through A. For explanation, the figure is drawn simply. Since Case 1 is strategic substitutes in which goods are perfect substitutes; that is, R_i slopes down. Point A is the equilibrium solution with no WRCP as a strategic commitment device.

If firm 1 offers WRCP, its marginal cost of production increases and thus it decreases its output (Lemma 4). In Figure 1, if firm 1 chooses q_1^* and offers WRCP, then its reaction curve shifts down for $q_1>q_1^*$ and becomes the kinked bold lines. Therefore, firm 1's unilateral solution can occur at a point like D. From Figure 1, it is seen that firm 1's profit is lower at

D than at A while firm 2's objective function value is higher at D than at A. Here, if firm 2 chooses q_2^* and offers WRCP, then its reaction curve shifts to the left for $q_2 > q_2^*$ and becomes the kinked bold broken lines. Therefore, the bilateral WRCP solution can become B. Firm 2's objective function value is lower at B than at D, while firm 1's profit is higher at B than at D. Firm 2's unilateral solution can occur at C. Firm 2's objective function value is lower at C than at A, while firm 1's profit is higher at C than at A. In addition, firm 1's profit at B is lower than at C, while firm 2's objective function value is higher at B than at C.

Each firm's WRCP adoption decreases its optimal output. Given the output level of firm i, decreasing firm *j*'s output decreases the total market output. Furthermore, since firm *i*'s optimal strategy increases its output because of strategic substitutes, firm *j*'s payoff decreases. Since firm *j*'s WRCP adoption decreases its output and payoff, it never offers WRCP. Hence, the equilibrium occurs at A in Figure 1.

The equilibrium of this case can be stated in the following proposition.

Proposition 1: In $\theta < \left(2q_1 + q_2\right)/2\left(q_1 + q_2\right)$, there exists an equilibrium point in which neither firm offers WRCP as a strategic commitment.

Proof: Lemmas 1 and 2 (i) state that $R_i(q_j)$ and $R_i^t(q_j)$ are downward-sloping, namely strategic substitutes. Lemma 4 states that firm *i*'s optimal output is lower when it offers WRCP than when it does not. Decreasing firm *i*'s output increases firm *j*'s amount of demand because of substitute goods, and the optimal strategy increases firm *j*'s output because of strategic substitutes. Increasing firm *j*'s output decreases firm *i*'s amount of demand and payoff. Firm *j*'s reaction function gives its optimal output for each output of firm *i*. In firm *j*'s reaction function, decreasing firm *i*'s output never decreases firm *j*'s output. Even if firm *i*'s output decreases, since firm *j*'s output never decreases, firm *i*'s payoff decreases because of substitute goods. Thus, Proposition 1 is proved. Q.E.D

Proposition 1 says that neither firm offers WRCP in the case of $\theta < (2q_1 + q_2)/2(q_1 + q_2)$. Since firm *i*'s WRCP adoption increases its marginal cost of production, it decreases its optimal output; that is, firm i that adopts WRCP becomes a less aggressive competitor. If the adoption of WRCP makes firm i behave less aggressively, then firm j has an incentive to be more aggressive, and therefore firm i loses share in the market. Thus, neither firm offers WRCP.

4.2. Case 2

Case 2 is illustrated in Figure 2. In this case, firm 2's output is a strategic complement. Point E is the equilibrium solution with no WRCP offered. Since firm *i*'s WRCP adoption increases its marginal cost of production, it decreases its optimal output (Lemma 4). On R_2, firm 1's profit is highest at S. If firm 1 chooses q_1^S and offers WRCP, then its reaction curve shifts down for $q_1 > q_1^S$ and becomes the kinked bold lines. Hence, firm 1's unilateral solution can occur at S. From Figure 2, it is seen that firm 2's objective function value is lower at S than at E. On the other hand, if firm 2 chooses q_2^S and offers WRCP, then its reaction curve shifts to the right for $q_2 > q_2^S$ and becomes the kinked bold broken lines. Therefore, firm 2's unilateral solution can occur at F. From Figure 2, it is seen that firm 1's profit is higher at F than at S. Hence, firm 1 has no incentive to choose q_2^S and offer WRCP.

The following result differs notably from that of Case 1.

Proposition 2: In $\theta > (2q_1 + q_2)/2(q_1 + q_2)$, there exists an equilibrium point in which only firm 2 offers WRCP as a strategic commitment.

Proof: I first consider firm 1's Stackelberg leader output when neither firm offers WRCP. If firm 1 is the Stackelberg leader, then it maximizes $\pi_1(q_1, R_2(q_1))$ with respect to q_1. Therefore, firm 1's Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial \pi_1}{\partial q_1} + \frac{\partial \pi_1}{\partial q_2}\frac{\partial R_2}{\partial q_1} = 0 .$$

Here $\partial \pi_1 / \partial q_2$ is negative, and $\partial R_2 / \partial q_1$ is positive (Lemma 2 (ii)). To satisfy the first-order condition, $\partial \pi_1 / \partial q_1$ must be positive. Hence, firm 1's Stackelberg leader output is smaller than its Cournot output. At equilibrium, q_1 coincides with q_1^* (Lemma 3). Furthermore, firm 1's profit-maximizing output is lower when it adopts WRCP than when it does not (Lemma 4). A decrease in firm 1's output is decided by the value of t_1. Let t_1 be a variable that can take any value more than zero. In R_2, firm 1's profit is highest at its Stackelberg leader point. Therefore, if firm 2 does not offer WRCP, then firm 1 selects its Stackelberg solution and maximizes its profit. However, firm 2's objective function value at firm 1's Stackelberg solution is lower than at the Cournot-Nash solution with no WRCP. Hence, firm 2 does not want firm 1 to offer WRCP.

Next, I consider firm 2's Stackelberg leader output when neither firm offers WRCP. If firm 2 is the Stackelberg leader, then it maximizes $V_2(R_1(q_2), q_2)$ with respect to q_2. Therefore, firm 2's Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial V_2}{\partial q_2} + \frac{\partial V_2}{\partial q_1}\frac{\partial R_1}{\partial q_2} = 0 .$$

Here, $\partial V_2 / \partial q_1$ is positive, and $\partial R_1 / \partial q_2$ is negative (Lemma 1). To satisfy the first-order condition, $\partial V_1 / \partial q_1$ must be positive. Hence, firm 2's Stackelberg leader output is lower than its Cournot output. At equilibrium, q_2 coincides with q_2^* (Lemma 3). Furthermore, Lemma 4 shows that firm 2's payoff-maximizing output is lower when it offers WRCP than when it does not. A decrease in firm 2's output is decided by the value of t_2. Let t_2 be a variable that can take any value more than zero. In R_1, firm 2's profit is highest at its Stackelberg leader point. Therefore, firm 2 chooses q_2^{*S} corresponding to its Stackelberg solution and offers WRCP. Hence, firm 1's profit will increase. Our equilibrium concept is subgame perfection and all information in the model is common knowledge. Thus, firm 1 has no incentive to offer WRCP, and only firm 2 adopts WRCP. Q.E.D.

5. Conclusion

I have examined two cases of a game model with a concave demand function where a profit-maximizing firm competes with a socially concerned firm. Each firm is allowed to offer WRCP as a strategic commitment. I have presented the respective equilibrium solutions of the two cases. I have found that the equilibrium of Case 2 may be profitable for both firms.

References

Baron, D. P., 2001. Private politics, corporate social responsibility, and integrated strategy. Journal of Economic Management and Strategy 10, 7-45.

Besley, T., Ghatak, M., 2007. Retailing public goods: the economics of corporate social responsibility. Journal of Public Economics 91, 1645-1663.

Bulow, J. I., Geanakoplos, J. D., Klemperer, P. D., 1985. Multimarket oligopoly: Strategic Substitutes and Complements. Journal of Political Economy 93, 488-511.

Cooper, T. E., 1986. Most-favored-customer pricing and tacit collusion. Rand Journal of Economics 17, 377-388.

Cracau, D., 2015. The effect of strategic firm objectives on competition. In: Ohnishi, K. (Ed.), Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 1 (pp. 170-181). Sharjah, UAE: Bentham Science Publishers.

Delbono, F., Scarpa, C., 1995. Upward-sloping reaction functions under quantity competition in mixed oligopolies. Bulletin of Economic Research 47, 341-346.

Fanti, L., Buccella, D., 2018. Profitability of corporate social responsibility in network industries. International Review of Economics 65, 271-289.

Flores, D., García, A., 2016. On the output and welfare effects of a non-profit firm in a mixed duopoly: A generalization. Economic Systems 40, 631-637.

García, A., Leal, M., Lee, S.-H., 2019. Endogenous timing with a socially responsible firm. Korean Economic Review 35, 345-370.

Goering, G. E., 2007. The strategic use of managerial incentives in a non-profit firm mixed duopoly. Managerial and Decision Economics 28, 83-91.

Han, L., 2019. Partial ownership for a public firm and corporate social responsibility. Theoretical Economics Letters 9, 2447-2455.

Kim, H.-S., 1996. Strategic excess capacity and first-mover advantage under variable demand. Seoul Journal of Economics 9, 105-122.

Königstein, M., Müller, W., 2001. Why firms should care for customers. Economics Letters 72, 47-52.

Kopel, M., 2015. Price and quantity contracts in a mixed duopoly with a socially concerned firm. Managerial and Decision Economics 36, 559-566.

Kopel, M., Brand, B., 2012. Socially responsible firms and endogenous choice of strategic incentives. Economic Modelling 29, 982-989.

Kopel, M., Lamantia, F., Szidarovszky, F., 2014. Evolutionary competition in a mixed market with socially concerned firms. Journal of Economic Dynamic & Control 48, 394-409.

Kotchen, M. J., 2006. Green markets and private provision of public goods. Journal of Political Economy 114, 816-834.

Lambertini, L., Tampieri, A., 2012. Corporate social responsibility and firms' ability to collude. In: Boubaker, S., Nguyen, D. K. (Eds.), Board Directors and Corporate Social Responsibility (pp. 167-178). London: Palgrave Macmillan.

Ohnishi, K., 2003. A model of a price-setting duopoly with a wage-rise contract. Australian Economic Papers 42, 149-157.

Ohnishi, K., 2007. Wage-rise contract and Cournot competition with labor-managed firms. Rivista di Politica Economica 97, 17-31.

Ohnishi, K., 2008. Mixed duopoly with wage-rise contract as strategic commitment. Ekonomia 11, 35-49.

Ohnishi, K., 2010. Lifetime employment contract and quantity competition with profit-maximizing and joint-stock firms. Journal of Institutional and Theoretical Economics 166, 462-478

Ohnishi, K., 2021. Lifetime employment and reaction functions of socially concerned firms under quantity competition. MPRA paper number 110867.

Ohnishi, K., 2022. Socially concerned duopolies with lifetime employment as a strategic commitment. MPRA paper number 111625.

Planer-Friedrich, L., Sahm, M., 2018. Why firms should care for all consumers. Economics Bulletin 38, 1603-1612.

White, M. D., 2001. Managerial incentives and the decision to hire managers in markets with public and private firms. European Journal of Political Economy 17, 877-896.

Xu, Y., 2014. CSR impact on hospital duopoly with price and quality competition. Journal of Applied Mathematics 2014, 152060.

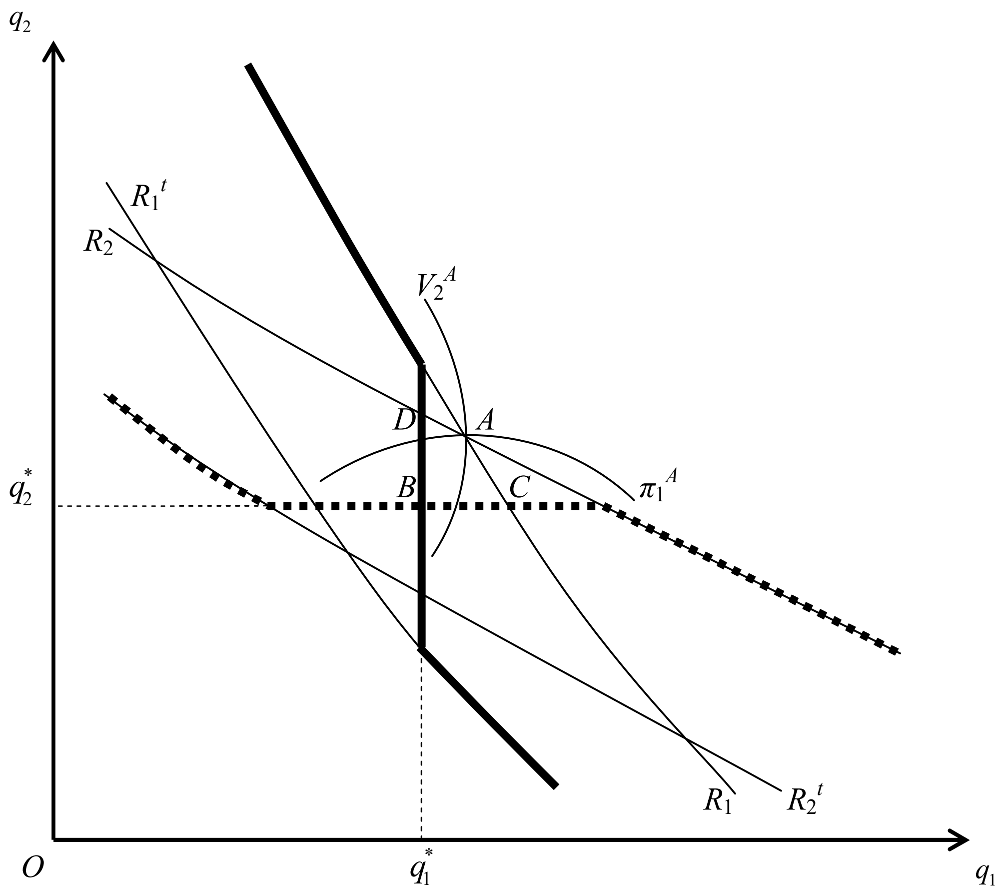

Figure 1: $\theta < (2q_1 + q_2)/2(q_1 + q_2)$

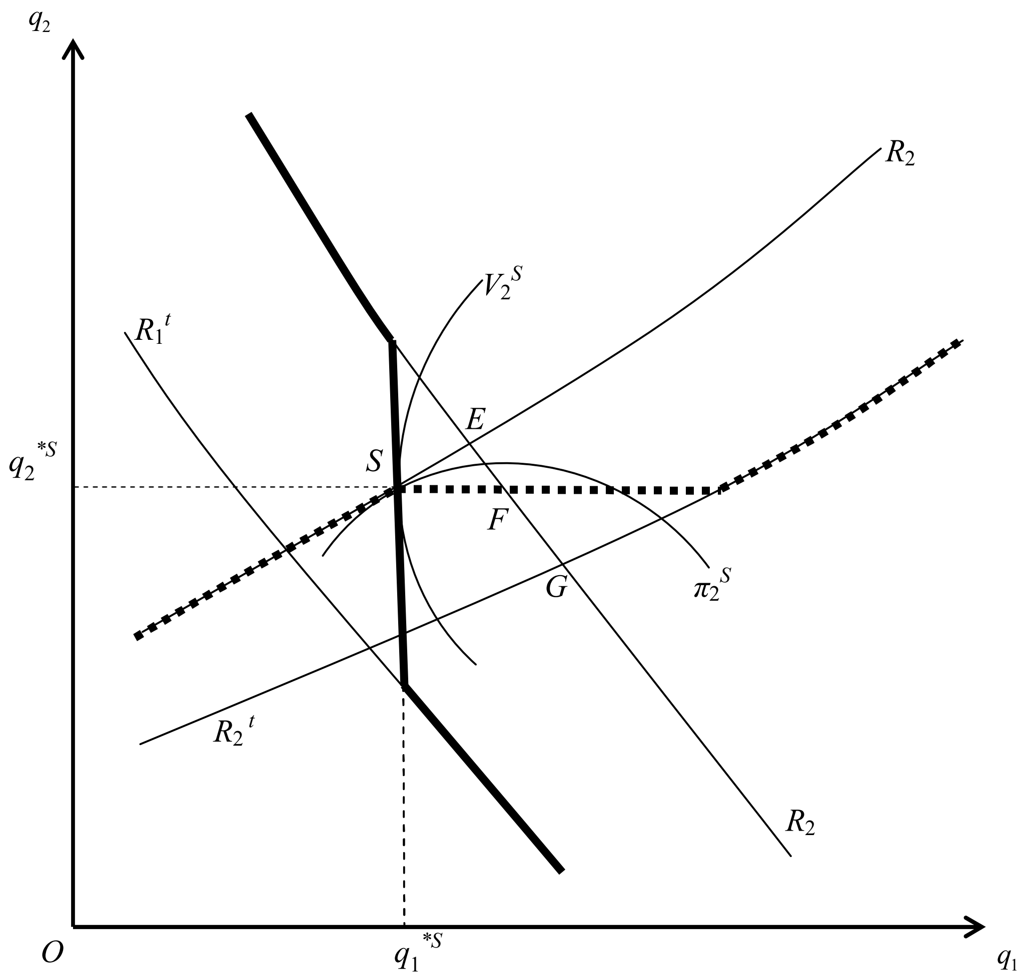

Figure 2: $\theta > (2q_1 + q_2)/2(q_1 + q_2)$