Collective Learning and Distributive Uncertainty

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Abstract

I study a committee that is considering a costly project whose distributive consequences are unknown. The committee is divided into two factions. Support of both factions is required for the project to be approved. By delaying approval, the committee can gradually learn which faction benefits from the project. I show that a project that gives a lower payoff to everyone is more likely to be approved than a more socially efficient project. Furthermore, the equilibrium amount of learning is excessive, and a deadline on adopting the project is socially optimal in a wide range of settings.

Keywords: voting, learning, reform adoption, collective experimentation, distributive uncertainty

JEL codes: D72, D83.

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1 Introduction

Reforms and other collective decisions often involve distributive consequences that are initially unknown but gradually revealed. Consider, for example, a regional legislature that can choose whether to offer a subsidy to a foreign firm. If the subsidy is offered, the firm will open a production facility in the region. The legislature does not know whether the firm will place the facility in city A or in city B. Both cities would have to share the cost of the subsidy, but only the city where the facility is placed will reap the benefits. City B has better transport connections, so it is a more likely candidate. However, the national government is considering whether to build a railway line to city A – if it announces its intention to do so, the firm will prefer to build the facility there. By delaying the vote on the subsidy, the legislature can update its belief about the government's likely decision, and hence about the identity of the city that would benefit from the subsidy.

Collective decisions on such projects, in which the distribution of benefits is initially unknown but gradually revealed, are common in politics and economics. They include countries choosing to join or leave free trade zones without knowing the direction of the resulting change in relative prices; firms deciding to jointly lobby for increased patent protection prior to knowing which of them wins a patent race; and legislatures choosing to increase subsidies for alternative energy sources without knowing which technology will prove most promising (and hence which firms will benefit from the subsidy). In this paper, I explore the implications of such distributive uncertainty on voting outcomes, and their implications for optimal decision rules.

In more detail, the paper models a committee that needs to decide whether to implement a costly project. The committee consists of two factions, called A and B. The project can be of two types. One type of the project brings a positive payoff to faction A, while the other – to faction B. The type is initially unknown, and there is a common belief about it. Time is continuous. At any point in time, the committee can either approve the project, which ends the game, or continue waiting. As long as the committee waits, at any point a public signal can arrive and reveal the project's type. Arrival of a signal revealing each type corresponds to a jump in a Poisson process. The type that favours faction A is more likely to be revealed sooner – hence, as long as no signal has arrived, the common probability that the project benefits faction A is decreasing.

Approval of the project requires support of both factions.[1] Because of this, once the type of the project is revealed, the faction that does not gain from the project will vote against it forever, blocking its approval. More generally, the project can only be approved if for each faction the probability that the project benefits it is sufficiently high. Hence, the collective decision is driven by the common belief about the project's type; and the project is approved when this belief reaches a certain intermediate value.

That value is determined by the asymmetry in the factions' behaviour. Because the belief is only becoming worse for faction A, it does not gain from learning. Instead it simply votes for the project until its expected payoff from approving it becomes negative. Faction B, on the other hand, gains from delaying approval, because this makes it less likely that a project from which it does not gain is approved. However, it knows that after a certain point, faction A will cease to support the project. Anticipating this, faction B switches its decision and votes for the project.

This logic leads to two main results. First, for a significant range of parameter values, the project is more likely to be approved whenever its cost is higher. Hence, a project that is ex ante and ex post *worse for everyone* has a higher probability of being implemented than a more efficient project. The reason is that, when the cost of the project is higher, faction A stops supporting it earlier. This forces faction B to also switch its decision earlier, or else the project would never be approved. Because both factions agree on

[1] If one faction has sufficient weight to be able to force approval on its own, the problem becomes equivalent to a standard model of individual experimentation as in Keller et al. (2005).

the project at an earlier time, there is a lower probability that the project's type is revealed (preventing approval) before that.

Second, I show that, unless the size of faction A is very small, the equilibrium amount of information acquisition is excessive – in other words, the committee delays the decision for too long. Because of this, it is socially optimal to impose a deadline, forcing the committee to adopt the project before the deadline or never at all. The reason is that adopting a project earlier makes it less likely that nature reveals the type and prevents an ex ante socially efficient project from being approved. Because a deadline results in the project being adopted at a time when it is more likely to benefit faction A, it also redistributes surplus from faction B to faction A – hence, the efficiency gain from it is higher when the weight of faction A is greater.

The rest of this section discusses the related literature. Section 2 introduces the basic model, in which only one type can be revealed, and players do not discount their future payoffs. It then derives the equilibria, and establishes the main results. Section 3 analyses a more general setting with discounting and a possibility of either type being revealed. Section 4 concludes. All proofs are in the appendix.

Related literature. The idea that the payoff consequences of collective decisions are ex ante uncertain dates back to at least Fernandez and Rodrik (1991). More recently, researchers have looked at settings in which decision-making bodies can also vote to learn these consequences. One strand of literature applies the idea of individual experimentation to a voting framework. In Strulovici (2010), each voter is unsure about her preferences over a proposal. Voters are ex ante identical, and have identically distributed types that indicate whether they stand to gain or to lose if the proposal is adopted. As the decision is being delayed, each voter can learn about her type. A key result is that the committee stops learning too early compared to the social optimum – a conclusion that is the opposite of the result of this paper. In a related framework, Messner and Polborn (2012) study the effect of different voting rules, showing that a supermajority rule is optimal. Hudja (2019) and Freer et al. (2020) study a similar setting in laboratory experiments.

Another approach models collective learning as acquiring information about a common state. In these models, voters choose between two alternatives, one of which is preferred by all voters in one state, while another is preferred by all voters in the other state. A classic example is a jury deciding whether to convict or acquit the defendant, with all jurors agreeing that the defendant should be convicted if he is guilty, and acquitted if he is innocent. In Chan et al. (2018), a committee is deciding on whether to approve one of two alternatives or continue gathering information about a state of the world. All members want to select the alternative that matches the state of the world, although preference intensities differ. The reason why the committee, at some point, chooses to stop acquiring information is that information acquisition is costly. Anesi and Bowen (2018) analyse policy experimentation in which a committee, at each stage, is voting on a tax rate, on a redistribution scheme, as well as choosing between a risky reform and a safe alternative; selecting the reform enables learning whether it is good or bad. As in Chan et al. (2018), all members agree that a good reform is better than the status quo, and a bad reform is worse; the reason why information acquisition stops is also due to cost: acquiring information requires selecting the reform, which is costly if the reform is bad. Anesi and Safronov (2021) analyse the impact of rules that allow costly deliberation to stop. In their model, at each stage, the committee can vote to stop information acquisition; doing so enables it to make a vote on approving or rejecting the reform. The paper shows how such deliberation rules can bring Paerto-inefficient outcomes.[2]

The key difference between these two lines of research and this paper is that in my model, voters are learning about the distributional consequences of

[2]A number of papers also study policy experimentation by multiple alternating principals, but without voting. See, for example, Callander and Hummel (2014).

a proposal. Thus, unlike models of individual experimentation, in my paper a signal reveals the payoffs of all voters from the project. Hence, there is a common belief about the project's type, which drives the collective decision.[3] At the same time, unlike models of jury decisions, payoffs need not have the same sign for all voters – instead, voters of different factions have opposing preferences.[4] This information and payoff structure underlies the results of the paper. Specifically, it implies that when information is fully revealed (and, more generally, when the common belief is close to zero or one) the project is always rejected even if it is socially optimal to adopt it.[5] Hence, the project can only be approved at an intermediate range of beliefs, which implies that a costlier project is more likely to be approved. The fact that in each state the two factions receive opposing payoffs means that, unlike earlier models in which costless information never hurts, in this model full revelation of information is not socially optimal, as it blocks welfare-improving projects from being approved. This implies the optimality of a deadline.

Overall, models of individual experimentation are relevant for settings in which ex ante similar voters are unsure about their individual payoffs from adopting the proposal, but not about the nature of the proposal itself. Models of jury decisions apply to situations in which voters are uncertain about whether the proposal is good or bad for everyone. My paper, on the other hand, applies to settings in which voters have known factional identities and are learning about the distribution of the payoffs from the project across factions.

[3] In the language of models of experimentation, in my model the type of each voter is perfectly positively correlated with the types of members of the same faction, and perfectly negatively correlated with the types of members of the other faction. Note that while a faction acts as if it was a single player, the size of each faction matters for the welfare analysis.

[4] For a model of voting under similar adversarial preferences in a setting without collective learning, see Kim and Fey (2007).

[5] In experimentation models, in contrast, if all information is revealed the decision depends on whether the majority of voters gains or loses from the project. In jury models, if information is fully revealed all voters are in favour of the socially optimal alternative.

A related paper that similarly studies information acquisition by a divided committee with conflicting preferences is Ginzburg and Guerra (2019). In it, a committee decides whether to learn the state before choosing to accept or reject a proposal. Learning the state of the world is a one-shot decision, and both states are equally likely to be revealed if the committee votes in favour of learning. Here, on the other hand, the state is revealed gradually, and one state is revealed faster than the other – hence, the "intensive margin" of learning becomes an object of analysis.

The results of the paper have a parallel in the literature on legislative bargaining (see Eraslan et al., 2020 for an overview). In particular, in Austen-Smith et al. (2019), inefficient policies are more likely to be chosen because they would be easier to repeal. A similar effect emerges in my paper: more costly proposals are more likely to be adopted. However, this happens not because of concerns over ability to repeal (once adopted, the proposal is irreversible), but because a more costly project forces the committee to come to an agreement earlier.

2 Basic Model and Results

2.1 Model setup

A committee consisting of factions A and B is considering whether to implement a project. There is continuous time t. At every point in time, each faction decides between voting for and against the project. Neither faction has sufficient weight to force the committee to approve the project. Hence, the project is implemented once both factions vote in favour of it – this means, in particular, that any faction can block the project by voting against it forever. When the project is approved, the game ends and payoffs (discussed below) are realised.

Implementing the project will impose a cost $c \in (0, 1)$ on each of the factions. At the same time, if implemented, the project will generate a benefit whose value is normalised to 1. The project has an unknown type $\theta \in \{a, b\}$. The type of the project corresponds to the faction that receives the benefit. Thus, if a project of type a is approved at time t, faction A receives a payoff of $1 - c > 0$, while faction B receives a payoff of $-c < 0$. Similarly, if a project of type b is approved, faction A receives a payoff of $-c$, while faction B receives a payoff of $1 - c$. Until the project is approved, each faction receives a payoff of zero.

If the project has type $\theta = a$, at any time a public signal can arrive and reveal its type. The arrival of a signal corresponds to a jump time of a Poisson process with intensity λ. As long as the signal has not arrived, players are updating their beliefs about the type. Let p_0 be the common prior probability that $\theta = a$, and let p_t be the probability that $\theta = a$ conditional on no signal arriving before time t. In the subsequent text, I will refer to p_t as "the belief". All aspects of the game except for the type are common knowledge.

I will focus on Markov strategies with p_t as a Markov state. For each faction $i \in \{A, B\}$, a Markov strategy implies a set $S_i \in [0, 1]$ of beliefs, such that faction i votes to approve the project if and only if $p_t \in S_i$. Thus, S_i fully describes faction i's strategy. Note that, once a signal arrives, p_t remains constant at zero or at one forever. Hence, after a signal arrives, the project will never be approved, as one of the factions strictly prefers to vote against it.

As usual in voting games, the game has many trivial equilibria, for example, one in which each faction votes against the project at all beliefs. Hence, I will restrict attention to equilibria with the following characteristics: at the equilibrium a faction votes to approve the project at belief p_t if and only if its expected payoff when the project is approved at p_t is larger than its expected payoff from delaying the project by any (possibly infinite) amount of time before returning to playing according to the equilibrium. This eliminates weakly dominated strategies in way similar to the standard refinement applied in voting games, and assumes that a player votes as if she was pivotal.[6]

Formally, consider strategies S_A, S_B, and take any belief $p_t \in (0,1)$. Let $u_i(p_t)$ be faction i's expected payoff if the project is approved at belief p_t.[7] Given the strategy S_{-i} of the other faction, for each $T > 0$, let $V_i(p_t, T, S_{-i})$ be faction i's expected payoff at belief p_t from voting against the project for T units of time before switching to voting in favour of it if no signal has arrived during that time. I will assume that strategies S_A, S_B constitute an equilibrium whenever for each $i \in \{A, B\}$ and all $p_t \in [0,1]$, we have $p_t \in S_i$ if and only if $u_i(p_t) \geq V_i(p_t, T, S_{-i})$ for all finite or infinite $T > 0$.

2.2 Equilibrium

Consider a belief p_t. The payoffs of factions A and B from approving the project immediately equal

$$u_A(p_t) = p_t(1-c) - (1-p_t)c = p_t - c,$$

and

$$u_B(p_t) = -p_t c + (1-p_t)(1-c) = 1 - c - p_t.$$

Note that, at a given belief, any faction can ensure a payoff of zero by voting against the project forever, as doing so will mean that the project is never adopted. Consequently, faction $i \in \{A, B\}$ will not vote for the project when its instantaneous payoff $u_i(p_t)$ is negative. Therefore, faction A will vote against the project at all $p_t < c$, while faction B will vote against the project at all $p_t > 1 - c$. As a consequence, if $c > \frac{1}{2}$, then at any belief at least one of the factions will vote against the project, and hence the project will never be approved. If $c \leq \frac{1}{2}$, the project can only be approved when $p_t \in [c, 1-c]$.

[6] See also Strulovici (2010) for a similar refinement in a continuous-time voting game.

[7] In the language of the literature on strategic experimentation (Keller et al., 2005), $u_i(p_t)$ is the myopic payoff of faction i from adopting the project at belief p_t.

As the approval is being delayed, players are updating their beliefs about the type of the project. If no signal arrives by time t, the belief equals

$$p_t = \frac{p_0 e^{-\lambda t}}{p_0 e^{-\lambda t} + (1 - p_0)} = \frac{1}{1 + \frac{1-p_0}{p_0} e^{\lambda t}}. \tag{1}$$

As p_t is decreasing with time, faction A becomes increasingly more pessimistic about its payoff from the project. Hence, it receives no benefit from learning, and has no incentive to delay voting for the project in order to learn its type. Instead, it votes for the project whenever its myopic payoff $u_A(p_t)$ from adopting the project is greater than zero – that is, as long as $p_t \geq c$. The following proves this formally:

Lemma 1. *At any equilibrium, faction A votes for the project if and only if $p_t \in [c, 1]$.*

This means, in particular, that if the initial belief p_0 is below c, the project is never approved.

For faction B, delaying the decision makes it more likely that a signal arrives and reveals the project's type to be a. Hence, delaying adoption of the project benefits faction B, as it makes it less likely that a project of type a is approved. However, once p_t falls to c, any further delay means that the project is never approved, as faction A would switch to opposing it. Hence, at $p_t = c$, faction B has to choose between approving the project immediately, or never adopting it. It chooses the former if c is low enough that its payoff from approving the project at $p_t = c$ is positive. Otherwise, the project is never approved. The following result captures this intuition and describes the time at which the project is approved at the equilibrium:

Lemma 2. *If $c \leq \min\left\{p_0, \frac{1}{2}\right\}$, the project is approved at time $t^* = \frac{1}{\lambda} \ln\left(\frac{p_0}{1-p_0}\frac{1-c}{c}\right)$ when the belief equals c, unless a signal arrives before that. If $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project is never approved.*

Since approval of the project requires support of both factions, neither can receive a strictly negative payoff at the equilibrium. At the same time, Lemma 2 implies that approval of the project, if it happens at all, takes place when faction A is indifferent between approving and rejecting the project. Hence, its expected payoff is zero in equilibrium. This will happen even when p_0 is high, i.e. when the project is ex ante likely to favour it. Faction B, however, can receive a positive expected payoff. Hence, the faction whose preferred type tends to be revealed with time is worse off than the faction whose preferred type is not revealed.

2.3 Project quality and approval chance

We can now turn to the first main result of the paper, which relates the project's efficiency to its chance of being approved. Recall that a project, if approved, imposes a cost c on each faction. Thus, given the belief, a project with a lower cost Pareto-dominates a project with a higher cost. Does the outcome of the vote reflect this efficiency ranking?

If $c \leq \min\left\{p_0, \frac{1}{2}\right\}$, Lemma 2 implies that a project of type b is approved with certainty. A project of type a is approved once the belief reaches c, as long as no signal arrives by then. Hence, approval is more likely if it takes less time for the belief to reach c, which happens if c and the initial belief p_0 are closer to each other. This implies the following result:

Proposition 1. *If $c \leq \min\left\{p_0, \frac{1}{2}\right\}$, the ex ante probability that the project is approved is $\frac{1-p_0}{1-c}$, which is decreasing in p_0 and increasing in c. If $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project is never approved.*

Thus, the probability of approval is strictly increasing in c for any project, except those that have zero chance of acceptance. In other words, a project that is ex ante and ex post less Pareto-efficient has a *higher* chance of being approved. The reason for this is that faction A is more reluctant to support a project with a higher cost. Hence it switches from supporting to opposing

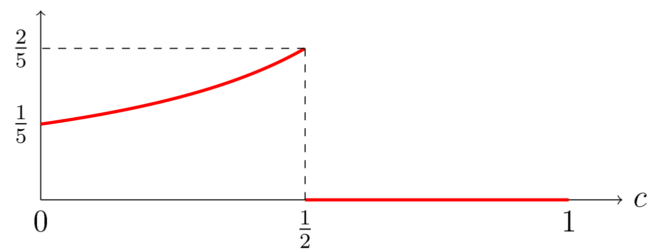

Figure 1: Probability that the project is approved as a function of c, for $p_0 = 0.8$.

the project earlier. As a result, faction B will wait less before switching to support the project. Because a shorter delay makes it less likely that the type is revealed, it implies that the project has a higher ex ante chance of being approved. Figure 1 shows the probability of approval as a function of the project's cost c.

Furthermore, the probability that the project is approved is also decreasing in p_0. Thus, although the voting rule is symmetric in the sense of requiring consent of both factions for approval, a project has a higher chance of being approved if it is ex ante more likely to favour faction B.

2.4 Optimal amount of learning

How can the decision-making procedure be modified to improve efficiency? In many decision-making bodies there is a minimum waiting time between the submission of a proposal and a final decision on it. For example, parliaments often require several readings to approve a law, with a minimum time between them. Such arrangements impose a minimum amount of learning before a project can be adopted. Conversely, some decision-making procedures impose a deadline after which the proposal cannot be adopted – this has the effect of limiting the maximum length of learning. Under the utilitarian welfare criterion, when are such rules optimal?

Suppose the share of faction A's members is α, and the share of faction

B's members is $1-\alpha$. Thus, α is the weight of faction A's utility in the utilitarian welfare function. If $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project is never approved. A deadline or a minimum waiting time has no effect in this case. Otherwise, the effect of planner's interventions depends on α, as the following result shows:

Proposition 2. *Suppose $c \leq \min\left\{p_0, \frac{1}{2}\right\}$. If $c \geq \alpha$, not imposing a decision rule is socially optimal. If $c < \alpha$, it is socially optimal to impose a deadline of*

$$\bar{T} = \frac{1}{\lambda} \ln\left(\frac{p_0}{1-p_0}\frac{1-p^*}{p^*}\right), \text{ where } p^* = \min\{p_0, 1-c\}.$$

Hence, a minimum waiting time is never socially optimal, while a deadline is optimal if the share of faction A is sufficiently large.

Intuitively, a minimum waiting time that is smaller than t^* as defined in Lemma 2 has no effect, since the committee will anyway wait until t^*. A minimum waiting time greater than t^* means that by that time either the type is revealed, or the belief p_t falls below c. In both of these cases, the project will not be approved, and each faction will receive a payoff of zero. At the equilibrium, however, the ex ante expected payoff is positive for faction B, and zero for faction A – hence, imposing such a minimum waiting time is suboptimal.

A deadline, on the other hand, can force the committee to agree on approving the project at an earlier time than t^*. This has two effects. First, it reduces the probability that the type is revealed before the committee agrees on approving the project. This makes approval more likely, increasing welfare. Second, by forcing approval when the belief is higher, a deadline additionally benefits faction A at the expense of faction B. Because of this, whenever the ex ante welfare gain from the project is sufficiently high, and the share of faction A is large – that is, when c is small and α is large – the earliest deadline at which the project can still be adopted is socially optimal. That earliest deadline is the time at which the belief equals $\min\{p_0, 1-c\}$.

Proposition 2 implies that when $c < \alpha$, the equilibrium amount of learning is excessive. A benevolent social planner should then reduce the amount of information that the committee acquires by imposing a deadline. In particular, when the cost of the project is sufficiently low relative to other parameters of the model, it is socially optimal to set the tightest possible deadline – that is, to force the committee to choose between approving the project immediately or never at all. The following result shows this formally:

Corollary 1. *If $c < \min\left\{\alpha, 1 - p_0, p_0, \frac{1}{2}\right\}$, it is socially optimal to set a deadline $\bar{T} = 0$.*

Hence, for low-cost projects, not acquiring any information is socially optimal.

3 General case

This section generalises the preceding analysis to the case when future payoffs are discounted and both types can be revealed by nature.

Suppose that the committee discounts payoffs at an exponential rate $r > 0$. Suppose further that types a and b are revealed at rates λ_a and λ_b, respectively. Assume without loss of generality that $\lambda_a \geq \lambda_b$. If $\lambda_a = \lambda_b$, the belief does not evolve with time until a type is revealed. Hence, at a Markov equilibrium the project is either adopted immediately, or never adopted. The former happens when both factions are in favour of the project ex ante, that is, when $p_0 \in [c, 1 - c]$; otherwise, the latter happens.

From now on I will focus on the more interesting case when $\lambda_a > \lambda_b$. The belief that the project is of type a conditional on no signal arriving before time t equals

$$p_t = \frac{p_0 e^{-\lambda_a t}}{p_0 e^{-\lambda_a t} + (1 - p_0)\, e^{-\lambda_b t}} = \frac{1}{1 + \frac{1-p_0}{p_0} e^{(\lambda_a - \lambda_b)t}}, \tag{2}$$

which, as before, is decreasing with time. Hence, faction A, as before, does not gain from learning, and votes for the project if and only if its myopic payoff from the project is positive. Therefore, the statement of Lemma 1 continues to hold, as the following result shows:

Lemma 3. *At any equilibrium, faction A votes for the project if and only if $p_t \in [c, 1]$.*

However, faction B now faces a tradeoff. On the one hand, a delay reduces the probability that the project of type a is adopted. On the other hand, it also reduces the payoff from the project due to discounting. Furthermore, type b can now be revealed with time – hence, waiting now also reduces the probability that a project of type b is approved. This tradeoff determines the point at which faction B starts voting in favour of the project – that is, the time at which the project is adopted if no signal arrives by then.

When $p_t < c$, the project will never be adopted, since faction A will vote against it. Thus, as before, the project cannot be approved if $c > p_0$. Furthermore, again as before, if $c > \frac{1}{2}$, there is no belief at which both factions agree to approve the project, so the project also cannot be approved.

Otherwise, at all $p_t \geq c$, faction B can choose to approve the project at any time before p_t reaches c. It then faces a standard optimal stopping problem, with the restriction that if the project is not approved when the belief reaches c, it will never be approved, and faction B will receive a payoff of zero.

If without the restriction the solution to this stopping problem involves stopping at the belief that is greater than c, then it also optimal for faction B to switch to approving the project at that belief. On the other hand, if the unrestricted solution involves stopping at a belief below c, then at $p_t = c$ faction B knows that continuing to oppose the project means that its payoff is zero. Hence, at $p_t = c$ it will support the project if its instantaneous payoff from it is positive – that is, if $c \leq \frac{1}{2}$.

This reasoning implies that the equilibrium looks as follows:

Lemma 4. *Let* $\bar{c} := \frac{\sqrt{\lambda_b + r}}{\sqrt{\lambda_a + r} + \sqrt{\lambda_b + r}} < \frac{1}{2}$, *and let*

$$p^* := \begin{cases} \frac{1}{1 + \frac{\lambda_a + r}{\lambda_b + r}\frac{c}{1-c}} \geq c & \text{if } c \in (0, \bar{c}]\,, \\ c & \text{if } c \in \left(\bar{c}, \frac{1}{2}\right]\,. \end{cases}$$

If $c \leq \min\left\{p_0, \frac{1}{2}\right\}$ *and* $p_0 > p^*$, *then at the equilibrium the project is approved at belief* p^* *and at time* $t^* = \frac{1}{\lambda_a - \lambda_b} \ln\left(\frac{p_0}{1-p_0}\frac{1-p^*}{p^*}\right)$, *unless a signal arrives before that. If* $c \leq \min\left\{p_0, \frac{1}{2}\right\}$ *and* $p_0 \leq p^*$, *then at the equilibrium the project is approved immediately with probability one. If* $c > \min\left\{p_0, \frac{1}{2}\right\}$, *then at the equilibrium the project is never approved.*

To see the intuition, consider the case when $c \leq \min\left\{p_0, \frac{1}{2}\right\}$ and the initial belief p_0 is greater than p^*. When $c \leq \bar{c}$, the solution of the aforementioned unrestricted stopping problem (which in this case equals p^*) is weakly greater than c. In this case, faction B switches to supporting the project when the belief reaches that solution. On the other hand, if $c > \bar{c}$, the unrestricted solution is smaller than c. Then faction B delays approval until the belief reaches c, and at that point votes for the project.

At the same time, if $c \leq \min\left\{p_0, \frac{1}{2}\right\}$ and p_0 is already below the optimal solution, the project is adopted at the start. Finally, when $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project, as discussed above, is never approved.

Note that when $\lambda_b = r = 0$ as in the baseline model, we have $\bar{c} = 0$ and $p^* = c$. Hence the equilibrium becomes identical to the one described in Proposition 1.

When $c \leq \bar{c}$, by Lemma 4 the belief p^* at which the project is approved is decreasing in c. Thus, an increase in the cost of the project means the committee waits longer, making approval less likely. On the other hand, if $c > \bar{c}$, the project is approved at belief $p^* = c$, as in the baseline model. Then an increase in c makes the committee wait less, increasing the chance of approval. Formally, we have the following result:

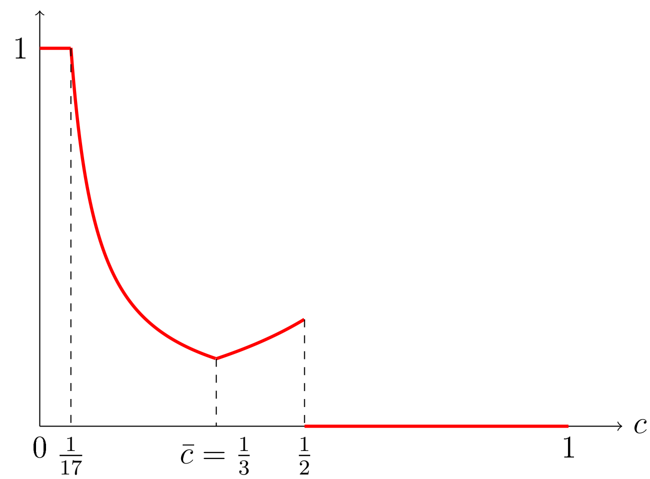

Figure 2: Probability that the project is approved as a function of c, for $r = 1$, $\lambda_a = 15$, $\lambda_b = 3$, and $p_0 = 0.8$.

Proposition 3. *If $c > \min\left\{p_0, \frac{1}{2}\right\}$, or if $p_0 \leq p^*$, then the probability that the project is approved does not depend on p_0 or c. Otherwise, the probability that the project is approved is decreasing in p_0; furthermore, it is decreasing in c when $c < \bar{c}$, and increasing in c if $c > \bar{c}$.*

As before, the project is more likely to be approved if it is more likely to favour faction B. Furthermore, for a certain range of costs, a less socially efficient project is more likely to be approved. This result generalises the result of Proposition 1 – in particular, when λ_b and r approach zero, then $\bar{c}$ approaches zero as well, and the probability of approval is increasing in the project's cost c for almost all values of c.

Figure 2 shows the probability of approval as a function of c. As one can see from the figure, if the cost of the project is very low, then $p^* \geq p_0$, and the project is approved with probability one. As the cost increases, the probability of approval falls. Once the cost exceeds $\bar{c}$, the probability of approval starts to increase with the cost. Finally, if the cost is exceeds $\frac{1}{2}$, the project is never approved.

Consider now the optimal decision rules. As before, a minimum waiting time increases the amount of learning, while a deadline reduces it. Furthermore, as before, a deadline makes approval more likely and redistributes surplus from faction B to faction A. Recall that α denotes the weight of faction A in the social welfare function. Then the following result describes the optimal decision rules:

Proposition 4. *Suppose* $c \leq \min\left\{p_0, \frac{1}{2}\right\}$. *If* $p_0 \leq p^*$, *or if* $c > \max\left\{\alpha, \frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}\right\}$, *not imposing a decision rule is weakly optimal. In all other cases, a deadline is optimal.*

Hence, when $\alpha > c$ the equilibrium amount of information acquisition is inefficiently large, and a deadline is required to correct this. This result mirrors the result of Proposition 2. In addition, the equilibrium amount of learning is also too high if $\frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}} > c$. This can hold when λ_b is large (so learning is slow), or when r is large (so delaying reduces the present value of payoffs by a large amount.

Note that when $\lambda_b = r = 0$ as in the baseline model, then $p^* = c$. Hence, the condition $p_0 \geq p^*$ automatically holds when $c \leq \min\left\{p_0, \frac{1}{2}\right\}$, while the second condition is equivalent to $c > \alpha$ – hence, the result becomes identical to the one in Proposition 2.

4 Conclusions

Collective decisions that involve uncertainty are common. This paper has looked at settings in which the uncertainty concerns distributive consequences of a decision, and the amount of learning can be endogenously chosen. For these situations, the paper has shown two results. First, in a large range of settings, the equilibrium amount of learning is excessive. Second, the need for consensus between factions with opposing interests ensures that less efficient projects are more likely to be approved.

Appendix

Proof of Lemma 1. If $p_t < c$, then for any S_B, $u_A(p_t) < V_A(p_t, \infty, S_B) = 0$, where $V_A(p_t, \infty, S_B)$ is faction A's payoff from voting against the project forever. Hence, any $p_t < c$ does not belong to S_A.

Consider now any $p_t \geq c$, and fix S_B. Suppose faction A votes against the project for T units of time before voting in favour of it. Doing this implies a (possibly infinite) time $\tau \geq T$ such that the project is approved at time τ if no signal arrives by then. The probability that no signal arrives by the time τ equals $e^{-\lambda\tau}$ for a project of type a, and equals 1 for type b. Hence, for any $T > 0$ and any S_B, we have

$$\begin{aligned} V_A(p_t, T, S_B) =& p_t(1-c)e^{-\lambda\tau} - (1-p_t)c \\ <& p_t(1-c) - (1-p_t)c \\ =& u_A(p_t). \end{aligned}$$

Hence, if $p_t \geq c$, then $u_A(p_t) > V_A(p_t, T, S_B)$ for all $T > 0$, and thus $p_t \in S_A$. $\square$

Proof of Lemma 2. Consider first the case when $c > \frac{1}{2}$. Then at any belief $p_t \geq c$, the payoff of faction B if the project is approved immediately is

$$u_B(p_t) = (1-p_t)(1-c) - p_t c = 1 - p_t - c < 0,$$

and hence B votes against the project at p_t. Hence, there is no p_t at which both A and B vote for the project, so the project is never approved. Similarly, if $c > p_0$, then the project is never approved, because faction A will never vote in favour of it except when the type is revealed to be a, in which case faction B will vote against the project.

Now consider the case when $c \leq \frac{1}{2}$ and $c \leq p_0$. Take any $p_t > c$. By continuity of (1), there exit sufficiently small $\varepsilon > 0$ such that $p_{t+\varepsilon} \geq c$. Suppose faction B votes against the project for ε units of time before switching to voting in favour of the project. Then the project is adopted ε units of time later unless its type is a and a signal reveals it during this period. The payoff of faction B from then equals

$$V_B\left(p_t, \varepsilon, S_A\right) = \left(1 - p_t\right)\left(1 - c\right) - p_t c e^{-\lambda\varepsilon} > \left(1 - p_t\right)\left(1 - c\right) - p_t c = u_B\left(p_t\right).$$

Thus, faction B votes against the project at p_t.

Now take $p_t = c$. Then $u_B\left(c\right) = \left(1 - c\right)^2 - c^2 \geq 0$. Given the strategy of faction A, if faction B votes against the project for any $T > 0$ units of time, the project will not be adopted. Hence, $u_B\left(c\right) \geq V_B\left(c, T, S_A\right) = 0, \forall T > 0$. Therefore, faction B votes in favour of the project.

Hence, the project is adopted when the belief p_t reaches c. Substituting $p_t = c$ into (1) and solving for t yields the expression for the time t^*. $\square$

Proof of Proposition 1. By Lemma 2, if $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project is never approved. If $c \leq \min\left\{p_0, \frac{1}{2}\right\}$, the project is approved at time t^* if no signal arrives. This happens with probability $e^{-\lambda t^*}$. Substituting $t = t^*$ into (1) and using $p_t = c$ yields $e^{-\lambda t^*} = \frac{1-p_0}{p_0}\frac{c}{1-c}$. Then the ex ante probability that the project is approved equals

$$p_0 e^{-\lambda t^*} + \left(1 - p_0\right) = \left(1 - p_0\right)\left(\frac{c}{1 - c} + 1\right),$$

which is equivalent to the expression in the proposition. $\square$

Proof of Proposition 2. Let $c \leq \min\left\{p_0, \frac{1}{2}\right\}$. By Lemma 2, the project is approved at time t^* and at belief $p_t = c$, if no signal arrives by then. Setting a minimum waiting time $T \leq t^*$ has no effect. Setting a minimum waiting time $T > t^*$ means that the project is not approved. Recall that at the equilibrium the project is only approved when each faction's payoff from approval is weakly positive. Hence, setting a minimum waiting time $T > t^*$ weakly reduces welfare. We can conclude that a minimum waiting time is always weakly welfare-reducing.

Consider now a deadline $\bar{T}$. If $\bar{T} \geq t^*$, the deadline has no effect. Suppose now that $\bar{T} < t^*$. If $\bar{T}$ is such that the project is never approved, this intervention is not optimal by the same logic as above. Suppose instead that $\bar{T}$ is such that the project is approved at $\bar{T}$ unless a signal arrives by then. In this case, if $\theta = b$, the project is approved with certainty; and if $\theta = a$, the project is approved with probability $e^{-\lambda \bar{T}}$. Hence, the expected utility of faction A equals

$$U_A = p_0 e^{-\lambda \bar{T}} (1-c) - (1-p_0) c,$$

while the expected utility of faction B equals

$$U_B = -p_0 e^{-\lambda \bar{T}} c + (1-p_0)(1-c).$$

Then the utilitarian welfare of the project equals

$$\begin{aligned} W =& \alpha U_A + (1-\alpha) U_B \\ =& p_0 (\alpha - c) e^{-\lambda \bar{T}} + (1-p_0)(1-\alpha-c). \end{aligned}$$

If $\alpha < c$, welfare is increasing in $\bar{T}$, so it is optimal to let $\bar{T} \to t^*$, that is, not to impose a deadline. If $\alpha = c$, welfare is constant in $\bar{T}$, so it is also weakly socially optimal not to impose a deadline.

If $\alpha > c$, welfare is decreasing in $\bar{T}$, so it is optimal to impose a deadline $\bar{T}$ equal to the earliest time at which both factions can vote for the project. If $p_0 \leq 1-c$, this corresponds to $\bar{T} = 0$. If $p_0 > 1-c$, this corresponds to $\bar{T}$ such that $p_{\bar{T}} = 1-c$; using (1), that time equals $\frac{1}{\lambda} \ln \left(\frac{p_0}{1-p_0} \frac{c}{1-c} \right)$. Combining these two observations yields the result. □

Proof of Corollary 1. If $c < \min \left\{ \alpha, 1-p_0, p_0, \frac{1}{2} \right\}$, then $c < \min \left\{ p_0, \frac{1}{2} \right\}$ and $\alpha > c$, hence by Proposition 2 it is optimal to impose a deadline of $\bar{T}$.

Furthermore, the condition on c implies that $p_0 < 1 - c$, and hence $p^* = p_0$, so $\bar{T} = 0$. $\square$

Proof of Lemma 3. If $p_t < c$, then $u_A(p_t) = e^{-rt}(p_t - c) < 0$. Thus, for any S_B, $u_A(p_t) < V_i(p_t, \infty, S_B) = 0$. Hence, any $p_t < c$ does not belong to S_A.

Consider now any $p_t \geq c$, and fix S_B. Suppose faction A votes against the project for T units of time before voting in favour of it. Doing this implies a (possibly infinite) time $\tau \geq T$ such that the project is approved at time τ no signal arrives by then; otherwise, it is never approved. The probability that no signal arrives by the time τ equals $e^{-\lambda_a \tau}$ for a project of type a, and $e^{-\lambda_b \tau}$ for type b. Hence, for any $T > 0$ and any S_B, we have

$$\begin{aligned}
V_A(p_t, T, S_B) &= \left[p_t(1-c)\,e^{-\lambda_a \tau} - (1-p_t)\,ce^{-\lambda_b \tau}\right] e^{-r[t+\tau]} \\
&< \left[p_t(1-c)\,e^{-\lambda_b \tau} - (1-p_t)\,ce^{-\lambda_b \tau}\right] e^{-r[t+\tau]} \\
&= \left[p_t(1-c) - (1-p_t)\,c\right] e^{-(\lambda_b + r)\tau - rt} \\
&= u_A(p_t)\,e^{-(\lambda_b + r)\tau - rt} \\
&\leq u_A(p_t),
\end{aligned}$$

where the last inequality follows from the fact that $u_A(p_t) = p_t(1-c) - (1-p_t)\,c \geq 0$ for all $p_t \geq c$. Hence, if $p_t \geq c$, then $u_A(p_t) > V_A(p_t, T, S_B)$ for all $T > 0$, and therefore $p_t \in S_A$. $\square$

Proof of Lemma 4. As discussed above, if $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project is never approved. For the rest of the proof, consider the case when $c \leq \min\left\{p_0, \frac{1}{2}\right\}$. Let t be the time at which faction B switches to voting for the project. Let $W_B(t)$ be the expected payoff of faction B as a function of t. Then faction B solves

$$\max_t W_B(t) \text{ subject to } p_t \geq c. \tag{3}$$

Note that $W_B(t) = -p_0 c e^{-(\lambda_a+r)t} + (1-p_0)(1-c)e^{-(\lambda_b+r)t}$, and

$$\frac{\partial W_B(t)}{\partial t} = (\lambda_a + r)\, p_0 c e^{-(\lambda_a+r)t} - (\lambda_b + r)(1-p_0)(1-c)\, e^{-(\lambda_b+r)t}.$$

This derivative is positive if and only if

$$\begin{aligned} & (\lambda_b + r)(1-p_0)(1-c)\, e^{-(\lambda_b+r)t} < (\lambda_a + r)\, p_0 c e^{-(\lambda_a+r)t} \\ \iff & e^{(\lambda_a-\lambda_b)t} < \frac{\lambda_a + r}{\lambda_b + r}\frac{p_0}{1-p_0}\frac{c}{1-c}. \end{aligned} \tag{4}$$

Suppose that $p_0 \le p^*$; note that this can only hold together with $c \le \min\left\{p_0, \frac{1}{2}\right\}$ when $p^* \ge c$, that is, when $p^* = \frac{1}{1+\frac{\lambda_a+r}{\lambda_b+r}\frac{c}{1-c}}$. Then we have

$$\begin{aligned} & p_0 \le \frac{1}{1+\frac{\lambda_a+r}{\lambda_b+r}\frac{c}{1-c}} \\ \iff & \frac{p_0}{1-p_0} \le \frac{\lambda_b + r}{\lambda_a + r}\frac{1-c}{c} \\ \iff & \frac{\lambda_a + r}{\lambda_b + r}\frac{p_0}{1-p_0}\frac{c}{1-c} \le 1, \end{aligned}$$

and hence (4) cannot hold, because $\lambda_a > \lambda_b$ implies that $e^{(\lambda_a-\lambda_b)t} > 1$. Thus, $W_B(t)$ is decreasing in t, and hence the optimal t equals zero, so the project is adopted immediately.

Suppose instead that $p_0 > p^*$. Then (4) holds if and only if

$$t < \frac{1}{\lambda_a - \lambda_b}\ln\left(\frac{\lambda_a + r}{\lambda_b + r}\frac{p_0}{1-p_0}\frac{c}{1-c}\right) := \hat{t}.$$

Thus, $\hat{t}$ is the solution to the unrestricted stopping problem. At time $\hat{t}$, the belief equals $p_{\hat{t}} = \frac{1}{1+\frac{\lambda_a+r}{\lambda_b+r}\frac{c}{1-c}}$.

If $c \in \left(0, \frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}\right]$, then $\frac{\lambda_a+r}{\lambda_b+r} \le \left(\frac{1-c}{c}\right)^2$, so $p_{\hat{t}} \ge c$, and the constraint in (3) is satisfied. Then faction B switches to voting for the project at $t^* = \hat{t}$, when the belief equals $p^* = p_{\hat{t}}$. Note also that $c \in \left(0, \frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}\right]$ implies that $\frac{\lambda_a+r}{\lambda_b+r} \leq \left(\frac{1-c}{c}\right)^2$, and hence

$$p^* = p_{\hat{t}} = \frac{1}{1+\frac{\lambda_a+r}{\lambda_b+r}\frac{c}{1-c}} \geq \frac{1}{1+\frac{1-c}{c}} = c.$$

If $c \in \left(\frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}, \frac{1}{2}\right]$, then $\frac{\lambda_a+r}{\lambda_b+r} > \left(\frac{1-c}{c}\right)^2$, so $p_{\hat{t}} < c$. Then $\frac{\partial W_B(t)}{\partial t} > 0$ for all t such that $p_t \geq c$, and we have a corner solution at which faction B switches to voting for the project at a belief $p^* = c$. That belief is reached at time $t^* = \frac{1}{\lambda_a-\lambda_b}\ln\left(\frac{p_0}{1-p_0}\frac{1-p^*}{p^*}\right)$. □

Proof of Proposition 3. If $p_0 \leq p^*$, or if $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project is either approved immediately with probability one, or is never approved. In either case, the probability that the project is approved does not depend on p_0 and on c.

For all other cases, note that by Lemma 4, the project is approved at time $t^* = \frac{1}{\lambda_a-\lambda_b}\ln\left(\frac{p_0}{1-p_0}\frac{1-p^*}{p^*}\right)$ if no signal arrives before that. The ex ante probability of approval then equals

$$\begin{aligned}
&p_0 e^{-\lambda_a t^*} + (1-p_0)\, e^{-\lambda_b t^*} \\
=&p_0\left(\frac{p_0}{1-p_0}\frac{1-p^*}{p^*}\right)^{-\frac{\lambda_a}{\lambda_a-\lambda_b}} + (1-p_0)\left(\frac{p_0}{1-p_0}\frac{1-p^*}{p^*}\right)^{-\frac{\lambda_b}{\lambda_a-\lambda_b}} \\
=&p_0^{-\frac{\lambda_b}{\lambda_a-\lambda_b}}(1-p_0)^{\frac{\lambda_a}{\lambda_a-\lambda_b}}\left[\left(\frac{1-p^*}{p^*}\right)^{-\frac{\lambda_a}{\lambda_a-\lambda_b}} + \left(\frac{1-p^*}{p^*}\right)^{-\frac{\lambda_b}{\lambda_a-\lambda_b}}\right].
\end{aligned}$$

The above expression is decreasing in p_0. Furthermore, it is increasing in p^*, and by Lemma 4, p^* is decreasing in c if $c \leq \frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}$, and increasing in c otherwise. This implies the result. □

Proof of Proposition 4. When $c > \min\left\{p_0, \frac{1}{2}\right\}$, the project can never be approved, so decision rules have no effect.

The rest of the proof will focus on the case when $c \leq \min\left\{p_0, \frac{1}{2}\right\}$. If $p_0 > p^*$, then by Lemma (4), without a decision rule the project is approved at time t^* if no signal arrives by then. If $p_0 \leq p^*$, without a decision rule the project is approved at time zero.

When the project is approved, both factions receive a weakly positive payoff, and at least one faction receives a strictly positive payoff. Since approval of the project happens with a strictly positive probability, the project is ex ante welfare improving. Hence, any deadline or a minimum waiting time that ensures that the project will not be approved reduces social welfare. Thus, we can without loss of generality focus on decision rules at which the project is approved with positive probability.

Such a decision rule implies a time T such that the project is approved at T if no signal arrives by then. In particular, a project of type $\theta = a$ is approved with probability $e^{-\lambda_a T}$, while a project of type $\theta = b$ is approved with probability $e^{-\lambda_b T}$. Let $W(T)$ denote the utilitarian social welfare for a decision rule with a given T. It is given by

$$\begin{aligned} W(T) =& p_0 \left[\alpha(1-c) - (1-\alpha)c\right] e^{-(\lambda_a + r)T} + (1-p_0)\left[(1-\alpha)(1-c) - \alpha c\right] e^{-(\lambda_b + r)T} \\ =& p_0 (\alpha - c) e^{-(\lambda_a + r)T} + (1-p_0)(1-\alpha-c) e^{-(\lambda_b + r)T}. \end{aligned}$$

For the project to be approved at time T, the belief at that time must satisfy $p_T \in [c, 1-c]$. At the same time, we must have $p_T \leq p_0$. Hence, the set of feasible values of T is given by $p_T \in [c, \min\{1-c, p_0\}]$. Using (2), this is equivalent to

$$e^{(\lambda_a - \lambda_b)T} \in \left[\max\left\{\frac{p_0}{1-p_0}\frac{c}{1-c}, 1\right\}, \frac{p_0}{1-p_0}\frac{1-c}{c}\right].$$

Hence, the optimal T is given by

$$\arg\max_T W(T) \text{ subject to } e^{(\lambda_a - \lambda_b)T} \in \left[\max\left\{\frac{p_0}{1-p_0}\frac{c}{1-c}, 1\right\}, \frac{p_0}{1-p_0}\frac{1-c}{c}\right].$$

Differentiating, we obtain

$$\frac{\partial W(T)}{\partial T} = (\lambda_a + r) p_0 (c - \alpha) e^{-(\lambda_a + r)T} - (\lambda_b + r)(1 - p_0)(1 - \alpha - c) e^{-(\lambda_b + r)T}.$$

We have four cases.

Case 1: $\alpha \in [c, 1-c]$**.** Then $\frac{\partial W(T)}{\partial T} < 0$ for all T. Hence, the smallest possible T is optimal. Therefore, a deadline is optimal if $p_0 > p^*$, and no intervention is needed if $p_0 \leq p^*$.

Case 2: $\alpha > 1 - c \geq c$**.** Then $\frac{\partial W(T)}{\partial T} > 0$ if and only if

$$e^{(\lambda_a - \lambda_b)T} > \frac{(\lambda_a + r) p_0 (\alpha - c)}{(\lambda_b + r)(1 - p_0)(\alpha - [1 - c])}.$$

Hence, the optimal T is such that either $e^{(\lambda_a - \lambda_b)T} = \max\left\{\frac{p_0}{1-p_0}\frac{c}{1-c}, 1\right\}$, or $e^{(\lambda_a - \lambda_b)T} = \frac{p_0}{1-p_0}\frac{1-c}{c}$. We can prove that welfare is higher in the former case. If $\max\left\{\frac{p_0}{1-p_0}\frac{c}{1-c}, 1\right\} = 1$, welfare is higher in the former case if and only if

$$\begin{aligned} & p_0(\alpha - c) + (1 - p_0)(1 - \alpha - c) \\ \geq & p_0(\alpha - c)\left(\frac{p_0}{1-p_0}\frac{1-c}{c}\right)^{-\frac{\lambda_a + r}{\lambda_a - \lambda_b}} + (1 - p_0)(1 - \alpha - c)\left(\frac{p_0}{1-p_0}\frac{1-c}{c}\right)^{-\frac{\lambda_b + r}{\lambda_a - \lambda_b}}, \end{aligned}$$

which always holds, because $p_0 \geq c$ implies that $\frac{p_0}{1-p_0}\frac{1-c}{c} \geq 1$.

On the other hand, if $\max\left\{\frac{p_0}{1-p_0}\frac{c}{1-c}, 1\right\} = \frac{p_0}{1-p_0}\frac{c}{1-c}$, welfare is higher when

$e^{(\lambda_a-\lambda_b)T}=\max\left\{\frac{p_0}{1-p_0}\frac{c}{1-c},1\right\}$ if and only if

$$
\begin{aligned}
&p_0\,(\alpha-c)\left(\frac{p_0}{1-p_0}\frac{c}{1-c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}+(1-p_0)\,(1-\alpha-c)\left(\frac{p_0}{1-p_0}\frac{c}{1-c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\\
\geq &p_0\,(\alpha-c)\left(\frac{p_0}{1-p_0}\frac{1-c}{c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}+(1-p_0)\,(1-\alpha-c)\left(\frac{p_0}{1-p_0}\frac{1-c}{c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\\
\Longleftrightarrow\ &p_0^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\,(1-p_0)^{\frac{\lambda_a+r}{\lambda_a-\lambda_b}}\left[(\alpha-c)\left(\frac{c}{1-c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}+(1-\alpha-c)\left(\frac{c}{1-c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\right]\\
\geq &p_0^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\,(1-p_0)^{\frac{\lambda_a+r}{\lambda_a-\lambda_b}}\left[(\alpha-c)\left(\frac{1-c}{c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}+(1-\alpha-c)\left(\frac{1-c}{c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\right]\\
\Longleftrightarrow\ &(\alpha-c)\left[\left(\frac{c}{1-c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}-\left(\frac{1-c}{c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}\right]\\
\geq &(1-\alpha-c)\left[\left(\frac{1-c}{c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}-\left(\frac{c}{1-c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\right],
\end{aligned}
$$

which holds because $c\leq 1-c$ implies that $\left(\frac{c}{1-c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}-\left(\frac{1-c}{c}\right)^{-\frac{\lambda_a+r}{\lambda_a-\lambda_b}}\geq 0$ and $\left(\frac{1-c}{c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}-\left(\frac{c}{1-c}\right)^{-\frac{\lambda_b+r}{\lambda_a-\lambda_b}}\leq 0$.

Therefore, when $\alpha>1-c\geq c$, welfare is the highest when $e^{(\lambda_a-\lambda_b)T}=\max\left\{\frac{p_0}{1-p_0}\frac{c}{1-c},1\right\}$, that is, at the smallest feasible T. Hence, a deadline is optimal if $p_0>p^*$, and no intervention is needed if $p_0\leq p^*$.

Case 3: $\alpha<c\leq 1-c$ **and** $p_0\leq p^*$**.** When $\alpha<c\leq 1-c$, we have $\frac{\partial W(T)}{\partial T}>0$ if and only if

$$
e^{(\lambda_a-\lambda_b)T}<\frac{\lambda_a+r}{\lambda_b+r}\frac{p_0}{1-p_0}\frac{c-\alpha}{1-c-\alpha}. \tag{5}
$$

As $p_0\leq p^*$, without intervention the project is approved at time 0. A deadline has no effect. A minimum waiting time is optimal if and only if welfare is strictly increasing in T at $T = 0$ – that is, if and only if

$$\begin{aligned} 1 &< \frac{\lambda_a + r}{\lambda_b + r} \frac{p_0}{1 - p_0} \frac{c - \alpha}{1 - c - \alpha} \\ \iff p_0 &> \frac{1}{1 + \frac{\lambda_a + r}{\lambda_b + r} \frac{c - \alpha}{1 - c - \alpha}}. \end{aligned} \tag{6}$$

By Lemma 4, either $p^* = c$, or $p^* = \frac{1}{1 + \frac{\lambda_a + r}{\lambda_b + r} \frac{c}{1 - c}}$. Since $c \leq \min\left\{p_0, \frac{1}{2}\right\}$, the former case cannot hold when $p_0 \leq p^*$. On the other hand, if $p^* = \frac{1}{1 + \frac{\lambda_a + r}{\lambda_b + r} \frac{c}{1 - c}}$, then, using the fact that $c < 1 - c$, we have $\frac{c}{1 - c} > \frac{c - \alpha}{1 - c - \alpha}$ for all $\alpha \in (0, 1)$, and thus $p^* < \frac{1}{1 + \frac{\lambda_a + r}{\lambda_b + r} \frac{c - \alpha}{1 - c - \alpha}}$. Together with the fact that $p_0 \leq p^*$, this implies that (6) never holds, and hence a minimum waiting time is never optimal.

To summarise, no decision rule is optimal in this case.

Case 4: $\alpha < c \leq 1 - c$ and $p_0 > p^*$. In this case a deadline is optimal if $e^{(\lambda_a - \lambda_b)t^*}$ is greater than the right-hand side of (5), and a minimum waiting time is optimal if $e^{(\lambda_a - \lambda_b)t^*}$ is smaller than the right-hand side of (5).

If $c \leq \frac{\sqrt{\lambda_b + r}}{\sqrt{\lambda_a + r} + \sqrt{\lambda_b + r}}$, then Lemma 4 implies that

$$e^{(\lambda_a - \lambda_b)t^*} = \frac{p_0}{1 - p_0} \frac{1 - p^*}{p^*} = \frac{p_0}{1 - p_0} \frac{\lambda_a + r}{\lambda_b + r} \frac{c}{1 - c},$$

and deadline is optimal whenever $\frac{c}{1 - c} > \frac{c - \alpha}{1 - c - \alpha}$. This is true for any $\alpha \in (0, 1)$.

If $c \in \left(\frac{\sqrt{\lambda_b + r}}{\sqrt{\lambda_a + r} + \sqrt{\lambda_b + r}}, \frac{1}{2}\right]$, then Lemma 4 implies that

$$e^{(\lambda_a - \lambda_b)t^*} = \frac{p_0}{1 - p_0} \frac{1 - p^*}{p^*} = \frac{p_0}{1 - p_0} \frac{1 - c}{c},$$

which is greater than the right-hand side of (5) if and only if

$$\frac{1 - c}{c} > \frac{\lambda_a + r}{\lambda_b + r} \frac{c - \alpha}{1 - c - \alpha}. \tag{7}$$

If $\alpha = 0$, (7) is equivalent to $\frac{1-c}{c} > \sqrt{\frac{\lambda_a+r}{\lambda_b+r}}$, which does not hold when $c \in \left(\frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}, \frac{1}{2}\right]$. Furthermore, as the right-hand side of (7) is increasing in α, (7) does not hold for any $\alpha \in [0, c]$. Hence, a deadline is never optimal when $\alpha < c < 1-c$ and $c \in \left(\frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}, \frac{1}{2}\right]$. At the same time, when $c \in \left(\frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}, \frac{1}{2}\right]$, Lemma 4 implies that without intervention, the project is approved at belief $p^* = c$, that is, when $e^{(\lambda_a-\lambda_b)T} = \frac{p_0}{1-p_0}\frac{1-c}{c}$. A minimum waiting time cannot extend the waiting time further without preventing project approval, which, as we have seen, is suboptimal. Therefore, no intervention is optimal in this case.

To summarise, neither a deadline nor a minimum waiting time is optimal if $c > \min\left\{p_0, \frac{1}{2}\right\}$; if $p_0 \leq p^*$; or if $\alpha < c < 1-c$ and $c \in \left(\frac{\sqrt{\lambda_b+r}}{\sqrt{\lambda_a+r}+\sqrt{\lambda_b+r}}, \frac{1}{2}\right]$. Otherwise, a deadline is optimal. $\square$

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