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Abstract

The aim of this paper is to investigate the impact of public sentiment on tail risk forecasting. In this framework, we extend the Realized Exponential GARCH model to directly incorporate information from realized volatility measures and exogenous variables. Several indices related to social media and journal articles regarding the economy and stock market volatility are considered as potential drivers of volatility dynamics. An application to the prediction of daily Value at Risk and Expected Shortfall for the Standard & Poor's 500 index provides evidence that combining the information content of realized volatility and sentiment measures can lead to significant accuracy gains in forecasting tail risk.

Keywords: Realized Exponential GARCH; sentiment indices; economic policy uncertainty; tail risk forecasting; risk management.
JEL Codes: C22, C53, C58, D80, E66, G32.

1. Introduction

The global financial crisis of 2007-08 emphasized how quantitative financial risk management has become a key tool in investment decisions, capital allocation and regulation. Although there are several methods for estimating the risk of an investment in capital markets, since its implementation by the Basel Committee on Banking Supervision (BCBS) in 1996 (Basel Committee, 1996), Value-at-Risk (VaR) has become the standard measure of market risk, as it is used for both internal control of financial institutions and regulatory purposes.

The most recent financial crisis revealed substantial weaknesses in the risk models used by national supervisors and the Basel Accords. The changing nature of financial risk requires accurate risk measures and models that react quickly to the impulses of the latest events. This has led to extensive changes in financial market regulation and banking supervision. In this vein, the Basel III accord aims to achieve benefits from the financial stability of the banking system with sustainable costs for both credit institutions and economy, requiring financial institutions to accurately assess their exposure to financial and market risks.

The BCBS has veered toward the Expected Shortfall (ES) as the primary measure of market risk (Basel Committee, 2013, 2016, 2017, 2019), thus complementing, and in part replacing, VaR

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as a key measure for international banking regulation. Also, the BCBS has proposed a transition from VaR with a 99% confidence level to ES with a 97.5% confidence level.

Changes in market risk capital accords are showing an increasing focus on ES mainly because VaR does not address the implications regarding the distribution of losses beyond the risk quantile threshold and, at the same time, it is not a coherent measure of risk as it does not satisfy the property of subadditivity, i.e., the VaR of a portfolio can be greater than the sum of the VaRs of the individual assets in the portfolio (Artzner et al., 1999). The ES, which gives the expected loss conditional on returns exceeding the corresponding VaR threshold, is known to be a coherent measure of risk (Acerbi and Tasche, 2002) and it has been suggested as an alternative to VaR in risk management applications. While superior theoretical properties favor ES over VaR as a risk measure, a major drawback of the ES is that it is not elicitable, namely, there is no scoring or loss function that can be minimized by the true ES (Gneiting, 2011). However, Fissler and Ziegel (2016) found that ES is jointly elicitable with VaR, providing a class of consistent scoring functions that can be used to jointly evaluate VaR and ES forecasts. This emphasized the importance of generating reliable VaR and ES forecasts, which are considered core metrics in financial risk management.

Concurrently, volatility plays a central role in risk management, portfolio allocation and pricing of financial instruments. The widespread availability of databases providing intraday prices of several financial assets has led to new developments in daily volatility modeling using nonparametric estimators that exploit the information contained in high-frequency data. Such ex-post volatility measures can be used directly for modelling and forecasting volatility dynamics, which in turn can be essential for improving the predictive accuracy of risk measures. In this regard, the volatility forecasting literature provides extensive evidence that the use of realized volatility (RV) measures, can be beneficial in improving the accuracy of volatility forecasts (Andersen and Bollerslev, 1998; Andersen et al., 2003; Hansen and Lunde, 2011), with several papers also supporting the usefulness of including realized measures in risk forecasting models (Gerlach et al., 2020; Naimoli et al., 2022).

The role of public sentiment and its impact on the financial system and the economy have recently become increasingly relevant, as policy uncertainty increases market volatility and negatively affects corporate outcomes (Chahine et al., 2021). Also, economic policy uncertainty increases the cost of raising equity capital, especially when the economy is weak (Chan et al., 2021). Accordingly, economic policy uncertainty, which is uncertainty related to monetary policy, fiscal policy, and other relevant policies, could have significant impacts on equity markets. For example, Pástor and Veronesi (2013) analyzing the effects of political uncertainty on stock prices in the context of a general equilibrium model showed that political uncertainty increases risk premia and makes stocks more volatile and more correlated, especially in relatively weaker economic conditions. Antonakakis et al. (2013), relying on a DCC-GARCH model, found that correlations between U.S. stock market returns, volatility, and economic policy uncertainty vary over time and that a rise in policy uncertainty increases stock market return uncertainty. Liu and Zhang (2015) investigate the predictability of economic policy uncertainty to stock market volatility by using extensions of the Heterogeneous Autoregressive RV (HAR-RV) of Corsi (2009), with the out-of-sample results revealing that economic policy uncertainty plays a significant role in forecasting realized market volatility. Audrino et al. (2020) applying a novel and extensive dataset incorporating social media (Twitter and Stocktwits), news articles (RevenPack News Analytics), information consumption, and search engine data (Google trends and Wikipedia) found evidence that sentiment and attention variables can improve the accuracy of HAR-RV forecasting models. Xu et al. (2021) provide a quantile GARCH-MIDAS model approach to examine the influence of monthly economic policy uncertainty on daily VaR in the West Texas Intermediate crude oil spot and futures markets, finding that an increase in economic policy uncertainty leads to greater WTI crude oil market risk.

This paper aims to investigate the effects of public sentiment indices on tail risk forecasting. Specifically, we analyze the impact of uncertainty-related sentiment on VaR and ES forecasts using a large dataset combining social media and newspaper articles concerning the economy and stock market volatility. As a proxy for economic policy uncertainty, we use the Economic Policy Uncertainty (EPU) index developed by Baker et al. (2016), which relies on newspaper coverage to measure uncertainty about a range of economic policy factors such as government spending, taxation, regulatory changes, and monetary policies. Along the same lines, Baker et al. (2019) introduced the Equity Market Volatility (EMV) Index that moves with VIX and the RV of returns on the Standard & Poor's 500 (S&P 500). Finally, social media has become not only a "newspaper" of the new era, but also a possible tool for measuring the mood or feeling of individuals. Given the popularity of Twitter, we also refer to several Twitter-based Economic Uncertainty (TEU) indices proposed by Baker et al. (2021).

In light of the existing literature in this field, the impact on stock market volatility from exogenous predictors is predominantly modelled in a HAR or GARCH-MIDAS framework. For example, Liu and Zhang (2015) found that including EPU index as an additional predictor variable in HAR-RV specifications significantly improves the predictive ability of the models for U.S. stock market volatility. Also, Ma et al. (2018) investigated whether and how the EPU increases the accuracy of HAR-RV type models in forecasting the volatility of crude oil futures. On the other hand, the class of generalized autoregressive conditional heteroskedasticity-mixed-data sampling (GARCH-MIDAS) models offers a convenient approach to combine data that are sampled at different frequencies, avoiding information loss. In particular, to relate low-frequency macroeconomic variables to financial market volatility, Engle et al. (2013) proposed the GARCH-MIDAS model specifying the low-frequency component as a MIDAS filter of macroeconomic variables. This modelling framework has been applied in several empirical applications to explore the potential impact of EPU (Yu and Huang, 2021), text-counting EMV trackers (Zhu et al., 2019) and Twitter-based uncertainty measures (Lang et al., 2021) on volatility.

Several articles have highlighted the importance of incorporating sentiment indices into return and volatility forecasting models but without paying due attention to VaR and ES. In investigating whether exogenous uncertainty predictors can explain financial market volatility and improve tail risk forecasts, this paper contributes to the literature on analyzing financial risk management under economic and financial policy uncertainty in several ways. First, we examine the impact that recently developed newspaper-based uncertainty indices have on tail risk forecasting by using the EPU, EMV and four TEU indicators observed on a daily basis rather than a lower frequency, such as weekly, monthly or quarterly. Second, from a methodological point of view, since our focus is on daily exogenous indicators, there is no need for approaches that combine information sampled at different frequencies, such as MIDAS-type specifications. In this direction, this paper seeks to further investigate how to optimally use exogenous variables and improve the accuracy of tail risk forecasting by extending the Realized Exponential GARCH (REGARCH) of Hansen and Huang (2016). Specifically, this is done by directly incorporating the exogenous covariates into the volatility dynamics, but also by adding a measurement equation that simultaneously links the latent conditional variance to the exogenous variables. The proposed modelling approach allows to parsimoniously and simultaneously exploit the information content of multiple realized volatility and exogenous measures in forecasting volatility, VaR and ES, while controlling for noisy signals arising from different volatility sources. Finally, we evaluate the accuracy of VaR and ES forecasts at different risk levels and market conditions by means of consistent loss functions. This allows us to test the extent to which public sentiment influences financial risk management, which type of sentiment measure is most relevant in forecasting tail risk, and what their predictive power is at different risk levels.

An empirical application on the S&P 500 index reveals that incorporating newspaper-based uncertainty indices as additional predictor variables in a Realized GARCH framework leads to significant accuracy gains in forecasting tail risk compared to the specification based only on the use of the 5-minute RV. In particular, the out-of-sample results provide evidence that combining realized volatility, economic policy uncertainty and equity market volatility measures in a REGARCH model can significantly improve the quality of VaR and ES forecasts at different levels of risk. On the other hand, Twitter-based uncertainty indices turn out to be less influential in forecasting tail risk. The ability of competing models to generate accurate VaR forecasts is assessed through backtesting procedures along with the Quantile Loss function (González-Rivera et al., 2004). Also, we rely on strictly consistent loss functions to jointly evaluate VaR and ES forecasts (Fissler and Ziegel, 2016; Patton et al., 2019). To test the statistical significance of performance gaps of the competing models, the Model Confidence Set of Hansen et al. (2011) is used.

The remainder of this paper is organized as follows. Section 2 briefly reviews the Realized GARCH and Realized Exponential GARCH models. The proposed extensions of this class of models are discussed in Section 3. Section 4 focuses on the estimation procedure of the models under analysis. Section 5 provides information on the data and descriptive statistics. The forecasting results are reported in Section 6. Finally, Section 7 concludes the paper.

2. Realized GARCH models

2.1. Realized GARCH

The Realized GARCH (RGARCH) model by Hansen et al. (2012) represents a flexible framework for jointly modelling returns and realized volatility measures. The RGARCH differs from standard GARCH model in several respects. First, in volatility dynamics, the squared returns are replaced by a more efficient proxy, i.e., a realized volatility measure. Second, it relates the realized measure to the latent volatility through a measurement equation, also including an asymmetric response to shocks. Finally, the RGARCH model is very flexible and dynamically complete, allowing the generation of multi-step volatility forecasts.

Here, the focus is on the log-linear specification of RGARCH, which is defined by the following three equations

$$r_t = \sqrt{h_t}\, z_t \tag{1}$$
$$log(h_t) = \omega + \beta\, log(h_{t-1}) + \gamma\, log(y_{t-1}) \tag{2}$$
$$log(y_t) = \xi + \varphi\, log(h_t) + \tau(z_t) + u_t\,, \tag{3}$$

where r_t and y_t denote the daily log-return and realized volatility measure, respectively. Letting $\mathcal{F}_{t-1}$ be the information set at time $t-1$, then $h_t = var(r_t|\mathcal{F}_{t-1})$, with $z_t \overset{iid}{\sim} \mathcal{D}(0,1)$ and $u_t \overset{iid}{\sim}$

$\mathcal{D}(0, \sigma_u^2)$ mutually independent. The Equation 1 is the return equation, while Equation 2 denotes the GARCH or volatility equation. Finally, the Equation 3 is the measurement equation, which is designed to capture the contemporaneous dependence between latent volatility and the realized measure, where the function $\tau(z_t) = \tau_1 z_t + \tau_2(z_t^2 - 1)$ allows for leverage-type effects.

The log-linear specification is very flexible since no specific assumptions on the parameters are required to guarantee the positivity of the conditional variance, which automatically holds by construction, whereas the restriction $\beta + \varphi\gamma < 1$ is required for the stationarity of the RGARCH process (Li et al., 2019).

2.2. *Realized Exponential GARCH*

In order to benefit from the multiple information characterizing volatility measures, Hansen and Huang (2016) extended the RGARCH through the possibility of adding a measurement equation for each realized measure of interest, resulting in the Realized Exponential GARCH (REGARCH).

In this framework, let $\boldsymbol{y}_t = (y_{1,t}, \cdots, y_{K,t})'$ be a vector of K realized measures, the REGARCH is defined as

$$r_t = \sqrt{h_t}\, z_t \tag{4}$$

$$log(h_t) = \omega + \beta\, log(h_{t-1}) + \delta(z_{t-1}) + \boldsymbol{\gamma}'\, \boldsymbol{u}_{t-1} \tag{5}$$

$$log\,(y_{j,t}) = \xi_j + \varphi_j\, log(h_t) + \tau_j(z_t) + u_{j,t} \qquad j = 1, ..., K\,, \tag{6}$$

where $\boldsymbol{\gamma} = (\gamma_1, ..., \gamma_K)'$ and $\boldsymbol{u}_t = (u_{1,t}, ..., u_{K,t})'$. The innovation terms z_t and $\boldsymbol{u}_t$ are mutually and serially independent, with $z_t \overset{iid}{\sim} \mathcal{D}(0, 1)$ and $\boldsymbol{u}_t \overset{iid}{\sim} \mathcal{D}(0, \Sigma)$. Finally, the functions $\tau(z_t) = \tau_1 z_t + \tau_2(z_t^2 - 1)$ and $\delta(z_t) = \delta_1 z_t + \delta_2(z_t^2 - 1)$ are used to model leverage-type effects, capturing the dependence between returns and future volatility.

Compared to RGARCH, the REGARCH model is specified with respect to u_t instead of y_t; it provides an autoregressive (AR) representation for the conditional variance with innovations given by $\delta(z_t) + \boldsymbol{\gamma}'\, \boldsymbol{u}_t$, such that $\beta < 1$ measures the persistence of the conditional variance; it accommodates an additional leverage term in the GARCH equation, i.e., $\delta(z_t)$; finally, it allows for the inclusion of multiple realized volatility measures.

3. Including exogenous variables in Realized Exponential GARCH models

To better model and forecast financial time series volatility, in addition to using multiple realized volatility measures, it may be useful to include exogenous regressors in the specification of volatility dynamics. For example, several studies have analyzed the ability of different economic variables to improve in-sample and out-of-sample volatility forecasts (Engle and Rangel, 2008; Engle et al., 2013; Asgharian et al., 2013; Conrad and Loch, 2015; Asgharian et al., 2015; Dorion, 2016). Therefore, the use of additional covariates that explain the "economic source" of volatility can lead to better performance in terms of both in-sample fit and out-of-sample forecasts. In this context, a particularly popular model is the so-called GARCH-X model in which the standard GARCH specification is augmented by adding exogenous regressors to the volatility equation.

Along these lines, a natural way to incorporate exogenous covariates into the REGARCH model is to directly add explanatory variables in the volatility equation[1]

$$
\begin{aligned}
log(h_t) &= \omega + \beta\, log(h_{t-1}) + g(\boldsymbol{\lambda}, \boldsymbol{x}_{t-1}) + \delta(z_{t-1}) + \boldsymbol{\gamma}'\, \boldsymbol{u}_{t-1} && (7)\\
log\,(y_{j,t}) &= \xi_j + \varphi_j\, log(h_t) + \tau_j(z_t) + u_{j,t} \qquad j = 1, ..., K\,, && (8)
\end{aligned}
$$

where $g(\cdot)$ is a linear or non-linear function of the exogenous variables $\boldsymbol{x}_t = (x_{1,t}, \cdots, x_{M,t})'$ and a parameter vector $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_M)'$. Accordingly, the inclusion of exogenous variables (X) in the volatility dynamics leads to the Realized Exponential GARCH-X (REGARCH-X) model.

The main advantage of the REGARCH-X is that it accounts for the forecasting information provided by multiple exogenous variables. Also, in analyzing the properties of a GARCH-X specification with an explanatory variable that enters additively into the conditional variance equation, Han (2015) provided evidence that the GARCH-X process more adequately explains stylized financial time series facts such as long-memory and leptokurtosis, where these properties heavily depend on the degree of persistence of the exogenous covariate. On the other hand, a drawback of this class of models is that the dynamics of the exogenous covariates are not specified. Therefore, since the future values of the covariates are not known, the REGARCH-X allows only one-step-ahead volatility forecasts.

In practice, exogenous variables may exhibit values that move dynamically in light of a functional relationship with volatility or stock returns. Therefore, in the spirit of Hansen and Huang (2016), it is reasonable to consider the additional regressors as endogenous variables within the model by specifying a measurement equation even for $\boldsymbol{x}_t$. A practical implementation of these approach leads to the Complete Realized Exponential GARCH-X (REGARCH-CX) model. Assuming, for simplicity, that $g(\cdot)$ is specified as a logarithmic function of the exogenous variables $\boldsymbol{x}_t$, the REGARCH-CX is defined by the following equations

$$
\begin{aligned}
log(h_t) &= \omega + \beta\, log(h_{t-1}) + \delta(z_{t-1}) + \boldsymbol{\gamma}'\, \boldsymbol{u}_{t-1} + \boldsymbol{\lambda}'\, \boldsymbol{v}_{t-1} && (9)\\
log\,(y_{j,t}) &= \xi_j + \varphi_j\, log(h_t) + \tau_j(z_t) + u_{j,t} \qquad j = 1, ..., K && (10)\\
log\,(x_{i,t}) &= \zeta_i + \phi_i\, log(h_t) + \kappa_i(z_t) + v_{i,t} \qquad i = 1, ..., M\,. && (11)
\end{aligned}
$$

Because the effects of exogenous variables can affect market volatility in different ways, the appeal of this modelling approach lies in its ability to capture complex dynamics through a flexible parameter structure. Specifically, the distinctive measurement equations relate the realized volatility measures $\boldsymbol{y}_t$ and regressors $\boldsymbol{x}_t$ to the conditional variance, where this set of variables contributes to model the volatility dynamics through the coefficients $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ in the GARCH equation.

Here, it is worth noting that the GARCH equation is an AR(1) model for the log-conditional variance, with innovations given by $\delta(z_{t-1}) + \boldsymbol{\gamma}'\, \boldsymbol{u}_{t-1} + \boldsymbol{\lambda}'\, \boldsymbol{v}_{t-1}$. Also, note that ex-post volatility measures may differ from the conditional variance of returns due to sampling error affecting realized measures and volatility shocks given by the difference between ex-post and ex-ante volatility, which in the standard REGARCH model correspond to $\delta(z_t) + \boldsymbol{\gamma}'\, \boldsymbol{u}_t$ (Hansen and Huang, 2016). Therefore, it follows that including the term $\boldsymbol{\lambda}'\, \boldsymbol{v}_t$ in the GARCH equation can help smooth noisy signals originating from different volatility sources, such as economic drivers of financial volatility.

[1]As the return equation is standard, in the following, we focus only on the GARCH and measurement equations.

From a different perspective, the explanatory variables $\boldsymbol{x}_t$ are assumed to be proportional to the conditional variance and volatility shocks. This specification is driven by the link between uncertainty in financial markets and economic fluctuations. In particular, movements in financial volatility can be extremely informative about future economic activity (see e.g. Bloom, 2009; Fornari and Mele, 2013, among others). However, if we expect the realized measures to be proportional to h_t, with $\xi \approx 0$ and $\varphi \approx 1$, this would not be the case for the exogenous economic variables that provide a much more noisy signal about financial volatility. In this respect, it is worth remarking the idea behind the specification of the measurement equation for $\boldsymbol{x}_t$. Because news can have different effects on volatility depending on the state of the economy, directly modelling interactions between h_t and the "filtered" exogenous variables through $\boldsymbol{v}_t$ can potentially contribute to mitigate the measurement error. Finally, the existence of a measurement equation for each covariate in the model makes the REGARCH-CX dynamically complete, allowing multi-period volatility forecasting to be performed iteratively according to the jointly estimated empirical dynamics.

As different news may have different impacts on financial markets, in our empirical analysis several specifications combining realized volatility measures and the information content from daily text-count indices are considered to forecast VaR and ES.

4. Estimation

In this section, we discuss the estimation procedure for REGARCH-CX. The estimation of the model parameters can be easily performed by the maximum likelihood (ML) approach, making appropriate assumptions on the model error terms. The log-likelihood of the REGARCH-CX model is given by

$$\mathcal{L}(\boldsymbol{r}, \boldsymbol{y}, \boldsymbol{x}; \boldsymbol{\theta}) = \sum_{t=1}^{T} log\, f(r_t, y_t, x_t | \mathcal{F}_{t-1}),$$

where $\boldsymbol{\theta}$ is the parameter vector characterizing the volatility and measurement equations, with $f(r_t, y_t, x_t|\mathcal{F}_{t-1})$ the joint conditional density that, by assumptions, can be factorized as

$$f(r_t|\mathcal{F}_{t-1})\, f(y_t, x_t|r_t, \mathcal{F}_{t-1}).$$

Since the main focus is on VaR and ES, to more accurately capture the tail behavior of stock returns, differently from Hansen and Huang (2016) who assume an underlying Gaussian QMLE structure for the log-linear REGARCH model, here we consider a Student-t distribution for z_t (Gerlach et al., 2020; Naimoli et al., 2022), while assuming a multivariate Normal distribution for the measurement errors. Under the stated distributional assumptions, it can be easily shown that the log-likelihood function can be written as

$$\mathcal{L}(\boldsymbol{r}, \tilde{\boldsymbol{u}}; \boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T} -\mathcal{V}(\nu) + log(h_t) + (1+\nu)\, log\left(1 + \frac{r_t^2}{h_t(\nu-2)}\right) \tag{12}$$

$$-\frac{1}{2}\sum_{t=1}^{T} N log(2\pi) + log\left(|\Sigma_{\tilde{\boldsymbol{u}}}|\right) + \tilde{\boldsymbol{u}}_t'\, \Sigma_{\tilde{\boldsymbol{u}}}^{-1}\, \tilde{\boldsymbol{u}}_t\,, \tag{13}$$

where $\tilde{\boldsymbol{u}}_{\boldsymbol{t}} = (\boldsymbol{u}_t', \boldsymbol{v}_t')'$, with $N = (K + M)$ and $\mathcal{V}(\nu) = log\left[\Gamma\left(\frac{\nu+1}{2}\right)\right] - log\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{1}{2} log[\pi(\nu-2)]$.

The overall log-likelihood allows for both the contribution of the returns from (12) and explanatory variables (realized volatility measures and exogenous regressors) from (13). The partial log-likelihood of returns in (12) can be used to compare specifications using different sets of realized measures or RGARCH-type models with GARCH-type models. Finally, the log-likelihood of the REGARCH-X closely follows that of REGARCH-CX by setting $\tilde{\boldsymbol{u}}_t = \boldsymbol{u}_t$.

5. Data

To investigate the impact of sentiment indices on tail risk forecasting, we cross-reference information provided by two main categories of data. First, we account for dynamic financial market signals through daily observations of open-to-close log-returns and the 5-minute RV (RV5) of the S&P 500 index, publicly accessible at https://realized.oxford-man.ox.ac.uk. The realized volatility can be easily calculated as the sum of the squared intraday returns $r_{t,i} : RV_t = \sum_{i=1}^{N} r_{t,i}^2$. Second, to measure the impulse generated by the socially perceived uncertainty, we refer to some sentiment indices, i.e., daily indicators based on text counts of newspaper articles that include several keywords related to the economy or stock market volatility.

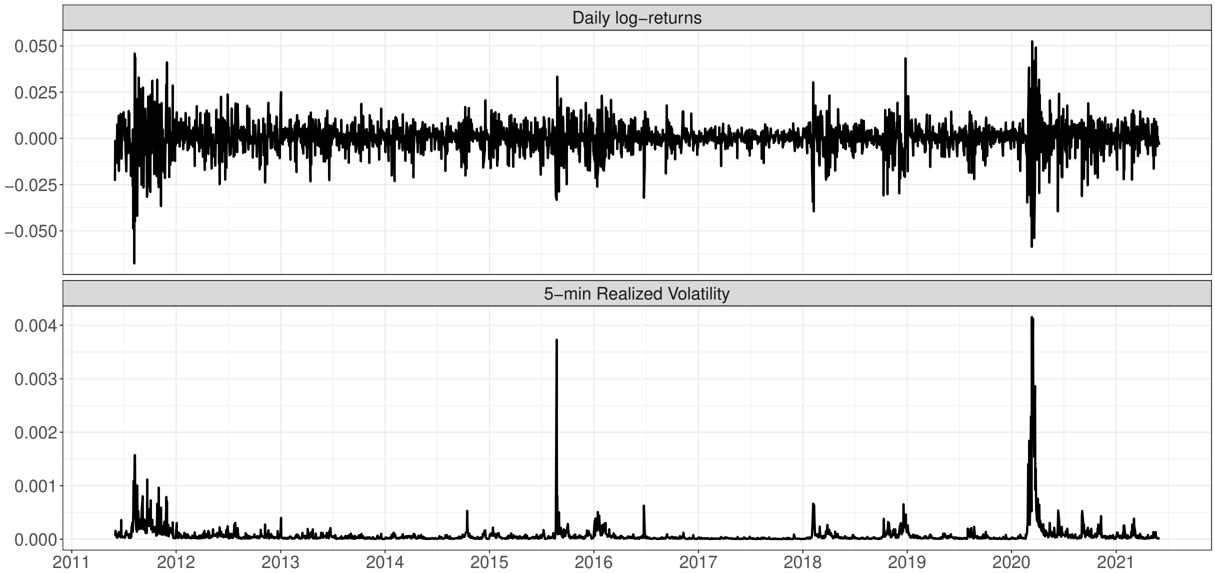
To investigate the role of economic policy uncertainty, we resort to the daily Economic Policy Uncertainty index for the United States. The EPU reflects the frequency of articles in 10 leading U.S. newspapers (USA Today, the Miami Herald, the Chicago Tribune, the Washington Post, the Los Angeles Times, the Boston Globe, the San Francisco Chronicle, the Dallas Morning News, the Houston Chronicle, and the WSJ) that contain a triple of terms related to economy (E), policy (P), and uncertainty (U). Specifically, the index is based on newspaper coverage frequency containing the terms *uncertainty* or *uncertain*, *economic* or *economy* and one or more of the following terms: *congress*, *legislation*, *white house*, *regulation*, *federal reserve*, or *deficit* (for further details on how the index is constructed see Baker et al., 2016).

On the other hand, to account for the public attention on future market volatility, we consider the Equity Market Volatility Index (Baker et al., 2019). This is because the EMV moves with the CBOE Volatility Index (VIX) and with the realized volatility of returns on the S&P 500. The EMV tracker is calculated through an analysis of articles from eleven major U.S. newspapers (the Boston Globe, Chicago Tribune, Dallas Morning News, Houston Chronicle, Los Angeles Times, Miami Herald, New York Times, San Francisco Chronicle, USA Today, Wall Street Journal, and Washington Post) containing terms related to stock market uncertainty, i.e., by counting the following keywords: E (*economic*, *economy*, *financial*); M (*stock market*, *equity*, *equities*, *Standard and Poors* (and variants)) and V (*volatility*, *volatile*, *uncertain*, *uncertainty*, *risk*, *risky*).

Finally, we also consider measures to track perceptions of economic uncertainty using text messages posted by users and journalists on the social network Twitter. TEU indicators are based on counts of messages (tweets) about the economy (E) and uncertainty (U). To construct these indicators, a database of tweets containing a keyword related to economics (*economic*, *economical*, *economically*, *economics*, *economies*, *economist*, *economists*, *economy*) and uncertainty (*uncertain*, *uncertainly*, *uncertainties*, *uncertainty*) is employed. Following Baker et al. (2021), four variants of daily TEU indices are considered in our analysis.

The TEU.ENG, based on all tweets in English regardless of the user's location, consists of the total number of daily English-language tweets that contain terms related to uncertainty and economy. Aiming to provide a daily Twitter economic uncertainty indicator for the United States, the TEU.USA is constructed by isolating the number of tweets originating from users in the U.S. using a geotagged-based Random Forest classifier. Along these lines, two other variations of

Figure 1: Daily log-returns (top panel) and 5-minute Realized Volatility (bottom panel) of the S&P 500 index for the full sample period June 01, 2011 – June 01, 2021.



the TEU.USA are considered. Since the number of retweets of a message can be considered as a measure of influence, the TEU.WGT is a variant of TEU.USA that weights each tweet by considering the number of retweets of each message (1+log(1+# of retweets)). Finally, to track changes in Twitter usage intensity over time, the TEU.SCA index scales the number of tweets each day by the number of tweets on that day containing the word *have*. The data and further details on these uncertainty indices can be found at https://www.policyuncertainty.com/index.html.

Given the characteristics of the daily Twitter data, the sampling period is from June 01, 2011 to June 01, 2021, for a total of 2507 observations. Also, to have the same observations as the financial variables, all uncertainty measures were adjusted by excluding non-trading days. To make the financial and text-count measures comparable, but also to reduce computational issues, in our out-of-sample forecasting study, the uncertainty indices were adjusted to have the same scale as the RV5.

Figure 1 illustrates the time plots of the daily log-returns (top panel) and RV5 (bottom panel) of the S&P 500 index, highlighting several high-volatility periods. Specifically, the effects of the 2011-12 sovereign debt crisis in Europe were then followed by the 2015-16 stock market sell-off, which was triggered by several events including the Chinese stock market turbulence, the Greek debt default in 2015, the end of quantitative easing in the U.S. in late 2014, the 2016 United Kingdom European Union membership referendum (Brexit) and the U.S. presidential election. High volatility in 2018 stems from a variety of issues, such as trade tensions between the U.S. and China, weakening of the technology sector, concerns about slowing global growth and the increase in interest rates by the Federal Reserve. Finally, recent volatility in global financial markets is largely induced by the current Covid-19 pandemic.

More insights on the features of the variables can be found in Table 1, which reports the main descriptive statistics of daily log-returns, RV5, and all daily indicators based on keywords related to economic and policy uncertainty. The daily log-returns show a standard deviation around 1%

Table 1: Summary statistics for the full sample period June 01, 2011 – June 01, 2021. Key to table. r_t: daily open-to-close log-returns; RV5: daily 5-min Realized Volatility; EPU: Economic Policy Uncertainty Index; EMV: Equity Market Volatility Index; TEU.$*$: Twitter-based Economic Uncertainty (TEU) Index, where $* \in$(ENG, USA, WGT, SCA); $\dagger = \times 100$; $\ddagger = /1000$.

	mean	sd	median	min	max	skew	kurtosis
r_t	0.000	0.009	0.000	–0.068	0.052	–0.550	6.675
RV5†	0.008	0.022	0.003	0.000	0.415	10.951	156.775
EPU‡	0.119	0.092	0.091	0.003	0.808	2.469	8.415
EMV‡	0.057	0.083	0.028	0.005	0.940	4.043	24.076
TEU.ENG‡	0.102	0.090	0.075	0.008	1.476	3.406	26.751
TEU.USA‡	0.102	0.109	0.070	0.003	1.560	3.864	25.107
TEU.WGT‡	0.102	0.118	0.067	0.002	1.662	4.129	27.119
TEU.SCA‡	0.102	0.099	0.073	0.005	2.038	6.026	77.917

Table 2: Correlation matrix of log-transformed variables.

	log(RV5)	log(EPU)	log(EMV)	log(TEU.ENG)	log(TEU.USA)	log(TEU.WGT)	log(TEU.SCA)
log(RV5)	1.000	–	–	–	–	–	–
log(EPU)	0.322	1.000	–	–	–	–	–
log(EMV)	0.415	0.575	1.000	–	–	–	–
log(TEU.ENG)	0.312	0.560	0.534	1.000	–	–	–
log(TEU.USA)	0.383	0.626	0.562	0.916	1.000	–	–
log(TEU.WGT)	0.360	0.612	0.554	0.921	0.988	1.000	–
log(TEU.SCA)	0.388	0.419	0.333	0.512	0.699	0.650	1.000

and are negatively skewed. In addition, the pronounced excess kurtosis provides evidence of a non-Gaussian r_t distribution, as expected. The RV5 and uncertainty indices are characterized by positive skewness and excess kurtosis, although more moderate values are observed after log-transformation.

Along these lines, Table 2 reports the correlation matrix for the log-transformed variables. This is natural choice as we consider a log-volatility specification in the REGARCH framework but, at the same time, the log-transformation also makes the relationship between RV5 and the exogenous variables more linear. It is worth noting that all measures are positively correlated, where RV5 exhibits a correlation between 0.312 (TEU.ENG) and 0.415 (EMV) with the uncertainty indices, indicating that market volatility and sentiment indices tend to move in the same direction.

6. Tail-risk forecasting

6.1. Forecasting design, backtesting and scoring rules

Model parameters are recursively estimated daily via ML over a 1500-day rolling window. As a result, the out-of-sample period runs from May 17, 2017 to June 01, 2021, covering the stock market volatility in 2018 and the turbulence period driven by the ongoing Covid-19 pandemic. To accurately determine regulatory capital requirements and assess exposure to different degrees of risk of financial institutions, one-step-ahead VaR and ES forecasts are generated for the risk levels of 0.5%, 1% and 2.5%.

Formally, let $\mathcal{F}_t$ be the information set available at time t and

$$F_{t,r}(x) = Pr(r_t \leq x | \mathcal{F}_{t-1})$$

be the Cumulative Distribution Function (CDF) of r_t conditional on $\mathcal{F}_{t-1}$. Assuming that $F_{t,r}(\cdot)$ is strictly increasing and continuous on the real line $\mathbb{R}$, the one-step-ahead α-level VaR at time t can be defined as

$$VaR_{t+1}(\alpha) = F^{-1}_{t+1,r}(\alpha) = h_{t+1}\, F^{-1}_z(\alpha) = h_{t+1}\, z_\alpha, \qquad 0 < \alpha < 1,$$

where $F_z(\cdot)$ is the CDF of the returns innovations z_t. Similarly, the one-step-ahead α-level ES can be defined (see, e.g., Acerbi and Tasche, 2002) as the tail expectation of r_{t+1} conditional on $VaR_{t+1}(\alpha)$ violations

$$ES_{t+1}(\alpha) = E\left(r_{t+1} | \mathcal{F}_t, r_{t+1} \leq VaR_{t+1}(\alpha)\right) = h_{t+1}\, E(z_{t+1} | z_{t+1} \leq z_\alpha).$$

The adequacy of VaR forecasts is evaluated through backtesting, using both the empirical Violation Rate (VR), defined as the proportion of VaR violations over the forecast period, along with the Conditional Coverage (CC) test of Christoffersen (1998) and the Dynamic Quantile (DQ) test of Engle and Manganelli (2004).

On the other hand, in order to rank the models according to their ability to accurately predict extreme VaR losses and statistically assess the differences in the forecasting performance of different models, we resort to the Quantile Loss (QL) function (González-Rivera et al., 2004)

$$QL_t(\alpha) = (\alpha - l_t)(r_t - VaR_t(\alpha))\,, \tag{14}$$

where l_t is a dummy variable such that $l_t = \mathcal{I}_{(r_t < VaR_t(\alpha))}$. Being an asymmetric loss function, the QL is particularly suitable for evaluating quantile risk measures, as it imposes a higher penalty, with weight $(1-\alpha)$, for observations below the α-quantile level, i.e., when returns exceeding VaR are observed. Also, the QL loss function is well known to be a strictly consistent scoring rule for VaR.

Finally, to assess the ability of the proposed models to jointly forecast VaR and ES, we resort to the results of Fissler and Ziegel (2016) on the joint elicitability of the (VaR,ES) pair. Specifically, a risk measure is *elicitable* if there exists a scoring function such that the risk measure is the solution to minimizing the expected loss (see, e.g., Gneiting, 2011; Fissler and Ziegel, 2016; Fissler et al., 2016; Patton et al., 2019, among others). Specifically, Fissler and Ziegel (2016) show that VaR and ES are jointly elicitable with respect to the following class of loss functions

$$FZ_t = (l_t - \alpha)\left(G_1(v_t) - G_1(r_t) + \frac{1}{\alpha}G_2(e_t)v_t\right) - G_2(e_t)\left(\frac{1}{\alpha}l_t r_t - e_t\right) - \mathcal{G}_2(e_t)\,, \tag{15}$$

where $G_1(\cdot)$ is weakly increasing, $G_2(\cdot)$ is strictly increasing and strictly positive and $\mathcal{G}_2'(\cdot) = G_2(\cdot)$. It can be shown that the expected value of the loss in (15) is uniquely minimized by setting v_t and e_t equal to the VaR(α) and ES(α) series, respectively. Assuming VaR and ES to be strictly negative and $ES_t(\alpha) \leq VaR_t(\alpha) < 0$, the zero-degree homogeneous loss function (Patton et al., 2019) is obtained from (15) by setting $G_1(x) = 0$ and $G_2(x) = -1/x$

$$FZ^0_t = \frac{1}{\alpha ES_t(\alpha)} l_t\,(r_t - VaR_t(\alpha)) + \frac{VaR_t(\alpha)}{ES_t(\alpha)} + log(-ES_t(\alpha)) - 1\,. \tag{16}$$

The significance of differences in forecasting performance of the models is tested by the Model Confidence Set (MCS) approach of Hansen et al. (2011), considering different confidence levels and the Semi-Quadratic (SQ) statistic.

6.2. *Empirical results*

This section investigates the impact of sentiment indices in generating out-of-sample one-step-ahead forecasts of VaR and ES. In this regard, the standard REGARCH based on the 5-minute RV is compared with different specifications of REGARCH-X and REGARCH-CX that combine the RV5 with the uncertainty indices under analysis. Regarding the naming of the models, for example, REG-CX(RV5,EPU,EMV) represents the Complete REGARCH-X model employing RV5, EPU and EMV as covariates.

Table 3: Value-at-Risk backtesting. VR shows the Violation Rate as percentage of returns smaller than VaR during the forecast period (1007 days) at the risk levels of 0.5%, 1% and 2.5%. In **bold** models showing the VR closest to the nominal α-level. CC and DQ report the p-values for the Conditional Coverage test and Dynamic Quantile test, respectively. Boxes indicate evidence against the null hypothesis of correct model specification at the 5% significance level.

	$\alpha = 0.5\%$			$\alpha = 1\%$			$\alpha = 2.5\%$		
	VR	CC	DQ	VR	CC	DQ	VR	CC	DQ
REG(RV5)	1.390	$\boxed{0.004}$	$\boxed{0.032}$	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.674	0.079	0.297
REG-X(RV5,EPU)	1.192	$\boxed{0.027}$	0.281	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.674	0.079	0.302
REG-X(RV5,EMV)	1.291	$\boxed{0.011}$	0.101	2.185	$\boxed{0.004}$	$\boxed{0.007}$	3.674	0.070	0.200
REG-X(RV5,TEU.USA)	1.490	$\boxed{0.001}$	$\boxed{0.006}$	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.774	$\boxed{0.048}$	0.165
REG-X(RV5,TEU.SCA)	1.390	$\boxed{0.004}$	$\boxed{0.032}$	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.774	0.051	0.194
REG-X(RV5,TEU.WGT)	1.390	$\boxed{0.004}$	$\boxed{0.032}$	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.774	$\boxed{0.048}$	0.164
REG-X(RV5,TEU.ENG)	1.490	$\boxed{0.001}$	$\boxed{0.006}$	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.774	$\boxed{0.048}$	0.164
REG-X(RV5,EPU,EMV)	1.291	$\boxed{0.011}$	0.101	2.085	$\boxed{0.008}$	$\boxed{0.027}$	3.674	0.070	0.199
REG-X(RV5,EPU,TEU.USA)	1.390	$\boxed{0.004}$	$\boxed{0.025}$	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.873	$\boxed{0.032}$	0.135
REG-X(RV5,EMV,TEU.USA)	1.390	$\boxed{0.004}$	$\boxed{0.035}$	2.085	$\boxed{0.008}$	$\boxed{0.027}$	3.774	$\boxed{0.048}$	0.164
REG-X(RV5,EPU,EMV,TEU.USA)	1.390	$\boxed{0.004}$	$\boxed{0.035}$	2.085	$\boxed{0.008}$	$\boxed{0.027}$	3.774	$\boxed{0.048}$	0.166
REG-CX(RV5,EPU)	0.993	0.134	0.501	1.986	$\boxed{0.015}$	0.050	3.674	0.079	0.310
REG-CX(RV5,EMV)	1.291	$\boxed{0.011}$	0.104	1.986	$\boxed{0.015}$	0.056	3.376	0.175	0.521
REG-CX(RV5,TEU.USA)	1.192	$\boxed{0.027}$	0.100	2.085	$\boxed{0.008}$	$\boxed{0.009}$	3.873	$\boxed{0.032}$	0.141
REG-CX(RV5,TEU.SCA)	1.291	$\boxed{0.010}$	$\boxed{0.033}$	2.085	$\boxed{0.008}$	$\boxed{0.026}$	3.774	$\boxed{0.048}$	0.164
REG-CX(RV5,TEU.WGT)	1.291	$\boxed{0.010}$	$\boxed{0.027}$	2.085	$\boxed{0.008}$	$\boxed{0.009}$	3.972	$\boxed{0.021}$	0.105
REG-CX(RV5,TEU.ENG)	1.192	$\boxed{0.027}$	0.060	1.986	$\boxed{0.015}$	$\boxed{0.005}$	3.376	0.175	0.289
REG-CX(RV5,EPU,EMV)	**0.596**	0.884	0.999	**1.192**	0.726	0.986	**3.178**	0.417	0.609
REG-CX(RV5,EPU,TEU.USA)	0.794	0.446	0.711	1.887	$\boxed{0.028}$	$\boxed{0.004}$	3.376	0.180	0.169
REG-CX(RV5,EMV,TEU.USA)	1.092	0.063	0.503	1.887	$\boxed{0.027}$	$\boxed{0.021}$	3.575	0.099	0.232
REG-CX(RV5,EPU,EMV,TEU.USA)	0.794	0.446	0.842	1.589	0.116	0.144	3.476	0.134	0.356

The results of the backtesting procedure of the 1-step-ahead VaR forecasts for the risk levels of 0.5%, 1%, and 2.5% are reported in Table 3, showing the empirical VR and the evidence of correct model specification under the CC and DQ tests. A VR close to the nominal risk level is a necessary but not sufficient condition for an accurate forecasting model. In this vein, the VaR violation rate is used as a first metric to evaluate the accuracy of VaR forecasting, where models showing the VR closest to nominal quantile levels tend to be preferred. Our findings provide evidence that the REG-CX(RV5,EPU,EMV) always returns the VR closest to the nominal α-risk level. Overall, several models fail the CC test, while fewer problems are found for the DQ test. In particular, specifications incorporating the EPU and EMV uncertainty indices along with the RV5 are generally less likely to be rejected by the backtesting at the usual 5% significance level compared to other models. On the other hand, for specifications adopting TEU measures, it is more challenging to pass the DQ test, particularly at $\alpha = 0.5\%; 1\%$. Finally, the standard

Table 4: QL and FZ^0 loss functions. Ratios of the QL and FZ^0 loss functions of all competing models to those of the REG(RV5) (benchmark model) at the risk levels of 0.5%, 1% and 2.5%. In bold models minimizing the losses. Gray and light gray shaded boxes indicate models included in the 75% and 90% MCS, respectively.

	$\alpha = 0.5\%$		$\alpha = 1\%$		$\alpha = 2.5\%$	
	QL	FZ^0	QL	FZ^0	QL	FZ^0
REG(RV5)	100.00	100.00	100.00	100.00	100.00	100.00
REG-X(RV5,EPU)	98.87	99.34	99.58	99.65	100.00	99.88
REG-X(RV5,EMV)	98.02	99.03	98.90	99.32	99.62	99.71
REG-X(RV5,TEU.USA)	99.72	99.93	100.13	100.05	100.16	100.03
REG-X(RV5,TEU.SCA)	99.89	100.01	99.84	99.98	100.11	100.01
REG-X(RV5,TEU.WGT)	100.06	100.08	100.19	100.10	100.16	100.04
REG-X(RV5,TEU.ENG)	99.89	100.00	100.23	100.09	100.11	100.04
REG-X(RV5,EPU,EMV)	98.02	99.09	98.87	99.39	99.61	99.74
REG-X(RV5,EPU,TEU.USA)	99.38	99.66	99.90	99.85	100.25	99.98
REG-X(RV5,EMV,TEU.USA)	98.87	99.55	99.26	99.67	100.02	99.89
REG-X(RV5,EPU,EMV,TEU.USA)	98.87	99.55	99.22	99.64	100.13	99.91
REG-CX(RV5,EPU)	95.88	97.66	96.41	97.98	99.23	99.28
REG-CX(RV5,EMV)	95.26	97.63	96.89	98.23	99.07	99.33
REG-CX(RV5,TEU.USA)	97.80	98.78	97.57	98.87	99.79	99.63
REG-CX(RV5,TEU.SCA)	99.38	99.86	99.58	99.92	100.15	99.98
REG-CX(RV5,TEU.WGT)	98.70	99.25	98.22	99.21	100.16	99.77
REG-CX(RV5,TEU.ENG)	104.29	101.65	102.17	100.93	100.75	100.43
REG-CX(RV5,EPU,EMV)	**92.32**	**96.38**	**93.14**	**96.44**	**97.74**	**98.52**
REG-CX(RV5,EPU,TEU.USA)	103.84	101.28	102.69	100.67	102.34	100.63
REG-CX(RV5,EMV,TEU.USA)	93.79	97.04	95.18	97.50	98.94	99.03
REG-CX(RV5,EPU,EMV,TEU.USA)	95.65	98.03	96.54	98.12	98.22	99.02

REG(RV5), although it passes the becktesting at $\alpha = 2.5\%$, fails the CC and DQ tests at the most extreme risk levels of 0.5% and 1%. These findings reveal that through the inclusion of uncertainty indices, improvements in the VaR forecasting accuracy are observed. This is particularly evident for REGARCH-CX models that include a measurement equation also for exogenous variables.

Focusing on the ability of the competing models to generate accurate VaR and ES forecasts, Table 4 reports the ratio of the QL and FZ^0 loss functions of all models over the REG(RV5), which is taken as the benchmark model, together with the MCS results at the 75% and 90% confidence levels. Values smaller than 100 denote improvements over the benchmark.

Again, it emerges that the REGARCH specifications combining RV5 with EPU and EMV tend to produce more accurate VaR and ES forecasts, where the gains are particularly evident at the more extreme risk levels. Also, it is worth noting that the REG-CX(RV5,EPU,EMV) minimizes the QL and FZ^0 at each risk level and is the only model always entering the 75% MCS. Specifically, in terms of QL, the gain over the benchmark is greater than 7.5% at $\alpha = 0.5\%$ and 6.5% at $\alpha = 1\%$. When jointly evaluating the quality of VaR and ES forecasts through the FZ^0 loss function, the REG-CX(RV5,EPU,EMV) shows improvements of more than 3.5% over the REG(RV5) for both $\alpha = 0.5\%$ and $\alpha = 1\%$. Moving to the 2.5% risk level, the gains in predictive performance relative to the benchmark are less pronounced, but still remain around 2% and 1.5% for QL and FZ^0, respectively. Although not directly analyzed here, these gaps to the benchmark are substantially preserved even at the 5% risk level.

The benefits of introducing sentiment measures into REGARCH-CX models are remarkable even when considering specifications with two measurement equations associating RV5 with EPU or EMV, but are mitigated when considering sentiment estimates derived from Twitter. In this case, the least performing specifications are those including the TEU.ENG index, using all English tweets that contain both economic and uncertainty terms, regardless of the user's geolocation. This may be because the effect generated by the socially perceived uncertainty when considering all tweets in English, including those from users outside the United States (e.g., Canada, the United Kingdom, Australia, India, etc.) could be biased by significant events occurring outside the United States (such as the Brexit) that are likely to negatively affect the tail risk estimate. However, even when considering indices that rely only on tweets posted by users located in the United States, we find that modelling approaches based on the combination of RV5 with the different Twitter-based measures of economic uncertainty (TEU.USA, TEU.SCA, and TEU.WGT) lead to less significant accuracy gains in forecasting daily VaR and ES than specifications using EPU and EMV. This is also evident from the fact that the REGARCH-CX model with four measurement equations defined for RV5, EPU, EMV, and TEU.USA, respectively, shows higher losses for QL and FZ^0 than the specification using three measurement equations that only merges information from RV5, EPU, and EMV.

In addition, our results confirm that the benefits of including exogenous factors are more moderate when working in a REGARCH-X framework that uses uncertainty measures directly in volatility dynamics without "filtering" their information content through the corresponding measurement equations. The class of REGARCH-X models appears to be less sensitive to the choice of exogenous regressors, with average losses for QL and FZ^0 closer to each other than REGARCH-CX specifications.

Finally, the REG(RV5) is never included in the MCS at $\alpha = 1\%$; for FZ^0 it enters the MCS only at the 90% confidence level at the risk levels of 0.5% and 2.5%; for QL it enters the 75% MCS only in the case of $\alpha = 2.5\%$.

Summarizing, our findings point out that: extending the standard REGARCH model by incorporating public sentiment indices leads to improvements in the out-of-sample forecasting of tail risk measures; the REGARCH-CX combining the information content of RV5, EPU and EMV minimizes the considered losses in all cases and is the only model always entering the 75% MCS; this reflects the advantage of adding a measurement equation in the REGARCH-X model that simultaneously links latent conditional variance to exogenous variables; the TEU measures appear to be less influential in forecasting VaR and ES than EPU and EMV; the main benefits in combining economic and financial uncertainty indices with RV5 occur at the more extreme risk levels of 0.5% and 1%.

7. Conclusion

In this paper, we introduce a novel approach to account for the impact of sentiment indices on financial tail risk forecasting. Specifically, the REGARCH is extended to jointly modelling the information from realized volatility measures and exogenous variables leading to the REGARCH-X and REGARCH-CX.

In our empirical analysis, several variables related to social media and journal articles regarding the economy and stock market volatility are considered. This class of variables includes the EPU, EMV, and four Twitter-based indices of economic uncertainty observed at a daily frequency. Several interesting results emerge from the out-of-sample evaluation of models that include exogenous covariates compared to the standard REGARCH that relies only on the 5-minute RV. With respect to the backtesting of VaR forecasts, the proposed specifications are generally less likely to be rejected than conventional REGARCH, showing an empirical VR closer to the nominal quantile level. Turning to the loss functions for VaR and ES, the REGARCH-CX models show consistently lower losses than the other models considered, especially the specification combining the dynamics of RV5, EPU and EMV. This is also confirmed by the MCS results. The major benefits of combining the information content of realized volatility measures and uncertainty indices are particularly evident at the more extreme risk levels of 0.5% and 1%, with an average improvement of 7% over the benchmark in terms of VaR, using the QL function. Similarly, for ES, according to the FZ^0, the average gains are around 3.5% compared to REGARCH using only RV5.

On the other hand, the benefits of incorporating Twitter-based uncertainty indices are less evident, if any. The mitigated effects of Twitter-based measures on tail risk forecasting could be due to several factors. First, differently from EPU and EMV indices that are constructed using information provided by leading U.S. newspapers, Twitter-based uncertainty measures could include information from a broader pool of users, who may express their perception of uncertainty in different ways. Second, social media influencers create viral movements and hashtags, tweeting new information to a large network of followers who, in turn, retweet the information. As a result, Twitter-based uncertainty indices might be noisier than measures based on major U.S. journals. In addition, the recent decline in Twitter popularity and usage could have increased the volatility of these uncertainty indices.

Finally, we found that it is reasonable to specify a measurement equation also for exogenous variables. A functional relationship between exogenous variables and volatility allows filtering noisy signals arising from different sources of volatility, leading to significant improvements in tail risk forecasting. From a risk management perspective, this could allow financial institutions to more effectively allocate capital under the Basel Capital Accord.

A natural extension of the current work would be to use both different modelling approaches, such as semiparametric models for VaR and ES, and alternative sentiment and attention variables.

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References

Acerbi, C. and D. Tasche (2002). On the coherence of expected shortfall. *Journal of Banking & Finance 26*(7), 1487 – 1503.

Andersen, T. G. and T. Bollerslev (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review 39*(4), 885–905.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys (2003). Modeling and forecasting realized volatility. *Econometrica 71*(2), 579–625.

Antonakakis, N., I. Chatziantoniou, and G. Filis (2013). Dynamic co-movements of stock market returns, implied volatility and policy uncertainty. *Economics Letters 120*(1), 87–92.

Artzner, P., F. Delbaen, J.-M. Eber, and D. Heath (1999). Coherent measures of risk. *Mathematical finance 9*(3), 203–228.

Asgharian, H., C. Christiansen, and A. J. Hou (2015). Effects of macroeconomic uncertainty on the stock and bond markets. *Finance Research Letters 13*, 10–16.

Asgharian, H., A. J. Hou, and F. Javed (2013). The importance of the macroeconomic variables in forecasting stock return variance: A garch-midas approach. *Journal of Forecasting 32*(7), 600–612.

Audrino, F., F. Sigrist, and D. Ballinari (2020). The impact of sentiment and attention measures on stock market volatility. *International Journal of Forecasting 36*(2), 334–357.

Baker, S. R., N. Bloom, S. Davis, and T. Renault (2021). Twitter-derived measures of economic uncertainty.

Baker, S. R., N. Bloom, and S. J. Davis (2016). Measuring economic policy uncertainty. *The quarterly journal of economics 131*(4), 1593–1636.

Baker, S. R., N. Bloom, S. J. Davis, and K. J. Kost (2019). Policy news and stock market volatility. Technical report, National Bureau of Economic Research.

Basel Committee (1996). Overview over the amendment to the capital accord to incorporatemarket risks, technical report, baselcommittee on banking supervision. Available athttps://www.bis.org/publ/bcbs23.htm.

Basel Committee (2013). Fundamental review of the trading book: A revised market risk framework. technical report, bank for international settlements. Available athttps://www.bis.org/publ/bcbs265.htm.

Basel Committee (2016). Minimum capital requirements for market risk. technical report, bank for international settlements. Available athttps://www.bis.org/bcbs/publ/d352.htm.

Basel Committee (2017). Pillar 3 disclosure requirements - consolidated and enhanced framework. technical report, basel committee on banking supervision. Available athttps://www.bis.org/bcbs/publ/d400.htm.

Basel Committee (2019). Minimum capital requirements for market risk. technical report, bank for international settlements. Available athttps://www.bis.org/bcbs/publ/d457.htm. Revised in Februrary 2019.

Bloom, N. (2009). The impact of uncertainty shocks. *econometrica 77*(3), 623–685.

Chahine, S., M. Daher, and S. Saade (2021). Doing good in periods of high uncertainty: Economic policy uncertainty, corporate social responsibility, and analyst forecast error. *Journal of Financial Stability 56*, 100919.

Chan, Y.-C., W. Saffar, and K. J. Wei (2021). How economic policy uncertainty affects the cost of raising equity capital: Evidence from seasoned equity offerings. *Journal of Financial Stability 53*, 100841.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review 39*(4), 841–862.

Conrad, C. and K. Loch (2015). Anticipating long-term stock market volatility. *Journal of Applied Econometrics 30*(7), 1090–1114.

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics 7*(2), 174–196.

Dorion, C. (2016). Option valuation with macro-finance variables. *Journal of Financial and Quantitative Analysis 51*(4), 1359–1389.

Engle, R. F., E. Ghysels, and B. Sohn (2013). Stock market volatility and macroeconomic fundamentals. *Review of Economics and Statistics 95*(3), 776–797.

Engle, R. F. and S. Manganelli (2004). Caviar: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics 22*(4), 367–381.

Engle, R. F. and J. G. Rangel (2008). The spline-garch model for low-frequency volatility and its global macroeconomic causes. *The review of financial studies 21*(3), 1187–1222.

Fissler, T. and J. F. Ziegel (2016, 08). Higher order elicitability and osband's principle. *Ann. Statist. 44*(4), 1680–1707.

Fissler, T., J. F. Ziegel, and T. Gneiting (2016). Expected shortfall is jointly elicitable with value at risk-implications for backtesting. *Risk Magazine*, 58–61.

Fornari, F. and A. Mele (2013). Financial volatility and economic activity. *Journal of Financial Management, Markets and Institutions 1*(2), 155–198.

Gerlach, R., A. Naimoli, and G. Storti (2020). Time-varying parameters realized garch models for tracking attenuation bias in volatility dynamics. *Quantitative Finance 20*(11), 1849–1878.

Gneiting, T. (2011). Making and evaluating point forecasts. *Journal of the American Statistical Association 106*(494), 746–762.

González-Rivera, G., T.-H. Lee, and S. Mishra (2004). Forecasting volatility: A reality check based on option pricing, utility function, value-at-risk, and predictive likelihood. *International Journal of Forecasting 20*(4), 629–645.

Han, H. (2015). Asymptotic properties of garch-x processes. *Journal of Financial Econometrics 13*(1), 188–221.

Hansen, P. and A. Lunde (2011). Forecasting volatility using high frequency data. *The Oxford Handbook of Economic Forecasting, Oxford: Blackwell*, 525–556.

Hansen, P. R. and Z. Huang (2016). Exponential garch modeling with realized measures of volatility. *Journal of Business & Economic Statistics 34*(2), 269–287.

Hansen, P. R., Z. Huang, and H. H. Shek (2012). Realized garch: a joint model for returns and realized measures of volatility. *Journal of Applied Econometrics 27*(6), 877–906.

Hansen, P. R., A. Lunde, and J. M. Nason (2011). The model confidence set. *Econometrica 79*(2), 453–497.

Lang, Q., X. Lu, F. Ma, and D. Huang (2021). Oil futures volatility predictability: Evidence based on twitter-based uncertainty. *Finance Research Letters*, 102536.

Li, Y.-N., Y. Zhang, and C. Zhang (2019). Statistical inference for measurement equation selection in the log-realgarch model. *Econometric Theory 35*(5), 943–977.

Liu, L. and T. Zhang (2015). Economic policy uncertainty and stock market volatility. *Finance Research Letters 15*, 99–105.

Ma, F., M. Wahab, J. Liu, and L. Liu (2018). Is economic policy uncertainty important to forecast the realized volatility of crude oil futures? *Applied Economics 50*(18), 2087–2101.

Naimoli, A., R. Gerlach, and G. Storti (2022). Improving the accuracy of tail risk forecasting models by combining several realized volatility estimators. *Economic Modelling 107*, 105701.

Pástor, L. and P. Veronesi (2013). Political uncertainty and risk premia. *Journal of financial Economics 110*(3), 520–545.

Patton, A. J., J. F. Ziegel, and R. Chen (2019). Dynamic semiparametric models for expected shortfall (and value-at-risk). *Journal of Econometrics 211*(2), 388 – 413.

Xu, Y., X. Wang, and H. Liu (2021). Quantile-based garch-midas: Estimating value-at-risk using mixed-frequency information. *Finance Research Letters 43*, 101965.

Yu, X. and Y. Huang (2021). The impact of economic policy uncertainty on stock volatility: Evidence from garch–midas approach. *Physica A: Statistical Mechanics and its Applications 570*, 125794.

Zhu, S., Q. Liu, Y. Wang, Y. Wei, and G. Wei (2019). Which fear index matters for predicting us stock market volatilities: Text-counts or option based measurement? *Physica A: Statistical Mechanics and its Applications 536*, 122567.