The paces of fiscal consolidations, fiscal sustainability, and welfare: An overlapping generations approach

Maebayashi, Noritaka

The University of Kitakyushu

5 August 2021

The pace of fiscal consolidations, fiscal sustainability, and welfare: An overlapping generations approach

Noritaka Maebayashi*

Abstract

This study investigates expenditure- and tax-based consolidations under the rule of reductions in debt-to-GDP ratios to the target level and the effects of these consolidations on fiscal sustainability and welfare, using an overlapping generations model with exogenous growth settings. We derive (i) the global transition dynamics of the economy, (ii) a threshold (ceiling) of public debt to ensure fiscal sustainability, (iii) sustainable paces of these consolidations, and (iv) the optimal pace of consolidations from viewpoints of both social welfare and fairness of each generation's welfare. We find that higher paces or lower targets of debt-to-GDP ratio make fiscal policies more sustainable. The pace of tax-based consolidation required to ensure fiscal sustainability is higher than that required of expenditure-based consolidation. As for welfare, countries may differ in their choice of the type of consolidation, which depends on the size of outstanding debts relative to capital, productivity of the economy, tax rate levels, and the extent of utility derived by individuals from public goods and services. More importantly, it may also depend on whether policymakers emphasize social welfare or fairness of welfare distribution between generations. By contrast, a common result from the viewpoints of both social welfare and fair distribution of welfare across generations is that rapid paces of fiscal consolidation are supported.

JEL classification: E62; H60; H40
Keywords: Fiscal consolidation, Consolidation pace, Fiscal sustainability, Welfare

*Faculty of Economics and Business Administration, The University of Kitakyushu, 4-2-1 Kitagata, Kokura Minami-ku, Kitakyushu, Fukuoka 802-8577, JAPAN; e-mail: non818mn@kitakyu-u.ac.jp. This research was supported by grants-in-aid from the Ministry of Education, Culture, Sports, Science, and Technology of the Japanese government (grant number 20K13455).

1 Introduction

The default risk on Greek government debt in the 2008–2009 global financial crisis raised concern over the sustainability of public debts or deficits among countries with large public debts. Long-term sustainability is one of the largest concerns of both policymakers and academics (e.g., Fatás and Mihov, 2010; D'Erasmo et al., 2016 for recent studies). In fact, the Stability and Growth Pact (SGP) in the EU identifies fiscal sustainability as the main goal of its fiscal framework. The fiscal consolidation rule in the SGP has two directives; it sets the (i) target of debt levels, and (ii) the pace of reduction in debt. The rule states that member states whose current debt-to-GDP ratio is above the 60% threshold must reduce their ratios to 60% at an average rate of one-twentieth per year.

Although the need for fiscal consolidation prevails in OECD countries with high debt, there is little consensus on the paces of fiscal consolidation (see e.g., Rawdanowicz, 2014)[1] Why is determining the pace of fiscal consolidation difficult? Consider the case of countries with very large outstanding debts (e.g., Japan, Greece, Italy, Portugal, and the US). At a very slow pace, fiscal consolidations may fail to sustain fiscal policy due to the large interest payment of public debt as well as the crowding-out effect of public debt on capital accumulation (for references to the theoretical literature, see, e.g., Diamond, 1965 and Chalk, 2000; Mankiw and Elmendorf, 1999 provide a survey of the empirical literature). Furthermore, it postpones the burden of debt payment on future generations, which might not be fair to them. By contrast, a very rapid pace of consolidation may lead to a burden on the current and earlier generations and result in a large loss of social welfare. Then, a common pace of consolidation under the SGP in the EU might be the *hard coordinated consolidation regime*, as classified by Panico and Purificato (2013), for countries with extremely high debts (e.g., Greece, Italy, and Portugal).

When considering the pace of consolidation, it is also important to know as to the type of consolidation that is more effective between spending cuts and tax increases. According to the literature survey by Molnar (2012), earlier studies are not conclusive on this issue. Some studies (e.g., Alesina and Ardagna, 1998, 2009; von Hagen et al., 2002; Guichard et al., 2007) indicate that consolidation based on expenditure cuts are found to be more effective while others (e.g., Alesina and Perotti, 1995; Tsibouris et al., 2006) find that revenue-based consolidations can be effective.

Accordingly, we tackle the following research questions. (i) How does the pace of fiscal consolidation affect the transition paths of the economy? (ii) How rapid should the pace of fiscal consolidation be to ensure fiscal sustainability? (iii) How does the pace of fiscal consolidation impact each generation's welfare or social welfare? Is there a trade-off between the two? (iv) What is the different impact on the abovementioned three questions under expenditure- and tax-based fiscal consolidations?

To that end, we study a standard overlapping-generations (OLG) model of a closed economy developed by Diamond (1965). We introduce a debt policy rule under which the government debt relative to the size of the economy is adjusted gradually to a targeted debt/GDP level in the long

[1] Rawdanowicz (2014) states as follows: "While there is generally little controversy about the need for fiscal consolidation, its optimal pace is ..., posing a key dilemma for policymakers in many OECD countries. Some argue for postponing consolidation as a large, frontloaded adjustment that can reduce GDP growth with negative fallout for the fiscal situation ... The choice of optimal consolidation path depends crucially on the ultimate long-term objective of fiscal policy and market conditions. Estimating optimal consolidation pace is challenging given the nexus of interactions between fiscal policy, financial markets and economic growth."

run.[2] Under expenditure-based (resp. tax-based) consolidations, governments adjust their spending (resp. income tax rates) with fixed income and consumption tax rates (resp. the fixed ratio of government spending to GDP). In OLG models, fiscal sustainability means that the ratio of public debt to GDP (or capital) converges to a stable level in the long run and Ponzi games by the government is possible (e.g., Chalk, 2000; de la Croix and Michel, 2002; Yakita, 2008). Thus, we investigate the global transition dynamics and check whether the transition paths converge globally to the steady state. To shed light on global transitional dynamics, we employ analytically tractable settings with inelastic labor supply, log utility, and the Cobb–Douglas production functions. We also calibrate the model to the data of Japan, the US, Greece, Italy, and Portugal as examples of countries with very high debt-to-GDP ratios.

In this study, the pace of consolidation plays an important role in turning unsustainable transition paths into sustainable ones. Then, welfare effects of the transition from unsustainable to sustainable paths are highlighted in this study while a large body of previous studies focuses only on the steady states between pre and post policy changes or transitions between these two. We also judge the welfare effect of the fiscal consolidation based on both social welfare and fairness of welfare distribution between each generation. We extend these welfare analyses by introducing CRRA utility, increases in consumption tax during tax-based consolidation, and the consolidation regime mixed expenditure cut with tax increases.

The main findings of this study are summarized as follows.

(i) A unique stable steady state exists under both, expenditure- and tax-based consolidations with the debt policy rule. Properties of global transition paths are derived analytically and represented in two two-dimensional phase diagrams, each under two types of consolidation plans.

(ii) There is a threshold of public debt for each level of capital in order for the government to sustain fiscal policy, and the threshold of public debt increases in proportion to the size of capital under each type of consolidation plan. A higher pace or lower target of debt-to-GDP ratio makes fiscal policies more sustainable.

(iii) The minimal pace of tax-based consolidation that ensures fiscal sustainability is higher than that of expenditure-based consolidation, indicating that expenditure-based consolidation is more likely to make fiscal policy sustainable.

(iv) Numerical investigations show that Japan, Greece, Italy, and Portugal cannot sustain fiscal policy either without reducing debt or with very low paces of reduction in debts. By contrast, the US economy may sustain its fiscal policy even without reducing debt.

(iv) Social welfare improves in all countries (Japan, the US, Greece, Italy, and Portugal) by fiscal consolidation. The choice of the type of consolidation (between tax-based or expenditure-based) may differ among countries. It depends on how large the outstanding debts relative to capital are, productivity of the economy, tax rate levels, and how large the utility that individuals derive from public goods and services is. More importantly, it also depends on whether policymakers emphasize social welfare or fairness of welfare distribution between

[2] A recent empirical study by Molnar (2012) finds that fiscal rules are associated with a greater probability of stabilizing debt. Many empirical analyses show that better-designed rules are more likely to reduce fiscal deficits (see the survey by Eyraud et al., 2018).

generations. By contrast, a common result from the viewpoints of both social welfare and fair distribution of welfare is that rapid paces of fiscal consolidation are supported.

Related literature

Fiscal consolidation is shown to be productive in the medium and long term in the literature of exogenous growth models. Some studies (e.g., Coenen et al., 2008; Forni et al., 2010; Bi et al., 2013; Cogan et al., 2013; Erceg and Lindé, 2013; Philippopoulos et al., 2017) use new Keynesian dynamic stochastic general equilibrium (DSGE) models while others (e.g., Papageorgiou, 2012; Hansen and İmrohoroğlu, 2016) use real business-cycle (RBC) models. Common features of these studies are that the focal point is the effect of fiscal consolidation on transitional dynamics.

Growth models that examine optimal paces of consolidation include Maebayashi et al. (2017), Morimoto et al. (2017), and Futagami and Konishi (2018). These studies consider a debt policy rule in line with the SGP's 60% rule of the debt-to-GDP ratio for welfare analyses of fiscal consolidations, and show that a faster pace of consolidation drives larger welfare gains.[3] Rawdanowicz (2014) also sheds light on the pace of consolidation plan to reduce debt from 90% to 60% of GDP within 20 years and to maximize a discounted sum of real GDP growth (or minimize a discounted sum of squared output gaps).

In the literature, Erceg and Lindé (2013) compare spending-based versus tax-based consolidation in a two-country new Keynesian model. They show that spending-based consolidation has far less costly effects on output than tax-based consolidation in the longer-term. Erceg and Lindé (2013) demonstrate that this finding is consistent with the supply side effects emphasized in Uhlig (2010). Maebayashi et al. (2017) show that spending-based consolidations have larger welfare gains than the tax-based consolidations in an endogenous growth model. Morimoto et al. (2017) assess both sustainability and social welfare in a small open endogenously growing economy and show that expenditure-based consolidation can be preferable for both fiscal sustainability and welfare.[4]

However, previous studies on transitional dynamics, optimal paces of fiscal consolidation, and spending-based versus tax-based consolidation assume an infinitely lived agent, and therefore ignore intergenerational welfare losses or gains and the possibility of a Ponzi game by governments.

Chalk (2000), de la Croix and Michel (2002), and Yakita (2008) investigate the sustainability of public debt (global transitional dynamics of debt) in OLG models and concluded that a Ponzi game by the governments is possible. The sustainability in OLG models is often defined as the convergence of the public debt to a sustainable level in the long term under some fiscal rules. Constant deficit (or deficit to GDP) rules are examined in Chalk (2000) and Yakita (2008), while a constant debt-to-GDP ratio is imposed in de la Croix and Michel (2002).[5] They show that a debt above the threshold level can explode and cannot sustain fiscal policy. These studies however do not conduct analyses of fiscal consolidations that encompass the timeline effect of reduction in debt-to-GDP ratio.

[3]These studies use endogenous growth models with productive government spending and public debt, which are similar to Greiner and Semmler (2000) and Ghosh and Mourmouras (2004), for example.

[4]Other recent studies on the sustainability of debt in some endogenous growth models include Greiner (2007, 2012), Kamiguchi and Tamai (2012), and Miyazawa et al. (2019). These studies consider infinitely lived agent economies.

[5]In Chalk (2000), de la Croix and Michel (2002), and Yakita (2008), factor prices are endogenous, while other studies on fiscal sustainability by Bräuninger (2005), Arai(2011), Teles and Mussolini (2014), and Maebayashi and Konishi (2021) use endogenously growing OLG models with constant factor prices. Oguro and Sato (2014) examine the relationship between interest rates on government bonds and the fiscal consolidation rule by using an OLG model with endogenous and stochastic growth settings.

There are additional merits of studying fiscal consolidation policy in an OLG model with finitely lived agents rather than an infinitely lived agent model. As emphasized by Uhlig and Yanagawa (1996), because labor income is paid mostly to the young and capital income accrues mostly to the old in a lifecycle of finitely lived agents, lower productivity of labor and higher labor income tax means that younger people in an economy are left with less income out of which to save and to buy capital stock. In this study, therefore, lower productivity of the economy and high wage income tax rates may fail to increase GDP, consumption, and public spending even after consolidation. Such a possibility is paid little attention in many studies mentioned above that assume infinitely lived agents who are in essence always young.

When we regard pay-as-you-go social security as an implicit debt, privatization of public pension is like a problem of reductions in debt. Recently, Nishiyama and Smetters (2007) (by a simulation methodology in line with Auerbach and Kotlikoff, 1987) and Andersen and Bhattacharya (2020) (by a Diamond-type OLG model) show that decreasing debt can achieve Pareto improvement if the present value of these net resources can be distributed to future generations. However, these analyses sacrifice global transitional dynamics in the sense that they focus only on the steady states with between pre- and post-policy changes or transitions between the two. As a result, these studies ignore how to prevent unsustainable ways of consolidation.

Building on these previous studies, we examine the global transition dynamics of both expenditure- and tax-based consolidations, a sustainable pace of these consolidations, welfare effects, and an optimal pace of consolidations from the viewpoints of both social welfare and fairness of welfare across generations in an OLG economy.

2 The Model

2.1 Model Setting

The framework is based on an OLG model following Diamond (1965). There are L_t individuals who live for two periods. We assume population grows at an exogenous rate, n, so that $L_t = (1+n)L_{t-1}$. They supply one unit of labor in youth inelastically and retire in old age.

A single final good, Y_t, is produced by using capital, K_t, and labor, L_t, according to a constant-returns-to-scale technology, $Y_t = F(K_t, L_t)$. The intensive form of this function is $y(k_t) = f(k_t)$, where $f'(\cdot) > 0$ and $f''(\cdot) < 0$ with respect to the capital to labor ratio: $k_t \equiv K_t/L_t$. We assume that capital depreciates fully after one period. Profit maximization under perfect competition yields the interest rate, $R_t = f'(k_t) \equiv R(k_t)$, and the wage rate, $w_t = f(k_t) - f'(k_t)k_t \equiv w(k_t)$.

Individuals consume private goods and services when young c_t (old d_{t+1}) and utilize public goods and services provided by the government in both periods S_t^g and S_{t+1}^g. We assume that public goods and services are denoted by $S_t^g = G_t/(L_t + L_{t-1}) = ((1+n)/(2+n))g_t$, where $g_t \equiv G_t/L_t$ and G_t is public spending.[6] The lifetime utility function of an individual born in period t is

$$U_t = \ln c_t + \theta \ln S_t^g + \beta(\ln d_{t+1} + \theta \ln S_{t+1}^g), \tag{1}$$

where β and θ denote the subjective discount factor and the preference weight on public goods and services, respectively. Let s_t be the saving in youth. The lifetime budget constraints of generation t are $(1+\tau_t^c)c_t + s_t = (1-\tau_t^w)w_t$ and $(1+\tau_{t+1}^c)d_{t+1} = (1-\tau_t^R)R_{t+1}s_t$, where τ_t^w, τ_t^R, and τ_t^c, are tax rates on wage income, capital income, and consumption, respectively. The utility maximization yields

$$s_t = \frac{\beta(1-\tau_t^w)}{1+\beta} w(k_t). \tag{2}$$

Next, we move onto the fiscal policy. The governments face their budget constraint, $B_{t+1} = R(k_t)B_t + G_t - T_t$, where B_t, G_t, and $T_t (= \tau_t^w w(k_t)L_t + \tau_t^R R(k_t)(B_t + K_t) + \tau_t^c(c_t L_t + d_t L_{t-1}))$ are government bonds, government expenditure and tax revenue, respectively. Dividing this constraint by L_t, we obtain

$$(1+n)b_{t+1} = R(k_t)b_t + g_t - \tau_t^w w(k_t) - \tau_t^R R(k_t)(b_t + k_t) - \tau_t^c\left(c_t + \frac{d_t}{1+n}\right), \tag{3}$$

where $b_t \equiv B_t/L_t$ and $g_t \equiv G_t/L_t$. Additionally, fiscal policy is subject to the following debt policy rule:

$$b_{t+1} - b_t = -\phi\left(b_t - \bar{b}y(k_t)\right), \tag{4}$$

where, $\phi(>0)$ and $\bar{b}$ stand for the adjustment coefficient of the rule and the target level of debt-to-GDP ratio, respectively. We consider the case of $\bar{b} > 0$. We rewrite (4) into $\frac{b_{t+1}-b_t}{y_t} = -\phi\left(\frac{b_t}{y_t} - \bar{b}\right)$ to interpret it. If the ratio of debt-to-GDP ratio ($b_t/y_t (= B_t/Y_t)$) is larger than $\bar{b}$, the government has to reduce b_t/y_t by making fiscal surplus a percentage of GDP ($(b_{t+1} - b_t)/y_t$), according to the difference between the current and target levels of debt-to-GDP ratio ($b_t/y_t - \bar{b}$).

[6] This study considers government spending by destination, which includes expenditures for individual consumption ($G_t/(L_t + L_{t-1})$) (e.g., healthcare, housing, and education), incurred by the government for the benefit of individual households.

If the adjustment coefficient (ϕ) takes a large (or small) value, the government adjusts b_t/y_t to the target level ($\bar{b}$) at a fast (or slow) pace. This policy rule is in line with the SGP rule in the EU, which we refer to in Section 1.[7]

There are three notable cases as to the values of ϕ. First, when $\phi = 1$, (4) leads to $b_{t+1} - b_t = -(b_t - \bar{b}y(k_t))$. Here, the government will reduce public debt by the difference between the current and target levels of the debt-to-GDP ratio ($b_t/y_t - \bar{b}$) in one period. If we regard one period as 30 years, the fiscal consolidation in the EU may ask for such a tight plan because the plan achieves a fiscal reconstruction in 20 years (within 30 years). Second, when $0 < \phi < 1$, it takes more than one period to achieve a fiscal reconstruction because public debt decreases more gradually. Finally, when applying $\phi = 0$ to (4), we obtain $b_{t+1} = b_t = b_0$, which indicates that the government does not reduce outstanding public debts but keeps its debts at the initial level b_0. Throughout this study, we treat $\phi \leq 1$ as the case where fiscal consolidations are implemented and $\phi = 0$ as the one without fiscal consolidations. We summarize these points in the following Remark 1.

Remark 1. *(i) When $\phi = 1$, a fiscal reconstruction is achieved in one period. (ii) When $0 < \phi < 1$, it takes more than one period to accomplish a fiscal reconstruction. A larger (or lower) ϕ leads to a more rapid (or slower) fiscal consolidation. (iii) when $\phi = 0$, no fiscal consolidations to reduce outstanding debt are implemented, that is, $b_{t+1} = b_t = b_0$.*

The government implements fiscal consolidations with (4) unexpectedly at time 0. The tax rates at $t = 0$ before consolidations are supposed to be given by $(\tau^w_{init}, \tau^R_{init}, \tau^c_{init})$.

2.2 Equilibrium

Asset market clears as $B_{t+1} + K_{t+1} = (1+n)(b_t + k_t)L_t = s_t L_t$. This together with (2) yields

$$b_{t+1} + k_{t+1} = \frac{\beta(1-\tau^w_t)}{(1+\beta)(1+n)} w(k_t). \tag{5}$$

By substituting (4) into (5), we can derive the difference equation of k_t as

$$k_{t+1} = \Phi(k_t, b_t, \tau^w_t) \equiv \frac{\beta(1-\tau^w_t)w(k_t)}{(1+\beta)(1+n)} - \left[b_t - \phi\left(b_t - \bar{b}y(k_t)\right)\right]. \tag{6}$$

The goods market equilibrium condition is given by $K_{t+1} = Y_t - G_t - c_t L_t - d_t L_{t-1}$ and is rewritten into $c_t + d_t/(1+n) = y(k_t) - g_t - (1+n)k_{t+1}$. This, together with (6), yields the tax revenues from consumption (per capita) as

$$\tau^c_t \left(c_t + \frac{d_t}{1+n}\right) = \tau^c_t[y(k_t) - g_t - (1+n)\Phi(k_t, b_t, \tau^w_t)]. \tag{7}$$

[7] This policy rule is also in line with Bohn (1998), who shows empirically that the primary (non-interest) budget surplus is an increasing function of the debt-GDP ratio in the US economy. Bohn (1998) considers neither the targeted debt-to-GDP ratio nor the consolidation in response to the distance between current and the targeted debt to GDP, both of which are helpful for the long-run commitment to fiscal consolidation.

From (3), (4), (6), and (7), we obtain

$$
\begin{aligned}
(1+\tau_t^c)g_t =&(1+n)\left[b_t-\phi\left(b_t-\bar{b}y(k_t)\right)\right]+\tau_t^w w(k_t)+\tau_t^R R(k_t)k_t+\tau_t^c y(k_t)\\
&-(1-\tau_t^R)R(k_t)b_t-(1+n)\tau_t^c\Phi\left(k_t,b_t,\tau_t^w\right).
\end{aligned}
\tag{8}
$$

The following condition must be satisfied to sustain (keep) fiscal policy (providing public services): $g_t>0$ for all t:

$$
\begin{aligned}
b_t <&\frac{\phi\bar{b}(1+n)(1+\tau_t^c)y(k_t)+\tau_t^w w(k_t)+\tau_t^R R(k_t)k_t+\tau_t^c y(k_t)-\tau_t^c\frac{\beta(1-\tau_t^w)}{1+\beta}w(k_t)}{(1-\tau_t^R)R(k_t)-(1+n)(1+\tau_t^c)(1-\phi)}\\
&\equiv\Omega\left(k_t,\tau_t^w,\tau_t^R,\tau_t^c\right),
\end{aligned}
\tag{9}
$$

otherwise ($g_t\leq 0$), $g_t=0$ binds, meaning that fiscal policy cannot be sustained.[8]

In this study, we consider a dynamically-efficient economy: $R(k_t)>1+n\Leftrightarrow k_t<k_{GR}$, where k_{GR} is the golden rule capital stock. Furthermore, we pay attention to fiscal policies with positive debts: $\Omega\left(k_t,\tau_t^w,\tau_t^R,\tau_t^c\right)>0\Leftrightarrow k_t<\hat{k}$, where $\hat{k}_t$ satisfies $(1-\tau_t^R)R\left(\hat{k}_t\right)=(1+n)(1+\tau_t^c)(1-\phi)$.

Condition 1. $k_t<\min\{k_{GR},\hat{k}_t\}$, *where* $(1-\tau_t^R)R\left(\hat{k}_t\right)=(1+n)(1+\tau_t^c)(1-\phi)$.

In the next section, we consider fiscal consolidations ($\phi\in(0,1]$: Remark 1-(i) and-(ii)) by adjusting expenditure g_t as

$$
\begin{aligned}
(1+\tau^c)g_t =&(1+n)\left[b_t-\phi\left(b_t-\bar{b}y(k_t)\right)\right]+\tau^w w(k_t)+\tau^R R(k_t)k_t+\tau^c y(k_t)\\
&-(1-\tau^R)R(k_t)b_t-(1+n)\tau^c\Phi\left(k_t,b_t\right)
\end{aligned}
\tag{10}
$$

with the constant tax rates (i.e., $\tau_t^w=\tau^w$, $\tau_t^R=\tau^R$, and $\tau_t^c=\tau^c$), termed *the expenditure-based consolidation*, hereafter. Furthermore, let us denote $\Phi\left(k_t,b_t,\tau_t^w\right)$ in (6) and $\Omega\left(k_t,\tau_t^w,\tau_t^R,\tau_t^c\right)$ in (9) simply as $\Phi(k_t,b_t)$ and $\Omega(k_t)$, respectively.

Equations (4) and (6) combined with (9) characterize the dynamic system of the economy under the expenditure-based consolidation.

Before moving onto the following sections, we mention the case of no fiscal consolidation ($\phi=0$: Remark 1-(iii)) with the tax rates fixed at the level before consolidations ($\tau^w=\tau^w_{init}$, $\tau^R=\tau^R_{init}$, and $\tau^c=\tau^c_{init}$ for all t). Applying $\phi=0$ and $b_{t+1}=b_t=b_0$, $\forall t$ into (6), we obtain the following dynamic system:

$$
k_{t+1}=\Phi(k_t,b_0)=\frac{\beta(1-\tau^w)w(k_t)}{(1+\beta)(1+n)}-b_0\quad\text{and}\quad b_{t+1}=b_t=b_0
\tag{11}
$$

[8] If $g_t=0$ binds at a certain period, fiscal policy can no longer follow the rule of (4), because large issuance of public bonds is necessary to meet net interest payment of debt: $\tau_t^w w(k_t)+\tau_t^R R(k_t)k_t+\tau_t^c[y(k_t)-(1+n)k_{t+1}]+(1+n)b_{t+1}=(1-\tau_t^R)R(k_t)b_t$. By this government budget constraint and (5), the issuance of public bonds and accumulation of capital are derived as $(1+n)b_{t+1}=(1+\tau_t^c)^{-1}\{(1-\tau_t^R)R(k_t)b_t-\tau_t^w w(k_t)-\tau_t^R R(k_t)k_t-\tau_t^c[y(k_t)-(\beta/(1+\beta))(1-\tau_t^w)w(k_t)]\}$ and $(1+n)k_{t+1}=(1/(1+\tau_t^c))[(\beta/(1+\beta))(1-\tau_t^w)w(k_t)-(1-\tau_t^R)R(k_t)b_t+\tau_t^w w(k_t)+\tau_t^R R(k_t)k_t+\tau_t^c y(k_t)]$, respectively. However, these dynamics under $g_t=0$ are outside the main scope of our investigation.

for a given $b_0 > 0$. Assuming that $w(k_t)$ is concave in k_t: i.e., $w''(k_t) < 0$, we can derive the following facts.

Remark 2 *Consider the case of no fiscal consolidation ($\phi = 0$, $b_{t+1} = b_t = b_0$, and $\tau^w = \tau^w_{init}$ $\forall t$) and define*

$$b^{upper} \equiv [\beta(1-\tau^w)/(1+\beta)(1+n)]\, w\left(\bar{k}_{no}\right) - \bar{k}_{no}.$$

(i) If the initial public debt b_0 is large enough to satisfy $b_0 > b^{upper}$, where $\bar{k}_{no}$ satisfies $w'\left(\bar{k}_{no}\right) = (1+\beta)(1+n)/\beta(1-\tau^w)$, k_t decreases monotonically and eventually takes zero, meaning that the economy goes bankrupt and public debt cannot be sustainable (Figure 1-(a)).

(ii) If $b_0 < b^{upper}$, public debt can (cannot) be sustainable when $k_0 \geq (<) \underline{k}_{no}$, where $\underline{k}_{no}$ satisfies $\Phi\left(\underline{k}_{no}, b_0, \tau^w\right) = \underline{k}_{no}$ and $[\beta(1-\tau^w)/(1+\beta)(1+n)]\, w'\left(\underline{k}_{no}\right) > 1$. When $k_0 > \underline{k}_{no}$, k_t converges on a steady-state value, k^*_{no}, which satisfies $\Phi\left(k^*_{no}, b_0, \tau^w\right) = k^*_{no}$ and $[\beta(1-\tau^w)/(1+\beta)(1+n)]\, w'\left(k^*_{no}\right) < 1$ (Figure 1-(b)).

Remark 2 indicates that fiscal consolidations should be implemented in an economy in which current debt b_0 is larger than b^{upper}. As we discuss later, the current level of public debt: b_0 in Japan, Greece, Italy, Portugal may be the case with the one in Remark 2.[9] In the following sections, we study the economy under Remark 2 mainly and examine the effects of fiscal consolidation on the transition paths of the economy, fiscal sustainability, and welfare.

[Figure 1]

3 Expenditure-based consolidation

3.1 Dynamics under expenditure-based consolidation

In this section, we derive the global transitional dynamics of the economy under the expenditure-based consolidation. For the tractability of analyses, we consider the case of Cobb–Douglas production function: $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$ $(0 < \alpha < 1)$. Then, equations (4), (6), (9), and Condition 1 for $\phi \in (0, 1]$ are written as follows:

$$b_{t+1} - b_t = -\phi\left(b_t - \bar{b}Ak_t^{\alpha}\right), \tag{12}$$

$$k_{t+1} = \Phi(k_t, b_t) = \eta A k_t^{\alpha} - \left[b_t - \phi\left(b_t - \bar{b}Ak_t^{\alpha}\right)\right], \tag{13}$$

$$b_t < \Omega(k_t) = \frac{Ak_t^{\alpha}\left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right]}{(1-\tau^R)\alpha A k_t^{\alpha-1} - (1+n)(1+\tau^c)(1-\phi)}, \tag{14}$$

$$k_t < \min\{k_{GR}, \hat{k}\}, \;\; k_{GR} = \left(\frac{\alpha A}{1+n}\right)^{\frac{1}{1-\alpha}}, \;\; \hat{k} = \left[\frac{(1-\tau^R)\alpha A}{(1+n)(1+\tau^c)(1-\phi)}\right]^{\frac{1}{1-\alpha}}, \tag{15}$$

where $\eta \equiv \frac{\beta(1-\alpha)(1-\tau^w)}{(1+\beta)(1+n)}$, $1-(1+n)\eta = 1 - \frac{\beta(1-\alpha)(1-\tau^w)}{1+\beta} > 0$, and $\tilde{\tau} \equiv \tau^w(1-\alpha) + \tau^R\alpha$.

We start with the derivation of the steady-state values of k_t and b_t. Applying $k_{t+1} = k_t$ and $b_{t+1} = b_t$ to (12) and (13) and solving yields

$$(k^*, b^*) = \left(\left[\left(\eta - \bar{b}\right)A\right]^{\frac{1}{1-\alpha}}, \;\; \bar{b}\left(\eta - \bar{b}\right)^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}}\right), \tag{16}$$

[9] We should not ignore other countries whose outstanding public debts are growing rapidly.

where we assume $\bar{b} < \eta$ to ensure the existence of this steady state. (16) leads to the following proposition:

Proposition 1. *A unique steady state (k^*, b^*) exists if and only if $\bar{b} < \eta$. Both k^* and b^* are independent on the pace of fiscal consolidation ϕ.*

For the tractability of later analyses, we define $y_t/k_t = Ak_t^{\alpha-1} \equiv q(k_t)$ and $b_t/k_t \equiv x_t$ and prepare the expressions of (14), (15), and (16) with $(q(k_t), x_t)$ as follows:

$$x_t < \tilde{\Omega}(q(k_t)) \equiv \frac{q(k_t)\left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right]}{(1-\tau^R)\alpha q(k_t) - (1+n)(1+\tau^c)(1-\phi)}, \tag{17}$$

$$q(k_t) > \max\left\{q(k_{GR}), q\left(\hat{k}\right)\right\}, \quad q(k_{GR}) = \frac{1+n}{\alpha}, \quad q\left(\hat{k}\right) = \frac{(1+n)(1+\tau^c)(1-\phi)}{(1-\tau^R)\alpha}, \tag{18}$$

$$(q(k^*), x^*) = \left(\frac{1}{\eta - \bar{b}}, \frac{\bar{b}}{\eta - \bar{b}}\right). \tag{19}$$

Next, we derive the $k_{t+1} = k_t$ and $b_{t+1} = b_t$ loci on the (k_t, b_t) plane. On the one hand, from (12), $b_{t+1} = b_t$ locus is given by

$$b_t = \bar{b}Ak_t^{\alpha}. \tag{20}$$

It is the concave and strictly increasing function of k_t that takes $k_t = b_t = 0$.

[Figure 2]

On the other hand, $k_{t+1} = k_t$ locus is $k_t = \eta Ak_t^{\alpha} - \left[b_t - \phi\left(b_t - \bar{b}Ak_t^{\alpha}\right)\right]$, which is rewritten as

$$b_t = \frac{\left(\eta - \phi\bar{b}\right)Ak_t^{\alpha} - k_t}{1-\phi} \equiv Z(k_t) \qquad \text{for} \quad \phi \in (0,1), \tag{21}$$

$$k_t = k^* \quad \forall b_t \qquad \text{for} \quad \phi = 1. \tag{22}$$

Here, keep in mind that $\bar{b} < \eta$. Furthermore, $Z(k_t)$ has the following properties. First, $Z(0) = Z(\tilde{k}) = 0$, where $\tilde{k} \equiv [(\eta - \phi\bar{b})A]^{\frac{1}{1-\alpha}}$. Second, $Z(k_t) > 0$ holds for $0 < k_t < \tilde{k}$, and $Z'(k_t) = \frac{(\eta-\phi\bar{b})\alpha q(k_t)-1}{1-\phi} \geq (<) 0$ for $0 \leq k_t \leq \bar{k}$ ($\bar{k} < k \leq \tilde{k}_t$), where $\bar{k} \equiv [(\eta - \phi\bar{b})\alpha A]^{\frac{1}{1-\alpha}}$.

Thus, we obtain the following lemma.

Lemma 1. *Suppose that $\bar{b} < \eta$. The $b_{t+1} = b_t$ and $k_{t+1} = k_t$ loci have the following properties.*

(i) The $b_{t+1} = b_t$ locus is a concave and upward-sloping curve that takes $k_t = b_t = 0$.

(ii) The shape of $k_{t+1} = k_t$ locus depends on the value of ϕ.

(a) When $\phi < 1$, it is an inverted-U shaped curve that takes $k_t = 0$ and $\tilde{k}$ (> 0) when $b_t = 0$.

(b) When $\phi = 1$, $k_{t+1} = k_t$ locus is a perpendicular line: $k_t = k^* = \left[\left(\eta - \bar{b}\right)A\right]^{\frac{1}{1-\alpha}}$.

From (12) and Lemma 1-(i), $b_{t+1} > (\leq) b_t$ holds below (above) the $b_{t+1} = b_t$ locus at each point of the (k_t, b_t) plane as depicted in Figure 2. Furthermore, from (13) and Lemma 1-(ii), when $0 < \phi < 1$, $k_{t+1} > (\leq) k_t$ holds below (or above) the inverted-U shaped $k_{t+1} = k_t$ locus at each point of the (k_t, b_t) as depicted in Figure 2- (a). When $\phi = 1$, $k_{t+1} > (\leq) k_t$ at the left (right) side of $k_{t+1} = k_t$ locus represented as a perpendicular line in Figure 2 -(b).

For the later use, we express $b_{t+1} = b_t$ locus ((20)) and $k_{t+1} = k_t$ locus for $\phi \in (0,1)$ ((21)) with $(q(k_t), x_t)$ as follows:

$$x_t = \bar{b}q(k_t), \tag{23}$$

$$x_t = \tilde{Z}(q(k_t)) \equiv \frac{\left(\eta - \phi\bar{b}\right) q(k_t) - 1}{1 - \phi}. \tag{24}$$

Finally, we examine the region in which $g_t \geq 0$ on the (k_t, b_t) plane. Eq. (9), associate with the value of $\hat{k}_t$ (see (15)), yield the following:

Lemma 2. *$g_t = 0$ locus has the following properties.*

(i) *$g_t = 0$ locus is a convex upward-sloping curve that takes $k_t = b_t = 0$ and has asymptote $\lim_{k_t \to \hat{k}} \Omega(k_t) = +\infty$.*

(ii) *$g_t = 0$ locus intersect with $b_{t+1} = b_t$ locus at a unique point $H(k_H, b_H)$, where $k_H > 0$ and $b_H > 0$ are given by*

$$(k_H, b_H) = \left(\left[\tfrac{\bar{b}(1-\tau^R)\alpha A}{\bar{b}(1+\tau^c)(1+n)+\tilde{\tau}+\tau^c(1-\eta(1+n))} \right]^{\frac{1}{1-\alpha}}, (\bar{b}A)^{\frac{1}{1-\alpha}} \left[\tfrac{(1-\tau^R)\alpha}{\bar{b}(1+\tau^c)(1+n)+\tilde{\tau}+\tau^c(1-\eta(1+n))} \right]^{\frac{\alpha}{1-\alpha}} \right). \tag{25}$$

(iii) *$g_t = 0$ locus intersect with $k_{t+1} = k_t$ locus at a unique point $P(k_P, b_P)$, where $k_P > 0$ and $b_P > 0$. When $\phi = 1$, $k_P = k^*$ holds.*

Proof: See Appendix A.

From (14) and Lemma 2, fiscal policy above $g_t = 0$ locus in the (k_t, b_t) plane (shaded area in Figure 2) cannot be sustainable because $g_t = 0$ binds there. All transition paths that lead to this area should be avoided and be regarded as unsustainable.

For later use, we notify the properties of $x_t = \tilde{\Omega}(q(k_t))$ ($g_t = 0$ locus expressed by $(q(k_t), x_t)$: see (17)). This together with (23) and (24) yield the following facts. First, at the point $H(k_H, b_H)$, $q(k_H) = \frac{\bar{b}(1+\tau^c)(1+n)+\tilde{\tau}+\tau^c(1-(1+n)\eta)}{\bar{b}(1-\tau^R)\alpha}$ and $x_H (\equiv b_H/k_H) = \frac{\bar{b}(1+\tau^c)(1+n)+\tilde{\tau}+\tau^c(1-(1+n)\eta)}{(1-\tau^R)\alpha}$. Second, at the point $P(k_P, b_P)$, $q(k_P)$ and $x_P \equiv b_P/k_P$ satisfy

$$\frac{\left(\eta - \phi\bar{b}\right) q(k_P) - 1}{1 - \phi} = \frac{q(k_P)\left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right]}{(1-\tau^R)\alpha q(k_P) - (1+n)(1+\tau^c)(1-\phi)} \tag{26}$$

and

$$\begin{aligned}&(1-\phi)x_P\left[(1-\tau^R)\alpha x_P - \tilde{\tau} - \tau^c - \eta(1+n)\right] \\ &\quad = \phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta) - (1-\tau^R)\alpha x_P,\end{aligned} \tag{27}$$

$$\begin{aligned}&\underline{x}_P < x_P < \bar{x}_P, \\ &\underline{x}_P \equiv \left[\tilde{\tau} + \tau^c + \eta(1+n)\right]/(1-\tau^R)\alpha, \\ &\bar{x}_P \equiv \left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right]/(1-\tau^R)\alpha.\end{aligned} \tag{28}$$

Appendix B shows the uniqueness of k_P and derivation of (28).

From (14), (16), and Lemma 2, the following condition must be satisfied to ensure fiscal sustainability in the steady state when $\phi \in (0, 1)$.

Condition 2. $k^* > k_P$ *if and only if*

$$\bar{b} < \bar{b}_1 \equiv \frac{\zeta_1 + \sqrt{\zeta_1^2 + 4(1+n)(1+\tau^c)\zeta_2}}{2(1+n)(1+\tau^c)} \in (0, \eta),$$

where, $\zeta_1 \equiv \tilde{\tau} + \tau^c(1-(1+n)\eta) + (1-\tau^R)\alpha - (1+\tau^c)(1+n)\eta$ *and* $\zeta_2 \equiv \eta[\tilde{\tau} + \tau^c(1-(1+n)\eta)](> 0)$.

Appendix C derives Condition 2 by (26) and shows that $\bar{b}_1$ is increasing in τ^w, τ^R, and τ^c.[10] Condition 2 indicates that the target level of debt-to-GDP ratio ($\bar{b}$) must be lower than $\bar{b}_1$ (the ceiling level of $\bar{b}$). $\bar{b}_1$ is increasing in the income and consumption tax rates because a rise in tax revenue loosens government's budget.

Figure 2 illustrates a phase diagram of the economy, highlighting that the steady state S is stable. Let us start with the case of $\phi \in (0, 1)$ (case (a)). The saddle arm converging to $P(k_P, b_P)$, labeled "*Threshold*," represents the threshold of public debt for each level of k_t. An economy, whose initial state is below the threshold curve as represented by Q_1, converges gradually to the steady state S. At the steady state S, the state variables (k_t, b_t) take constant values of (k^*, b^*), and the government can run its fiscal policy with its positive debt $b^* > 0$ permanently.

By contrast, an economy whose initial state is above the threshold curve, will bind $g_t = 0$ and fiscal policy cannot be sustainable. The point Q_2 represents the case where the initial public level is so large that the economy will not converge to any steady states. In such situations, expenditure cut even under the debt policy rule (4) can no longer eliminate outstanding public debts. In particular, in the early stage of fiscal consolidation, a large public debt crowds out capital accumulation, shrinks the economy, and exacerbates a fiscal condition seriously.

Next, we move onto the case of $\phi = 1$. Applying $\phi = 1$ into (12) leads to $b_{t+1} - b_t = -(b_t - \bar{b}Ak_t^\alpha)$. Then, a reduction in public debt in each period is the distance between the debt revel b_t and $b_{t+1} = b_t$ locus. Furthermore, a fall in debt is greater as the current outstanding debt is larger, indicating that the fiscal policy is sustainable as long as the initial state is outside of $g_t \leq 0$ (the shaded area).

In summary, we can state the following proposition.

Proposition 2. *Fiscal policy and public debt are unsustainable in either of (i) or (ii).*

(i) The target level of public debt-to-GDP ratio: $\bar{b}$ *is larger than the certain level* $\bar{b}_1$.

(ii) Initial public debt is large enough to exceed the threshold level that is represented by the positive function of k_t.

Next, we focus on the properties of the sustainable transition path during the expenditure-based consolidation. They depend on the initial state of the economy and the pace of debt reduction.

[10] The signs of $d\bar{b}_1/d\tau^R$ and $d\bar{b}_1/d\tau^c$ are always positive, while that of $d\bar{b}_1/d\tau^w$ is not. However, $d\bar{b}_1/d\tau^w > 0$ holds if $1 - \alpha - \tilde{\tau} + \tau^c\left[\frac{(1-\alpha)\beta}{1+\beta}(2-\tau^w) - 1\right] > 0$. This sufficient condition for $d\bar{b}_1/d\tau^w > 0$ is satisfied under the realistic parameter values of $(\alpha, \beta, \tau^w, \tau^c, \tau^R)$ for Japan, the US, Greece, Italy, and Portugal in Table 1 and Section 3.3.

We begin with the case of $0 < \phi < 1$. When the initial public debt is large relative to the capital stock (size of the economy), as represented by the point Q_1, a large public debt crowds out capital accumulation (*crowding out of capital*, hereafter). Accordingly, capital decreases in the early stage of fiscal consolidation. However, as b_t steadily declines, capital begins to increase (*crowding in of capital*, hereafter), eventually exceeding its initial level in the long run. Next, when the initial debt lies in the region of HPS, as represented by Q_3, fiscal consolidation reduces b_t and crowds in capital accumulation both in the short and long run. Finally, when capital is large relative to the debt (as in Q_4), public debt becomes low relative to GDP, leading to a small gap between the current and target debt-to-GDP ratio by (12). Therefore, the size of expenditure cut is small enough to make fiscal policy sustainable.

We move onto the case of $\phi = 1$. As in the initial state represented by Q_5, a strong effect of debt reduction would promote capital accumulation both in the short and long run unless its accompanying expenditure cut would induce $g_t = 0$ to bind.

3.2 Changes in ϕ and $\bar{b}$ under the expenditure-based consolidation

In this section, we investigate how the policy variables $(\bar{b}, \phi)$ that characterize the fiscal consolidation strategy ((12)) affect the steady-state (long-run effects) and fiscal sustainability (short- and medium- run effects).

3.2.1 Effects on the steady state $S(k^*, b^*)$

Recall that ϕ is neutral to the steady state (by Proposition 1), and then we focus on the effect of $\bar{b}$ on the steady state values: k^* and b^* here. By (16), we obtain the following immediately.

Proposition 3. *(i) A fall in $\bar{b}$ increases the steady-state capital stock per capita k^*. (ii) A fall(or rise) in $\bar{b}$ decreases (or increases) the steady-state public debt per capita b^* for $\bar{b} \leq (>)(1-\alpha)\eta$.*

Intuitive reasons for Proposition 3 are as follows. First, lowering $\bar{b}$ causes a larger gap between the current and the target debt-to-GDP ratio by (12) and then a larger amount of debt reduction accompanies. Thus, a larger crowding in of capital increases k^*. Next, a reduction in $\bar{b}$ has two opposite effects on b^*. Lowering $\bar{b}$ decreases the long-run public debt level directly while it increases $b^*(= \bar{b}y(k^*))$ indirectly through its positive effect on k^* in the long run. Unless $\bar{b}$ is large enough to satisfy $\bar{b} > (1-\alpha)\eta$, the direct effect of decreasing $\bar{b}$ dominates the indirect effect and decreases b^*.

3.2.2 Effects on fiscal sustainability

Next, we investigate how a rise in the pace of consolidation (ϕ) or a fall in the targeted debt-to-GDP ratio ($\bar{b}$) affects fiscal sustainability.

We start by considering the impact of a rise in ϕ on the sustainability of fiscal policy. Using (14) and (21), we obtain the following.

Lemma 3. *An increase in ϕ shifts the $k_{t+1} = k_t$ locus upward (or downward) for $k_t < (>)k^*$ and the $g_t = 0$ locus downward (or upward) for $k_t > (<)k_H$.*
Proof: See Appendix D.

From Lemma 3, we obtain $dk_P/d\phi > 0$ immediately. Furthermore, differentiating (27) by ϕ, we obtain

$$\frac{dx_P}{d\phi} = \frac{x_P[(1-\tau^R)\alpha x_P - \tilde{\tau} - \tau^c - \eta(1+n)]}{(1-\phi)[(1-\tau^R)\alpha x_P - \tilde{\tau} - \tau^c - \eta(1+n)] + (1-\tau^R)\alpha + (1-\phi)(1-\tau^R)\alpha x_P} \tag{29}$$

$dx_P/d\phi > 0$ from $x_P > \underline{x}_P$. This together with $db_P/dk_P > 0$ leads to the following proposition:

Proposition 4. *A rise in ϕ shifts $P(k_P, b_P)$ to the upper right direction as in Figure 3-(a), indicating that a rise in ϕ makes fiscal policy more sustainable.*

As ϕ increases, the decline in public debt (b_t) in the early stage of the transition is large. Then, the government can extend fiscal space more rapidly through (i) decreases in interest payment and (ii) increases in tax revenues by the crowding in of capital, making fiscal policy more sustainable.

We next examine a fall in $\bar{b}$. From (14), (20), and (21), a fall in $\bar{b}$ shifts $g_t = 0$ locus $b_{t+1} = b_t$ locus, and $k_{t+1} = k_t$ locus downward, downward, and upward, respectively. Thus, a fall in $\bar{b}$ increases k_P ($\partial k_p/\partial \bar{b} < 0$), as represented in Figure 3-(b). Furthermore, Appendix B shows that (27) and the definition of $x_P \equiv b_P/k_P$ yields

$$\begin{aligned}
\frac{db_P}{d\bar{b}} &= \overbrace{\frac{\partial b_P}{\partial k_P}\frac{\partial k_P}{\partial \bar{b}}}^{\text{indirect effect}} + \overbrace{\frac{\partial b_P}{\partial \bar{b}}}^{\text{direct effect}} = \frac{\overbrace{[\omega_1 - \omega_2 x_P]}^{(+)\ \text{from (27)}}\overbrace{(\partial k_P/\partial \bar{b})}^{(-)} + \phi(1+n)(1+\tau^c)k_P}{2(1-\phi)(1-\tau^R)\alpha x_P + \omega_2} \\
\omega_1 &\equiv 2\left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right] > 0, \\
\omega_2 &\equiv (1-\tau^R)\alpha - (1-\phi)\left[\tilde{\tau} + \tau^c + \eta(1+n)\right] > 0,
\end{aligned} \tag{30}$$

where $\omega_1 - (1-\tau^R)\alpha x_P > 0$ and $\omega_2 > 0$ hold from (28). The sign of the direct effect of changing $\bar{b}$ is positive ($\partial b_P/\partial \bar{b} > 0$) while that of the indirect effect (through the effect of $\bar{b}$ on k_P) is negative ($\frac{\partial b_P}{\partial k_P}\frac{\partial k_P}{\partial \bar{b}} < 0$). Then, the total effect is ambiguous. Accordingly, Appendix E conducts numerical analyses in the cases of Japan, the US, Greece, Italy, and Portugal and yields the following results. A fall in $\bar{b}$ shifts $P(k_P, b_P)$ to the upper right direction, as in Figure 3-(b), indicates that a reduction in $\bar{b}$ makes fiscal policy more sustainable.

A lower $\bar{b}$ causes a larger gap between the current and the target debt-to-GDP ratio by (12) and then a larger amount of debt reduction accompanies in the short and medium run.[11] This extends the fiscal space through (i) decreases in interest payment and (ii) increases in tax revenues by the crowding in of capital, making fiscal policy more sustainable.

[Figure 3]

3.3 Numerical studies under expenditure-based consolidation

We calibrate the model to the date of Japan, Greece, Italy, Portugal, and the US as examples of countries whose debt-to-GDP ratios are very high among OECD countries. We consider the following scenarios. Expenditure-based consolidation starts at period 0 unexpectedly for given (k_0, b_0). Constant tax rates are assumed to be at the initial levels, $\tau^R = \tau^R_{init}$ $\tau^w = \tau^R_{init}$, $\tau^c = \tau^c_{init}$.

[11] Here, note that a fall in $\bar{b}$ makes fiscal policy unsustainable if the initial state (k_t, b_t) is already near the region of $g_t \leq 0$ ((14)). However, since we have focused mainly on a somewhat mature economy without capital shortage, we could ignore such a rare case throughout this study.

3.3.1 Parameter choices

The targeted debt-to-output ratio is set at 0.6 as the benchmark (the target value of the SGP in the EU). Since B_t is a stock while Y_t is a flow, an appropriate measure of the targeted debt-to-output ratio in the model is $\bar{b} = 0.6/30(= 0.02)$, taking one period as 30 years.[12] The subjective discount factor is set at $\beta = (0.973)^{30}$ as in Song et al. (2012). We adapt $A^{JPA} = 20$ to the Japanese economy. [13]

We next move onto each country's specific parameter values.

[Table 1]

Japan

We choose $\alpha^{JPA} = 0.38$ following Hansen and İmrohoroğlu (2016).[14] Capital and wage income tax rates are set to $\tau^R_{JPA} = 0.46$ and $\tau^w_{JPA} = 0.31$ based on the estimated values in Gunji and Miyazaki (2011), overall statutory tax rates on dividend income, and average personal income tax and social security contribution rates on gross labor income at the OECD tax database. The (2000–2007) average capital income tax rate by Gunji and Miyazaki (2011) is around 0.53 while the (2000–2007) overall private income tax (PIT) on dividend plus corporate income tax rate (CIT) is around 0.56. Since the (2000–2020) overall PIT plus is around 0.49, the adjusted value of τ^R by Gunji and Miyazaki (2011) from 2000 to 2020 is 0.46. We use the (1995–2007) average wage income tax rate of around 0.31 by Gunji and Miyazaki (2011) since the average personal income tax and social security contribution rates do not change drastically between 2000 and 2019.[15] Consumption tax rate is set to the latest value of $\tau^c_{JPA} = 0.1$ in 2020. The average annual population growth rate between 1990 and 2018 was 0.09% according to the World Development Indicators, and thus we set $n^{JPA} = 0$.[16] The output to capital ratio $(Y/K(= q(k)))$ in Japan from 1990 to 2020 is around 0.32 on average according to the AMECO database.[17] Since K_t is a stock while Y_t is a flow, an appropriate measure of the output to capital ratio in the model is $q(k_0^{JPA}) = 0.32 \times 30 = 9.6$. Solving $q(k_0^{JPA}) = A^{JPA}(k_0^{JPA})^{\alpha^{JPA}-1} = 20(k_0^{JPA})^{0.38-1} = 9.6$ yields $k_0^{JPA} \approx 3.27$. We obtain the output per capita: $y(k_0^{JPA}) = q(k_0^{JPA})k_0^{JPA} \approx 31.36$ and interest rate: $R(k_0^{JPA}) \approx 3.65$ (the annual rate of around 4.4%). Next, let us use the (2014–2018) debt-to-output ratio of 2.37 (OECD, 2021) as the current level. Then, we obtain $b_0^{JPA}/y_0^{JPA} = 2.37/30$ in the model. From $b_0^{JPA} = (b_0^{JPA}/y_0^{JPA})(k_0^{JPA}/q(k_0^{JPA}))$, we obtain $b_0^{JPA} \approx 2.48$.

The US, Greece, Portugal, and Italy

The value of α in the US: $\alpha^{US} = 0.35$, in Greece: $\alpha^{GRE} = 0.4$, in Italy: $\alpha^{ITA} = 0.39$, and in Portugal: $\alpha^{PRT} = 0.39$ follow the values in Trabandt and Uhlig (2011). The average annual population growth rate between 2000 and 2019 was 0.97% in the US, 0.21% in

[12] This adjustment between a stock and a flow is in line with Song et al. (2012) and Andersen and Bhattacharya (2020). They employ OLG models where one period corresponds to 20 or 30 years.

[13] A is simply a scale parameter when the production is Cobb–Douglas and the utility is log-linear (see e.g., the Appendix A.5 of de la Croix and Michel (2002)).

[14] Hansen and İmrohoroğlu (2016) set to 0.3783: the sample (1981–2010) average of the annual ratio of capital income in Japan.

[15] The (2000–2007) total tax wage of a single person (without dependent) at 100% of the average wage is around 29% while the (2000–2019) total tax wage is around 30% according to the OECD tax base (accessed on February 9, 2021).

[16] I retrieved from https://data.worldbank.org/indicator/SP.POP.GROW (accessed on October 5, 2020).

[17] We use the data of GDP at constant market prices per unit of net capital stock at the AMECO database (accessed on February 13, 2021).

Greece, 0.22% in Italy, and 0.09% in Portugal (World Development Indicators), and thus we set $(n^{US}, n^{GRE}, n^{ITA}, n^{PRT}) = (0.01, 0, 0, 0)$.

We employ the values of tax rates (τ^R, τ^w, τ^c) in these four countries: $(0.34, 0.28, 0.05)$ in the US, $(0.16, 0.41, 0.15)$ in Greece, $(0.30, 0.47, 0.15)$ in Italy, and $(0.23, 0.31, 0.23)$ in Portugal based on the estimated values in Trabandt and Uhlig (2011), and the overall statutory tax rates on dividend income, and average personal income tax and social security contribution rates on gross labor income at OECD tax database. The (1995–2007) average capital income tax rate by Trabandt and Uhlig (2011) is around 0.36 in the US, 0.16 in Greece, 0.34 in Italy, and 0.23 in Portugal while the (1995–2007) overall private income tax (PIT) on dividend plus corporate income tax rate (CIT) is around 0.61 in the US, 0.34 in Greece, 0.53 in Italy, and 0.49 in Portugal. Since the (1995–2020) overall PIT plus is around 0.57 in the US, 0.35 in Greece, 0.47 in Italy, 0.48 in Portugal, the adjusted value of τ^R by Trabandt and Uhlig (2011) from 2000 to 2020 is 0.16 in Greece and 0.30 in Italy. Since the average personal income tax and social security contribution rates do not change drastically between 2000 and 2019 (OECD tax base), we adapt the values of τ^w in Trabandt and Uhlig (2011). Consumption tax rate in Greece, Italy and Portugal are based on the actual value in 2020 (OECD tax data base), while the value in the US is based on Trabandt and Uhlig (2011).

The (1990–2020) average output to capital ratios ($q(k)$) in the US, Greece, Italy, and Portugal are 0.41, 0.28, 0.30, and 0.36, respectively (AMECO database), indicating that values of $q(k_0)$ in the model (30 years in one period) are given by $q(k_0^{US}) = 30 \times 0.41 = 12.3$ in the US, $q(k_0^{GRE}) = 30 \times 0.28 = 8.4$ in Greece, $q(k_0^{ITA}) = 30 \times 0.30 = 9.0$ in Italy, and $q(k_0^{PRT}) = 30 \times 0.36 = 10.8$ in Portugal. The (2015–2019) debt-to-output ratio of 1.36 in the US, 1.93 in Greece, 1.53 in Italy, 1.42 in Portugal (OECD, 2021) are adjusted to $b_0^{US}/y_0^{US} = 1.36/30$, $b_0^{GRE}/y_0^{GRE} = 1.93/30$, $b_0^{ITA}/y_0^{ITA} = 1.53/30$, and $b_0^{PRT}/y_0^{PRT} = 1.42/30$ in the model.

Here, we normalize the Japanese economy as the baseline. From data of the actual public debt per capita in 2015 and in 2018 (OECD, 2017, 2019), the ratios of the public debt per capita in country j to those in Japan are calculated as $(b_0^{US}/b_0^{JPA}) = 0.67$, $(b_0^{GRE}/b_0^{JPA}) = 0.55$, $(b_0^{ITA}/b_0^{JPA}) = 0.66$, and $(b_0^{PRT}/b_0^{JPA}) = 0.50$. Since the public debt per capita in country j in the model is given by $b_0^{JPA} \times (b_0^i/b_0^{JPA})$ $(i = US, GRE, ITA, PRT)$ and $b_0^{JPA} = 2.48$, we obtain $b_0^{US} = 2.48 \times 0.67 \approx 1.66$, $b_0^{GRE} = 2.48 \times 0.55 \approx 1.36$, $b_0^{ITA} = 2.48 \times 0.66 \approx 1.64$, and $b_0^{PRT} = 2.48 \times 0.50 \approx 1.24$. From the data of GDP per capita between 1990 and 2019 (World Development Indicators), the ratios of the output per capita in country j to those in Japan are calculated as $(y_0^{US}/y_0^{JPA}) \approx 1.13$, $(y_0^{GRE}/y_0^{JPA}) \approx 0.49$, $(b_0^{ITA}/b_0^{JPA}) \approx 0.77$, and $(y_0^{PRT}/y_0^{JPA}) \approx 0.45$. These together with $y(k_0^{JPA}) = 31.36$ yield $y_0^i (i = US, GRE, ITA, PRT)$ in the model as $(y_0^{US}, y_0^{GRE}, y_0^{ITA}, y_0^{PRT}) \approx (35.66, 15.46, 24.30, 14.11)$. From $k_0^i = b_0^j \times \left(\frac{q(k_0^j)}{b_0^i/y_0^i}\right)$, we have $k_0^{US} = 1.66 \times \left(\frac{12.3}{1.36/30}\right) \approx 2.98$, $k_0^{GRE} = 1.36 \times \left(\frac{8.4}{1.93/30}\right) \approx 2.52$, $k_0^{ITA} = 1.64 \times \left(\frac{9.0}{1.53/30}\right) \approx 3.57$, and $k_0^{PRT} = 1.24 \times \left(\frac{10.8}{1.42/30}\right) \approx 2.43$. Substituting the values of y_0^j, k_0^i, and α^j into $y_0^i = A^j (k_0^j)^{\alpha^j}$ yields $(A^{US}, A^{GRE}, A^{ITA}, A^{PRT}) \approx (24.34, 10.68, 14.80, 9.99)$.

These parameter choices yield the plausible values of interest rate of $R(k_0^{JPA}) \approx 3.65$, $R(k_0^{US}) \approx 4.31$, $R(k_0^{GRE}) = 3.36$, $R(k_0^{ITA}) = 3.51$, and $R(k_0^{PRT}) \approx 4.21$ and the ratio of government spending to GDP of $g_0^{JPA}/y(k_0^{JPA}) = 0.3619$, $g_0^{US}/y(k_0^{US}) = 0.2519$, $g_0^{GRE}/y(k_0^{GRE}) = 0.3162$, $g_0^{ITA}/y(k_0^{ITA}) = 0.4270$, and $g_0^{PRT}/y(k_0^{PRT}) = 0.3528$.[18] Here, we evaluate $g_0^i/y(k_0^i)$

[18] Annual (long-run) interest rate of between 4% ($(1+0.04)^{30} \approx 3.24$) and 5% ($(1+0.05)^{30} \approx 4.32$). Government consumption + investment + transfer to GDP (the data value in Trabandt and Uhlig (2011)) is 0.26 in the US, 0.35 in

($i = JPA$, US, GRE, ITA, and PRT) as if no fiscal consolidation is implemented in period 0 (i.e., $\phi = 0$ and $b_1 = b_0$).[19]

3.3.2 Results

Figures 4, 5, 6, 7, and 8 illustrate the transitional dynamics for Japan, the US, Greece, Italy, and Portugal, respectively. Table 2 focuses on the unsustainable paces of expenditure-based consolidation and Table 3 represents the values of steady-state variables. These figures and tables show the following results.

[Figures 4, 5, 6, 7, and 8] [Tables 2 and 3]

Sustainable transition paths

First, we confirm that the properties of transitional dynamics of (k_t, b_t) in five countries are in line with those in Section 3.1, that is, crowding out of capital by a large initial debt and crowding in of capital by fiscal consolidation. From (10), larger (resp. smaller) initial cuts in public expenditures g_0, with a faster (resp. slower) pace of fiscal consolidation, ϕ (the first term in the RHS of (10)) extend fiscal space more rapidly (resp. slowly), leading to increases (resp. further decrease) in public expenditure. From (31) and (32) below, consumption of both the young and old in time 0 (c_0 and d_0) are not affected by the initial cuts in public expenditures g_0. Since c_t is increasing in k_t ((32)), the dynamics of c_t reflects the dynamics of k_t. Finally, (33) shows that d_t decreases (resp. increases) in b_t because asset income from bonds decreases (resp. increases).

$$d_0 = \frac{(1-\tau^R)R(k_0)(k_0+b_0)}{1+\tau^c} \quad \text{(generation } -1 \text{ in period } 0), \tag{31}$$

$$c_t = \frac{(1-\tau^w)w(k_t) - \left(\Phi(k_t, b_t) + (1-\phi)b_t + \phi\bar{b}y(k_t)\right)}{1+\tau^c} \quad \text{(generations } t \geq 0 \text{ in period } t), \tag{32}$$

$$d_{t+1} = \frac{(1-\tau^R)R(k_{t+1})(k_{t+1}+b_{t+1})}{1+\tau^c} \quad \text{(generations } t \geq 0 \text{ in period } t+1). \tag{33}$$

In Japan, a large initial public debt decreases capital in the short and medium runs when $0 < \phi < 1$ (crowding out of capital) but begins to increase in the latter stages, eventually exceeding its initial level in the long run (crowding in of capital). When $\phi = 1$, even with a large initial public debt, capital does not decrease in the short run, because large crowding in of capital by a rapid pace debt reduction surpasses the crowding out of capital by large initial public debt. The steady-state levels of capital, k^*, government spending, g^*, and consumption by the young, c^*, exceed the current/initial ones, k_0, g_0, and c_0, respectively. By contrast, the steady-state consumption by the old d^* becomes lower than the initial consumption d_0, because assets from bonds decreases by fiscal consolidation.

In Greece, Italy, and Portugal, capital stock decreases during fiscal consolidation both when $0 < \phi < 1$ and $\phi = 1$. Eventually, the steady-state levels of capital, k^*, government spending,

Greece, 0.40 in Italy, and 0.34 in Portugal, respectively. In Japan, government production costs (% of GDP) between 2007 and 2019 were around 0.21 on average (OECD data accessed on June 30, 2021) and transfer payment from 2000 to 2010 ranged between around 0.15 and 0.17 (see Hansen and İmrohoroğlu, 2016).

[19] From (3), we obtain $g_0/y(k_0) = [(1+n)b_0 + \tau^w w(k_0) + \tau^R R(k_0)k_0 + \tau^c y(k_0) - (1-\tau)R(k_0)b_0 - (1+n)\tau^c\Phi(k_0, b_0)]/[(1+\tau^c)y(k_0)]$.

g^*, consumption by the young c^*, and consumption by the old d^* are lower than the current/initial ones, k_0, g_0, c_0, and d_0, respectively. Therefore, fiscal consolidations fail to be productive. This reason is as follows. Because labor income is paid mostly to the young and capital income accrues mostly to the old in a lifecycle of finitely lived agents, lower productivity of labor A and higher labor income tax in Greece, Italy, and Portugal lead to younger people having less income to save and to buy capital stock. In addition, a large initial debt in such low productivity economies enlarges the crowding out of capital. These negative effects on capital surpass the crowding in of capital by fiscal consolidation. Notably, our result here is in contrast to those of Papageorgiou (2012) and Maebayashi et al. (2017), because the latter studies show that fiscal consolidation in Greece is productive. However, Papageorgiou (2012) and Maebayashi et al. (2017) assume infinitely lived agents who are in essence always young, and the initial state is assumed to be in the stable steady state.

A common fact in Japan and these European countries is that a larger decline in capital in the early stage of consolidation is associated with a slower pace of fiscal consolidation. In contrast to these four countries, capital in the US increases in the entire process of fiscal consolidation. Capital in the US increases rapidly with a faster pace of consolidation. Furthermore, fiscal policy in the US can be sustainable even without consolidation (i.e., $\phi = 0$ and $b_{t+1} = b_t = b_0$). This result for the US is attributable to high productivity A and low tax burden on wage income.

Unsustainable transition paths

Next, we examine the cases in which fiscal policies are unsustainable. Fiscal policy is unsustainable without decreasing outstanding debt: $\phi = 0$ in Japan ($g_2 = 0$), Greece ($g_1 = 0$), Italy ($g_1 = 0$), and Portugal ($g_1 = 0$). The lowest pace of the example consolidation plans ($\phi = 0.1$) cannot make fiscal policy sustainable in Japan ($g_3 = 0$), Greece ($g_1 = 0$), Italy ($g_1 = 0$), and Portugal ($g_2 = 0$). Only in Greece does fiscal consolidation even under $\phi = 0.3$ not succeed ($g_1 = 0$ in Greece), indicating that the current/initial fiscal condition in Greece is the worst of the five countries. Outstanding public debts in Japan are so large relative to the size of the economy that the very low paces of consolidation plans cannot ensure their fiscal sustainability. In Greece, Italy, and Portugal, large outstanding debts under low productivity of the economy and a high tax burden on the young deter capital accumulation seriously and therefore, these three European countries face an even worse situation than Japan.

In unsustainable transition paths, debt increases monotonically, so does the asset income from bonds and consumption by the old (d_t). Meanwhile, large crowding out of capital by a large debt decreases wage income and consumption by the young (c_t). It increases the interest rate and cost of debt repayment for the government. Then, g_t decreases to zero.[20] From (1) when $g_t = 0$ is binding (no public goods and services) in period t, utility of generation $t-1$ takes asymptotically to $-\infty$, we regard the generation that faces this situation as the non-surviving generation (Table 2). These generations can occur in Japan, Greece, Italy, and Portugal but not in the US.

[20] Transition paths of (k_t, b_t) in the region of $g_t = 0$ on the phase diagram in Figures 4, 5, 6, 7, and 8, result from the dynamic systems under $(\tau_t^w, \tau_t^R, \tau_t^c) = (\tau^w, \tau^R, \tau^c)$ that we have shown in footnote 8.

4 Tax-based consolidation

4.1 Dynamics under expenditure-based consolidation

In this section, we examine a fiscal consolidation by adjusting the income tax rates, termed *the tax-based consolidation* hereafter. In tax-based consolidation, the government is assumed to secure its spending by a rate proportional to the rate of GDP: $G_t = \lambda Y_t$ ($\lambda \in (0, 1)$), but to adjust the income tax rates (τ_t^w and τ_t^R) endogenously to follow the debt policy rule: (4).

To study a global transition path analytically and examine the differences from expenditure-based consolidations essentially, we adopt the following simple ways of tax instruments and assumptions. First, as for endogenous income tax rates, we assume that $\tau_t^R = \delta\tau_t^w$ and $\delta > 0$ and simply denote $\tau_t^w = \tau_t$ and $\tau_t^R = \delta\tau_t$, respectively. This specification induces both tax rates to move in the same direction.[21] Second, we consider the same timeline $\phi \leq 1$, constant consumption tax $\tau_t^c = \tau^c$, and the Cobb–Douglas production function ($Y_t = AK_t^\alpha L_t^{1-\alpha}$), as in the case of expenditure-based consolidation. In Section 6, we relax some of these assumptions and introduce a variable consumption tax and other ways of income tax adjustment.

Substituting $G_t = \lambda Y_t$ ($g_t = \lambda A k_t^\alpha$), $\tau_t^w = \tau_t$, and $\tau_t^R = \delta\tau_t$ into (8) leads to

$$\begin{aligned}\tau_t =& \frac{[(1+\tau^c)\lambda - \tau^c(1-\tilde{\eta})]k_t + \alpha b_t}{[1+\alpha(\delta-1)+\tilde{\eta}\tau^c]k_t + \delta\alpha b_t} - \frac{(1+\tau^c)(1+n)\left[b_t - \phi\left(b_t - \bar{b}Ak_t^\alpha\right)\right]}{q(k_t)\left\{[1+\alpha(\delta-1)+\tilde{\eta}\tau^c]k_t + \delta\alpha b_t\right\}} \\ &\equiv \tau(k_t, b_t), \end{aligned} \tag{34}$$

where $\tilde{\eta} \equiv \frac{\beta(1-\alpha)}{1+\beta} \in (0, 1)$ and recall that $q(k)(= Ak^{\alpha-1})$.

Substituting (34) and (12) into (5), we obtain

$$\begin{aligned} k_{t+1} = \tilde{\Phi}(k_t, b_t) \equiv& \frac{(1+\tau^c)(1-\lambda) + \alpha(\delta-1) + (\delta-1)\alpha(b_t/k_t)}{1+\alpha(\delta-1)+\tilde{\eta}\tau^c + \delta\alpha(b_t/k_t)} \cdot \frac{\tilde{\eta}}{1+n} A k_t^\alpha \\ &- \frac{1-\tilde{\eta}+\alpha(\delta-1)+\delta\alpha(b_t/k_t)}{1+\alpha(\delta-1)+\tilde{\eta}\tau^c+\delta\alpha(b_t/k_t)}\left[b_t - \phi\left(b_t - \bar{b}Ak_t^\alpha\right)\right] \qquad \text{for } k_t > 0. \end{aligned} \tag{35}$$

Eqs. (12) and (35) characterize the dynamic system of the economy under the tax-based consolidation.

Using (12) and (35), we first investigate the existence of the steady states. Applying $k_{t+1} = k_t = k$ and $b_{t+1} = b_t = \bar{b}Ak^\alpha$ ($\bar{b} > 0$) into (12) and (35) we obtain

$$\begin{aligned} &\mu_1 q(k)^2 + \mu_2 q(k) + \mu_3 = 0, \\ &\mu_1 \equiv \alpha\bar{b}\left[(1+n)\delta\bar{b} - \tilde{\eta}(\delta-1)\right], \\ &\mu_2 \equiv (1+n)[\underbrace{1-\tilde{\eta}+\delta\alpha+\alpha(\delta-1)}_{=\frac{1-\alpha}{1+\beta}+2\alpha\delta(>0)}]\bar{b} - \tilde{\eta}[(1+\tau^c)(1-\lambda)+\alpha(\delta-1)], \\ &\mu_3 \equiv (1+n)[1+\alpha(\delta-1)+\tilde{\eta}\tau^c] > 0. \end{aligned} \tag{36}$$

(36) with $\mu_1 > 0$ for $0 < \delta \leq 1$ leads directly to the following proposition:

---

[21]This specification of tax adjustment encompasses those of tax adjustments in many studies (e.g., Bräuninger, 2005; Yakita, 2008; Morimoto et al., 2017), all of which consider the case when $\delta = 1$: $\tau_t^R = \tau_t^w$.

Proposition 5.

(i) When $0 < \delta \leq 1$, two steady states exist if and only if $\mu_2 < 0$ and $\mu_2^2 - 4\mu_1\mu_3 > 0$.

(ii) When $\delta > 1$ and $\mu_1 > 0$, no steady state exists if $1 \geq (1+\tau^c)\beta$.

(iii) When $\delta > 1$ and $\mu_1 \leq 0$, a unique steady-state exists.

Proof: See Appendix F.

Note the following three points. First, when $0 < \delta \leq 1$, we assume that $\mu_2 < 0$ and $\mu_2^2 - 4\mu_1\mu_3 > 0$ to ensure the existence of the steady state. Accordingly, $\mu_2 < 0 \Leftrightarrow \bar{b} < \bar{b}_2 \equiv \frac{\tilde{\eta}[(1+\tau^c)(1-\lambda)+\alpha(\delta-1)]}{(1+n)[1-\tilde{\eta}+\delta\alpha+\alpha(\delta-1)]}$ and $(1+\tau^c)(1-\lambda)+\alpha(\delta-1) > 0$ from $\bar{b} > 0$ must be satisfied.

Second, when $\delta > 1$ and $\mu_1 > 0$ no steady state exists since $1 \geq (1+\tau^c)\beta$ holds for reasonable range of parameter sets (β, τ^c), demanding $(\mu_1 \leq 0 \Leftrightarrow)\ \bar{b} \leq \bar{b}_3 \equiv \tilde{\eta}(\delta-1)/(\delta(1+n))$ to avoid this case.[22] Hereafter, we impose $\mu_1 \leq 0$ ($\bar{b} \leq \bar{b}_3$) and focus on the case of Proposition 5-(iii) when $\delta > 1$.

Finally, since (36) are independent on ϕ, the pace of tax-based consolidation does not affect the steady state values of k_t and b_t as in the case of expenditure-based consolidation.

Next, we derive $k_{t+1} = k_t$ locus as the function of $b_t = m(k_t)$. From (35), we obtain

$$\begin{aligned} & k_{t+1} - k_t = \tilde{\Phi}(k_t, b_t) - k_t = 0 \\ & \Leftrightarrow h(b_t, k_t) \equiv a_1 b_t^2 + a_2(q(k_t))k_t b_t + a_3(q(k_t))k_t^2 = 0 \ \text{ for } \ k_t > 0, \end{aligned} \tag{37}$$

where, for $\phi \in (0,1]$, $a_1 \equiv (1+n)(1-\phi)\delta\alpha \geq 0$, $a_2(q(k_t)) \equiv a_{21} + a_{22}q(k_t)$, $a_3(q(k_t)) \equiv a_{31} + a_{32}q(k_t)$, $a_{21} \equiv (1+n)\left\{(1-\phi)\left[\frac{1-\alpha}{1+\beta} + \alpha\delta\right] + \alpha\delta\right\} > 0$, $a_{22} \equiv \left[(1+n)\bar{b}\phi\delta - \tilde{\eta}(\delta-1)\right]\alpha$, $a_{31} = \mu_3 > 0$, and $a_{32} \equiv (1+n)\left[\frac{1-\alpha}{1+\beta} + 2\delta\alpha\right]\bar{b}\phi - \tilde{\eta}\left[(1+\tau^c)(1-\lambda) + \alpha(\delta-1)\right]$. Here, we notify the following condition on the parameters:

Condition 3.

When $\delta > 1$, i.e., $\mu_1 \leq 0$ (Proposition 5-(iii)),

$a_{32} \leq 0$ and $a_{21}a_{32} - a_{22}a_{31} \leq 0$ if $\frac{(1+\tau^c)(1-\lambda)\delta}{(\delta-1)} > \max\left\{\frac{1-\alpha}{1+\beta} + \alpha\delta, 1-\alpha+\tilde{\eta}\tau^c\right\}$

for $0 < \phi \leq 1$ and $(0 <)\bar{b} \leq \min\left\{\bar{b}_3, \bar{b}_4\right\}$, where $\bar{b} = \bar{b}_4$ satisfies $a_{21}a_{32} = a_{22}a_{31}$.

See Appendix G for the derivation of Condition 3. Since values of $(\tau^c, \delta, \lambda, \alpha, \beta)$ in countries where $\delta > 1$ (the UK and Denmark in the EU 14 countries[23], Japan, and the US) satisfy Condition 3, we impose it in this study.[24] [25] Let us summarize the facts on the ceiling of $\bar{b}$ as follows.

[22] $1 \geq (1+\tau^c)\beta$ is satisfied under $0 < \tau^c \leq 1$ and $\beta = 0.973^{30}$.

[23] See Trabandt and Uhlig (2011).

[24] $(\tau^c, \alpha) = (0.16, 0.36)$ in the UK while $(\tau^c, \alpha) = (0.35, 0.40)$ in Denmark (see Trabandt and Uhlig, 2011). $\beta = 0.973^{30}$ is taken in both countries. δ in the UK and Denmark are $\tau^R/\tau^w = 0.46/0.28 \approx 1.6$ and $0.51/0.47 \approx 1.1$, respectively (see Trabandt and Uhlig, 2011).

[25] Even if Condition 3 is relaxed when $\delta > 1$, $\mu_1 \leq 0$ and add the cases of (i) $a_{32} \leq 0$ and $a_{21}a_{32} - a_{22}a_{31} > 0$ and (ii) $a_{32} > 0$, we can characterize the $b_t = m(k_t)$ on the (k_t, b_t) plane and obtain the same results on policy effects of changes in $\bar{b}$ and ϕ (as we examine in Section 4.2) qualitatively. These are available upon request.

Lemma 4.

(i) *When* $0 < \delta \leq 1$, $\bar{b} < \bar{b}_2 \equiv \frac{\tilde{\eta}[(1+\tau^c)(1-\lambda)+\alpha(\delta-1)]}{(1+n)[1-\tilde{\eta}+\delta\alpha+\alpha(\delta-1)]}$, *where* $(1+\tau^c)(1-\lambda)+\alpha(\delta-1) > 0$.

(ii) *When* $\delta > 1$ *(and* $\mu_1 \leq 0$*),* $\bar{b} \leq \min\left\{\bar{b}_3, \bar{b}_4\right\}$, *where* $\bar{b}_3 \equiv \frac{\tilde{\eta}(\delta-1)}{\delta(1+n)}$ *and* $\bar{b} = \bar{b}_4$ *satisfies* $a_{21}a_{32} = a_{22}a_{31}$.

We move to examine (37). Appendix H shows that $k_{t+1} = k_t$ locus takes zero when $k_t = 0$ and $\breve{k} \equiv -a_{31}/a_{32} > 0$ for $a_{32} < 0$. To reveal more properties of $k_{t+1} = k_t$ locus, we rewrite (37) into

$$q(k_t) = \Gamma_k(x_t) \equiv -\frac{a_1 x_t^2 + a_{21}x_t + a_{31}}{a_{22}x_t + a_{32}} \quad \text{for} \quad a_{22}x_t + a_{32} \neq 0, \tag{38}$$

where, recall that $x_t \equiv b_t/k_t$. The first derivative of $\Gamma_k(x_t)$ is $\Gamma_k'(x_t) = \Lambda(x_t)/(a_{22}x_t + a_{32})^2$, where $\Lambda(x_t) \equiv -a_1 x_t(a_{22}x_t + 2a_{32}) - (a_{21}a_{32} - a_{22}a_{31})$.

From (37) and (38), we obtain the following properties of $b_t = m(k_t)$ on the (k_t, b_t) plane.

Lemma 5.

(i) *When* $0 < \delta \leq 1$, $b_t = m(k_t)$ *satisfies* $m(0) = m(\breve{k}) = 0$ *and takes the inverted-U shaped curve for* $\phi \in (0,1]$ *as in Figure 9-(a).*

(ii) *When* $\delta > 1$ *(and* $\mu_1 \leq 0$*), (a)* $b_t = m(k_t)$ *for* $\phi \in (0,1)$ *satisfies* $m(0) = m(\breve{k}) = 0$ *and takes the inverted-U shaped curve as in Figure 9-(b), while (b)* $b_t = m(k_t)$ *for* $\phi = 1$ *is monotonically decreasing in* k_t *that satisfies* $m(\breve{k}) = 0$ *(*$\breve{k} = q^{-1}(-\mu_3/\mu_2)$*) and has asymptote* $\lim_{q(k_t)\to -\frac{(1+n)\alpha\delta}{\bar{b}^{-1}\mu_1}} m(k_t) = +\infty$ *(as in Figure 10).*

Proof: See Appendix H.

[Figures 9 and 10]

From (37), (38) and Lemma 5, $k_{t+1} > (\leq) k_t$ holds below (above) $k_{t+1} = k_t$ locus both in $(x_t, q(k_t))$ and (k_t, b_t) planes. See Appendix H in more details.

Next, we move onto $b_{t+1} = b_t$ locus in the tax-based consolidation. This is common to the case of expenditure-based consolidation, $b_t = \bar{b}Ak_t^{\alpha}$ (see 20). $b_{t+1} = b_t$ locus is written into $x_t = \bar{b}q(k_t)$ (see 23), which is equivalent to $q(k_t) = \Gamma_b(x_t) \equiv \bar{b}^{-1}x_t$. $b_{t+1} > (\leq) b_t$ holds below (above) $b_{t+1} = b_t$ locus both in $(x_t, q(k_t))$ and (k_t, b_t) planes.

Figure 9-(a), on the one hand, illustrates the phase diagrams of the economy when $0 < \delta \leq 1$ and $\phi \in (0,1]$, highlighting that the steady state $S(k_S^*, b_S^*)$ is stable and the steady state $U(k_U^*, b_U^*)$ is saddle-point stable. In those cases, the knife-edge saddle arm converging to $U(k_U^*, b_U^*)$ represents the threshold of the public debt in order for the government to sustain fiscal policy. As we explain below, if (k_t, b_t) is above this threshold curve, $k_{t+1} = 0$ binds at a certain period t and $\tau_{t+1}^w = 1$ binds at the next period $t+1$, making fiscal policy unsustainable.

Figure 9-(b) (resp. Figure 10), on the other hand, illustrates the phase diagram of the economy for $\phi \in (0,1)$ (resp. $\phi = 1$) when $\delta > 1$ (and $\mu_1 \leq 0$), highlighting that the unique steady state $S(k_S^*, b_S^*)$ is stable. The knife-edge saddle arm converging to $k_t = b_t = 0$ represents the threshold curve for $\phi \in (0,1)$ while no threshold arm exists for $\phi = 1$. As we explain below, if (k_t, b_t) is above this threshold curve, $k_{t+1} = 0$ binds at a certain period t and $\tau_{t+1}^R = 1$ binds at the next period $t+1$, making fiscal policy unsustainable.

When $0 < \delta \leq 1$ fiscal policy cannot be sustainable either if $k_{t+1} = 0$ or $\tau_t^w = \tau_t = 1$ binds (i.e., $k_{t+1} \leq 0$ or $\tau_t \geq 1$). $k_{t+1} = 0$ is equivalent to $\tilde{\Phi}(k_t, b_t) = 0$, which we call $k_{t+1} = 0$ locus hereafter, is written by

$$q(k_t) = \Theta(x_t) \equiv -\frac{a_1 x_t^2 + [a_{21} - (1+n)\alpha\delta]x_t}{a_{32} + a_{22}x_t} \quad \text{for} \quad a_{32} + a_{22}x_t \neq 0. \tag{39}$$

Appendix I shows that $k_{t+1} = 0$ locus is always above the $k_{t+1} = k_t$ locus (i.e., $\Gamma_k(x_t) > \Theta(x_t)$ for $0 \leq x_t < -a_{32}/a_{22}$). Next, $k_{t+1} = 0$ locus is above the threshold curve since $k_{t+1} = 0$ realizes eventually only when (k_t, b_t) is above the threshold curve. Furthermore, from (34) and (35), if $\tau_t^w = \tau_t = 1$, $k_{t+1} = 0$ always binds whereas $\tau_t = 1$ does not always bind if $k_{t+1} = 0$. Then, the condition of $\tau_t = 1$ is above $k_{t+1} = 0$ locus. These positional relationships between the condition of $\tau_t = 1$ and $k_{t+1} = 0$ locus and the threshold curve indicate that if (k_t, b_t) is above the threshold curve, the economy faces $k_{t+1} = 0$ at a certain period t. Then, fiscal policy cannot be sustainable in the next period $t+1$, where $\tau_{t+1} = 1$ and $\tau_{t+1}^R = \delta\tau_{t+1} = 1$ also bind, since (34) with $k_{t+1} = 0$ yields $\tau_{t+1} = 1/\delta > 1$.

When $\delta > 1$, fiscal policy cannot be sustainable either if $k_{t+1} = 0$ or $\tau_t^R = \delta\tau_t = 1$ binds (i.e., $k_{t+1} \leq 0$ or $\delta\tau_t \geq 1$). Appendix I shows that the $k_{t+1} = 0$ locus ((39)) is always above the $k_{t+1} = k_t$ locus (i.e., $\Gamma_k(x_t) > \Theta(x_t)$ for $x_t > 0$). Furthermore, Appendix J shows that $\delta\tau_t = 1$ does not bind above the $b_{t+1} = b_t$ locus or in the steady state $S(k_S^*, b_S^*)$. Thus, if (k_t, b_t) is above the threshold curve, $k_{t+1} = 0$ binds at a certain period t, and fiscal policy cannot be sustainable in the next period $t+1$ where $\tau_{t+1}^R = \delta\tau_{t+1} = 1$ also binds, since (34) with $k_{t+1} = 0$ yields $\delta\tau_{t+1} = \delta \cdot (1/\delta) = 1$. We summarize the results in the following proposition.

Proposition 6.

(i) When $0 < \delta \leq 1$ and $\phi \in (0, 1]$, the steady state $S(k_S^*, b_S^*)$ is stable while the steady state $U(k_U^*, b_U^*)$ is saddle-point stable, the saddle arm converging to $U(k_U^*, b_U^*)$ represents the threshold of the public debt in order for the government to sustain fiscal policy.

(ii) When $\delta > 1$ (with $\mu_1 \leq 1$) and $\phi \in (0, 1)$, the unique steady state $S(k_S^*, b_S^*)$ is stable and the arm converging to $k_t = b_t = 0$ represents the threshold of the public debt to sustain fiscal policy.

(iii) When $\delta > 1$(with $\mu_1 \leq 1$) and $\phi = 1$, the unique steady state $S(k_S^*, b_S^*)$ is stable and fiscal policy is sustainable unless initial state (k_0, b_0) is above the $k_{t+1} = 0$ locus.

4.2 Changes in $\bar{b}$ and ϕ under the tax-based consolidation

In this section, we examine the effects of changes in $\bar{b}$ and ϕ on fiscal sustainability and the steady states. From (38), we obtain

$$\left.\frac{\partial \Gamma_k(x_t)}{\partial \bar{b}}\right|_{q(k_t)=\Gamma_k(x_t)} = \frac{(1+n)\phi \overbrace{\left(a_1 x_t^2 + a_{21} x_t + a_{31}\right)}^{(+)}}{(a_{22}x_t + a_{32})^2} \left(\delta \alpha x_t + \underbrace{1 - \tilde{\eta} + \alpha(\delta - 1)}_{=\frac{1-\alpha}{1+\beta}+\delta\alpha(>0)} \right) > 0 \quad (40)$$

$$\left.\frac{\partial \Gamma_k(x_t)}{\partial \phi}\right|_{q(k_t)=\Gamma_k(x_t)} = \frac{(1+n)\bar{b}\left(\delta \alpha x_t + \frac{1-\alpha}{1+\beta} + \alpha\delta\right)\left[\Gamma_b(x_t) - \Gamma_k(x_t)\right]}{a_{22}x_t + a_{32}} > (\leq) 0$$
$$\text{for } \Gamma_b(x_t) > (\leq) \Gamma_k(x_t). \quad (41)$$

Thus, a fall in $\bar{b}$ shifts $q(k_t) = \Gamma_k(x_t)$ ($k_{t+1} = k_t$ locus) downward (resp. upward) whereas $q(k_t) = \Gamma_b(x_t)$ ($b_{t+1} = b_t$ locus) upward (resp. downward) as depicted in Figure 11. Furthermore, a rise in ϕ shifts $q(k_t) = \Gamma_k(x_t)$ ($k_{t+1} = k_t$ locus) downward (resp. upward) for $k_U^* < k_t \leq k_S^*$ (resp. $k_t \leq k_U^*$ and $k_t > k_S^*$) while $q(k_t) = \Gamma_b(x_t)$ ($b_{t+1} = b_t$ locus) and the steady states remain unchanged (as we have seen below Proposition 5) as depicted in Figure 12. These facts together with (36) (see Appendix K in more details) show the following.

Proposition 7.

(i) A fall in $\bar{b}$ or a rise in ϕ shifts the threshold curve leftward, indicating that these policy changes make fiscal policy more sustainable.

(ii) A fall in $\bar{b}$ increases capital stock per capita in the steady state S: k_S^*.

These effects of $\bar{b}$ and ϕ under tax-based consolidation are similar to those under expenditure-based consolidation (Section 3.2).

[Figures 11 and 12]

A lower $\bar{b}$ causes a larger gap between the current and the target debt-to-GDP ratio by (12) and then a larger amount of debt reduction accompanies it in the early stage of the transition. An increase in ϕ also causes a larger decline in public debt in the early stage of the transition by (12). To achieve a larger amount of debt reduction, income tax rate τ_t is increased in contrast to the case of the expenditure-based consolidation. Large burdens of tax hamper capital accumulation in the early stages of consolidations.

In the long run, reduction in debt extends the fiscal space through the following two channels. First, the interest payment of public debt decreases. Second, crowding in of capital enhances tax revenues. However, the latter effects are weakened by increases in distortionary tax rates under the tax-based consolidation. Therefore, readers may imagine that fiscal policy is more likely to be sustainable under expenditure-based consolidation than tax-based consolidation. The next section numerically investigates the transitional dynamics of tax-based consolidation in the five countries and identifies which plan is preferable from the viewpoint of fiscal sustainability.

4.3 Numerical studies under tax-based consolidations

In this section, we calibrate the model in the case of tax-based consolidation to the data of the five countries in Section 3.3.

4.3.1 Parameter choices and scenarios

Benchmark parameters and variables follow the ones in Section 3.3. λ is set to satisfy $g_{init} = \lambda A k_0^{\alpha}$, where $g_{init} = (1+n)b_0 + \tau^w_{init} w(k_0) + \tau^R_{init} R(k_0)k_0 + \tau^c y(k_0) - (1-\tau^R_{init})R(k_0)b_0 - (1+\tau^c)\Phi(k_0, b_0, \tau^w_{init})$ and both τ^w_{init} and τ^R_{init} take the values in Table 1. Then, we can calibrate the value of λ in each country as $\lambda^{JPA} = 0.3619$, $\lambda^{US} = 0.2519$ $\lambda^{GRE} = 0.3162$, $\lambda^{ITA} = 0.4270$, and $\lambda^{PRT} = 0.3528$. We consider the following scenario. Governments implement tax base consolidations at period 0 unexpectedly before decision-making of the young in period 0. Then, τ_0 and g_0 follows (34) and $g_0 = \lambda A k_0^{\alpha}$, respectively. Consumptions in period 0 are given by $d_0 = \frac{(1-\delta\tau_0)R(k_0)(k_0+b_0)}{1+\tau^c}$ (consumption of the old) and $c_0 = \frac{(1-\tau_0)w(k_0)}{(1+\beta)(1+\tau^c)}$ (consumption of the young). For τ_t ($t \geq 0$) given by (34), we have $g_t = \lambda A k_t^{\alpha}$, $d_t = \frac{(1-\delta\tau_t)R(k_t)(k_t+b_t)}{1+\tau^c}$ $c_t = \frac{(1-\tau_t)w(k_t)}{(1+\beta)(1+\tau^c)}$ with $\{k_t, b_t\}_{t=0}^{\infty}$ following the dynamic equations (12) and (35).

4.3.2 Results

Figures 13, 14, 15, 16, and 17 illustrate the transitional dynamics for Japan, the US, Greece, Italy, and Portugal, respectively. Table 4 focuses on the unsustainable paces of tax-based consolidation and Table 5 represents the values of steady-state variables. These figures and tables show the following results.

[Figures 13, 14, 15, 16, and 17] [Tables 4 and 5]

Sustainable transition paths

First, we confirm that the properties of transitional dynamics of (k_t, b_t) in the five countries are in line with those in Section 4 (Figures 9-(b) and 10 are the cases for Japan and the US while Figures 9-(a) for Greece, Italy, and Portugal). Income tax rates, $\tau^w_t(=\tau_t)$ and $\tau^R(=\delta\tau_t)$, increase just after the implementation of tax-based consolidation. When the pace of consolidation in Japan, Greece, Italy, and Portugal is high (even when low in the US), the income tax rates turn to decrease in the short run and keep decreasing into the steady state values. By contrast, under a slow pace of consolidation, the income tax rates in these four countries keep increasing in the short and medium run and turn to decrease into the steady state values. The steady-state values of income tax rates (both $\tau^*(=\tau^{w*})$ and $\delta\tau^*(=\tau^{R*})$) are lower than the initial levels: $\tau^w_{init}(=\tau^w)$ and $\tau^R_{init}(=\tau^R)$ in all five countries (see Tables 3 and 5). This is because fiscal space created by fiscal consolidation allow the long-run income tax rates to be lower than the initial levels to keep the ratio of government spending to GDP a constant value, λ.

In contrast to the expenditure-based consolidation, tax-based consolidation decreases c_t and d_t just after the implementation of fiscal consolidations with g_t unchanged. Faster (resp. slower) paced consolidations crowd in resources to the private and public sectors, c_t, d_t, and g_t strongly (resp. weakly) in the medium run. However, faster (resp. slower) paced consolidation decreases d_t strongly (resp. weakly) in the short run because cuts in debt reduce asset income. The steady-state levels of capital, k^*, government spending, g^*, and consumption by the young c^* exceed the current/initial ones, k_0, g_0, c_0. in Japan and the US but fall behind in Greece, Italy, and

Portugal. The steady-state consumption by the old d^* becomes lower than the initial one d_0 in Japan, Greece, Italy, and Portugal, because assets from bonds decrease by fiscal consolidation.

Notice here the following results on consumptions c^* and d^* and public spending g^* in the long run. c^* and d^* (resources in the private sector) are larger under tax-based consolidation than under expenditure-based consolidation while public spending g^* is lower under tax-based consolidation than under expenditure-based consolidation. The reason for these results is as follows. Fiscal space created by tax-based fiscal consolidation allows the long-run income tax rates to decrease. Decreases in the long-run tax rates under tax-based fiscal consolidation strengthen (resp. weaken) the crowding in of resources into the private sector, c^* and d^* (resp. into the public sector g^*). Owing to high productivity in the US, d^* only in the US can exceed d_0.

Unsustainable transition paths

Next, we examine the cases in which fiscal policies are unsustainable. Fiscal policy is unsustainable without decreasing outstanding debt: $\phi = 0$ in Japan, Greece, Italy, and Portugal but sustainable in the US, which is similar to the results under expenditure-based consolidation. Consolidation plans with a very slow pace, $\phi = 0.1$ in Japan, $\phi = 0.1, 0.3, 0.5$ in Greece and Italy, $\phi = 0.1, 0.3$ in Portugal, cannot sustain fiscal policy. These results are similar to those in the expenditure-based consolidation.

In unsustainable transition paths, increases in tax rates reduce disposable income, consumption, and capital accumulation, and then decrease public spending g_t. Unlike the expenditure-based consolidation, consumption by the old (d_t) decreases, since the tax rate of asset income rises as debt increases monotonically.

Sustainable (or unsustainable) pace of between expenditure- and tax-based consolidation

Finally, we examine under which consolidation plan the fiscal policy is more likely to be sustainable between expenditure- and tax-based consolidations. We compare the minimal pace of tax-based consolidation with that of expenditure- based consolidation in the four countries whose fiscal policy can be unsustainable under the low paces of consolidation plans. Tables 6 and 7 show that the minimal pace of tax-based consolidation that ensures fiscal sustainability is higher than that of expenditure-based consolidation for all the four countries, indicating that expenditure-based consolidation is more likely to make fiscal policy sustainable.

[Tables 6 and 7]

This finding is attributable to the following reasons. As Proposition 7 shows, the tax-based plan requires a steep hike in income tax rate τ_t to achieve a larger amount of debt reduction, which deters capital accumulation in the early stages of fiscal consolidation. Additionally, a large distortionary income tax can afford to release less resources that enlarge fiscal capacity under tax-based consolidation.

5 Welfare of each generation and social welfare

In this section, we examine the welfare effects of both expenditure- and tax-based consolidation. Let us begin with the welfare of each generation. The welfare of the initial old (generation -1) is $U^{old}_{init} \equiv \ln d_0 + \theta \ln g_0$ and that of generation $t(\geq 0)$ is given by (1). We set $\theta = 0.8$ as a benchmark in the sense that utility from public goods and services is relatively high. From, Figures 18, 19, 20, 21, and 22, the following two common facts are observed among the five countries.

First, both under expenditure- and tax-based consolidation, slower paces of consolidations keep welfare losses of early generations (generation -1 and generation 0) smaller while they cause larger welfare loss of later generations. The reason for this result is as follows. Slower paces of consolidation induce large outstanding debts to persist for longer periods, which not only crowds out capital accumulation strongly but also causes later generations to suffer from lower government services by a large interest repayment. A rapid pace of consolidation avoids such a situation and makes later generations better off. Furthermore, welfare loss of the initial old (generation -1) is small even with a rapid pace of consolidation, because they can obtain asset income from a large initial public debt. For these reasons, we can predict that the social welfare under more rapid paces of consolidations is larger, as shown later in this section.

Second, comparing welfare under expenditure-based consolidation, U_t^{exp} with that under tax-based consolidation, U_t^{tax}, we find the following results for the cases of rapid paces of consolidations (e.g., $\phi = 1, 0.9$). $U_{-1}^{exp} < U_{-1}^{tax}$ holds for generation -1, but the relationship turns to the opposite $U_1^{exp} > U_1^{tax}$ drastically for the welfare of generation 1. In the long run, $U_t^{exp} - U_t^{tax}$ turns to a decrease.

The reason for this result is as follows. Rapid fiscal consolidations decrease a large amount of debt in the short run. Under a rapid expenditure-based consolidation, the cut in g_0 is strong while c_0 and d_0 are not affected (Section 3.3). By contrast, under a rapid tax-based consolidation, both c_0 and d_0 decreases while g_0 is not affected (see Section 4.3). Therefore, $c_0^{exp} - c_0^{tax} > 0$, and $d_0^{exp} - d_0^{tax} > 0$, and $g_0^{exp} - g_0^{tax} < 0$ hold initially, where the difference in c_t, g_t, and d_t between the two types of consolidations for $\phi = 1$ denote $c_t^{exp} - c_t^{tax}$, $g_t^{exp} - g_t^{tax}$, and $d_t^{exp} - d_t^{tax}$, respectively. A large cut in g_0 under expenditure-based consolidation can induce $U_{-1}^{exp} < U_{-1}^{tax}$. However, resources released to the private sector in the early stages of expenditure-based consolidation bring a large (resp. small) amount of government spending with constant tax rates in Japan and the US (resp. in Greece, Italy, and Portugal) while a large tax burden in the early stage of tax-based consolidations does not. This induces a large gap in g_1 in period 1 ($g_1^{exp} - g_1^{tax}$) and results in a large difference in the welfare of generation 1 between the two types of consolidation ($U_1^{exp} > U_1^{tax}$). As income tax rates decrease in the later stage of tax-based consolidation, the gap in g_t decreases and the signs of $c_t^{exp} - c_t^{tax}$ and $d_t^{exp} - d_t^{tax}$ turn negative. Thus, $U_t^{exp} - U_t^{tax}$ turns to decrease in the long run.

[Figures 18, 19, 20, 21, and 22]

Next, we evaluate the effect of fiscal consolidation by social welfare. Social welfare is defined as $W \equiv U_{init}^{old} + \sum_{t=0}^{\infty} \lambda^t U_t$, where $\lambda \in (0, 1)$ is a social discount factor. We set $\lambda = 0.7$. Tables 8 and 9 show the results for $\theta = 0.8$ (high utility from public goods and services) and $\theta = 0.2$ (low utility from public goods and services), respectively.

Fiscal consolidations improve social welfare in all countries in the following two senses. Fiscal consolidations in Japan, Greece, Italy, and Portugal turn unsustainable transition paths and welfare levels into sustainable ones. Meanwhile, fiscal consolidations in the US cause larger welfare gains, compared with the case of no consolidation ($\phi = 0$). When $\theta = 0.8$ (Table 8), tax-based consolidation should be chosen in Japan, Greece, and Portugal while expenditure-based consolidation should be chosen in the US and Italy. The optimal pace of consolidation is $\phi = 1$ in all countries. The former result changes in the US, Greece, and Portugal when $\theta = 0.2$ (Table 9). Tax-based consolidation is better in the US while expenditure-based consolidation is better in Greece and Portugal. The latter result, in which the optimal paces is given by $\phi = 1$, is robust even when $\theta = 0.2$, and is in line with the prediction seen earlier in this section.

The difference in social welfare between the two types of consolidation depends on the following difference in each generation's utility level, as we have seen above. First, $U_{-1}^{exp} < U_{-1}^{tax}$, just after the fiscal consolidation. Second, $U_1^{exp} > U_1^{tax}$ for generation 1. Finally, $U_t^{exp} - U_t^{tax}$ turns to a decrease in the long run. The total effects can differ among the five countries, for the following reasons.

In a high productivity economy, like the US, public services increase more under expenditure-based consolidation while consumption of private goods increase more under tax-based consolidation. This is because decreases in long-run income tax rates have a negative (resp. positive) effect on public spending (resp. consumptions of private goods). When utility from public services is high (resp. low), the former (resp. latter) effect is more important and therefore, expenditure-based (resp. tax-based) consolidation is better. In Japan, even with somewhat high productivity of the private sector, the initial public debt is very large and so is the initial cut in public services under expenditure-based consolidation. Thus, expenditure-based consolidation cannot be chosen even when utility from public services is low. In Greece, Italy, and Portugal, because the productivity of the private sector is low and the tax burden on wages is relatively high, capital stock decreases during the consolidation process. In Italy, the initial size of government is relatively large owing to low productivity in the private sector and very high tax burden, indicating that keeping a large government would be a better choice. Constant high tax rates under expenditure-based consolidation release more resources into the public sector than under the tax-based consolidation. Therefore expenditure-based consolidation is better in Italy. In Greece and Portugal, the size of the public sector is smaller than that in Italy, and a large initial cut in government spending under expenditure-based consolidation is harmful when $\theta = 0.8$ but not so harmful when $\theta = 0.2$.

[Tables 8 and 9]

However, social welfare is somewhat problematic if fiscal consolidation causes large welfare inequality between generations, and it obscures this inequality. Then, we need to pay attention to the fairness of welfare distribution between generations for the evaluation of the fiscal consolidation strategy. To gauge the fairness of welfare distribution, we calculate the Gini coefficient of each generation's welfare.[26]

Both Tables 10 ($\theta = 0.8$) and 11 ($\theta = 0.2$) show that tax-based consolidation should be chosen in Greece, Italy, and Portugal while expenditure-based consolidation should be chosen in Japan and the US from the viewpoint of fairness of welfare distribution between generations. The fairest pace of consolidation is $\phi = 1$ in all countries.[27] Between fairness of welfare and social welfare, the choice of consolidation type (expenditure-based or tax-based consolidation) is different (resp. same) in Japan and Italy (resp. in the US, Greece, and Portugal) while the pace of consolidation is the same in all countries ($\phi = 1$).

[Tables 10 and 11]

[26] We have taken account of from generation -1 to 19 in a practical calculation.

[27] No consolidation ($\phi = 0$) leads to the fairest outcome in the US, and therefore, no consolidation should be chosen in the US from the viewpoint of fairness of welfare distribution. However, we focus on the cases and effects of fiscal consolidations here.

Recall that a slower consolidation keeps welfare losses of early generations small while causing large welfare loss of later generations. Then, we find that a more rapid pace of fiscal consolidation induces fairer welfare distribution between generations and that the fairest pace is $\phi = 1$. Next, why is expenditure-based consolidation fairer in the US and Japan, while tax-based consolidation is fairer in Greece, Italy, and Portugal? To consider the reasons, we note that both types of consolidations generate differences in welfare distribution during these consolidations but no differences after the consolidations end (in the steady state). Thus, the important point is which consolidation ends faster, or equivalently, which steady state between the two types of consolidations is closer to the initial state of the economy. In the US, and Japan, $k_0 < k^*$, and then, a smaller steady state value is closer to the initial state. Because k^* under expenditure-based consolidation is smaller than that under tax-based consolidation in the US and Japan, expenditure-based consolidation ends faster and is chosen from the viewpoint of fairness of welfare distribution. By contrast, in Greece, Italy, and Portugal, $k_0 > k^*$, and a larger steady state value is closer to the initial state. Because k^* under tax-based consolidation is larger than that under expenditure-based consolidation, tax-based consolidation is chosen.

In summary, the choices of consolidation type between tax- or expenditure-based may differ among countries and depend on the initial outstanding debts and capital, productivity of the economy, tax rate levels, and the extent of the utility derived by individuals from public goods and services. Furthermore, choices of consolidation type in each country can differ depending on whether policymakers emphasize social welfare or fairness of welfare distribution between generations. By contrast, a common result from the viewpoints of both social welfare and fairness of welfare distribution is that a very slow pace of fiscal consolidation cannot be supported.

6 Welfare analyses with more general utility function and the target-based tax instruments

In the previous sections, we use a log linear utility function and consider the tax-based consolidation under which capital income tax rate is proportional to wage income tax rate and consumption tax rate is constant. The tractability of analyses from these assumptions yield clear intuitions behind how fiscal consolidations affect fiscal sustainability and welfare. Here, we use a more general utility function, introduce increases in consumption tax and the consolidation plan based on both expenditure cuts and tax increases, and consider some case studies on welfare under sustainable paces of consolidation plans. We begin to change utility function into the CRRA form as follows:

$$\tilde{U}_t = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \theta \frac{(S_t^g)^{1-\sigma} - 1}{1-\sigma} + \beta \left[\frac{d_{t+1}^{1-\sigma} - 1}{1-\sigma} + \theta \frac{(S_{t+1}^g)^{1-\sigma} - 1}{1-\sigma} \right], \tag{42}$$

where $\tilde{U}_t = U_t$ when $\sigma = 1$ (see (1)). Maximization of (42) subject to the same budget constraint $(1+\tau_t^c)c_t + s_t = (1-\tau_t^w)w_t$ and $(1+\tau_{t+1}^c)d_{t+1} = (1-\tau_t^R)R_{t+1}s_t$ as before (see below (1)), together with the asset market-clearing condition $(1+n)(b_{t+1}+k_{t+1}) = s_t L_t$ and debt policy rule, (4), yields

$$k_{t+1} = \frac{[(1-\tau_t^w)/(1+n)]w(k_t)}{1 + \beta^{-\frac{1}{\sigma}} \left\{ \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) (1-\tau_{t+1}^R) R(k_{t+1}) \right\}^{1-\frac{1}{\sigma}}} - \left[b_t - \phi \left(b_t - \bar{b} y(k_t) \right) \right]. \tag{43}$$

6.1 Dynamic systems

Expenditure-based consolidation
Applying $\tau_t^w = \tau_{init}^w = \tau^w$, $\tau_t^R = \tau_{init}^R = \tau^R$, and $\tau_t^c = \tau_{init}^c = \tau^c$, $\forall t$, we obtain

$$\begin{aligned} k_{t+1} &= \frac{[(1-\tau^w)/(1+n)]w(k_t)}{1+\beta^{-\frac{1}{\sigma}}\left\{(1-\tau^R)R(k_{t+1})\right\}^{1-\frac{1}{\sigma}}} - \left[b_t - \phi\left(b_t - \bar{b}y(k_t)\right)\right] \\ \Leftrightarrow k_{t+1} &= \varphi_1(k_t, b_t). \end{aligned} \tag{44}$$

(44) and (4) characterize the dynamics of the economy under expenditure-based consolidation. Replacing $\Phi\left(k_t, b_t, \tau_t^w\right)$ in (8) into $\varphi_1(k_t, b_t)$ and applying $\tau_t^w = \tau_{init}^w = \tau^w$, $\tau_t^R = \tau_{init}^R = \tau^R$, and $\tau_t^c = \tau_{init}^c = \tau^c$, $\forall t$, the government constraint is

$$\begin{aligned} (1+\tau^c)g_t =& (1+n)\left[b_t - \phi\left(b_t - \bar{b}y(k_t)\right)\right] + \tau^w w(k_t) + \tau^R R(k_t)k_t + \tau^c y(k_t) \\ &- (1-\tau^R)R(k_t)b_t - (1+n)\tau^c\varphi_1\left(k_t, b_t\right), \end{aligned} \tag{45}$$

where g_t adjusts to satisfy (45).

Tax-based consolidation
Next, we consider tax-based consolidation. Forward-looking policy variables τ_{t+1}^c and τ_{t+1}^R in (43) causes complicated adjustments of tax rates and it may be difficult to practically implement the tax-based consolidation. To resolve this problem, we introduce the following target-based adjustment of tax rates for τ_t^c and τ_t^R when we consider tax-based consolidation.

$$\tau_t^R = \tau_{init}^R\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_R} \quad (\text{where } \tau_{init}^R = \tau^R) \tag{46}$$

$$\tau_t^c = \tau_{init}^c\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_c} \quad (\text{where } \tau_{init}^c = \tau^c) \tag{47}$$

Both capital income and consumption tax rates increase from their initial levels (τ^R and τ^c) according to the distance between the current ($b_t/y(k_t)$) and the targeted debt-to-GDP ratio ($\bar{b}$) and eventually return to the initial levels ($\tau_t^R = \tau^R$ and $\tau_t^R = \tau^R$) when the consolidations end ($b_t/y(k_t) = \bar{b}$). ρ^R $(= \frac{b_t/\bar{b}y(k_t)}{\tau_t^R}\frac{d\tau_t^R}{d(b_t/\bar{b}y(k_t))})$ and ρ^c $(= \frac{b_t/\bar{b}y(k_t)}{\tau_t^c}\frac{d\tau_t^c}{d(b_t/\bar{b}y(k_t))})$ represent the elasticity of the increase in tax on capital income and consumption in response to the distance between the current and the targeted debt-to-GDP ratio. In contrast to τ_t^R and τ_t^c, wage income tax τ_t^w is adjusted to satisfy $g_t = \lambda y(k_t)$ and the government budget constraint as in the previous tax-based consolidation.

Substituting (46) and (47) into (43) and using (4), we obtain

$$\begin{aligned} k_{t+1} =& \frac{[(1-\tau_t^w)/(1+n)]w(k_t)}{1+\beta^{-\frac{1}{\sigma}}\left\{\left[\frac{1+\tau^c(b_t/(\bar{b}y(k_t))^{\rho_c}}{1+\tau^c([b_t-\phi(b_t-\bar{b}y(k_t))]/(\bar{b}y(k_{t+1})))^{\rho_c}}\right]\left[1-\tau^R\left(\frac{[b_t-\phi(b_t-\bar{b}y(k_t))]}{\bar{b}y(k_{t+1})}\right)^{\rho_R}\right]R(k_{t+1})\right\}^{1-\frac{1}{\sigma}}} \\ &- \left[b_t - \phi\left(b_t - \bar{b}y(k_t)\right)\right] \\ \Leftrightarrow k_{t+1} =& \varphi_2(k_t, b_t, \tau_t^w). \end{aligned} \tag{48}$$

Replacing $\Phi\left(k_t, b_t, \tau_t^w\right)$ in (8) into $\varphi_2(k_t, b_t)$ and inserting $g_t = \lambda y(k_t)$, the government constraint is

$$
\begin{aligned}
&\left[1+\tau^c\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_c}\right]\lambda y(k_t)-(1+n)\left[b_t-\phi\left(b_t-\bar{b}y(k_t)\right)\right]\\
&=\tau_t^w w(k_t)+\tau^R\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_R}R(k_t)k_t+\tau^c\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_c}y(k_t)\\
&\quad-\left[1-\tau^R\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_R}\right]R(k_t)b_t-(1+n)\tau^c\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_c}\varphi_2(k_t,b_t,\tau_t^w),
\end{aligned}
\tag{49}
$$

where τ_t^w is adjusted to satisfy (49) and is defined as $\tau_t^w \equiv \tau^w(k_t, b_t)$. Substituting $\tau^w(k_t, b_t)$ into (48), we obtain

$$
k_{t+1}=\varphi_2(k_t,b_t,\tau^w(k_t,b_t)). \tag{50}
$$

(50) and (4) characterize the dynamics of the economy under tax-based consolidation.

Expenditure- and tax-based consolidation

Finally we consider the consolidation plan based on both expenditure cuts and tax increases (expenditure- and tax-based consolidation, hereafter). We introduce the target-based adjustment of the wage income tax rate together with those of capital income tax ((46)) and consumption tax ((47)).

$$
\tau_t^w=\tau_{init}^w\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_w}\qquad(\text{where }\ \tau_{init}^w=\tau^w) \tag{51}
$$

Substituting (46), (47), and (51) into (43) and using (4), we obtain

$$
\begin{aligned}
k_{t+1}=&\frac{\left[1-\tau^w\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_w}\right]\frac{w(k_t)}{1+n}}{1+\beta^{-\frac{1}{\sigma}}\left\{\left[\frac{1+\tau^c(b_t/(\bar{b}y(k_t))^{\rho_c}}{1+\tau^c([b_t-\phi(b_t-\bar{b}y(k_t))]/(\bar{b}y(k_{t+1})))^{\rho_c}}\right]\left[1-\tau^R\left(\frac{[b_t-\phi(b_t-\bar{b}y(k_t))]}{\bar{b}y(k_{t+1})}\right)^{\rho_R}\right]R(k_{t+1})\right\}^{1-\frac{1}{\sigma}}}\\
&-\left[b_t-\phi\left(b_t-\bar{b}y(k_t)\right)\right]\\
\Leftrightarrow k_{t+1}=&\ \varphi_3(k_t,b_t)
\end{aligned}
\tag{52}
$$

Under the consolidation here, g_t is adjusted to satisfy the following government constraint in which changes in tax rates are based on (46), (47), and (51).

$$
\begin{aligned}
&\left[1+\tau^c\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_c}\right]g_t-(1+n)\left[b_t-\phi\left(b_t-\bar{b}y(k_t)\right)\right]\\
&=\tau^w\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_w}w(k_t)+\tau^R\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_R}R(k_t)k_t+\tau^c\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_c}y(k_t)\\
&\quad-\left[1-\tau^R\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_R}\right]R(k_t)b_t-(1+n)\tau^c\left(\frac{b_t}{\bar{b}y(k_t)}\right)^{\rho_c}\varphi_3(k_t,b_t),
\end{aligned}
\tag{53}
$$

(52) and (4) characterize the dynamics of the economy under consolidation based on expenditure cuts with tax increases. During this expenditure- and tax-based consolidation, cuts in public spending can be mitigated while the tax burden increases relative to expenditure-based consolidation. We evaluate these effects of expenditure- and tax-based consolidation numerically.

6.2 Calibration and results

We use $\sigma = 2$, a commonly used value in the literature (see e.g., Andersen and Bhattacharya, 2020), and baseline parameters in Table 1. As for the choices of (ρ^w, ρ^R, ρ^c), we set $\rho^R = 0.18$ to realize a similar magnitude of changes in τ_t^R to those in some rapid tax-based consolidations before ($\phi = 0.7, 0.9, 1$) while the choices of $\rho^w = 0.08$ and $\rho^c = 0.05$ ensure moderate changes in τ_t^w and τ_t^c. The scenarios of consolidations are in line with those in the previous sections.

Transitional dynamics

Figures 23, 24, 25, 26, and 27, represent the transitional dynamics under the three types of consolidations ((a) expenditure-based consolidation, (b) tax-based consolidation, and (c) expenditure- and tax-based consolidation) in Japan, the US, Portugal, Greece, and Italy, respectively. Transitional dynamics both under expenditure- and tax-based consolidations are qualitatively similar to those in the previous sections except that τ_t^R under the latter cannot be below the initial level in the steady state.

In contrast to the case in which $\sigma = 1$, tax-based consolidations in Greece and Italy even for $\phi = 1$ cannot converge to the steady state and therefore, are unsustainable. The reason for this is as follows. Both τ_0^R and τ_0^c increases according to large distances between $b_0/y(k_0)$ and $\bar{b}$, and τ_t^R and τ_t^c begin to decrease as outstanding debt decreases. When $\sigma = 2(> 1)$, the income effect of the term $\left\{\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(1-\tau_{t+1}^R\right)R_{t+1}\right\}^{1-\frac{1}{\sigma}}$ (see (43)) dominates its substitution effect. The intertemporal decline in consumption tax and $\frac{1+\tau_t^c}{1+\tau_{t+1}^c}$ and capital income tax rate τ_{t+1}^R (i.e., an increase in $\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(1-\tau_{t+1}^R\right)R_{t+1}$) decrease saving and capital accumulation. Under low productivity A and high baseline wage income tax rate τ^w in Greece and Italy, these declines in capital accumulation are seriously deep, making fiscal policy unsustainable.

Next, we focus on expenditure- and tax-based consolidation. As expected, during this consolidation, cuts in public spending are mitigated while the tax burden increases relative to expenditure-based consolidation. Only in Italy is this type of consolidation not sustainable when $\phi = 0.7$ for the same reason above (low productivity high and wage income tax rate with negative intertemporal effect on capital accumulation, i.e., an increase in $\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(1-\tau_{t+1}^R\right)R_{t+1}$)

[Figures 23, 24, 25, 26, and 27]

Social welfare

Table 12 represents the results on social welfare. As for social welfare, the results for the choices of consolidation type between expenditure- and tax-based consolidation and the optimal pace of consolidation are robust to those in the previous section in Japan, the US, and Portugal for both $\theta = 0.8$ and $\theta = 0.2$. When $\theta = 0.8$, tax-based consolidation with $\phi = 1$ is chosen in Japan and Portugal while expenditure-based consolidation with $\phi = 1$ is chosen in the US. When $\theta = 0.2$, tax-based consolidation with $\phi = 1$ is chosen in Japan and the US while expenditure-based consolidation with $\phi = 1$ is chosen in Portugal. In Greece and Italy, there is no choice of tax-based consolidation, because it is unsustainable, leading to select expenditure-based consolidation with $\phi = 1$.

Even with the third option, namely, expenditure- and tax-based consolidation, these results are robust except for the case of the US for $\theta = 0.8$. Recall that high productivity in the US economy leads the wage income tax rate to fall much more under the tax-based consolidation, making government size too low when utility from public services is high. Expenditure- and tax-based consolidation can avoid this situation by tax increases during the consolidation process. Although expenditure- and tax-based consolidation in the US is dominated by tax-based consolidation, it dominates expenditure-based consolidation when $\theta = 0.8$. This is because mitigating a cut in public spending is more important than increases in the tax burden when utility from public services is high.

In Japan and Portugal, expenditure- and tax-based consolidation can dominate expenditure-based consolidation as in the US when $\theta = 0.8$ but is dominated by tax-based consolidation. Because very large initial outstanding debts in Japan and Portugal require a large cut in expenditure, expenditure-based consolidation is avoided when utility from public services is high. Tax-based consolidation is better than expenditure- and tax-based consolidation, because the former yields benefits from falls in wage income tax in the medium and long runs while the latter does not. When $\theta = 0.2$, expenditure- and tax-based consolidation is dominated by expenditure- based consolidation in all five countries, because the burden of tax increases is larger than the benefits from mitigating a cut in public spending. The same applies in Greece and Italy, even when $\theta = 0.8$, because the burden of wage income tax is already high.

[Table 12]

Fairness of welfare

Table 13 represents the results on the Gini coefficient of each generation's welfare.[28] In Japan, expenditure-based consolidation with $\phi = 1$ is chosen under two options, expenditure-based and tax-based consolidation. This is robust to the result in the previous section both when $\theta = 0.8$ and $\theta = 0.2$. Under the three options, expenditure- and tax-based consolidation is selected when $\theta = 0.8$ under $\phi = 1$. The reason is as follows. When $\theta = 0.8$, how evenly g_t is distributed is important for the fairness of welfare distribution. Expenditure- and tax-based consolidation can make a cut in g_0 very small and g_t increases gradually, while expenditure-based consolidation causes a sharp decline in g_0 and steep recovery of g_t. Thus, expenditure- and tax-based consolidation is chosen. By contrast, when $\theta = 0.2$, smooth transitions of c_t and d_t are important for the fairness of welfare distribution. Because the transitions of c_t and d_t under expenditure-based consolidation (without the distortionary effect of tax increases) are both smoother than under the expenditure- and tax-based consolidations, expenditure-based consolidation is chosen.

In the US, the result that expenditure-based consolidation with $\phi = 1$ is chosen when $\theta = 0.2$ is robust to the case in the previous section. However, the result that tax-based consolidation with $\theta = 1$ is chosen is different from that in the previous section. This is because, in contrast to the case of $\sigma = 1$, increases in g_t (at the early stages of tax-based consolidation) under $\sigma = 2$ are small and induce a very smooth transition of g_t. The reason is attributable to the intertemporal term of $\left\{\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(1-\tau_{t+1}^R\right)R_{t+1}\right\}^{1-\frac{1}{\sigma}}$ (see (43)). From (46) and (47), a steep reduction in b_t decreases τ_{t+1}^c and τ_{t+1}^R. This positive income effect discourages saving, erodes some capital accumulation (from the crowding-in effects of reduction in debt), and make increases in $g_t(=\lambda y(k_t))$ small.

In Portugal, the result that tax-based consolidation is chosen both when $\theta = 0.8$ and 0.2 is robust to the case in the previous section. However, the fairest pace of consolidation is not $\phi = 1$ but $\phi = 0.9$. This is also the case for both Greece and Italy. In these countries, capital stock decreases monotonically and increases R_{t+1} in the intertemporal term of (43). This positive income effect of R_{t+1} on both c_t and d_t weakens the negative effect of slightly slow paces of consolidation and makes the transition of c_t and d_t flat. Furthermore, the income effect from

[28] We take account of generation -1–19 in a practical calculation.

a rise in R_{t+1} decreases capital accumulation and government spending $g_t(=\lambda y(k_t))$. This negative effect on g_t is weakened by a positive effect on g_t through high tax burdens by slightly slow paces of consolidation, and makes the transition of g_t flat as well. For these reasons, $\phi = 0.9$ is selected in these countries. These reasons also provide a clear explanation for why expenditure- and tax-based consolidation is better than expenditure-based consolidation in Greece and Italy.

[Table 13]

7 Conclusion

This study investigates the effects of expenditure- and tax-based consolidations on fiscal sustainability and welfare by using an OLG model with exogenous growth settings. Under the debt policy rule of reductions in debts to the targeted debt-to-GDP ratio, we investigate global transition dynamics of the economy and obtain the following results.

First, a unique stable steady state exists both under the expenditure- and tax-based consolidations with the debt policy rule. Properties of global transition paths are derived analytically and represented in two two-dimensional phase diagrams under each of the two types of consolidation plans.

Second, there is a threshold of public debt for each level of capital in order for the government to sustain fiscal policy, and the threshold of public debt is increasing in the size of capital under each of the two types of consolidation plans. A higher pace or lower target of debt-to-GDP ratio makes fiscal policies more sustainable.

Third, the minimal pace of tax-based consolidation that ensures fiscal sustainability is higher than that ensured by expenditure-based consolidation, indicating that expenditure-based consolidation is more likely to make fiscal policy sustainable. Numerical investigations show that Japan, Greece, Italy, and Portugal cannot sustain fiscal policy either without reducing debt or with very low paces of reduction in debt. By contrast, the US economy may sustain its fiscal policy even without reducing debt.

Finally, social welfare improves in all countries (Japan, the US, Greece, Italy, and Portugal) by fiscal consolidation. Choices of consolidation type between tax-based or expenditure-based may differ among countries depending on the size of large outstanding debts relative to capital, productivity of the economy, tax rate levels, and the extent of the utility derived by individuals from public goods and services. More importantly, it may depend on whether policymakers emphasize social welfare or fairness of welfare distribution between generations. By contrast, a common result from the viewpoints of both social welfare and fairness of welfare is that rapid paces of fiscal consolidation are supported.

Notwithstanding the relevance of this study's findings, our analyses are subject to several limitations that present avenues for future research. First, although this study considers a closed economy, extending its framework to consider open economies with internationally mobile capital is vital. As shown by Chang (1990) and Azzimonti et al. (2014), the integrated financial market faces a lower interest rate elasticity with respect to government borrowing and allows the government to increase its debt. Internationally cooperative fiscal consolidation may be necessary to counteract this problem. The result indicating that a rapid pace of debt reduction is preferable may provide a basis for addressing this issue. Furthermore, questions still exist as to the differences in the choice of fiscal consolidation type (expenditure- or tax-based) among open economies with overlapping generations.

Second, this study ignores the role of productive government spending (e.g., public education and infrastructure) in enhancing productivity and growth. Although Maebayashi et al. (2017) and Futagami and Konishi (2018) investigate the role of productive government spending in the fiscal consolidation policy, they do not consider the aspect of intergenerational welfare distribution. By contrast, Anderson and Bhattacharya (2020) investigate the welfare effect of public debt incurred to finance public education expenditure (productive debt) in an OLG model and show that productive debt policies can lead to a Pareto improvement. However, as stated in Section 1, they focus only on the steady states pre- and post-policy changes or on transitions between the two. Furthermore, in their study, public debt is limited to productive debt. This study does not focus on the sustainability of actual large outstanding unproductive debt (attributable to social security and public health expenditure for older adults) or its decrease owing to the externality of productive spending. These aspects should be addressed in future research.

References

[1] Alesina, A., Ardagna S., 1998. Fiscal adjustments—Why they can be expansionary?, Economic Policy 13, 488–545.

[2] Alesina, A., Ardagna S., 2009. Large changes in fiscal policy: Taxes versus spending, NBER Working Paper, No. 15438.

[3] Alesina, A., Perotti R., 1995. Fiscal expansions and adjustments in OECD countries, Economic Policy 21, 205—248.

[4] Andersen, T.M., Bhattacharya D., 2020. Intergenerational debt dynamics without tears, Review of Economic Dynamics 35, 192–219.

[5] Arai, R., 2011 Productive government expenditure and fiscal sustainability. FinanzArchiv: Public Finance Analysis 67, 327–351.

[6] Auerbach, A., Kotlikoff L., 1987. Dynamic Fiscal Policy. Cambridge University Press, Cambridge.

[7] Azzimonti, M., de Francisco., E., Quadrini, V., 2014. Financial globalization, inequality, and the rising public debt. American Economic Review 104, 2267–2302.

[8] Bi, H.E., Leeper, M., Leith C., 2013. Uncertain fiscal consolidations, Economic Journal 123, F31–F63.

[9] Bohn, H., 1998. The behavior of U.S. public debt and deficits, Quarterly Journal of Economics 113, 949–963.

[10] Bräuninger, M., 2005. The budget deficit, public debt, and endogenous growth. Journal of Public Economic Theory 7, 827–840.

[11] Chalk, N.A., 2000. The sustainability of bond-financed deficits: An overlapping generations approach, Journal of Monetary Economics 45, 293–328.

[12] Chang, R., 1990. International coordination of fiscal deficits. Journal of Monetary Economics 25, 347–366

[13] Coenen, G., Mohr, M., Straub R., 2008. Fiscal consolidation in the Euro area: Long-run benefits and short-run costs, Economic Modeling 25, 912–932.

[14] Cogan, J.F., Taylor, J.B. Wieland, V., Wolters, M.H., 2013. Fiscal consolidation strategy, Journal of Economic Dynamics and Control 37, 404–421.

[15] de la Croix, D., Michel, P., 2002. A Theory of Economic Growth. Cambridge University Press, Cambridge.

[16] D'Erasmo, P., Mendoza, E.G., Zhang J., 2016. What is a sustainable public debt? In: Taylor, J., Uhlig, H. (Eds.), Handbook of Macroeconomics, vol. 2B, Elsevier, North-Holland, Chapter 32, pp. 2493—2597.

[17] Diamond, P.A., 1965. National debt in a neoclassical growth model, American Economic Review 55, 1126—1150.

[18] Erceg, C.J., Lindé, J., 2013. Fiscal consolidation in a currency union: Spending cuts vs. tax hikes. Journal of Economic Dynamics and Control 37, 422—445.

[19] Eyraud, L., Debrun, X., Hodge, A., Lledó, V., Pattillo, C., 2018. Second-generation fiscal rules: Balancing simplicity, flexibility, and enforceability, IMF Staff Discussion Notes 2018/004, International Monetary Fund.

[20] Ghosh, S., Mourmouras, I.A., 2004. Endogenous growth, welfare, and budgetary regimes. Journal of Macroeconomics 26, 623—635.

[21] Fatás, A., Mihov I., 2010. The Euro and fiscal policy NBER chapters, In: Europe and the Euro—, National Bureau of Economic Research, Inc., pp. 287—324.

[22] Forni, L., Gerali, A., Pisani, M., 2010. The macroeconomics of fiscal consolidations in Euro Area countries. Journal of Economic Dynamics and Control 34, 1791–1812.

[23] Futagami, K., Konishi, K., 2018. Dynamic analysis of budget policy rules in Japan. Journal of the Japanese and International Economies 50, 72–88.

[24] Greiner, A., 2007. An endogenous growth model with public capital and sustainable government debt, Japanese Economic Review 58, 345–361.

[25] Greiner, A., 2012. Public capital, sustainable debt, and endogenous growth. Research in Economics 66, 230–238.

[26] Greiner, A., Semmler W., 2000. Endogenous growth, government debt and budgetary regimes. Journal of Macroeconomics 22, 363–384.

[27] Guichard, S., Kennedy, M., Wurzel, E., André, C., 2007. What promotes fiscal consolidation: OECD country experiences, OECD Economics Department Working Paper, No. 553.

[28] Gunji, H., Miyazaki K., 2011. Estimates of average marginal tax rates on factor incomes in Japan. Journal of the Japanese and International Economies 25, 81–106.

[29] Hansen, G.D., İmrohoroğlu, S., 2016. Fiscal reform and government debt in Japan: A neoclassical perspective. Review of Economic Dynamics 21, 201–224.

[30] Kamiguchi, A., Tamai T., 2012. Are fiscal sustainability and stable balanced growth equilibrium simultaneously attainable?, Metroeconomica 63, 443–457.

[31] Maebayashi, N., T. Hori., Futagami, K., 2017. Dynamic analysis of reductions in public debt in an endogenous growth model with public capital, Macroeconomic Dynamics 21, 1454–1483.

[32] Maebayashi, N., Konishi K., 2021. Sustainability of public debt inequality in a general equilibrium model, Macroeconomic Dynamics 25, 874–895.

[33] Mankiw, N.G., Elmendorf D.W., 1999. Government debt. In Taylor, J.B., Woodford M. (Eds.). Handbook of Macroeconomics, vol. 1C, Elsevier, Amsterdam, pp. 1615–1663.

[34] Miyazawa, K., Ogawa, H., and Tamai, T., 2019. Capital market integration and fiscal sustainability. European Economic Review 120, 103305.

[35] Molnar, M., 2012 Fiscal consolidation: Part 5. What factors determine the success of consolidation efforts?, OECD Economics Department Working Papers No. 936 https://dx.doi.org/10.1787/5k9h28mzp57h-en.

[36] Morimoto, K., Hori, T., Maebayashi, N., Futagami K., 2017. Debt policy rules in an open economy, Journal of Public Economic Theory 19, 158–177.

[37] Nishiyama, S., Smetters K., 2007. Does social security privatization produce efficiency gains?, The Quarterly Journal of Economics 122, 1677–1719

[38] OECD, 2017. Government at a glance 2017, OECD Publishing, Paris http://dx.doi.org/10.1787/gov_glance-2017-en.

[39] OECD, 2019. Government at a glance 2019, OECD Publishing, Paris, https://doi.org/10.1787/8ccf5c38-en.

[40] OECD, 2021. General government debt (indicator). DOI: 10.1787/a0528cc2-en (Accessed on February 5, 2021)

[41] Oguro, K., Sato, M., 2014. Public debt accumulation and fiscal consolidation, Applied Economics 46, 663–673.

[42] Panico C., Purificato F., 2013. Policy coordination, conflicting national interests and the European debt crisis, Cambridge Journal of Economics 37, 585–608.

[43] Papageorgiou, D., 2012. Fiscal policy reforms in general equilibrium: The case of Greece. Journal of Macroeconomics 34, 504–522.

[44] Philippopoulos, A., Varthalitis, P., Vassilatos V., 2017. Fiscal consolidation and its cross-country effects, Journal of Economic Dynamics and Control 83, 55–106.

[45] Rawdanowicz, Ł., 2014. Choosing the pace of fiscal consolidation. OECD Journal: Economic Studies 2013/1. http://dx.doi.org/10.1787/eco_studies-2013-5k3tq96mbr44.

[46] Song, Z., Storesletten, K., Zilibotti F., 2012. Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt, Econometrica 80, 2785–2803.

[47] Teles, V.K., Mussolini, C.C., 2014. Public debt and the limits of fiscal policy to increase economic growth, European Economic Review 66, 1–15.

[48] Trabandt, M., Uhlig H., 2011. The Laffer curve revisited, Journal of Monetary Economics 58, 305–327.

[49] Tsibouris, G.C., Horton, M.A., Flanagan, M.J., Maliszewski, W.S., 2006. Experience with large fiscal adjustment, IMF Occasional Paper, No. 246.

[50] Uhlig, H., 2010. Some fiscal calculus, American Economic Review Papers & Proceedings 100, 30–34.

[51] Uhlig, H., Yanagawa, N., 1996. Increasing the capital income tax may lead to faster growth, European Economic Review 40, 1521–1540.

[52] Von Hagen, J., Hallett A.H., Strauch R.R., 2002. Budgetary consolidation in Europe: Quality, economic conditions and persistence, Journal of the Japanese and International Economies 16, 512–535.

[53] Yakita, A., 2008 Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting, Journal of Public Economics 92, 897–914.

Appendix

A Proof of Lemma 2

(i) From (14), $g_t = 0$ locus: $b_t = \Omega(k_t)$ satisfies $\Omega(0) = 0$ and has asymptote $\lim_{k_t \to \hat{k}} \Omega(k_t) = +\infty$, where $\hat{k}$ is defined by (15). Additionally, the first and second derivatives of $g_t = 0$ locus: $b_t = \Omega(k_t)$ are as follows.

$$\begin{aligned}
\Omega'(k_t) =& \frac{\left[\phi\bar{b}(1+\tau^c)(1+n) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right]}{\left[(1-\tau^R)\alpha q(k_t) - (1+n)(1+\tau^c)(1-\phi)\right]^2} \\
& \times \left[(1-\tau^R)q(k_t) - (1+n)(1+\tau^c)(1-\phi)\right] q(k_t) > 0 \quad \text{for} \quad k_t < \hat{k}, \qquad \text{(A.1)} \\
\Omega''(k_t) =& \frac{\left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right]\alpha(1-\alpha)(1+n)(1-\phi)Ak_t^{\alpha-2}}{[(1-\tau^R)\alpha q(k_t) - (1+n)(1-\phi)]^3} \\
& \times \left\{2(1-\alpha)q(k_t) + \left[(1-\tau^R)\alpha q(k_t) - (1+n)(1+\tau^c)(1-\phi)\right]\right\} > 0 \quad \text{for} \quad k_t < \hat{k}. \qquad \text{(A.2)}
\end{aligned}$$

These results prove (i).

(ii) In the intersection point between $b_{t+1} = b_t$ and $g_t = 0$ loci, $\bar{b}q(k_t) = \tilde{\Omega}(k_t)$ holds by (17) and (23). Therefore, we have

$$\bar{b}q(k_t) = \frac{q(k_t)\left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right]}{(1-\tau^R)\alpha q(k_t) - (1+n)(1+\tau^c)(1-\phi)}. \qquad \text{(A.3)}$$

From (A.3), we obtain

$$k_H = \left[\frac{\bar{b}(1-\tau^R)\alpha A}{\bar{b}(1+\tau^c)(1+n) + \tilde{\tau} + \tau^c(1-\eta(1+n))}\right]^{\frac{1}{1-\alpha}}.$$

Inserting the value of k_H into (20) yields

$$b_H = \left(\bar{b}A\right)^{\frac{1}{1-\alpha}} \left[\frac{(1-\tau^R)\alpha}{\bar{b}(1+\tau^c)(1+n) + \tilde{\tau} + \tau^c(1-\eta(1+n))}\right]^{\frac{\alpha}{1-\alpha}}.$$

(iii) The result is evident from Lemma 1-(ii) and Lemma 2-(i).

B Derivations of (26), (27), (28), and (30)

In the intersection point between $k_{t+1} = k_t$ locus and $g_t = 0$ locus under $\phi < 1$, $\tilde{Z}(q(k_t)) = \tilde{\Omega}(q(k_t))$ holds. Therefore, we have (26).

Next, we move onto the value of x_P. Because $x_t = \tilde{\Omega}(q(k_t))$ and $x_t = \left[(\eta - \phi\bar{b})q(k_t) - 1\right]/(1-\phi)$ are written as

$$q(k_t) = \frac{(1+n)(1+\tau^c)(1-\phi)x_t}{(1-\tau^R)\alpha x_t - \phi\bar{b}(1+n)(1+\tau^c) - \tilde{\tau} - \tau^c(1-(1+n)\eta)} \qquad \text{(B.1)}$$

and

$$q(k_t) = \frac{(1-\phi)x_t + 1}{\eta - \phi\bar{b}}, \tag{B.2}$$

respectively, we have (27) in $P(q(k_P), x_P)$. Here, we define

$$p_L(x) \equiv (1-\phi)x\left[(1-\tau_R)\alpha x - \tilde{\tau} - \tau^c - \eta(1+n)\right], \tag{B.3}$$

$$p_R(x) \equiv \phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta) - (1-\tau^R)\alpha x. \tag{B.4}$$

$p_L(x)$ is an downward-sloping line that satisfies $p_L(\bar{x}_P) = 0$, where $\bar{x}_P = \frac{\tilde{\tau}+\tau^c+\eta(1+n)}{(1-\tau^R)\alpha}$ while $p_R(x)$ is a quadratic function of x that satisfies $p_R(x) > (=)0$ for $x > (=)\underline{x}_P$, where $\underline{x}_P = \frac{\phi\bar{b}(1+n)(1+\tau^c)+\tilde{\tau}+\tau^c(1-(1+n)\eta)}{(1-\tau^R)\alpha}$ and $p'_R(x) > 0$ and $p''_R(x) > 0$ for $x \geq \underline{x}_P$. Therefore, the value of x_P is represented by the intersection between $p_L(x)$ and $p_R(x)$ as represented in Figure 28. Thus, we have (28): $\underline{x}_P < x_P < \bar{x}_P$ and

$$p'_R(x_P) > p'_L(x_P). \tag{B.5}$$

[Figure 28]

Finally, we derive (30). To do this, let us rewrite (27), using the definition of $x_P \equiv b_P/k_P$ into

$$\begin{aligned}&(1-\phi)(1-\tau^R)\alpha b_P^2 + \left\{(1-\tau^R)\alpha - (1-\phi)\left[\tilde{\tau} + \tau^c + \eta(1+n)\right]\right\} k_P b_P \\ &= \left[\phi\bar{b}(1+n)(1+\tau^c) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right] k_P^2.\end{aligned} \tag{B.6}$$

Taking the total differentials of (B.6) yields (30).

C Derivation of Condition 2

We rearrange (26) into

$$\begin{aligned}&(1-\phi)^{-1}\left[(\eta-\phi\bar{b})q(k_P) - 1\right]\underbrace{\left[(1-\tau^R)\alpha q(k_P) - (1+n)(1+\tau^c)(1-\phi)\right]}_{>0 \text{ from Condition 1}} \\ &= \left[\phi\bar{b}(1+\tau^c)(1+n) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right] q(k_P),\end{aligned} \tag{C.1}$$

Let us define

$$P_L(q(k)) \equiv (1-\phi)^{-1}\left[(\eta-\phi\bar{b})q(k) - 1\right]\left[(1-\tau^R)\alpha q(k) - (1+n)(1+\tau^c)(1-\phi)\right] \tag{C.2}$$

$$P_R(q(k)) \equiv \left[\phi\bar{b}(1+\tau^c)(1+n) + \tilde{\tau} + \tau^c(1-(1+n)\eta)\right] q(k). \tag{C.3}$$

$P_L(q(k))$ satisfies $P_L(q(\hat{k})) = P^{LHS}(q(\tilde{k})) = 0$ and is strictly increasing in $q(k)$ for $q(k) \geq q(\tilde{k})$ ($k_t < \tilde{k}$), while $P_R(q(k))$ is upward-sloping line that satisfies $\lim_{k_t \to +\infty} P_R(q(k)) = P_R(0) = 0$. $q(k_P)$ in (C.1) is given by the intersection point between $P_L(q(k))$ and $P_R(q(k))$ as represented in Figure 28.

$k^* > k_P$ if and only if $P_R(q(k^*)) - P_L(q(k^*)) > 0$ from (C.1), (C.2), and (C.3), where $q(k^*) = [\eta - \bar{b}]^{-1}$, $P_R(q(k^*)) = \frac{\phi\bar{b}(1+n)(1+\tau^c)+\tilde{\tau}+\tau^c(1-(1+n)\eta)}{\eta-\bar{b}}$, and $P_L(q(k^*)) = \frac{\bar{b}[(1-\tau^R)\alpha-(1+\tau^c)(1+n)(1-\phi)(\eta-\bar{b})]}{(\eta-\bar{b})^2}$.

Therefore, we have

$$\begin{aligned}
&P_R(q(k^*)) - P_L(q(k^*)) > 0 \\
&\Leftrightarrow \frac{(\eta-\bar{b})[\tilde{\tau}+\tau^c(1-(1+n)\eta)+\bar{b}(1+\tau^c)(1+n)] - \bar{b}(1-\tau^R)\alpha}{(\eta-\bar{b})^2} > 0 \\
&\Leftrightarrow (\eta-\bar{b})[\tilde{\tau}+\tau^c(1-(1+n)\eta)+\bar{b}(1+\tau^c)(1+n)] > \bar{b}(1-\tau^R)\alpha. \qquad \text{(C.4)}
\end{aligned}$$

Solving this inequality (C.4) with respect to $\bar{b}$ yields

$$\begin{aligned}
&\bar{b} < \bar{b}_1 \equiv \frac{\zeta_1 + \sqrt{\zeta_1^2 + 4(1+n)(1+\tau^c)\zeta_2}}{2(1+n)(1+\tau^c)}, \\
&\zeta_1 \equiv \tilde{\tau} + \tau^c(1-(1+n)\eta) + (1-\tau^R)\alpha - (1+\tau^c)(1+n)\eta, \\
&\zeta_2 \equiv \eta[\tilde{\tau}+\tau^c(1-(1+n)\eta)](>0). \qquad \text{(C.5)}
\end{aligned}$$

Dividing (C.4) by $\eta - \bar{b}$ (>0) rewrite (C.4) into

$$\tilde{\tau}+\tau^c(1-(1+n)\eta)+\bar{b}(1+\tau^c)(1+n) > \frac{\bar{b}(1-\tau^R)\alpha}{\eta-\bar{b}}. \qquad \text{(C.6)}$$

The LHS of (C.6) is an upward-sloping line with respect to $\bar{b}$, which takes $\tilde{\tau}$ at $\bar{b}=0$. By contrast, the RHS of (C.6) is a strictly increasing and convex function of $\bar{b}$ that takes the value zero at $\bar{b}=0$ and has asymptote $+\infty$ when $\bar{b}\to\eta$. Therefore, $\bar{b}_1 \in (0,\eta)$. Furthermore, because the LHS of (C.6) is increasing in τ^R and τ^c while the RHS is decreasing (resp. neutral) in (resp. to) τ^R (resp. τ^c), thus $\bar{b}_1$ is increasing in both τ^R and τ^c. From (C.5) and $\eta \equiv \frac{\beta(1-\alpha)(1-\tau^w)}{(1+\beta)(1+n)}$, we obtain $\frac{d\zeta_1}{d\tau^w} = (1-\alpha)\left[1+\frac{\tau^c\beta}{1+\beta}+\frac{(1+\tau^c)\beta}{1+\beta}\right] > 0$ and $\frac{d\zeta_2}{d\tau^w} = \frac{\beta(1-\alpha)}{1+\beta}\left[1-\alpha-\tilde{\tau}+\tau^c\left[\frac{\beta(1-\alpha)}{1+\beta}(2-\tau^w)-1\right]\right] > 0$ if $1-\alpha-\tilde{\tau}+\tau^c\left[\frac{\beta(1-\alpha)}{1+\beta}(2-\tau^w)-1\right] > 0$. Thus, $\frac{d\bar{b}_1}{d\tau^w} > 0$ if $1-\alpha-\tilde{\tau}+\tau^c\left[\frac{\beta(1-\alpha)}{1+\beta}(2-\tau^w)-1\right] > 0$.

D Proof of Lemma 3

From (21), we obtain

$$\left.\frac{\partial Z(k_t)}{\partial \phi}\right|_{b_t=Z(k_t)} = \frac{\left[\left(\eta-\bar{b}\right)Ak_t^{\alpha-1}-1\right]k_t}{(1-\phi)^2} = \frac{\left[\left(\eta-\bar{b}\right)q(k_t)-1\right]k_t}{(1-\phi)^2} \gtreqless 0 \quad \text{for} \quad k_t \lesseqgtr k^*. \qquad \text{(D.1)}$$

From (14), we obtain

$$\begin{aligned}
&\left.\frac{\partial \Omega(k_t)}{\partial \phi}\right|_{b_t=\Omega(k_t)} \\
&= \frac{(1+\tau^c)(1+n)Ak_t^{\alpha}\left[\bar{b}(1-\tau^R)\alpha q(k_t) - \bar{b}(1+\tau^c)(1+n) - \tilde{\tau} - \tau^c(1-(1+n)\eta)\right]}{\left[(1-\tau^R)\alpha q(k_t) - (1+\tau^c)(1+n)(1-\phi)\right]^2}. \qquad \text{(D.2)}
\end{aligned}$$

(D.2) together with (25) yields

$$\left.\frac{\partial\Omega(k_t)}{\partial\phi}\right|_{b_t=\Omega(k_t)} \gtreqless 0 \qquad \text{for} \quad k_t \lesseqgtr k_H. \tag{D.3}$$

E Effect of changes in $\bar{b}$ on fiscal sustainability (numerical analyses)

Under the baseline parameter values with current (initial) values of k_0 and b_0 of Japan (JPA), the US, Greece (GRE), Italy (ITA), and Portugal (PRT) (see Table 1 and Section 3.3), how a fall in $\bar{b}$ affects the point $P(k_P, b_R)$ and the threshold of fiscal sustainability is represented in Figure 29.

[Figure 29]

Figure 29 shows that for all candidate value of ϕ, $P(k_P, b_P)$ shifts to the upper right direction, and so is the threshold curve at the same time. Therefore, a fall in $\bar{b}$ improves fiscal sustainability.

F Proof of Proposition 5

Both (i) and (iii) are evident. (ii) When $\delta > 1$ and $\mu_1 > 0$, steady states exist if and only if $\mu_2 < 0$ and $\mu_2^2 - 4\mu_1\mu_3 > 0$. From the definitions of μ_2 (see (36)) and $\tilde{\eta} \equiv \beta(1-\alpha)/(1+\beta)$

$$\begin{aligned}\mu_2 =&(1+n)\frac{1-\alpha}{1+\beta} - \tilde{\eta}(1+\tau^c)(1-\lambda) + (1+n)\alpha\delta\bar{b} + \alpha\mu_1 \\ >& \frac{1-\alpha}{1+\beta} - \tilde{\eta}(1+\tau^c) + (1+n)\alpha\delta\bar{b} + \alpha\mu_1 \\ =& \frac{1-\alpha}{1+\beta}[1-\beta(1+\tau^c)] + (1+n)\alpha\delta\bar{b} + \alpha\mu_1 > 0\end{aligned} \tag{F.1}$$

if $1 \geq (1+\tau^c)\beta$. Thus, if $1 \geq (1+\tau^c)\beta$, no steady state exists when $\delta > 1$ and $\mu_1 > 0$.

G Derivation of Condition 3

Applying $\mu_1 \leq 0 \Leftrightarrow \bar{b} \leq \bar{b}_3 \equiv \frac{\tilde{\eta}(\delta-1)}{(1+n)\delta}$ into a_{32} (for $0 < \phi \leq 1$), we have

$$\begin{aligned}a_{32} &= (1+n)\left(\frac{1-\alpha}{1+\beta} + 2\alpha\delta\right)\bar{b}\phi - \tilde{\eta}\left[(1+\tau^c)(1-\lambda) + \alpha(\delta-1)\right] \\ &\leq \frac{\tilde{\eta}(\delta-1)}{\delta}\left(\frac{1-\alpha}{1+\beta} + 2\alpha\delta\right) - \tilde{\eta}\left[(1+\tau^c)(1-\lambda) + \alpha(\delta-1)\right] \\ &= \frac{\tilde{\eta}}{\delta}\left[(\delta-1)\left(\frac{1-\alpha}{1+\beta} + \alpha\delta\right) - (1+\tau^c)(1-\lambda)\delta\right]\end{aligned} \tag{G.1}$$

Thus $a_{32} \leq 0$ if

$$(1+\tau^c)(1-\lambda)\delta \geq (\delta-1)\left(\frac{1-\alpha}{1+\beta} + \alpha\delta\right). \tag{G.2}$$

Next, we can calculate $a_{21}a_{32}$ and $a_{22}a_{31}$ as

$$a_{21}a_{32} = (1+n)\left[(1-\phi)\left(\frac{1-\alpha}{1+\beta}+\alpha\delta\right)+\alpha\delta\right]\left\{\left(\frac{1-\alpha}{1+\beta}+2\alpha\delta\right)(1+n)\bar{b}\phi-\tilde{\eta}\left[(1+\tau^c)(1-\lambda)+\alpha(\delta-1)\right]\right\}, \tag{G.3}$$

$$a_{22}a_{31} = (1+n)\alpha\left[(1+n)\delta\bar{b}\phi-\tilde{\eta}(\delta-1)\right][1+\alpha(\delta-1)+\tilde{\eta}\tau^c], \tag{G.4}$$

and find that both $a_{21}a_{32}$ and $a_{22}a_{31}$ are increasing in $\bar{b}$. When $\bar{b}=0$, $a_{21}a_{32} < a_{22}a_{31}$ for $0<\phi\leq 1$ if and only if

$$\left[(1-\phi)\left(\frac{1-\alpha}{1+\beta}+\alpha\delta\right)+\alpha\delta\right][(1+\tau^c)(1-\lambda)+\alpha(\delta-1)] > \alpha(\delta-1)[1+\alpha(\delta-1)+\tilde{\eta}\tau^c], \tag{G.5}$$

indicating that $a_{21}a_{32}-a_{22}a_{31}<0$ for $(0<)\bar{b}\leq\min\left\{\bar{b}_3,\bar{b}_4\right\}$, where $\bar{b}=\bar{b}_4$ satisfies $a_{21}a_{32}=a_{22}a_{31}$.

(G.5) together with

$$\left[(1-\phi)\left(\frac{1-\alpha}{1+\beta}+\alpha\delta\right)+\alpha\delta\right][(1+\tau^c)(1-\lambda)+\alpha(\delta-1)] > \alpha\delta\left[(1+\tau^c)(1-\lambda)+\alpha(\delta-1)\right], \tag{G.6}$$

leads to the following:

$$a_{21}a_{32}-a_{22}a_{31}<0 \qquad \text{if} \qquad (1+\tau^c)(1-\lambda)\delta \geq (\delta-1)(1-\alpha+\tilde{\eta}\tau^c). \tag{G.7}$$

From (G.2) and (G.7), $a_{32}\leq 0$ and $a_{21}a_{32}-a_{22}a_{31}<0$ if $(1+\tau^c)(1-\lambda)\delta > (\delta-1)\cdot\max\left\{\frac{1-\alpha}{1+\beta}+\alpha\delta, 1-\alpha+\tilde{\eta}\tau^c\right\}$.

H Proof of Lemma 5 and phase diagram under the tax-based consolidation

From (35), we have

$$k_{t+1}-k_t \lesseqgtr 0 \Leftrightarrow h(b_t,k_t)\equiv a_1 b_t^2 + a_2(q(k_t))k_t b_t + a_3(q(k_t))k_t^2 \lesseqgtr 0 \;\text{ for }\; k_t>0, \tag{H.1}$$

where

$$\begin{aligned}
&a_1 \equiv (1+n)(1-\phi)\delta\alpha \geq 0, \quad a_2(q(k_t))\equiv a_{21}+a_{22}q(k_t), \quad a_3(q(k_t))\equiv a_{31}+a_{32}q(k_t)\\
&a_{21}\equiv (1+n)\{(1-\phi)\underbrace{[1-\tilde{\eta}+\alpha(\delta-1)]}_{=\frac{1-\alpha}{1+\beta}+\alpha\delta}+\alpha\delta\}>0, \quad a_{22}\equiv\left[(1+n)\bar{b}\phi\delta-\tilde{\eta}(\delta-1)\right]\alpha,\\
&a_{31}=\mu_3>0, \quad a_{32}\equiv (1+n)\underbrace{[1-\tilde{\eta}+\alpha(\delta-1)]}_{=\frac{1-\alpha}{1+\beta}+\alpha\delta(>0)}\bar{b}\phi-\tilde{\eta}[(1+\tau^c)(1-\lambda)+\alpha(\delta-1)].
\end{aligned}$$

Here, we define $k_{t+1}=k_t$ locus which is derived from $h(b_t,k_t)=0$ as $b_t=m(k_t)$.

H.1 The case of $0<\phi<1$ ($a_1>0$)

$b_t=m(k_t)$ satisfies $b_t=m(0)=0$ and $b_t=m(\breve{k})=0$, where $\breve{k}\equiv A^{\frac{1}{1-\alpha}}\left[-(a_{31}/a_{32})\right]^{\frac{1}{\alpha-1}}$ The former is obvious from $\lim_{k_t\to 0}h(b_t,k_t)=a_1 b_t^2=0$. The latter is shown as follows. From

$a_3(q(\breve{k})) = a_{31} + a_{32}q(\breve{k}) = 0$, we have $h\left(b_t, \breve{k}_t\right) = \left[a_1 b_t + a_2(q(\breve{k}_t))\right] b_t = 0$. This together with $a_1 b_t + a_2(q(\breve{k}_t)) \neq 0$ lead to $b_t = 0$. Thus, the $k_{t+1} = k_t$ locus takes zero when $k_t = 0$ and $\breve{k}$.

To reveal more properties of $k_{t+1} = k_t$ locus and the dynamics of k_t for $k_t > 0$, we rewrite (H.1) into $k_{t+1} \gtreqless k_t \Leftrightarrow a_1 x_t^2 + a_2(q(k_t))x_t + a_3(q(k_t)) \lesseqgtr 0$ for $k_t > 0$ (recall that $x_t \equiv b_t/k_t$), which leads to

$$k_{t+1} \gtreqless k_t \Leftrightarrow q(k_t) \gtreqless \left(\lesseqgtr\right) \Gamma_k\left(x_t\right) \equiv -\frac{\overbrace{a_1}^{(+)} x_t^2 + \overbrace{a_{21}}^{(+)} x_t + \overbrace{a_{31}}^{(+)}}{a_{22}x_t + a_{32}}$$
$$\text{for} \quad a_{22}x_t + a_{32} < (>)0, \tag{H.2}$$

and the derivative of $\Gamma_k(x_t)$ with respect to x_t is given by

$$\Gamma_k'\left(x_t\right) = \frac{\Lambda\left(x_t\right)}{\left(a_{22}x_t + a_{32}\right)^2},$$
$$\Lambda\left(x_t\right) \equiv -\underbrace{a_1}_{(+)} x_t\left(a_{22}x_t + 2a_{32}\right) - \left(a_{21}a_{32} - a_{22}a_{31}\right). \tag{H.3}$$

$k_{t+1} = k_t$ locus is represented by the relationship between $q(k_t)$ and x_t. In addition to this, by (12) and (23), the motion of debt is

$$b_{t+1} \gtreqless b_t \Leftrightarrow q(k_t) \gtreqless \Gamma_b(x_t) \equiv \bar{b}^{-1}x_t. \tag{H.4}$$

Obviously, $\Gamma_b(x_t)$ is positive linear and takes the value zero when $x_t = 0$. Finally, (H.2) and (H.4) show that the steady states given in Proposition 5 are represented by the intersection points between $q(k_t) = \Gamma_k(x_t)$ and $q(k_t) = \Gamma_b(x_t)$.

Step1: Representation of (H.2) **and** (H.4) **into the** $(x_t, q(k_t))$ **plane**
Examining (H.2), (H.3), and (H.4) yields the following cases (i) and (ii).

(i) When $0 < \delta \leq 1$, we obtain the followings. First, $a_{22} > 0$ is satisfied. Second, $\mu_2 < 0$ (from Proposition 5-(i)) ensures $a_{32} < 0$ and the existence of two steady states, indicating that $q(k_t) = \Gamma_k(x_t)$ and $q(k_t) = \Gamma_b(x_t)$ intersect at the steady states denoted by $S(x_S^*, q(k_S^*))$ and $U(x_U^*, q(k_U^*))$. From these facts,

$$-\underbrace{a_{32}}_{(-)}/\underbrace{a_{22}}_{(+)} > 0, \quad q(\breve{k}) = -\underbrace{a_{31}}_{(+)}/\underbrace{a_{32}}_{(-)} > 0, \quad \underbrace{a_{21}}_{(+)}\underbrace{a_{32}}_{(-)} - \underbrace{a_{22}}_{(+)}\underbrace{a_{31}}_{(+)} \leq 0.$$

Then, $\Gamma_k(x_t) > 0$ for $0 \leq x_t \leq -a_{32}/a_{22}$ from (H.2). Furthermore, applying $a_{21}a_{32} - a_{22}a_{31} \leq 0$ and $-a_{32}/a_{22} > 0$ to $\Lambda(x_t)$ in (H.3), we find that sign $\Gamma_k'(x_t) = \Lambda(x_t) > 0$ for $0 \leq x_t \leq -a_{32}/a_{22}$.

These facts indicate that $q(k_t) = \Gamma_k(x_t)$ is monotonically increasing in x_t and satisfies $\Gamma_k(0)(= q(\breve{k})) = -a_{31}/a_{32} > 0$ and $\lim_{x_t \to -\frac{a_{32}}{a_{22}}} \Gamma_k(x_t) = +\infty$. Then, $q(k_t) = \Gamma_k(x_t)$ and an upward-sloping line $q(k_t) = \Gamma_b(x_t)$ intersect at x_S^* and x_U^*. both x_S^* and x_U^* lie between 0 and $-a_{32}/a_{22}(> 0)$ as represented in Figure 9-(a).

(ii) When $\delta > 1$ and $\mu_1 \leq 0$ hold, we obtain the followings. First, $a_{22} < 0$ is satisfied. Second,

$$a_{21}a_{32} - a_{22}a_{31} \leq 0 \text{ (by } \textbf{Condition 3}\text{)},$$
$$-\underbrace{a_{32}}_{(-)}/\underbrace{a_{22}}_{(-)} < 0, \;\; \Gamma_k(0)(= q(\breve{k})) = -\underbrace{a_{31}}_{(+)}/\underbrace{a_{32}}_{(-)} > 0 \;\; (a_{32} < 0 \text{ by } \textbf{Condition 3}).$$

Third, $\Gamma_k(x_t) > 0$ for $x_t \geq 0 > -a_{32}/a_{22}$. Finally, from Proposition 5-(iii) ensures the uniqueness of steady state, inducing $q(k_t) = \Gamma_k(x_t)$ and $q(k_t) = \Gamma_b(x_t)$ to intersect at $S(x_S^*, q(k_S^*))$. Thus, (H.3) with $-a_{32}/a_{22} < 0$, and $\Lambda(0) = -(a_{21}a_{32} - a_{22}a_{31}) \geq 0$ implies that $\Lambda(x_t) > 0$ for $x_t \geq 0$. Then, $\Gamma_k(x_t)$ is positive and monotonically increasing in x_t for $x_t \geq 0$ and satisfies $\Gamma_k(0)(= q(\breve{k})) = -a_{31}/a_{32}(> 0)$ and $\lim_{x_t \to +\infty} \Gamma_k(x_t) = \lim_{x_t \to +\infty} -2a_1 x_t/a_{22} = +\infty$ (Figure 9-(b)).

Step 2: translation of (H.2) and (H.4) into the (k_t, b_t) planes

We translate these relationships between x_t and $q(k_t)$ into the (k_t, b_t) planes.

(i) When $0 < \delta \leq 1$, since $q(k_t) \geq 0$ for $0 \leq x_t < -a_{32}/a_{22}(> 0)$, $q(k_t) = \Gamma_k(x_t)$ is transformed into $k_{t+1} = k_t$ locus: $b_t = m(k_t)$ as follows. The point $(x_t, q(k_t)) = (0, -a_{31}/a_{32})$ (in the LHS of the Figures 9) which corresponds to the point $(b_t, k_t) = (0, \breve{k})$ (in the RHS of the Figures 9). The trajectory of (b_t, k_t), when $(x_t, q(k_t))$ moves from $(0, -a_{31}/a_{32})$ to the final destination $(x_t, q(k_t)) \to (-a_{32}/a_{22}, +\infty)$ along $q(k_t) = \Gamma_k(x_t)$, represents the $k_{t+1} = k_t$ locus: $b_t = m(k_t)$. As x_t increases from 0 through x_S and x_U to $-a_{32}/a_{22}$ along $q(k_t) = \Gamma_k(x_t)$, $q(k_t)$ increases from $-a_{31}/a_{32}(= q(\breve{k})$ through $q(k_S^*)$ and $q(k_U^*)$ to $+\infty$. At the same time, as k_t decreases from $\breve{k}$ through k_S^* and k_U^* to 0 along the $b_t = m(k_t)$, b_t increases from 0 through b_S^* to the upper level and turns to a decrease so as to go through b_U^*, and finally takes 0, as shown in the RHS of Figure 9.

Furthermore, $a_{22}x_t + a_{32} < 0$ is satisfied because of $0 \leq x_t < -a_{32}/a_{22}(> 0)$. Then, (H.2) implies that $k_{t+1} \gtreqless k_t$ if and only if $q(k_t) \gtreqless \Gamma_k(x_t)$ for $0 \leq x_t < -a_{32}/a_{22}$. Thus, $k_{t+1} > (<) k_t$ holds above (bellow) $q(k_t) = \Gamma_k(x_t)$, which satisfies $k_{t+1} > (<) k_t$ bellow (above) $b_t = m(k_t)$ ($k_{t+1} = k_t$ locus) correspondingly.

The translation of (H.2) into $b_t = m(k_t)$ in the rest case (ii) $\delta > 1$ and $\mu_1 \leq 0$ follows that in (i).

H.2 The case of $\phi = 1$ ($a_1 = 0$)

Because of $a_1 = 0$, (H.1) with $a_{21} = (1+n)\alpha\delta > 0$, $a_{22} = \mu_1/\bar{b}$, $a_{31} = \mu_3 > 0$ and $a_{32} = \mu_2$ leads to

$$k_{t+1} \gtreqless k_t \Leftrightarrow q(k_t) \gtreqless (\lesseqgtr)\, \Gamma_k(x_t) = -\frac{(1+n)\alpha\delta x_t + \mu_3}{\bar{b}^{-1}\mu_1 x_t + \mu_2} \quad \text{for} \quad \bar{b}^{-1}\mu_1 x_t + \mu_2 < (>) 0, \quad \text{(H.5)}$$

and

$$\Gamma_k'(x_t) = -\frac{(1+n)\alpha\delta\mu_2 - b^{-1}\mu_1\mu_3}{(b^{-1}\mu_1 x_t + \mu_2)^2}. \quad \text{(H.6)}$$

(i) When $0 < \delta \leq 1$, $\mu_1 \bar{b}^{-1} > 0$ and $\mu_2 < 0$ yields

$$-\underbrace{\mu_2}_{(-)}/\underbrace{\mu_1\bar{b}^{-1}}_{(+)} > 0, \quad q(\breve{k}) = -\underbrace{\mu_3}_{(+)}/\underbrace{\mu_2}_{(-)} > 0, \quad (1+n)\alpha\delta\underbrace{\mu_2}_{(-)} - \underbrace{\bar{b}^{-1}\mu_1}_{(+)}\underbrace{\mu_3}_{(+)} \leq 0.$$

Then $\Gamma_k(x_t) > 0$ for $0 \leq x_t \leq -\mu_2/(\mu_1 \bar{b}^{-1})$. From Proposition 5-(i), the existence of two steady states indicates that $q(k_t) = \Gamma_k(x_t)$ and $q(k_t) = \Gamma_b(x_t)$ intersect at the steady states denoted by $S(x_S^*, q(k_S^*))$ and $U(x_U^*, q(k_U^*))$. Furthermore, applying $a_{21}a_{32} - a_{22}a_{31} = (1+n)\alpha\delta\mu_2 - \bar{b}^{-1}\mu_1\mu_3 \leq 0$ and $-a_{32}/a_{22} = -\mu_2/(\mu_1\bar{b}^{-1}) > 0$ to (H.6), we find that $q(k_t) = \Gamma_k(x_t)$ is monotonically increasing in x_t for $0 \leq x_t \leq -\mu_2/(\mu_1\bar{b}^{-1})$ and satisfies $\Gamma_k(0) = q(\breve{k}) = -\mu_3/\mu_2 > 0$ and $\lim_{x_t \to \frac{-\mu_2}{\mu_1 \bar{b}^{-1}}} \Gamma_k(x_t) = +\infty$ (similar to the case in Figure 9-(a)).

(ii) When $\delta > 1$ and $\mu_1 \leq 0$,

$$a_{21}a_{32} - a_{22}a_{31} = (1+n)\alpha\delta\mu_2 - \bar{b}^{-1}\mu_1\mu_3 \leq 0 \text{ (by **Condition 3**)},$$
$$-\underbrace{\mu_2}_{(-)}/\underbrace{\mu_1\bar{b}^{-1}}_{(-)} < 0,\ \Gamma_k(0)(= q(\breve{k})) = -\underbrace{\mu_3}_{(+)}/\underbrace{\mu_2}_{(-)} > 0\ (a_{32} = \mu_2 < 0 \text{ by **Condition 3**}).$$

Then, $\Gamma_k(x_t) > 0$ for $x_t \geq 0 > -\mu_2/(\mu_1\bar{b}^{-1})$. Applying $(1+n)\alpha\delta\mu_2 - \bar{b}^{-1}\mu_1\mu_3 \leq 0$ and $-\mu_2/(\mu_1\bar{b}^{-1}) < 0$ to (H.6) we find that $q(k_t) = \Gamma_k(x_t)$ is monotonically increasing in x_t for $x_t \geq 0$ and $\Gamma_k(0) = q(\breve{k}) = -\mu_3/\mu_2 > 0$ and $\lim_{x_t\to+\infty}\Gamma_k(x_t) = \lim_{x_t\to+\infty} -\frac{(1+n)\alpha\delta}{\bar{b}^{-1}\mu_1} > 0$. The trajectory of (b_t, k_t), when $(x_t, q(k_t))$ moves from $(0, -\mu_3/\mu_2)$ to the final destination $(x_t, q(k_t)) \to (+\infty, -(1+n)\alpha\delta/(\bar{b}^{-1}\mu_1))$ along $q(k_t) = \Gamma_k(x_t)$, represents the $k_{t+1} = k_t$ locus: $b_t = m(k_t)$. Thus, as k_t decreases from $\breve{k} = q^{-1}(-\mu_3/\mu_2)$ through k_S^* to $q^{-1}(-(1+n)\alpha\delta/(\bar{b}^{-1}\mu_1))$, b_t increases from 0 through b_S^* to $+\infty$ as represented in the RHS of Figure 10.

I The condition of $k_{t+1} \geq 0$ and $k_{t+1} = 0$ locus

(35) indicates that because $1 + \alpha(\delta-1) + \tilde{\eta}\tau^c + \delta\alpha(b_t/k_t) > 0$ for $k_t \geq 0$ and $b_t \geq 0$, $k_{t+1} = \tilde{\Phi}(k_t, b_t) \geq 0$ is written as

$$[(1+\tau^c)(1-\lambda)+\alpha(\delta-1)+(\delta-1)\alpha x_t]\tilde{\eta}Ak_t^\alpha - [1-\tilde{\eta}+\alpha(\delta-1)+\delta\alpha x_t][b_t - \phi(b_t - \bar{b}Ak_t^\alpha)] \geq 0. \tag{I.1}$$

Dividing (I.1) by $k_t(>0)$, we have

$$q(k_t) \geq (<) \Theta(x_t) \equiv -\frac{\overbrace{a_1}^{(+)\text{ or }0} x_t^2 + [(1-\phi)(1-\alpha)(1+\beta)+\alpha\delta]x_t}{a_{32}+a_{22}x_t} \quad \text{for} \quad a_{32}+a_{22}x_t \leq (>) 0, \tag{I.2}$$

where, $a_1 > 0$ for $(0<)\phi < 1$ while $a_1 = 0$, $a_{22} = \mu_1/\bar{b}$, and $a_{32} = \mu_2$ for $\phi = 1$. $k_{t+1} \geq 0$ is satisfied as long as $q(k_t) \geq (<)\Theta(x_t)$ for $a_{32} + a_{22}x_t \leq (>)0$. Furthermore, by (H.2) and (I.2), the difference between $\Gamma_k(x_t)$ and $\Theta(x_t)$ is derived as

$$\Gamma_k(x_t) - \Theta(x_t) = -\frac{\overbrace{a_{31}}^{(+)} + (1+n)\alpha\delta x_t}{a_{32}+a_{22}x_t} > (<) 0 \quad \text{for} \quad a_{32}+a_{22}x_t \leq (>)0, \tag{I.3}$$

where, $a_{31} = \mu_3 > 0$ for $\phi = 1$. Therefore, we obtain the followings for $0(<)\phi \leq 1$.
(i) When $0 < \delta \leq 1$ ($a_{22} > 0$, and $a_{32} < 0$), $0 \leq x_t < -a_{32}/a_{22}(> 0)$ holds along $q(k_t) = \Gamma_k(x_t)$ (in (H.2)). Applying $0 \leq x_t < -a_{32}/a_{22}(> 0)$, $a_{22} > 0$, and $a_{32} < 0$ into (I.2) and (I.3), we find that $k_{t+1} \geq 0$ is satisfied as long as $q(k_t) \geq \Theta(x_t)$ and that $\Gamma_k(x_t) > \Theta(x_t)$ for

$0 \leq x_t < -a_{32}/a_{22}$, respectively.[29]

(ii) When $\delta > 1$ and $\mu_1 \leq 0$ ($a_{22} < 0$ and $a_{32} < 0$ (from **Condition 3**)), $\Gamma_k(x_t) > 0$ and $a_{32} + a_{22}x_t \leq 0$ hold for $x_t > 0 > -a_{32}/a_{22}$. Then, (I.2) and (I.3) with $a_{32} + a_{22}x_t \leq 0$ imply that $k_{t+1} \geq 0$ is satisfied as long as $q(k_t) \geq \Theta(x_t)$ and that $\Gamma_k(x_t) > \Theta(x_t)$ for $x \geq 0$, respectively.

J Conditions when $\delta\tau_t = 1$ (for $\delta > 1$) binds

$\delta\tau_t = 1$ binds if and only if $\delta\tau_t \geq 1$. $\delta\tau_t \geq 1$ is rewritten by using (34) and the definition of μ_3 into

$$\left\{\delta\left[(1+\tau^c)\lambda - \tau^c(1-\tilde{\eta})\right] - \delta(1+\tau^c)(1+n)\phi\bar{b} - \frac{\mu_3}{1+n}\right\} q(k_t) \geq \delta(1+\tau^c)(1+n)(1-\phi)x_t. \tag{J.1}$$

Thus, $\delta\tau_t \geq 1$ for $(0 <)\phi < 1$ if and only if

$$\delta\left[(1+\tau^c)\lambda - \tau^c(1-\tilde{\eta})\right] > \delta(1+\tau^c)(1+n)\phi\bar{b} + \frac{\mu_3}{1+n} \tag{J.2}$$

and

$$q(k_t) \geq \frac{\delta(1+\tau^c)(1+n)(1-\phi)x_t}{\delta\left[(1+\tau^c)\lambda - \tau^c(1-\tilde{\eta})\right] - \delta(1+\tau^c)(1+n)\phi\bar{b} - \frac{\mu_3}{1+n}} \equiv \Upsilon(x_t). \tag{J.3}$$

To ensure $\delta\tau_t < 1$ in the steady state S for $(0 <)\phi < 1$, the RHS of (J.3) must satisfy

$$\begin{aligned} &\Upsilon(x_t) > \Gamma_b(x_t) = b^{-1}x_t \\ &\Leftrightarrow \delta(1+\tau^c)(1+n)\bar{b} + \frac{\mu_3}{1+n} > \delta\left[(1+\tau^c)\lambda - \tau^c(1-\tilde{\eta})\right]. \end{aligned} \tag{J.4}$$

From (J.2) and (J.4), $\delta\tau_t < 1$ always holds in the steady state S,

When $(0 <)\phi < 1$ we arrive at the following facts. First, $\delta\tau_t = 1$ binds if and only if (J.2) and (J.3). Second, $q(k_t) \geq \Upsilon(x_t)$ is above $q(k_t) = \Gamma_b(x_t)$ in the $(x_t, q(k_t))$ plane. Third, $q(k_t) = \Upsilon(x_t)$ is transformed into the function :$b_t = \upsilon(k_t)$ in the (k_t, b_t) plane and $q(k_t) \geq \Upsilon(x_t) \Leftrightarrow b_t \leq \upsilon(k_t)$. Since $b_t = \upsilon(k_t)$ is increasing in k_t and $b_t \leq \upsilon(k_t)$ is always below $b_{t+1} = b_t$ locus, $\delta\tau_t \geq 1$ does not bind above $b_{t+1} = b_t$ locus or in the steady state S.

When $\phi = 1$, from (J.1), if $\delta\left[(1+\tau^c)\lambda - \tau^c(1-\tilde{\eta})\right] \leq \delta(1+\tau^c)(1+n)\phi\bar{b} + \frac{\mu_3}{1+n}$ which is equivalent to (J.4), $\delta\tau_t \geq 1$ does not bind.

[29] $\Theta(x_t)$ has the following properties. $\Theta(0) = 0$, $\Theta(x_t) > (\leq) 0$ for $0 \leq x_t < -a_{32}/a_{22}$, and $\lim_{x_t \to -\frac{a_{32}}{a_{22}}} \Theta(x_t) = +(-)\infty$ if $a_{21} - (1+n)\alpha\delta = (1+n)(1-\phi)[1-\tilde{\eta}+\alpha(\delta-1)] > 0$ or $a_{21} \leq (1+n)\alpha\delta$ and $-\frac{a_{32}}{a_{22}} < -\frac{a_{21}-(1+n)\alpha\delta}{a_1}$ (if $a_{21} \leq (1+n)\alpha\delta$ and $-\frac{a_{32}}{a_{22}} \geq -\frac{a_{21}-(1+n)\alpha\delta}{a_1}$).

K Effects of $\bar{b}$ on $S(k_S^*, b_S^*)$ when $\delta > 1$, $\mu_1 \leq 0$

Taking the total differentials of (36) yields

$$\frac{dq(k_S^*)}{d\bar{b}} = -\frac{q(k_S^*)\left\{\alpha\left[2(1+n)\delta\bar{b} - \tilde{\eta}(\delta-1)\right]q(k_S^*) + (1+n)\left(\frac{1-\alpha}{1+\beta} + 2\delta\alpha\right)\right\}}{\underbrace{2\mu_1 q(k_S^*) + \mu_2}_{(-)}}. \tag{K.1}$$

Let us define the LHS of (36) as $\Xi(q(k)) \equiv \mu_1 q(k)^2 + \mu_2 q(k) + \mu_3$. $q(k_S^*)$ is given by the intersection point between the $q(k_t)$ axis and the inverted U-shaped quadratic function $\Xi(q(k))$ that takes $\Xi(0) = \mu_3 > 0$ and satisfies $\Xi'(q(k_S^*)) = 2\mu_1 q(k_S^*) + \mu_2 < 0$ and $\Xi'(-\mu_2/(2\mu_1)) = 0$. From $\delta > 0$, $\mu_1 \leq 0$, and $\mu_2 = (1+n)\bar{b}\left(\frac{1-\alpha}{1+\beta} + 2\delta\alpha\right) - \tilde{\eta}[(1+\tau^c)(1-\lambda) + \alpha(\delta-1)]$, we have

$$\begin{aligned}
&\alpha\left[2(1+n)\delta\bar{b} - \tilde{\eta}(\delta-1)\right]q(k_t) + (1+n)\left(\frac{1-\alpha}{1+\beta} + 2\delta\alpha\right) \\
&> \frac{2\mu_1}{\bar{b}}q(k_t) + (1+n)\left(\frac{1-\alpha}{1+\beta} + 2\delta\alpha\right) \\
&> \frac{2\mu_1}{\bar{b}}q(k_t) + \frac{\mu_2}{\bar{b}}.
\end{aligned} \tag{K.2}$$

Evaluating (K.2) at $q(k_t) = -\mu_2/(2\mu_1)$, we have $\alpha\left[2(1+n)\delta\bar{b} - \tilde{\eta}(\delta-1)\right](-\mu_2/2\mu_1) + (1+n)\left(\frac{1-\alpha}{1+\beta} + 2\delta\alpha\right) > 0$. Since $q(k_S^*) > -\mu_2/(2\mu_1)$, we have $\alpha\left[2(1+n)\delta\bar{b} - \tilde{\eta}(\delta-1)\right]q(k_S^*) + (1+n)\left(\frac{1-\alpha}{1+\beta} + 2\delta\alpha\right) > 0$. Thus, $dq(k_S^*)/d\bar{b} > 0$.

Parameter or variable	JPA	US	GRE	ITA	PRT	Source
α	0.38	0.35	0.40	0.39	0.39	Hansen and İmrohoroğlu (2016) and Trabandt and Uhlig (2011)
$\bar{b}$	0.02	0.02	0.02	0.02	0.02	Set $(0.6/30 = 0.02)$
τ^R	0.46	0.34	0.16	0.30	0.23	Data average with Gunji and Miyazaki (2011) and Trabandt and Uhlig (2011)
τ^w	0.31	0.28	0.41	0.47	0.31	Gunji and Miyazaki (2011) and Trabandt and Uhlig (2011)
τ^c	0.10	0.05	0.15	0.15	0.23	Data and Trabandt and Uhlig (2011)
n	0	0.01	0	0	0	Data average
Y_0/K_0	0.32	0.41	0.28	0.30	0.36	Data average
B_0/Y_0	2.37	1.36	1.93	1.53	1.42	Data average
A	20	24.34	10.68	14.80	9.99	Calibrated (A of JPA: Set)
k_0	3.27	2.98	2.52	3.57	2.43	Calibrated
b_0	2.48	1.66	1.36	1.64	1.24	Calibrated

Table 1: Benchmark parameters and variables

Expenditure-based consolidation					
		$g_t = 0$ binds	$k_{t+1} = 0$ binds	$d_t = 0$ binds	non-surviving generation
JPA	$\phi = 0$	period 2	period 2 ($k_3 = 0$)	period 3	generation 1
	$\phi = 0.1$	period 3	period 3 ($k_4 = 0$)	period 4	generation 2
GRE	$\phi = 0$	period 1	period 1 ($k_2 = 0$)	period 2	generation 0
	$\phi = 0.1$	period 1	period 1 ($k_2 = 0$)	period 2	generation 0
	$\phi = 0.3$	period 1	period 1 ($k_2 = 0$)	period 2	generation 0
ITA	$\phi = 0$	period 2	period 1 ($k_2 = 0$)	period 2	generation 1
	$\phi = 0.1$	period 2	period 1 ($k_2 = 0$)	period 2	generation 1
PRT	$\phi = 0$	period 1	period 1 ($k_2 = 0$)	period 2	generation 0
	$\phi = 0.1$	period 2	period 2 ($k_3 = 0$)	period 3	generation 1

Table 2: Unsustainable paths under expenditure-based consolidations

Expenditure-based consolidation					
Benchmark case	JPA	US	GRE	ITA	PRT
k^*	3.4069	5.3068	0.9044	1.2861	1.1428
b^*	0.6373	0.8731	0.2052	0.3265	0.2105
g^*	13.0454	13.5972	3.4841	6.9928	3.8376
c^*	8.3572	13.5122	2.1932	3.1874	2.5010
d^*	6.4150	11.2954	3.6775	4.8596	3.0426

Table 3: Values of the steady-state variables under expenditure-based consolidation

Tax-based consolidation					
		$\tau_t = 0$ or $\delta\tau_t = 1$binds	$k_{t+1} = 0$ binds	$d_t = 0$ binds	non-surviving generation
JPA	$\phi = 0$	$\delta\tau_3 = 1$ binds	period 2 ($k_3 = 0$)	period 3	generation 2
	$\phi = 0.1$	$\delta\tau_3 = 1$ binds	period 2 ($k_3 = 0$)	period 3	generation 2
GRE	$\phi = 0$	$\tau_1 = 1$ binds	period 1 ($k_2 = 0$)	period 2	generation 1
	$\phi = 0.3$	$\tau_2 = 1$ binds	period 1 ($k_2 = 0$)	period 2	generation 1
	$\phi = 0.5$	$\tau_2 = 1$ binds	period 2 ($k_3 = 0$)	period 3	generation 2
ITA	$\phi = 0$	$\tau_2 = 1$ binds	period 1 ($k_2 = 0$)	period 2	generation 1
	$\phi = 0.3$	$\tau_2 = 1$ binds	period 1 ($k_2 = 0$)	period 2	generation 1
	$\phi = 0.5$	$\tau_4 = 1$ binds	period 3 ($k_4 = 0$)	period 4	generation 3
PRT	$\phi = 0$	$\tau_2 = 1$ binds	period 1 ($k_2 = 0$)	period 2	generation 1
	$\phi = 0.3$	$\tau_3 = 1$ binds	period 2 ($k_3 = 0$)	period 3	generation 2

Table 4: Unsustainable paths under tax-based consolidations

Tax-based consolidation					
Benchmark case	JPA	US	GRE	ITA	PRT
k^*	3.8420	6.0811	1.0338	1.2957	1.1952
b^*	0.6671	0.9157	0.2165	0.3275	0.2142
g^*	12.0696	11.5344	3.4222	6.9915	3.7785
c^*	9.3178	15.2983	2.4713	3.2084	2.6045
d^*	8.1257	12.9357	3.8957	4.8779	3.1311
$\tau^*(=\tau^{w*})$	0.2863	0.2228	0.3698	0.4681	0.2939
$\delta\tau^*(=\tau^{R*})$	0.3991	0.2705	0.1443	0.2988	0.2180

Table 5: Values of the steady-state variables under tax-based consolidation

Expenditure-based consolidation					
Benchmark case	JPA	US	GRE	ITA	PRT
sustainable	$\phi \in [0.12, 1]$	$\phi \in [0, 1]$	$\phi \in [0.42, 1]$	$\phi \in [0.30, 1]$	$\phi \in [0.27, 1]$
unsustainable	$\phi \in [0, 0.11]$	-	$\phi \in [0, 0.41]$	$\phi \in [0, 0.29]$	$\phi \in [0, 0.26]$

Table 6: Pace of expenditure-based fiscal consolidation ϕ and sustainability of public debt

Tax-based consolidation					
Benchmark case	JPA	US	GRE	ITA	PRT
sustainable	$\phi \in [0.19, 1]$	$\phi \in [0, 1]$	$\phi \in [0.63, 1]$	$\phi \in [0.53, 1]$	$\phi \in [0.42, 1]$
unsustainable	$\phi \in [0, 0.18]$	-	$\phi \in [0, 0.62]$	$\phi \in [0, 0.52]$	$\phi \in [0, 0.41]$

Table 7: Pace of tax-based fiscal consolidation ϕ and sustainability of public debt

Social welfare W (the benchmark case: $\theta = 0.8$)							
		$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$	$\phi = 0.5$	$\phi = 0.3$	$\phi = 0$
JPA	expenditure base	20.4351	20.3981	20.2720	20.0234	19.4505	-
	tax base	20.5533	20.4959	20.3308	20.0395	19.3681	-
US	expenditure base	22.3933	22.3845	22.3543	22.3000	22.2014	21.7678
	tax base	22.3249	22.3087	22.2672	22.2066	21.9237	21.7282
GRE	expenditure base	10.0736	9.9570	9.4970	8.1172	-	-
	tax base	10.1045	9.8797	8.8688	-	-	-
ITA	expenditure base	15.4012	15.3242	15.0654	14.4908	11.1238	-
	tax base	15.2607	15.1194	14.6091	-	-	-
PRT	expenditure base	10.0559	9.9918	9.7599	9.2349	7.1164	-
	tax base	10.0725	9.9714	9.6414	8.8007	-	-

Table 8: Social welfare W (the benchmark case: $\theta = 0.8$)

Social welfare W (when $\theta = 0.2$)							
		$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$	$\phi = 0.5$	$\phi = 0.3$	$\phi = 0$
JPA	expenditure base	14.2955	14.2793	14.2288	14.1353	13.9314	-
	tax base	14.4488	14.4171	14.3227	14.1495	13.7323	-
US	expenditure base	16.4194	16.4158	16.4046	16.3857	16.3528	16.2136
	tax base	16.6469	16.6386	16.6165	16.5836	16.5297	16.3090
GRE	expenditure base	7.8706	7.8227	7.6701	7.2382	-	-
	tax base	7.7988	7.6449	6.9291	-	-	-
ITA	expenditure base	10.5665	10.5376	10.4438	10.2450	9.2437	-
	tax base	10.3461	10.2521	9.9035	-	-	-
PRT	expenditure base	7.6227	7.5971	7.5111	7.3286	6.6711	-
	tax base	7.4913	7.4290	7.2187	6.6581	-	-

Table 9: Social welfare W (when $\theta = 0.2$)

	Gini coefficient of welfare Δ (the benchmark case: $\theta = 0.8$)						
		$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$	$\phi = 0.5$	$\phi = 0.3$	$\phi = 0.1$
JPA	expenditure base	0.0172	0.0177	0.0194	0.0233	0.0334	-
	tax base	0.0194	0.0202	0.0225	0.0272	0.0402	-
US	expenditure base	0.0252	0.0253	0.0257	0.0265	0.0279	0.0199
	tax base	0.0259	0.0261	0.0266	0.0274	0.0287	0.0216
GRE	expenditure base	0.0330	0.0297	0.0368	0.0997	-	-
	tax base	0.0210	0.0319	0.0878	-	-	-
ITA	expenditure base	0.0172	0.0156	0.0150	0.0313	0.1708	-
	tax base	0.0138	0.0138	0.0293	-	-	-
PRT	expenditure base	0.0169	0.0147	0.0176	0.0430	0.1715	-
	tax base	0.0123	0.0138	0.0306	0.0811	-	-

Table 10: Gini coefficient of welfare Δ (the benchmark case: $\theta = 0.8$)

	Gini coefficient of welfare Δ (when $\theta = 0.2$)						
		$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$	$\phi = 0.5$	$\phi = 0.3$	$\phi = 0$
JPA	expenditure base	0.0132	0.0135	0.0148	0.0177	0.0250	-
	tax base	0.0193	0.0200	0.0220	0.0263	0.0384	-
US	expenditure base	0.0214	0.0215	0.0217	0.0221	0.0229	0.0178
	tax base	0.0250	0.0251	0.0255	0.0262	0.0273	0.0207
GRE	expenditure base	0.0385	0.0368	0.0378	0.0671	-	-
	tax base	0.0192	0.0299	0.0828	-	-	-
ITA	expenditure base	0.0208	0.0199	0.0198	0.0292	0.0939	-
	tax base	0.0150	0.0155	0.0333	-	-	-
PRT	expenditure base	0.0212	0.0202	0.0210	0.0350	0.0933	-
	tax base	0.0106	0.0129	0.0296	0.0775	-	-

Table 11: Gini coefficient of welfare Δ (when $\theta = 0.2$)

Social welfare W							
		$\theta = 0.8$			$\theta = 0.2$		
		$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$	$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$
JPA	expenditure base	8.9555	8.9506	8.9273	6.0612	6.0591	6.0501
	tax base	9.0040	8.9976	8.9768	6.0921	6.0897	6.0809
	expenditure & tax base	8.9583	8.9492	8.9170	6.0435	6.0378	6.0214
US	expenditure base	9.1170	9.1169	9.1137	6.2656	6.2654	6.2644
	tax base	9.1105	9.1082	9.1022	6.2839	6.2832	6.2814
	expenditure & tax base	9.1312	9.1301	9.1246	6.2643	6.2635	6.2609
GRE	expenditure base	5.5746	5.4033	3.2802	4.3003	4.2380	3.5861
	tax base	-	-	-	-	-	-
	expenditure & tax base	5.5585	5.3577	2.5041	4.2234	4.1346	2.7577
ITA	expenditure base	7.7605	7.7063	7.4196	5.2740	5.2514	5.1391
	tax base	-	-	-	-	-	-
	expenditure & tax base	7.7096	7.6403	-	5.2036	5.1664	-
PRT	expenditure base	6.1890	6.1384	6.8913	4.5008	4.4808	4.3911
	tax base	6.2698	6.2107	5.9180	4.4920	4.4601	4.2944
	expenditure & tax base	6.1915	6.1265	5.8438	4.4496	4.4178	4.2971

Table 12: Social welfare W (the case of CRRA utility function)

Gini coefficient of welfare Δ							
		$\theta = 0.8$			$\theta = 0.2$		
		$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$	$\phi = 1$	$\phi = 0.9$	$\phi = 0.7$
JPA	expenditure base	0.0171	0.0173	0.0180	0.0154	0.0156	0.0161
	tax base	0.0174	0.0175	0.0179	0.0180	0.0181	0.0185
	expenditure & tax base	0.0170	0.0173	0.0183	0.0160	0.0163	0.0171
US	expenditure base	0.0199	0.0198	0.0198	0.0175	0.0175	0.0176
	tax base	0.0176	0.0176	0.0177	0.0179	0.0179	0.0180
	expenditure & tax base	0.0197	0.0195	0.0195	0.0175	0.0176	0.0177
GRE	expenditure base	0.0936	0.0836	0.2142	0.0592	0.0545	0.0795
	tax base	-	-	-	-	-	-
	expenditure & tax base	0.0914	0.0799	0.4234	0.0539	0.0470	0.2669
ITA	expenditure base	0.0278	0.0253	0.0220	0.0250	0.0234	0.0206
	tax base	-	-	-	-	-	-
	expenditure & tax base	0.0252	0.0221	-	0.0221	0.0195	-
PRT	expenditure base	0.0250	0.0215	0.0183	0.0209	0.0190	0.0167
	tax base	0.0157	0.0137	0.0235	0.0159	0.0137	0.0255
	expenditure & tax base	0.0213	0.0174	0.0213	0.0184	0.0158	0.0174

Table 13: Gini coefficient of welfare Δ (the case of CRRA utility function)

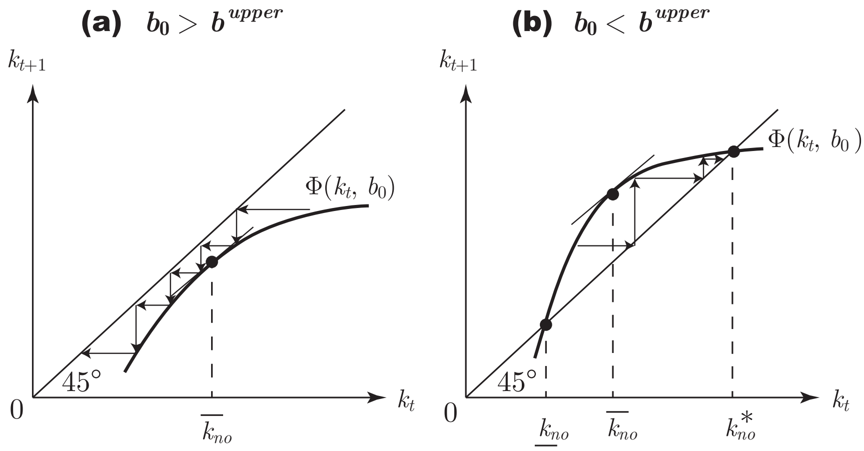

Figure 1: The dynamics of k_t under no fiscal consolidation

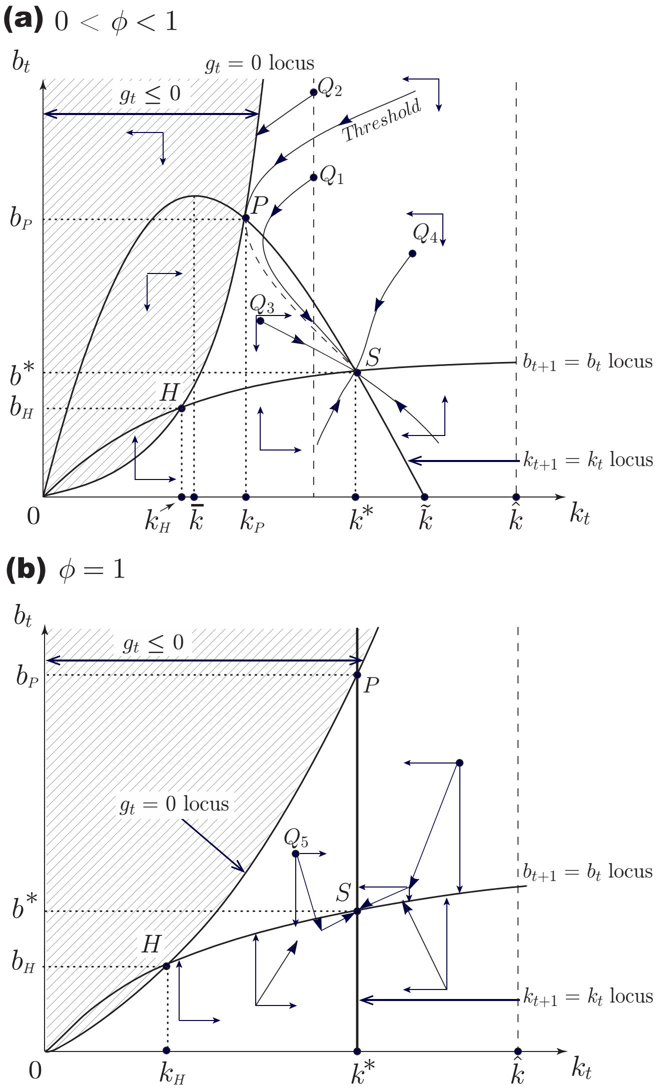

Figure 2: The dynamics of k_t and b_t under expenditure-based consolidation

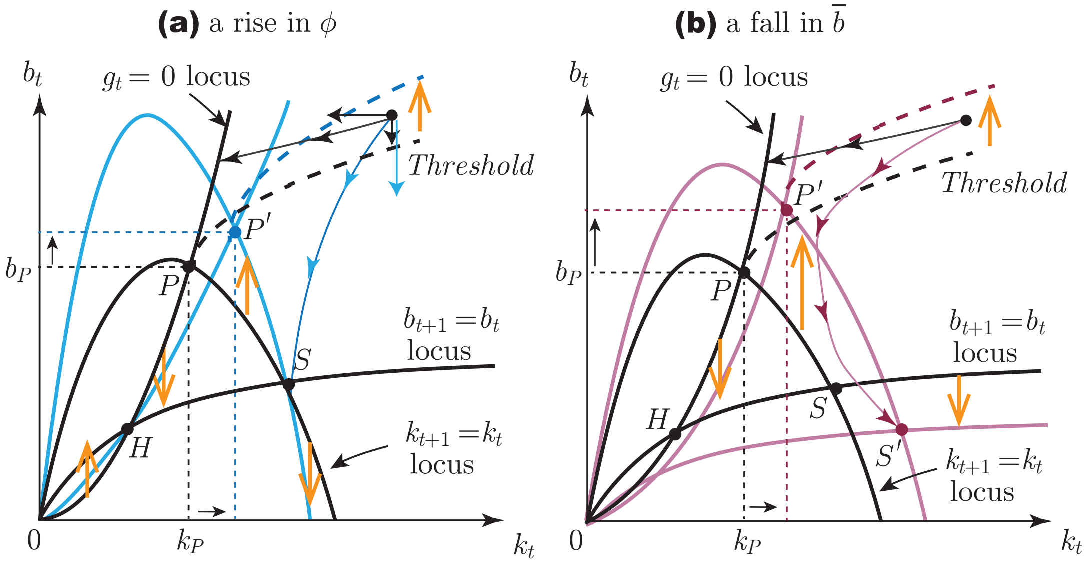

Figure 3: Effects of a fall in $\bar{b}$ and a rise in ϕ in the case of expenditure-based consolidation

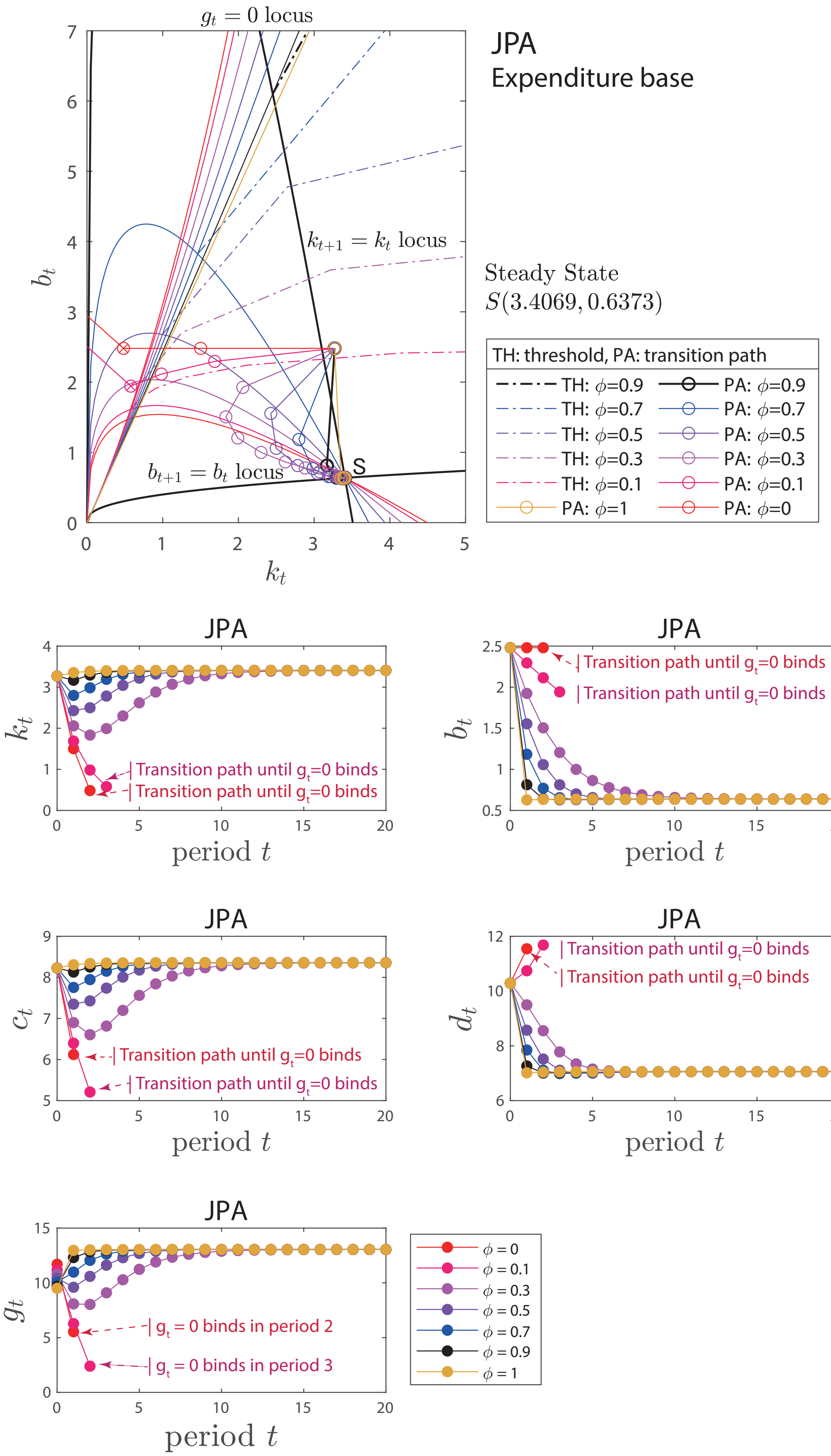

Figure 4: Transitional dynamics for Japan under expenditure-based consolidations

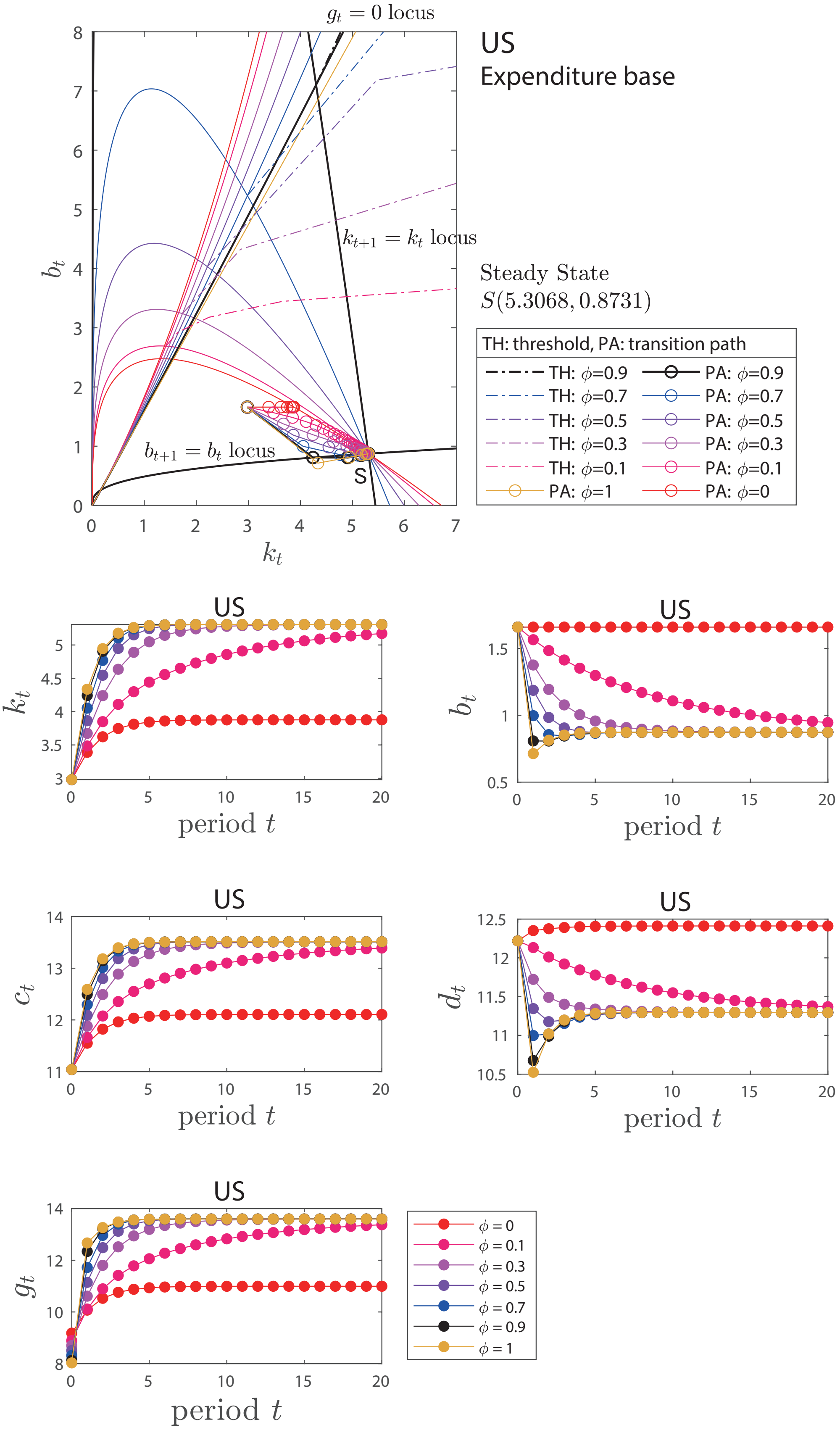

Figure 5: Transitional dynamics for the US under expenditure-based consolidations

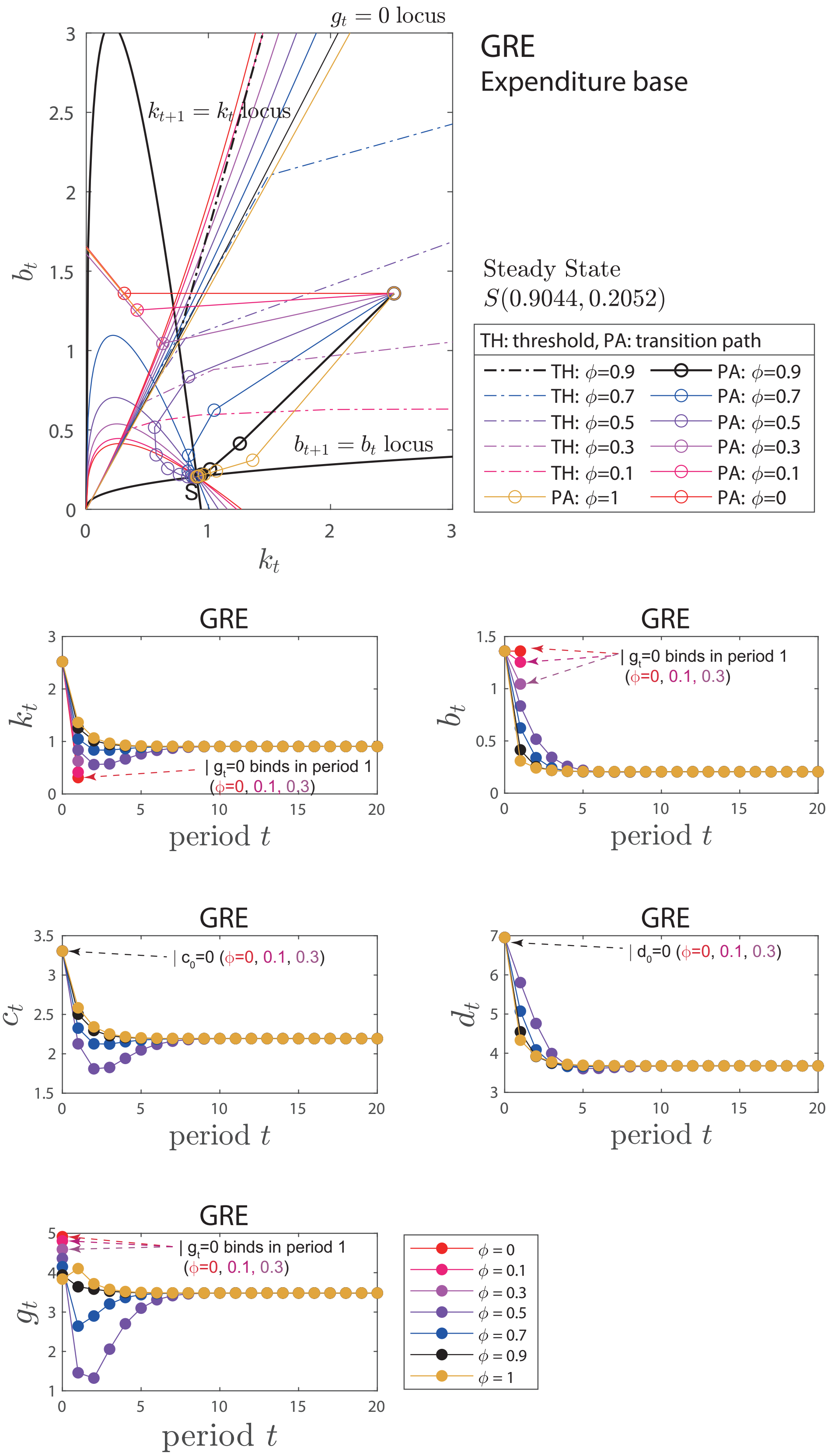

Figure 6: Transitional dynamics for Greece under expenditure-based consolidation

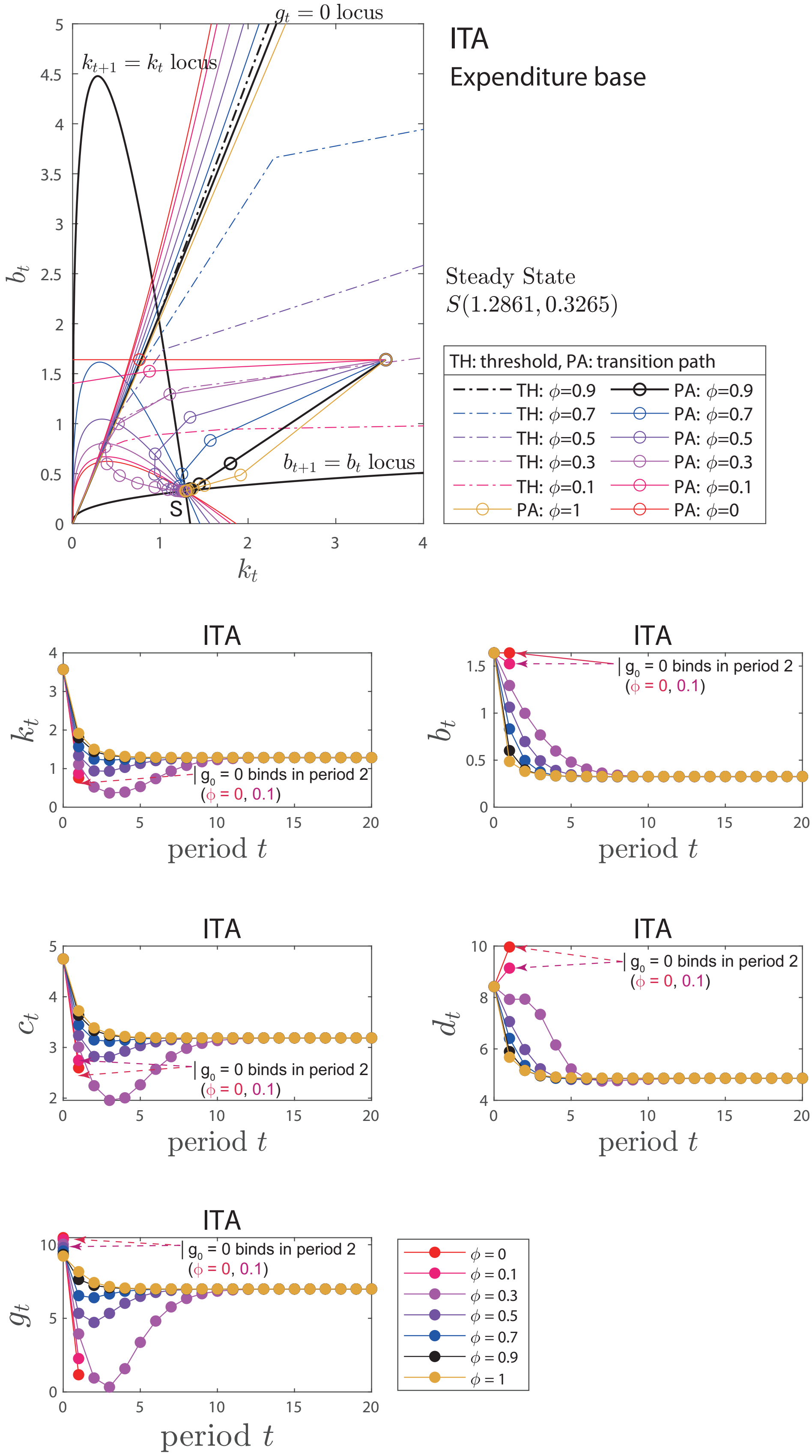

Figure 7: Transitional dynamics for Italy under expenditure-based consolidations

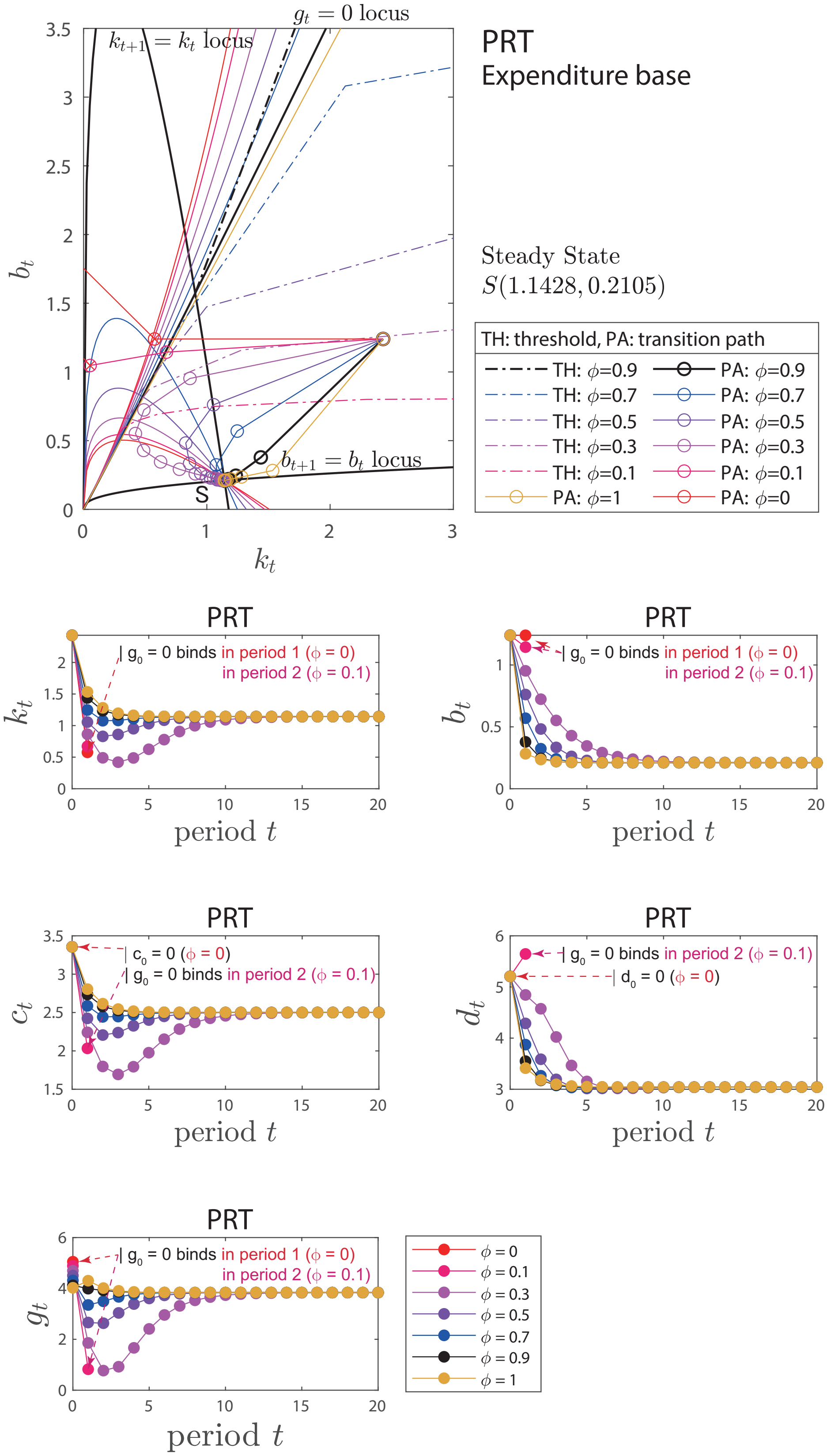

Figure 8: Transitional dynamics for Portugal under expenditure-based consolidations

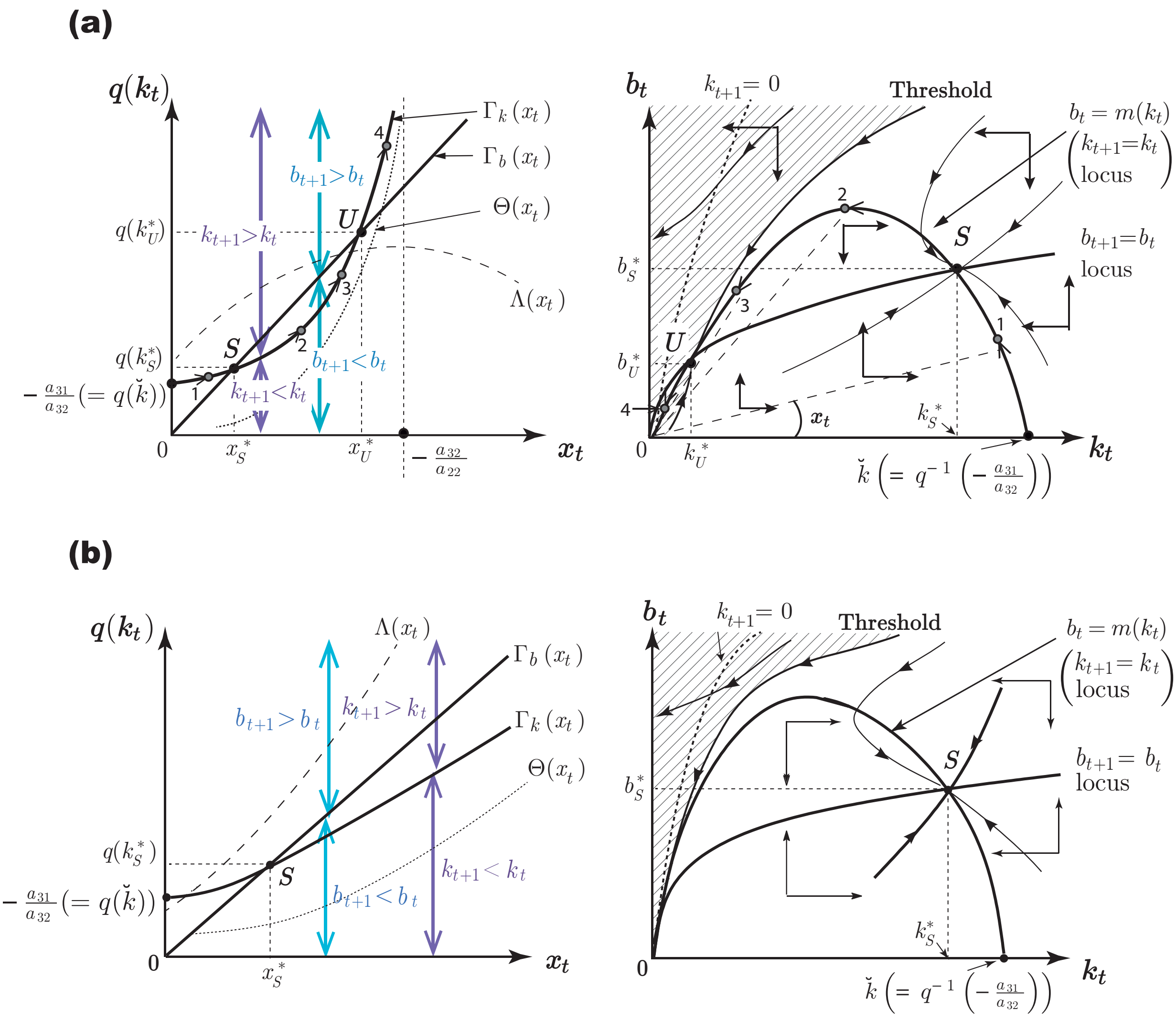

Figure 9: Phase diagram for $0<\phi<1$ in the case of tax-based consolidation: **(a)** $0<\delta\leq 1$ **(b)** $\delta>1$ and $\mu_1\leq 0$

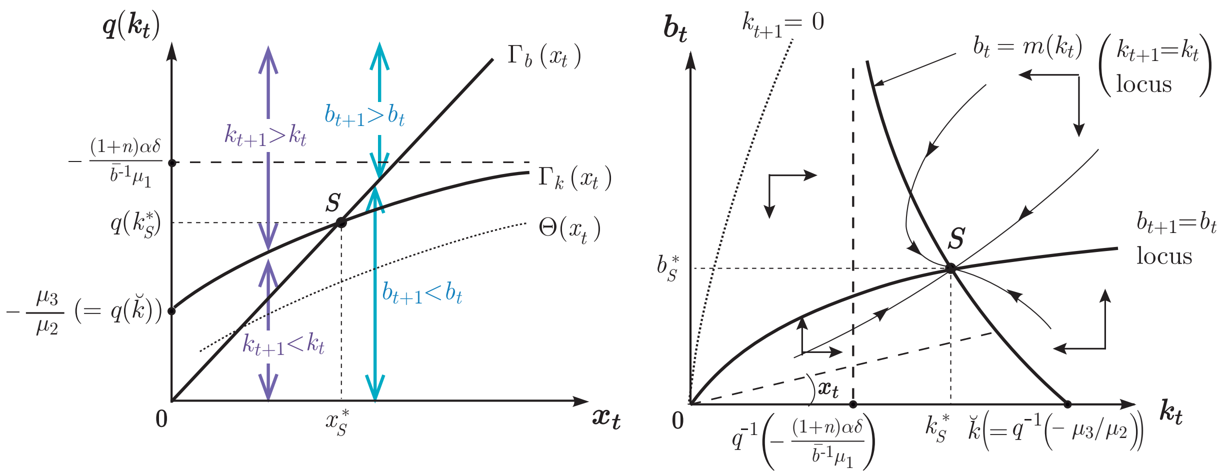

Figure 10: Phase diagram for $\phi=1$: the case of $\delta>1$ and $\mu_1\leq 0$ under tax-based consolidation (Note: the case when $0<\delta\leq 1$ is similar to Figure 9-(a)).

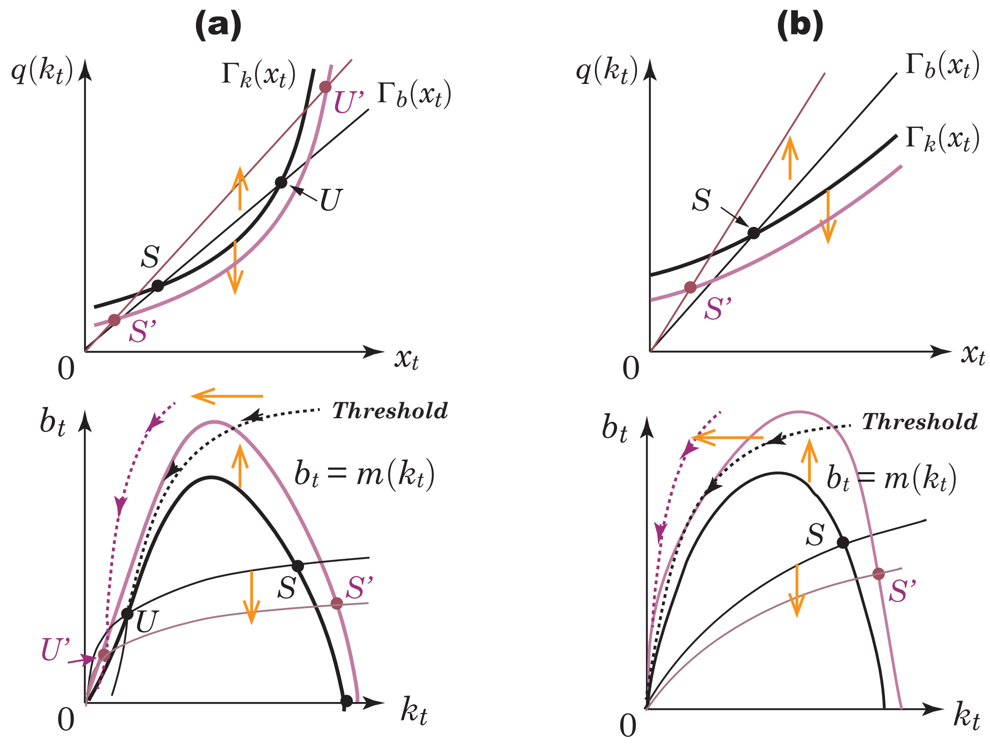

Figure 11: Effects of a reduction in $\bar{b}$ on the sustainability of public debt and the steady state under tax-base consolidation: **(a)** $0 < \delta \leq 1$ **(b)** $\delta > 1$ and $\mu_1 \leq 0$

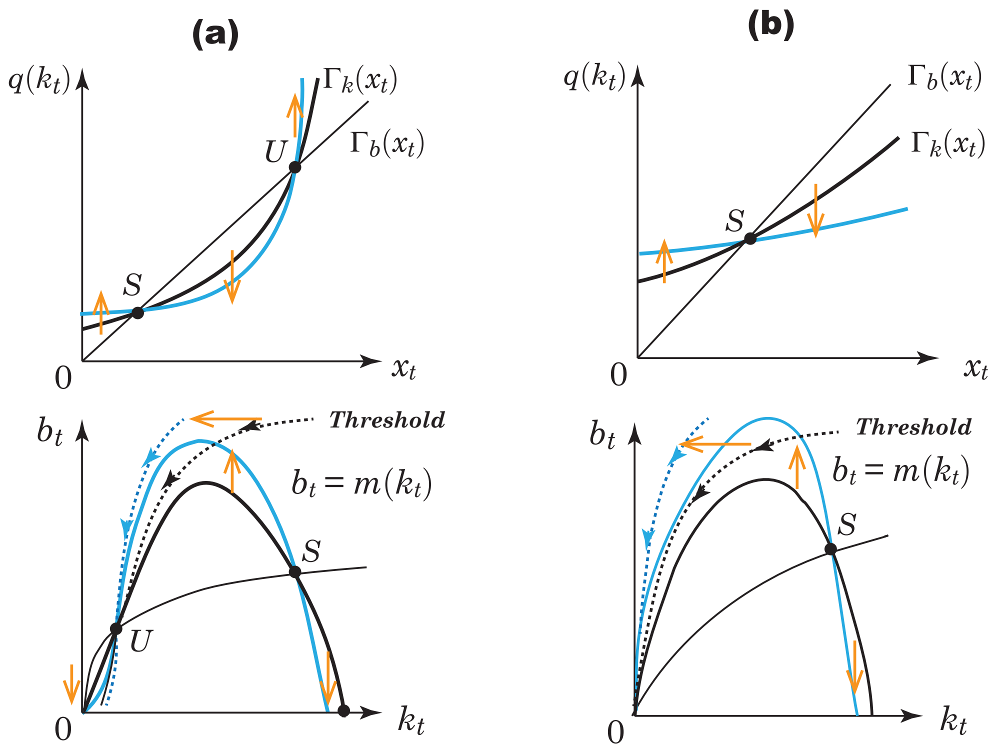

Figure 12: Effects of an increase in ϕ on the sustainability of public debt under tax-based consolidation: **(a)** $0 < \delta \leq 1$ **(b)** $\delta > 1$ and $\mu_1 \leq 0$

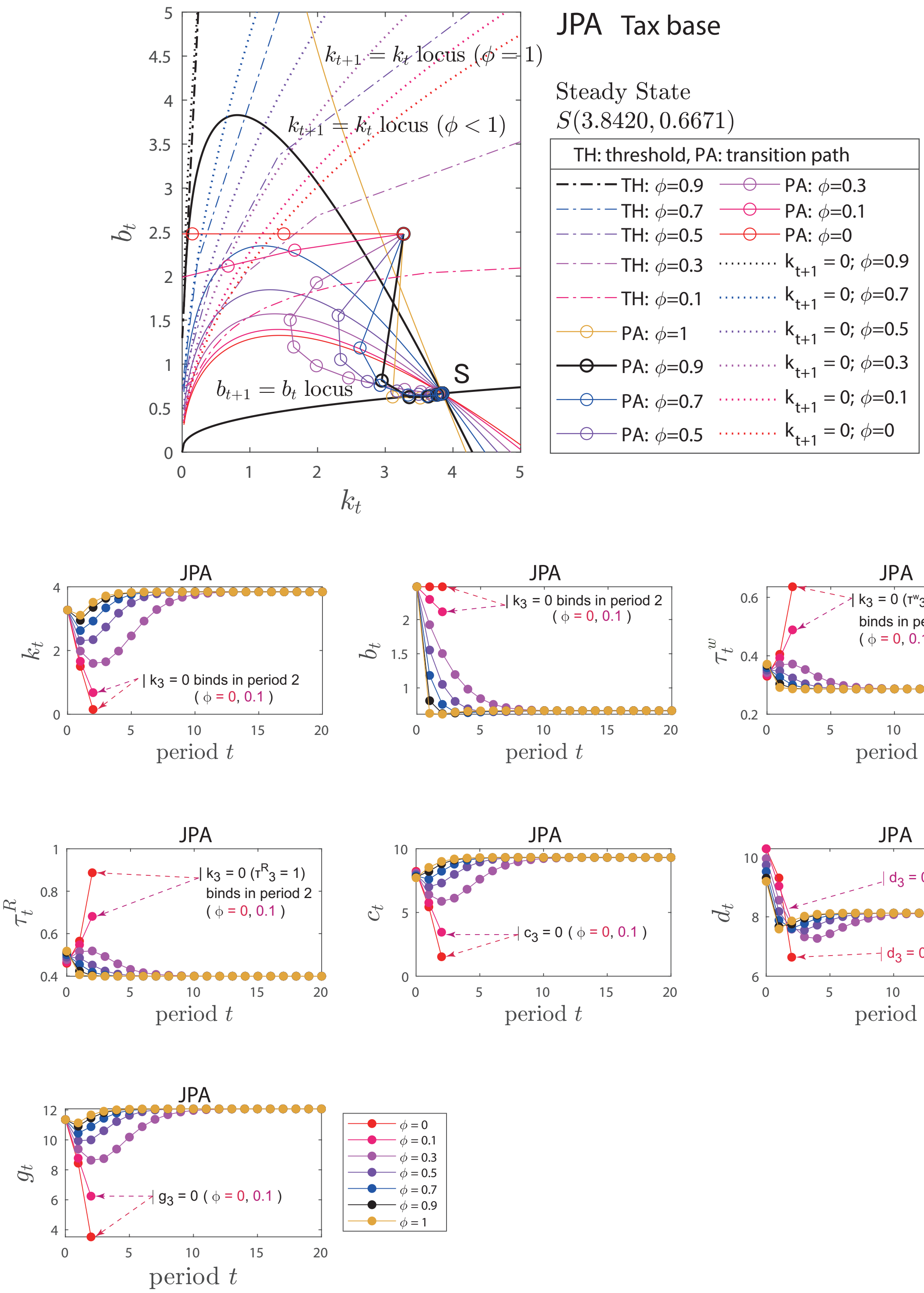

Figure 13: Transitional dynamics for Japan under tax-based consolidations

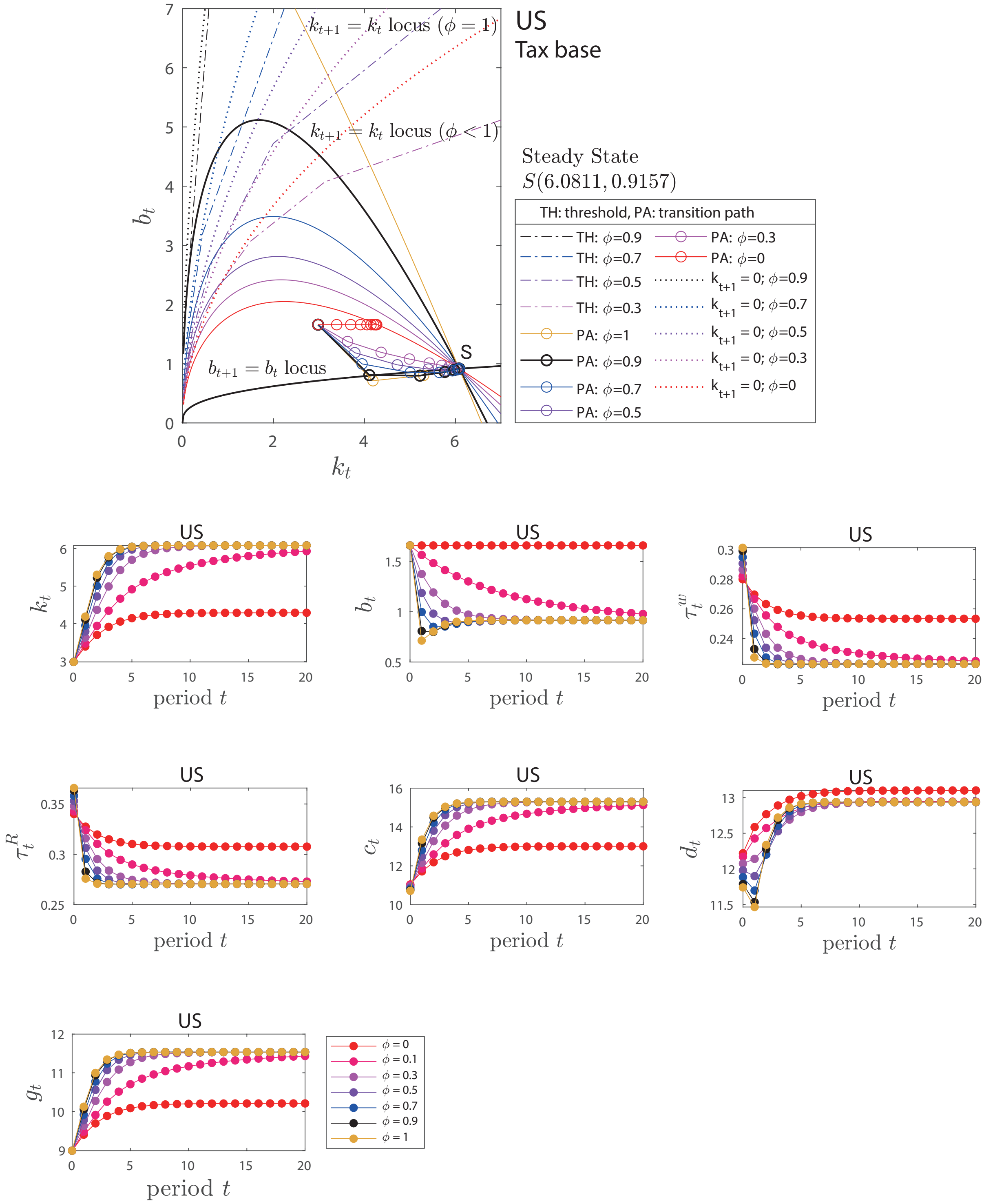

Figure 14: Transitional dynamics for the US under tax-based consolidations

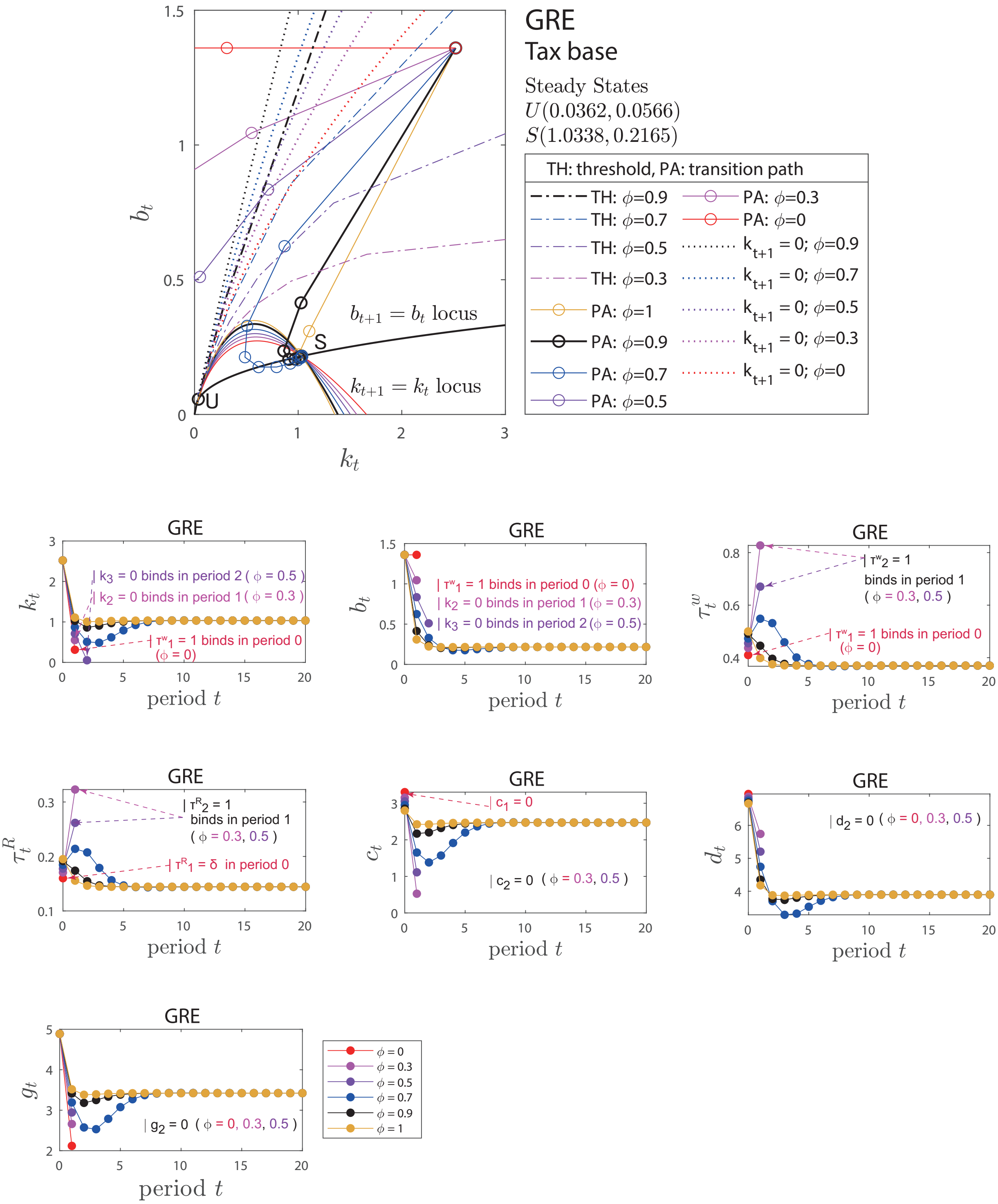

Figure 15: Transitional dynamics for Greece under tax-based consolidations

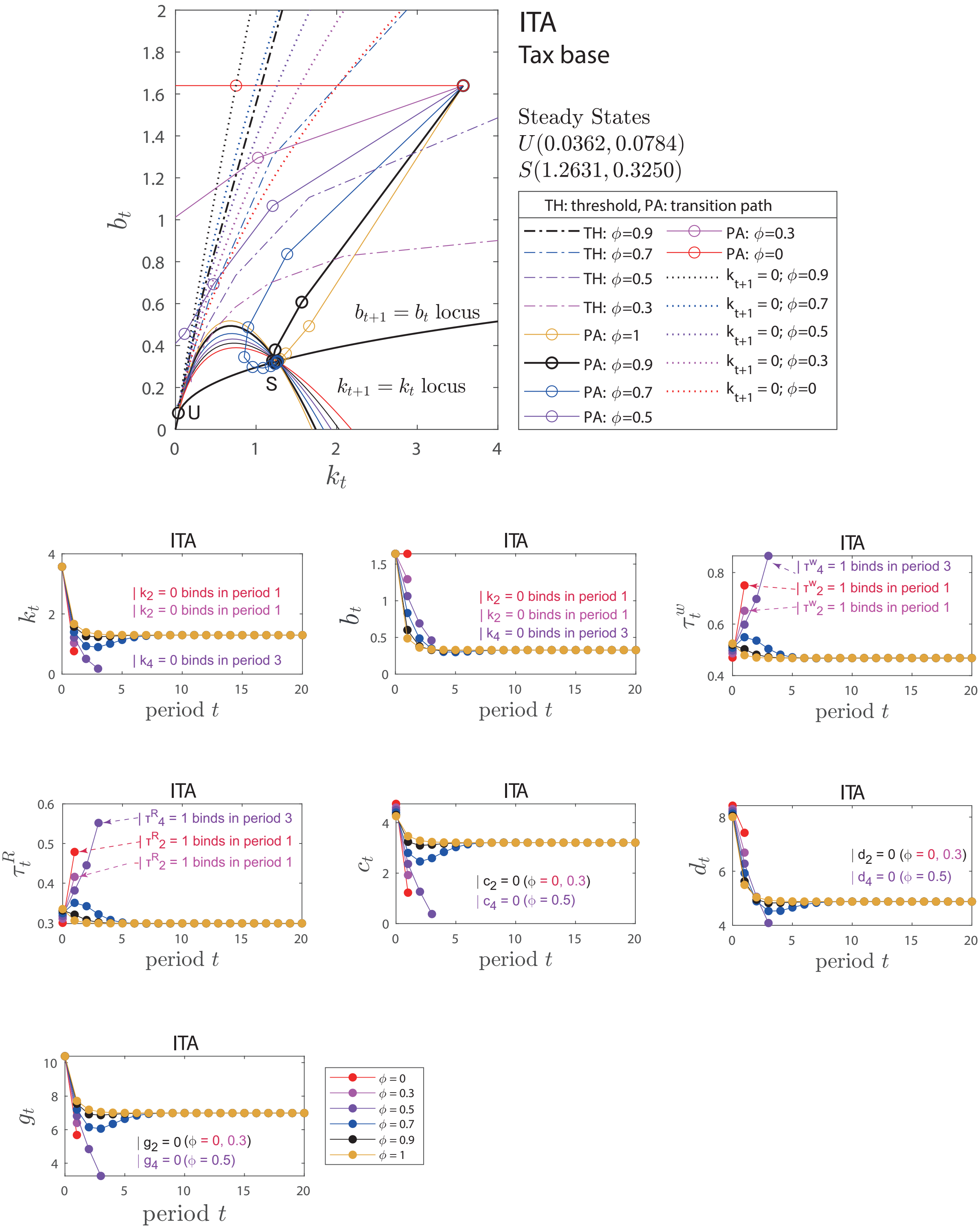

Figure 16: Transitional dynamics for Italy under tax-based consolidations

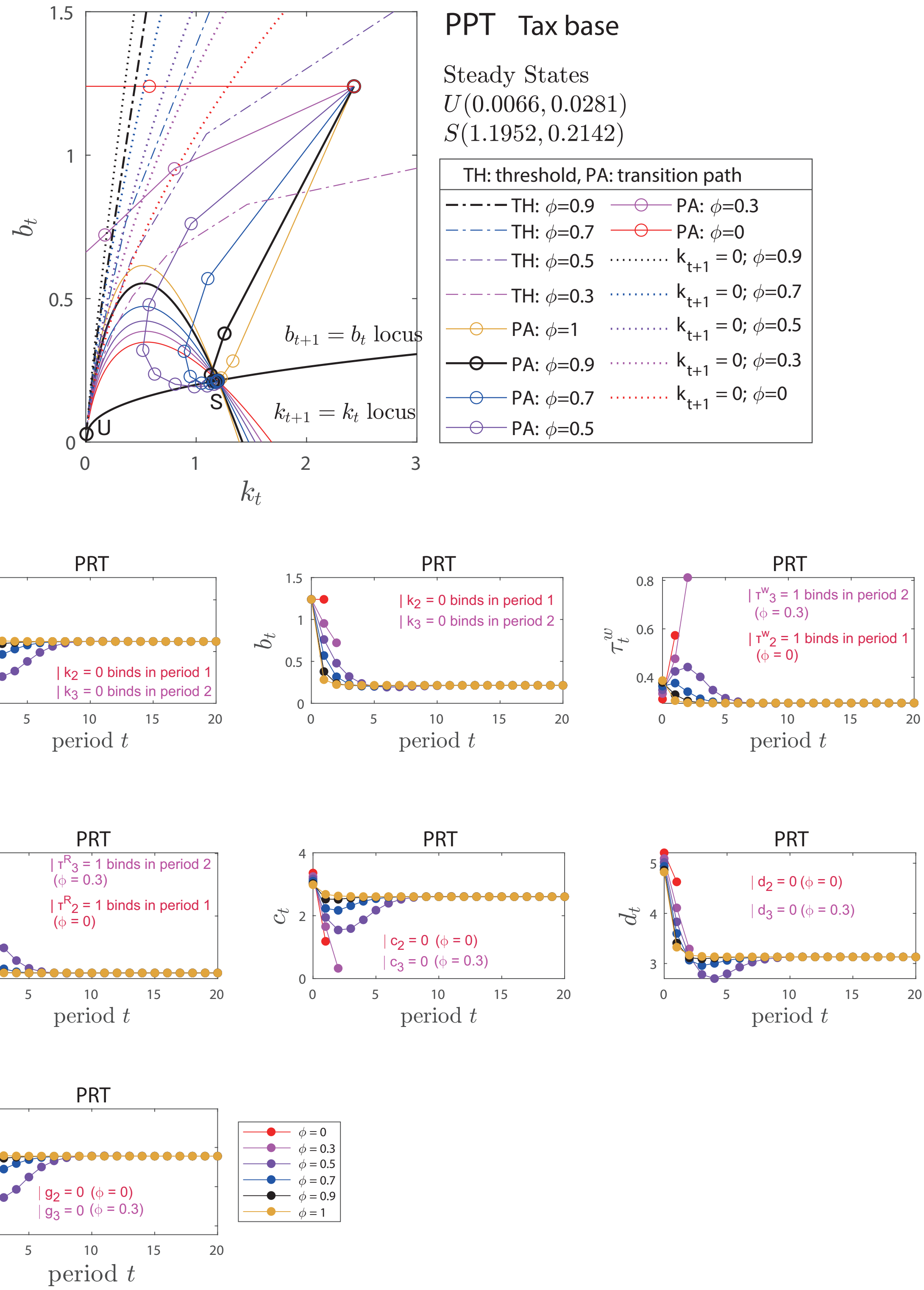

Figure 17: Transitional dynamics for Portugal under tax-based consolidations

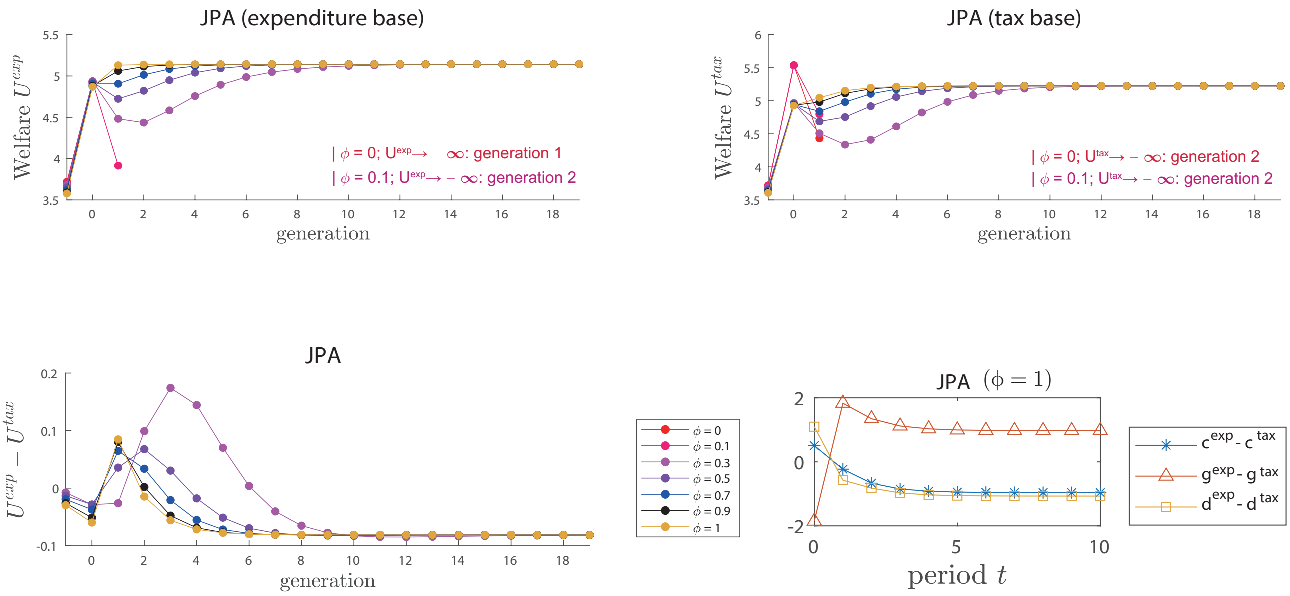

Figure 18: Welfare of each generation: the case of Japan ($\theta = 0.8$)

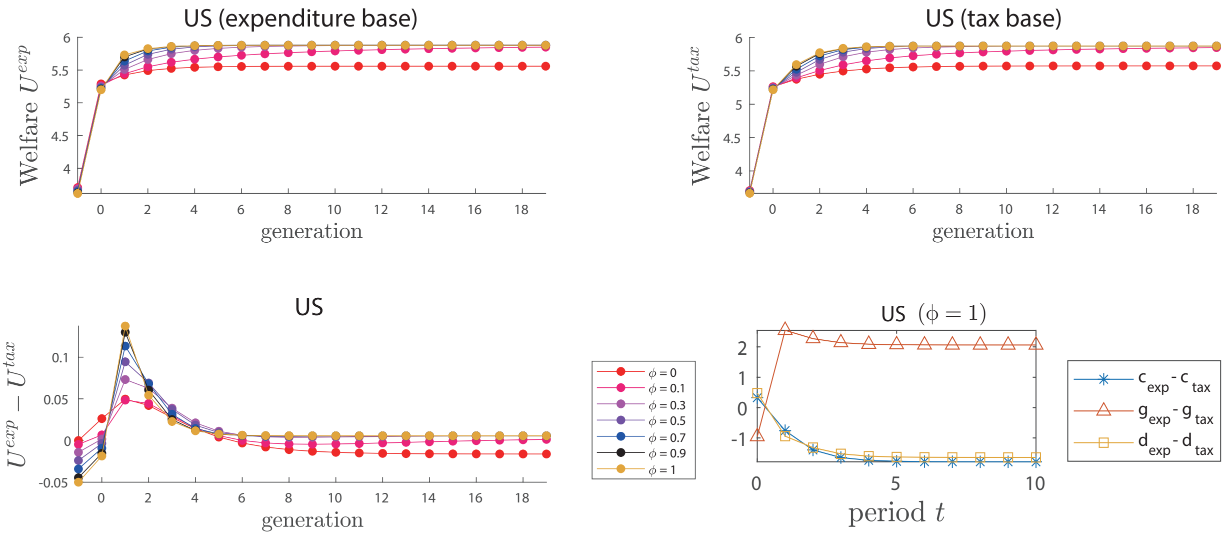

Figure 19: Welfare of each generation: the case of the US ($\theta = 0.8$)

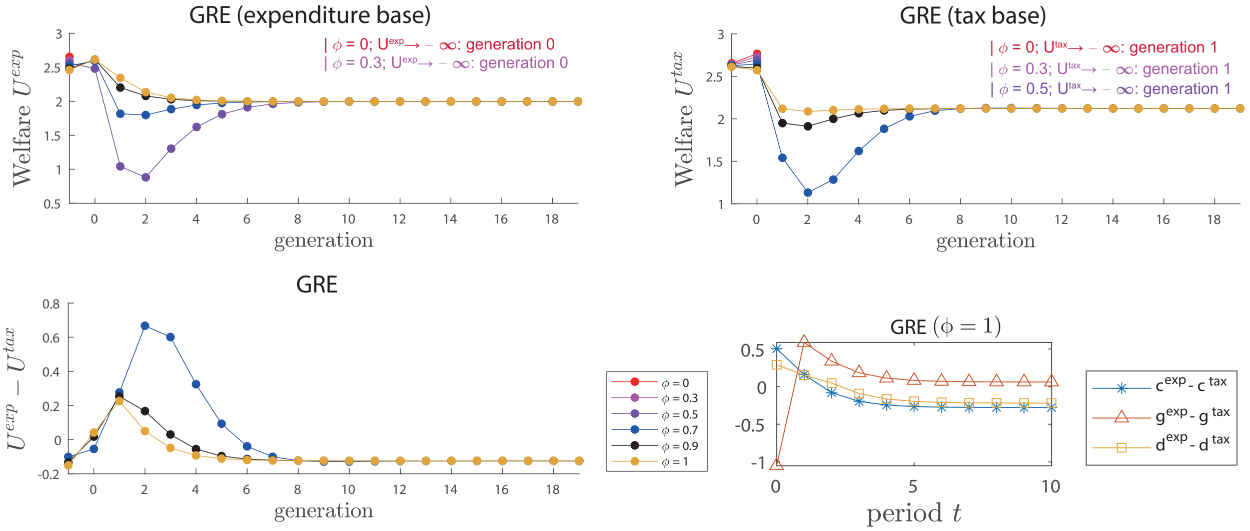

Figure 20: Welfare of each generation: the case of Greece ($\theta = 0.8$)

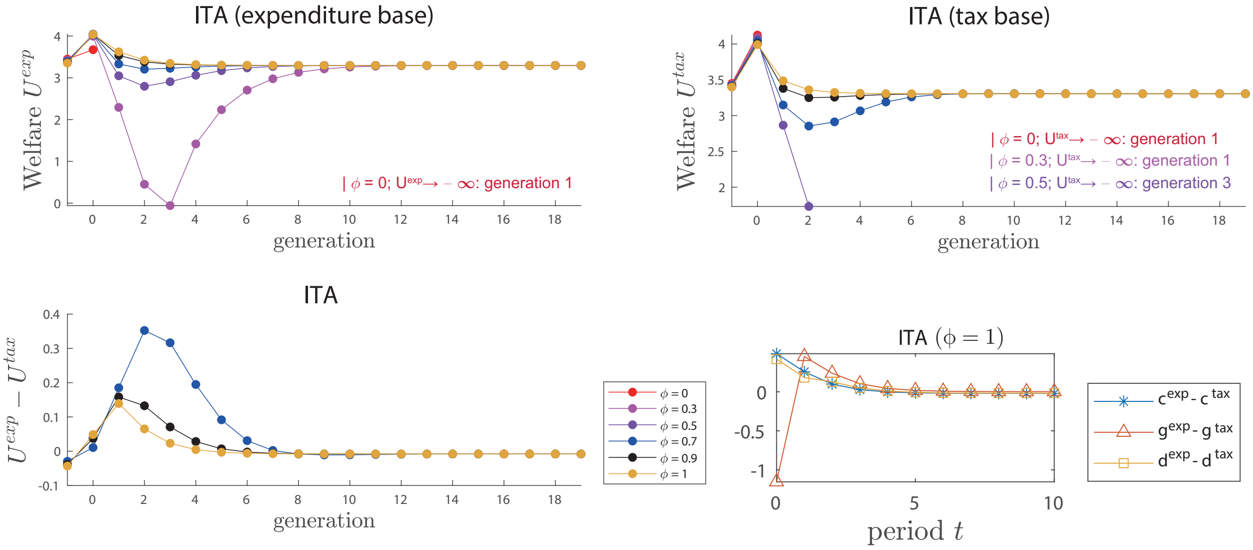

Figure 21: Welfare of each generation: the case of Italy ($\theta = 0.8$)

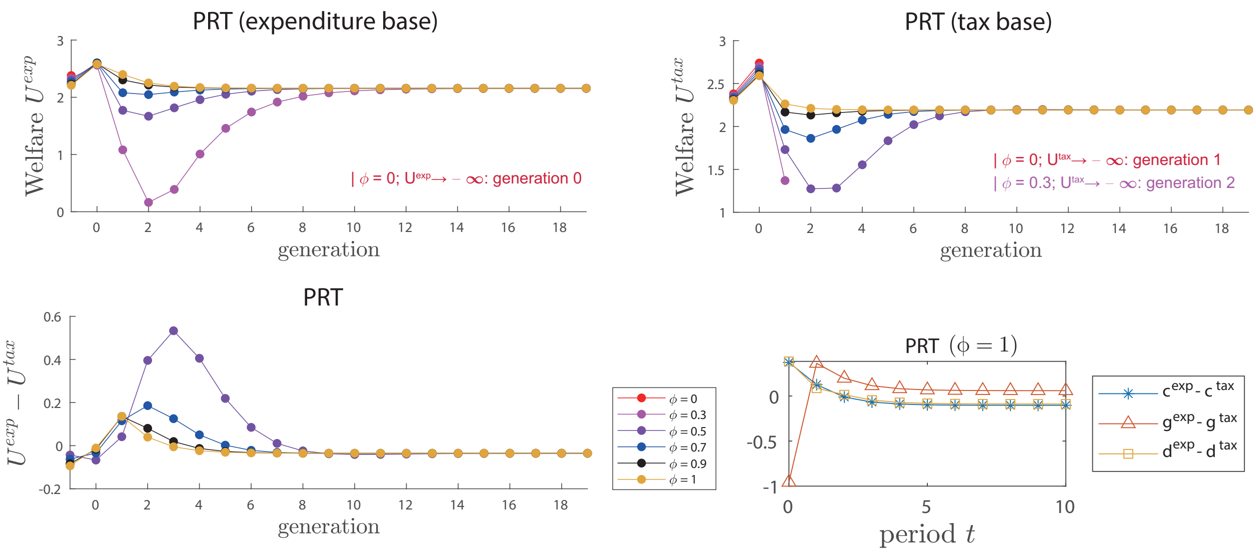

Figure 22: Welfare of each generation: the case of Portugal ($\theta = 0.8$)

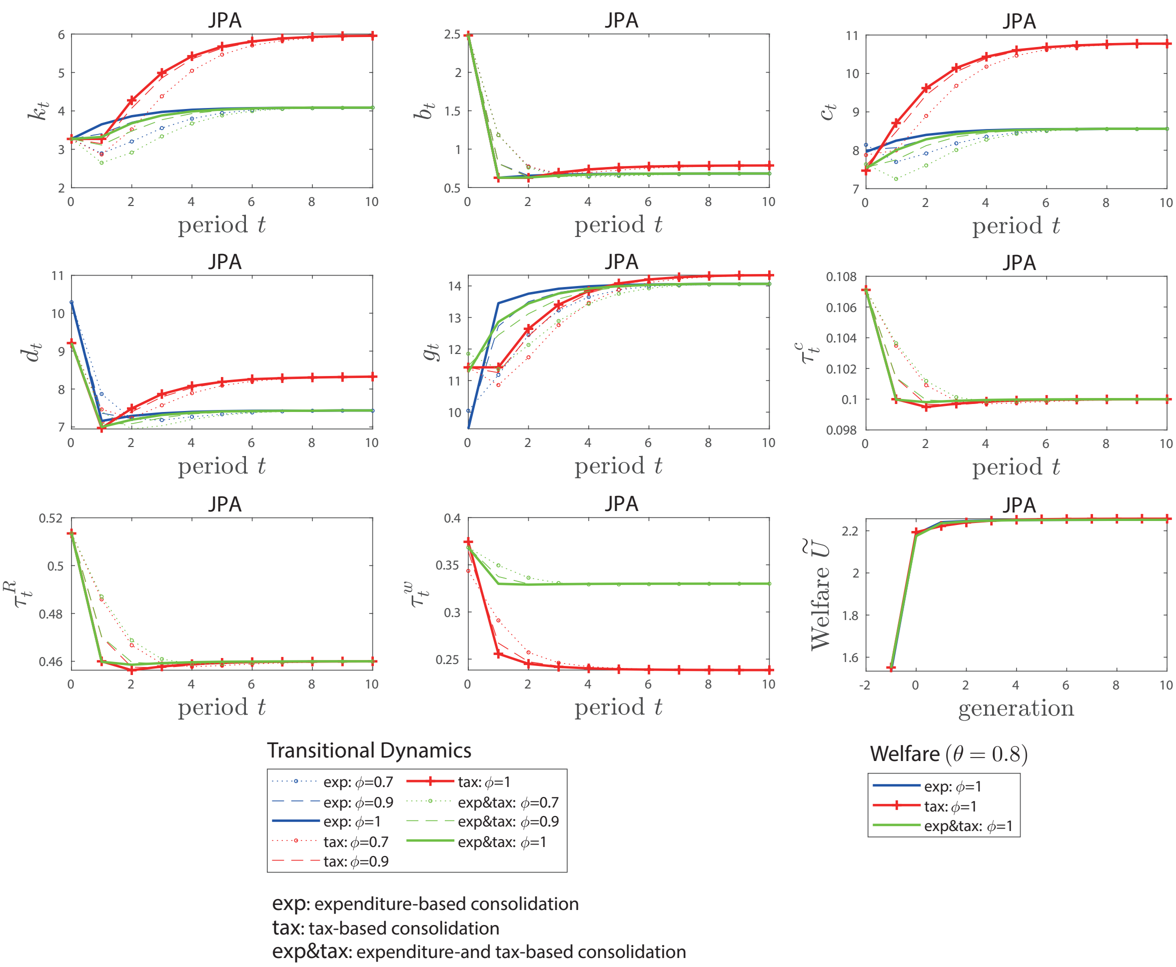

Figure 23: Transitional dynamics and welfare for Japan (CRRA utility)

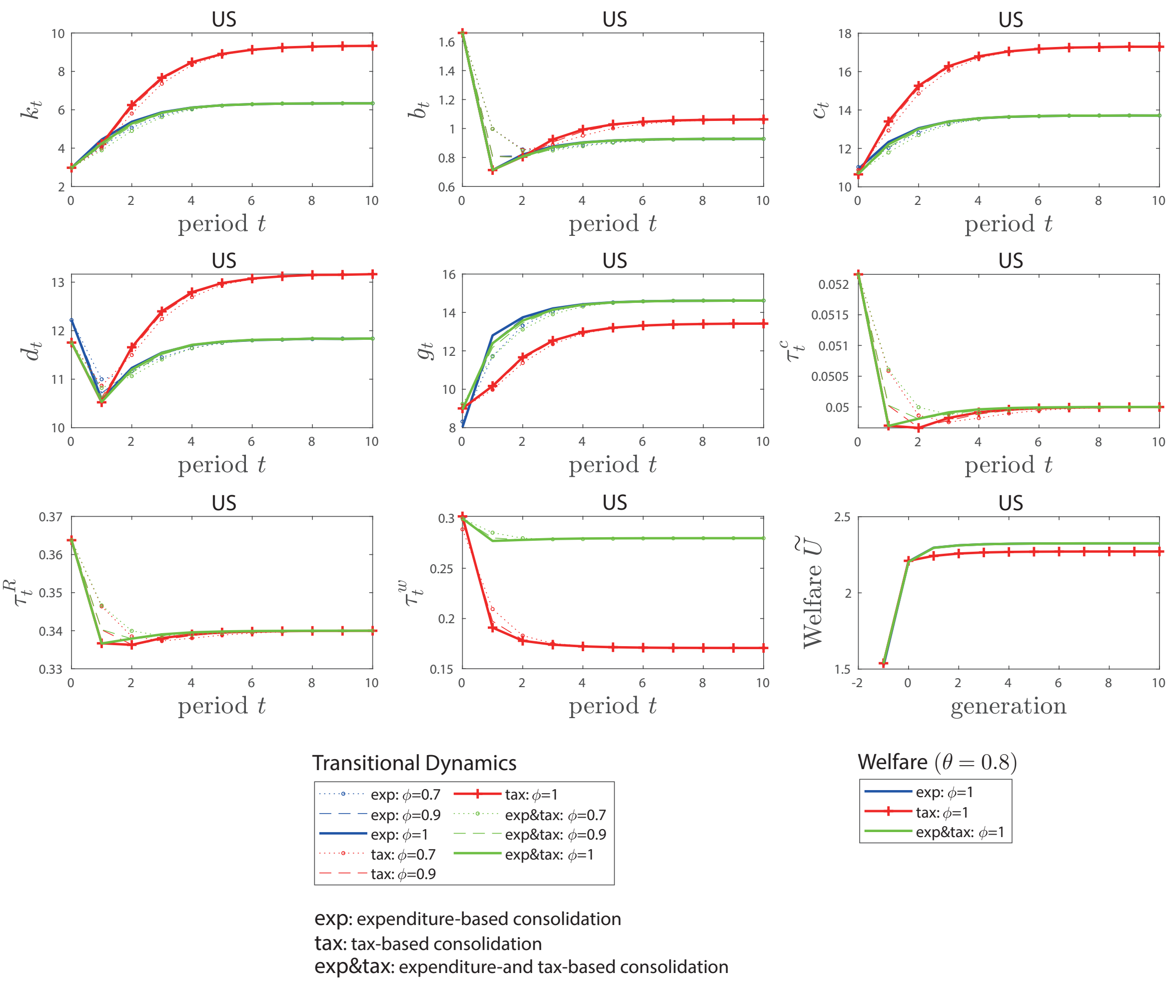

Figure 24: Transitional dynamics and welfare for the US (CRRA utility)

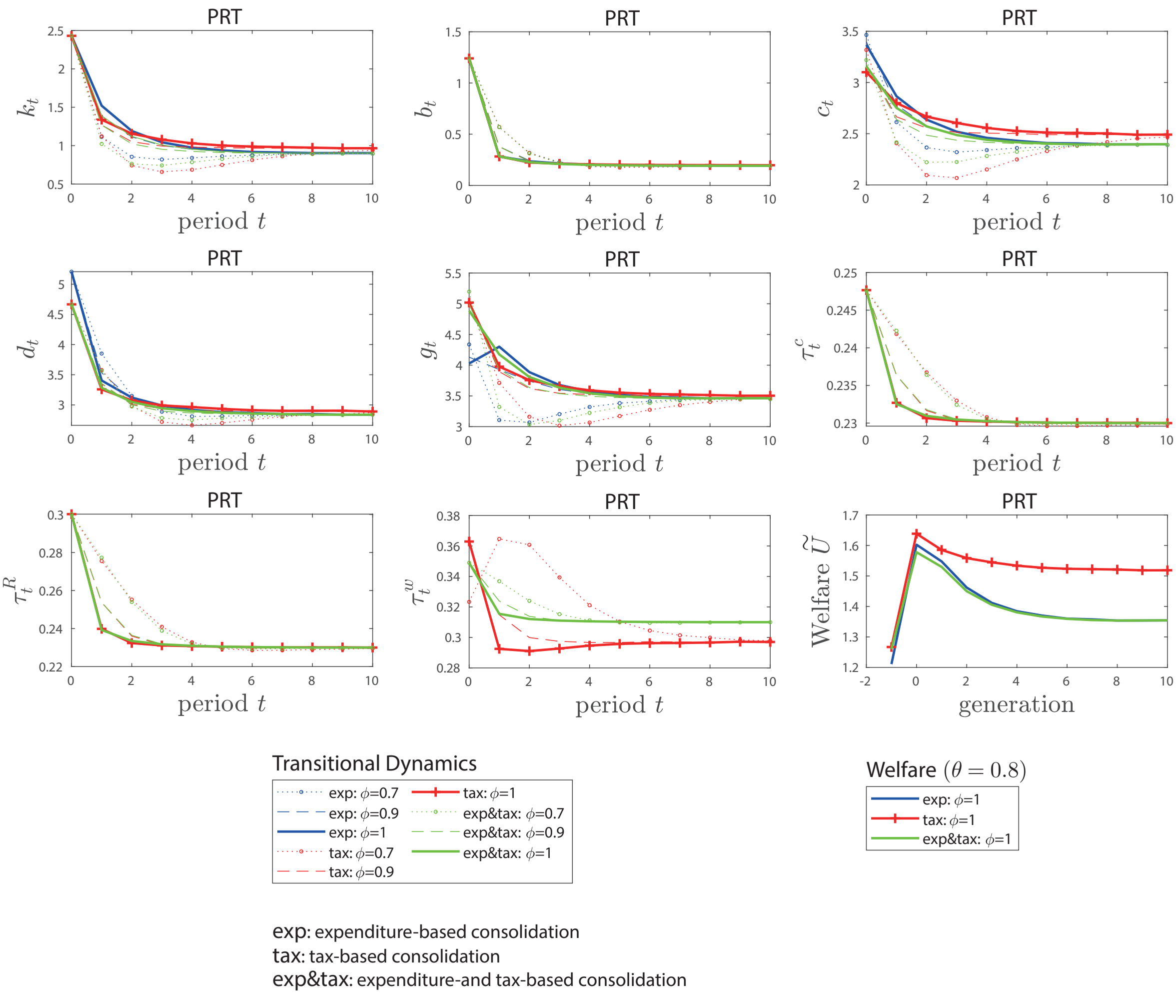

Figure 25: Transitional dynamics and welfare for Portugal (CRRA utility)

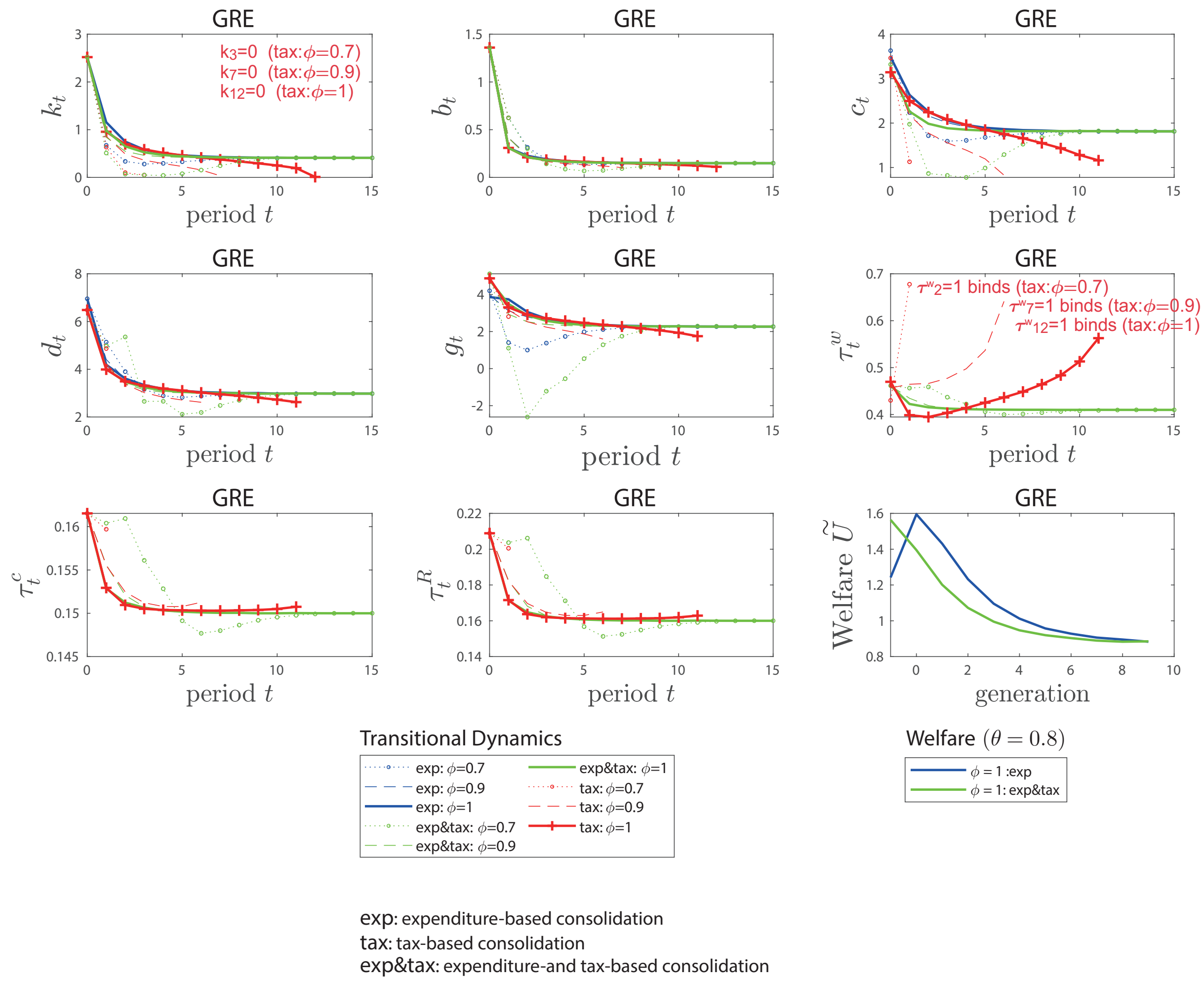

Figure 26: Transitional dynamics and welfare for Greece (CRRA utility)

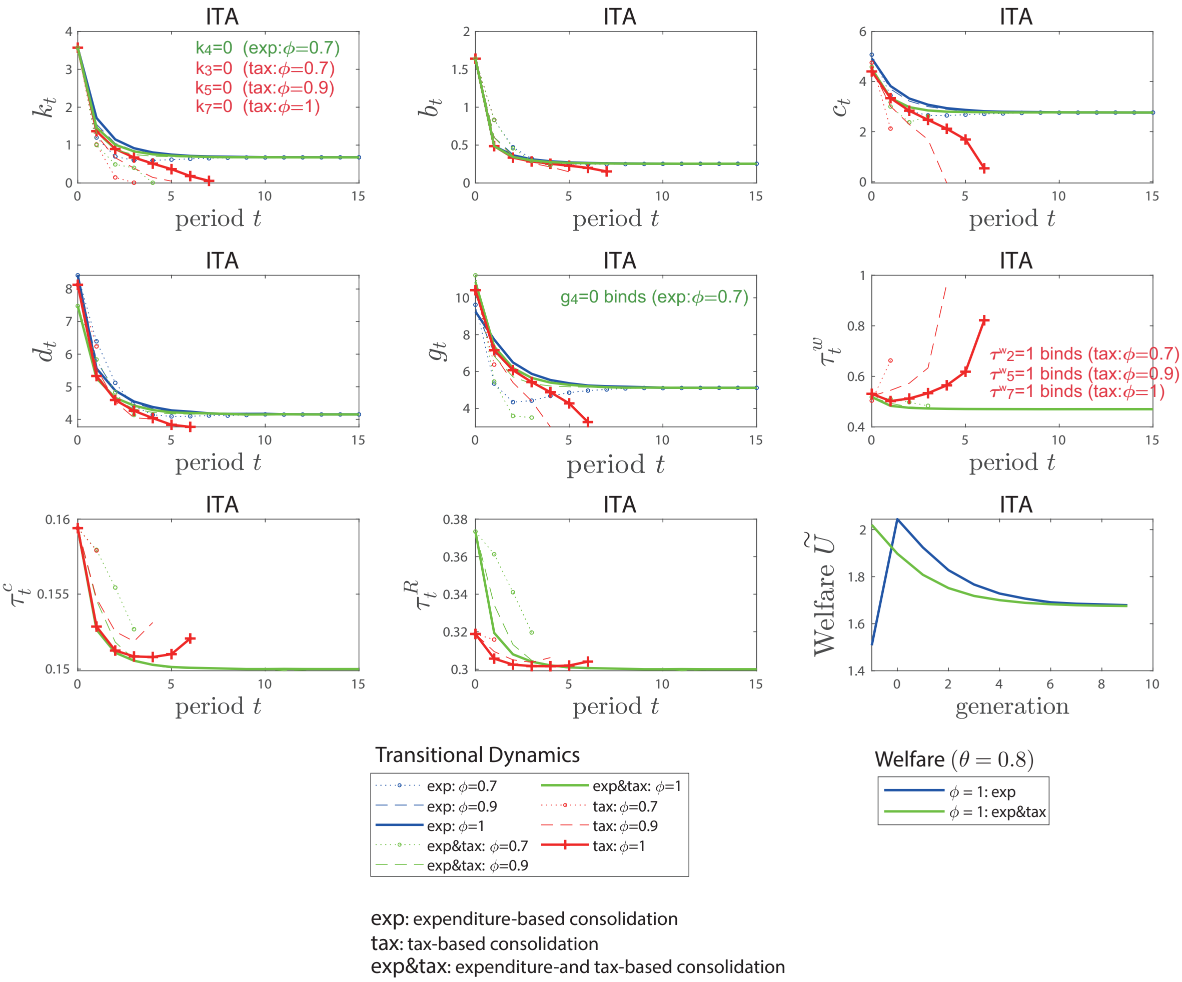

Figure 27: Transitional dynamics and welfare for Italy (CRRA utility)

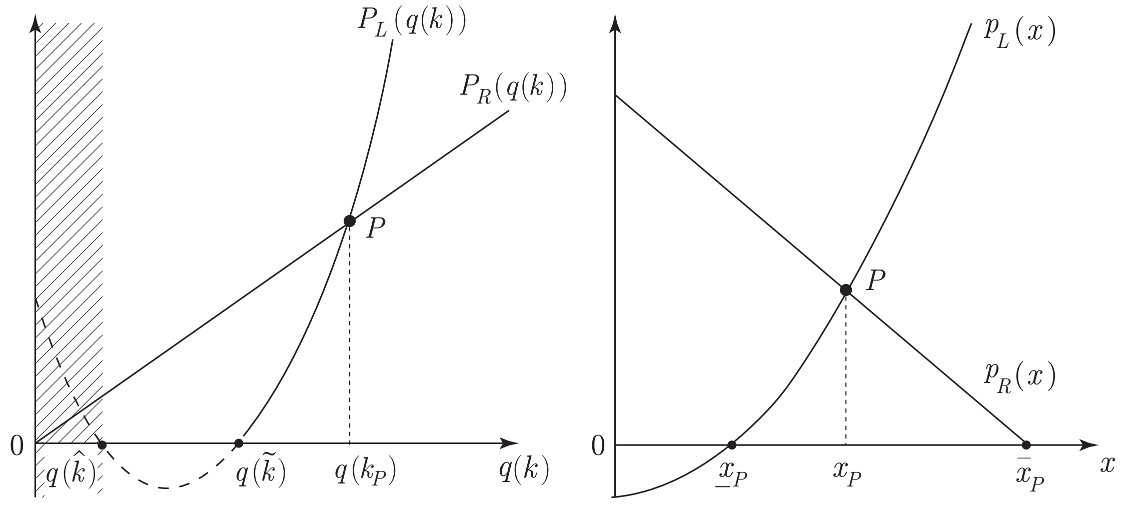

Figure 28: $q(k_P)$ and x_P

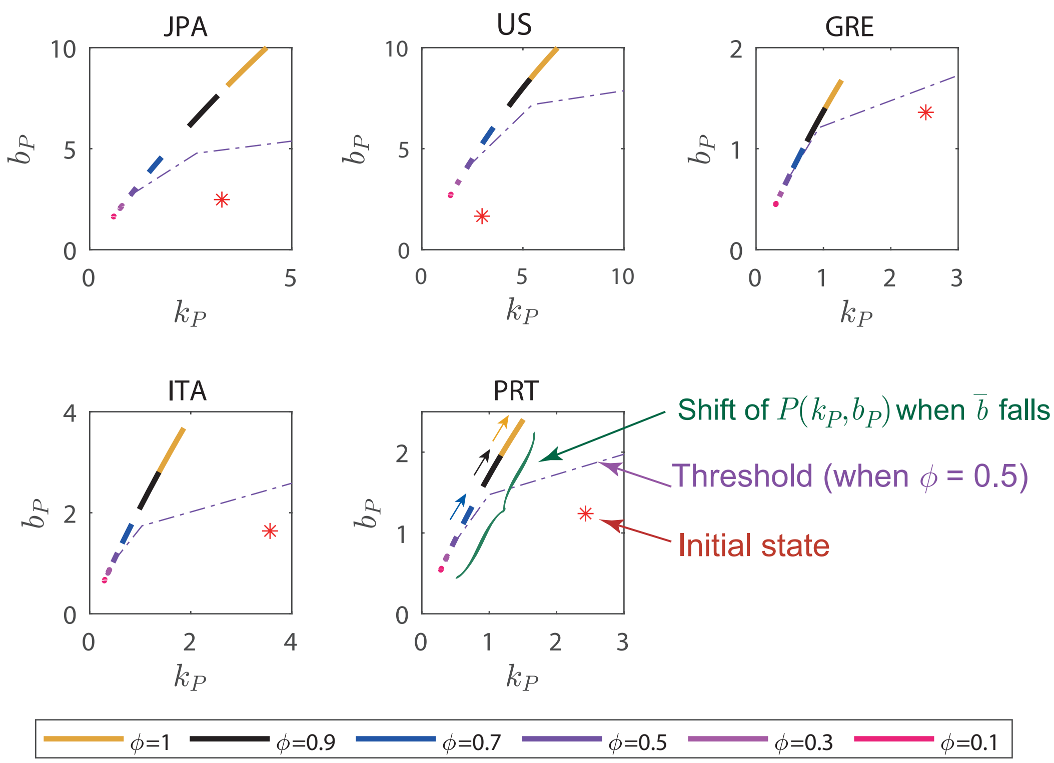

Figure 29: Effects of a fall in $\bar{b}$ on fiscal sustainability