Politics of Public Education and Pension Reform with Endogenous Fertility

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Abstract

We demonstrate the interaction between short-lived governments' decisions on education and pension policies and parents' decisions on fertility in an overlapping generations growth model. Our analysis shows that increased expected life expectancy lowers fertility, decreases the ratio of education expenditure to GDP, and increases the ratio of pension benefits to GDP as well as per capita GDP growth rate. We also consider a reform that reduces pension benefits designed by a long-lived planner and show that the reduction is optimal from a social welfare perspective when the planner gives a large weight to future generations.

- Keywords: Fertility; Public Pension; Public Education; Probabilistic Voting; Overlapping Generations
- JEL Classification: D70, E62, H52, H55

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Highlight

Households make decisions on fertility to maximize lifetime utility.

A government representing living generations decides on education and pension policies.

Households' decisions on fertility affect the government's policy choice.

The government's choice could be suboptimal owing to its short-sightedness.

Reducing pension benefits might be supported from a social welfare viewpoint.

1 Introduction

Declining birth rates and increasing life expectancy, observed in most Organisation for Economic Co-operation and Development (OECD) member countries in the past decades, have led to an increase in the share of older adults in the voting population (OECD, 2016); this trend is projected to continue over the next few decades (Rouzet et al., 2019). As a result, pension benefits for older adults are expected to increase (Gonzalez-Eiras and Niepelt, 2012), while government spending on schemes that may not directly benefit directly older adults, such as public education, (Poterba, 1997; Cattaneo and Wolter, 2009) could decrease. At the same time, an aging population reduces the willingness of the working middle-aged population to pay higher taxes to meet the government's growing pension burden (Razin et al., 2002). However, older adults may not object to education spending because of altruistic concerns for the younger generations or because such spending may enhance productivity and ensure a higher level of tax revenues (Gradstein and Kaganovich, 2004). Therefore, these opposing effects lead to the following question: how does the government allocate its limited budget each period to pension for older adults and education for the younger generation in response to population aging?

Recent studies on the topic include those of Gonzalez-Eiras and Niepelt (2012), Lancia and Russo (2016), Ono and Uchida (2016), and Bishnu and Wang (2017). They use probabilistic voting (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) to describe intergenerational conflicts in the allocation of government revenues to public pension and education. In this voting environment, each office-seeking candidate proposes a policy platform in terms of maximizing its probability of winning, resulting in the selection of policies that maximize an objective function that weights the utility of each generation by its share of the population. The advantage of assuming probabilistic voting is that it allows us to handle votes for multiple policies and avoid the problem of voting cycles that may arise in majority voting for multiple policy variables (Persson and Tabellini, 2000).

Probabilistic voting further allows us to capture the impact of marginal changes in population composition on equilibrium policy through changes in generational weights in the objective function. The studies mentioned above assume exogenous fertility, and look at the impact of its exogenous decline on the equilibrium policy and the resulting welfare distribution across generations. However, in reality, fertility is endogenously determined from the optimizing behavior of households (Becker, 1991). In particular, an increase in expected life expectancy affects the fertility decisions of households (Ehrlich and Lui, 1991; Zhang et al., 2001; Zhang and Zhang, 2005), and this in turn affects the population share, or the political weight, of older adults in the next period. Thus, as emphasized by Gonzalez-Eiras and Niepelt (2012) and Bishnu and Wang (2017), the interaction of pension and education policy choices with fertility decisions is an important issue in analyzing the determination of fiscal policies and assessing the optimality of the resulting allocation.[1]

To demonstrate the interaction between determination of education and pension policies and parents' decisions on fertility, we utilize the overlapping-generation model with physical and human capital accumulation developed by Gonzalez-Eiras and Niepelt (2012) and Ono and Uchida (2016). We follow de la Croix and Doepke (2003, 2004) and extend the model by introducing quantity-quality trade-off in the decisions on children developed by Becker and Lewis (1973). Specifically, parents care about consumption, the number of children, and the human capital (i.e., quality) of their children. Parents vote for public education that affects the formation of their children's human capital as well as pension provisions that benefit retired older adults. Parents spends a part of their lives raising children. Given the education and pension policies, parents choose consumption, saving, and the number of children to maximize their lifetime utility.[2]

Within this framework, we show that the expected life expectancy has a direct effect on an individual's decision on fertility for a given set of policy variables, as shown by Ehrlich and Lui (1991) and the literature that follows them. We also show that the expected life expectancy has an indirect effect through political decisions on the level of pension benefits for older adults, which is new in the literature. In particular, the indirect effect comes through the following four routes: the political weight on older adults, labor tax, private saving, and pension benefits. Overall, we find that the net effect is negative in the present framework. We also show that this negative effect of increased expected life expectancy on fertility works to increase the ratio of pension benefits to GDP, decrease the ratio of education expenditure to GDP, and increase the per capita GDP growth rate.

The result of the positive association between expected life expectancy and pension expenditures is consistent with the evidence observed in developed countries. To curb the projected increase in expenditures in the future, many developed countries are working to reduce pension benefits (OECD, 2020). However, such a move would reduce consumption by the current older adult generation and would therefore not gain political support in the current environment where older adults are gaining increasing political power. Therefore, this leads us to the

[1] Gonzalez-Eiras and Niepelt (2012) say: "With endogenous fertility, the demographic structure would turn into an endogenous state variable, rendering an analytical solution of the policy game considered in the present paper infeasible. ... We leave an analysis of these feedback effects for future research." Bishnu and Wang (2017) add: "As the intergenerational distribution of political power is tied to the demographic change, which in turn is determined by the changing pattern of fertility and longevity, a natural extension of this study is to accommodate individual choice of fertility and longevity. We leave this for future study."

[2] Expected life expectancy could be controllable through health investment (Grossman, 1972). However, in this study, we assume it to be exogenous and focus on the interaction between fertility and policy choices.

second question: how would you justify the reduction in pension benefits being considered by some developed countries? One answer could be that the cuts help internalize intergenerational externalities through physical and human capital accumulation. A decrease in pension benefits reduces the tax burden on middle-aged individuals, thus increasing their savings and promoting physical capital accumulation. In addition, the reduction in tax burden allows them to increase their educational expenditures, which promotes human capital accumulation. These positive external effects on future generations through physical and human capital might provide benefits from a long-term perspective and thus justify the pension cuts.

To assess the internalization of intergenerational externalities through pension cuts, we assume a long-lived planner who has the power to impose ceiling constraints on pension benefits. The government, representing the living generations, determines education and pension expenditures and the tax on labor income subject to the constraint introduced by the long-lived planner. The decision of this planner can be seen as a kind of enactment of a law that restricts pension benefits from a long-term perspective. We first look at how education and pension expenditures, labor tax, fertility, and economic growth would change if the planner introduces a ceiling on pension benefits. We then derive the optimal ceiling on pension benefits in terms of maximizing social welfare that aggregates the utility of all generations. Based on the characterization of the optimal ceiling, we clarify the conditions under which the equilibrium allocation in the absence of the pension ceiling fails to achieve social welfare maximization, and what level of pension ceiling should be imposed in such a case.

We show that the optimal ceiling depends on the degree to which the long-lived planner discounts future generations. In particular, a critical value of the social discount factor represents the degree; it is optimal to set a ceiling (no ceiling) on pension when the discount factor is above (below) the critical value from the viewpoint of social welfare maximization. The mechanism behind this result is that a pension cut benefits future generations at the expense of the current older generation, and the planner attaches a larger weight to the benefit and a smaller weight to the cost as the discount factor becomes larger. A side-effect of this result is that the pension cut creates a trade-off between fertility and growth. A reduction in pension benefits promotes savings and economic growth, but discourages fertility. This effect suggests the difficulty of reconciling the two goals of improving fertility and economic growth, which are key issues for many aging countries.

The paper proceeds as follows. The next section reviews the related literature. Section 3 presents the model. Section 4 characterizes political equilibrium. Section 5 considers pension reforms and provides the optimal pension ceiling. Section 6 concludes with brief remarks. All proofs are given in the Appendix.

2 Related Literature

The literature on public education and pension begins with Pogue and Sgontz (1977), who show that pay-as-you-go (PAYG) social security creates incentives for public investment in education. Such an incentive is also indicated by Becker and Murphy (1988), who demonstrate the role of PAYG social security in garnering political support from the current working population for public investment in education. Later, there have been studies by Cremer et al. (1992); Kaganovich and Zilcha (1999); Pecchenino and Utendorf (1999); Boldrin and Montes (2005); Poutvaara (2006); Cremer et al. (2011); and Andersen and Bhattacharya (2017) that focus on how households behave when public education and pensions are provided by the government; decisions on these policies through voting are therefore abstracted away from their analyses.

Early studies on the political economy of public education and pensions include those by Bearse et al. (2001), Soares (2006), Iturbe-Ormaetxe and Valera (2012), Kaganovich and Meier (2012), Kaganovich and Zilcha (2012), and Naito (2012). A common feature of these studies is that the two-dimensional voting aspect is reduced to one dimension for simplicity of analysis. In other words, they consider a vote on public education for a given pension benefit, or a vote on the allocation of tax revenue for a given tax rate. Therefore, these studies do not indicate how the size of the government (i.e., the tax rate) and the allocation of government spending between education and pensions are determined jointly through voting in the presence of generational conflict.

This problem is resolved by introducing two-dimensional voting based on altruism (Tabellini, 1991), party competition (Levy, 2005), issue-by-issue voting (Poutvaara, 2006), and reputation (Bellettini and Ceroni, 1999; Boldrin and Rustichini, 2000; Rangel, 2003). However, these studies abstract from physical and/or human capital formation, and thus, show nothing about the interaction between policy and capital formation. Capital formation is introduced by Kemnitz (2000), Gradstein and Kaganovich (2004), Holtz-Eakin et al. (2004), Tosun (2008), and Bernasconi and Profeta (2012). These studies assume myopic voting, in which the current voters take future policy as a given. In other words, the forward-looking decisions of voters are absent in the analysis of these studies. Therefore, they abstract from the feedback mechanism between current and future redistribution policies through physical and/or human capital accumulation, which plays a crucial role in shaping fiscal policies.

The feedback mechanism is demonstrated by Beauchemin (1998), Forni (2005), Bassetto (2008), Mateos-Planas (2008), Gonzalez-Eiras and Niepelt (2012), Song (2011), Chen and Song (2014), and Arcalean (2018).[3] In particular, the present study is closely related to Gonzalez-Eiras and Niepelt (2012), Lancia and Russo (2016), Ono and Uchida (2016), and Bishnu and Wang (2017), who analyze the politics of public education and pensions in the presence of the

[3]The studies of multiple policy instruments other than education spending in the presence of the feedback mechanism include Hassler et al. (2003, 2005, 2007); Arawatari and Ono (2009, 2013); Song et al. (2012); Müller et al. (2016); Röhrs (2016); Arai et al. (2018); and Uchida and Ono (2021).

feedback mechanism in the overlapping generations model. As mentioned earlier, these studies look at the impact of exogenous declines in population growth rates on policy decisions, thus ignoring the interaction between policy decisions and fertility decisions. This study contributes to the literature by showing the importance of this interaction in assessing the impact of aging on policy decisions and economic growth as well as the optimality of pension reforms.

From a methodological point of view, this study contributes to the literature on aging and intergenerational conflict over policy making through probabilistic voting (Grossman and Helpman, 1998; Hassler et al., 2005; Gonzalez-Eiras and Niepelt, 2008; Song, 2011; Song et al., 2012; Arai et al., 2018; Uchida and Ono, 2021). The study, to the best of our knowledge, is the first to obtain a closed-form solution of policy functions in a dynamic setting with endogenous fertility. de la Croix and Doepke (2009) and Kimura and Yasui (2009) have analyzed the politics of education when fertility is endogenous, but their models are static in nature and thus assume away intertemporal interaction between fertility and policy choices via physical/human capital accumulation. The present study overcomes this limitation and demonstrates the dynamic impact of fertility on policy decisions and the resulting resource allocation across generations.

3 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live at most for three periods: young, middle, and old age; they face uncertainties in the third period of life. Let $\pi \in [0,1]$ denote life expectancy (i.e., the probability of living in old age). This is idiosyncratic for all individuals and is constant across periods. Each middle-aged individual gives birth to n children. The middle-aged population for period t is N_t and the population grows at a rate of n_t: $N_{t+1} = n_t N_t$. The gross population growth rate n_t depends on fertility decisions of the middle-aged, which is described below.

Individuals

Individuals have the following economic behavior during their life cycle. In their youth, individuals do not make any economic decisions and depend on their parents for their livelihood. In middle age, individuals work, receive market wages, pay taxes, and make fertility and saving decisions. In old age, they retire and receive and consume returns from savings.

Consider middle-aged individuals in period t. Each of them is endowed with one unit of time. Raising one child takes fraction $\phi \in (0,1)$ of time. Each individual devotes ϕn_t units of time to raise children and supplies the remaining time, $1-\phi n_t$, to the labor market. Each middle-aged individual obtains labor income $(1-\phi n_t)\, w_t h_t$, where w_t is the wage rate per unit of labor and h_t is the human capital endowment. After paying tax $\tau_t w_t h_t (1-\phi n_t)$, where τ_t is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings held as an annuity and invested in physical capital, s_t. Therefore, the period-t budget constraint for the middle-aged becomes

$$c_t + s_t \leq (1-\tau_t) w_t h_t (1-\phi n_t)\,. \tag{1}$$

The period $t+1$ budget constraint in old age is

$$d_{t+1} \leq \frac{R_{t+1}}{\pi} s_t + b_{t+1}, \tag{2}$$

where d_{t+1} is consumption, R_{t+1} is the gross return from savings, and b_{t+1} is the pay-as-you-go public pension benefit. If an individual dies at the end of the middle-age period, the annuitized wealth is transferred, via annuity markets, to individuals who live throughout the old age. Therefore, the return from savings becomes R_{t+1}/π under the assumption of perfect annuity markets.

The children's human capital over period $t+1$, h_{t+1}, is a function of per capita government spending on public education, x_t, and the parents' human capital, h_t. In particular, h_{t+1} is formulated using the following equation:

$$h_{t+1} = h\left(h_t, x_t\right) \equiv D\left(h_t\right)^{1-\eta}\left(x_t\right)^{\eta}, \tag{3}$$

where $D(>0)$ is a scale parameter and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to education spending.

Two remarks are in order. First, we abstract private education away from the analysis to simplify the presentation of the model. Second, we do not distinguish between spending on K-12 and higher education. In other words, we consider that x, investment in public education, includes investment in both K-12 and higher education. In a real economy, the benefits received from public education spending vary from person to person because some people receive higher education while others do not. The model does not explicitly depict such intra-generational heterogeneity. Instead, we focus on a representative agent in order to show the extent to which each individual within a generation benefits from public education investment in K-12 and higher education on an average.

The middle-aged individuals care about consumption, c_t and d_{t+1}, their number of children, n_t, and the human capital of children, h_{t+1}. The preferences of the middle-aged in period t are specified by the following expected utility function à la de la Croix and Doepke (2003, 2004):

$$\ln c_t + \delta \ln n_t h_{t+1} + \beta\pi \ln d_{t+1}, \tag{4}$$

where $\beta \in (0,1)$ is a discount factor, and $\delta(>0)$ is the degree of preference for the children's quantity and quality.

We substitute the budget constraints (1) and (2) into the expected utility function in (4) to form the unconstrained maximization problem:

$$\max_{\{s_t, n_t\}} \ln\left((1-\tau_t) w_t h_t \left(1-\phi n_t\right) - s_t\right) + \delta \ln n_t h_{t+1} + \beta\pi \ln\left(\frac{R_{t+1}}{\pi} s_t + b_{t+1}\right).$$

By solving this problem, we obtain the following fertility, savings, and consumption functions:

$$n_t = n\left(\tau_t, w_t h_t, b_{t+1}\right) \equiv \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{(1-\tau_t)w_t h_t + \frac{b_{t+1}}{R_{t+1}/\pi}}{(1-\tau_t)w_t h_t}, \tag{5}$$

$$s_t = s\left(\tau_t, w_t h_t, b_{t+1}\right) \equiv \frac{\beta\pi}{1+\delta+\beta\pi}\left[(1-\tau_t)w_t h_t - \frac{1+\delta}{\beta\pi} \cdot \frac{b_{t+1}}{R_{t+1}/\pi}\right], \tag{6}$$

$$c_t = c\left(\tau_t, w_t h_t, b_{t+1}\right) \equiv \frac{1}{1+\delta+\beta\pi}\left[(1-\tau_t)w_t h_t + \frac{b_{t+1}}{R_{t+1}/\pi}\right], \tag{7}$$

$$d_{t+1} = d'\left(\tau_t, w_t h_t, b_{t+1}\right) \equiv \frac{\beta R_{t+1}}{1+\delta+\beta\pi}\left[(1-\tau_t)w_t h_t + \frac{b_{t+1}}{R_{t+1}/\pi}\right], \tag{8}$$

where we drop the argument R_{t+1} from the expressions of $s(\cdot)$, $c(\cdot)$, and $d'(\cdot)$ since R_{t+1} becomes constant as we demonstrate below.

Firms

There is a continuum of identical firms that are perfectly competitive profit maximizers, and produce the final output Y_t with a constant-returns-to-scale Cobb–Douglas production function, $Y_t = A_t\left(K_t\right)^{\alpha}\left(L_t\right)^{1-\alpha}$. Here, $A_t(>0)$ is total factor productivity, K_t is aggregate capital, L_t is aggregate labor in efficiency units, and $\alpha \in (0,1)$ is a constant parameter representing the capital share in production.

In each period, a firm chooses capital and labor to maximize its profit, $A_t\left(K_t\right)^{\alpha}\left(L_t\right)^{1-\alpha} - R_t K_t - w_t L_t$, where R_t is the gross return on physical capital and w_t is the wage rate. The firm's profit maximization leads to

$$K_t : R_t = \alpha A_t\left(K_t\right)^{\alpha-1}\left(L_t\right)^{1-\alpha}, \tag{9}$$

$$L_t : w_t = (1-\alpha)A_t\left(K_t\right)^{\alpha}\left(L_t\right)^{-\alpha}. \tag{10}$$

Capital fully depreciates in a single period.

The productivity parameter A_t is assumed to be proportional to the per labor capital: $A_t = Q\left(K_t/L_t\right)^{1-\alpha}$, where $Q(>0)$ is constant. Thus, capital investment involves a technological externality of the type often used in endogenous-growth theories. This assumption, called the "AK" technology, results in a constant interest rate across periods, as demonstrated below. This approach enables us to obtain an analytical solution for the model. Under this assumption, the first-order conditions in (9) and (10) are rewritten as follows:

$$R_t = R \equiv \alpha Q, \tag{11}$$

$$w_t = (1-\alpha)Q\frac{K_t}{L_t}. \tag{12}$$

Government Budget Constraint

Government expenditures include public education spending and public pension payments. They are financed by taxes on labor income. The government budget constraint in period t is $\tau_t w_t L_t = \pi N_{t-1} b_t + x_t N_{t+1}$, where $\tau_t w_t L_t$ is the aggregate labor income tax revenue, $\pi N_{t-1} b_t$ is the public pension payment, and $x_t N_{t+1}$ is the aggregate public expenditure on education.

Let $k_t \equiv K_t/N_t$ denote per capita capital. By using (12) and dividing both sides of the constraint by N_t, we can obtain a per capita expression of the government budget constraint:

$$\tau_t(1-\alpha)Qk_t = \frac{\pi b_t}{n_{t-1}} + n_t x_t. \tag{13}$$

Market Clearing

The market-clearing condition for capital is $K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged in period t, $N_t s_t$, to the stock of aggregate physical capital at the beginning of period $t+1$. We can rewrite the capital market clearing condition as

$$n_t k_{t+1} = s\left(\tau_t, w_t h_t, b_{t+1}\right), \tag{14}$$

where $s(\cdot)$ is defined in (6).

The market-clearing condition for labor is

$$L_t = \left(1-\phi n_t\right) N_t h_t, \tag{15}$$

which expresses the equality of the aggregate labor demand, L_t, to the aggregate supply, $\left(1-\phi n_t\right) N_t h_t$. Using (12) and (15), we can define the effective wage income as follows:

$$w_t h_t = \bar{w}\left(n_t, k_t\right) \equiv (1-\alpha)Q\frac{K_t}{\left(1-\phi n_t\right) N_t h_t} h_t = \frac{(1-\alpha)Q}{1-\phi n_t} k_t. \tag{16}$$

Thus, the effective labor income, $w_t h_t = \bar{w}\left(n_t, k_t\right)$, depends on n_t and k_t, but is independent of h_t.

Using (16), we can reformulate the fertility function in (5) as $n_t = n\left(\tau_t, \bar{w}\left(n_t, k_t\right), b_{t+1}\right)$, or:

$$n_t = n\left(\tau_t, b_{t+1}, k_t\right) \equiv \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{(1-\tau_t)(1-\alpha)Qk_t + \frac{b_{t+1}}{R/\pi}}{(1-\tau_t)(1-\alpha)Qk_t + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{b_{t+1}}{R/\pi}}. \tag{17}$$

Using (16) and (17), we can also reformulate the saving function in (6) as

$$s_t = s\left(\tau_t, \bar{w}\left(n\left(\tau_t, b_{t+1}, k_t\right), k_t\right), b_{t+1}\right),$$

or:

$$s_t = s\left(\tau_t, b_{t+1}, k_t\right) \equiv \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau_t)(1-\alpha)Qk_t - \frac{1}{\beta\pi} \cdot \frac{b_{t+1}}{R/\pi}\right]. \tag{18}$$

Indirect Utility

In the present framework, there are three state variables in period t: physical capital, k_t, human capital, h_t, and the fertility rate, n_{t-1}. We can express the indirect utility of the middle-aged over period t, V_t^M, and that of older adults over period t, V_t^O, as functions of the three state variables, as follows:

$$\begin{aligned} V_t^M &= \ln c\left(\tau_t, \bar{w}\left(n\left(\tau_t, b_{t+1}, k_t\right), k_t\right), b_{t+1}\right) + \delta \ln n\left(\tau_t, b_{t+1}, k_t\right) \cdot h\left(h_t, x_t\right) \\ &\quad + \beta\pi \ln d'\left(\tau_t, \bar{w}\left(n\left(\tau_t, b_{t+1}, k_t\right), k_t\right), b_{t+1}\right), \end{aligned} \tag{19}$$

$$V_t^O = \ln d\left(b_t, n_{t-1}, k_t\right), \tag{20}$$

where $h(\cdot)$, $c(\cdot)$, $d'(\cdot)$, $\bar{w}(\cdot)$, and $n(\cdot)$ are defined in (3), (7), (8), (16), and (17), respectively, and $d(\cdot)$, representing the consumption of the older adult, is defined as follows:

$$d\left(b_t, n_{t-1}, k_t\right) \equiv \frac{R}{\pi} n_{t-1} k_t + b_t.$$

4 Political Equilibrium

In this section, we consider voting on fiscal policy. We employ probabilistic voting à la Lindbeck and Weibull (1987), in which there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government's budget constraint. As Persson and Tabellini (2000) demonstrate, the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

In the current framework, both older adults and the middle-aged have an incentive to vote. Thus, the political objective in period t is the weighted sum of the utility of older adults and the middle-aged, given by $\pi\omega V_t^O + n_{t-1}\left(1-\omega\right) V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on older adults and the middle-aged, respectively. A larger value of ω implies a greater political power of older adults. We use gross population growth rate n_{t-1} to adjust the weight of the middle-aged, and life expectancy (i.e., the probability of living in old age) π to adjust the weight of older adults, to reflect their share of the population. To obtain the intuition behind this result, we divide the objective function by $n_{t-1}\left(1-\omega\right)$ and redefine it, denoted by Ω, as follows:

$$\Omega_t = \frac{\pi\omega}{n_{t-1}(1-\omega)} V_t^O + V_t^M, \tag{21}$$

where V_t^M and V_t^O are defined in (19) and (20), respectively, and the coefficient $\pi\omega/n_{t-1}(1-\omega)$ of V_t^O represents the relative political weight of older adults.

The political objective function in (21) suggests that the current policy choice of (τ_t, b_t, x_t) affects future policy decisions via fertility choice and physical capital accumulation. In particular, the current choice of τ_t, b_t, and x_t affect the fertility decision and formation of physical capital in the next period, and thus influences political decision making on pension payments, b_{t+1}, in the next period. Conversely, as seen in (6) and (17), the level of pension benefits in the next period also affects the economic decisions on savings and fertility in the current period.

To demonstrate this mutual interaction between economic and political decisions, we employ the Markov-perfect equilibrium concept, in which today's fiscal policy depends on the current payoff-relevant state variables. In the current framework, the payoff-relevant state variables in period t are the fertility rate, n_{t-1}, and physical capital, k_t; the human capital, h_t, is a payoff-irrelevant state variable due to the specification of the human capital formation function in (3) and assumption of the logarithmic utility function in (4). Thus, the expected provision of public pension in period $t+1$, b_{t+1}, could be given by the functions of the period $t+1$ state variables, k_{t+1} and n_t: $b_{t+1} = \bar{B}(k_{t+1}, n_t)$. Using the notation, with z' and z_- denoting the next period and previous period z, respectively, we can define a Markov-perfect political equilibrium in the current framework as follows.

Definition 1 *A* Markov-perfect political equilibrium *is a five-tuple* $\left(\bar{T}, \bar{B}, \bar{X}, \bar{S}, \bar{N}\right)$*, where* $\bar{T}: \Re_+ \times \Re_+ \to [0,1]$ *is the tax rule,* $\tau = \bar{T}(k, n_-)$*;* $\bar{B}: \Re_+ \times \Re_+ \to \Re_+$ *is the pension rule,* $b = \bar{B}(k, n_-)$*;* $\bar{X}: \Re_+ \times \Re_+ \to \Re_+$ *is the education expenditure rule,* $x = \bar{X}(k, n_-)$*;* $\bar{S}: [0,1] \times \Re_+ \to \Re_+$ *is the optimal private saving rule,* $s = \bar{S}\left(\tau, k \mid \bar{B}\right)$*; and* $\bar{N}: [0,1] \times \Re_+ \to \Re_+$ *is the optimal private fertility rule,* $n = \bar{N}\left(\tau, k \mid \bar{B}\right)$*, such that (i) for a given* τ*,* k*, and* $\bar{B}$*, the optimal private saving and fertility rules are the maps,* $\bar{S}$ *and* $\bar{N}$*, respectively, that solve*

$$\bar{S}\left(\tau, k \mid \bar{B}\right) = s\left(\tau, \bar{w}\left(\bar{N}\left(\tau, k \mid \bar{B}\right), k\right), \bar{B}\left(\bar{S}\left(\tau, k \mid \bar{B}\right), \bar{N}\left(\tau, k \mid \bar{B}\right)\right)\right),$$
$$\bar{N}\left(\tau, k \mid \bar{B}\right) = n\left(\tau, \bar{B}\left(\bar{S}\left(\tau, k \mid \bar{B}\right), \bar{N}\left(\tau, k \mid \bar{B}\right)\right), k\right),$$

where $b' = \bar{B}(k', n)$ *with* $n = \bar{N}\left(\tau, k \mid \bar{B}\right)$ *and* $nk' = \bar{S}\left(\tau, k \mid \bar{B}\right)$*; (ii) given the set of initial conditions,* (k, n_-)*, and the political objective function*

$$\Omega\left(b, \tau, x, n_-, k, h \mid \bar{B}\right) \equiv \frac{\pi\omega}{n_-(1-\omega)} V^O\left(b, n_-, k\right) + V^M\left(\tau, x, b', k, h\right),$$

where $b' = \bar{B}(k', n)$ *with* $n = \bar{N}\left(\tau, k \mid \bar{B}\right)$ *and* $nk' = \bar{S}\left(\tau, k \mid \bar{B}\right)$*, the equilibrium fiscal policies solve*

$$\left(\bar{T}(k, n_-), \bar{B}(k, n_-), \bar{X}(k, n_-)\right) = \arg\max \Omega\left(b, \tau, x, n_-, k, h \mid \bar{B}\right)$$

subject to the government budget constraint

$$\bar{T}(k, n_-)(1-\alpha) Qk = \frac{\pi \bar{B}(k, n_-)}{n_-} + \bar{N}\left(\bar{T}(k, n_-), k \mid \bar{B}\right) \bar{X}(k, n_-).$$

Part (i) defines functional equations that map current tax and physical capital stock to optimal private savings and fertility, $s = \bar{S}\left(\tau, k \mid \bar{B}\right)$ and $n = \bar{N}\left(\tau, k \mid \bar{B}\right)$. This set of rules describes the private sector's response to changes in τ under the expectation that future pensions will be set according to the equilibrium rule $\bar{B}(k', n)$. Part (ii) describes the government's problem. In each period, the government sets fiscal policies subject to its budget constraint and the private sector's response, consistent with the expectation that future governments will follow the Markov-perfect political equilibrium rule.

4.1 Characterization of Political Equilibrium

To obtain the set of policy functions in Definition 1, we conjecture the following policy function of pension benefits in the next period:

$$\begin{aligned} b' &= \frac{\frac{\pi\omega}{n(\tau,b',k)(1-\omega)} B + C}{\frac{\pi\omega}{n(\tau,b',k)(1-\omega)} E + F} \cdot \frac{R}{\pi} n\left(\tau, b', k\right) k' \\ &= \frac{\frac{\pi\omega}{n(\tau,b',k)(1-\omega)} B + C}{\frac{\pi\omega}{n(\tau,b',k)(1-\omega)} E + F} \cdot \frac{R}{\pi} s\left(\tau, \bar{w}\left(n\left(\tau, b', k\right), k\right), b'\right), \end{aligned} \quad (22)$$

where B, C, E, and F are constant parameters, and the equality in the second line comes from the capital-market-clearing condition in (14). Equation (22) implies that the amount of the pension benefit, b', is set to match a certain proportion of the savings, $nk' = s$. The proportion depends on the relative weight given to older adults, $\pi\omega/n(1-\omega)$, in the political objective function.

The conjecture in (22) suggests that there is a mutual interaction between pension benefits, b', and the fertility rate, n. To see the interaction, recall the fertility function in (5), which is rewritten as follows:

$$n = \frac{\frac{\delta}{1+\delta+\beta\pi}\left[(1-\tau)wh + \frac{b'}{R/\pi}\right]}{\phi\,(1-\tau)\,wh} = \frac{\frac{\delta}{1+\delta+\beta\pi}\left[(1-\tau)(1-\alpha)Q\frac{k}{(1-\phi n)} + \frac{b'}{R/\pi}\right]}{\phi\,(1-\tau)\,(1-\alpha)Q\frac{k}{(1-\phi n)}}, \tag{23}$$

where the second equality comes from the labor-market-clearing condition in (16). Note that solving (23) for n results in the expression in (17).

An increase in the fertility rate n leads to a decrease in the labor supply because of the time required for births and child rearing. The decrease in labor supply leads to an increase in the equilibrium wage in the labor market, thereby improving fertility through the income effect, as seen in the numerator of the right-hand side of (23). On the other hand, an increase in wages reduces fertility through the increased opportunity cost of fertility, as seen in the denominator of the right-hand side of (23). In the current framework, the negative effect of the latter exceeds the positive effect of the former, so that the right-hand side of (23) is decreasing in the fertility rate n.

Given the property of the right-hand side of (23), we consider the effect of an increase in pension benefits b' on the right-hand side and thus the determination of n that satisfies (23). An increase in pension benefits b' raises the lifetime income of an individual, thereby improving fertility through the income effect. This is the economic effect of pension benefits on the fertility rate. Conversely, an increase in the fertility rate leads to a decrease in the relative political weight of older adults, as seen in the conjecture in (22), which affects the determination of pension benefits. This is the political effect of fertility on pension benefits. Thus, through economic and political effects, pension benefits and fertility interact with each other.

There is also a mutual interaction between savings, s, and pension benefits, b'. To see the interaction, recall the savings function in (6). As can be seen in the second term in parentheses, pension benefits b' discourage the middle-aged from saving. On the other hand, pension benefits encourage them to have more children as seen in (23). This in turn leads to an increase in the labor market equilibrium wage through a decrease in the labor supply, as described in the paragraph above. This creates a positive income effect on savings. Thus, pension benefits have two opposing effects on savings, with the net effect being negative. This is the economic effect of pension benefits on savings. A decrease in savings then leads to a decrease in pension benefits from the conjecture of b' in (22): this is the political effect. Thus, savings and pension benefits also interact through the economic and political effects.

Considering these two interactions, we substitute (6) and (23) into (22) and solve for b' to obtain the following equation:

$$b' = G \cdot (1-\tau)\, k, \tag{24}$$

where G is defined as

$$G \equiv \frac{-G_2 + \sqrt{(G_2)^2 - 4G_1G_3}}{2G_1},$$

and G_1, G_2, and G_3 are defined as

$$\begin{aligned}
G_1 &\equiv \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{1}{R/\pi} \cdot \left[\left(\frac{\pi\omega}{1-\omega} E + \frac{F}{\phi} \right) + \left(\frac{\pi\omega}{1-\omega} B + \frac{C}{\phi} \right) \frac{1}{1+\beta\pi} \right], \\
G_2 &\equiv (1-\alpha)\, Q \left\{ \left(\frac{\pi\omega}{1-\omega} E + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{F}{\phi} \right) \right. \\
&\quad \left. + \frac{\beta\pi}{1+\beta\pi} \cdot \left[-\frac{\delta}{1+\delta+\beta\pi} \cdot \left(\frac{\pi\omega}{1-\omega} B + \frac{C}{\phi} \right) + \frac{1}{\beta\pi} \cdot \left(\frac{\pi\omega}{1-\omega} B + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{C}{\phi} \right) \right] \right\}, \\
G_3 &\equiv (-1) \left(\frac{\pi\omega}{1-\omega} B + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{C}{\phi} \right) \cdot \frac{R}{\pi} \cdot \frac{\beta\pi}{1+\beta\pi} \cdot [(1-\alpha)\, Q]^2 .
\end{aligned}$$

Equation (24) shows that a higher labor income tax rate is associated with a lower level of pension benefits. An increase in the tax rate lowers the relative price of births. This gives individuals an incentive to increase fertility, which in turn affects the level of pension benefits through the conjecture of b' in (22). At the same time, an increase in the tax rate has the effect of reducing savings because it reduces disposable income. This has the effect of reducing pension benefits through the conjecture of b' in (22). The net effect of both is negative.

We substitute the policy function of b' in (24) into the fertility function in (17) and obtain

$$n = \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{(1-\alpha)Q + \frac{G}{R/\pi}}{(1-\alpha)Q + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{G}{R/\pi}}. \tag{25}$$

Equation (25) shows that the fertility rate is independent of the labor income tax rate and state variables, and thus is constant across periods. This implies that the direct effect of the tax on the fertility is offset by the indirect effect of the tax through public pension benefits, b'. This result might hinge on the assumption of the logarithmic utility function. In other words, the two effects may not necessarily cancel each other out if we generalize the utility function. However, there still remains the endogenous determination of fertility through the utility-maximizing behavior of individuals and its response to changes in structural parameters. Thus, we maintain the assumption of logarithmic utility in the following analysis.

Using the pension benefits in (24) and a constant fertility rate in (25), we can reformulate the political objective function in (21) as follows:

$$\Omega = \frac{\pi\omega}{n_{-1}(1-\omega)} \ln d\,(b, n_{-}, k) + \ln c\,(\tau, k) + \delta \ln n \cdot h\,(x, h) + \beta\pi \ln d'\,(\tau, k)\,,$$

where $h(x,h) \equiv D\,(h)^{1-\eta}\,(x)^{\eta}$ as in (3), and $d(\cdot)$, $c(\cdot)$, and $d'(\cdot)$ are defined as follows:

$$\begin{aligned}
d\,(b, n_-, k) &\equiv \frac{R}{\pi} n_- k + b, \\
c\,(\tau, k) &\equiv \frac{1}{1+\delta+\beta\pi} \cdot \left[(1-\tau)\,\bar{w}\,(n,k) + \frac{G\cdot(1-\tau)\,k}{R/\pi} \right], \\
d'\,(\tau, k) &\equiv \frac{\beta R}{1+\delta+\beta\pi} \cdot \left[(1-\tau)\,\bar{w}\,(n,k) + \frac{G\cdot(1-\tau)\,k}{R/\pi} \right].
\end{aligned}$$

Given the government budget constraint in (13), we can derive the first-order conditions with respect to τ, x, and b as follows:

$$\tau : \frac{c_\tau}{c} + \beta\pi\frac{d'_\tau}{d'} + \lambda(1-\alpha)Qk = 0, \tag{26}$$

$$x : \delta\frac{h'_x}{h'} - \lambda n = 0, \tag{27}$$

$$b : \frac{\pi\omega}{n_{-1}(1-\omega)} \cdot \frac{d_b}{d} - \lambda\frac{\pi}{n_-} = 0, \tag{28}$$

where λ is the Lagrangian multiplier associated with the government budget constraint and p_q $(p = c, d', h', d; q = \tau, x, b)$ denotes the derivative of p with respect to q. We should note that in deriving the first-order conditions in (26) – (28), we use the fact that the fertility rate is constant and independent of policy variables.

According to the expressions in (26) – (28), the government chooses each policy to equate its marginal benefits with its marginal costs. A detailed interpretation of each condition is as follows. Equation (26) shows that the government chooses the labor-income tax rate, τ, to equate its marginal costs and benefits. The first term of (26) represents the marginal costs of the tax and includes the following two effects on middle-age consumption. First, an increase in the tax rate leads to a decrease in the disposable income of the middle-aged, which in turn leads to a decrease in their consumption. Second, an increase in the tax rate leads to a decrease in the amount of pension benefits in old age, as shown in (24), which in turn leads to a decrease in consumption in the middle age. The second term of (26) also shows the marginal costs of the tax on old-age consumption and includes the similar two effects as in the first term. The third term shows the marginal benefits from increased tax revenues.

Equation (27) shows that the government chooses the education expenditures, x, to equate its marginal benefits arising from human capital accumulation represented by the first term, with the marginal costs from increased spending on x represented by the second term. Equation (28) shows that the government chooses the pension benefits, b, to equate the marginal benefits arising from increased consumption by older adults, represented by the first term, with the marginal costs from increased spending on b, represented by the second term.

Using the conditions in (26) – (28), alongside the government budget constraint in (13), we verify the conjecture in (22), and obtain the following result.

Proposition 1 *There is a unique Markov-perfect political equilibrium such that the policy functions of τ, b, and x are given by*

$$\tau = \frac{\frac{\pi\omega}{n_{-}(1-\omega)} + \left[\delta\eta - (1+\beta\pi)\frac{\alpha}{1-\alpha}\right]}{\frac{\pi\omega}{n_{-}(1-\omega)} + (1+\delta\eta+\beta\pi)}, \tag{29}$$

$$b = \frac{\frac{\pi\omega}{n_{-}(1-\omega)}\frac{1-\alpha}{\alpha} - (1+\delta\eta+\beta\pi)}{\frac{\pi\omega}{n_{-}(1-\omega)} + (1+\delta\eta+\beta\pi)}\frac{R}{\pi}n_{-}k, \tag{30}$$

$$x = \frac{1}{n(\pi)} \cdot \frac{\delta\eta}{\frac{\pi\omega}{n_{-}(1-\omega)} + (1+\delta\eta+\beta\pi)}Qk, \tag{31}$$

where $n(\pi)$ is the fertility rate given by

$$n(\pi) \equiv \frac{-(1+\delta+\alpha\beta\pi) + \sqrt{(1+\delta+\alpha\beta\pi)^2 + 4\alpha\beta(1+\delta\eta+\beta\pi)\frac{1-\omega}{\omega}\frac{\delta}{\phi}}}{2\alpha\beta(1+\delta\eta+\beta\pi)\frac{1-\omega}{\omega}}. \tag{32}$$

Proof. See Appendix A.1.

Proposition 1 implies that the fertility and policy functions have the following features. First, the fertility rate is independent of the state variables and is constant across periods. This is because, as mentioned above, the direct effect of the tax on fertility is offset by the indirect effect of the tax through public pension benefits. Second, the levels of public pension benefits, b', and public education expenditure, x, are linear functions of output, Qk. This property is necessary for generating a balanced growth path for the economy. Third, the labor-income tax rate is independent of the state variables and is constant across periods. This property is necessary for the government's budget to be balanced each period.

4.2 Effects of Expected Life Expectancy

The result in Proposition 1 suggests that increased life expectancy affects the choice of fertility and policies, and the effects also extend to economic growth. This subsection analyzes these effects in turn.

First, consider the effect of expected life expectancy on the fertility rate in (32), which is summarized in the following proposition.

Proposition 2 *A higher expected life expectancy is associated with a lower fertility rate: $\partial n/\partial\pi < 0$.*

Proof. See Appendix A.1.

To understand the mechanism behind the result in Proposition 2, recall the fertility function in (17) that expresses the privately optimal fertility rate derived from the individual utility maximization for a given policy set. The expression in (17) indicates that there are two types of effects of expected life expectancy on fertility: one is the direct effect on an individual's decision on fertility for a given set of policy variables, and the other is the indirect effect on fertility through political decisions about the level of pension benefits.[4] In what follows, we examine various factors that contribute to these two effects.

First, consider the direct effect that comes from the following two routes. First, an increase in expected life expectancy increases the weight of old-age consumption utility. This in turn strengthens the incentive for individuals to save, and thus increases the costs of raising children. This has a negative effect on fertility. Second, an increase in expected life expectancy lowers the return on savings and thus reduces the incentive for individuals to save. This has a positive effect on fertility.

Next, consider the indirect effect that comes from the following four routes. First, an increase in expected life expectancy leads to an increase in the political weight of older adults. This works to increase the level of pension benefits through voting, and thus has a positive effect on fertility. Second, an increase in expected life expectancy increases the weight in old-age consumption utility. This leads to a reduction in the labor-income tax rate, which in turn lowers the level of pension benefits. This has a negative effect on fertility. Third, an increase in the weight of old-age consumption utility increases the incentive for individuals to save. This leads to an increase in the level of pension benefits, as can be seen from the policy function of pension benefits. This has a positive effect on fertility. Finally, per capita pension benefits are reduced to keep the pension-GDP ratio constant against the increase in expected life expectancy. This has a negative effect on fertility. Overall, there are six conflicting effects, but the net effect is negative in the current framework.

From the policy function of pension benefits presented in Proposition 1, we find that the provision of public pensions is also affected by the expected life expectancy, as summarized in the following Corollary.

Corollary 1 *The government provides pension and thus $b > 0$ holds when the following conditions hold:*

$$1 + \delta\eta + \beta\pi < \begin{cases} \frac{\pi\omega}{n_{-1}(1-\omega)} \cdot \frac{1-\alpha}{\alpha} & \textit{for } t = 0, \\ \frac{\pi\omega}{n(\pi)(1-\omega)} \cdot \frac{1-\alpha}{\alpha} & \textit{for } t \geq 1. \end{cases} \tag{33}$$

Proof. See Appendix A.1.

Condition in (33) indicates that there are three effects of expected life expectancy on the government's provision of public pensions. First, the higher the expected life expectancy, the easier it is for the government to provide pensions because of the greater political weight of older adults. This effect is captured by the term π on the right-hand side of (33). Second, the higher the expected life expectancy, the higher the weight of consumption utility in old age. This provides an incentive for the government to lower the labor- income tax rate to maintain the level of consumption in old age. This effect is captured by the term π on the left-hand side.

In period $t = 0$, if the effect of the former exceeds that of the latter, pension benefits will be paid to older adults. From period $t = 1$ onward, there is a third effect in addition to the two

[4]Notice that fertility is independent of the labor-income tax rate as noted in the paragraph following (25); hence, there is no indirect effect on fertility through the labor-income tax rate.

effects described above. An increase in expected life expectancy leads to a decline in the fertility rate, which in turn increases the relative political weight of older adults and thus strengthens the government's incentive to provide pension benefits. This effect is represented by the term $n(\pi)$ on the right-hand side of (33). Thus, due to this additional effect, the increase in the expected life expectancy gives a stronger incentive for the government to provide pension benefits from period 1 onward than in period 0.

As described above, the expected life expectancy affects the government's policy choice. This implies that the expected life expectancy affects the share of each expenditure to GDP, as shown in the following proposition.

Proposition 3 *An increase in the expected life expectancy results in (i) an increase in the ratio of pension benefits to GDP; and (ii) a decrease in the ratio of education expenditure to GDP:* $\partial\left(\pi b N_{-}/QK\right)/\partial\pi > 0$ *and* $\partial\left(xN'/QK\right)/\partial\pi < 0$*.*

Proof. See Appendix A.2.

First, consider the effect of the expected life expectancy on the pension-GDP ratio. As mentioned above, there are two positive and two negative effects of the expected life expectancy on the per capita pension benefits. In addition, given the per capita pension benefits, an increase in the expected life expectancy has the effect of increasing the total pension benefits. Overall, there are three positive effects and two negative effects, and the former exceeds the latter; hence, an increase in the expected life expectancy leads to an increase in the pension GDP ratio.

Next, consider the education expenditure-GDP ratio. As the expected life expectancy increases, the political weight of older adults increases. The decrease in fertility brought about by the increase in the expected life expectancy further increases the relative political weight of older adults. These have the effect of reducing education spending through voting. Furthermore, an increase in the expected life expectancy implies an increase in the weight of the utility of old-age consumption, which has the effect of lowering the current labor-income tax rate and thus leads to a decrease in education spending. Due to these two negative effects, an increase in the expected life expectancy leads to a decrease in the ratio of education expenditure to GDP.

As already discussed, there are several effects of the expected life expectancy on the labor-income tax rate, which can be summarized as follows. First, an increase in the expected life expectancy raises the weight of utility form consumption in old age for middle-aged individuals. This has the effect of lowering the tax rate to maintain consumption or savings. Second, an increase in the expected life expectancy raises the political weight of older adults and thus gives the government an incentive to increase pension benefits. This has the effect of raising the tax rate. Finally, since an increase in the expected life expectancy leads to a decrease in the fertility rate (Proposition 2), this further raises the political weight of older adults, reinforcing the second effect. In summary, there are two positive and one negative effects of expected life expectancy on the labor- income tax rate, and the net effect is positive or negative depending on the structural parameter values.

Finally, based on the result presented in Proposition 1, we derive the growth rate of the economy and investigate how this is affected by an increased life expectancy. To this end, we consider per capita output, $y = Qk$. Then the growth rate of per capita output is

$$\frac{y'}{y} = \frac{Qk'}{Qk} = \frac{s/n}{k}. \tag{34}$$

Equation (34) indicates that the expected life expectancy affects the growth rate via the fertility rate, n, as well as savings, s.

Proposition 4 *An increase in expected life expectancy leads to an increase in per capita GDP growth rate:* $\partial\left(y'/y\right)/\partial\pi > 0$.

Proof. See Appendix A.3.

As we have already shown, an increase in the expected life expectancy leads to a decrease in the fertility rate. Thus, this has a positive effect on the growth rate. To see the growth effect through savings, recall the savings function in (18), which is restated as follows:

$$s = \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right]$$

Expected life expectancy affects saving through three terms: a coefficient $\beta\pi/(1+\beta\pi)$, representing the propensity to save, the labor-income tax rate, τ, and pension benefits, b'.

As the expected life expectancy increases, the weight on the utility of old-age consumption increases. This in turn strengthens the incentive for individuals to save, and thus has a positive effect on savings. Second, an increase in the expected life expectancy increases the political weight of older adults, which strengthens the incentive for the government to increase pension payments. This works to raise the labor-income tax rate. On the other hand, since a higher expected life expectancy increases the weight of the utility of old-age consumption, the government lowers the labor-income tax rate in order to maintain the level of consumption in old age. Finally, the increase in the expected life expectancy has two positive effects and two negative effects on the determination of pension benefits, as discussed above. In the end, the sum of the positive effects exceeds the sum of the negative effects in the current present framework, so an increase in the expected life expectancy leads to an increase in the per capita GDP growth rate.

5 Pension Reforms

The aging of the population due to the increase in the expected life expectancy, and the associated decline in fertility (Proposition 2), will lead to an increase in the political weight of older adults. This gives the short-lived government, representing the current living generations, an incentive to increase the pension benefits (Proposition 3). While increased pension benefits discourage individuals from saving, they also influence the behavior of the government, which ultimately leads to an increase in the GDP per capita (Proposition 4). However, since future generations cannot participate in current policy decisions, they cannot internalize the impact of the current period policy on them through physical and human capital accumulation.

To internalize these externalities, we consider a long-lived planner who has the power to impose ceiling constraints on pension benefits. The decision of this planner can be seen as a kind of enactment of a law that restricts pension benefits from a long-term perspective. We first look at how the corresponding tax rate, education expenditures, fertility rates, and economic growth rates would change if the planner introduced a ceiling on pension benefits in an economy that does not have such a ceiling, such as the one we characterized in Section 4. We then derive the optimal ceiling on pension benefits in terms of maximizing social welfare that aggregates the utility of all generations, and identify when it is desirable to impose the ceiling.

5.1 Effects of Pension Cuts

Recall the pension benefits in the absence of the ceiling in (22), which is restated as:

$$b = \frac{\frac{\pi\omega}{n_-(1-\omega)}B + C}{\frac{\pi\omega}{n_-(1-\omega)}E + F} \cdot \frac{R}{\pi} n_- k.$$

Based on the equilibrium policy function of b in (30), B, C, E, and F are defined by

$$B \equiv \frac{1-\alpha}{\alpha}, C \equiv -(1+\delta\eta+\beta\pi), E \equiv 1, F \equiv 1+\delta\eta+\beta\pi.$$

Let ε and ε' denote the ceiling on pension benefits for the current and next periods, respectively. Then we can write b and b' in the presence of the ceiling as follows:

$$b = \varepsilon \cdot \frac{\frac{\pi\omega}{n_-(1-\omega)}B + C}{\frac{\pi\omega}{n_-(1-\omega)}D + E} \cdot \frac{R}{\pi} n_- k, \tag{35}$$

$$b' = \acute{\varepsilon} \cdot \frac{\frac{\pi\omega}{n(1-\omega)}B + C}{\frac{\pi\omega}{n(1-\omega)}D + E} \cdot \frac{R}{\pi} n k', \tag{36}$$

where $\varepsilon,\ \varepsilon' \in [0,1]$. In other words, when $\varepsilon = \varepsilon' = 1$ holds, we have a situation where there are no restrictions on pension benefits, and we obtain the same policy functions and corresponding allocation as in Section 3. Since $\varepsilon = \varepsilon' = 1$ is optimal for the government that aims to maximize the political objective function, when $\varepsilon, \varepsilon' < 1$, it chooses to provide pension benefits up to the upper limit.

In the presence of the ceiling, the conjecture of the pension benefits in (36) is reformulated by using the fertility function in (17) and saving function in (18) as follows:

$$b' = \tilde{G}\left(\varepsilon'\right) \cdot (1-\tau)k, \tag{37}$$

where the definition of $\tilde{G}$ is given in Appendix A.4. Equation (37) is analogous to (24) in the absence of the ceiling, and thus $G = \tilde{G}$ holds if $\varepsilon' = 1$. Following the same manner as in the absence of the ceiling, we can derive the fertility and consumption functions and the associated political objective function as follows:

$$\Omega = \frac{\pi\omega}{n_-(1-\omega)} \ln \tilde{d}\left(n_-, k, \varepsilon\right) + \ln \tilde{c}\left(\tau, k, \varepsilon'\right) + \delta \ln \tilde{n}\left(\varepsilon'\right) h(x,h) + \beta\pi \ln \tilde{d}'\left(\tau, k, \varepsilon'\right), \tag{38}$$

where n, c, d', and d are

$$\tilde{n}\left(\varepsilon'\right) \equiv \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{(1-\alpha)Q + \frac{\tilde{G}(\varepsilon')}{R/\pi}}{(1-\alpha)Q + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{\tilde{G}(\varepsilon')}{R/\pi}}, \tag{39}$$

$$\tilde{c}\left(\tau, k, \varepsilon'\right) \equiv \frac{1}{1+\delta+\beta\pi} \cdot \left[(1-\tau)\,\bar{w}\left(\tilde{n}\left(\varepsilon'\right), k\right) + \frac{\tilde{G}\left(\varepsilon'\right) \cdot (1-\tau)\,k}{R/\pi}\right], \tag{40}$$

$$\tilde{d}'\left(\tau, k, \varepsilon'\right) \equiv \frac{\beta R}{1+\delta+\beta\pi} \cdot \left[(1-\tau)\,\bar{w}\left(\tilde{n}\left(\varepsilon'\right), k\right) + \frac{\tilde{G}\left(\varepsilon'\right) \cdot (1-\tau)\,k}{R/\pi}\right], \tag{41}$$

$$d = \tilde{d}\left(n_-, k, \varepsilon\right) \equiv \frac{R}{\pi} n_- k + \varepsilon \cdot \frac{\frac{\pi\omega}{n_-(1-\omega)} B + C}{\frac{\pi\omega}{n_-(1-\omega)} E + F} \cdot \frac{R}{\pi} n_- k. \tag{42}$$

The government in each period chooses a set of policy variables, (τ, x, b), to maximize Ω in (38) subject to the government budget constraint in (13), taking the state variables, k and n_- as well as the ceilings, ε and ε', as a given. The derivation of the following solutions is given in Appendix A.5.

The government's choice of the labor-income tax rate becomes

$$\tau = \tilde{\tau}\left(n_-, \varepsilon\right) \equiv \frac{\delta\eta(1-\alpha)\left[\frac{\pi\omega}{n_-(1-\omega)} + (1+\delta\eta+\beta\pi)\right] + \varepsilon\,(1+\beta\pi)\left[\frac{\pi\omega}{n_-(1-\omega)}(1-\alpha) - (1+\delta\eta+\beta\pi)\,\alpha\right]}{(1+\delta\eta+\beta\pi)\,(1-\alpha)\left[\frac{\pi\omega}{n_-(1-\omega)} + (1+\delta\eta+\beta\pi)\right]}. \tag{43}$$

Equation (43) indicates that tighter pension rules (i.e., a lower ε) leads to lower pension benefits. This means less tax revenue is needed to pay for pension benefits, so the government has an incentive to lower the labor-income tax rate. The associate education expenditure, x, is obtained by substituting the tax rate in (43) and the pension benefits in (35) into the government budget constraint:

$$x = \tilde{X}\left(\varepsilon, \tilde{n}\left(\varepsilon\right), \tilde{n}\left(\varepsilon'\right)\right) \cdot Qk, \tag{44}$$

where $\tilde{X}(\cdot)$ is defined by

$$\tilde{X}\left(\varepsilon, \tilde{n}\left(\varepsilon\right), \tilde{n}\left(\varepsilon'\right)\right) \equiv \frac{1}{\tilde{n}\left(\varepsilon'\right)} \cdot \frac{\delta\eta}{1+\delta\eta+\beta\pi} \cdot \frac{\frac{\pi\omega}{\tilde{n}(\varepsilon)(1-\omega)}(1-\alpha)\,(1-\varepsilon) + (1+\delta\eta+\beta\pi)\,(1-\alpha(1-\varepsilon))}{\frac{\pi\omega}{\tilde{n}(\varepsilon)(1-\omega)} + (1+\delta\eta+\beta\pi)}.$$

Figure 1 numerically shows how the policy variables and the corresponding economic growth rates are affected when the current ceiling on pension benefits, ε, is reduced, taking the future ceiling, ε', as a given. For numerical illustration, we estimate the parameters of the model using some key statistics of average OECD countries during 1995 and 2015 (see Appendix A.6 in details). Since a reduction in the ceiling implies a decrease in the financial resources needed, the labor-income tax rate decreases along with the pension benefit-GDP ratio, as illustrated in panels (a) and (b), respectively. However, the ratio of education expenditure to GDP raises as in Panel (c) because the decrease in pension benefits leaves more financial resources available for education. In addition, the income effect of the lower tax rate works to increase savings as in Panel (d), which turns into an increase in the economic growth rate as depicted in Panel (e).

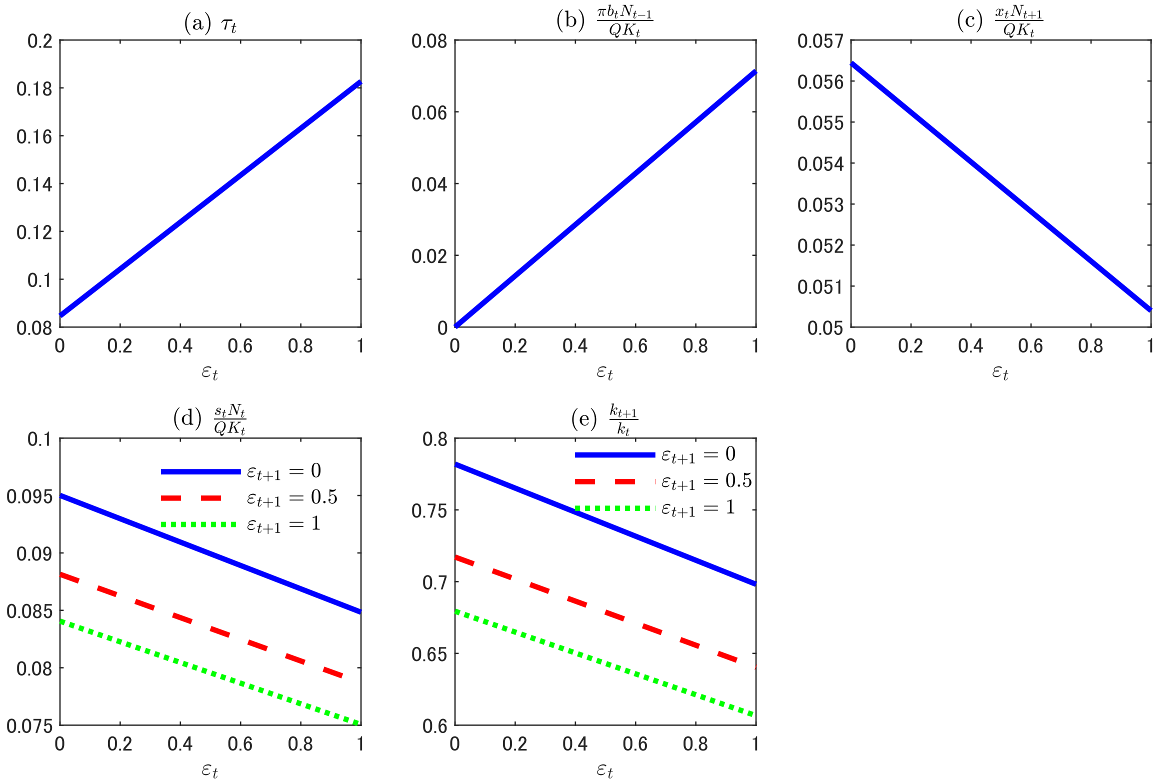

Figure 1: Effects of the current ceiling on pension benefits on (a) the labor-income tax rate, (b) ratio of pension benefits to GDP, (c) ratio of public education expenditure to GDP, (d) ratio of saving to GDP, and (e) per capita capital growth rate

Panels (d) and (e) also show the effects of the future ceiling, ε' on the growth rate. The result in panels (d) and (e) imply that a reduction in future pension benefits strengthens the incentive for individuals to save, which stimulates physical capital accumulation and thus increases the growth rate. We use the properties of these policy functions in the analysis of the next section.

5.2 Optimal Pension Reform

As mentioned in the introduction to this section, governments representing the currently living generations do not consider intergenerational externalities in their policy choices. To solve this problem, we have introduced pension ceilings and shown that by manipulating this ceiling, we can control the government's policy choices and the corresponding economic outcomes. Taking this result into account, we now aim to maximize the social welfare, which is the aggregate sum of the utility of each generation, by manipulating the pension ceiling. By doing so, we clarify under what conditions the equilibrium allocation in the absence of the pension ceiling fails to achieve social welfare maximization, and what level of pension ceiling should be imposed in such a case.

The social welfare function, denoted by SW, is

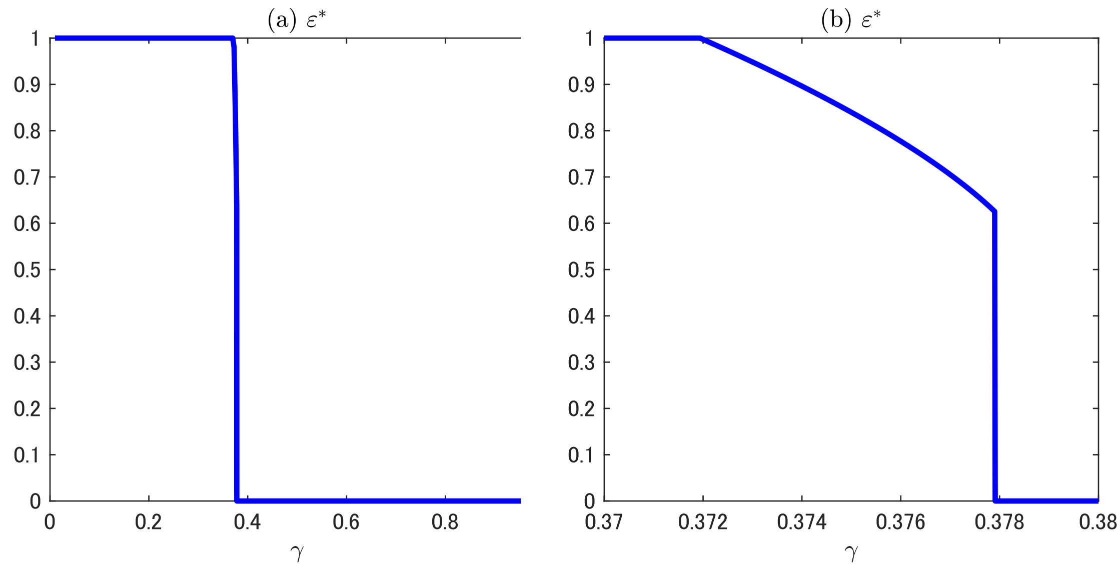

Figure 2: (a) The optimal ε that maximizes the social welfare; (b) an enlarged view of the figure in panel (a) around $\gamma = 0.372$.

$$SW = \gamma^{-1} V_0^O + \sum_{t=0}^{\infty} \gamma^t V_t^M, \tag{45}$$

where $\gamma \in (0,1)$ is the social discount factor. Reverse discounting, $1/\gamma(>1)$, must be applied to V_0^o to preserve dynamic consistency. The long-lived planner, whose objective is to choose the sequence of the pension ceiling, maximizes SW subject to the successive short-lived governments' choice of policies in (37), (43), and (44), and the associated fertility and consumption functions in (39) – (42). We assume that $\varepsilon_t = \varepsilon$ for all t, and that in the initial period, the long-lived planner imposes the ceiling that is invariant across periods. Under this assumption, SW is a function of ε as well as the initial conditions, n_{-1}, k_0, and h_0: $SW = SW(\varepsilon; n_{-1}, k_0, h_0)$. The derivation of $SW(\cdot)$ is included in Appendix A.7.

We derive the optimal ε that maximizes SW based on the numerical approach introduced in the previous subsection. The choice of ε is independent of the initial conditions of physical and human capital, k_0 and h_0, because of the assumption of the logarithmic utility function. The initial condition of fertility, n_{-1}, is set to match the fertility rate in the absence of the pension ceiling in (32). Figure 2 takes the social discount factor γ on the horizontal axis and plots the optimal ε. The figures show that there is a critical value of γ around 0.372. When γ is below the critical value, it is optimal to set $\varepsilon = 1$; no restrictions are required on the short-lived government's choice of pension benefits from the perspective of social welfare maximization. However, if γ is above the critical value, it is optimal to limit or prohibit the provision of pension benefits from the view of social welfare maximization.

To understand the mechanism behind the above-mentioned result, recall the effect of pension benefit cut we investigated in the previous subsection. The cut (i.e., a reduction of ε) has the following two opposing effects, as illustrated in Figure 3. One is the negative effect of the reduced

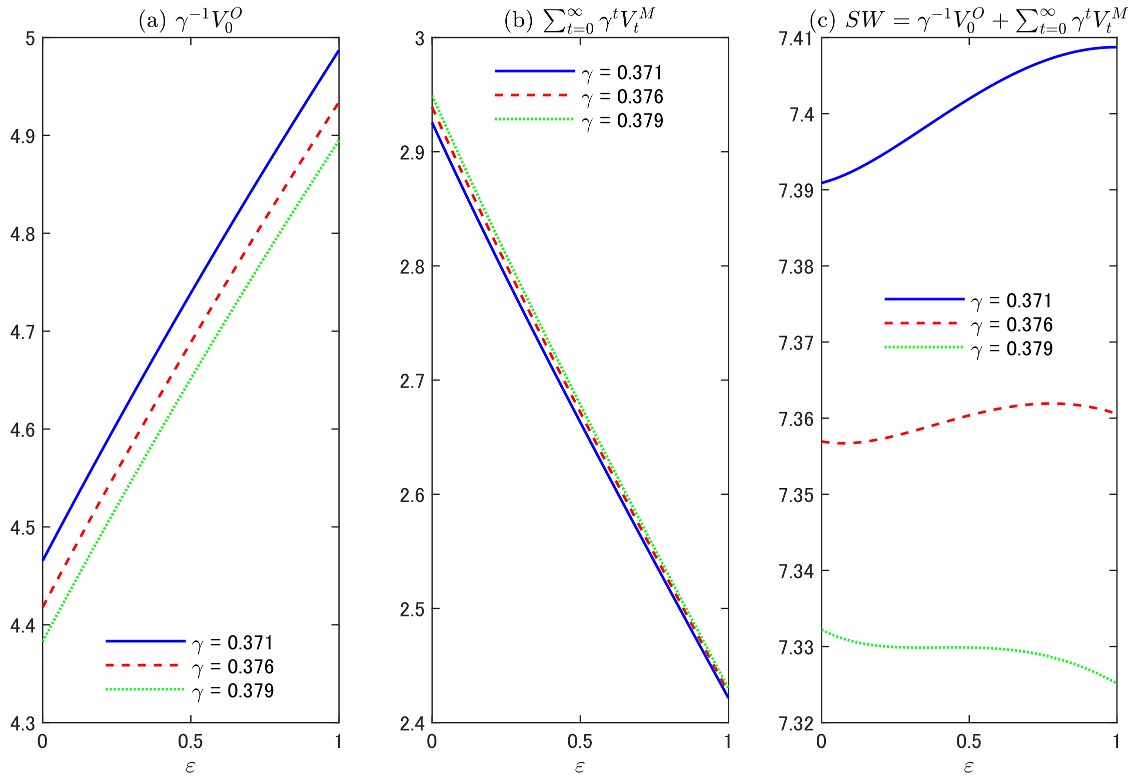

Figure 3: Effects of the pension cut on (a) the welfare of the initial old; (b) welfare of the generations born in period 0 onward; and (c) social welfare.
Note: Figure 3 compare the three cases: $\gamma = 0.371$, 0.376, and 0.379. Their relative order and the associated result remain unchanged even if we assume $\gamma < 0.371$ and $\gamma > 0.379$ for the first and third cases, respectively.

pension benefits received by the initial older adults. The larger the γ, the smaller this effect, as depicted in Panel (a). The other is the positive effect of the tax cut associated with the reduction in the pension payments, which includes the following two impacts: an increase in education spending, which stimulates human capital accumulation, and an increase in savings, which promotes physical capital accumulation. The larger the γ, the more highly the planner values these impacts, as depicted in Panel (b). Therefore, when the γ is small, the positive effect outweighs the negative one, so it is best to set the γ to 1 to maximize the social welfare, as observed from the blue solid curve in Panel (c). However, when the γ is large, the opposite holds: setting γ to 0 is optimal as observed from the green dotted curve in Panel (c). Finally, when γ is moderate, such that γ is set around 0.376, the optimal ε is between 0 and 1, as depicted by the red dotted curve in Panel (c). These results imply that whether to cut pension benefits from the viewpoint of social welfare depends on how much importance society attaches to its future generations.

The results of the optimal pension ceiling show a trade-off between fertility and growth. A reduction in pension benefits promotes savings, that is, capital accumulation, by reducing the tax burden on the working middle-aged, and thus increases the growth rate of per capita GDP. However, a reduction in pension benefits affects the fertility behavior of households through policy changes, leading to a decline in the fertility rate. In many developed countries, declining fertility is one of the major policy issues, and at the same time, dealing with an increasing pension burden, which hampers growth, is also a major policy challenge. Our results suggest the difficulty of reconciling the two goals of improving fertility and economic growth, which are likely to be urgent issues for many aging countries.

6 Conclusion

In this study, we addressed the following two questions: (1) how, in each period, the government allocates its limited budget to pension for older adults and education for the younger generation in response to population aging; and (2) from which economic perspective are the cuts in pension benefits being considered by developed countries justified. To examine these questions, we used the overlapping-generation model with physical and human capital accumulation and extended it by introducing quantity-quality trade-off in the decisions on having children. The novelty of our approach is that by endogenizing parents' fertility decisions, we could shed light on the interaction between political decisions on education and pension expenditures and parents' decisions on fertility.

Previous studies on the political economy of education and pensions assumed the population growth rate as an exogenous variable, and looked at the impact of changes in this exogenous variable (i.e., a decline in the population growth rate) on policy formation. Our study, on the other hand, modeled the endogenous fertility decisions of households and found that the decisions have an important impact on government policy choices and associated allocations. This study, however, assumes inelastic labor supply, so one of the issues to be addressed is the endogenization of labor supply. With this extension, we can look at household fertility and labor supply decisions and their impact on policymaking. We expect that the approach presented in this study will provide a basis for addressing this issue in the future.

A Appendices

A.1 Proofs of Propositions 1 and 2 and Corollary 1

Recall the conjecture of b' in (22), which is restated as

$$b' = \frac{\frac{\pi\omega}{n(1-\omega)}B + C}{\frac{\pi\omega}{n(1-\omega)}E + F}\frac{R}{\pi}s. \tag{A.1}$$

In addition, with the use of (5) and (16), the saving function in (6) is rewritten as

$$s = \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right]. \tag{A.2}$$

We substitute the fertility function in (17) and the saving function in (A.2) into the conjecture of b' in (A.1) and obtain

$$b' = \frac{\phi\frac{1+\delta+\beta\pi}{\delta}\frac{(1-\tau)(1-\alpha)Qk+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}}{(1-\tau)(1-\alpha)Qk+\frac{b'}{R/\pi}}\frac{\pi\omega}{1-\omega}B + C}{\phi\frac{1+\delta+\beta\pi}{\delta}\frac{(1-\tau)(1-\alpha)Qk+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}}{(1-\tau)(1-\alpha)Qk+\frac{b'}{R/\pi}}\frac{\pi\omega}{1-\omega}E + F}\cdot\frac{R}{\pi}\cdot\frac{\beta\pi}{1+\beta\pi}\cdot\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right], \tag{A.3}$$

or,

$$G_1\cdot\left(b'\right)^2 + G_2(1-\tau)k\cdot b' + G_3\cdot((1-\tau)k)^2 = 0, \tag{A.4}$$

where G_1, G_2, and G_3 are defined in the text. Assuming $G_1 \neq 0$, we solve (A.4) for b' and obtain

$$b' = G(1-\tau)k = \frac{-G_2 + \sqrt{(G_2)^2 - 4G_1G_3}}{2G_1}(1-\tau)k, \tag{A.5}$$

where G is defined in the text.

We substitute V_t^M in (19) and V_t^O in (20) into Ω_t in (21) and obtain

$$\begin{aligned}\Omega_t \simeq{}& \frac{\pi\omega}{n_{-1}(1-\omega)}\ln\left(\frac{R}{\pi}n_{-1}k + b\right) + (1+\delta+\beta\pi)\ln\left[(1-\tau)(1-\alpha)Qk + \frac{b'}{R/\pi}\right]\\ &- \delta\ln\left[(1-\tau)(1-\alpha)Qk + \frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}\right] + \delta\eta\ln x.\end{aligned} \tag{A.6}$$

We use the notation $\simeq$ in (21) because the irrelevant terms are omitted from the expression of Ω_t. We further substitute (A.5) and the government budget constraint in (13) into the political objective function in (A.6) and obtain

$$\Omega \simeq \frac{\pi\omega}{n_{-}(1-\omega)}\ln\left(\frac{R}{\pi}n_{-}k + b\right) + (1+\beta\pi)\ln(1-\tau) + \delta\eta\ln\left[\tau(1-\alpha)Qk - \frac{\pi b}{n_{-1}}\right]. \tag{A.7}$$

The first-order conditions with respect to b and τ are:

$$b: \frac{\pi\omega}{n_{-}(1-\omega)}\cdot\frac{1}{\frac{R}{\pi}n_{-}k + b} - \delta\eta\frac{\frac{\pi}{n_{-}}}{\tau(1-\alpha)Qk - \frac{\pi b}{n_{-}}} \leq 0, \tag{A.8}$$

$$\tau: \frac{(-1)(1+\beta\pi)}{1-\tau} + \frac{\delta\eta(1-\alpha)Qk}{\tau(1-\alpha)Qk - \frac{\pi b}{n_{-}}} = 0. \tag{A.9}$$

Equation (A.9) is rewritten as

$$\tau = \frac{\delta\eta(1-\alpha)Qk + (1+\beta\pi)\frac{\pi b}{n_-}}{(1+\delta\eta+\beta\pi)(1-\alpha)Qk}. \tag{A.10}$$

Substitution of (A.10) into (A.8) leads to the policy function of pension benefits:

$$b = \frac{\frac{\pi\omega}{n_-(1-\omega)}\frac{(1-\alpha)}{\alpha} - (1+\delta\eta+\beta\pi)}{\frac{\pi\omega}{n_-(1-\omega)} + (1+\delta\eta+\beta\pi)}\frac{R}{\pi}n_-k, \tag{A.11}$$

verifying the conjecture in (22). Equation (A.11) indicates that the public pension is provided, that is, $b > 0$ holds, if the fertility rate is below the following critical value:

$$b > 0 \Leftrightarrow 1+\delta\eta+\beta\pi < \frac{\pi\omega}{n_-(1-\omega)} \cdot \frac{1-\alpha}{\alpha}. \tag{A.12}$$

The policy function of τ is derived by substituting (A.11) into (A.9):

$$\tau = \frac{\frac{\pi\omega}{n_-(1-\omega)} + \left[\delta\eta - (1+\beta\pi)\frac{\alpha}{1-\alpha}\right]}{\left[\frac{\pi\omega}{n_-(1-\omega)} + (1+\delta\eta+\beta\pi)\right]}. \tag{A.13}$$

Recall that the fertility rate for a given set of policy variables is (17). We need to replace τ and b' in (17) with n_{-1} and k by using the policy functions in (A.11) and (A.13). Taking one period lag of (A.11), we have

$$b' = \frac{\frac{\pi\omega}{n(1-\omega)}\frac{1-\alpha}{\alpha} - (1+\delta\eta+\beta\pi)}{\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)}\frac{R}{\pi}nk' = \frac{z_1(n)}{z_0(n)}\frac{R}{\pi}nk', \tag{A.14}$$

where $z_0(\cdot)$ and $z_1(\cdot)$ are defined as

$$z_0(n) \equiv \frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi), \tag{A.15}$$

$$z_1(n) \equiv \frac{\pi\omega}{n(1-\omega)}\frac{1-\alpha}{\alpha} - (1+\delta\eta+\beta\pi). \tag{A.16}$$

The policy function b' in (A.14) is reformulated as follows:

$$\begin{aligned} b' &= \frac{z_1(n)}{z_0(n)}\frac{R}{\pi}s \\ &= \frac{z_1(n)}{z_0(n)}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right] \end{aligned}$$

where the equality in the first line comes from (14), and the equality in the second line comes from (A.2). By rearranging the terms, we have

$$\begin{aligned} \left[1 + \frac{z_1(n)}{z_0(n)}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1}{\beta\pi}\frac{1}{R/\pi}\right]b' &= \frac{z_1(n)}{z_0(n)}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}(1-\tau)(1-\alpha)Qk \\ &= \frac{z_1(n)}{z_0(n)}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1+\beta\pi}{(1-\alpha)z_0(n_{-1})}(1-\alpha)Qk, \end{aligned}$$

where equality in the second line comes from (A.13). Thus, we obtain

$$b' = \frac{\frac{z_1(n)}{z_0(n)}\frac{R}{\pi}\frac{\beta\pi}{(1-\alpha)z_0(n_{-1})}}{1+\frac{z_1(n)}{z_0(n)}\frac{1}{1+\beta\pi}}(1-\alpha)Qk. \tag{A.17}$$

We substitute (A.13) and (A.17) into the fertility function in (17) and obtain the equation that characterizes the equilibrium fertility rate:

$$n = \frac{1}{\phi}\cdot\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{1+\frac{z_1(n)}{z_0(n)}}{1+\frac{z_1(n)}{z_0(n)}\cdot\frac{1+\delta}{1+\delta+\beta\pi}}. \tag{A.18}$$

We further reformulate the expression in (A.18) by substituting $z_0(\cdot)$ in (A.15) and $z_1(\cdot)$ in (A.16) into (A.18) and rearranging the terms to obtain:

$$n = \frac{\delta}{\phi}\cdot\frac{1}{(1+\delta+\alpha\beta\pi)+\alpha\beta\pi\,(1+\delta\eta+\beta\pi)\,\frac{n(1-\omega)}{\pi\omega}}. \tag{A.19}$$

Equation (A.19) shows that there is a unique $n > 0$ satisfying (A.19). We should note that (A.19) is rewritten as a quadratic equation for n :

$$\alpha\beta\,(1+\delta\eta+\beta\pi)\,\frac{1-\omega}{\omega}\,(n)^2 + (1+\delta+\alpha\beta\pi)\,n - \frac{\delta}{\phi} = 0.$$

Thus, we can solve this equation for n as

$$n = n\,(\pi) \equiv \frac{-\,(1+\delta+\alpha\beta\pi)+\sqrt{(1+\delta+\alpha\beta\pi)^2+4\alpha\beta\,(1+\delta\eta+\beta\pi)\,\frac{1-\omega}{\omega}\frac{\delta}{\phi}}}{2\alpha\beta\,(1+\delta\eta+\beta\pi)\,\frac{1-\omega}{\omega}}, \tag{A.20}$$

where $n'(\pi) < 0$ holds.

To obtain the policy function of x, we substitute the policy functions of b in (A.11) and τ in (A.13) into the government budget constraint in (13). Then, we have

$$x = \frac{1}{n\,(\pi)}\cdot\frac{\delta\eta}{\frac{\pi\omega}{n_{-}(1-\omega)}+(1+\delta\eta+\beta\pi)}Qk, \tag{A.21}$$

where $n = n\,(\pi)$ comes from the fertility rate in (A.20).

■

A.2 Proof of Proposition 3

The aggregate expenditure for public pension is πbN_{-}, and the aggregate GDP is QK. Thus, the ratio of public pension to GDP is

$$\begin{aligned}\frac{\pi bN_{-}}{QK} &= \frac{\pi b}{Qkn_{-}}\\ &= \frac{\frac{\pi\omega}{n_{-}(1-\omega)}(1-\alpha)-(1+\delta\eta+\beta\pi)\,\alpha}{\frac{\pi\omega}{n_{-}(1-\omega)}+(1+\delta\eta+\beta\pi)},\end{aligned}$$

where the second equality comes from the policy function of b in (30). After some manipulation, we have

$$\frac{\pi b N_-}{QK} = \begin{cases} (1-\alpha) - \left[\frac{1}{\frac{1+\delta\eta}{\pi\omega}+\frac{\beta}{\omega}} \cdot \frac{1}{n_{-1}(1-\omega)} + 1\right]^{-1} & \text{for } t = 0, \\ (1-\alpha) - \left[\frac{1}{\frac{1+\delta\eta}{\pi\omega}+\frac{\beta}{\omega}} \cdot \frac{1}{n(\pi)(1-\omega)} + 1\right]^{-1} & \text{for } t \geq 1. \end{cases}.$$

Since $n'(\pi) < 0$, we have $\partial\left[\pi b N_-/QK\right]/\partial\pi > 0$.

The ratio of public education expenditure to GDP is

$$\begin{aligned} \frac{xN'}{QK} &= \frac{xn}{Qk} \\ &= \frac{\delta\eta}{\frac{\pi\omega}{n_-(1-\omega)} + (1+\delta\eta+\beta\pi)}, \end{aligned}$$

where the second equality comes from the policy function of x in (31). Thus, we have

$$\frac{xN'}{QK} = \begin{cases} \frac{\delta\eta}{\frac{\pi\omega}{n_{-1}(1-\omega)}+(1+\delta\eta+\beta\pi)} & \text{for } t = 0, \\ \frac{\delta\eta}{\frac{\pi\omega}{n(\pi)(1-\omega)}+(1+\delta\eta+\beta\pi)} & \text{for } t \geq 1. \end{cases}$$

Since $n'(\pi) < 0$, we have $\partial\left(xN'/QK\right)/\partial\pi < 0$.

■

A.3 Proof of Proposition 4

The aggregate output is QK, so the per capita output is $QK/N = Qk$. Thus, the gross growth rate of the per capita output is $Qk'/Qk = k'/k$. To compute k'/k, recall the capital market clearing condition in (14), which is rewritten as follows:

$$\begin{aligned} nk' &= \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right] \\ &= \frac{\beta\pi}{1+\beta\pi}\left[\frac{1+\beta\pi}{(1-\alpha)z_0(n_-)}(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{1}{R/\pi}\cdot\frac{\frac{z_1(n)}{z_0(n)}\frac{Q}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1+\beta\pi}{(1-\alpha)z_0(n_-)}(1-\alpha)Qk}{1+\frac{z_1(n)}{z_0(n)}\frac{Q}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1}{\beta\pi}\frac{1}{R/\pi}}\right], \end{aligned}$$

where the first equality is derived by using (A.2), and the second equality is derived by using (A.13) and (A.17). Rearranging the terms and substituting $n = n(\pi)$ in (A.20) into the above expression, we obtain

$$\frac{k'}{k} = \frac{\alpha Q}{\left(1 - \frac{\beta\pi}{1+\beta\pi}(1-\alpha)\right)\frac{\omega}{(1-\omega)\beta} + \left(1+\frac{\delta\eta}{1+\beta\pi}\right)\alpha n(\pi)}. \tag{A.22}$$

Notice that the first term in the denominator on the right-hand side is decreasing in π, and the second term in the denominator is also decreasing in π since $n'(\pi) < 0$. Thus, we obtain $\partial\left(k'/k\right)/\partial\pi > 0$.

■

A.4 Derivation of (37)

Recall the conjecture of b' in (36). By using the fertility function in (17) and the saving function in (18), we can reformulate (36) as follows:

$$
\begin{aligned}
b' = \varepsilon' \cdot & \frac{B\left[(1-\tau)(1-\alpha)Qk + \frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}\right] + C\frac{1}{\phi}\cdot\frac{\delta}{1+\delta+\beta\pi}\cdot\left[(1-\tau)(1-\alpha)Qk + \frac{b'}{R/\pi}\right]}{E\left[(1-\tau)(1-\alpha)Qk + \frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}\right] + F\frac{1}{\phi}\cdot\frac{\delta}{1+\delta+\beta\pi}\cdot\left[(1-\tau)(1-\alpha)Qk + \frac{b'}{R/\pi}\right]} \\
& \times \frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right].
\end{aligned}
$$

This is further reformulated as

$$
\tilde{G}_1 \cdot \left(b'\right)^2 + \tilde{G}_2 \cdot (1-\tau)\,kb' + \tilde{G}_3 \cdot \left[(1-\tau)\,k\right]^2 = 0, \tag{A.23}
$$

where $\tilde{G}_1$, $\tilde{G}_2$, and $\tilde{G}_3$ are defined by

$$
\begin{aligned}
\tilde{G}_1 &\equiv \frac{\delta}{1+\delta+\beta\pi}\frac{1}{R/\pi}\left[\left(\frac{\pi\omega}{1-\omega}E + \frac{F}{\phi}\right) + \varepsilon'\left(\frac{\pi\omega}{1-\omega}B + \frac{C}{\phi}\right)\frac{1}{1+\beta\pi}\right], \\
\tilde{G}_2 &\equiv (1-\alpha)Q\left\{\left(\frac{\pi\omega}{1-\omega}E + \frac{\delta}{1+\delta+\beta\pi}\frac{F}{\phi}\right)\right. \\
&\quad \left. + \varepsilon'\frac{\beta\pi}{1+\beta\pi}\left[-\frac{\delta}{1+\delta+\beta\pi}\left(\frac{\pi\omega}{1-\omega}B + \frac{C}{\phi}\right)\right] + \frac{1}{\beta\pi}\left(\frac{\pi\omega}{1-\omega}B + \frac{\delta}{1+\delta+\beta\pi}\frac{C}{\phi}\right)\right\}, \\
\tilde{G}_3 &\equiv (-1)\varepsilon'\left(\frac{\pi\omega}{1-\omega}B + \frac{\delta}{1+\delta+\beta\pi}\frac{C}{\phi}\right)\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\left[(1-\alpha)Q\right]^2.
\end{aligned}
$$

Solving (A.23) for b' leads to

$$
b' = \tilde{G}\left(\varepsilon'\right)\cdot(1-\tau)k,
$$

where $\tilde{G}\left(\cdot\right)$ is defined by

$$
\tilde{G}\left(\cdot\right) \equiv \frac{-\tilde{G}_2 + \sqrt{\left(\tilde{G}_2\right)^2 - 4\tilde{G}_1\tilde{G}_3}}{2\tilde{G}_1}.
$$

■

A.5 Derivation of (43) and (44)

We substitute the fertility and consumption functions shown in Subsection 4.1 into the political objective function in (38) and obtain

$$
\begin{aligned}
\Omega &= \frac{\pi\omega}{n_-(1-\omega)}\ln\tilde{d}\left(n_-,k,\varepsilon\right) + \ln\tilde{c}\left(\tau,k,\varepsilon'\right) + \delta\ln\tilde{n}\left(\varepsilon'\right)h(x,h) + \beta\pi\ln\tilde{d}'\left(\tau,k,\varepsilon'\right) \\
&\simeq \frac{\pi\omega}{n_-(1-\omega)}\ln\left(1 + \varepsilon\cdot\frac{\frac{\pi\omega}{n_-(1-\omega)}B + C}{\frac{\pi\omega}{n_-(1-\omega)}E + F}\right) + (1+\beta\pi)\ln\left(1-\tau\right) + \delta\eta\ln x.
\end{aligned}
$$

By using the government budget constraint, $\tau(1-\alpha)Qk = \pi b/n_- + nx$, we can reformulate the expression of Ω above as

$$\Omega \simeq \frac{\pi\omega}{n_-(1-\omega)} \ln\left(1+\varepsilon\cdot\frac{\frac{\pi\omega}{n_-(1-\omega)}B+C}{\frac{\pi\omega}{n_-(1-\omega)}E+F}\right) + (1+\beta\pi)\ln(1-\tau)$$
$$+\delta\eta\ln\frac{1}{\tilde{n}(\varepsilon')}\left[\tau(1-\alpha)-\varepsilon\frac{\frac{\pi\omega}{n_-(1-\omega)}B+C}{\frac{\pi\omega}{n_-(1-\omega)}E+F}\alpha\right].$$

The first-order condition with respect to τ is

$$\frac{1+\beta\pi}{1-\tau} = \frac{\delta\eta(1-\alpha)}{\tau(1-\alpha)-\varepsilon\frac{\frac{\pi\omega}{n_-(1-\omega)}B+C}{\frac{\pi\omega}{n_-(1-\omega)}D+E}\alpha},$$

which is rewritten as

$$\tau = \tilde{\tau}(n_-,\varepsilon) \equiv \frac{\delta\eta(1-\alpha)\left[\frac{\pi\omega}{n_-(1-\omega)}+(1+\delta\eta+\beta\pi)\right]+\varepsilon(1+\beta\pi)\left[\frac{\pi\omega}{n_-(1-\omega)}(1-\alpha)-(1+\delta\eta+\beta\pi)\alpha\right]}{(1+\delta\eta+\beta\pi)(1-\alpha)\left[\frac{\pi\omega}{n_-(1-\omega)}+(1+\delta\eta+\beta\pi)\right]}. \tag{A.25}$$

Substituting (A.25) and (39) into the government budget constraint in (13), we have

$$\tilde{\tau}(n_-,\varepsilon)(1-\alpha)Qk = \frac{\pi}{n_-}\varepsilon\cdot\frac{\frac{\pi\omega}{n_-(1-\omega)}B+C}{\frac{\pi\omega}{n_-(1-\omega)}D+E}\cdot\frac{R}{\pi}n_-k+\tilde{n}(\varepsilon')x,$$

or,

$$x = X(\varepsilon,\tilde{n}(\varepsilon),\tilde{n}(\varepsilon'))\cdot Qk,$$

where,

$$\tilde{X}(\varepsilon,\tilde{n}(\varepsilon),\tilde{n}(\varepsilon')) \equiv \frac{1}{\tilde{n}(\varepsilon')}\cdot\frac{\delta\eta}{1+\delta\eta+\beta\pi}\cdot\frac{\frac{\pi\omega}{\tilde{n}(\varepsilon)(1-\omega)}(1-\alpha)(1-\varepsilon)+(1+\delta\eta+\beta\pi)(1-\alpha(1-\varepsilon))}{\frac{\pi\omega}{\tilde{n}(\varepsilon)(1-\omega)}+(1+\delta\eta+\beta\pi)},$$

■

A.6 Model Calibration

We describe here how to estimate the parameters in the model to illustrate the results in Figures 1, 2 and 3. Our strategy is to calibrate the model economy in the absence of the pension ceiling, such that the political equilibrium with $b > 0$ matches some key statistics of average OECD countries during 1995 and 2015. We fix the share of capital at $\alpha = 1/3$ following Song et al. (2012) and Lancia and Russo (2016). Each period lasts 30 years; this assumption is standard in the quantitative analyses of the two- or three-period overlapping-generation model (e.g., Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2016). Our selection of R is 1.04 per year (e.g., Song et al., 2012; Lancia and Russo, 2016). The productivity parameter is $Q = 3.12$ since $Q = R/\alpha$.

The discount factor β is set at 0.99 per quarter, which is standard in the literature (e.g., Kydland and Prescott, 1982; de la Croix and Doepke, 2003). Since the individuals in the present model plan over generations that span 30 years, we discount the future by $(0.99)^{4\times 30} = (0.99)^{120}$. The opportunity cost of raising a child is about 20% of the parent's time (Kimura and Yasui, 2009). By assuming that the duration of parenthood is 18 years, ϕ is $0.2 \times (18/30) = 0.12$. The probability of living in old age, π, is taken from the average life expectancy at birth. The average life expectancy in OECD countries is 78.474 years,[5] so individuals will, on an average, live $78.474 - 60 = 18.474$ years into old age. In other words, individuals are expected to live 18.474/30 of their 30 years of old age, so $\pi \simeq 0.616$.

To determine the remaining three parameters, δ, η, and ω, we focus on the fertility rate in (32), ratio of education expenditure to GDP, xN'/Y, and ratio of pension benefits to GDP, $\pi bN_-/Y$ in the absence of the pension ceiling. The two ratios are given by

$$\frac{xN'}{Y} = \frac{\delta\eta}{\frac{\pi\omega}{n(1-\omega)} + (1 + \delta\eta + \beta\pi)}, \tag{A.26}$$

$$\frac{\pi bN_-}{Y} = \frac{\frac{\pi\omega}{n(1-\omega)}(1-\alpha) - (1 + \delta\eta + \beta\pi)\,\alpha}{\frac{\pi\omega}{n(1-\omega)} + (1 + \delta\eta + \beta\pi)}. \tag{A.27}$$

We use the data of the OECD average during 1995–2015 to solve the three equations (32), (A.26), and (A.27) for δ, η, and ω. The annual population growth rate of 1.0064. This implies that the gross population growth rate for 30 years is $(1.0064)^{30}$. The average ratio of education expenditure to GDP is 0.0504, and average ratio of pension benefits to GDP is 0.0714.[6] We substitute these data and the values of α, β, ϕ, and π into (32), (A.26), and (A.27). Then we obtain $\delta = 0.1958$, $\eta = 0.5594$, and $\omega = 0.6336$.

■

A.7 Derivation of Social Welfare Function

Recall the pension benefits in the presence of the ceiling in (35) and (36), which are restated as follows:

$$b_t = \varepsilon_t \cdot \frac{\frac{\pi\omega}{n_{t-1}(1-\omega)}B + C}{\frac{\pi\omega}{n_{t-1}(1-\omega)}E + F} \cdot \frac{R}{\pi} n_{t-1} k_t, \tag{A.28}$$

$$b_{t+1} = \varepsilon_{t+1} \cdot \frac{\frac{\pi\omega}{n_t(1-\omega)}B + C}{\frac{\pi\omega}{n_t(1-\omega)}E + F} \cdot \frac{R}{\pi} n_t k_t.$$

For the tractability of the following analysis, the period is specified using subscripts. From (37), we have

$$b_{t+1} = \tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t. \tag{A.29}$$

[5]Source: OECD Stat, https://stats.oecd.org/ (Accessed on August 27, 2021). The OECD average data collected below are from the same source, unless otherwise noted.

[6]Sources: For the average ratio of education expenditure to GDP: World Development Indicators, https://databank.worldbank.org/source/world-development-indicators (Accessed on August 27, 2021). For the average ratio of pension benefits to GDP: OECD (2001), Pension spending (indicator). doi: 10.1787/a041f4ef-en (Accessed on September 14, 2021).

The fertility functions, with period specified, can be written as follows:

$$n_t = \begin{cases} n_{-1} & \text{for } t = -1 \\ \tilde{n}\left(\varepsilon_{t+1}\right) & \text{for } t \geq 0. \end{cases} \tag{A.30}$$

Recall the indirect utility function of the middle-aged individuals in (19). By specifying period, we can write (19) as

$$\begin{aligned} V_t^M &= \ln c\left(\tau_t, \bar{w}\left(n_t, k_t\right), b_{t+1}\right) + \delta \ln n_t \cdot h\left(h_t, x_t\right) \\ &\quad + \beta\pi \ln d'\left(\tau_t, \bar{w}\left(n_t, k_t\right), b_{t+1}\right). \end{aligned}$$

Plugging the pension benefits in period $t+1$ in (A.29), we have

$$\begin{aligned} V_t^M &= \ln c\left(\tau_t, \bar{w}\left(n\left(\tau_t, \tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t, k_t\right), k_t\right), \tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t\right) \\ &\quad + \delta \ln n\left(\tau_t, \tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t, k_t\right) \cdot h\left(h_t, x_t\right) \\ &\quad + \beta\pi \ln d'\left(\tau_t, \bar{w}\left(n\left(\tau_t, \tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t, k_t\right), k_t\right), \tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t\right). \end{aligned} \tag{A.31}$$

With (A.30), we can write the tax rate in period t as

$$\tau_t = \tilde{\tau}\left(\tilde{n}\left(\varepsilon_t\right), \varepsilon\right) = \begin{cases} \tilde{\tau}\left(n_{-1}, \varepsilon_0\right) & \text{for } t = 0, \\ \tilde{\tau}\left(\tilde{n}\left(\varepsilon_t\right), \varepsilon_t\right) & \text{for } t \geq 1. \end{cases} \tag{A.32}$$

The education expenditure x_t, given by (44), is

$$x_t = \tilde{X}\left(\cdot\right) k_t, \tag{A.33}$$

where $\tilde{X}\left(\cdot\right)$ is:

$$\tilde{X}\left(\cdot\right) = \begin{cases} \tilde{X}\left(\varepsilon_0, n_{-1}, \tilde{n}\left(\varepsilon_1\right)\right) \equiv \frac{1}{\tilde{n}(\varepsilon_1)} \cdot \frac{\delta\eta}{1+\delta\eta+\beta\pi} \cdot \frac{\frac{\pi\omega}{n_{-1}(1-\omega)}(1-\alpha)(1-\varepsilon_0)+(1+\delta\eta+\beta\pi)(1-\alpha(1-\varepsilon_0))}{\frac{\pi\omega}{n_{-1}(1-\omega)}+(1+\delta\eta+\beta\pi)} Q & \text{for } t = 0, \\ \tilde{X}\left(\varepsilon_t, \tilde{n}\left(\varepsilon_t\right), \tilde{n}\left(\varepsilon_{t+1}\right)\right) \equiv \frac{1}{\tilde{n}(\varepsilon_{t+1})} \cdot \frac{\delta\eta}{1+\delta\eta+\beta\pi} \cdot \frac{\frac{\pi\omega}{\tilde{n}(\varepsilon_t)(1-\omega)}(1-\alpha)(1-\varepsilon_t)+(1+\delta\eta+\beta\pi)(1-\alpha(1-\varepsilon_t))}{\frac{\pi\omega}{\tilde{n}(\varepsilon_t)(1-\omega)}+(1+\delta\eta+\beta\pi)} Q & \text{for } t \geq 1. \end{cases} \tag{A.34}$$

We replace k_t appeared in (A.31) and (A.33) by k_0 and $(\varepsilon_t)_{t=0}$. From the capital market clearing condition, we have

$$\begin{aligned} k_{t+1} &= \frac{s_t}{n_t} \\ &= \frac{1}{n_t} \cdot \frac{\beta\pi}{1+\delta+\beta\pi}\left[(1-\tau_t)\frac{(1-\alpha)Q}{1-\phi n_t}k_t - \frac{1+\delta}{\beta\pi} \cdot \frac{b_{t+1}}{R/\pi}\right] \\ &= \frac{1}{n_t} \cdot \frac{\beta\pi}{1+\delta+\beta\pi}\left[\frac{(1-\alpha)Q}{1-\phi n_t} - \frac{1+\delta}{\beta\pi} \cdot \frac{\tilde{G}\left(\varepsilon_{t+1}\right)}{R/\pi}\right](1-\tau_t)k_t, \end{aligned} \tag{A.35}$$

where the second line comes from the saving function in (18) and the effective wage income in (16), and the third line comes from (A.29). By substituting (A.30) and (A.32) into (A.35), we obtain

$$k_{t+1} = \tilde{K}\left(\cdot\right) k_t, \tag{A.36}$$

where $\tilde{K}(\cdot)$ is defined by

$$\tilde{K}(\cdot)=\begin{cases}\begin{array}{c}\tilde{K}\left(\varepsilon_0,n_{-1},\varepsilon_1,\tilde{n}\left(\varepsilon_1\right)\right)\\ \equiv\frac{1}{\tilde{n}(\varepsilon_1)}\cdot\frac{\beta\pi}{1+\delta+\beta\pi}\left[\frac{(1-\alpha)Q}{1-\phi\tilde{n}(\varepsilon_1)}-\frac{1+\delta}{\beta\pi}\cdot\frac{\tilde{G}(\varepsilon_1)}{R/\pi}\right]\left(1-\tau\left(n_{-1},\varepsilon_0\right)\right)\end{array} & \text{for } t=0,\\ \begin{array}{c}\tilde{K}\left(\varepsilon_t,\tilde{n}\left(\varepsilon_t\right),\varepsilon_{t+1},\tilde{n}\left(\varepsilon_{t+1}\right)\right)\\ \equiv\frac{1}{\tilde{n}(\varepsilon_{t+1})}\cdot\frac{\beta\pi}{1+\delta+\beta\pi}\left[\frac{(1-\alpha)Q}{1-\phi\tilde{n}(\varepsilon_{t+1})}-\frac{1+\delta}{\beta\pi}\cdot\frac{\tilde{G}(\varepsilon_{t+1})}{R/\pi}\right]\left(1-\tau\left(\tilde{n}\left(\varepsilon_t\right),\varepsilon_t\right)\right)\end{array} & \text{for } t\geq 1.\end{cases} \tag{A.37}$$

Thus, from (A.36) and (A.37), we can write k_t as a function of k_0 and $(\varepsilon_t)_{t=0}$ as follows:

$$\begin{aligned}k_t&=\tilde{K}\left(\varepsilon_{t-1},\tilde{n}\left(\varepsilon_{t-1}\right),\varepsilon_t,\tilde{n}\left(\varepsilon_t\right)\right)k_{t-1}\\&=\tilde{K}\left(\varepsilon_{t-1},\tilde{n}\left(\varepsilon_{t-1}\right),\varepsilon_t,\tilde{n}\left(\varepsilon_t\right)\right)\tilde{K}\left(\varepsilon_{t-2},\tilde{n}\left(\varepsilon_{t-2}\right),\varepsilon_{t-1},\tilde{n}\left(\varepsilon_{t-1}\right)\right)k_{t-2}\\&\ \vdots\\&=\tilde{K}\left(\varepsilon_{t-1},\tilde{n}\left(\varepsilon_{t-1}\right),\varepsilon_t,\tilde{n}\left(\varepsilon_t\right)\right)\tilde{K}\left(\varepsilon_{t-2},\tilde{n}\left(\varepsilon_{t-2}\right),\varepsilon_{t-1},\tilde{n}\left(\varepsilon_{t-1}\right)\right)\cdots\\&\cdots\tilde{K}\left(\varepsilon_1,\tilde{n}\left(\varepsilon_1\right),\varepsilon_2,\tilde{n}\left(\varepsilon_2\right)\right)\tilde{K}\left(\varepsilon_0,n_{-1},\varepsilon_1,\tilde{n}\left(\varepsilon_1\right)\right)k_0\\&=\prod_{j=1}^{t-1}\tilde{K}\left(\varepsilon_j,\tilde{n}\left(\varepsilon_j\right),\varepsilon_{j+1},\tilde{n}\left(\varepsilon_{j+1}\right)\right)\tilde{K}\left(\varepsilon_0,n_{-1},\varepsilon_1,\tilde{n}\left(\varepsilon_1\right)\right)k_0.\end{aligned} \tag{A.38}$$

Recall the indirect utility function of the period-t middle-aged in (A.31), which is reformulated as

$$\begin{aligned}V_t^M&\simeq(1+\delta+\beta\pi)\ln\left[(1-\tau_t)(1-\alpha)Qk_t+\frac{\tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t}{R/\pi}\right]\\&-\delta\ln\left[(1-\tau_t)(1-\alpha)Qk_t+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{\tilde{G}\left(\varepsilon_{t+1}\right)(1-\tau_t)k_t}{R/\pi}\right]+\delta\eta\ln x_t\\&=(1+\delta+\beta\pi)\ln\left[(1-\alpha)Q+\frac{\tilde{G}\left(\varepsilon_{t+1}\right)}{R/\pi}\right]-\delta\ln\left[(1-\alpha)Q+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{\tilde{G}\left(\varepsilon_{t+1}\right)}{R/\pi}\right]\\&+(1+\beta\pi)\ln(1-\tau_t)k_t+\delta\eta\ln x_t,\end{aligned}$$

or,

$$V_t^M\simeq\tilde{V}\left(\varepsilon_{t+1}\right)+(1+\beta\pi)\ln(1-\tau_t)k_t+\delta\eta\ln x_t, \tag{A.39}$$

where $\tilde{V}\left(\varepsilon_{t+1}\right)$ is defined by

$$\tilde{V}\left(\varepsilon_{t+1}\right)\equiv(1+\delta+\beta\pi)\ln\left[(1-\alpha)Q+\frac{\tilde{G}\left(\varepsilon_{t+1}\right)}{R/\pi}\right]-\delta\ln\left[(1-\alpha)Q+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{\tilde{G}\left(\varepsilon_{t+1}\right)}{R/\pi}\right]. \tag{A.40}$$

We substitute (A.32), (A.33), and (A.38) into (A.39) and reformulate the indirect utility function of the period-0 middle-aged as

$$\begin{aligned}V_0^M&\simeq\tilde{V}\left(\varepsilon_1\right)+(1+\beta\pi)\ln(1-\tilde{\tau}\left(n_{-1},\varepsilon_0\right))k_0+\delta\eta\ln\tilde{X}\left(\varepsilon_0,n_{-1},\tilde{n}\left(\varepsilon_1\right)\right)k_0\\&\simeq\tilde{V}\left(\varepsilon_1\right)+(1+\beta\pi)\ln(1-\tilde{\tau}\left(n_{-1},\varepsilon_0\right))+\delta\eta\ln\tilde{X}\left(\varepsilon_0,n_{-1},\tilde{n}\left(\varepsilon_1\right)\right)\end{aligned} \tag{A.41}$$

The indirect utility of the period-t middle-aged is

$$\begin{aligned}
V_t^M &\simeq \tilde{V}(\varepsilon_{t+1}) + (1+\beta\pi)\ln(1-\tilde{\tau}(\tilde{n}(\varepsilon_t),\varepsilon_t))k_t + \delta\eta\ln\tilde{X}(\varepsilon_t,\tilde{n}(\varepsilon_t),\tilde{n}(\varepsilon_{t+1}))k_t \\
&= \tilde{V}(\varepsilon_{t+1}) + (1+\beta\pi)\ln(1-\tilde{\tau}(\tilde{n}(\varepsilon_t),\varepsilon_t)) + \delta\eta\ln\tilde{X}(\varepsilon_t,\tilde{n}(\varepsilon_t),\tilde{n}(\varepsilon_{t+1})) + (1+\delta\eta+\beta\pi)\ln k_t \\
&\simeq \tilde{V}(\varepsilon_{t+1}) + (1+\beta\pi)\ln(1-\tilde{\tau}(\tilde{n}(\varepsilon_t),\varepsilon_t)) + \delta\eta\ln\tilde{X}(\varepsilon_t,\tilde{n}(\varepsilon_t),\tilde{n}(\varepsilon_{t+1})) \\
&+ (1+\delta\eta+\beta\pi)\ln\prod_{j=1}^{t-1}\tilde{K}(\varepsilon_j,\tilde{n}(\varepsilon_j),\varepsilon_{j+1},\tilde{n}(\varepsilon_{j+1}))\tilde{K}(\varepsilon_0,n_{-1},\varepsilon_1,\tilde{n}(\varepsilon_1)).
\end{aligned} \tag{A.42}$$

From (20), the indirect utility of the period-0 older adults is

$$V_0^o = \ln\left(\frac{R}{\pi}n_{-1}k_0 + b_0\right) = \ln\left(\frac{R}{\pi} + \varepsilon_0\frac{\frac{\pi\omega}{n_{-1}(1-\omega)}B + C}{\frac{\pi\omega}{n_{-1}(1-\omega)}D + E}\frac{R}{\pi}\right)n_{-1}k_0. \tag{A.43}$$

The social welfare function is the sum of the lifecycle utility of all current and future generations,

$$SW = \gamma^{-1}V_0^o + \sum_{t=0}^{\infty}\gamma^t V_t^M, \tag{A.44}$$

where $\gamma \in (0,1)$ is the planner's discount factor. Reverse discounting, γ^{-1}, must be applied to V_0^o (i.e., the utility of older adults in period 0) to preserve dynamic consistency.

To reformulate the second term on the right-hand side of (A.44), we substitute (A.41) and (A.42) into it and obtain:

$$\begin{aligned}
\sum_{t=0}^{\infty}\gamma^t V_t^M &= V_0^M + \sum_{t=1}^{\infty}\gamma^t V_t^M \\
&= \tilde{V}(\varepsilon_1) + (1+\beta\pi)\ln(1-\tilde{\tau}(n_{-1},\varepsilon_1)) + \delta\eta\ln\tilde{X}(\varepsilon_0,n_{-1},\tilde{n}(\varepsilon_1)) \\
&+ \sum_{t=1}^{\infty}\gamma^t\Bigg[\tilde{V}(\varepsilon_{t+1}) + (1+\beta\pi)\ln(1-\tilde{\tau}(n(\varepsilon_t),\varepsilon_t)) + \delta\eta\ln\tilde{X}(\varepsilon_t,\tilde{n}(\varepsilon_t),\tilde{n}(\varepsilon_{t+1})) \\
&+ (1+\delta\eta+\beta\pi)\ln\prod_{j=1}^{t-1}\tilde{K}(\varepsilon_j,\tilde{n}(\varepsilon_j),\varepsilon_{j+1},\tilde{n}(\varepsilon_{j+1}))\tilde{K}(\varepsilon_0,n_{-1},\varepsilon_1,\tilde{n}(\varepsilon_1))\Bigg],
\end{aligned}$$

that is,

$$\begin{aligned}
\sum_{t=0}^{\infty}\gamma^t V_t^M &= (1+\beta\pi)\ln(1-\tilde{\tau}(n_{-1},\varepsilon_0)) + \delta\eta\ln\tilde{X}(\varepsilon_0,n_{-1},\tilde{n}(\varepsilon_1)) \\
&+ \sum_{t=0}^{\infty}\gamma^t\tilde{V}(\varepsilon_{t+1}) + \sum_{t=1}^{\infty}\gamma^t\left[(1+\beta\pi)\ln(1-\tilde{\tau}(\tilde{n}(\varepsilon_t),\varepsilon_t)) + \delta\eta\ln\tilde{X}(\varepsilon_t,\tilde{n}(\varepsilon_t),\tilde{n}(\varepsilon_{t+1}))\right] \\
&+ (1+\delta\eta+\beta\pi)\sum_{t=1}^{\infty}\gamma^t\ln\prod_{j=1}^{t-1}\tilde{K}(\varepsilon_j,\tilde{n}(\varepsilon_j),\varepsilon_{j+1},\tilde{n}(\varepsilon_{j+1})) + \tilde{K}(\varepsilon_0,n_{-1},\varepsilon_1,\tilde{n}(\varepsilon_1)).
\end{aligned} \tag{A.45}$$

We assume $\varepsilon_t = \varepsilon$ for all t. Under this assumption, we have

$$\sum_{t=0}^{\infty}\gamma^t\tilde{V}(\varepsilon) = \frac{1}{1-\gamma}\tilde{V}(\varepsilon),$$

and we also have

$$\sum_{t=1}^{\infty}\gamma^{t}\left[(1+\beta\pi)\ln(1-\tilde{\tau}\left(\tilde{n}\left(\varepsilon\right),\varepsilon\right))+\delta\eta\ln\tilde{X}\left(\varepsilon,\tilde{n}(\varepsilon),\tilde{n}(\varepsilon)\right)\right]$$
$$=\gamma\left(1+\gamma+\gamma^{2}+\cdots\right)\left[(1+\beta\pi)\ln(1-\tilde{\tau}\left(\tilde{n}\left(\varepsilon\right),\varepsilon\right))+\delta\eta\ln\tilde{X}\left(\varepsilon,\tilde{n}(\varepsilon),\tilde{n}(\varepsilon)\right)\right]$$
$$=\frac{\gamma}{1-\gamma}\left[(1+\beta\pi)\ln(1-\tilde{\tau}\left(\tilde{n}\left(\varepsilon\right),\varepsilon\right))+\delta\eta\ln\tilde{X}\left(\varepsilon,\tilde{n}(\varepsilon),\tilde{n}(\varepsilon)\right)\right],$$

In addition,

$$\begin{aligned}(1+\delta\eta+\beta\pi)\sum_{t=1}^{\infty}\gamma^{t}\ln\prod_{j=1}^{t-1}\tilde{K}\left(\varepsilon,\tilde{n}\left(\varepsilon\right),\varepsilon,\tilde{n}\left(\varepsilon\right)\right)&=(1+\delta\eta+\beta\pi)\sum_{t=1}^{\infty}\gamma^{t}\ln\tilde{K}\left(\varepsilon,\tilde{n}\left(\varepsilon\right),\varepsilon,\tilde{n}\left(\varepsilon\right)\right)^{t-1}\\&=(1+\delta\eta+\beta\pi)\sum_{t=1}^{\infty}\gamma^{t}\left(t-1\right)\ln\tilde{K}\left(\varepsilon,\tilde{n}\left(\varepsilon\right),\varepsilon,\tilde{n}\left(\varepsilon\right)\right)\\&=(1+\delta\eta+\beta\pi)\left(\gamma\cdot0+\gamma^{2}\cdot1+\gamma^{3}\cdot2+\cdots\right)\\&\times\ln\tilde{K}\left(\varepsilon,\tilde{n}\left(\varepsilon\right),\varepsilon,\tilde{n}\left(\varepsilon\right)\right)\\&=(1+\delta\eta+\beta\pi)\left(\frac{\gamma}{1-\gamma}\right)^{2}\ln\tilde{K}\left(\varepsilon,\tilde{n}\left(\varepsilon\right),\varepsilon,\tilde{n}\left(\varepsilon\right)\right),\end{aligned}$$

$$(1+\delta\eta+\beta\pi)\sum_{t=1}^{\infty}\gamma^{t}\ln\tilde{K}\left(\varepsilon,n_{-1},\varepsilon,\tilde{n}\left(\varepsilon\right)\right)=(1+\delta\eta+\beta\pi)\frac{\gamma}{1-\gamma}\ln\tilde{K}\left(\varepsilon,n_{-1},\varepsilon,\tilde{n}\left(\varepsilon\right)\right).$$

With the use of the results established thus far, we can reformulate (A.45) as follows

$$\begin{aligned}\sum_{t=0}^{\infty}\gamma^{t}V_{t}^{M}&=(1+\beta\pi)\ln(1-\tilde{\tau}\left(n_{-1},\varepsilon\right))+\delta\eta\ln\tilde{X}\left(\varepsilon,n_{-1,}\tilde{n}\left(\varepsilon\right),\right)\\&+\frac{1}{1-\gamma}\tilde{V}\left(\varepsilon\right)+\frac{\gamma}{1-\gamma}\left[(1+\beta\pi)\ln(1-\tilde{\tau}\left(\tilde{n}\left(\varepsilon\right),\varepsilon\right))+\delta\eta\ln\tilde{X}\left(\varepsilon,n(\varepsilon),n(\varepsilon)\right)\right]\\&+(1+\delta\eta+\beta\pi)\left(\frac{\gamma}{1-\gamma}\right)^{2}\ln\tilde{K}\left(\varepsilon,\tilde{n}\left(\varepsilon\right),\varepsilon,\tilde{n}\left(\varepsilon\right)\right)+(1+\delta\eta+\beta\pi)\frac{\gamma}{1-\gamma}\ln\tilde{K}\left(\varepsilon,n_{-1},\varepsilon,\tilde{n}\left(\varepsilon\right)\right),\end{aligned}$$

or

$$\begin{aligned}\sum_{t=0}^{\infty}\gamma^{t}V_{t}^{M}&=(1+\beta\pi)\ln(1-\tilde{\tau}\left(n_{-1},\varepsilon_{1}\right))+\delta\eta\ln\tilde{X}\left(\varepsilon,n_{-1,}\tilde{n}\left(\varepsilon\right)\right)\\&+(1+\delta\eta+\beta\pi)\frac{\gamma}{1-\gamma}\ln\tilde{K}\left(\varepsilon,n_{-1},\varepsilon,\tilde{n}\left(\varepsilon\right)\right)\\&+\frac{\gamma}{1-\gamma}\left[\frac{1}{\gamma}\tilde{V}\left(\varepsilon\right)+(1+\beta\pi)\ln(1-\tilde{\tau}\left(\tilde{n}\left(\varepsilon\right),\varepsilon\right))\right.\\&\left.+\delta\eta\ln\tilde{X}\left(\varepsilon,\tilde{n}(\varepsilon),\tilde{n}(\varepsilon)\right)+(1+\delta\eta+\beta\pi)\frac{\gamma}{1-\gamma}\ln\tilde{K}\left(\varepsilon,\tilde{n}\left(\varepsilon\right),\varepsilon,\tilde{n}\left(\varepsilon\right)\right)\right].\end{aligned}\tag{A.46}$$

We substitute (A.43) and (A.46) into (A.44) and obtain

$$
\begin{aligned}
SW \simeq\; & \frac{1}{\gamma} \ln \left(\frac{R}{\pi} + \varepsilon \frac{\frac{\pi\omega}{n_{-1}(1-\omega)} B + C}{\frac{\pi\omega}{n_{-1}(1-\omega)} D + E} \frac{R}{\pi} \right) + (1+\beta\pi) \ln(1 - \tilde{\tau}\left(n_{-1}, \varepsilon\right)) \\
& + \delta\eta \ln \tilde{X}\left(\varepsilon, n_{-1}, \tilde{n}\left(\varepsilon\right)\right) + (1 + \delta\eta + \beta\pi) \frac{\gamma}{1-\gamma} \ln \tilde{K}\left(\varepsilon, n_{-1}, \varepsilon, \tilde{n}\left(\varepsilon\right)\right) \\
& + \frac{\gamma}{1-\gamma} \left[\frac{1}{\gamma} \tilde{V}\left(\varepsilon\right) + (1+\beta\pi) \ln(1 - \tilde{\tau}\left(\tilde{n}\left(\varepsilon\right), \varepsilon\right)) \right. \\
& \left. + \delta\eta \ln \tilde{X}\left(\varepsilon, \tilde{n}(\varepsilon), \tilde{n}(\varepsilon)\right) + (1 + \delta\eta + \beta\pi) \frac{\gamma}{1-\gamma} \ln \tilde{K}\left(\varepsilon, \tilde{n}\left(\varepsilon\right), \varepsilon, \tilde{n}\left(\varepsilon\right)\right) \right]. \qquad \text{(A.47)}
\end{aligned}
$$

■

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