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Two Conditions Which Induce Giffen Behavior In Any Numerical Analysis When Applied To The Wold-Juréen (1953) Utility Function

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Abstract: The present paper extends the existing literature on the relationship between the Wold-Juréen (1953) utility function and Giffen behavior, by extending the recent contribution by Sproule (2020) to the domain of numerical methods. In particular, this paper offers an analytical framework, by which a numerical analysis can used to induce the Wold-Juréen (1953) utility function to exhibit Giffen behavior. Our framework also demonstrates the instructional value to the Wold-Juréen (1953) utility function, especially in those microeconomics courses, in which calculus is not employed or is not emphasized.

Keywords: Wold-Juréen (1953) utility function, Slutsky decomposition, Giffen paradox, pedagogy.

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1. Introduction

In 2011, Wim Heijman and Pierre van Mouche (2011a) released an edited collection of contributions with the title, "New Insights into the Theory of Giffen Behaviour". The sole motivation for this tome was the further exploration and study of Giffen behavior. In the introductory chapter, Heijman and van Mouche (2011b) reported that the genesis for their book sprang from a solitary question – a question which was posed by one of the volume's contributors, Peter Moffatt. That one question is this:

> "(T)o what extent are Giffen goods, for a single consumer, theoretically possible in the neoclassical framework of utility maximization under a budget restriction?" [Heijman and van Mouche (2011b, pages 1-2)].

One major object-of-analysis within this monograph is a particular utility function due to Herman Wold and Lars Juréen -- a utility function which we shall hereafter refer to as the Wold-Juréen (1953) utility function. The main reason for the widespread interest in the Wold-Juréen (1953) utility function is that it has proven pivotal to the study of Giffen behavior – so much so, that Sproule (2020, page 2) proclaimed the Wold-Juréen (1953) utility function the progenitor of all theoretical research on Giffenity using a two-good utility function. Sproule's proclamation represents no more that a crystalization a related statement by Peter Moffatt (2011), viz.,

> "Ever since Wold and Jureen's attempt to illustrate the Giffen paradox by specifying a particular direct utility function, there has been a stream of contributions from authors pursuing similar objectives, for example Spiegel (1994), Weber (1997 and 2001), Moffatt (2002), and Sørensen (2007).[2] One of the lessons learned from this strand of literature is that it is not easy to specify a direct utility function that predicts 'Giffen behaviour' and simultaneously satisfies the basic axioms of consumer theory." [Moffatt (2011, page 127)]

While the Wold-Juréen (1953) utility function has proven pivotal to the study of Giffen behavior, so too has a related paper by Christian Weber (1997), in that Weber's paper

[2] Other examples might include: Haagsma (2012), Landi (2015), and Nachbar (1998).

provides the first thorough enumeration of the essential properties of the Wold-Juréen (1953) utility function.

Since the appearance of the tome by Heijman and van Mouche (2011a), and the prior appearance of the paper by Weber (1997), no other significant contribution to this literature has been made; that is, until the appearance of the paper by Sproule (2020). In this paper, Sproule (2020) presented two main arguments:

- **Argument 1:** Using the Wold-Juréen (1953) utility function, Weber (1997) demonstrated that the Giffenity of Good 1 depends upon the relative magnitudes of the decision maker's (hereafter DM's) income vis-à-vis the price of Good 2. In particular, Weber stated that: "Giffen behavior is more likely for higher … incomes" and that the Giffenity of Good 1 "is more likely at lower values of the price for Good 2" [Weber (1997, page 40)] In response to these two claims, Sproule (2020, page 2) stated: "Weber's precondition is so vague that it lacks broad appeal" and that Weber's precondition does not accord "with a core tenet of microeconomics, which is that economic decision-making is predicated on (changes in) relative prices."
- **Argument 2**: Sproule (2020) then went on to offer a new precondition for Giffen behavior under the Wold-Juréen (1953) utility function – one which does accord with the core tenet of microeconomics regarding relative prices. In particular, Sproule's (2020) precondition (or prediction) is this: if the DM has the Wold-Juréen (1953) utility function, and if the price of Good 1 is greater than or equal to the price of Good 2, then Good 1 is a Giffen good."

The purpose of this paper is to extend Sproule's (2020) analysis of the Wold-Juréen (1953) utility function and Giffen behavior to the domain of numerical methods. In particular, the purpose of this paper is to provide the instructor of the typical undergraduate course in microeconomics with an analytical model for a particular two-good utility function, in which one of the goods is an inferior good, so that this same good (when placed in a numerical domain) can be shown to exhibit Giffen behavior, when two restrictions are imposed on the selection of the numerical values. That utility function is the Wold-Juréen (1953) utility function. The origins of these two restrictions are: (a) the core properties of the Wold-Juréen (1953) utility function itself, and (b) the analysis by Sproule (2020).

This paper is organized as follows. Section 2 provides a terse overview of the core literature. In particular, Section 2 highlights the key points reported in two papers, viz., Weber (1997) and Sproule (2020). Section 3 presents our theoretical framework. In particular, in Section 3, we present the two conditions which (when combined) induce Giffen behavior in any numerical analysis when they are applied to the Wold-Juréen (1953) utility function. In Section 4, we undertake a numerical demonstration of the validity of the analytical framework presented in Section 3. In particular, we show that if the set of numerical values for the exogenous variables (viz., the two prices and our DM's income) satisfy the two conditions defined in Section 3, then that requirement alone is sufficient to induce the Wold-Juréen (1953) utility function to exhibit Giffen behavior. Summary remarks are offered in Section 5.

2. Previous Research

Suppose that our DM resides in a two-good world, $\left(x_1, x_2\right)$, where x_1 and x_2 denote the quantities of Goods 1 and 2. In this world, suppose that p_1 and p_2 denote the market prices of these two goods, and m denotes our DM's income.

Finally suppose that our DM's utility function is the Wold-Juréen (1953) utility function. That is, suppose that his utility function is defined as $U = \frac{(x_1 - 1)}{(x_2 - 2)^2}$ where $x_1 > 1$ and $0 < x_2 < 2$ [see Wold and Juréen (1953), Weber (1997), and Sproule (2020)]. If so, then our DM's ordinary or Marshallian demand functions are as follows:

$$x_1^* = 2 + \frac{2p_2 - m}{p_1} \tag{1}$$

$$x_2^* = 2\left(\frac{m - p_1}{p_2} - 1\right) \quad (2)$$

where $x_1^* > 1$ and $0 < x_2^* < 2$. [See Weber (1997) and Sproule (2020).]

As mentioned above, just two papers are needed to summarize the relationship between the Wold-Juréen (1953) utility function and Giffen behavior or (in common parlance) "Giffenity". These are Weber (1997) and Sproule (2020). Key elements of these two papers are defined next in Propositions 1 and 2.

Proposition 1 [Weber (1997)]: If the DM has the Wold-Juréen (1953) utility function, then:

(a) $\text{sign}(\text{TE}) = \text{sign}\left(\frac{\partial x_1^*}{\partial p_1}\right) = \text{sign}(m-2p_2)$ where TE denotes the total effect of a change in the price of Good 1 [see Weber (1997, page 40)].

(b) If m is large and if p_2 is small, then $\text{TE} = \frac{\partial x_1^*}{\partial p_1} > 0$ and Good 1 is a Giffen good [see Weber (1997, page 40)].

Sproule (2020) found that Weber's precondition for Giffenity [viz., $m > 2p_2$ in Proposition 1] to be intuitively indefensible. As a consequence, Sproule proposed an alternative precondition, which is this:

Proposition 2 [Sproule (2020)]: If the DM has the Wold-Juréen (1953) utility function, and if $p_1 > p_2$, then x_1^* is a Giffen good [that is, $\frac{\partial x_1^*}{\partial p_1} = \text{TE} > 0$].

As mentioned already in Arguments 1 and 2 above, Sproule (2020) argued that his precondition [viz., $p_1 > p_2$] is more appealing than Weber's precondition [viz., $m-2p_2 > 0$] because Sproule's precondition accords with a core tenet of micro-economics, which is that economic decision-making is predicated on relative prices or on changes in relative prices.

3. Our Model for Inducing Giffen Behavior In Any Numerical Analysis When Applied To The Wold-Juréen (1953) Utility Function

Our strategy for inducing Giffen behavior in any numerical analysis when applied to the Wold-Juréen (1953) utility function flows from two propositions. These are reported next in Propositions 3 and 4:[3]

Proposition 3: If our DM has the Wold-Juréen (1953) utility function, then

$p_1 + p_2 < m < p_1 + 2p_2$

Proof: Since by definition, $x_1^* = 2 + \frac{2p_2 - m}{p_1}$ for $x_1^* > 1$ [see Equation (1)], it then follows that

$$p_1 + 2p_2 > m \qquad (3)$$

And since by definition, $x_2^* = 2\left(\frac{m - p_1}{p_2} - 1\right)$ for $0 < x_2^* < 2$ [see Equation (2)], it also follows that

$$p_1 + p_2 < m < p_1 + 2p_2 \qquad (4)$$

Finally, we note that, given Equation (4), Equation (3) is redundant. •

Proposition 4: When using numerical methods to determine the presence of Giffen behavior in the context of the Wold-Juréen (1953) utility function, success requires that the numerical values be chosen so that they satisfy these two conditions,

$p_1 + p_2 < m < p_1 + 2p_2$ and $p_1 > p_2$.

Proof: See Proposition 2 and Proposition 3 above**.** •

4. The Confirmation of Proposition 4 by A Numerical Example

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[3] The reader ought to keep in mind that, henceforth, we shall employ one of two possible variants of the substitution effect in the Slutsky equation, viz., we shall use the "Slutsky substitution effect" rather than the "Hicksian substitution effect". For further details, see Chapter 8 of Varian (2014).

In this section, we confirm that the model outlined in Section 3 is sufficiently simple that it can be used in a "traditional classroom" lecture, or (in the common parlance) a "chalk and talk" lecture [Becker and Watts (1996).], especially in those instances in which it cannot be presumed that all students have a command of calculus. In this section, we show that if a given set of numerical values fully accord with Proposition 4 above, then the numerical values for the component parts of the Slutsky Equation (viz., the total effect, substitution effect, and income effect for the Wold-Juréen (1953) utility function) can be deduced and that the sign of numerical value of the TE is positive..

Remark 1: One set of numerical values which satisfy the two conditions defined in Proposition 4 is this set:

$$
\begin{aligned}
&p_1 = 10 \\
&p_1' = 9 \\
&p_2 = 5 \\
&p_1 + p_2 = 15 \\
&p_1 + 2p_2 = 20 \\
&m = 18
\end{aligned}
$$

Remark 2: To verify that the set of numerical values contained in Remark 1 accords with Proposition 4, we present these two numerical tests:

$$
\begin{aligned}
&\text{Test 1: } p_1 + p_2 = 15 < m = 18 < p_1 + 2p_2 = 20 \\
&\text{Test 2: } p_1 = 10 > p_2 = 5
\end{aligned}
$$

Remark 3: The numerical values reported in Remark 1, and then vetted in Remark 2, yield the three points of equilibrium which are integral to the determination of the magnitudes of the associated total effect, substitution effect, and income effect for for Good 1 when the DM's utility function is the Wold-Juréen (1953) utility function. These three points we term here as Point A, Point B, and Point C.[4]

[4] The graphical counterpart to the present description or analysis can be found in Figure 8.1 of Varian (2014).

- **Point A:** Point A is the point of initial equilibrium; that is, Point A is the equilibrium point where the exogenous variables are our DM's income and the initial values of the price of Good 1 and Good 2. Hence, we denote the general value of Good 1 at Point A as:

$$x_1^A = x_1^A(p_1, p_2, m) = 2 + \frac{2p_2 - m}{p_1}.$$

- **Point C:** Point C is the point of final equilibrium; that is, Point C is the equilibrium point where the exogenous variables are our DM's income, the initial price for Good 2, and the final price for Good 1. In summary, we denote the general value of Good 1 at Point C as:

$$x_1^C = x_1^C(p_1', p_2, m) = 2 + \frac{2p_2 - m}{p_1'}.$$

- **Point B:** Point B is the third point of equilibrium; that is, Point B is the equilibrium point where the exogenous variables are the initial price for Good 2, the final price for Good 1, and our DM's income is adjusted so that the equilibrium values at Point B, $\left(x_1^B(p_1', p_2, m'), x_2^B(p_1', p_2, m')\right)$, are equally affordable as the equilibrium values at Point A, $\left(x_1^A(p_1, p_2, m), x_2^A(p_1, p_2, m)\right)$, where $m' = \Delta p_1 . x_1^A + m$. For more on the adjustment of the DM's income, m', see Varian (2014, Chapter 8). With this. we can now denote the general value of Good 1 at Point B as:

$$x_1^B = x_1^B(p_1', p_2, m') = 2 + \frac{2p_2 - m'}{p_1'}$$

Remark 4: Given Remark 1 and Remark 3, we can now state the numerical values for Good 1 at Points A, B, and C as follows:

- **Point A:** $x_1^A = x_1^A(p_1, p_2, m) = 2 + \dfrac{2p_2 - m}{p_1} = 2 + \dfrac{2(5) - 18}{10} = 1.2000$

- **Point B:** Since $m' = \Delta p_1 . x_1^A + m = (-1)(1.2) + 18 = 16.8$, therefore

$$x_1^B = x_1^B(p_1', p_2, m') = 2 + \frac{2p_2 - m'}{p_1'} = 2 + \frac{2(5) - 16.8}{9} = 1.2444$$

- **Point C:** $x_1^C = x_1^C(p_1^{'}, p_2, m) = 2 + \dfrac{2p_2 - m}{p_1^{'}} = 2 + \dfrac{2(5) - 18}{9} = 1.1111$

Remark 5: With Remark 4 in tow, we are now in a position to define the numerical values of the three component parts of the Slutsky Equation, viz., the total effect (TE), the substitution effect (SE), and the income effect (IE). In view of Remarks 1 and 4, these numerical values are as follows.

- **Total Effect:** The numerical value of the TE is:

$$TE = \frac{x_1^C - x_1^A}{\Delta p_1} = \frac{1.1111 - 1.2000}{-1} = \frac{-0.0889}{-1} = 0.0889 > 0$$

  We note here that the numerical value of the TE is positive, which indicates that Good 1 is a Giffen good, just as predicted by Proposition 4 and Remark 2.

- **Substitution Effect:** The numerical value of the SE is:

$$SE = \frac{x_1^B - x_1^A}{\Delta p_1} = \frac{1.2444 - 1.2000}{-1} = \frac{0.0444}{-1} = -0.0444 < 0$$

- **Income Effect:** The numerical value of the IE is:

$$IE = \frac{x_1^C - x_1^B}{\Delta p_1} = \frac{1.1111 - 1.2444}{-1} = \frac{-0.1333}{-1} = 0.1333 > 0$$

  We note here that the numerical value of the IE is positive, which indicates that Good 1 is an inferior good and which is accord with a feature of the Wold-Juréen (1953) utility function.

Remark 6: The results reported in Remark 5 accord with all expectations for the Slutsky decomposition under the Wold-Juréen (1953) utility function, viz.,

- $SE = -0.0444 < 0$

- $0 < IE = 0.1333$.

- $\text{TE} = \text{SE} + \text{IE} \Leftrightarrow 0.0889 = -0.0444 + 0.1333$

- $\text{TE} = 0.0889 > 0$ because $|\text{SE}| < |\text{IE}|$

In summary, we note (once again) that the TE is positive (which indicates that Good 1 is a Giffen good), and this property arises because the IE is positive and because the IE dominates the SE.

5. Conclusion

As noted at the outset, the Wold-Juréen (1953) utility function is the progenitor of all research inquiries into the identification of those two-good utility functions, which have the potential to exhibit Giffen behavior [Moffatt (2011) and Sproule (2020)]. The present paper extends the existing research on the relationship between the Wold-Juréen (1953) utility function and Giffen behavior, by building upon the contribution by Sproule (2020). In particular, the present paper offers an analytical framework, by which a numerical analysis alone can be used to induce the Wold-Juréen (1953) utility function to exhibit Giffen behavior. As a consequence, the present paper offers new evidence of the pedagogical value of the Wold-Juréen (1953) utility function, especially in those microeconomics courses in which calculus is not emphasized or not required.

This paper is organized as follows. In Section 2, we offered a terse overview of the salient features of two core papers on Giffenity under the Wold-Juréen (1953) utility function [viz., Weber (1997) and Sproule (2020)]. In Section 3, we first presented our theoretical framework, which contains two conditions, and then we argued that these two conditions (when combined) induce Giffen behavior in any numerical analysis when they are applied to the Wold-Juréen (1953) utility function. In Section 4, we undertook a numerical confirmation of this claim. In particular, we showed by way of example that if the set of numerical values of the exogenous variables (viz., the set comprised of the price of Good 1, the price of Good 2, and the DM’s income) satisfy the two conditions defined in Section 3, then that fact alone will induce the Wold-Juréen (1953) utility function to exhibit Giffen behavior.

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