Developing an Income-Distribution-Sensitive Taylor Rule: An Application to South Africa

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Introduction

Between the late 1980s and early 1990s, monetary policy moved from being implemented in an almost discretionary manner to being implemented based on calibrating a central bank's policy reaction to fluctuations in the macroeconomy using more objective means (Taylor, 1993).

The reactions of central banks to such fluctuations can be captured through the estimation of so-called "monetary policy rules". These rules are tools to simply communicate central banks' otherwise complex strategies and reactions, ultimately aiding in chaperoning inflation or investment sentiment into the future by being at least somewhat transparent in their expected reactions.

Monetary policy rules come in many forms and often relate to how the interest rate or money supply change based on targeted macroeconomic indicators (Salter, 2014). Arguably the most often quoted of these monetary policy rules is the Taylor Rule, developed by John Taylor in 1993 before being refined in 1999 (Taylor, 1993; Taylor, 1999). These rules assess how a central bank's policy rate responds to output and inflation volatility in their simplest form.

Since its original publication in 1993, the Taylor Rule has been examined and adapted *ad nauseam*. Most of this analysis is, however, rooted in mainstream literature. The rules estimated to date usually opt to augment the seminal 1993 rule to include a central bank's preference for interest rate smoothing (Rudebusch, 2002; Orphanides, 2003), moderating exchange rate fluctuation (Taylor, 2001; Mohanty & Klau, 2005), and stabilising financial conditions (Castro, 2011), *inter alia*. In other cases, research uses the Taylor Rule as a stabilising component in a larger macroeconomic model (see Goodfriend & King, 1997; Clarida et al., 1999; Woodford, 2003; Gali & Monacell, 2005; Carlin & Soskice, 2005, among many others).

In more recent years, the general basis for studying monetary policy has also been widened to the issue of income inequality. In particular, since the mid and late 2010s, much research has been done on the impact that monetary policy might have on the distribution of income within particular countries (see Coibion et al., 2017; Mumtaz & Theophilopoulou, 2017; Samarina & Nguyen, 2019).

With this said, there is very little work available regarding the converse- while much has been explored regarding the effect of monetary policy on income distribution, very little has been done regarding the reaction of monetary policy to changes in income distribution. Although this topic has seen some interest in Post-Keynesian circles (most prominently based on works by Taylor (2004) and Ocampo et al. (2009)), a more nuanced understanding of the reaction of central banks to changes in income distribution is warranted.

Given this foothold, the current research first provides a theoretical foothold to estimate such an income-distribution-sensitive Taylor Rule before estimating the rule for South Africa to assess whether the South African Reserve Bank reacts to changes to movements in the country's income distribution. Therefore, this short article moves from proposing a theoretical framework to estimating such an interest rate rule before concluding.

Theoretical Framework

As glossed over before, to usher expectations on future inflation in a particular direction or bolster investor sentiment, central banks use non-discretionary monetary policy rules governing either the movement in the interest rate or monetary aggregates (McCallum, 1988; Taylor, 1993; Salter, 2014).

In general practice, the derivation of one such rule comes with an assumption regarding the central bank's preferences. A central bank usually concerns itself with prices and output growth rates to varying degrees. In this vein, a simple central bank loss function governing its preferences could be of the form:

$$(1)\quad \ell_t = \frac{\alpha_\pi(\pi_t)^2}{2} + \frac{\alpha_y(y_t)^2}{2}$$

Where the losses of a central bank in a particular period (ℓ_t) are governed by either an increase or a decrease in the growth rate of prices (inflation- π_t) or output (GDP - y_t) (Carlin & Soskice, 2015). α_π and α_y are, in this instance, the weights attached to the central bank's distaste for inflation and output volatility, respectively.

However, a central bank is constrained by the very nature of the economy in which it operates- its preferences are constrained by what drives inflation and what drives output growth.

The evolution of inflation is usually governed by a form of the Phillips Curve (Phillips, 1958), relating either price or wage inflation to output growth or unemployment and inflation inertia.

From the perspective of the price Phillips Curve, one could posit a parsimonious backward-looking relation linking current inflation (π_t^p) to its previous value (π_{t-1}^p) and the value of previous output growth (y_{t-1}), as shown in equation 2 (and adapted from Romer (2012)):

$$(2)\quad \pi_t^p = \gamma_p \pi_{t-1}^p + \varphi_p y_{t-1}$$

In line with the notion that price inflation feeds into wage inflation (and vice versa), the wage Phillips curve (looking at wage inflation- π_t^w) is also critical to understand when developing robust monetary policy (even though such a focus for central banks is indirect through the price inflation channel) and can be specified as follows to take into account a proxy for income distribution (Taylor, 2004):

$$(3)\quad \pi_t^w = \vartheta_p \pi_t^p + \varphi_w y_{t-1} \pm \theta_s s_{t-1}$$

Beyond the wage-price spiral effect (ϑ_p) and wage inertia being driven by previous output growth (φ_w), what is perhaps Structuralism-centric is the inclusion of a variable attempting to underpin the dynamics of class struggle and income distribution- the labourer's share of income (s_t in its contemporaneous form).

Although an imperfect proxy (in an imperfect data world), the labour share attempts to capture the degree of wealth concentration for individuals whose main income comes in the form of salaries or wages vis-à-vis those who earn the majority of their income through profits and rent. As the labour share increases, the portion of national income going to labourers increases (the largest portion of a workforce in an economy), arguably improving the distribution of income (Marx, 1867).

Importantly, to ensure agnosticism in the analysis, the sign on the coefficient attached to the labour share can either be positive or negative depending on the inflation regime of the country in question (Taylor, 2004).

If, for example, the labour share increased (perhaps due to a contraction in national income), employers might seek to lay off workers and, in so doing, control their respective wage bills. Doing so *en masse* would depress wages, implying negativity in θ_s. On the other hand, and as is more often the case, an increase in the labour share (driven, for example, by the implementation of a minimum wage) drives up future wage inflation, implying positivity in θ_s (Goodwin (1967), Desai (1973) and Barbosa-Filho & Taylor (2006))

Beyond price and wage constraints, central banks are also constrained by investment and savings behaviour within the economy (Taylor, 2009), as shown in equation 4:

$$(4)\quad y_t = \sigma_y y_{t-1} - \sigma_i (i_t - \pi_{t-1}^p) \pm \sigma_s s_{t-1}$$

Where output is related to its one-period lagged counterpart, the labourer's share in income, and the (backward-looking) real interest rate ($i_t - \pi_{t-1}^p$).

Focusing once more on the labourer's share of income, its inclusion in the relationship governing output dynamics is also dependent on the output regime in question. If, for instance, the labourer's share of income had to increase and output was to grow- economic growth is bolstered when the financial position of the working class is improved upon, thus classifying the economy in question as "wage-led" in economic growth and implying positivity in σ_s.

If, however, an increase in the labourer's share of national income relates to a decrease in economic growth, it is argued that the output dynamics of a particular country are "profit-led". Accordingly, if output dynamics are profit-led, σ_s is expected to be negative (Marglin & Bhaduri, 1991).

Before further analysis, one should first substitute the price Phillips relation (2) into its wage counterpart (3) to simplify these constraints as follows:

$$(5)\ \ \pi_t^w = \vartheta_w \gamma_p \pi_{t-1}^p + (\varphi_w + \vartheta_w \varphi_p) y_{t-1} \pm \theta_s s_{t-1}$$

However, before continuing, what is important to note is the structure of the labourer's share of income. Classically, the labourer's share of income is calculated as the ratio between the wage bill across the working-class economy $(W_t L_t)$ and the level of output/national income[1] (Y_t) and the current price level (P_t). More formally:

$$(6)\ \ s_t = \frac{W_t L_t}{Y_t} * \frac{1}{P_t}$$

Considering that wage inflation is calculated as the growth rate of per unit labour costs ($\frac{W_t L_t}{Y_t}$ $or\ NULC_t$) equation 6 can be rewritten as follows:

$$(7)\ \ s_t = NULC_t * \frac{1}{P_t}$$

Applying a simple growth accounting decomposition to the labour share as defined in (7) produces the following:

$$(8)\ \ s_t - s_{t-1} = \pi_t^w - \pi_t^p$$

[1] Assuming that the economy is in equilibrium

Solving for wage inflation in equation (8) and substituting this result into equation 5 yields the following:

$$(9)\ \pi_t^p = \vartheta_w \gamma_p \pi_{t-1}^p + (\varphi_w + \vartheta_w \varphi_p) y_{t-1} - s_t + (1 \pm \theta_s) s_{t-1}$$

Equations 4 and 9 are the two major constraints to a central bank's loss function. As such, one should look to minimise the loss of the central bank subject to these constraints in the Lagrangian format as outlined below:

$$(10)\mathcal{L}_{t+j} = \sum_{j=0}^{\infty} \beta^j \left[\begin{array}{l} \frac{\alpha_\pi \left(\pi_{t+j}^p\right)^2}{2} + \frac{\alpha_y (y_{t+j})^2}{2} + \lambda_{1,t+j} \{\pi_{t+j}^p - \vartheta_w \gamma_p \pi_{t+j-1}^p - (\varphi_w + \vartheta_w \varphi_p) y_{t+j-1} + s_{t+j} - (1 \pm \theta_s) s_{t+j-1}\} \\ \qquad + \lambda_{2,t+j} \{y_{t+j} - \sigma_y y_{t+j-1} + \sigma_i (i_{t+j} - \pi_{t+j-1}^p) \pm \sigma_s s_{t+j-1}\} \end{array} \right]$$

Finding first-order conditions for inflation, output growth, labourer's share of income and the interest rate yield:

a. $\mathcal{L}_{\pi_t^p} = 0 = \alpha_\pi \pi_t^p + \lambda_{1,t} - \beta \vartheta_w \gamma_p \lambda_{1,t+1} - \beta \sigma_i \lambda_{2,t+1}$
b. $\mathcal{L}_{y_t} = 0 = \alpha_y y_t - \beta(\varphi_w + \vartheta_w \varphi_p) \lambda_{1,t+1} + \lambda_{2,t} - \beta \sigma_y \lambda_{2,t+1}$
c. $\mathcal{L}_{s_t} = 0 = \lambda_{1,t} - \beta\, (1 \pm \theta_s) \lambda_{1,t+1} \pm \beta \sigma_s \lambda_{2,t+1}$
d. $\mathcal{L}_{i_t} = 0 = \sigma_i \lambda_{2,t}$

Considering condition d, and noting that σ_i is a non-zero component of the Investment-Savings relationship, it stands to reason that $\lambda_{2,t}$ (and thus, $\lambda_{2,t+1}$) are constrained to 0. With this information, one could solve for $\lambda_{1,t+1}$ from condition c as follows:

$$(11)\lambda_{1,t+1} = \frac{\lambda_{1,t}}{\beta\,(1 \pm \theta_s)}$$

Substituting (11) into condition a and solving for $\lambda_{1,t}$ yields the following:

$$(12)\lambda_{1,t} = -\frac{\alpha_\pi}{(1-\frac{\vartheta_w \gamma_p}{(1\pm\theta_s)})}\ \pi_t^p$$

Similarly, substituting (11) into condition b and solving for $\lambda_{1,t}$ will result in the following:

$$(13)\lambda_{1,t} = \frac{\alpha_y(1\pm\theta_s)}{(\varphi_w+\vartheta_w\varphi_p)}\ y_t$$

Equating (12) and (13) yields a rule for the optimal trade-off between output growth and inflation for any central bank working under the same constraints and exhibiting the same loss function. This rule, summarised in equation 14, is different from most trade-off equations in the literature, given the inclusion of the labour share in the constraint equations:

$$(14)\frac{\alpha_y(1\pm\theta_s)}{(\varphi_w+\vartheta_w\varphi_p)}\ y_t \quad = -\frac{\alpha_\pi}{(1-\frac{\vartheta_w \gamma_p}{(1\pm\theta_s)})}\ \pi_t^p$$

For simplicity, this can be rewritten in reduced form as follows:

$$(15)\ \mu_y y_t == -\ \mu_\pi \pi_t^p$$

Making output growth the subject of the formula and substituting this result into equation (4) yields:

$$(16)\ -\frac{\mu_\pi}{\mu_y}\pi_t^p = \ \sigma_y y_{t-1} - \ \sigma_i(i_t - \ \pi_{t-1}^p)\ \pm \sigma_s s_{t-1}$$

Solving for the interest rate, one obtains a naïve version of the final interest rate rule that can be estimated almost immediately:

$$(17)i_t = \ \frac{\mu_\pi}{\mu_y \sigma_i}\pi_t^p + \ \frac{\sigma_y}{\sigma_i}\ y_{t-1} \pm \frac{\sigma_s}{\sigma_i} s_{t-1} + \pi_{t-1}^p$$

However, because inertia from the previous variation in the interest rate is important in defining the current interest rate (Rudebusch, 2002), one should augment equation 17 further for robustness' sake.

Such an augmentation requires using the optimal trade-off equation outlined in equation (15) once more. Lagging this optimal trade-off by one period, solving for inflation, and substituting this result into equation 17 yields the following:

$$(18) i_t = \frac{\mu_\pi}{\mu_y \sigma_i} \pi_t^p \pm \frac{\sigma_s}{\sigma_i} s_{t-1} + \left[\frac{\sigma_y}{\sigma_i} - \frac{\mu_y}{\mu_\pi}\right] y_{t-1}$$

Pushing equation 4 back by one period and substituting the result into equation 18 yields an interest rate rule inclusive of inertia:

$$(19)\ i_t = \frac{\mu_\pi}{\mu_y \sigma_i} \pi_t^p \pm \frac{\sigma_s}{\sigma_i} s_{t-1} + \left[\frac{\sigma_y}{\sigma_i} - \frac{\mu_y}{\mu_\pi}\right] \left[\sigma_y y_{t-2} - \sigma_i (i_{t-1} - \pi_{t-2}^p) \pm \sigma_s s_{t-2}\right]$$

In reduced form, an income-distribution-sensitive monetary policy rule is proposed using the following form:

$$(20) i_t = \rho_1 i_{t-1} + \rho_2 \pi_t^p \pm \rho_3 \pi_{t-2}^p \pm \rho_4 s_{t-1} \pm \rho_5 s_{t-2} \pm \rho_6 y_{t-2}$$

A few stylised facts are important to clarify related to equation 20.

First, it is highly likely that ρ_1 is positive and close to 1; this is due to a central bank's general tendency towards a stable interest rate (Rudebusch, 2002).

Second, it is no surprise that an inflation-targeting central bank will increase the interest rate in response to inflationary pressure (especially if that pressure threatens a target breach). ρ_2 is, therefore, positive. Depending on how far in advance inflationary pressure is met with changes in the interest rate, it is also likely that ρ_3[2] will be positive.

Depending on the stance of a central bank on changes in the labour share, both ρ_4 and ρ_5 could be positive or negative. Responses by an inflation-targeting central

[2] Given the mathematical manipulations, there is a case for negativity in the coefficient as well. In the case of an inflation-targeting central bank, such negativity is highly unlikely.

bank to changes in the functional distribution of income (as proxied by the labour share) vary depending on:

- **The inflation regime of the labour share**. An increase in the labour share leading to wage, and therefore, price inflation, would be met with a central bank increasing the repo rate to stave off inflationary pressure, with the converse also applying, implying positivity in ρ_4 and ρ_5. Should the converse hold (where the labour share negatively relates to wage inflation and therefore transitively to price inflation), a central bank may decrease the interest rate if the labour share increases.
- **The output regime of the labour share.** An increase in the labour share leading to an increase in output might signal an overheating economy, prompting a central bank to tighten its policy stance, implying positivity in ρ_4 and ρ_5. Conversely, a profit-led output regime (where an increase in the labour share leads to worsened economic growth) may lead to a central bank loosening its policy rate.
- **Whether the central bank inherently aims to protect income distribution dynamics or not.** Beyond the obvious channels mentioned above, central banks may want to affect income distribution between classes for other reasons. These reactions are often related to stabilising the macroeconomic system within which a central bank operates by moving an economy towards a particular equilibrium[3]. Depending on how a central bank wishes to stabilise the economy (at least in the short-term), an increase in the labour share can either be met with an increase or a decrease in the interest rate (see work done by Flaschel and Krolzig (2005), and Franke et al. (2006)).

Estimation Framework and Analysis

The analysis makes use of the following data (South African Reserve Bank, 2022), with the subsequent analysis beginning at the start of the repo rate series (quarter 4 of 2001):

[3] See work done by Franke, et al. (2006) on the destabilising nature of income distribution shocks in a macroeconomy

Table 1: Data sources

Indicator	Electronic Source	KBP Code (where applicable)
Repurchase rate[4]	https://www.resbank.co.za/en/home/what-we-do/statistics/key-statistics/selected-historical-rates	
Compensation of employees to GDP at factor cost (labour share)	https://www.resbank.co.za/en/home/what-we-do/statistics/releases/online-statistical-query	KBP6295L
Total consumer prices (All urban areas)		KBP7170Q
Gross domestic product at market prices		KBP6006K

As is found in most empirical Structuralist literature (see, for example, Gordon (2011) or Malikane (2013)), the mathematical structure of the labour share induces simultaneity in any related regression analysis. In particular, the labour share at any point in time contains both output/national income and the price level at that same point in time). Therefore, simply including the labour share in an analysis without considering this simultaneity impacts the validity of the analysis.

As a result, equation 20 is estimated using two methods.

First of these is a simple Ordinary Least Squares (OLS) regression, done as a baseline to assess whether the aforementioned simultaneity meaningfully impacts coefficient size and significance.

Second, I apply a Generalised Method of Moments (GMM) framework to estimate equation 20. The GMM framework instruments the labour share for lagged values that are not contemporaneous with the GDP and inflation terms in equation 20. Selecting the appropriate number of instruments to include in the analysis requires the use of the method outlined by Scheufele (2010) and Malikane (2013).

In essence, this entails specifying a high number of lagged instruments before running a GMM estimation on this artificially high number of lags. The number of lagged instruments is slowly whittled down until GMM-estimated parameters become statistically insignificant.

Based on this approach, I find that using lags 3-15 of the labour share to proxy for the lagged and twice-lagged labour share is appropriate. With this said, it is important to flag that the significance of each labour share coefficient does not taper off completely, irrespective of the lag structure of the instruments. Further, the coefficient sizes and signs remain relatively similar and are, thus, not necessarily dependent on instrument lag structure.

[4] It is important to flag that the quarterly average of the repurchase rate was taken

The results from these estimations are summarised below:

Table 2: Estimates for the Income-Distribution-Sensitive Taylor Rule for South Africa (Q4 2001 - Q4 2021)

Equation 20: $i_t = \rho_1 i_{t-1} + \rho_2 \pi_t^p \pm \rho_3 \pi_{t-2}^p \pm \rho_4 s_{t-1} \pm \rho_5 s_{t-2} \pm \rho_6 y_{t-2}$		
	Ordinary Least Squares	**Generalised Method of Moments**
ρ_1	0.902*** (20.37)	0.926*** (24.98)
ρ_2	0.034* (1.71)	0.046*** (2.95)
ρ_3	0.084*** (3.86)	0.061*** (3.61)
ρ_4	-0.389*** (-3.66)	-0.416*** (-2.63)
ρ_5	0.430*** (3.83)	0.484*** (2.76)
ρ_6	0.074*** (2.59)	0.093*** (2.83)
C	-0.027** (-2.29)	-0.044*** (-3.22)
R squared	0.97	0.97
F statistic/Wald Chi	550.06***	8395.73***
Significant at 1%***, 5%**, and 10%*		
Robust standard errors (Z-statistics in parentheses)		

Before focusing on the novel inclusion of both labour share terms, it is important to stress that all other signs and levels of significance are as *a priori* anticipated. These estimates also fair relatively well in comparison to other estimates of the South African Taylor rule (see Ncube & Tshuma (2010), or Loate, et al. (2021)), both in terms of R-squared and in terms of coefficient size regarding all other similarly characterised parameters.

With that in mind, an interesting feature of the current specification of the Taylor rule is the size (and significance) of the coefficient on both labour share terms. In absolute terms, the central bank reacts more to changes in the labour share than to changes in inflation or output in the short run. As Flaschel and Krolzig (2005), and Franke, et al. (2006) imply, this is because, even with inflation and GDP volatility, unless such volatility is extreme, the macroeconomic system under which a central bank operates is at least somewhat stable (in the dynamical sense). However, changes in the labour share brew macroeconomic instability, requiring more sizeable central bank intervention at any point in time, as this estimation confirms.

It is unclear, at this stage, whether this reaction is due to the inflationary pressure or output swings likely caused by changes in the labour share or whether this is simply a stabilisation mechanism used by the SARB to steer the economy to some form of equilibrium. The main drivers of such a large and as yet unexplored reaction are room for future research.

Concluding Remarks

This paper has sought to contribute two things to the broader discussion on monetary policy.

First, the article has developed a simple Taylor-type rule applicable to all central banks with similar loss functions and constraints. Instead of continuing along the mainstream, this Taylor-type rule is “income distribution sensitive” and takes into account the labourer’s share of national income. The so-called labour share is a macro-aggregate that proxies, however imperfectly, the distribution of income within an economy.

Second, the article estimates this newly developed Taylor rule by using South African data as a proof of concept using both OLS and GMM estimation techniques.

I find that this new distribution-sensitive rule fairs well relative to similar one-equation estimates of the variability in the repo rate in South Africa, irrespective of how it is estimated.

Beyond robust specification, I also find that the central bank in South Africa reacts to changes in the labour share moreso than to changes in either inflation or output at any point in time. As discussed by Flaschel and Krolzig (2005) and Franke et al. (2006), this is likely because changes in macro-aggregates like inflation and GDP, while distortionary, do not have a destabilising impact on an economic system unless those changes are extreme. Conversely, a shift in income distribution between classes has a larger destabilising impact, requiring the intervention of the South African Reserve Bank.

Whether this instability manifests in inflation or output volatility which prompts the Reserve Bank to react, or whether the inherent shift in income distribution is worrisome to the SARB is difficult to tell given such a high-level analysis.

Nevertheless, as is evident from the analysis, the SARB does, in fact, react to changes in functional income distribution. These changes are often glossed over in mainstream analysis but are relatively straightforward to introduce into common economic modelling- an exclusion, which I believe, can be redressed in future research.