On Labor Productivity Growth and the Wage Share with Endogenous Size and Direction of Technical Change

Zamparelli, Luca

'Sapienza' University of Rome

April 2022

On Labor Productivity Growth and the Wage Share with Endogenous Size and Direction of Technical Change

Luca Zamparelli

April 8, 2022

Abstract

This paper combines induced innovation and endogenous growth to investigate both the relation between the wage share and labor productivity growth and the long-run determinants of the wage share. We assume that myopic competitive firms choose the size and direction of technical change to maximize the growth rate of profits. We first prove that the optimal choice of labor productivity growth may be either a positive or a negative function of the wage share, depending on specific restrictions on the innovation technology. Next, by embedding the microeconomic problem into a Classical growth model, we show that a rise in the saving rate may reduce the steady state wage share. Both results conflict with the standard findings of the induced innovation literature, where labor productivity growth is always a positive function of the wage share and where the steady state labor share is independent of the saving rate.

1 Introduction

The notion that high real wages or real wage growth may foster labor productivity growth is well-established both in economic history and in economic theory. The Habakkuk hypothesis (Habakkuk, 1962) maintains that in the nineteenth century the pace of labor-saving technical change was faster in the

United States than in Britain because of scarcer and more expensive labor. Allen (2009) singled out the high price of labor relative to energy costs as one of the fundamental forces that triggered the British industrial revolution.

From a theoretical standpoint, this connection is rooted in the incentive to introduce labor-saving innovations for competitive, profit-maximizing, firms that face high labor costs. It has been formally developed and investigated within different analytical frameworks. The theory of induced technical change traces back to Hicks's conjecture that "a change in the relative prices of the factors of production is itself a spur to invention....directed to economizing the use of a factor which has become relatively expensive" (Hicks, 1932, p.124). This result was later proved independently by Kennedy (1964) and von Weizsäcker (1962). They assumed the existence of an innovation possibility frontier (IPF hereafter), which describes the trade-off between freely available capital- and labor-augmenting innovations. The IPF is decreasing and strictly concave so that substituting capital- to labor- saving technical change becomes progressively harder as capital productivity growth increases. Myopic competitive firms choose a point on the IPF, that is the *direction* of technical change, in order to maximize the rate of unit cost reduction, or equivalently the rate of growth of the profit rate, given the level of labor and capital employed. The firms' optimal choice produces a relation between the direction, or bias, of technical change and functional income distribution: labor- (capital-) productivity growth becomes a positive function of the wage (profit) share. At the macroeconomic level, the mechanism of induced innovation, also known as induced innovation hypothesis (Funk, 2002), has been implemented both in neoclassical (Drandakis and Phelps, 1965; von Weizsacker, 1966) and Classical (Shah and Desai, 1981; van der Ploeg, 1987; Foley, 2003; Julius, 2005) growth models with exogenous labor supply. One important implication of these models concerns long-run income distribution. In steady state, the wage share only depends on the shape of the innovation possibility frontier; it is 'exogenous' in the sense that changes in the economy's saving preferences do not affect it. In particular, the curvature of the IPF at the point where capital productivity growth is zero uniquely determines the long-run level of the wage share.

The same positive relation between the wage share and labor productivity growth can be found in a recent literature, which has introduced endogenous, costly, technical change in Classical models of growth. In these contributions (Foley et al., 2019, Ch.9; Tavani and Zamparelli, 2021), competitive firms choose the intensity, or *size*, of technical change rather than its direction. In fact, capital productivity is fixed and firms can only augment the productivity of labor. Specifically, they need to decide how to allocate resources between the alternative uses of physical capital accumulation and labor-saving R&D investment. In this context, a higher wage share makes R&D investment relatively more profitable so that firms divert funds from physical to R&D investment thus raising labor productivity growth. Contrary to the induced innovation theory, the saving rate affects long-run income distribution in Classical growth models with endogenous intensity of technical change and exogenous labor supply. In this framework, the wage share is not constrained by the slope of the IPF when capital productivity growth is zero and it will adjust to balance the warranted and the natural growth rate, both of which are affected in different ways by the saving rate. In Tavani and Zamparelli (2021), a higher propensity to save raises capital accumulation (the warranted growth rate) more than labor productivity growth (the natural growth rate): the wage share increases as a result of higher labor demand relative to its fixed supply.

This paper offers a synthesis of induced and endogenous technical change to investigate both the relation between the wage share and labor productivity growth and the long-run determinants of the wage share. In line with to the induced innovation tradition, we assume that myopic competitive firms maximize the instantaneous rate of growth of profits subject to an innovation technology. The set of capital- and labor- saving innovations, however, is not freely available to firms but depends on the amount of R&D investment they perform. Accordingly, firms maximize their objective function by simultaneously choosing the allocation of funds between capital accumulation and R&D investment, which determines the size of technical change, and whether to direct technological progress relatively more toward capital- or labor- saving innovations, the direction of technical change. This integration is relevant because the emerging relation between the wage share and labor productivity growth is not necessarily positive, contrary to both the literatures we reviewed, and because this possibility affects the relation between the saving rate and the wage share in steady state. In particular, we make the following two contributions. First, we devise the restrictions on the innovation technology necessary for a 'perverse', negative, relation between the wage share and labor productivity growth; and we discuss this result in light of the original 1960s debate that followed the emergence of the induced innovation theory. Secondly, we embed our microeconomic analysis into a Classical growth model with exogenous labor supply. We show that the saving rate affects the long-run distribution of income and that its rise may reduce the steady state wage share. This conflicts with the long-run results obtained both in the original induced innovation literature, where the labor share is a mere function of the slope of the IPF and thus independent of the saving rate, and in Classical growth models with constant output-capital ratio, where the saving rate and the wage share always move in the same direction. It is useful to anticipate the intuition underlying these results. They both depend on a specific feature of the innovation technology, that is the possibility that the level of R&D investment affect the trade-off between labor- and capital- saving innovations. In the original induced innovation literature, a rise in the wage share enhances labor productivity growth because firms' optimality condition will make them move along the IPF to points where its slope (in absolute terms) is lower. These points are necessarily associated to higher (lower) labor (capital) productivity growth. In our setting, on the contrary, restoring optimality after an increase in the wage share may not require an increase in labor productivity growth if the higher wage share, through its effect on R&D investments, bends the trade-off between labor and capital productivity growth. The logic is similar with respect to our second result. When firms only choose the direction of technical change, the steady state wage share is determined by the slope of the IPF irrespective of the saving rate. However, when they also simultaneously choose the intensity of innovation, the saving rate influences long-run income distribution if its effect on R&D investments also deforms the trade-off between labor and capital saving innovations.

At the onset of the induced innovation literature, a number of contributions have investigated the simultaneous choice of direction and intensity of technical change. Kamien and Schwartz (1969) explored the problem from a microeconomic point of view under the alternative assumptions of myopic and forward looking competitive firms. Nordhaus (1967) solved the infinite horizon problem of a benevolent planner who maximizes the discounted value of consumption per capita. von Weizsäcker (1966) analyzed a competitive two-sector economy. The innovation technology they adopt, however, produces the standard positive relation between the wage share and labor productivity growth (see Kamien and Schwartz, 1969, p. 676, eq. 36). From this point of view, the present contribution can be seen as an inquiry into the consequences of generalizing their assumptions on technology.

More recently, the joint determination of intensity and direction of technical change has also been analyzed by Acemoglu (2002, 2003, 2007) within the endogenous growth framework based on monopolistic competition developed at the beginning of the 1990s (see Segerstrom et al. (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). He focuses more on the relation between relative factors scarcity, rather than relative factors share, and factors productivity growth. He shows that the factors elasticity of substitution is crucial in determining the sign of this relation. When the elasticity is lower (higher) than one, a scarcer labor supply will favor labor (capital) augmenting innovations. Our contribution shows that even with zero factors elasticity of substitution the relation between labor productivity growth and relative factors shares can be either positive or negative.

Finally, Zamparelli (2015) has introduced the endogenous direction and intensity of technical change in a Classical growth model with exogenous labor supply. On the one hand, he does not find an explicit relation between labor productivity growth and the wage share; on the other, even though he finds that the saving rate affects the wage share, he does not discuss the technological assumptions necessary for this result.

The rest of the paper is organized as follows. Section 2 presents the microeconomic problem of the firm and derives the relation between the wage share and labor productivity growth. Section 3 analyzes the macroeconomic long-run equilibrium of the model with a specific focus on the connection between the saving rate and the wage share. Section 4 concludes.

2 The Model

2.1 Households and firms

The economy is populated by a fixed number (normalized to one) of identical households, who are endowed with one unit of homogeneous labor (L) and own a certain share of the capital stock (K). Households supply labor inelastically and, if employed, earn the real wage rate w; they also earn profit income on the capital they own. They save a constant fraction (s) of their total income, which they transfer to a representative firm. The firm invests aggregate savings to either increase its physical capital stock or to improve technology[1].

2.2 Technology

The final good Y is produced using labor and capital in fixed proportions. Letting A and B denote, respectively, labor and capital productivity, the production function is

$$Y = \min\{AL, BK\}. \tag{1}$$

The modeling of technological change includes insights from both the induced innovation literature and endogenous growth theory. As anticipated in the Introduction, the former represented the evolution of technology through an IPF, which states an inverse relation between the freely available maximum growth rates of labor and capital productivity. The frontier is decreasing and strictly concave in order to capture the increasing complexity in the trade-off between labor-augmenting and capital-augmenting innovations. On the other hand, the endogenous growth literature (see for example Aghion, 2010) posited

[1]The assumption of a representative firm may appear restrictive, but it is equivalent to assuming a fixed number of firms, each of which has access to the same technology and to the same fraction of aggregate savings.

that technical change is a costly activity, which requires the investment of physical or human resources. If we let g_x be the growth rate of variable x, we have

$$g_A = f(g_B, b), \tag{2}$$

where $b \equiv R/Y$ and R is the amount of physical output invested in R&D. f is twice continuously differentiable, and we incorporates the labor- capital- productivity trade-off described by the IPF by assuming $f^{'}_{g_B} < 0, f^{''}_{g_B,g_B} < 0$. On the other hand, $f^{'}_b > 0$ makes the position of the IPF dependent on R&D investment: higher investment raises the highest achievable labor productivity growth rate for any given capital productivity growth rate. We make the additional assumption $f^{''}_{b,b} < 0$, which implies decreasing returns to R&D. Notice also that the normalization of R&D investment by total output is imposed in order to rule out explosive growth; this is a standard result in endogenous growth models when R&D inputs consist of an accumulable factor such as physical output, and it is typically justified with the increasing complexity of discovering new ideas. We do not place restrictions on the sign of the second-order mixed partial derivatives, but we know from Young's theorem that $f^{''}_{g_B,b} = f^{''}_{b,g_B}$.

2.3 Income distribution, saving allocation and optimal productivity growth

The representative firm has no incentive to keep spare capacity or hire unproductive labor, therefore $AL = BK$, so that the number of employed workers in the economy is $L = BK/A$. We denote the wage share as $\omega \equiv wL/Y = w/A$, equal to the unit labor cost. Accordingly, total profits are $\Pi = Y - wL = Y(1-\omega) = BK(1-\omega)$. The next step is the description of how savings are allocated to physical capital accumulation and R&D investment. From the standpoint of a profit-seeking firm, the two types of investment pose a trade-off.

They both increase total profits. While capital accumulation increases the size of a firm, innovations raise its profits per unit of capital by reducing unit costs. Letting δ be the share of savings invested in R&D, the R&D investment share of output is:

$$b = R/Y = \delta sY/Y = \delta s. \tag{3}$$

Physical capital accumulation, on the other end, obeys:

$$g_K = (1-\delta)sY/K = (1-\delta)sB. \tag{4}$$

We assume that the representative firm acts myopically and choose δ and g_B in order to maximize the instantaneous rate of growth of profits. In fact, this is the same objective function originally assumed by the induced innovation literature[2] (Kennedy, 1964). While in Kennedy's model firms choose only the direction of technical change given the position of the IPF and the factors employment, we let it choose both the intensity and the direction of technical change. In so doing, the firm also chooses how much to invest in physical capital. Differentiating total profits with respect to time we find $\dot{\Pi} = \dot{B}K(1-\omega) + \dot{K}B(1-\omega) + g_A\omega BK$, where the time derivative of variable x is denoted by $\dot{x}$. The corresponding rate of growth of profits is

$$g_\Pi \equiv \dot{\Pi}/\Pi = g_B + g_K + g_A\omega/(1-\omega). \tag{5}$$

Substituting from equations (2), (3) and (4), the firms' problem is to choose δ and g_B so as to maximize $g_\Pi = g_B + s(1-\delta)B + f(g_B, s\delta)\omega/(1-\omega)$. We study this problem by first assuming two specific functional forms for f and

[2] In the original induced innovation theory, to be precise, firms maximize the rate of growth of the profit *rate*, rather than the rate of growth of profits. But the two rates coincide when the level of capital stock is given, as commonly assumed in that literature.

later discussing the general case.

2.3.1 Two special cases

Let us start by positing $g_A = f(g_B, b) = h(g_B) + (s\delta)^{\alpha}$, with $\alpha \in (0,1)$ and $h', h'' < 0$. Under this functional form, R&D investments produce a vertical translation of the IPF and do not affect the terms of the trade-off between labor- and capital- productivity growth. If we denote the optimal level of a choice variable by $*$, the first order conditions with respect to g_B and δ are

$$h'(g_B^*) = -\frac{1-\omega}{\omega}, \tag{6}$$

and (after some manipulations)

$$\delta^* = \frac{1}{s}\left(\frac{\alpha}{B}\frac{\omega}{1-\omega}\right)^{\frac{1}{1-\alpha}}. \tag{7}$$

Equations (6) and (7) show that the choice of direction and intensity of technical change decomposes into two parts. Equation (6) demands the equality between the slope of the IPF and of the relative unit cost; this is the same exact tangency condition, which produced the positive relation between the wage share and labor productivity growth under the original induced innovation hypothesis. In fact, total differentiation of (6) yields $dg_B/d\omega = 1/\left(h'(g_B^*)\,(1-\omega)^2\right) < 0$: for a given amount of R&D investments (the position of the IPF), a rise in the wage share biases the direction of technical change away from capital productivity growth and in favor of labor productivity growth. Equation (7), on the other hand, shows that R&D investments are a positive function of the wage share because raising productivity growth becomes relatively more profitable than capital accumulation when unit labor costs increase. We can use the optimal values for δ and g_B to solve for the equilibrium labor productivity growth as

$$g_A^* = \left(\frac{\alpha}{B}\frac{\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}} + h(g_B^*),$$

which shows that an increase in the wage share unequivocally raises labor productivity growth given $h'(g_B^*)dg_B^*/d\omega > 0$. In other words, both effects of the labor share on the direction and the size of technical change move in the same direction to contribute to labor-saving technical change.

We can now consider the alternative specification $g_A = f(g_B, b) = h(g_B)\,(s\delta)^{\alpha}$. In this case, R&D investments dilates the IPF up and to the right. The IPF becomes steeper (in absolute terms) and the trade-off between labor- and capital-productivity growth changes. After some tedious but straightforward manipulations, the first order conditions with respect to g_B and δ are

$$\delta^* = -\frac{1}{s}\left(\frac{1-\omega}{\omega}\frac{1}{h'(g_B^*)}\right)^{\frac{1}{\alpha}}, \tag{8}$$

and

$$\delta^* = \frac{1}{s}\left(\frac{\alpha}{B}\frac{\omega}{1-\omega}h(g_B^*)\right)^{\frac{1}{1-\alpha}}. \tag{9}$$

The system made up of (8) and (9) shows that in this case the choice of direction of technical change and size of R&D investment does not decompose. Both equations are needed to investigate how the wage share affects δ^* and g_B^*. We can equate the right hand side of the two equations to study the dependence of g_B^* on the wage share

$$-\left(\frac{1-\omega}{\omega}\frac{1}{h'(g_B^*)}\right)^{\frac{1}{\alpha}} = \left(\frac{\alpha}{B}\frac{\omega}{1-\omega}h(g_B^*)\right)^{\frac{1}{1-\alpha}},$$

or

$$\left(\frac{\alpha}{B}h(g_B^*)\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1-\alpha}}h'(g_B^*)=-1. \tag{10}$$

Equation (10) implies that capital productivity growth is not necessarily a negative function of the wage share. In fact, we prove in Appendix A that $dg_B^*/d\omega > 0$ when $\frac{\alpha}{1-\alpha}\left(h'(g_B^*)\right)^2/h(g_B^*)+h''(g_B^*) > 0$, which may or may not be true since $h'' < 0$. This result has dramatic consequences for the relation between the wage share and labor productivity growth. In order to investigate this issue, let us also posit $h(g_B) = a(1-g_B)^{\gamma}$, with $a > 0, \gamma \in (0,1)$; and use it together with (9) to find

$$g_A^* = a^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{B}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}}\left(1-g_B^*(\omega)\right)^{\frac{\gamma}{1-\alpha}}, \tag{11}$$

where we emphasized the dependence of optimal capital productivity growth on the wage share. Equation (11) shows that the wage share affects labor productivity growth both directly and through its effect on capital productivity growth. While the first effect is always positive, we have just discussed that the second one may move in the opposite direction. In Appendix A we prove that $dg_B^*/d\omega > 0$ requires $\gamma > 1-\alpha$, and that the positive effect on capital productivity growth is strong enough to ensure $dg_A^*/d\omega < 0$. When the curvature of the IPF (a positive function of γ) is relatively high, labor productivity growth becomes sufficiently sensitive to changes in capital productivity growth to ensure that the 'perverse' relation between the wage share and labor productivity growth emerges. We discuss in more depth the intuition underlying this result in section 2.4 after presenting the general case, to which we now turn.

2.3.2 The general case

We now generalize our analysis by removing any specific functional form on the innovation function $g_A = f(g_B, b)$. In this case, choosing g_B and δ to maximize $g_\Pi = g_B + s(1-\delta)B + f(g_B, s\delta)\omega/(1-\omega)$ yields the following system of first order conditions

$$f'_{g_B}(g_B^*, s\delta^*) = -\frac{1-\omega}{\omega}$$

$$f'_b(g_B^*, s\delta^*) = B\frac{1-\omega}{\omega}.$$

We are interested in understanding under what conditions this system is compatible with a negative effect of the wage share on labor productivity growth. Notice that

$$\frac{dg_A^*}{d\omega} = f'_{g_B}(g_B^*, s\delta^*)\frac{dg_B^*}{d\omega} + sf'_b(g_B^*, s\delta^*)\frac{d\delta^*}{d\omega}.$$

Accordingly, since $f'_{g_B} < 0$ and $f'_b > 0$, a necessary condition for $\frac{dg_A^*}{d\omega} < 0$ is that either $\frac{dg_B^*}{d\omega} > 0$ or $\frac{d\delta^*}{d\omega} < 0$. If we totally differentiate the system with respect to g_B^*, δ^* and ω, after rearranging we find

$$f''_{g_B,g_B}(g_B^*, s\delta^*)\frac{dg_B^*}{d\omega} + sf''_{g_B,b}(g_B^*, s\delta^*)\frac{d\delta^*}{d\omega} = \frac{1}{\omega^2}$$

$$f''_{b,g_B}(g_B^*, s\delta^*)\frac{dg_B^*}{d\omega} + sf''_{b,b}(g_B^*, s\delta^*)\frac{d\delta^*}{d\omega} = -\frac{B}{\omega^2}.$$

Let us focus on the role played by the second-order mixed partial derivatives $f^{''}_{g_B,b} = f^{''}_{b,g_B}$. When $f^{''}_{g_B,b} = f^{''}_{b,g_B} = 0$, the system simplifies to

$$\frac{dg_B^*}{d\omega} = \frac{1}{\omega^2 f''_{g_B,g_B}(g_B^*, s\delta^*)} < 0$$

$$\frac{d\delta^*}{d\omega} = -\frac{B}{\omega^2 s f''_{b,b}(g_B^*, s\delta^*)} > 0.$$

This shows that both the alternative necessary conditions for the negative effect of the wage share on labor productivity growth are violated, and $\frac{dg_A^*}{d\omega} > 0$ follows necessarily. Our first example $f(g_B, b) = [s\delta]^\alpha + h(g_B)$ statisfies $f''_{g_B,b} = f''_{b,g_B} = 0$ and it confirms $\frac{dg_A^*}{d\omega} > 0$ always. When, on the other hand, $f''_{g_B,b} = f''_{b,g_B} \neq 0$, $\frac{dg_B^*}{d\omega} > 0$ and $\frac{d\delta^*}{d\omega} < 0$ cannot be simultaneously excluded so that $\frac{dg_A^*}{d\omega} < 0$ is in principle a possibility. Our second example, where $f(g_B, b) = (s\delta)^\alpha h(g_B)$ and $f''_{g_B,b} = f''_{b,g_B} = s^\alpha \alpha [\delta]^{\alpha-1} h'(g_B) < 0$, confirms that $\frac{dg_A^*}{d\omega} < 0$ is possible when $f''_{g_B,b} = f''_{b,g_B} \neq 0$.

2.4 Discussion

We have established that labor productivity growth is necessarily a positive function of the wage share when $f''_{g_B,b} = 0$, that is when the trade-off between capital- and labor- productivity growth is independent of the size of R&D investment. Let us now dig deeper into the economic intuition of this result. If we go back to the original induced innovation hypothesis, where firms only choose the direction of technical change, we know that firms optimize their plans when the slope of the IPF equals the ratio of capital to labor unit cost, which coincides with the relative factors share (see for example Samuelson (1965, p.344), or Drandakis and Phelps (1966, p.830)). The (absolute value of the) slope of the IPF is in fact the marginal rate of transformation between labor- and capital-productivity growth and it is increasing in g_B. A rise in the wage share produces a reduction in the unit factor cost ratio, which requires a lower slope of the IPF to preserve the optimality condition. This can only be achieved by reducing g_B, which, in turn, raises labor productivity growth. When we turn to our general innovation technology, we are in the exact same situation if $f''_{g_B,b} = 0$, as shown by equation (6). However, things may change if the size of technical change affects the trade-off between capital- and labor- productivity growth ($f''_{g_B,b} \neq 0$). In this case, a rise in the wage share affects R&D investments, which alters the productivity growth trade-off; if the result is a reduction in the marginal rate of transformation, a higher labor productivity growth may not be needed to reestablish the optimality condition. In other words, a rise in the labor share reduces the unit cost ratio. Optimality requires that the marginal rate of transformation between types of innovation also declines. In the original framework, this could only be achieved by changing the direction of technical change in favor of labor productivity growth; in our case, firms now have an additional tool to ensure that the marginal rate of transformation decreases, so that labor productivity growth does not necessarily need to increase.

The relevance of our result on income distribution and labor productivity growth thus depends on the plausibility of $f''_{g_B,b} \neq 0$. If we go back to the early stages of the development of the induced innovation theory, we can learn some insights from Nordhaus' (1973) radical rejection of the theory. The absence of path dependence in the innovation technology was one of his main concerns; he found problematic that the evolution of labor and capital productivity would not affect the relative difficulty of introducing factor augmenting innovations: '..the rate of capital-augmenting technological change is everywhere independent of the level of labor augmentation. Thus as technological change accumulates, there is no effect on the trade-off between labor and capital augmenting technological change.' (p. 215) He argued that an IPF such as $g_A = f(g_B)$ was just an extremely special case of the more general one $g_A = f(g_B, A, B)$, where the actual path followed by labor and capital productivity affects the innovation set available to firms. Now, the innovation technology we proposed is different from the one advocated by Nordhaus. However, the possibility that $f^{''}_{g_B,b} \neq 0$ goes in the direction of taking path dependence into account as the amount of actually performed technical change affects the marginal rate of transformation between labor- and capital- productivity growth. Just as an example, assume that $f^{''}_{g_B,b} = 0$; in this case, given the level of capital productivity growth g_B, we can raise R&D investment to improve labor productivity growth without labor-saving innovations ever becoming relatively harder to be discovered. If on the contrary we assumed $f^{''}_{g_B,b} < 0$ as in our second example, pursuing labor productivity growth would become increasingly harder in comparative terms. This shows that $f^{''}_{g_B,b} < 0$ may in fact be a more realistic assumption than $f^{''}_{g_B,b} = 0$, so that the possible 'perverse' relation between the wage share and labor productivity growth cannot be discarded as a simple theoretical curiosity.

3 Income distribution implications

As anticipated in the Introduction, the induced innovation hypothesis has been embedded both in neoclassical and Classical growth models with exogenous labor supply. An important result common to both frameworks is that long-run income distribution depends solely on technology, and specifically on the curvature of the IPF; this implies that the saving rate and fiscal policy do not affect the steady state wage share. In this section, we show how the generalization of innovation technology to simultaneously encompass the choice of direction and size of technical change has radical consequences in terms of the role played by the saving rate in the steady state equilibrium. We illustrate this result by implementing the induced innovation hypothesis into a Classical growth model. Shah and Desai (1981) offer an example of classical growth where firms choose the direction of technical change, but where innovations can be implemented with no cost. They do so by introducing the IPF into the classical Goodwin's (1967) growth cycle model. The aggregate economy is described by three differential equations, and the output-capital ratio, the labor share and the employment rate are the three state variables (see also Foley (2003) and Julius (2005)). Since according to the original IPF firms do not perform R&D investment, labor productivity growth only depends on capital productivity growth, say $g_A = j(g_B)$, while all savings are invested in physical capital accumulation. Notice also that when exogenous labor supply is normalized to one the employment rate coincides with total employment L. In our notation, the dynamical system is:

$$j'(g_B^*) = -\frac{1-\omega}{\omega}$$

$$g_L = g_B^* + sB - j(g_B^*)$$

$$g_\omega = g_w - j(g_B^*) = m(L) - j(g_B^*),$$

where $g_w = m(L)$ is a real wage Phillips curve describing the positive effect of labor market tightness on real wage growth. Steady states require that capital productivity growth be turned off, so that $g_B^* = 0$ determines the long run wage share. If we denote steady state values by ss, we can find ω_{ss} as solution to $j'(0) = -\frac{1-\omega_{ss}}{\omega_{ss}}$. The steady state wage share is determined by the slope of the IPF where capital productivity growth is zero, irrespective of the saving rate.

Le us now explore how the dynamical system changes when innovations are costly and require investment, that is when we adopt the innovation technology $g_A = f(g_B, b)$. In particular, in order to obtain analytical conclusions, let us focus on our second example $f(g_B, b) = h(g_B)\,(s\delta)^\alpha$. We can use (8) and (9) to find a differential equation for the output-capital ratio as $-\left(\frac{1-\omega}{\omega}\frac{1}{h'(g_B^*)}\right)^{\frac{1}{\alpha}} =$

$\left(\frac{\alpha}{B}\frac{\omega}{1-\omega}h(g_B^*)\right)^{\frac{1}{1-\alpha}}$. The rest of the model is

$$g_L = g_B^* + s(1-\delta^*)B - h(g_B^*)\left(s\delta^*\right)^{\alpha}$$

$$g_\omega = m(L) - h(g_B^*)\left(s\delta^*\right)^{\alpha},$$

where $\delta^* = \frac{1}{s}\left(\frac{\alpha}{B}\frac{\omega}{1-\omega}h(g_B^*)\right)^{\frac{1}{1-\alpha}}$ from (9). The system shows that the stability of the output-capital ratio $g_B^* = 0$ cannot determined the long-run wage share by itself anymore. In fact, $g_B^* = 0$, rather than solving for the equilibrium wage share, yields an isocline in the (ω_{ss}, B_{ss}) space: $-\left(\frac{1-\omega_{ss}}{\omega_{ss}}\frac{1}{h'(0)}\right)^{\frac{1}{\alpha}} = \left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{1}{1-\alpha}}$. If we also impose $g_L = 0$, while using $\delta_{ss}^* = \frac{1}{s}\left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{1}{1-\alpha}}$, we obtain an additional isocline in the (ω_{ss}, B_{ss}) plane: $s\left(1-\frac{1}{s}\left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{1}{1-\alpha}}\right)B_{ss} = \left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{\alpha}{1-\alpha}} s^{\alpha}h(0)$. The two isoclines jointly determine the long-run values of the wage share and the capital-output ratio. Since the saving rate enters the second isocline both through capital accumulation and through the size of R&D investment, it also affects the steady state wage share. In Appendix B we show that the two isoclines can be used to find ω_{ss} as solution to

$$\alpha\left(-h'(0)\right)^{1/\alpha}s = \alpha\left(\frac{1-\omega_{ss}}{\omega_{ss}}\right)^{\frac{1}{\alpha}} + s^{\alpha}\left(\frac{1-\omega_{ss}}{\omega_{ss}}\right)^{\frac{1+\alpha}{\alpha}}.$$

Total differentiation with respect to ω_{ss} and s, also developed in Appendix B, shows that $sign\frac{d\omega_{ss}}{ds} = sign\left(\left(\frac{1-\omega_{ss}}{\omega_{ss}}\right)^{\frac{1+\alpha}{\alpha}} - s^{1-\alpha}\left(-h'(0)\right)^{1/\alpha}\right)$. This condition shows that we cannot sign a priori the relation between the saving rate and the steady state equilibrium wage share. The reason is that we cannot establish how the saving rate affects R&D investment share in the long-run, that is $b_{ss} = s\delta_{ss}^*$; in fact, the optimal allocation of investment is itself some function of the steady state output-capital ratio and wage share $\delta^*_{ss} = \delta(B_{ss}, \omega_{ss})$, which in turn depend on the saving rate. In our specific example, where $f''_{g_B,b} < 0$, if a rise in the saving rate increased R&D investment then the trade-off between the two types of innovations would become steeper and the steady state wage share would be lower; if, on the contrary, the higher saving rate produced a reduction in R&D investment, the wage share would increase.

This result appears particularly significant once compared with the predictions of Classical labor-constrained growth models both with and without induced technical change. First, the mere possibility that a rise in the saving rate reduces the long-run equilibrium wage share is particularly remarkable because it contrasts with the model without technical change. In this case, the relation between the saving rate and the labor share is as follows. A rise in the saving rate produces an increase in capital accumulation. Since labor supply is fixed, or grows at a constant rate, higher accumulation tightens the labor market. Real wages grow relative to labor productivity and the wage share increases. In fact, this same mechanism is at work in Tavani and Zamparelli (2021) even though labor productivity growth is endogenous. Secondly, the other hand, we have seen that when the induced direction of technical change is added to this framework, the saving rate becomes irrelevant in determining the long-run wage share and the wage share is merely a function of the curvature of the IPF at $g_B = 0$. Under our generalization, on the contrary, the saving rate affects the steady state wage share when $f''_{g_B,b} \neq 0$. The sign of the relation will depend both on the actual sign of $f''_{g_B,b}$ and on whether R&D investments rise or fall after a rise in the saving rate.

4 Conclusions

Most advanced economies have recently experienced a slowdown in productivity growth (Dieppe, 2021). The notion that declining, or low, real wages may be contributing to this trend is becoming increasingly more popular in the public debate: 'Faced with reduced labour costs, employers have lesser incentives to substitute capital for labour, especially in labour intensive sectors, which hinders diffusion of artificial intelligence and other technologies.' (ILO, 2018). More in general, several commentators have suggested that rising income inequality is likely an important factor in explaining the present sluggish level of economy activity known a 'secular stagnation'. This relation may operate both through demand side factors, such as a higher average propensity to save (see for example Summers, 2014; Storm, 2017; and Kiefer et al. 2020), and by means of supply side elements, like the limited incentives to innovate due to low labor costs (Petach and Tavani, 2020).

Our paper has reviewed different strands of economic literature that, by focusing either on the direction or on the size on innovation, have provided strong microfoundations for a positive relation between the wage share and labor productivity growth. It has shown that this relation may also be present when the innovation technology allows firms to simultaneously choose both the direction and the size of innovation. However, it has proved that under specific technological restrictions the sign of the relation may change. In particular, it has established that labor productivity growth may be a negative function of the wage share when the level of R&D investment, that is the size of technical change, affect the trade-off between labor- and capital- productivity growth. Furthermore, it has shown that when this negative 'perverse' relation emerges, the saving rate influences the long-run distribution of income and its rise may reduce the steady state wage share. This result contrasts with the implications of most Classical labor constrained growth models.

5 References

Aghion, P. and Howitt, P. (1992), A Model of Growth Through Creative Destruction. *Econometrica*, 60(2): 323-351.

Aghion, P., and Howitt, P. (2010), *The Economics of Growth.* Cambridge, MA: MIT Press.

Allen, Robert C. (2009), *The British Industrial Revolution in Global Perspective.* New York: Cambridge Univ. Press.

Bester, H. and E. Petrakis (2003), Wages and Productivity Growth in a Competitive Industry. *Journal of Economic Theory* 109 (1): 52-69.

Dieppe, A. ed. (2021), *Global Productivity: Trends, Drivers, and Policies.* Washington, DC: World Bank.

Drandakis, E. M., and Phelps, E. S. (1965), A model of induced invention, growth and distribution. *Economic Journal* 76 (304): 823–40.

Foley, D. K. (2003), Endogenous technical change with externalities in a classical growth model. *Journal of Economic Behavior and Organization* 52 (2): 167-189.

Grossman, G. and Helpman, E. (1991), *Innovation and Growth in the Global Economy.* Cambridge, MA: MIT Press.

Habakkuk, H. J. (1962), *American and British Technology in the Nineteenth Century.* London: Cambridge Univ. Press.

Hicks, J.R. (1932[1960]), *The Theory of Wages.* London: Macmillan.

ILO (2018), Statement by Guy Ryder, ILO Director-General, to the International Monetary and Financial Committee, https://www.ilo.org/global/about-the-ilo/newsroom/statements-and-speeches/WCMS_626195/lang--en/index.htm.

Julius, A. J. (2005), Steady state growth and distribution with an endogenous direction of technical change. *Metroeconomica* 56 (1): 101-125.

Kennedy, C. (1964), Induced bias in innovation and the theory of distribution. *Economic Journal* 74 (295): 541-547.

Kiefer, D., Mendieta-Munoz, I., Rada, C. and Von Arnim, R. (2020), Secular stagnation and income distribution dynamics. *Review of Radical Political Economics* 52(2): 189-207.

Nordhaus, W. D. (1967), The Optimal Rate and Direction of Technical Change, in Shell, K., ed. *Essays on the Theory of Optimal Economic Growth.* Cambridge, MA: MIT Press.

Nordhaus, W. D. (1973), Some Skeptical Thoughts on the Theory of Induced Innovation. *The Quarterly Journal of Economics* 87 (2): 208-219.

Petach, L. and Tavani, D. (2020), Income shares, secular stagnation and the long-run distribution of wealth. *Metroeconomica* 71 (1): 235-55.

Segerstrom, P. S., Anant, T. and Dinopoulos, E. (1990), A Schumpeterian Model of the Product Life Cycle. *American Economic Review*, 80: 1077-1092.

Shah, A., and Desai, M. (1981), Growth cycles with induced technical change. *The Economic Journal* 91 (364): 1006-1010.

Storm, S. (2017), The New Normal: Demand, Secular Stagnation, and the Vanishing Middle Class. *International Journal of Political Economy* 46(4): 169-210.

Summers, L. (2014), U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound. *Business Economics*, 49(2): 65-73.

Tavani, D. and Zamparelli, L. (2020), Growth, income distribution, and the 'entrepreneurial state'. *Journal of Evolutionary Economics* 30 (1): 117-141.

van der Ploeg, F. (1987), Growth Cycles, Induced Techincal Change, and Perpetual Conflict over the Distribution of Income. *Journal of Macroeconomics* 9 (1): 1-12.

von Weizsäcker, C.C. (1962[2019]), A New Technical Progress Function. *German Economic Review* 11 (3): 248-265.

von Weizsäcker , C.C. (1966), Tentative notes on a two sector model with induced technical progress. *Review of Economic Studies* 33 (3): 245–51.

6 Appendices

6.1 Appendix A

In order to prove that $dg_B^*/d\omega > 0$ requires $\frac{\alpha}{1-\alpha}\left(h(g_B^*)\right)^{\frac{2\alpha-1}{1-\alpha}}\left(h'(g_B^*)\right)^2 + \left(h(g_B^*)\right)^{\frac{\alpha}{1-\alpha}} h''(g_B^*) > 0$, start by setting $-1/\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1-\alpha}} \equiv G(\omega)$ and $\left(h(g_B^*)\right)^{\frac{\alpha}{1-\alpha}} \equiv H(g_B^*)$ in (10) to find

$(\alpha/B)^{\frac{\alpha}{1-\alpha}} H(g_B^*)h'(g_B^*) = G(\omega)$. Total differentiation w.r.t. ω and g_B^* yields $dg_B^*/d\omega = G'(\omega)/\Big((\alpha/B)^{\frac{\alpha}{1-\alpha}}\left(H'(g_B^*)h'(g_B^*) + H(g_B^*)h''(g_B^*)\right)\Big)$. Since $G'(\omega) > 0$, then $dg_B^*/d\omega > 0$ requires $H'(g_B^*)h'(g_B^*) + H(g_B^*)h''(g_B^*) > 0$, that is to say $\frac{\alpha}{1-\alpha}\left(h(g_B^*)\right)^{\frac{2\alpha-1}{1-\alpha}}\left(h'(g_B^*)\right)^2 + \left(h(g_B^*)\right)^{\frac{\alpha}{1-\alpha}} h''(g_B^*) > 0$. If we divide both addends by $\left(h(g_B^*)\right)^{\frac{\alpha}{1-\alpha}}$, we find $dg_B^*/d\omega > 0$ when $\frac{\alpha}{1-\alpha}\left(h'(g_B^*)\right)^2/h(g_B^*) + h''(g_B^*) > 0$.

Let us now calculate $dg_B^*/d\omega$ under the specification $h(g_B^*) = a(1-g_B^*)^{\gamma}$. If we substitute for $h(g_B^*)$ and $h'(g_B^*) = -a\gamma(1-g_B^*)^{\gamma-1}$ into (10) we find $\left(\frac{\alpha}{B}\right)^{\frac{\alpha}{1-\alpha}} a^{1/(1-\alpha)}\gamma\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1-\alpha}}(1-g_B^*)^{\frac{\alpha+\gamma-1}{1-\alpha}} = 1$. Total differentiation of the latest equation yields $\left(\frac{\alpha}{B}\right)^{\frac{\alpha}{1-\alpha}} a^{1/(1-\alpha)}\gamma\left(\frac{\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}}(1-g_B^*)^{\frac{\alpha+\gamma-1}{1-\alpha}-1}\left(\frac{1-g_B^*}{(1-\omega)^2}d\omega - (\alpha+\gamma-1)\frac{\omega}{1-\omega}dg_B^*\right) = 0$. Hence $dg_B^*/d\omega = \frac{1-g_B^*}{\omega(1-\omega)}\frac{1}{\alpha+\gamma-1}$. It follows that $dg_B^*/d\omega > 0$ requires $\alpha+\gamma > 1$.

We now turn to study $dg_A^*/d\omega$. We can use (11) to find $dg_A^*/d\omega =$

$$a^{1/(1-\alpha)}\left(\frac{\alpha}{B}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\left(\frac{\omega}{1-\omega}\right)^{\frac{2\alpha-1}{1-\alpha}}\frac{1}{(1-\omega)^2}(1-g_B^*)^{\frac{\gamma}{1-\alpha}} - \left(\frac{\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}}\frac{\gamma}{1-\alpha}(1-g_B^*)^{\frac{\alpha+\gamma-1}{1-\alpha}} dg_B^*/d\omega\right)$$

$=$

$$\left(\frac{\alpha}{B}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{1-\alpha}a^{\alpha/(1-\alpha)}(1-g_B^*)^{\frac{\alpha+\gamma-1}{1-\alpha}}\left(\frac{\omega}{1-\omega}\right)^{\frac{2\alpha-1}{1-\alpha}}\frac{1}{1-\omega}\left(\alpha\frac{1-g_B^*}{1-\omega} - \gamma\omega dg_B^*/d\omega\right).$$

If we now plug $dg_B^*/d\omega = \frac{1-g_B^*}{\omega(1-\omega)}\frac{1}{\alpha+\gamma-1}$ into the previous expression we find $dg_A^*/d\omega < 0 \iff \alpha < \frac{\gamma}{\alpha+\gamma-1}$. If the necessary condition for $dg_B^*/d\omega > 0$ is satisfied, that is if $\alpha+\gamma-1 > 0$, then $dg_A^*/d\omega < 0$ always. We can prove it by multiplying both sides of $\alpha < \frac{\gamma}{\alpha+\gamma-1}$ by $(\alpha+\gamma-1)$ to find $\alpha(\alpha+\gamma-1) < \gamma$; we can further develop the inequality to find $\alpha(\alpha+\gamma) < (\alpha+\gamma)$, which is always satisfied since $\alpha < 1$. On the contrary, when $\alpha+\gamma-1 < 0$, the condition for $dg_A^*/d\omega < 0$ becomes $\alpha(\alpha+\gamma-1) > \gamma$: it is never true so that $dg_A^*/d\omega > 0$ necessarily.

6.2 Appendix B

Starting with the first isocline, $-\left(\frac{1-\omega_{ss}}{\omega_{ss}}\frac{1}{h'(0)}\right)^{\frac{1}{\alpha}} = \left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{1}{1-\alpha}}$, divide both members by $\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1}{1-\alpha}}$ and multiply them by $(h'(0))^{1/\alpha}$ to find $-\left(\frac{1-\omega_{ss}}{\omega_{ss}}\right)^{\frac{1}{\alpha(1-\alpha)}} = (h'(0))^{1/\alpha}\left(\frac{\alpha}{B_{ss}}h(0)\right)^{\frac{1}{1-\alpha}}$. Next, consider the second isocline $s\left(1-\frac{1}{s}\left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{1}{1-\alpha}}\right)B_{ss} = \left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{\alpha}{1-\alpha}}s^\alpha h(0)$. Develop the multiplication on the left hand side and rearrange to find $sB_{ss} = B_{ss}\left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{\alpha}{1-\alpha}}s^\alpha h(0) = (h(0))^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha\omega_{ss}}{1-\omega_{ss}}+s^\alpha\right)$. Hence $B_{ss}^{\frac{1}{1-\alpha}} = (h(0))^{\frac{1}{1-\alpha}}\frac{(\alpha\omega_{ss})^{\frac{\alpha}{1-\alpha}}}{(1-\omega_{ss})^{\frac{1}{1-\alpha}}}\left(\frac{\alpha\omega_{ss}+s^\alpha(1-\omega_{ss})}{1-\omega_{ss}}\right)/s$. Plug the latest result into $-\left(\frac{1-\omega_{ss}}{\omega_{ss}}\frac{1}{h'(0)}\right)^{\frac{1}{\alpha}} = \left(\frac{\alpha}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}h(0)\right)^{\frac{1}{1-\alpha}}$ and rearrange to find $(-h'(0))^{1/\alpha}s\alpha = \frac{(1-\omega_{ss})^{\frac{1}{\alpha}}}{(\omega_{ss})^{\frac{1+\alpha}{\alpha}}}(\alpha\omega_{ss} + s^\alpha(1-\omega_{ss})) = \alpha F(\omega_{ss})^{\frac{1}{\alpha}} + s^\alpha F(\omega_{ss})^{\frac{1+\alpha}{\alpha}}$, where we used $F(\omega_{ss}) \equiv \frac{1-\omega_{ss}}{\omega_{ss}}$. We can totally differentiate the previous equation w.r.t. ω_{ss} and s to find: $F'(\omega_{ss})\left(F(\omega_{ss})^{\frac{1-\alpha}{\alpha}} + s^\alpha\frac{1+\alpha}{\alpha}F(\omega_{ss})^{\frac{1}{\alpha}}\right)d\omega_{ss} = \alpha\left((-h'(0))^{1/\alpha} - s^{\alpha-1}F(\omega_{ss})^{\frac{1+\alpha}{\alpha}}\right)ds$. Hence we can find $\frac{d\omega_{ss}}{ds} = \frac{\alpha\left((-h'(0))^{1/\alpha}-s^{\alpha-1}F(\omega_{ss})^{\frac{1+\alpha}{\alpha}}\right)}{F'(\omega_{ss})\left(F(\omega_{ss})^{\frac{1-\alpha}{\alpha}}+s^\alpha\frac{1+\alpha}{\alpha}F(\omega_{ss})^{\frac{1}{\alpha}}\right)}$. Since $F'(\omega_{ss}) < 0$, we can conclude that $sign\frac{d\omega_{ss}}{ds} = sign\left(F(\omega_{ss})^{\frac{1+\alpha}{\alpha}} - s^{1-\alpha}(-h'(0))^{1/\alpha}\right)$.