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Estimating Illiquid Asset Class Alpha and Beta using Secondary Transaction Prices

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ABSTRACT

This paper introduces a novel method for estimating the true economic alpha and market beta of illiquid asset classes using secondary transaction prices. Furthermore, this approach can be used to measure the degree of stale pricing in the reported returns of such asset classes. We apply this methodology to private equity, venture capital, private real estate, and private natural resource asset classes. A significant degree of stale pricing was found with estimates of market beta significantly higher than those obtained using reported returns for all four of the asset classes considered. No asset class exhibited statistically significant positive alpha.

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Institutional investors have rapidly expanded their allocations to illiquid asset classes, such as private equity, venture capital, private real estate and private natural resources over the past 10 years. Assets managed in private market funds have recently reached $7 trillion[1], with US-based university endowments allocating over one-third of their portfolios to such strategies[2].

Investors gain access to these asset classes through investments in closed-end funds managed by third-party investment managers. These funds acquire illiquid assets which are held for a significant period (4.5 years for the median asset exited in 2020[3]). Between acquisition and sale these privately-owned assets have no market price. Even so, the investment manager typically reports a price (net asset value or NAV) of the fund on a quarterly or semi-annual basis based on estimated valuations for each asset in their portfolio.

Such estimated valuations are at risk of being stale, based on inputs that do not incorporate all recent information and market movements, or at worse managed, where the manager exercises discretion to smooth returns. Indeed, as presented in Table II, reported returns of these illiquid asset classes exhibit a significant degree of serial correlation, unlike liquid equity markets, indicative of such stale pricing.

Stale (or managed) pricing can lead to the understatement of market beta using reported returns and a consequent overstatement of alpha. This phenomenon has been identified by various studies on hedge funds, such as Asness, Krail, and Liew (2001), and

[1] McKinsey Global Private Markets Review 2021
[2] 2020 NACUBO-TIAA Study of Endowments
[3] The Private Equity Market in 2020: Escape from the Abyss, Bain & Company

poses a significant challenge for allocators who must use estimates of alpha and market beta to make informed investment decisions.

Researchers have puzzled over the low beta obtained using reported returns, which seem at odds with theoretical predictions. Axelson, Sorenson, and Stromberg (2014) argue that a simple application of the theory of Modigliani and Miller (1958) would imply a beta for private equity between 2.2 to 2.7, far higher than the reported beta (see Table II).

In response, a body of literature has attempted to introduce methods to estimate market beta that better reflect the true economic risk of these asset classes. Research has predominately focused on private equity and venture capital, while real estate and natural resources have been less covered.

Of the approximately $7.3 trillion managed in private market funds, $2.3 trillion is invested in private equity and $1.2 trillion in venture capital. However private real estate and natural resources also have substantial assets with $1.1 trillion and $0.9 trillion respectively [4]. These asset classes should therefore should not be overlooked when considering the alpha and beta of illiquid asset classes.

The main contribution of this paper is to introduce a novel method for estimating the true economic alpha and beta of four illiquid asset classes using secondary transaction prices. We apply this methodology to obtain, for the first time, consistent beta and alpha estimates for private equity, venture capital, real estate and natural resources. This approach could be used for any asset class for which secondary transaction data is available.

[4] A year of disruption in the private markets: McKinsey Global Private Markets Review 2021

Secondary transactions occur when an investor sells their interest in a private market fund to another investor. Approximately 1,600 secondary transactions take place per year, on average totalling $50bn of value[5]. While supply of transactions originates predominately from the investment manager (General Partner) and, to a less extent, existing institutional investors (Limited Partners) in the fund; demand is almost entirely from specialist investment managers[6]. Such specialist investors, focused solely on investing in private market funds, have raised over $339bn in capital across 529 funds in the past 10 years[7].

Secondary market participants are thus informed sophisticated investors operating in a market of sufficient scale and competition that it is reasonable to assume that secondary prices represent the true economic price of the fund interest at the point of transaction.

Two specialist data providers, Greenhill and Preqin, record data on secondary transactions. However, this data has been difficult to utilise for two reasons: firstly, only normalized prices are published, where secondary prices are given as a proportion of the reported price (NAV) of the fund interest being transacted; and secondly, these sources only disclose a trailing average of these normalized prices over semi-annual or annual periods for each illiquid asset class. The method outlined in this paper is able to utilise this data despite these difficulties.

We will also present an indicator of the degree of stale pricing in observable prices that can be applied to each asset class. This indicator can be applied consistently across all the illiquid asset classes considered, varying between a value of 0 in the absence of stale pricing to 1 when there pricing is entirely stale.

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[5] Secondaries 2021 – A Buyers' Market, Commonfund
[6] Setter Capital Volume Report H1 2021
[7] McKinsey Global Private Markets Review 2021

Applying this methodology to private equity, venture capital, real estate and natural resources, we estimate true economic betas of 2.1, 2.1, 1.8, and 1.8 respectively. These values are significant at the 0.01 threshold. We find that between 75% to 92% of the reported NAV of these illiquid investments is stale.

We find that private equity and venture capital have statistically insignificant alphas at all thresholds, while both real estate and natural resources have statistically significant negative alphas.

The remainder of this paper is organised as follows. Section I discusses related literature. Section II describes the data used in this paper. Section III describes the methodology for estimating beta and the degree of stale pricing for each illiquid asset class. Section IV describes the empirical results. The paper concludes with Section V.

I. Related Literature

This work is related to a substantial body of literature seeking to estimate the true economic alpha and market beta of illiquid asset classes. Prior studies have predominately focused on private equity and venture capital. There is no preceding research seeking to calculate the true economic market beta for either private real estate or private natural resource funds, although some literature on real estate risk is related.

A. Private Equity and Venture Capital

This study is most closely related to Boyer et al (2018) who similarly use secondary transaction data to estimate risk characteristics for private equity and venture capital. Boyer et al (2018) estimates a beta of 2.3 for private equity and 1.0 for venture capital. Estimates of alpha were statistically insignificant. Boyer et al does not estimate betas or alphas for either real estate or natural resources.

Unlike the method outlined in this paper, Boyer et al (2018) uses a proprietary data set of individual secondary transactions. While this data is more granular in nature, the data is not widely available and cannot be extended to other asset classes.

Other studies using market prices include Jegadeesh, Kraussl, and Pollet (2015) who use publicly-traded securities of fund of private equity funds and the management companies of private market investment managers to estimate risk and return attributes of private equity and venture capital. However publicly-traded investment managers (such as KKR, Blackstone, etc) have extensive operations outside of illiquid investment management (e.g. managing liquid hedge funds and fixed income strategies) which calls in to question how representative their stock price movements are to pure investing in private equity or venture capital.

The majority of prior studies have utilised cash flow data to estimate the risk attributes of private equity and Venture Capital. A significant number of recent studies (Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), Korteweg and Nagel (2016), and Robinson and Sensoy (2016)) use proprietary datasets of cash flows between private market funds and their investors to define public market equivalent (PME) benchmark for each fund (using a methodology developed by Kaplan and Schoar (2005) and extended by Korteweg and Nagel (2016)). Based on this PME analysis, these studies estimate betas of 1.2 and 1.1 to 1.4 for private equity and venture capital respectively. These approaches produce a wide range of alpha estimates, both statistically significant and insignificant.

This fund-level cash flow data is utilised in a range of other studies (Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), and Driessen, Lin, and Phalippou (2012)) to calculate standard or modified internal rate of returns (IRRs) for each illiquid investment fund. Using cross-sectional regression analysis on fund-level IRRs with the IRR of public equities, these papers arrive at a wide range of estimates for the beta of private equity and venture capital, from as low as 0.4 up to 2.7. We again find a wide range of alpha estimates using these techniques.

There are several studies that apply fund-level cash flow data to other techniques, including Ang et al (2018) and Buchner (2014). Ang et al (2018) estimates time-varying discount rates for fund-level cash flows, and uses these discount rates to estimate betas of 1.25 and 1.8 for private equity and venture capital. Buchner (2014) applies a dividend model to this cash flow data to arrive at betas of 2.2 for private equity and 2.6 for venture capital.

Caution is possibly warranted when using fund-level cash flow data due to the use of subscription lines of credit (where a fund uses a bridge loan to fund the entire purchase price of an asset and only later drawing capital from investors). A recent survey[8] suggested that over 90% of private investment managers utilise such credit lines that can distort the timing of cash flows and artificially boost internal rates of return.

Another branch of research utilises proprietary datasets of cash flows between the funds and their underlying illiquid portfolio assets (Frazoni, Nowark, and Philippou (2012) and Axelson, Sorensen, and Strömberg (2014)). Using IRR calculations on these cash flows

[8] Subscription Line of Credit: Benefits, Risks, and Distortions, 29 September 2020, CAIA.org

and comparing to similar IRRs for public equities, these studies estimate a beta of between 1.0 and 2.3 for private equity.

These datasets maybe less influenced by subscription lines of credit, but still rely on proprietary datasets not readily available to practitioners.

One approach adapted to venture capital is to use round-by-round valuation data along with exit events. This approach is used in a number of studies (Cochrane (2005), Ewens (2009), and Korteweg and Sorensen (2010)). The results of these studies vary widely from a beta of 0.6 to 2.8.

There are some drawbacks of this approach, including difficulty in assessing selection bias and correcting for missed rounds and other data issues. There are also potential conflict of interests where existing investors can make a small subsequent investment at a higher valuation and then mark-up their whole investment amount.

Overall, studies provide a wide range of both beta and alpha estimates. Alpha estimates range from significantly significant positive abnormal returns to negative alpha.

B. Private Real Estate

Existing literature on the risk characteristics of private real estate is far smaller and has primarily concentrated on reconciling the differing risk levels observed in indices of the reported returns of public and private real estate investments.

Studies use private real estate return indices calculated by NCREIF (the National Council of Real Estate Investment Fiduciaries). NCREIF indices represent the return of 'institutional quality' US real estate held by tax-exempt pension schemes, and are calculated without leverage or fees.

These indices represent low-risk investments in high quality buildings that are substantially let, offering stable income to their pension scheme owners. This is a good match to publicly-traded REITs, that are also owning stable income generating assets. However, according to McKinsey[9], this type of core strategy represents only 13% of assets managed in closed-end private real estate funds. The majority of funds pursue a higher risk value-added/opportunistic strategy that is typically investing in under-let buildings that require redevelopment and/or repositioning.

Several important studies (Barkham and Geltner (1995), Pagliari, Scherer, and Monopoli (2005), and Riddiough, Moriarty, and Yeatman (2005)) have compared NCREIF indices to indices of publicly-traded REIT returns. These papers attempt to correct for differences in leverage, fees and property type to ensure a like-for-like comparison. Once corrected, these studies conclude that the risk profile of public and private real estate are similar.

Given the substantial differences in the strategies deployed in the majority of closed-end Private Real Estate funds and those represented by REIT and NCREIF indices, these results can not be applied to the private real estate asset class.

[9] 'A turning point for real estate investment management', 2019, McKinsey

II. Data

A. Secondary Transaction Price Data

A secondary transaction occurs when an investor in a private market fund sells their interest to another investor. As previously outlined, both sides of these transactions are sophisticated informed investors who are operating in a market of sufficient size and competition to believe that secondary prices represent a true economic price of the fund interest being transacted.

Publicly available data on secondary transactions is available from two sources: Greenhill and Preqin. Greenhill is leading advisor in the secondary market, having advised on over 13,000 transactions representing approximately $300bn of value[10]. Preqin is a widely-used data provider focused on alternative investments, with 48,000 clients globally.[11]

These data sources record normalized secondary prices, where the secondary price is given as a proportion of the reported price (NAV) of the fund interest being transacted. These normalized secondary prices are aggregated to provide a series of semi-annual or annual average prices.

<FIGURE 1 HERE>

Greenhill and Preqin publish data on four illiquid asset classes: private equity, venture capital, real estate, and natural resources. As can be seen in Figure 1, Private equity, venture capital and private real estate have datasets of semi-annual values with varying lengths:

[10] Secondary Sale Advisory of LP Portfolios | Greenhill & Co, www.Greenhill.com, accessed 2 March 2022

[11] Who We Are | Preqin, www.preqin.com, accessed 2 March 2022

private equity and venture capital begin in 2004, while real estate begins in 2013. Natural resources only has annual values available since 2013, as can be seen in Figure 2. Table I presents a descriptive summary of the dataset.

<TABLE I HERE>

B. Reported Price Data

The reported returns of each illiquid asset class can be obtained from another data provider, Cambridge Associates, a global investment consultant with $389bn of assets under advisement. Cambridge Associates provides quarterly returns for private equity, venture capital, real estate and natural resource asset classes.

This data measures the average return of a pool of 4,395 private market funds split by asset class. Returns are based on changes to their reported net asset values. Table II presents summary statistics for each asset class.

<TABLE II HERE>

Indices can be constructed from these quarterly returns, can be seen in Figure 3, measuring the cumulative value over time from investing in each illiquid asset class, assuming any cash flows are reinvested. Such indices represent series of observable reported prices of each asset class over time.

<FIGURE 3 HERE>

C. Equity Market Price Data

We use the S&P 500 Index for our benchmark of equity market returns and prices.

III. Methodology

A. Estimating average (non-normalized) secondary prices

For each illiquid asset class, given an index of reported prices, p_t^O, we can define the relationship between an index of secondary prices, p_t, and normalized secondary prices, np_t, as follows:

$$\mathrm{np_t} = \frac{\mathrm{p_t}}{\mathrm{p_t^O}} \tag{1}$$

While neither p_t nor $np_t$is observable for any point in time t, publicly accessible data allows us to observe a series of n-period trailing averages of normalized prices:

$$ANP_n = \left\{anp_{nt,n} = \frac{1}{n}\sum_{i=1}^{n} np_{nt-i+1}\right\} \tag{2}$$

As reported prices are observable, trailing averages of reported prices can be calculated directly:

$$AP_n^O = \left\{ap_{nt,n}^O = \frac{1}{n}\sum_{i=1}^{n} p_{nt-i+1}^O\right\} \tag{3}$$

As p_t is not observable,

$$AP_n = \left\{ap_{nt,n} = \frac{1}{n}\sum_{i=1}^{n} p_{nt-i+1}\right\} \tag{4}$$

cannot be calculated directly. However, it can be estimated as the product of the average reported price and the average normalized price (which is the average of secondary prices divided by reported prices):

$$\widehat{AP}_n = \{\widehat{ap}_{nt,n} = ap^O_{nt,n} anp_{nt,n}\} \qquad (5)$$

This estimate will be unbiased when the covariance of ANP_n and AP^O_n is zero, which would be expected under the reasonable assumption that the expected return of the asset class is positive.

B. Estimating market beta

Since we consider the secondary price, p_t, to represent the true economic price of the asset class at time *t*, we can consider the series of secondary returns, $R = \{r_t\}$, to represent true economic returns.

The beta of secondary returns, R to the market returns, $R^M = \{r^M_t\}$, is then the true economic beta of the asset class. To estimate this beta, $\beta(R, R^M)$, we proceed by defining a series of growth rates of *n*-period trailing average of secondary prices (henceforth for simplicity referred to as average secondary growth):

$$\delta AP_n = \left\{\delta ap_{nt,n} = \frac{ap_{nt,n}}{ap_{n(t-1),n}} - 1\right\} \qquad (6)$$

This can be immediately reformulated in terms of secondary returns, r_t, as follows:

$$\delta ap_{nt,n} = \frac{\sum_{i=1}^{n} \prod_{j=1}^{2n-i+1} \left(1 + r_{nt-i-j+2}\right)}{\sum_{i=n+1}^{2n} \prod_{j=1}^{2n-i+1} \left(1 + r_{nt-i-j+2}\right)} - 1 \tag{7}$$

Now assuming R remains below 1 and therefore $\ln(1 + r_t) \approx r_t$, this can be approximated by:

$$\delta ap_{nt,n} \approx \sum_{i=1}^{2n-1} \theta_i r_{nt-i+1} \tag{8}$$

Where:

$$\theta_i = \begin{cases} i & i \leq n \\ 2n - i & i > n \end{cases}$$

Similarly, the growth rate of *n*-period trailing averages of market prices (henceforth referred to simply as average market growth) can be defined as:

$$\delta AP_n^M = \left\{ \delta ap_{nt,n}^M \approx \sum_{i=1}^{2n-1} \theta_i r_{nt-i+1,n}^M \right\} \tag{9}$$

We can now proceed to find an estimate of the true market beta, $\beta(R, R^M)$.

PROPOSITION 1: The beta of average secondary growth to average market growth is approximately equal to the beta of secondary returns to market returns.

$$\beta(\delta AP_n, \delta AP_n^M) \approx \beta(R, R^M) \tag{10}$$

Proof: This result is immediately clear using the approximations to δAP_n and δAP_n^M presented in equation (8) and equation (9), while assuming that R and R^M reflect informationally-efficient prices and therefore have no serial correlation nor lagged correlation with each other:

$$\beta(\delta AP_n, \delta AP_n^M) = \frac{Cov(\delta AP_n, \delta AP_n^M)}{Var(\delta AP_n^M)} \approx \frac{\sum_{i=1}^{2n-1} {\theta_i}^2 Cov(R, R^M)}{\sum_{i=1}^{2n-1} {\theta_i}^2 Var(R)} = \beta(R, R^M) \tag{11}$$

Since we have an approximation for the *n*-period trailing average of secondary prices, $\widehat{AP}_n$, we now have a method for estimating the true economic beta of the illiquid asset class with:

$$\hat{\beta} = \beta\left(\delta \widehat{AP}_n, \delta AP_n^M\right) \approx \beta(R, R^M) \tag{12}$$

C. Estimating the degree of stale pricing in observable prices

The basic econometric model of non-synchronous or stale pricing presented by Dimson (1979) relates reported returns R^O to true economic returns R as follows:

$$r_t^O = \sum_{i=1}^{k} a_i r_{t-i+1} \tag{13}$$

Where:

$a_i \in [0,1]$

$$\sum_{i=1}^{k} a_i = 1$$

This equation implies a relationship between the beta of reported returns, β^O, and true economic returns, β:

$$\beta^O = a_1 \beta \tag{14}$$

Using $\hat{\beta}$, the estimate of the true economic beta derived in equation (12), we can estimate $a_1 \approx \frac{\beta^O}{\hat{\beta}}$. When $a_1 = 1$, reported returns are exactly equal to contemporaneous true economic returns and therefore there is no stale pricing. When $a_1 = 0$, reported returns are entirely dependent on lagged true economic returns and therefore there is complete stale pricing.

Using this, we derive a stale pricing indicator, $\mathrm{SPI} \in [0,1]$, as:

$$\mathrm{SPI} = 1 - \frac{\beta^O}{\hat{\beta}} \tag{15}$$

D. Estimating illiquid asset class variance and volatility

Using equations (8) and (9), we can derive an estimate for the variance of each asset class as follows:

$$\mathrm{Var}(R) \approx \frac{\mathrm{Var}\left(\delta\widehat{\mathrm{AP}}_n\right)}{\mathrm{Var}(\delta \mathrm{AP}_n^M)} \mathrm{Var}(R^M) \tag{16}$$

The volatility of R, $\sigma(R)$, obviously follows clearly from this result.

E. Estimating illiquid asset class correlation

From equation (11), it is straightforward to see that the correlation of $\delta\widehat{\mathrm{AP}}_{\mathrm{n}}$ with $\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}}$,

$$\rho\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}},\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}}\right)=\frac{\mathrm{Covar}\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}},\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}}\right)}{\sqrt{\mathrm{Var}\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right)\mathrm{Var}(\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}})}}, \tag{17}$$

is an approximation for the correlation of the illiquid asset class with the market:

$$\rho\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}},\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}}\right)\approx\frac{\sum_{i=1}^{2n-1}{\theta_i}^2Cov(R,R^M)}{\sqrt{\sum_{i=1}^{2n-1}{\theta_i}^2Var(R)\sum_{i=1}^{2n-1}{\theta_i}^2Var(R^M)}}=\rho(R,R^M) \tag{18}$$

F. Estimating illiquid asset class alpha

From the proof of Proposition 1, we have

$$\delta ap_{nt,n}\approx\hat{\beta}\delta ap_{nt,n}^{M}+\alpha\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right)+\varepsilon_t{}' \tag{19}$$

$$\mathrm{r}_t\approx\hat{\beta}r_t^M+\alpha(R)+\varepsilon_t \tag{20}$$

From equations (8) and (9), with appropriate substitutions, we can see that:

$$\alpha\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right)\approx\left(\sum_{i=1}^{2n-1}\theta_i\right)\alpha(R) \tag{21}$$

Using the relationship between the means of R^M and $\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}}$, $\mu(\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}})=(\sum_{i=1}^{2n-1}\theta_i)\mu(R^M)$, we can reformulate equation (19) as:

$$\alpha(R)\approx\frac{\mu(R^M)}{\mu(\delta\mathrm{AP}_{\mathrm{n}}^{\mathrm{M}})}\alpha\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right) \tag{22}$$

IV. Results

A. Beta estimates

As presented in Table II, market betas obtained from reported returns are substantially below one, with values of 0.53, 0.37, 0.30, and 0.39 for private equity, venture capital, private real estate and natural resources respectively.

Our results, using secondary transaction prices, give betas of 2.08, 2.07, 1.82, and 1.82 for the four asset classes respectively. These results are significantly above the reported return beta at the 0.01 threshold. A summary of results can be found in Table III.

<TABLE III HERE>

The private equity beta estimate of 2.08 is comparable to the results of Axelson, Sorenson and Stromberg (2014), Buchner (2014), and Boyer et al (2018) but much higher than Franzoni, Nowak, and Phalippou (2012), Driessen, Lin and Phalippou (2012), and Ang et al (2018) whose results lie between 1.3 to 1.5. The estimate is close to the theoretical application of Modigliani and Miller (1958) obtained by Axelson, Sorenson and Stormberg (2014).

The venture capital beta estimate of 2.07 beta sits below Buchner (2014), Korteweg and Sorenson (2010), Driessen, Lin and Phalippou (2012), and Ewens (2009) who obtain estimates between 2.4 to 2.8. The closest result is Cochrane (2005) at 1.9, with Ang et al (2018) and Boyer et al (2018) coming in below at 1.7 and 1.0 respectively.

Our estimated beta for real estate and natural resources at 1.8 has no directly comparative estimates.

B. Alpha estimates

We find (annualized) alpha estimates for private equity and venture capital of 0.021 and -0.023 respectively. These estimates are statistically insignificant at all thresholds.

For real estate and natural resources, we estimate (annualized) alphas of -0.127 and -0.051 respectively. These estimate are statistically significant at all thresholds. Results are summarized in Table IV.

<TABLE IV HERE>

C. Stale pricing estimates

The values of the Stale Pricing Indicator are 0.75, 0.82, 0.83, and 0.79 for private equity, venture capital, real estate, natural resources respectively. As our beta estimates are significantly higher than the observable beta at the 0.01 threshold, it follows that these Stale Pricing Indicator values are likewise higher than 0 at the same threshold.

These values confirm that a significant degree of stale pricing is exhibited by the reported returns of all four asset classes. Table V outlines the Stale Pricing calculations.

<TABLE V HERE>

D. Volatility estimates

We estimate (annualized) volatility of 0.421, 0.392, 0.366, and 0.331 for private equity, venture capital, real estate and natural resources respectively. These values are greatly in excess of the volatility of reported returns. These results are summarized in Table VI.

<TABLE VI HERE>

E. Correlation estimates

We estimate correlations with equity markets of 0.7910, 0.8445, 0.8796, and 0.8559 for private equity, venture capital, real estate and natural resources respectively. These results are summarized in Table VII.

<TABLE VII HERE>

F. Conclusions

As illiquid asset classes have grown to become major components of institutional portfolios, it is important that allocators have an accurate understanding of the risk of such asset classes in order to make informed decisions.

Reported returns of such illiquid asset classes, based on estimated valuations, are at risk of being distorted by stale (or managed) prices. Such stale pricing can lead to an understatement of market beta.

It is indeed the case that the reported returns of such asset classes exhibit a high degree of serial correlation (unlike liquid equity markets) that is associated with stale pricing. Researchers have puzzled at the low betas obtained by using such returns.

While a body of literature has developed to present methods for calculating betas that better reflect true economic reality, this has been confined to private equity and venture capital.

In this paper, we present a methodology for utilizing secondary transaction prices to obtain estimates of alpha and beta of the true economic returns of illiquid asset classes. Using this methodology, we are able to estimate alpha and beta consistently for private equity, venture capital, private real estate and private natural resources.

Furthermore we derive a Stale Pricing Indicator, based on these beta estimates, that can provide a measure for the degree of stale pricing exhibited by the reported returns of each asset class.

Our results indicate that illiquid asset classes have true economic betas substantially higher than those obtained using reported returns. We estimates betas of 2.1, 2.1, 1.8 and 1.8 for private equity, venture capital, private real estate and private natural resources respectively. These estimates are significantly greater than reported betas at the 0.01 threshold.

Consequently, we estimate alphas that are either insignificant, in the case of private equity or venture capital, or significantly negative, in the case of real estate and natural resources.

Our estimates of (annualized) asset class volatility are also meaningfully higher than those obtained from reported returns, with estimates ranging between 0.331 and 0.421. All four asset classes exhibit a high degree of stale pricing, with between 75% to 82% of reported prices being stale.

References

Ang, Andrew, Bingxu Chen, William N. Goetzmann, and Ludovic Phalippou, 2018, Estimating Private Equity Returns from Limited Partner Cash Flows, *Journal of Finance* 73, 1751-1783.

Asness, Cliff S., Robert Krail, and John M. Liew, 2001, Do Hedge Funds Hedge?, *The Journal of Portfolio Management* 28, 6-19.

Axelson, Ulf, Morten Sorensen, and Per Stromberg, 2014, Alpha and beta of buyout deals: A jump CAPM for long-term illiquid investments, Unpublished working paper, London School of Economics.

Barkham, Richard, and David Geltner, 1995, Price Discovery in American and British Property Markets, *Real Estate Economics* 23, 21-44.

Boyer, Brian, Taylor D. Nadauld, Keith P. Vorkink, and Michael S. Weisbach, 2018, Private Equity Indices Based on Secondary Market Transactions, NBER Working paper 25207.

Buchner, Axel, 2014, The Alpha and Beta of Private Equity Investments. SSRN Electronic Journal.

Cochrane, John H., 2004, The Risk and Return of Venture Capital, *Journal of Financial Economics* 75, 3-52.

Dimson, Elroy, 1979, Risk Measurement When Shares Are Subject to Infrequent Trading, *Journal of Financial Economics* 7, 197-226.

Driessen, Joost, Tse-Chun Lin, and Ludovic Phalippou, 2012, A New Method to Estimate Risk and Return of Non-Traded Assets from Cash Flows: The Case of Private Equity Funds, *Journal of Financial and Quantitative Analysis* 47, 511-535

Ewens, Michael, 2009, A New Model of Venture Capital Risk and Return, Working paper, University of California San Diego.

Franzoni, Francesco, Eric Nowak and Ludovic Phalippou, 2012, Private Equity Performance and Liquidity Risk, *Journal of Finance* 67, 2341-2373

Harris, Robert S., Tim Jenkinson, and Steven Kaplan, 2014, Private Equity Performance: What Do We Know?, *Journal of Finance* 69, 1851-1882.

Higson, Chris, and Rüdiger Stucke, 2012, The Performance of Private Equity, Working paper, Collier Institute of Private Equity, London Business School.

Jegadeesh, Narasimhan, Roman Kräussl, and Joshua M. Pollet, 2015, Risk and Expected Returns of Private Equity Investments: Evidence Based on Market Prices, *The Review of Financial Studies* 28, 3269–3302.

Kaplan, Steven, and Antoinette Schoar, 2005, Private Equity Performance: Returns, Persistence, and Capital Flows, *Journal of Finance* 60, 1791-1823.

Korteweg, Arthur, and Stefan Nagel, 2016, Risk-Adjusting the Returns to Venture Capital, *Journal of Finance* 71, 1437-1470.

Korteweg, Arthur, and Martin Sorenson, 2011, Risk and Return Characteristics of Venture Capital-Backed Entrepreneurial Companies, *Review of Financial Studies* 23, 3738-3772.

Ljungqvist, Alexander, and Matthew Richardson, 2003, The Cash Flow, Return and Risk Characteristics of Private Equity, NBER Working paper 9454.

Modigliani, Franco, and Merton H. Miller, 1958, The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review* 48, 261-297.

Pagliari, Joseph, Kevin A. Scherer, and Richard T. Monopoli, 2005, Public Versus Private Real Estate Equities: A More Refined Long-Term Comparison, *Real Estate Economics* 33, 147-187.

Riddiough, Timothy J., Mark Moriarty, and P. J. Yeatman, 2005, Privately Versus Publicly Held Asset Investment Performance, *Real Estate Economics* 33, 121-146.

Robinson, David T., and Berk A. Sensoy, 2016, Cyclicality, performance measurement, and cash flow liquidity in private equity, *Journal of Financial Economics* 122, 521-543.

Table I: Secondary Transaction Prices. Descriptive information on the secondary transaction price data utilised in this study. Secondary prices are given by the data sources in normalised form, as a proportion of reported price (NAV) of the fund being transacted. Normalized prices are aggregated and provided as an average over semi-annual or annual periods.

	Private Equity	Venture Capital	Real Estate	Natural Resources
Frequency	Semi-Annual	Semi-Annual	Semi-Annual	Annual
Start	2004	2004	2008	2013
End	2020	2020	2020	2020
N	34	34	26	8
Data Sources				
Preqin	2004 to 2007	2004 to 2007		
Greenhill	2008 to 2020	2008 to 2020	2013 to 2020	2013 to 2020
Descriptive Statistics				
Mean	91.0	79.9	82.0	87.5
Max	109.7	96.3	94.0	93.0
Min	38.0	40.0	39.0	79.0
St. Deviation	13.3	11.9	11.3	4.6

Table II: Reported Returns. Descriptive information on the reported return data utilised in this study provided by Cambridge Associates. Returns are averages of the returns of a large pool of private market funds related to each asset class. Returns are based on changes in reported price (NAV). Beta is the excess returns of the asset class against the excess returns of the S&P 500 Total Return Index. Risk free rate is 3 month USD Libor.

	Private Equity	Venture Capital	Real Estate	Natural Resources
Frequency	Quarterly	Quarterly	Quarterly	Quarterly
Start	2004	2004	2004	2004
End	2020	2020	2020	2020
N	68	68	68	68
Descriptive Statistics				
Mean	0.037	0.033	0.022	0.028
Max	0.165	0.265	0.158	0.170
Min	-0.189	-0.124	-0.231	-0.234
St. Deviation	0.054	0.049	0.053	0.071
Beta, β^O	0.526	0.367	0.300	0.390
Serial Correlation	0.331	0.399	0.658	0.441

Table III: Market Beta Estimates. Table III presents the relationship between the growth of the n-period trailing average of secondary transaction prices with the growth of the n-period trailing average of market prices. The beta, $\hat{\beta}$, of this relationship is an approximation for the market beta of secondary returns to market returns. The alpha value of this relationship, $\alpha\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right)$, is a scalar multiple of the alpha of secondary returns to market returns, $\alpha(R)$, and can be used to estimate the latter (see Table IV). Significance testing of $\alpha\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right)$ can be used to test the significance of $\alpha(R)$. *** denotes significance at the 0.01 threshold. Standard errors are in brackets. Returns calculated on excess returns over USD 3mth LIBOR against the S&P 500 Total Return Index.

	Private Equity	Venture Capital	Real Estate	Natural Resources
Frequency	Semi-Annual	Semi-Annual	Semi-Annual	Annual
Start	2H 2004	2H 2004	2H 2008	2014
End	2H 2020	2H 2020	2H 2020	2020
$\hat{\beta}$	2.082***	2.068***	1.817***	1.821***
	(0.289)	(0.236)	(0.205)	(0.492)
$\alpha\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right)$	0.009	-0.010	-0.052***	-0.205***
	(0.028)	(0.023)	(0.022)	(0.059)
No. Obs.	33	33	25	7
R^2	62.56%	71.31%	77.38%	73.25%

Table IV: Alpha estimates. Table IV presents the calculation of the annualized alpha estimate, $\alpha(R)$, for each illiquid asset class. This estimate can be derived from the relationship between the growth of the n-period trailing average of secondary transaction prices, $\delta\widehat{\mathrm{AP}}_{\mathrm{n}}$, with the growth of the n-period trailing average of market prices, δAP_n^M. $\alpha(R)$ can be estimated with the alpha value of this relationship, $\alpha\left(\delta\widehat{\mathrm{AP}}_{\mathrm{n}}\right)$, scaled by the ratio of the mean return of the n-period trailing average of market prices, $\mu(\delta AP_n^M)$, and the mean return of the market, $\mu(R^M)$. This estimate is then annualized. Calculations are made on excess returns over USD 3mth LIBOR against the S&P 500 Total Return Index

	Private Equity	Venture Capital	Real Estate	Natural Resources
Annualized alpha estimates				
$\alpha\left(\delta\widehat{AP}_n\right)$	0.009	-0.010	-0.052	-0.205
$\mu(\delta AP_n^M)$	0.037	0.037	0.045	0.106
$\mu(R^M)$	0.022	0.022	0.028	0.031
$\alpha(R)$	0.021	-0.023	-0.127	-0.051

Table V: Stale Pricing estimates. Table V presents the calculation of the stale price indicator, SPI, for each illiquid asset class. This indicator is calculated by subtracting the ratio of the reported beta, β^O, with the estimate of the true economic beta, $\hat{\beta}$, from one. This indicator takes a value between 0 and 1. When the value is 0, the reported returns are identical to the contemporaneous true economic returns and there is no stale pricing; when the value is 1, the reported returns are entirely determined by lagged (or stale) returns and there is complete stale pricing.

	Private Equity	Venture Capital	Real Estate	Natural Resources
Stale Pricing Indicator				
β^O	0.526	0.367	0.300	0.390
$\hat{\beta}$	2.082	2.068	1.817	1.821
SPI	0.747	0.823	0.833	0.786

Table VI: Volatility estimates. Table VI presents the calculation of the annualized volatility estimate, $\sigma(R)$, for each illiquid asset class. This estimate can be derived from the variance of the growth of the n-period trailing average of secondary transaction prices, $Var(\delta\widehat{AP}_n)$. This variance is scaled by the ratio of the variance of market returns, $Var(R^M)$ to the variance of the growth of the n-period trailing average of market prices, $Var(\delta AP_n^M)$. The resulting estimated variance, $Var(R)$, can be used to estimate an annualized volatility. Calculations are made on excess returns over USD 3mth LIBOR against the S&P 500 Total Return Index

	Private Equity	Venture Capital	Real Estate	Natural Resources
Annualized volatility estimates				
$Var(\delta\widehat{AP}_n)$	0.056	0.048	0.042	0.017
$Var(\delta AP_n^M)$	0.008	0.008	0.010	0.004
$Var(R^M)$	0.006	0.006	0.008	0.006
$Var(R)$	0.044	0.038	0.033	0.027
$\sigma(R)$	0.421	0.392	0.366	0.331

Table VII: Correlation estimates. Table V presents the calculation of correlation of each illiquid asset class to equity markets. This estimate can be derived directly from the correlation of the growth of the n-period trailing average of secondary transaction prices to the growth of the n-period trailing average of market prices. Calculations are made on excess returns over USD 3mth LIBOR against the S&P 500 Total Return Index

	Private Equity	Venture Capital	Real Estate	Natural Resources
Correlation of illiquid asset class to equity markets				
ρ	0.7910	0.8445	0.8796	0.8559

Figure 1: Secondary Transaction Prices. The figure plots the semi-annual average of secondary transaction prices for private equity, venture capital and real estate. Prices are presented as a proportion of the most recent valuation-based net asset value of the fund interest being traded. Data is provided by Preqin (from 2004 to 2007) and Greenhill (from 2008 to 2020).

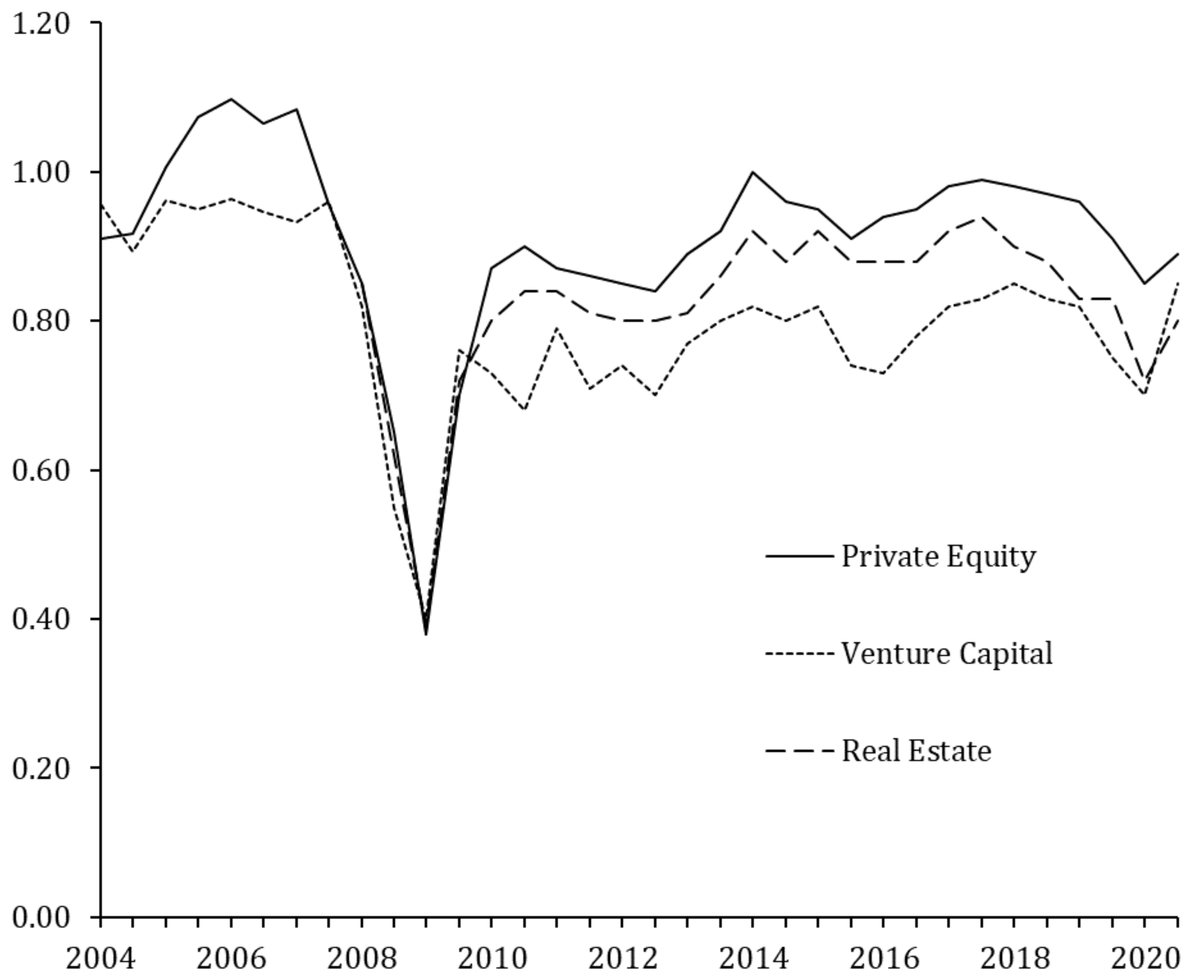

Figure 2: Secondary Transaction Prices of Natural Resources. The figure plots the annual average of secondary transaction prices for the private natural resources asset class. Prices are presented as a proportion of the most recent valuation-based net asset value of the fund interest being traded. Data is from Greenhill (from 2013 to 2020).

Figure 3: Reported Prices. The figure plots indices constructed using quarterly reported returns provided by Cambridge Associates, based at 100 on 31 December 2003. For each quarter, values are aggregate average returns across a pool of 4,000 private market funds split by asset class. Fund returns are based on changes in fund net asset values using estimate valuations of portfolio holdings.