# Democratic Victory and War Duration: Why Are Democracies Less Likely to Win Long Wars?

Nakao, Keisuke

24 April 2022

# Democratic Victory and War Duration: Why Are Democracies Less Likely to Win Long Wars?*

Keisuke Nakao†

April 12, 2022

**Abstract**

Using costly-process models of war with democratic citizens and soldiers, this article explores two contrasting claims on the negative association between the probability of democratic victory and the duration of war. As a claim holds, democracies are not militarily disadvantageous in long wars. Rather, they need long time to produce a surrender decision, because they incur audience costs if they break a prewar commitment too hastily. According to the other, democracies are less likely to win long wars, because their battlefield effectiveness declines over time. Although the underlying logic differs between the two claims, they offer analogous predictions as to military strategies consistent with empirical findings that while democracies could raise their chances of victory with a shortening strategy (e.g., maneuver), autocracies might have mixed incentives for shortening and protracting strategies (e.g., attrition). These results imply that both the mechanisms might be at work in a democracy's prosecution of war. (150 words)

JEL classifications: D74; F51.

Keywords: audience costs, battlefield effectiveness, democratic victory, military strategy, war duration.

*Acknowledgments: [TBW]

†College of Business and Economics. University of Hawaii at Hilo. 200 W. Kawili St., Hilo, HI 96720 USA. E-mail: keisuken@hawaii.edu. Phone: +1(808)932-7228.

# 1 Introduction

The negative association between the probability of democratic victory and the duration of war has been long recognized as a stylized fact (Bennett and Stam 1998). As it holds, democracies are more likely to win than are their autocratic opponents in the short run, but after roughly 18 months have passed, democracies become far more likely than autocracies to quit and more willing to settle for draws or losses. There have been two prominent claims that might explain this fact.

One was originally posed by Tocqueville ([1835, 1840] 2004) and later bolstered by Iklé (1971).[1] According to them, it is difficult for democracies to end a war especially in unfavorable terms, because restoring peace at the price of major concessions almost inevitably evokes a severe cleavage between hawks and doves within a government.[2] In light of such an intense internal struggle, anyone who advocates peace must bear the risk of being labeled as a "traitor." The fear of this taint naturally deters the leadership from taking initiatives toward peace and causes a delay in producing the decision to end a war. In Iklé's words: "fighting often continues long past the point where a rational calculation would indicate the war should be ended" (p. ix). However, as battle deaths accumulate, public support for war declines, and thus policymakers become pressured to make peace rather than continuing the war (Mueller 1973; Gartner, Segura, and Wikening 1997; Gartner and Segura 1998). Our theory delineates this dynamic of political influence by applying the concept of audience costs (Fearon 1994; Schultz 2001; Smith 1998a).

The other claim was offered by Reiter and Stam (1998b, 2002). They measure democracies' battlefield military effectiveness in two categories: one is individual soldiering that includes morale, leadership, and initiative; and the other organizational efficacy consisting of logistics, intelligence, and technology. Among these factors, as they found, the armies of democracies are superior in terms of leadership, initiative, and logistics. Moreover, all the three advantages dwindle as wars lengthen. These findings could be a powerful foundation for the fact shown above, because it

[1] In Tocqueville's words: "There are two things that will always be very difficult for a democratic people to do: to start a war and to finish it" (p. 765). Iklé concurs: "Warfare is such an all-absorbing enterprise [...] that after starting one, a government may lose sight of ending it" (p. viii). The concern with ending a war dates back to the times before modern democracies. Without referring to regime types, Machiavelli ([1526] 1901: 118) argues: "People may go to war when they will, but cannot always withdraw when they like."

[2] Tocqueville foresees armies to be a major opponent of restoring peace (p. 765).

is straightforward from their findings to conclude that democracies are less likely to win long wars, as their performance on the battlefield deteriorates.[3]

The Tocqueville-Iklé claim is sharply contrasting to the Reiter-Stam claim, because according to the former, the war outcomes influence the duration, whereas the latter indicates vice versa. Although both the claims sound sensible, they entail some empirical shortcomings. Namely, the former seems to contradict the historical tendency that wars initiated by democracies are likely to be short (Bennett and Stam 1996; Bueno de Mesquita et al. 1999), whereas the latter might suffer the selection effect—because democracy's typical war plan is to win shortly, only unsuccessful democracies fight long wars (Koch 2009; Lemke and Reed 2001; Reed and Clark 2000). Therefore, instead of further pursuing empirical investigation, we undertake theoretical assessments of whether these claims could withstand the rigor of microfoundation. For this end, we develop a series of game-theoretic models of war as a costly process (Wagner 2000).

The models are presented in three steps. In the first step, a baseline model is built upon the model of war of attrition (Maynard Smith 1974). The baseline model depicts a war fought between a democracy and an autocracy across discrete time periods. It critically differs from the canonical war-of-attrition model in that both the belligerents are uncertain about the relative strength that determines their risks of being defeated.[4] As the model presumes, the democracy is militarily advantageous at the war's onset, because of its strategic selection of winnable targets in initiating war (Bueno de Mesquita and Siverson 1995; Bueno de Mesquita et al. 1999; Clark and Reed 2003; Reiter and Stam 1998a). However, the democracy's failure to bring about a victory in a timely manner indicates that the democracy is weaker than the prior estimate. This change in the beliefs as to the relative strength induces the democracy to terminate the war.

In the second step, we advance the baseline model in two directions: one is to incorporate audience costs (Fearon 1994; Schultz 2001; Smith 1998a); and the other to endogenize battlefield effectiveness (Reiter and Stam 1998b, 2002). The model

[3] Tocqueville ([1835, 1840] 2004: 1170-1175) claims the opposite, contemplating: "What Makes Democratic Armies Weaker than Other Armies When Entering into a Campaign and More Formidable When War Is Prolonged."

[4] Unlike typical models of war with private information (Fearon 2007; Filson and Werner 2002, 2004; Powell 2004; Slantchev 2003b), our models do not assume informational asymmetry—neither side entertains informational advantage despite uncertainty (Smith and Stam 2004).

of audience costs highlights democratic citizens, who decide to support or oppose a war in every period. In doing so, they can exert collective influence on their leader's decisions to continue fighting or not. At the war's onset when citizens are optimistic about the war's prospects, they place a high value on a prewar commitment to the issue at stake. That means, the political cost of compromising original war aims is substantially large for the leader. However, as the democracy is revealed to be less advantageous over time, the citizenry support for the war withers. Accordingly, the political cost of surrendering inflicted by the domestic audience decreases as the war lengthens. If this cost is so large and persistent, the decision to end the war could be delayed.[5] Moreover, because longer wars are likely to be more costly, this delay in turn enhances the democracy's deterrability of an opponent. Therefore, unlike what the audience-costs literature commonly presumes, our model suggests that the deterrence effect of audience costs depends not just on the prewar civilian influence, but also on how tenaciously the audience keeps their support for the war after its outbreak. The durability of audience costs could matter for deterrence.

In the model of battlefield effectiveness, there are democratic soldiers, who decide to undertake or evade their military missions. The success in a mission depends on the relative strength, but conversely the soldiers can also collectively influence the democracy's performance on the battlefield. As the democracy fails to accomplish a short-run victory, less soldiers effectively undertake their missions, and its military performance gradually declines. In contrast to the audience-costs model, according to which democratic citizens can directly influence their leader's decisions, the battlefield-effectiveness model implies that direct influence by soldiers is limited to battle outcomes, but they can indirectly influence whether to continue or end a war. Both the models render some clues as to why democracies are less likely to win long wars. They suggest that domestic politics and military effectiveness can matter for the outcomes and duration of wars fought by democracies.

In the third step, we further extend these models by allowing the belligerents to choose a military strategy before they initiate fighting. Their options are those archetypal in the literature (Mearsheimer 1983; Reiter and Meek 1999; Stam 1996). One is to shorten the war by making battles more decisive (e.g., blitzkrieg or ma-

[5] Audience costs do not necessarily make a war longer. The audience may also urge their leader to stop fighting as soon as they perceive it hopeless. In this sense, audience costs function to reflect the public's will in foreign policies by reducing the leader's discretion.

neuver), and the other to protract the war by making battles less decisive (e.g., attrition).[6] Predictions based on our extended models are consistent with empirical findings that democracies tend to adopt the maneuver strategy, whereas the choices of autocracies are rather ambiguous (Reiter and Meek 1999; Reiter and Stam 2002). These results imply that both the mechanisms of audience costs and of battlefield effectiveness might be at work in a democracy's prosecution of war. Moreover, our theory renders a rationalist explanation for the open question as to why they choose so when the maneuver strategy is highly correlated with victory (Reiter and Meek 1999). With the maneuver strategy, a democracy could raise the chances of victory by terminating war before it loses public supports for fighting or before its forces in the battlefield become ineffective. On the other hand, an autocracy might have mixed incentives in that while the maneuver strategy shortens a war and thus reduces the cost of fighting, the attrition strategy could play on democracy's long-run weakness of losing public supports and deteriorating military performance in protracted warfare.

The rest of the article proceeds as follows. The second section reviews the related literature. The third section presents and solves the baseline model. Models in the fourth and fifth sections incorporate democratic citizens and soldiers into the baseline model. The sixth further extends these models to explore the relationship between regime types and military strategies. The seventh section concludes. Technical materials are left in Appendix.

[6] Blitzkrieg is frequently used to describe a quick victory on the battlefield, whereas with attrition, victory follows a series of set-piece battles and is not expected to be quick (Mearsheimer 1983: 34-35).

## 2 Theoretical Literature Review

Our theory pertains to (i) war termination, (ii) regime types, and (iii) military strategies. In the series of models presented in this article, one stream (Model I and its extension) relates regime types to (a) domestic politics, while another stream (Model II and its extension) relates them to (b) battlefield effectiveness. To the best of our knowledge, ours is the first theoretical attempt to incorporate all these aspects of wars in formal models.

As to (i) war termination, extant models have focused mostly on bargaining in dyadic contexts (Filson and Werner 2002; Langlois and Langlois 2009, 2012; Powell 2012; Slantchev 2003a, 2003b; Wolford, Reiter, and Carrubba 2011). According to them, two major causes of lengthy wars are the commitment problem (Fearon 2004, 2007) and private information (Powell 2004; Smith and Stam 2004; Wagner 2000). Our theory suggests that regime types can also matter for the outcomes and duration of war, holding either that democratic citizens affect their leader's decision to continue or end a war or that democratic soldiers influence battle outcomes through their military engagements.

As to (ii-a) domestic politics, a number of theoretical studies explain peace among democracies by institutional constraints (Baliga, Lucca, and Sjöström 2008; Debs and Goemans 2010; Bueno de Mesquita et al. 1999; Jackson and Morelli 2007) or by informational advantage (Fearon 1994; Guisinger and Smith 2002; Schultz 1998, 2001; Slantchev 2006; Smith 1998a; Tarar and Leventoğlu 2009, 2013). Others are concerned with wars caused by diversionary purposes (Smith 1996; Tarar 2006) or the agency problem (Downs and Rocke 1994). As shown above, the theoretical literature extensively investigate the influence of domestic politics on war's outbreak, but formal studies on the domestic influence on war termination remain limited. There are few exceptions (Filson and Werner 2004; Smith 1998b), but they capture the differences among regime types only with parameter values. For further theoretical sophistication, ours explicitly portrays democratic citizens as players in costly-process models of war.

As to (ii-b) battlefield effectiveness, some costly-process models of war portray shifts of the military balance as a result of developments on the battlefield (Langlois and Langlois 2009, 2012; Slantchev 2003b; Smith 1998; Smith and Stam 2003, 2004). We seek microfoundation for the shifts by incorporating individual soldiers who can influence the military balance, as they decide to undertake or evade military missions.

As to (iii) military strategies, theoretical studies are still sparse, although there are some formal work on indirect strategy (Lindsey 2015), fait accompli (Tarar 2016), punishment (Intriligator and Brito 1984; Nakao [forthcoming]), and concealment of strength (Baliga and Sjötröm 2008; Meirowitz and Sartori 2008; Slantchev 2010). Unlike those above, our models explore belligerents' choices between shortening and protracting strategies (i.e., maneuver and attrition) to assess whether our theoretical predictions are consistent with empirical findings (Reiter and Meek 1999; Reiter and Stam 2002). In doing so, we seek the rationales for each regime type to choose a certain military strategy.

# 3 Baseline Model

The baseline model is build upon the canonical war-of-attrition model (Maynard Smith 1974), but ours critically differs in threefold: (i) players can be defeated with some probabilities; (ii) these probabilities are determined by a state variable; (iii) the true state is unknown to both the players. Subsequently, the baseline model is advanced to endogenize the cost of surrendering (in Model I) and the probabilities of defeats (in Model II).

## 3.1 War of Attrition with Exogenous Defeats

Across discrete time periods $t \in \{1, 2, 3, \cdots\}$, a war is prosecuted between two belligerents—a democracy and an autocracy—indexed by $i, j \in \{D, A\}$. The belligerents simultaneously decide to "fight" or "surrender" in every period. If they both choose to "fight," a battle takes place and results in one of $D$'s "winning," "losing," and "indecisive" outcomes. In a battle, $i$'s "win" is identical to its opponent $j$'s "loss."

If the outcome is "indecisive", the war continues to the next period, and another battle is fought unless either $i$ surrenders. The war ends with $i$'s victory either when $i$ "wins" a battle or when its opponent $j$ "surrenders." If $i$ brings about a victory, $i$ seizes a lump-sum benefit $W_i > 0$, while $j$ gains nothing.[7] If both $D$ and $A$ "surrender" simultaneously, they both gain nothing. Throughout the war, each belligerent $i$ incurs a per-period cost of fighting $c_i > 0$.

## 3.2 Two-Sided Uncertainty about the Relative Strength

The probability distribution of battle outcomes is determined by a state variable $\theta$ that is binary, favoring either $D$ or $A$ ($\theta \in \{d, a\}$). Neither $D$ nor $A$ knows whether they are in state $d$ or in $a$, but they share the common prior probabilities $\Pr(d) > 0$ and $\Pr(a) > 0$ such that $\Pr(d) + \Pr(a) = 1$.[8]

[7] To draw a distinction between military defeat and conditional surrender, it also sounds reasonable to subtract $L_i > 0$ from $i$'s payoff (only) if $i$ is militarily defeated. The inclusion of $L_i$ will not change our main results.

[8] In the model, the priors are set to be common merely for simplicity, but they can differ (Smith and Stam 2004).

The probabilities of battle outcomes are defined as:

$$\begin{aligned}
\Pr(win_i|\theta) &\equiv \left(1-\delta^{\theta}\right)p_i^{\theta} \\
\Pr(ind|\theta) &\equiv \delta^{\theta},
\end{aligned}$$

where $\delta^{\theta} \in (0,1)$ is the per-period probability that a battle is "indecisive" in state $\theta$, and $p_i^{\theta} \in (0,1)$ the per-period probability of $i$'s "win" in $\theta$ given a battle is decisive (i.e., not "indecisive"), satisfying that $p_D^{\theta} + p_A^{\theta} = 1$ for each $\theta \in \{d,a\}$. Each $p_i^{\theta}$ represents belligerent $i$'s relative military strength in state $\theta$.[9] It is presumed that $p_D^d > p_D^a$ and $p_A^a > p_A^d$, so that each $i$ is more advantageous in its own state. To further refer to the states, $\theta \in \{d,a\}$ may be added to the superscript of relevant symbols, as with $\delta^{\theta}$ and $p_i^{\theta}$ above.

### 3.3 Erosion of Democracy's Confidence

As the war proceeds, the belligerents use Bayes' rule to learn the true state from developments on the battlefield. Put formally, as both $D$ and $A$ choose to "fight," they update their beliefs as to the two states, shown as:

$$\begin{aligned}
\Pr\left(d|ind^T\right) &\equiv \frac{\Pr(d)\Pr(ind|d)^T}{\Pr(d)\Pr(ind|d)^T + \Pr(a)\Pr(ind|a)^T} \\
&= \frac{1}{1+\frac{\Pr(a)}{\Pr(d)}\left(\frac{\Pr(ind|a)}{\Pr(ind|d)}\right)^T} \qquad (1) \\
\Pr\left(a|ind^T\right) &\equiv 1-\Pr\left(d|ind^T\right),
\end{aligned}$$

where $ind^T$ denotes $T \in \{0,1,2,\cdots\}$ times of "indecisive" battle outcomes. As democracies strategically select winnable targets when they begin wars (Bueno de Mesquita and Siverson 1995; Bueno de Mesquita et al. 1999; Clark and Reed 2003; Reiter and Stam 1998a), this war breaks out with the democracy's advantage, but lengthy battles indicate the match to be more equal than originally estimated.[10] The following assumption guarantees that the tide of war shifts against the democracy

[9] In a later model, $p_i^{\theta}$ is endogenized to explore how it changes over time and how the change affects war duration and outcomes.

[10] It is unnecessary but sounds reasonable to hold powerful pacifism (Lake 1992), which implies $\Pr(d) > \frac{1}{2}$ and $p_D^d > \frac{1}{2}$; In words, it is more likely that the democracy is advantageous to the autocracy than vice versa.

through fighting:

**Assumption 1** *A battle is more likely to remain indecisive in state $a$ than in $d$, or $\delta^d < \delta^a$.*

By Assumption 1, $\Pr(ind|a)/\Pr(ind|d)>1$, so that $\Pr\left(d|ind^T\right)$ decreases with $T$ (Equation (1)).

### 3.4 Payoff Analyses and Equilibrium

Based on the beliefs updated from $T$ periods of past "indecisive" outcomes, $D$ calculates its current-period payoff from fighting if $A$ also fights:

$$u_D\left(ind^T\right) \equiv \sum_{\theta\in\{d,a\}} \Pr\left(\theta|ind^T\right) u_D^\theta,$$

where for $i \in \{D, A\}$, $u_i^\theta$ is $i$'s per-period payoff from fighting in state $\theta$:

$$u_i^\theta \equiv \left(1-\delta^\theta\right) p_i^\theta W_i - c_i.$$

Particularly for the first period when no battles are precedented, $D$'s payoff from fighting is denoted as: $u_D \equiv u_D\left(ind^0\right)$.

As $D$ fights longer, it becomes more pessimistic about the war's prospect, and thus $u_D\left(ind^T\right)$ decreases with $T$.[11] For the state variable to influence $D$'s behavior, the following restriction is put on $D$'s payoff:

**Assumption 2** *$W_D$ is in the range where:*

$$\frac{c_D}{\sum_{\theta\in\{d,a\}} \Pr(\theta)\left(1-\delta^\theta\right) p_D^\theta} < W_D < \frac{c_D}{\left(1-\delta^a\right) p_D^a}.$$

By Assumption 2, $D$ is willing to fight at least once ($u_D > 0$) and unwilling to fight in state $a$ ($u_D^a < 0$). With Assumptions 1 and 2, $D$'s sequentially rational strategy against $A$'s fight forever can be uniquely identified:

[11] Trivially, $u_D^d > u_D^a$, because $p_D^d > p_D^a$ and $\delta^d < \delta^a$ (Assumption 1).

**Lemma 1** *There exists a unique* $T^* \geq 1$ *such that* $u_D\left(ind^{T^*-1}\right) > 0$ *and* $u_D\left(ind^{T^*}\right) \leq 0$.[12]

**Proof.** When $T = 0$, $u_D\left(ind^T\right)$ is positive (Assumption 2). As $T$ rises, $u_D\left(ind^T\right)$ monotonically decreases, because $\Pr\left(d|ind^T\right)$ decreases and $u_D^d > u_D^a$ (Assumption 1). As $T \to \infty$, $\Pr\left(a|ind^T\right)$ converges to one, so that $\lim_{T\to\infty} u_D\left(ind^T\right) = u_D^a$, which is negative (Assumption 2). Therefore, there must exist a unique time period $T^* \geq 1$ around which the sign of $u_D\left(ind^T\right)$ changes from being positive to become zero or negative. ■

Given $T^*$, $D$ chooses to fight for $T^*$ periods (based on up to $T^* - 1$ times of indecisive outcomes) and then surrenders in period $T^* + 1$ (based on $T^*$ times of indecisive outcomes).

Anticipating $D$'s surrender in period $T^* + 1$, $A$ estimates its continuation payoff from fighting, based on $T \in \{0, 1, 2, \cdots, T^* - 1\}$ "indecisive" periods:[13]

$$U_{A|T^*}\left(ind^T\right) \equiv \sum_{\theta \in \{d,a\}} \Pr\left(\theta|ind^T\right) U_{A|T^*}^{\theta}\left(ind^T\right),$$

where $U_{A|T^*}^{\theta}\left(ind^T\right)$ is $A$'s continuation payoff from fighting in $\theta$ after $T$ times of *ind* if $D$ surrenders in $T^* + 1$:

$$\begin{aligned} U_{A|T^*}^{\theta}\left(ind^T\right) &\equiv \left(\sum_{t=T+1}^{T^*} \Pr\left(ind|\theta\right)^{t-T-1} u_A^{\theta} + \Pr\left(ind|\theta\right)^{T^*-T}\right) W_A \\ &= \left(\left(1 - \left(\delta^{\theta}\right)^{T^*-T}\right) p_A^{\theta} + \left(\delta^{\theta}\right)^{T^*-T}\right) W_A - \frac{1 - \left(\delta^{\theta}\right)^{T^*-T}}{1 - \left(\delta^{\theta}\right)} c_A. \end{aligned}$$

In particular, for the first period, $U_{A|T^*} \equiv U_{A|T^*}\left(ind^0\right)$ and $U_{A|T^*}^{\theta} \equiv U_{A|T^*}^{\theta}\left(ind^0\right)$.

As the war evolves (with $T$ rising), two factors influence $A$'s payoff from fighting: the timing of $D$'s surrender approaches ($U_{A|T^*}^{\theta}\left(ind^T\right)$ increases for each $\theta \in \{d, a\}$); and it becomes more likely that $A$ is stronger ($\Pr\left(a|ind^T\right)$ increases). The assumption

[12] To rule out uninteresting mixed-strategy equilibria that may arise when either $i$'s expected payoff from fighting equals its payoff from surrendering, the model adopts the tie-breaking rule that $i$ chooses to surrender if it is indifferent between fighting and surrendering. This rule will hold for the rest of the article.

[13] To make a distinction, small letters ($u$, $v$) are assigned to per-period payoffs, and large letters ($U$, $V$) to continuation payoffs throughout the article.

below guarantees these two factors surely raise $A$'s payoff from fighting over time:[14]

**Assumption 3** *It is in A's interest to be stronger, or* $\left(1-\delta^d\right) p_A^d < (1-\delta^a)\, p_A^a$.

With Assumption 3, $A$ never surrenders once it starts fighting, because $U_{A|T^*}\left(ind^T\right)$ increases with $T$. As the war persists, while $D$'s incentive to fight diminishes, $A$'s incentive strengthens. Given these contrasting incentives, their decision problems can be interpreted that $D$ chooses when to surrender, but $A$ determines whether or not to fight in the first period.[15] The key condition that generates these contrasting incentives and characterizes the equilibrium behavior is the difference in the probabilities of indecisive outcomes between the two states $\Pr(ind|d) < \Pr(ind|a)$ (Assumption 1).

**Proposition 1** *(i) If* $U_{A|T^*} > 0$*, the baseline model holds a unique subgame perfect Nash equilibrium, where D fights for* $T^*$ *periods and surrenders in* $T^*+1$ *and all subsequent periods, while A fights forever.*[16] *(ii) If* $U_{A|T^*} \leq 0$*, A immediately surrenders in equilibrium.*[17]

**Proof.** (i) If $U_{A|T^*} > 0$, $A$ fights in the first period regardless of $D$'s strategy. It continues to fight in all the subsequent periods, or $U_{A|T^*}\left(ind^T\right)$ increases with $T$, because $U^\theta_{A|T^*}\left(ind^T\right)$ increases for each $\theta \in \{d, a\}$, $\Pr\left(a|ind^T\right)$ increases (Assumption 1), and $U^a_{A|T^*}\left(ind^T\right) > U^d_{A|T^*}\left(ind^T\right)$ for any $T$ (Claim 1 in Appendix B). Against $A$'s fight forever, $D$'s sequentially-rational strategy adopts $T^*$ (Lemma 1). Even off the equilibrium path ($t \geq T^*+2$), sequential rationality mandates $D$ to surrender and $A$ to fight. (ii) It is straightforward that $A$ surrenders in the first period if $U_{A|T^*} \leq 0$. ■

This model is further augmented to examine how domestic regime types matter for war by incorporating democratic citizens and soldiers.

---

[14] While the former factor raises $A$'s payoff for sure, the latter has an ambiguous effect without Assumption 3, because the relative size between $u_A^a$ and $u_A^d$ is indeterminate—in state $a$, although the victory is more likely ($p_A^a > p_A^d$), the war tends to last longer ($\delta^a > \delta^d$). Assumption 3 suffices that $U^a_{A|T^*}\left(ind^T\right) > U^d_{A|T^*}\left(ind^T\right)$.

[15] To put it in another way, $D$'s marginal benefit from fighting decreases over time, but $A$'s one increases. A more detailed discussion appears in Appendix A.

[16] Note that unlike games with private information, this game contains only one information set in each period. It has no distinction between separating or pooling equilibria. No separation of the two states is possible. Any equilibrium must pool the states.

[17] If $U_{A|T^*} \leq 0$, multiple equilibria can emerge, because subgame perfection allows several action profiles off the equilibrium path. Specifically, in any periods $T^*+k$ with $k \geq 1$ when their payoffs from fighting each other are both negative, either $D$ or $A$ fights, while the other surrenders.

# 4 Model I: Decreasing Audience Costs

The second model, Model I, aims to capture the Tocqueville-Iklé claim that democracies face political difficulties with producing the decision to end a war. It highlights democratic citizens, who collectively influence their leader's behavior by inflicting domestic political costs on him.

## 4.1 Public Opinion

Into the baseline model, Model I incorporates a number of democratic citizens $C$, whose population size is normalized to be one.[18] Citizens have the same preference as their leader $D$ except that they have diverse evaluations of the war $W_C \in (0, \infty)$, which follows a cumulative distribution function $F_C(\cdot)$ with its density $f_C(\cdot)$. A more hawkish citizen tends to have a larger $W_C$. As in the baseline model, all $D$, $A$, and $C$ are presumed to share the common priors $\Pr(d)$ and $\Pr(a)$.

Citizen $C$'s per-period payoff from fighting for $W'_C$ after $T$ periods of "indecisive" battle outcomes is:

$$u_C\left(W'_C|ind^T\right) \equiv \sum_{\theta \in \{d,a\}} \Pr\left(\theta|ind^T\right)\left(\left(1-\delta^\theta\right) p_D^\theta W'_C - c_D\right).$$

This citizen supports the war if $u_C\left(W'_C|ind^T\right) > 0$ and opposes it if $u_C\left(W'_C|ind^T\right) < 0$. Define $\overline{W}_C(T)$ with which $C$ is indifferent between "fight" and "surrender" given $ind^T$, or $u_C\left(\overline{W}_C(T)|ind^T\right) = 0$, so that

$$\overline{W}_C(T) \equiv \frac{c_D}{\sum_{\theta \in \{d,a\}} \Pr\left(\theta|ind^T\right)\left(1-\delta^\theta\right) p_D^\theta}. \tag{2}$$

Then, citizens with $W_C > \overline{W}_C(T)$ are those supporting the war, and they amount to:

$$\#cit(T) \equiv 1 - F_C\left(\overline{W}_C(T)\right),$$

which will determine the civilian influence on the democratic leader.

For citizens to influence the leader's decision, some restrictions are put on $F_C(\cdot)$ and $f_C(\cdot)$.

[18] To clarify, Model I will take over all Assumptions 1, 2, and 3 set for the baseline model.

**Assumption 4** *$F_C(\cdot)$ and $f_C(\cdot)$ satisfy the following three inequalities:*

$$\text{(i)} \qquad F_C\left(\frac{c_D}{\sum_{\theta\in\{d,a\}}\Pr(\theta)\left(1-\delta^\theta\right)p_D^\theta}\right) < 1/2$$

$$\text{(ii)} \qquad F_C\left(\frac{c_D}{\left(1-\delta^a\right)p_D^a}\right) > 1/2$$

$$\text{(iii)} \qquad f_C\left(W_C\right) > 0 \text{ for } W_C \in \left[\frac{c_D}{\sum_{\theta\in\{d,a\}}\Pr(\theta)\left(1-\delta^\theta\right)p_D^\theta}, \frac{c_D}{\left(1-\delta^a\right)p_D^a}\right).$$

By Assumption 4-(i), a majority of citizens support the war at the onset. By (ii), they oppose it in state $a$. By (iii), citizens have diverse evaluations on the war.

## 4.2 Domestic Influence on War

To delineate the civilian influence, suppose that by maintaining the war, the leader receives a per-period political payoff $ac(\#cit\,(T))$, which varies with the public approval for fighting $\#cit\,(T)$. Then $D$ chooses to fight after $T$ periods of $ind$ if and only if $u_D\left(ind^T\right) + ac(\#cit\,(T)) > 0$, or equivalently

$$u_D\left(ind^T\right) > -ac(\#cit\,(T)),$$

from which $ac\,(\#cit\,(T))$ can also be interpreted as "audience costs" that $D$ incurs when he surrenders despite a fraction $\#cit\,(T)$ of the opposition to surrendering. That is, the democracy continues to fight until the payoff from fighting falls below the cost of breaking the prewar commitment to the defense of national interests $W_D$. On $ac\,(\#cit\,(T))$, two restrictions are reasonably set: (i) $ac\,(\#cit\,(T))$ increases with $\#cit\,(T)$ (i.e., surrendering is more costly if more citizens support the war); and (ii) $ac\,(1/2) = 0$ (i.e., the sign of such costs depends on whether the median citizen supports or opposes the war).

As the democracy appears less and less advantageous over time, the civilian support for the war declines, and audience costs shrink.

**Lemma 2** *$ac(\#cit\,(T))$ decreases with $T$.*

**Proof.** As $T$ rises, $\Pr\left(d|ind^T\right)$ decreases (Assumption 1), $\overline{W}_C\,(T)$ increases (to maintain $u_C\left(\overline{W}_C\,(T)\,|ind^T\right) = 0$), $F_C\left(\overline{W}_C\,(T)\right)$ increases (Assumption 4-(iii)), $\#cit\,(T)$ decreases (by definition of $\#cit\,(\cdot)$), and then $ac\,(\#cit\,(T))$ decreases. ■

The timing of $D$'s surrender decision could be affected by the public opinion. By taking his political payoff of fighting into account, $D$ surrenders right after period $T^I$ when the payoff from fighting is outweighed by audience costs:

**Lemma 3** *There exists a unique* $T^I \geq 1$ *such that* $u_D(ind^{T^I-1}) > -ac\left(\#cit\left(T^I-1\right)\right)$ *and* $u_D(ind^{T^I}) \leq -ac(\#cit\left(T^I\right))$*.*

**Proof.** By Lemma 1, the left-hand side $u_D\left(ind^T\right)$ is originally positive, monotonically decreases with $T$ and converges to $u_D^a$, which is negative. In contrast, the right-hand side $-ac(\#cit\,(T))$ is originally negative (Assumptions 4-(i)), monotonically increases with $T$ (Lemma 2) and will turn to be positive (Assumption 4-(ii)). Thus, there must exist a unique time period $T^I \geq 1$ around which the left-hand side becomes lower than the right-hand side. ■

If audience costs persist even after the war's outbreak, a majority of citizens would disallow their government to easily revoke its original war aims. In response, expecting that the democracy is willing to fight tenaciously, the autocracy's decision to initiate the war could also be altered.

**Proposition 2** *If a majority of citizens still endorse the war in period* $T^*$ $(\#cit\,(T^*) > \frac{1}{2})$ *and their influence is sufficiently large* $(ac(\#cit\,(T^*)) > -u_D\left(ind^{T^*}\right))$*, then the democracy will postpone its surrender decision and fight longer* $(T^I > T^*)$*. In response, the autocracy which would fight forever without audience costs will immediately surrender if* $W_A$ *is in the range where* $U_{A|T^*} > 0$ *and* $U_{A|T^I} \leq 0$*.*[19]

**Proof.** By $\#cit\,(T^*) > \frac{1}{2}$, $ac(\#cit\,(T^*)) > 0$. By $ac\;(\#cit\,(T^*)) > -u_D\left(ind^{T^*}\right)$, $D$ is willing to fight at least in period $T^*+1$, and thus $T^I > T^*$, which suffices $U_{A|T^I} < U_{A|T^*}$. ■

In light of domestic audience, the democracy would keep on fighting even after its leader regards the war no longer worth fighting without domestic constraints. Put differently, democracies are less likely to lose short wars (Bennett and Stam 1998), because of the domestic opposition to a compromise in haste. However, as audience costs decrease over time, the political decision to end a war, which was difficult in

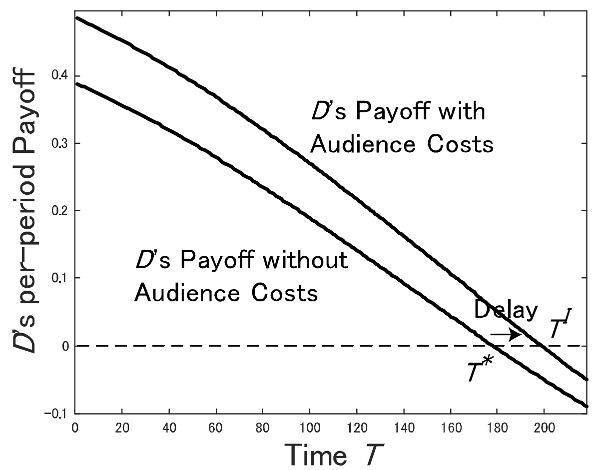


Figure 1: The delay in the democracy's surrendering caused by audience costs

the short run, can be made more easily in the long run. Decreasing audience costs can thus cause a delay to end wars (Figure 1).[20]

Furthermore, this delay might undermine the autocracy's willingness to fight, because longer wars are likely to inflict larger costs. In this sense, the logic of deterrence backed by domestic audience pertains even to war termination as well as onset. In contrast to models of war onset (Fearon 1994; Guisinger and Smith 2002; Slantchev 2006; Tarar and Leventoğlu 2009, 2013), which suggest the size of audience costs to matter for deterrence, our theory holds that the deterrent effect of domestic audience depends not just on their short-run influence, but also on how durably the public supports are expected to persist during wars.[21] In this regard, audience costs can matter even after wars' outbreak.

[19] Conversely, the decision can be made earlier if it is expedited by the majority (Bennett and Stam 1996). Put formally, $T^I < T^*$ if $\#cit\,(T^* - 1) < \frac{1}{2}$ and $ac(\#cit\,(T^* - 1)) < -u_D\left(ind^{T^*-1}\right)$.

[20] Figure 1 shows the delay by contrasting $D$'s per-period payoffs with and without audience costs ($u_D\left(ind^T\right)$ and $u_D\left(ind^T\right) + ac(\#cit\,(T))$), given the following parameters and functions: $\Pr(d) = \frac{8}{10}$; $\delta^d = \frac{98}{101}$; $\delta^d = \frac{98}{100}$; $p_D^d = \frac{8}{10}$; $p_D^a = \frac{4}{10}$; $W_D = 50$; $c_D = 1$; $W_C \sim U\left[\frac{c_D}{\sum_{\theta\in\{d,a\}}(1-\delta^\theta)p_D^\theta}, \frac{c_D}{(1-\delta^a)p_D^a}\right]$; $ac\,(\#cit\,(T)) = \frac{\#cit(T)}{10}$.

[21] In our model, key determinants of such durability are the public evaluation of war ($F_C\,(\cdot)$), their influence ($ac(\cdot)$), and the cost of fighting ($c_D$), but those factors that affect war onset might also play crucial roles in termination such as electoral institutions (Koch and Gartner 2005), opposition parties (Schultz 1998), and the media (Baum and Potter 2015; Slantchev 2006) that are all out of our scope.

# 5 Model II: Declining Battlefield Effectiveness

The next model, Model II, incorporates soldiers instead of citizens in democracy. As their performance on the battlefield changes over time, the soldiers collectively influence war outcomes (Reiter and Stam 1998b, 2002). Model II critically differs from the baseline model in that the probability distribution of defeats is endogenously determined.

## 5.1 Individual Soldiering

In Model II, the democracy has mobilized a number of soldiers $S$, whose manpower is normalized to be one. They share the same information on $\theta \in \{d, a\}$ with $D$ and $A$. In every period, each soldier decides to engage in his mission or evade it. If he engages, he incurs a per-period cost $c_S > 0$ and accomplishes the mission with per-period probability $\rho^\theta \in (0,1)$ in state $\theta$, for which $\rho^a < \rho^d$. In words, the democracy's military missions are more likely to be accomplished in state $d$ than in $a$. If a soldier evades, he earns nothing. Soldiers are heterogeneous in evaluation of mission accomplishment $W_S \in (0, \infty)$, which distributes according to its c.d.f. $F_S(\cdot)$ and p.d.f. $f_S(\cdot)$ with the following restriction:

**Assumption 4'** *$W_S$ distributes so broadly that*

$$f_S(W_S) > 0 \text{ for } W_S \in \left[\frac{c_S}{\sum_{\theta \in \{d,a\}} \Pr(\theta)\rho^\theta}, \frac{c_S}{\rho^a}\right).$$

A soldier's payoff from engaging in a mission with value $W'_S$ after $T$ "indecisive" periods can be shown as:

$$v_S\left(W'_S | ind^T\right) \equiv \sum_{\theta \in \{d,a\}} \Pr\left(\theta | ind^T\right) \rho^\theta W'_S - c_S.$$

A soldier engages in his mission (and is called "effective") if $v_S\left(W'_S | ind^T\right) > 0$ and evades it (being "ineffective") if $v_S\left(W'_S | ind^T\right) < 0$.[22] From the condition that $v_S\left(\overline{W}_S(T) | ind^T\right) = 0$, the threshold $\overline{W}_S(T)$ that demarcates the effective and

[22] In engaging in a mission, soldiers are made "effective" to accept the dangers of the battlefield and place themselves at risk for military objectives.

ineffective soldiers given $ind^T$ can be derived as:

$$\overline{W}_S(T) \equiv \frac{c_S}{\sum_{\theta\in\{d,a\}} \Pr\left(\theta|ind^T\right)\rho^\theta}. \tag{3}$$

Because those with $W_S > \overline{W}_S(T)$ are effective, the fraction of effective soldiers after $T$ periods of $ind$ is:

$$\#sol(T) \equiv 1 - F_S\left(\overline{W}_S(T)\right),$$

which in turn determines $pp_i^\theta(\#sol(T))$, or the per-period probability of $i$'s winning in state $\theta$ after $T$ periods of $ind$ such that $pp_D^\theta(\#sol(T)) + pp_A^\theta(\#sol(T)) = 1$ for $\theta \in \{d,a\}$ and $T \in \{0,1,2,\cdots\}$. If more soldiers effectively engage in their missions, the democracy is more likely to prevail in battles; so that $pp_D^\theta(\#sol(T))$ is assumed to increase with $\#sol(T)$ for each $\theta \in \{d,a\}$. Because $\Pr\left(d|ind^T\right)$ decreases with $T$, the soldiers reduce their confidence over time, and they undertake less and less missions in the battlefield:

**Lemma 4** *$pp_D^\theta(\#sol(T))$ decreases with $T$ for each $\theta \in \{d,a\}$.*

**Proof.** The proof resembles that of Lemma 2. As $T$ rises, $\Pr\left(d|ind^T\right)$ decreases (Assumption 1), $\overline{W}_S(T)$ increases, $F_S\left(\overline{W}_S(T)\right)$ increases (Assumption 4'), $\#sol(T)$ decreases, and finally $pp_D^\theta(\#sol(T))$ decreases. ■

By Lemma 4, the democracy becomes less advantageous, while the autocracy has more chances to prevail. Note that the decline of the democracy's battlefield effectiveness in Lemma 4 is different from what we observe in the raw data set, as it does not contain the selection effect—because strong democracies tend to win quickly and decisively, only weak democracies are left in long wars.[23] This effect generates a downward bias in the democracy's observed battlefield effectiveness ($\sum_{\theta\in\{d,a\}} \Pr\left(\theta|ind^T\right) pp_D^\theta(\#sol(T))$). Therefore, the actual decline ($\sum_{\theta\in\{d,a\}} \Pr(\theta)\, pp_D^\theta(\#sol(T))$) might not be so significant as it appears to be. In other words, the observed decline can be greater than in Lemma 4. Furthermore, this bias even widens as the war lengthens. Figure 2 shows the difference between observed battlefield effectiveness and actual (unbiased) effectiveness.[24]

[23] The effect is caused by the change in the probability distribution of states $\Pr\left(\theta|ind^T\right)$.

[24] For Figure 2, parameters and functions are set as: $\rho^d = \frac{8}{10}$; $\rho^a = \frac{4}{10}$; $c_S = 1$; $W_S \sim U\left[\frac{c_S}{\sum_{\theta\in\{d,a\}}\Pr(\theta)\rho^\theta}, \frac{c_S}{\rho^a}\right]$; $pp_D^d(\#sol(T)) = \frac{9\#sol(T)}{10}$; $pp_D^a(\#sol(T)) = \frac{9\#sol(T)}{20}$.

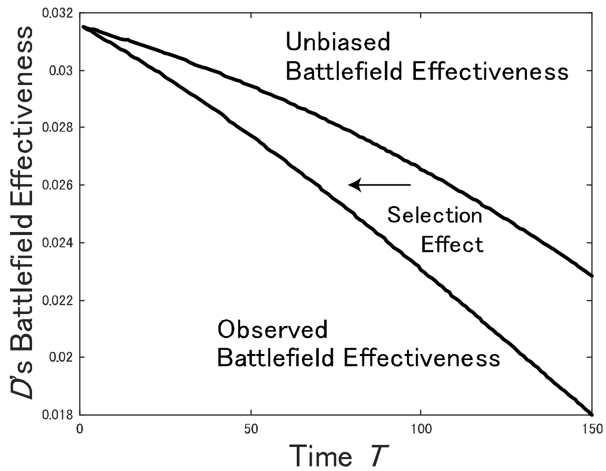


Figure 2: The downward bias in the democracy's battlefield effectiveness caused by the selection effect

## 5.2 Military Influence on Political Decisions

In light of changing battlefield effectiveness, the belligerents' payoffs are slightly modified—$p_i^\theta$ is replaced with $pp_i^\theta\left(\#sol\left(T\right)\right)$ in their payoffs (Appendix C). Let $v_D\left(ind^T\right)$ denote $D$'s per-period payoff from fighting after $T$ "indecisive" periods, and in particular $v_D \equiv v_D\left(ind^0\right)$. The assumption below replaces Assumption 2 of the baseline model, holding that $D$ is willing to fight at least once ($v_D > 0$) and unwilling to fight in the long run ($\lim_{T\to\infty} v_D\left(ind^T\right) < 0$).

**Assumption 2'** *$W_D$ is in the range where:*

$$\frac{c_D}{\sum_{\theta\in\{d,a\}}\left(1-\delta^\theta\right)pp_D^\theta\left(\#sol\left(0\right)\right)} < W_D < \lim_{T\to\infty}\frac{c_D}{\left(1-\delta^a\right)pp_D^a\left(\#sol\left(T\right)\right)}.$$

Assumption 2' guarantees that the timing of $D$'s surrender is uniquely determined:

**Lemma 5** *There exists a unique $T^{II} \geq 1$ such that $v_D(ind^{T^{II}-1}) > 0$ and $v_D(ind^{T^{II}})$ $\leq 0$.*

**Proof.** Because $pp_D^\theta\left(\#sol\left(T\right)\right)$ decreases with $T$ (Lemma 4), $D$'s per-period payoff from fighting in each $\theta$ ($v_D^\theta\left(ind^T\right)$ in Appendix C) also decreases. Therefore, the monotonicity of $v_D\left(ind^T\right)$ holds—$v_D\left(ind^T\right)$ decreases with $T$, because $\Pr\left(d|ind^T\right)$ decreases (Assumption 1). By Assumption 2', $T^{II}$ is greater than or equal to one but is finite. ■

Given $D$'s strategy of $T^{II}$, $A$'s continuation from fighting after $T$ "indecisive" periods can also be specified ($V_{A|T^{II}}\left(ind^{T}\right)$ in Appendix C and $V_{A|T^{II}} \equiv V_{A|T^{II}}\left(ind^{0}\right)$). Corresponding to Assumption 3 of the baseline model, the following restriction suffices that $A$ prefers state $a$ to $d$.

**Assumption 3'** *Regardless of* $T$, $\left(1-\delta^{d}\right) pp_{A}^{d}\left(\#sol\left(T\right)\right) < \left(1-\delta^{a}\right) pp_{A}^{a}\left(\#sol\left(T\right)\right)$.

Unlike the audience-costs argument of Model I, the inclusion of soldiers in Model II affects not only the timing of surrendering ($T^{II}$) but also the probabilities of defeats ($pp_{i}^{\theta}\left(\#sol\left(T\right)\right)$ for any $T \leq T^{II}$). In other words, soldiers influence the war at both the strategic and tactical levels. At the strategic level, when the democracy surrenders depends on how rapidly its battlefield effectiveness declines. At the tactical level, the democracy is more likely to win short wars, because higher military performance is expected in the early stage. In these senses, the negative association between the probability of democratic victory and the duration of war should be addressed in terms of changing battlefield effectiveness (Reiter and Stam 1998b) and also of the shifting incentives to surrender (Bennett and Stam 1998).

**Proposition 3** *If the fraction of effective soldiers* $\#sol\left(T^{*}\right)$ *is so large in period* $T^{*}$ *that* $v_{D}\left(ind^{T^{*}}\right) > 0$*, then* $D$ *will fight longer in Model II than in the baseline model* ($T^{II} > T^{*}$)*. In response, the autocracy which would fight forever without endogenous battlefield effectiveness will immediately surrender if* $W_{A}$ *is in the range where* $U_{A|T^{*}} > 0$ *and* $V_{A|T^{II}} \leq 0$*.*

**Proof.** While $u_{D}\left(ind^{T^{*}}\right) \leq 0$ (Lemma 1), a sufficiently large $\#sol\left(T^{*}\right)$ guarantees that $v_{D}\left(ind^{T^{*}}\right) > 0$. Given $ind^{T^{*}}$, $D$ fights at least one more period, and thus $T^{II} > T^{*}$. Because $A$ prefers state $a$ to $d$ (Assumption 3'), and because $pp_{A}^{\theta}\left(\#sol\left(T\right)\right)$ increases with $T$ in each $\theta$ (Lemma 4), $A$'s per-period payoff $v_{A}\left(ind^{T}\right)$ increases with $T$, and thus $A$ fights forever if $V_{A|T^{II}} > 0$ (Appendix A). Moreover, because $pp_{D}^{\theta}\left(\#sol\left(T\right)\right)$ decreases with $T$ in each $\theta$, $v_{D}\left(ind^{T}\right) > u_{D}\left(ind^{T}\right)$ for $T \leq T^{*}$. Because the sum of $D$ and $A$'s per-period payoffs is constant, this inequality is equivalent to $v_{A}\left(ind^{T}\right) < u_{A}\left(ind^{T}\right)$ for $T \leq T^{*}$, which suffices $V_{A|T^{II}} < U_{A|T^{*}}$ for $T^{II} > T^{*}$. ■

The democracy's short-run advantage and long-run disadvantage in the battlefield can affect when it surrenders ($T^{II}$). Moreover, its endogenous military effectiveness has marked effects on deterrence—whether the autocracy fights or not in the first place. In Model I, a delay in the democracy's surrendering caused by audience costs magnifies the deterrent effect—if $T^I > T^*$, then $U_{A|T^I} < U_{A|T^*}$. Model II also stipulates the corresponding effect if $T^{II} > T^*$. Moreover, because the democracy's declining battlefield effectiveness means that the autocracy is less advantageous in earlier battles, even greater deterrability can be expected in Model II. In other words, a democracy's short-run military advantage can serve to enhance its ability to deter opponents.

# 6 Regime Types and Military Strategies

The final analysis further extends Models I and II to seek implications toward military strategies. The purposes of the extensions are: (i) to test whether the choices of strategies predicted by these models are consistent with empirical findings (Reiter and Meek 1999); and once the consistency is confirmed, (ii) to deliver the rationales for the adoptions of the predicted strategies.

The military strategies addressed below are those archetypal in the literature (Mearsheimer 1983; Reiter and Stam 2002; Stam 1996). One is a shortening strategy that aims to end war sooner (e.g., blitzkrieg or maneuver), and the other a protracting strategy that tends to make war longer (e.g., attrition). The immediate effects of these strategies are captured by the difference in battle decisiveness in the models.

## 6.1 Shortening vs. Protracting Strategies

The extensions below apply to both Models I and II. In the extended models, the belligerents simultaneously choose either shortening or protracting strategy at the onset to maximize their own expected payoffs. The combination of their strategies (henceforth, called the "military strategy profile") determines the probability of the "indecisive" battle outcome in each state $\theta$. To be concrete, $\Pr(ind|\theta)$ equals $\varepsilon^\theta$ if both $D$ and $A$ choose the shortening strategy, $\eta^\theta$ if only either side chooses it, and $\delta^\theta$ if neither side chooses it, where $0 < \varepsilon^\theta < \eta^\theta < \delta^\theta < 1$ for each $\theta$. Regarding the protracting strategy as default, a belligerent can make battles more decisive by switching to the shortening strategy. The military strategy profile corresponding to each probability $\sigma^\theta \in \{\varepsilon^\theta, \eta^\theta, \delta^\theta\}$ is denoted in a reduced form $\sigma \in \{\varepsilon, \eta, \delta\}$.

The assumption below ensures that military strategies have no informational effects:

**Assumption 1'** $\varepsilon^a/\varepsilon^d = \eta^a/\eta^d = \delta^a/\delta^d > 1$.

With Assumption 1', Bayesian learning of the states is unaffected by the choices of strategies. Unless the purpose of choosing a certain strategy is informational, the assumption should make sense and can illuminate other possible effects of the strategies. In addition, the extended models inherit Assumptions 2-4 and 2'-4' regardless of $\sigma^\theta \in \{\varepsilon^\theta, \eta^\theta, \delta^\theta\}$.

## 6.2 Military Strategies in Light of Audience Costs or Endogenous Battlefield Effectiveness

Below we derive the rational military strategies in extended Models I and II. To address the choices of military strategies, it must be taken into account how the choices influence the belligerents' subsequent decisions to fight or surrender.

**Lemma 6** *In both extended Models I and II, $D$ can postpone its decision to surrender with the shortening strategy, regardless of $A$'s military strategy; i.e., $T^I(\delta) \leq T^I(\eta) \leq T^I(\varepsilon)$ and $T^{II}(\delta) \leq T^{II}(\eta) \leq T^{II}(\varepsilon)$, where $T^I(\sigma)$ and $T^{II}(\sigma)$ are the numbers of periods before $D$ surrenders given military strategy profile $\sigma$ in Models I and II, respectively.*

**Proof.** The proofs of Lemma 6 above and Proposition 4 below are left in Appendix D. ■

By choosing the shortening strategy, the democracy can increase the chance of short-run victory. Moreover, in extended Model I, the increased chance raises the audience costs of surrendering through promoting the public support of the war. In extended Model II, the shortening strategy assists the democracy in defeating its opponent before its battlefield effectiveness diminishes.[25] These factors altogether induce the democracy to fight longer (with larger $T^I(\sigma)$ and $T^{II}(\sigma)$) although battles are made more decisive (with a smaller $\sigma^\theta$).

While the democracy benefits from the shortening strategy, the autocracy has rather mixed incentives for shortening and protracting strategies.

**Proposition 4** *In both extended Models I and II, $D$'s optimal military strategy is the shortening strategy regardless of $A$'s military strategy, while $A$'s optimal military strategy is indeterminate.*

Even though the two models have distinct theoretical grounds, they share analogous predictions as to the choices of military strategies. In fact, these predictions are consistent with empirical findings that democracies tend to adopt the maneuver strategy, whereas the choices of autocracies are rather ambiguous (Reiter and Meek

[25] If the shortening strategy has some tactical advantage, it might raise $\rho^\theta$ (the probability that a soldier in $D$ accomplishes a mission). Extended Model II suggests that even without such advantage, the shortening strategy can benefit $D$.

| | Democracy ($D$) | Autocracy ($A$) |
|---|---|---|
| Shortening strategy (maneuver) | To reduce cost of fighting<br>To end war before its public support and/or battlefield effectiveness diminish<br>To postpone its surrendering | To reduce cost of fighting |
| Protracting strategy (attrition) | None | To fight long until $D$'s public support and/or battlefield effectiveness diminish<br>To hasten $D$'s surrendering |

Table 1: Rationales for the choices of military strategies

1999). These results suggest the possibility that both the mechanisms of audience costs and of battlefield effectiveness might be at work in democracies' prosecution of wars. For instance, democratic soldiers outperform autocratic counterparts in the short run if $pp_D^{\theta}(\#sol\,(T)) > 1/2$ for $T < T^{II}$ (Reiter and Stam 1998b), but democratic citizens refuse futile fighting in the long run if $ac(\#cit\,(T)) < 0$ for $T > T^{I}$ (Bennett and Stam 1998). These effects caused by regime types could shape wars fought by democracies.

According to our models, the shortening strategy surely advantages the democracy in having a higher probability of short-run victory, while reducing the risk of fighting long. A lengthy war is disadvantageous for the democracy, because the public opposes fighting long, and also because its forces cannot effectively fight long. In contrast, the effects of the shortening strategy on the autocracy are more complicated. On one hand, it can reduce the risk of fighting long. On the other hand, it can also lower the probability that the democracy ultimately surrenders, because it makes battles more decisive and the war more likely to end before the democracy surrenders, and also because the shortening strategy taken by the autocracy can induce the democracy to fight more durably (Lemma 6). These conflicting effects make the autocracy's optimal military strategy indeterminate. According to our models, it depends mainly on the per-period cost of fighting ($c_A$) and on how durably the democracy can fight ($T^{I}(\sigma)$ and $T^{II}(\sigma)$). That means, the protracting strategy would be desirable for the autocracy if the per-period cost of fighting is smaller and/or if the democracy is expected to give in earlier.

At bottom, the logic behind democracies' adoption of the shortening strategy is to have higher chances of short-run victory. With the shortening strategy, a democracy can end a war before the public withdraws their supports and before its soldiers reduce their performance on the battlefield. On the other hand, autocracies may benefit from either shortening or protracting strategy. While the shortening strategy helps to reduce the risk of prosecuting a lengthy war, the protracting strategy might also be functional if an autocracy is willing to fight until a democracy's long-run weakness materializes. The rationales for the choices of military strategies are summarized in Table 1.

## 7 Concluding Remark

Conceivably, a straightforward interpretation of the negative association between the probability of democratic victory and the duration of war is that democracies' war performance declines over time (Reiter and Stam 1998b, 2002). This decline seems to disadvantage democracies in long wars. We are concerned that this interpretation is based on the presumption that the duration of war is exogenously given. If democracies' military advantages dissipate in around eighteen months (Bennett and Stam 1998), why do not they stop fighting by then? If they do stop fighting, there should be few lengthy wars fought by democracies, generating a bias in the data set on past wars. If they do not, that is presumably because they cannot. Namely, the domestic opposition makes it difficult for democracies to end wars by compromising their war aims. A consequential delay in the process of ending wars is well exemplified by the closing phase of the Vietnam War—the U.S. decision to withdraw troops was postponed by the president's concern about his reelection in 1968 (Ellsberg 1972). This is what Tocqueville ([1835, 1840] 2004) foresaw and Iklé (1971) was afraid of.

Indeed, the duration of war is a choice of the belligerents. That is, they choose to fight until the condition for peace matures (Fearon 2007; Powell 2004; Slantchev 2003b; Smith and Stam 2004; Wagner 2000). Hence, the seeming decline of democracies' advantages may be a result of the selection effect—as strong democracies win swiftly and decisively, only weak democracies are likely to fight long. In this regard, the selection effect may operate even after a war's outbreak (Koch 2009; Lemke and Reed 2001; Reed and Clark 2000) as well as before (Fearon 2002; Reed 2000; Bueno de Mesquita 1999).[26] It might still hold true that democracies are strong especially at short wars, but the decline might not be so significant in light of the selection effect. In fact, we have demonstrated that observed battlefield effectiveness of democracies would contain a downward bias if the selection effect is not controlled for. It is then worth postulating the causality in reverse; i.e., war outcomes do affect the duration.

We have thus explored the alternative interpretation (Tocqueville [1835, 1840] 2004; Iklé 1971)—when democracies become disadvantageous, wars tend to be long—as well as the straightforward one shown above.[27] In doing so, we adopt the formal-

[26]Interestingly, the selection effect during wars works in the direction opposite to the selection effect before wars, with which only strong democracies initiate fighting.

[27]To clarify, the straightforward interpretation corresponds to the Reiter-Stam claim in Introduction, and the alternative interpretation to the Tocqueville-Iklé claim.

| | Model I | Model II |
|---|---|---|
| Key variables | Audience costs | Battlefield effectiveness |
| Proponents | Tocqueville ([1835, 1840] 2004) Iklé (1971) | Bennet and Stam (1998b) |
| Third players in $D$ | Citizens | Soldiers |
| Tactical influence on battle outcomes | No | Yes $\left(\begin{array}{c}\text{decreasing} \\ \Pr(win_D \mid \theta)\end{array}\right)$ |
| Strategic influence on war outcomes | Possible $(T^I \neq T^*)$ | Possible $(T^{II} \neq T^*)$ |
| Deterrence effect on autocracy (given $T^I = T^{II}$) | Weaker (larger $U_{A\mid T^I}$) | Stronger (smaller $V_{A\mid T^{II}}$) |
| Military strategies $\left(\begin{array}{c}\text{maneuver} \\ \text{vs. attrition}\end{array}\right)$ | $D$ : maneuver<br>$A$ : indeterminate | $D$ : maneuver<br>$A$ : indeterminate |

Table 2: Summary comparison between (extended) Models I and II

modeling approach that is helpful not only to delineate the strategic interactions across the key players (e.g., state leaders, citizens, and soldiers) but also to illuminate the causal link between the key variables (the probabilities of outcomes and the duration of war). With a series of models, we have tested these two interpretations by checking: (i) whether they possess behavioral rationalist foundation; and (ii) whether they are consistent with empirical findings of the choices of military strategies.[28] To our surprise, both the interpretations satisfy these two standards.[29] That means, our analysis suggests that the negative association can be explained by decreasing audience costs and declining battlefield effectiveness as well as by the selection effect. Our summary comparison between (extended) Models I and II appears in Table 2.

While empirical studies have increasingly emphasized the relevance of domestic politics to war termination, formal studies on this relevance have been limited presumably because of complexity with theoretical analyses. With formal models that

[28]Our theory is also consistent with the empirical finding that an opponent adopting the maneuver strategy is more difficult to deter (Mearsheimer 1983), because the expected cost of implementing the startegy is lower.

[29]In particular, we were originally doubtful that the alternative interpretation meets the latter standard, because if war outcomes affect the duration, any attempt to manipulate the duration seems ineffective on the outcomes. However, our theory holds that although citizens have no influence on the battlefields, they do change both the outcomes and duration through their influence on the government's decision to fight or surrender.

highlight a democracy's citizens and soldiers, we have attempted to depict the relevance. As the extant theoretical literature on domestic politics and international relations contributed to our understanding of war outbreak, future theoretical progress will, we hope, assist our understanding of war termination.[30]

[30] Our models assume away several key components on the topics, including bargaining (Powell 2004; Slantchev 2003b; Wagner 2000), shifts of military balance on the battlefield (Langlois and Langlois 2009, 2012), third parties (Nakao 2015), timing of election (Smith 1998b), and opposition parties (Schutz 1998). Inclusion of some of them into costly-process models of war could be a direction for theoretical innovation.

# References


[1] Baliga, Sandeep, David O. Lucca, and Tomas Sjöström. 2008. "Domestic Political Survival and International Conflict: Is Democracy Good for Peace?" *Review of Economic Studies*. 78(2): 458-486.

[2] Baliga, Sandeep, and Tomas Sjötröm. 2008. "Strategic Ambiguity and Arms Proliferation." *Journal of Political Economy.* 116(6): 1023-1057.

[3] Baum, Matthew A., and Philip B. K. Potter. 2015. *War and Democratic Constraint: How the Public Influences Foreign Policy.* New Jersey, NJ: Princeton University Press.

[4] Bennett, D. Scott, and Allan C. Stam III. 1996. "The Duration of Interstate Wars, 1816-1985." *American Political Science Review*. 90(2): 239-257.

[5] Bennett, D. Scott, and Allan C. Stam III. 1998. "The Declining Advantages of Democracy: A Combined Model of War Outcomes and Duration." *Journal of Conflict Resolution*. 42(3): 344-366.

[6] Bueno de Mesquita, Bruce, James D. Morrow, Randolph M. Siverson, and Alastair Smith. 1999. "An Institutional Explanation of the Democratic Peace." *American Political Science Review*. 93(4): 791-807.

[7] Bueno de Mesquita, Bruce, and Randolph M. Siverson. 1995. "War and the Survival of Political Leaders: A Comparative Study of Regime Types and Political Accountability." *American Political Science Review*. 89(4): 841-855.

[8] Clark, David H., and William Reed. 2003 "A Unified Model of War Onset and Outcome." *Journal of Politics*. 65(1): 69-91.

[9] Debs, Alexandre, and H.E. Goemans. 2010. "Regime Type, the Fate of Leaders, and War." *American Political Science Review*. 104(3): 430-445.

[10] Downs, George W., and David M. Rocke. 1994. "Conflict, Agency, and Gambling for Resurrection: The Principal-Agent Problem Goes to War." *American Journal of Political Science*. 38(2): 362-380.

[11] Ellsberg, Daniel. 1972. *Papers on the War*. New York: Simon and Schuster.

[12] Fearon, James D. 1994. "Domestic Political Audiences and the Escalation of International Disputes." *American Political Science Review*. 88(3): 577-592.

[13] Fearon, James D. 2004. "Why Do Some Civil Wars Last So Much Longer than Others?" *Journal of Peace Research*. 41(3): 275-301.

[14] Fearon, James D. 2007. "Fighting Rather Than Bargaining." Paper presented at the 2007 Annual Meetings of the American Political Science Association, Chicago, IL: 30 August-2 September.

[15] Filson, Darren, and Suzanne Werner. 2002. "A Bargaining Model of War and Peace: Anticipating the Onset, Duration, and Outcome of War." *American Journal of Political Science*. 46(4): 819-837.

[16] Filson, Darren, and Suzanne Werner. 2004. "Bargaining and Fighting: The Impact of Regime Type on War Onset, Duration, and Outcomes." *American Journal of Political Science*. 48(2): 296-313.

[17] Gartner, Scott Sigmund, and Gary M. Segura. 1998. "War, Casualties, and Public Opinion." *Journal of Conflict Resolution*. 42(3): 278-300.

[18] Gartner, Scott Sigmund, Gary M. Segura, and Michael Wikening. 1997. "All Politics Are Local: Local Losses and Individual Attitudes toward the Vietnam War." *Journal of Conflict Resolution*. 42(3): 669-694.

[19] Guisinger, Alexandra, and Alastair Smith. 2002. "Honest Threats: The Interaction of Reputation and Political Institutions in International Crises." *Journal of Conflict Resolution*. 46(2): 175-200.

[20] Intriligator, Michael D., and Dagobert L. Brito. 1984. "Can Arms Races Lead to the Outbreak of War?" *Journal of Conflict Resolution*. 28(1): 63-84.

[21] Iklé, Fred Charles. 1971. *Every War Must End*. New York: Columbia University Press.

[22] Jackson, Matthew O., and Massimo Morelli. 2007. "Political Bias and War." *American Economic Review*. 97(4): 1353-1373.

[23] Koch, Michael T. 2009. "Governments, Partisanship, and Foreign Policy: The Case of Dispute Duration." *Journal of Peace Research*. 46(6): 799-817.

[24] Koch, Michael, and Scott Sigmund Gartner. 2005. "Casualties and Constituencies: Democratic Accountability, Electoral Institutions, and Costly Conflicts." *Journal of Conflict Resolution*. 49(6): 874-894.

[25] Lake, David A. 1992. "Powerful Pacificts: Democratic States and War." *American Political Science Review*. 86(1): 24-37.

[26] Langlois, Jean-Pierre P., and Catherine C. Langlois. 2009. "Does Attrition Behavior Help Explain the Duration of Interstate Wars? A Game Theoretic and Empirical Analysis." *International Studies Quarterly.* 53(4): 1051-1073.

[27] Langlois, Jean-Pierre P., and Catherine C. Langlois. 2012. "Does the Principle of Convergence Really Hold? War, Uncertainty and the Failure of Bargaining." *British Journal of Political Science*. 42(3): 511-536.

[28] Lemke, Douglas, and William Reed. 2001. "The Relevance of Politically Relevant Dyads." *Journal of Conflict Resolution*. 45(1): 126-144.

[29] Lindsey, David. 2015. "Military Strategy, Private Information, and War." *International Studies Quarterly.* 59(4): 629-640.

[30] Machiavelli, Nicolo. [1526] 1901. *History of Florence and the Affairs of Italy: From the Earlies Times to the Death of Lorenzo the Magnificient*. London: M. Walter Dunne.

[31] Maynard Smith, John. 1974. "Theory of Games and the Evolution of Animal Conflicts." *Journal of Theoretical Biology*. 47(1): 209-221.

[32] Mearsheimer, John J. 1983. *Conventional Deterrence*. Ithaca, NY: Cornell University Press.

[33] Meirowitz, Adam, and Anne E. Sartori. 2008. "Strategic Uncertainty as a Cause of War." *Quarterly Journal of Political Science.* 3(4): 327-352.

[34] Mueller, John E. 1973. *War, Presidents and Public Opinion*. New York: John Wiley & Sons.

[35] Nakao, Keisuke. 2015. "Expansion of Rebellion: From Periphery to Heartland." *Journal of Peace Research*. 52(5): 591-606.

[36] Nakao, Keisuke. 2022. "Denial and Punishment in War." *Journal of Peace Research*. 59(2): 166-179.

[37] Powell, Robert. 2004. "Bargaining and Learning While Fighting." *American Journal of Political Science*. 48(2): 344-361.

[38] Powell, Robert. 2012. "Persistent Fighting and Shifting Power." *American Journal of Political Science*. 56(3): 620-637.

[39] Reed, William, and David H. Clark. 2000. "War Initiators and War Winners: The Consequences of Linking Theories of Democratic War Success." *Journal of Conflict Resolution*. 44(3): 378-395.

[40] Reiter, Dan, and Curtis Meek. 1999. "Determinants of Military Strategy, 1903-1994: A Quantitative Empirical Test." *International Studies Quarterly*. 43(2): 363-387.

[41] Reiter, Dan, and Allan C. Stam. 1998a. "Democracy, War Initiation, and Victory." *American Political Science Review*. 92(2): 377-389.

[42] Reiter, Dan, and Allan C. Stam. 1998b. "Democracy and Battlefield Military Effectiveness." *Journal of Conflict Resolution*. 42(3): 259-277.

[43] Reiter, Dan, and Allan C. Stam. 2002. *Democracies at War*. Princeton, NJ: Princeton University Press.

[44] Schultz, Kenneth. 1998. "Domestic Opposition and Signaling in International Crisis." *American Economic Review*. 92(4): 829-844.

[45] Schultz, Kenneth. 2001. "Looking for Audience Costs." *Journal of Conflict Resolution*. 45(1): 32-60.

[46] Slantchev, Branislav L. 2003a. "The power to Hurt: Costly Conflict with Completely Informed States." *American Political Science Review*. 97(1): 123-133.

[47] Slantchev, Branislav L. 2003b. "The Principle of Convergence in Wartime Negotiation." *American Political Science Review*. 97(4): 621-32.

[48] Slantchev, Branislav. 2006. "Politicians, the Media, and Domestic Audience Costs." *International Studies Quarterly*. 50(2): 445-477.

[49] Slantchev, Branislav. 2010. "Feigning Weakness." *International Organization.* 64(3): 357-388.

[50] Smith, Alastair. 1996. "Diversionary Foreign Policy in Democratic Systems." *International Studies Quarterly.* 40(1): 133-153.

[51] Smith, Alastair. 1998a. "International Crises and Domestic Politics." *American Political Science Review.* 92(3): 623-638.

[52] Smith, Alastair. 1998b. "Fighting Battles, Winning Wars." *Journal of Conflict Resolution.* 42(3): 301-320.

[53] Smith, Alastair, and Allan C. Stam. 2004. "Bargaining and the Nature of War." *Journal of Conflict Resolution.* 48(6): 783-813.

[54] Stam, Allan C., III. 1996. *Win, Lose, or Draw: Domestic Politics and the Crucible of War.* Ann Arbor, MI: University of Michigan Press.

[55] Tarar, Ahmer. 2006. "Diversionary Incentives and the Bargaining Approach to War" *International Studies Quarterly.* 50(1): 169-188.

[56] Tarar, Ahmer. 2016. "A Strategic Logic of the Military Fait Accompli." *International Studies Quarterly.* 60(4): 742-752.

[57] Tarar, Ahmer, and Bahar Leventoğlu. 2009. "Public Commitment in Crisis Bargaining." *International Studies Quarterly.* 53(3): 817-839.

[58] Tarar, Ahmer, and Bahar Leventoğlu. 2013. "Limited Audience Costs in International Crises." *Journal of Conflict Resolution.* 57(6): 1065-1089.

[59] Tocqueville, Alexis de. [1835, 1840] 2004. *Democracy in America.* Trans. Arthur Goldhammer. New York: Liberty of America.

[60] Wagner, R. Harrison. 2000. "Bargaining and War." *American Journal of Political Science.* 44(3): 469-84.

[61] Wolford, Scott, Dan Reiter, and Clifford J. Carrubba. 2011. "Information, Commitment, and War." *Journal of Conflict Resolution.* 55(4): 556-579.

# APPENDIX

## A Decision Rules to Fight or Surrender

In our models, each player decides whether to fight or surrender in every time period. This decision problem can be redefined as another problem of when to surrender across periods (or to fight forever). Given an information set, a player chooses the best timing of surrendering. This problem is tractable if monotonicity holds for per-period payoffs.

**Case 1** *If a player's per-period payoff from fighting decreases over time, and his payoff from surrendering is constant, he compares his current-period payoff from fighting with the payoff from surrendering to choose when to surrender.*

In Case 1, even if fighting is unworthy in future periods, it may be worth fighting in the current period. A player keeps on fighting as long as the current-period payoff from fighting is larger than the payoff from surrendering. He then stops fighting when it becomes unworthy. Case 1 applies to $D$ of all the three models, $C$ of Model I (for the support decision), and $S$ of Model II (for the mission decision).

**Case 2** *If a player's per-period payoff from fighting increases over time, he compares his continuation payoff from fighting forever with the payoff from surrendering to decide whether or not to fight in the first period.*

In Case 2, even if short fighting is unworthy, it may still be worth fighting long. In other words, if it is worth fighting in the first period, it must be worth fighting for all the future periods. A player thus takes all possible future payoffs from fighting into account, anticipating some benefits (e.g., his opponent's surrendering). Case 2 applies to $A$ of all the three models.

## B Claim and Its Proof for Baseline Model

**Claim 1** $U^a_{A|T^*}\left(ind^T\right) > U^d_{A|T^*}\left(ind^T\right)$ *for any* $T \in \{0, 1, 2, \cdots, T^* - 1\}$*.*

**Proof.** By definition, $U^\theta_{A|T^*}\left(ind^T\right) = U^\theta_{A|T^*-T}$. Also by definition and some algebra,

$$U^\theta_{A|T^*} = \left(1 - \left(\delta^\theta\right)^{T^*}\right) U^\theta_A + \left(\delta^\theta\right)^{T^*} W_A.$$

To assess the relative size between $U^a_{A|T^*-T}$ and $U^d_{A|T^*-T}$,

$$\begin{aligned} U^a_{A|T^*-T} - U^d_{A|T^*-T} &= \left(1 - \left(\delta^a\right)^{T^*-T}\right) U^a_A + \left(\delta^a\right)^{T^*-T} W_A \\ &\quad - \left(\left(1 - \left(\delta^d\right)^{T^*-T}\right) U^d_A + \left(\delta^d\right)^{T^*-T} W_A\right), \end{aligned}$$

which is positive, because $T^* - T \geq 1$ (or $T \leq T^* - 1$), $\delta^d < \delta^a$ (Assumption 1), and $U^d_A < U^a_A < W_A$ (Assumption 3). ■

## C Payoffs from Fighting in Model II

For $i \in \{D, A\}$, $i$'s per-period payoff from fighting after $T$ "indecisive" periods is:

$$v_i\left(ind^T\right) \equiv \sum_{\theta \in \{d,a\}} \Pr\left(\theta | ind^T\right) v^\theta_i\left(ind^T\right),$$

where

$$v^\theta_i\left(ind^T\right) \equiv \left(1 - \delta^\theta\right) pp^\theta_i\left(\#sol\left(T\right)\right) W_i - c_i. \tag{A1}$$

$A$'s continuation payoff from fighting $D$ with $T^{II}$ after $T$ "indecisive" periods is:

$$V_{A|T^{II}}\left(ind^T\right) \equiv \sum_{\theta \in \{d,a\}} \Pr\left(\theta | ind^T\right) V^\theta_{A|T^{II}}\left(ind^T\right),$$

where

$$V^\theta_{A|T^{II}}\left(ind^T\right) \equiv \sum_{t=T+1}^{T^{II}} \left(\delta^\theta\right)^{t-T-1} v^\theta_A\left(ind^{t-1}\right) + \left(\delta^\theta\right)^{T^{II}-T} W_A. \tag{A2}$$

**ONLINE APPENDIX**

## D Proofs for Extended Models I and II

For the proofs below, $\sigma \in \{\varepsilon, \eta, \delta\}$ will be added to the subscript of relevant symbols for denotation.

**Proof of Lemma 6.** For Model I, the existence of $T^I(\varepsilon)$, $T^I(\eta)$, and $T^I(\delta)$ is guaranteed by Assumptions 1', 2, and 4, as with the proof of Lemmas 3. For Model II, the existence of $T^{II}(\varepsilon)$, $T^{II}(\eta)$, and $T^{II}(\delta)$ is derived from Assumptions 1', 2', and 4' (Lemma 5). Moreover, because military strategies have no influence on $\Pr\left(\theta | ind^T\right)$ (Assumption 1'), $i$'s payoff with a certain military strategy is larger than the payoff with the other strategy if the former is larger in any states. In other words, a comparison of $i$'s payoffs across $i$'s military strategies in each $\theta$ suffices to determine $i$'s incentives.

For Model I, with a smaller $\sigma^\theta$, $u^\theta_{D|\sigma}$ is larger, where $u^\theta_{D|\sigma} \equiv \left(1 - \sigma^\theta\right) p^\theta_D W_D - c_D$. Thus, $u_{D|\sigma}\left(ind^T\right)$ is also larger. Also, with a smaller $\sigma^\theta$, $ac(\#cit_\sigma(T))$ is larger, because $\overline{W}_{C|\sigma}(T)$ is smaller (Equation (2)), and thus $\#cit_\sigma(T)$ larger. These two changes tighten the condition for $D$'s surrender (raising the left-hand side and lowering the right-hand side of $u_{D|\sigma}\left(ind^T\right) < -ac\left(\#cit_\sigma(T)\right)$) and therefore tend to increase $T^I(\sigma)$. With discrete time periods, $T^I(\varepsilon)$ ($T^I(\eta)$) is at least as large as $T^I(\eta)$ ($T^I(\delta)$).

For Model II, with a smaller $\sigma^\theta$, $v^\theta_{D|\sigma}\left(ind^T\right)$ is larger (Equation (A1) with $\sigma$ in Appendix C). So is $v_{D|\sigma}\left(ind^T\right)$. A larger $v_{D|\sigma}\left(ind^T\right)$ can hold the condition to fight ($v_{D|\sigma}\left(ind^T\right) > 0$) even with a larger $T$. Therefore, a smaller $\sigma^\theta$ tends to raise $T^{II}(\sigma)$. (Note that $\#sol(T)$ is independent of $\sigma$. Unlike $\overline{W}_C(T)$ of Equation (2), $\overline{W}_S(T)$ of Equation (3) does not contain $\delta^\theta$.) ■

**Proof of Proposition 4.** As in Propositions 2 and 3, $D$ and $A$'s decisions to fight or surrender are uniquely determined for a given military strategy profile $\sigma$ in both extended Models I and II, where regardless of $\sigma$, $D$'s per-period payoff from fighting monotonically decreases over time, while $A$'s payoff from fighting increases (Assumptions 3 and 3').

For Model I, $D$'s payoff comparison is immediate from the proof of Lemma 6: $U_{D|\delta} < U_{D|\eta} < U_{D|\varepsilon}$, where for $\sigma \in \{\varepsilon, \eta, \delta\}$, $U_{D|\sigma}$ is $D$'s continuation payoff from fighting given $\sigma$

$$U_{D|\sigma} \equiv \sum_{\theta \in \{d,a\}} \Pr(\theta) \sum_{t=1}^{T^I(\sigma)} \left(\sigma^\theta\right)^{t-1} \left(u^\theta_{D|\sigma} + ac(\#cit_\sigma(t-1))\right).$$

For $A$, a smaller $\sigma^\theta$ has both positive and negative effects on $U^\theta_{A|T^I(\sigma),\sigma}$. The positive effect is to reduce the duration of the war $((1-(\sigma^\theta)^{T^I(\sigma)})/\left(1-\sigma^\theta\right))$ for given $T^I(\sigma)$. The negative effect is to lower the probability of reaching $D$'s surrendering $((\sigma^\theta)^{T^I(\sigma)})$. The latter effect is even more significant with a larger $T^I(\sigma)$ (Lemma 6). Whether $U_{A|T^I(\sigma),\sigma}$ increases or decreases depends on which of these effects on $U^\theta_{A|T^I(\sigma),\sigma}$ dominates.

For $D$ of Model II, it is immediate from the proof of Lemma 6 that $V_{D|\delta} < V_{D|\eta} < V_{D|\varepsilon}$, where $V_{D|\sigma}$ is $D$'s continuation payoff from fighting given $\sigma$

$$V_{D|\sigma} \equiv \sum_{\theta \in \{d,a\}} \Pr(\theta) \sum_{t=1}^{T^{II}(\sigma)} \left(\sigma^\theta\right)^{t-1} v^\theta_{D|\sigma}\left(ind^{t-1}\right).$$

For $A$, the shortening strategy may raise or lower $V_{A|T^{II}(\sigma),\sigma}$, depending on the relative size between its positive and negative effects on $V^\theta_{A|T^{II}(\sigma),\sigma}$ (Equation (A2) with $\sigma$ and $ind^0$ in Appendix C). For the positive effect, a smaller $\sigma^\theta$ reduces the duration of war $((1-(\sigma^\theta)^{T^{II}(\sigma)})/\left(1-\sigma^\theta\right))$. For the negative effect, a smaller $\sigma^\theta$ makes $D$'s surrendering less likely to be reached (with smaller $(\sigma^\theta)^{T^{II}(\sigma)}$). It is even less likely with a larger $T^{II}(\sigma)$ (Lemma 6). ■

| $\delta^a$ | BM | | Model I | | | | | Model II | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $T^*$ | $U_{A\|T^*}$ | $T^I$ | $U_D$ | $U_{A\|T^I}$ | $P_A^I$ | $Dur^I$ | $T^{II}$ | $V_D$ | $V_{A\|T^{II}}$ | $P_A^{II}$ | $Dur^{II}$ |
| 0.96 | 300 | 6.84 | 294 | 16.83 | 6.84 | 0.28 | 21.16 | 174 | 18.79 | -1.04 | 0.20 | 21.15 |
| 0.97 | 178 | 1.26 | 198 | 11.66 | 1.20 | 0.2804 | 26.84 | 130 | 12.77 | -5.44 | 0.21 | 26.63 |
| 0.98 | 23 | 47.83 | 78 | 2.92 | 3.45 | 0.36 | 32.30 | 56 | 3.18 | 7.45 | 0.36 | 28.73 |

Table 3: Results in the numerical example

## E Numerical Example

With specific parameters and functions, we confirm our theoretical predictions. From the results summarized in Table 3, the following implications can be read:

- Audience costs influence the timing of $D$'s surrendering (Proposition 2). In particular, $D$'s surrendering is delayed ($T^I > T^*$) when $\delta^a = 0.97$ or $\delta^a = 0.98$. Moreover, this delay generates the deterrence effect on $A$ ($U_{A|T^I} < U_{A|T^*}$).

- Endogenous battlefield effectiveness also influences the timing (Proposition 3). $D$ fights longer ($T^{II} > T^*$) for $\delta^a = 0.98$. In Model II, $D$'s short-run advantage can enhance its ability to deter $A$ ($V_{A|T^{II}} < U_{A|T^*}$ regardless of $\delta^a$).

- With the shortening strategy, $D$'s surrendering is delayed (Lemma 6). As $\delta^a$ decreases, $T^I$ and $T^{II}$ increase.

- With the shortening strategy, $D$ can raise its continuation payoff (Proposition 4). With a smaller $\delta^a$, $U_D$ and $V_D$ tend to be larger.

- With the shortening strategy, $A$ does not necessarily raise its continuation payoff (Proposition 4). As $\delta^a$ falls, $U_{A|T^I}$, and $V_{A|T^{II}}$ change non-monotonically. Whether the shortening strategy raises or lowers $A$'s continuation payoff depends on its effects on the probability of $A$'s winning ($P_A^I$ and $P_A^{II}$) and the expected duration ($Dur^I$ and $Dur^{II}$).

The parameters and functions are set as follows: for the baseline model, $\Pr(d) = \frac{8}{10}$, $\frac{\delta^a}{\delta^d} = \frac{101}{100}$ (Assumption 1'), $p_D^d = \frac{8}{10}$, $p_D^a = \frac{4}{10}$, $W_D = 50$, $c_D = 1$, $W_A = 100$, and $c_A = 1$; in addition to those above, for Model I, $W_C \sim U\left[\frac{c_D}{\sum_{\theta\in\{d,a\}}\Pr(\theta)\left(1-\delta^\theta\right)p_D^\theta}, \frac{c_D}{(1-\delta^a)p_D^a}\right]$ and $ac\left(\#cit\left(T\right)\right) = \frac{1}{10}\#cit\left(T\right)$; and for Model II, $W_S \sim U\left[\frac{c_S}{\sum_{\theta\in\{d,a\}}\Pr(\theta)\rho^\theta}, \frac{c_S}{\rho^a}\right]$, $c_S = 1$, $pp_D^d\left(\#sol\left(T\right)\right) = \frac{9}{10}\#sol\left(T\right)$, and $pp_D^a\left(\#sol\left(T\right)\right) = \frac{9}{20}\#sol\left(T\right)$.