Agricultural Revolution and Industrialization

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Abstract

This study explores how agricultural technology affects the endogenous takeoff of an economy in the Schumpeterian growth model. Due to the subsistence requirement for agricultural consumption, an improvement in agricultural technology reallocates labor from agriculture to the industrial sector. Therefore, agricultural improvement expands firm size in the industrial sector, which determines innovation and triggers an endogenous transition from stagnation to growth. Calibrating the model to data, we find that without the reallocation of labor from agriculture to the industrial sector in the early 19th century, the takeoff of the US economy would have been delayed by about four decades.

JEL classification: O30, O40

Keywords: agricultural technology, endogenous takeoff, innovation, economic growth

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The spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it. [...] The introduction of the turnip [...] made possible a change in crop rotation which [...] brought about a tremendous rise in agricultural productivity. As a result, more food could be grown with much less manpower. Manpower was released for capital construction. The growth of industry would not have been possible without the turnip and other improvements in agriculture. Nurkse (1953, p. 52-53)

1 Introduction

According to Nurkse (1953), among many others, improvements in agricultural technology that released labor from agriculture were crucial for the industrial revolution. The industrial revolution in turn sparked centuries of sustained economic growth. History thus suggests that improvements in agricultural technology propagate pervasively throughout the economy and have momentous consequences that far exceed what one can see by looking at the sector in isolation.

Modern growth economics has investigated extensively the forces driving the growth process, typically building on the theory of endogenous technological change (Romer 1990). Since at its core the theory has dynamic increasing returns, it identifies the size of the market in which firms operate as a, if not the, crucial factor determining incentives to innovate. A spectacular application of these ideas is the Unified Growth Theory of Galor and Weil (2000); see also Galor (2005, 2011). Models in this tradition produce an endogenous takeoff and a transition from stagnation to growth. Following these two influential branches of growth economics, and to place industry solidly at the forefront of the analysis, Peretto (2015) has developed an IO-based Schumpeterian growth model with endogenous takeoff in which firm size determines the incentives to innovate; see, e.g., Cohen and Klepper (1996a, b) and Laincz and Peretto (2006) for evidence on this channel. We use this model to formalize Nurkse’s idea and then investigate the role that agriculture plays in shaping the growth path of the economy. This strikes us as a first-order question in light of studies like, among others, Voigtlander and Voth (2006), Vollrath (2011) and Lagakos and Waugh (2013) that document the important implications of productivity differences in agriculture for economic development across countries.¹

¹Dalgaard et al. (2020) provide empirical evidence that fishery productivity also has a persistent positive effect on economic development since pre-industrial times and causes an early takeoff of the economy.
an earlier takeoff with faster post-takeoff growth and final convergence to scale-invariant steady-state growth.

At the heart of the mechanism driving this result is a subsistence requirement for agricultural consumption, which yields that when agricultural productivity improves, labor moves from agriculture to industry. This reallocation alone can be sufficient to ignite industrialization. More generally, we have that: (i) for given agricultural technology, the model predicts a finite takeoff date with an associated wait time that is co-determined by initial firm size and decreasing in agricultural productivity; and (ii) for given firm size, the model identifies the minimum size of the improvement in agricultural technology—an Agricultural Revolution—that triggers an immediate Industrial Revolution. The combination of (i) and (ii) says that low agricultural productivity delays industrialization and creates a temporary drag on post-industrialization growth. The drag is only temporary and not permanent because our Schumpeterian growth model with endogenous market structure sterilizes the strong scale effect.

These properties provide a new lens for interpreting the empirical evidence. As mentioned, economies with large populations (e.g., China and India) failed to industrialize for many decades after smaller ones did (e.g., UK and USA). Growth theories based on increasing returns have problems explaining this fact. The typical argument is that they had bad institutions (e.g., Acemoglu and Robinson, 2012). Our analysis develops the complementary hypothesis that the allocation of labor to agriculture played an important role in determining their industrialization lags. Moreover, the scale-invariance of steady-state growth implies that while agricultural productivity does not affect income growth asymptotically, it has permanent and large effects on the overall time-profile of income. This property sheds new light on the debate about the role that agriculture (more generally, the primary sector) plays in shaping the dynamics of cross-country income differences.

We calibrate the model to US data to perform an illustrative quantitative analysis. The agricultural share of the US workforce was about 80% in the early 19th century (see Baten 2016) and decreased to about 70% in 1830 and 60% in 1840 (see Lebergott 1966 and Weiss 1986). We find that this reallocation of labor from agriculture to industry was a powerful push toward the takeoff of the US economy. In line with our analytical result, absent this reallocation the takeoff of the US economy would have occurred four decades later. Finally, we derive a formula that shows that a one-fifth increase in industrial employment reduces the wait time to takeoff by about a decade.

To illustrate further the properties of our model, in particular in a cross-country perspective, we develop three applications. The first explores the role of intellectual property rights as an example of a potentially important policy instrument. The second explores the role of a general-purpose technology as an example of extensions of the theoretical framework that speak to important issues debated in the literature. In this example, the model produces a great-divergence followed by great-convergence profile of growth rates due to two key properties: (i) the timing of takeoff depends on the level of the general-purpose technology and (ii) the steady-state growth rate does not depend on the level of the general-purpose technology because the model sterilizes the strong scale effect. This result illustrates the model’s ability to capture rich pattern of cross-country variation of income paths over time. The third application explores the role of frictions in the reallocation of labor across agricultural and industrial sectors.
This study relates to the literature on endogenous technological change. Romer (1990) develops the first R&D-based growth model driven by the invention of new products (horizontal innovation). Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom et al. (1990) develop the creative-destruction Schumpeterian growth model driven by the improvement of the quality of products (vertical innovation). Peretto (1994, 1998, 1999), Smulders (1994) and Smulders and van de Klundert (1995) combine the two dimensions of innovation to develop the creative-accumulation Schumpeterian growth model with endogenous market structure.² Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008, 2010) and Ang and Madsen (2011) provide early evidence for this class of models. Garcia-Macia et al. (2019) provide the latest evidence that growth is driven by the in-house innovation activity of existing firms. We contribute to this literature by incorporating an agricultural sector in the creative-accumulation model.

This study also relates to the literature on endogenous takeoff. The seminal contribution in this literature is Galor and Weil (2000). They develop unified growth theory and show that the quality-quantity trade-off in child rearing and the accumulation of human capital enable an economy to escape the Malthusian trap and experience an endogenous transition from stagnation to growth; see also Galor and Moav (2002), Galor and Mountford (2008), Galor et al. (2009) and Ashraf and Galor (2011). Galor (2011) provides a comprehensive review of unified growth theory. A recent study by Madsen and Strulik (2020) introduces land-biased technological change driven by education to the unified growth model and explores how it affects the endogenous takeoff of the economy and also the evolution of income inequality. We focus, instead, on the role of Schumpeterian technological progress driven by innovation as a complementary channel for the endogenous takeoff of the economy. Hansen and Prescott (2002) is another early study on endogenous takeoff. Gollin et al. (2002) introduce an agricultural sector into the Hansen-Prescott model, which features exogenous technological progress, to explore how agricultural technology affects industrialization. Our Schumpeterian growth model features multiple dimensions of innovation, which complement these perspectives by exploring the endogenous activation of endogenous technological progress. More generally, and in line with the overall thrust of this literature, we formalize the idea of Nurkse (1953), and the related big push idea of Murphy et al. (1989), in a very tractable dynamic general equilibrium model.³ Our model allows us to obtain analytical results and then quantify the effects of agricultural technology on the industrialization path of the economy—a path consisting of an endogenous takeoff followed by post-takeoff accelerating growth, with final convergence from below to scale-invariant innovation-led steady-state growth.

The rest of this paper is organized as follows. Section 2 presents some stylized facts. Section 3 describes the Schumpeterian growth model. Section 4 explores the effects of agricultural technology. Section 5 performs a quantitative analysis. Section 6 concludes.

²Howitt (1999) combines the two dimensions of technology to develop a creative-destruction version of the theory.
³Our model can be viewed as a modern version of the dual-sector model in Lewis (1954).
2 Stylized facts

In this section, we highlight some stylized facts from cross-country data. First, we look at available historical data in the 19th century for the following early industrialized countries: Belgium, Britain, France, Japan, Netherlands, Spain, Sweden, and the United States. We compute the average values of GDP per capita and the agricultural share of GDP in each country for each decade in the 19th century. Figure 1 plots the relationship between these two variables and shows a clearly negative relationship. In other words, around the time of its takeoff, when a country had a large agricultural sector in the economy, it tended to have a low income level. Then, as the size of its agricultural sector decreased over time, its income level tended to rise.

![Figure 1: Agricultural share and GDP per capita](image)

Data source: Roser (2013). GDP per capita is reported in log value. Agricultural share of GDP is defined as agricultural expenditures as a share of GDP. The data is from 1800 to 1899 based on the average value of each decade for the following early industrialized countries: Belgium, Britain, France, Japan, Netherlands, Spain, Sweden, and the United States.

A key component of our theoretical model is that the reduction in the size of the agricultural sector is driven by an increase in the level of agricultural productivity that leads to a reallocation of labor from agriculture to industry due to subsistence in agricultural consumption. Due to the lack of historical data on agricultural productivity, we look at recent data from developing countries, which only experienced industrialization recently or may even have yet to do so. Figure 2 plots the relationship between agricultural productivity and the agricultural share of labor in 1999-2001 and shows a clearly negative relationship.

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4 According to Rostow (1956), most of these countries experienced their takeoff in the 19th century, with Britain being the first one in as early as 1783-1802 and Japan being relatively late in 1878-1900.
across countries. In other words, a country that has a relatively high level of agricultural productivity tends to have a relatively low agricultural share of labor.

Figure 2: Agricultural productivity and labor share

Data source: FAO Statistical Yearbook by the Food and Agriculture Organization of the United Nations. Agricultural productivity is in log and defined as agricultural GDP per worker in the agricultural sector. Agricultural share of labor is defined as the share of workers in the agricultural sector. The sample is from 1999-2001 and for low-income and lower/upper middle-income countries as defined by the World Bank. The data that include high-income countries and from 1979-1981 and 1989-1991 show the same pattern.

Another important component of our theoretical model is that a higher level of agricultural productivity leads to a larger R&D share of output because the reallocation of labor from agriculture to industry increases industrial firm size, which in turn provides more incentives for R&D. Figure 3 plots the relationship between agricultural productivity and the R&D share of GDP in 1999-2001 and shows a clearly positive relationship for OECD countries. In other words, a country that has a relatively high level of agricultural productivity tends to have a relatively large R&D share of GDP.

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5See also Caselli (2005).
6Unfortunately, we only have R&D data for OECD countries.
These stylized facts support the mechanism in our theoretical model: higher agricultural productivity reallocates labor from agriculture to industry due to subsistence in agricultural consumption. The standard Schumpeterian mechanism then takes over: the reallocation of labor from agriculture to industry increases industrial firm size, which in turn provides more incentives for R&D and innovation. As stated in the introduction, we exploit this structure to investigate how agricultural productivity shapes the whole transition path of the economy from stagnation to growth.

Our theoretical model also predicts that a rise in agricultural productivity has a positive effect on economic growth at an early stage of economic development. Then, as the country develops over time, this positive effect on economic growth becomes smaller and eventually disappears. Therefore, we use the following empirical specification to examine our theory:  

\[ g_{it} = \vartheta_1 A_{it} + \vartheta_2 A_{it} \times y_{it} + \vartheta_3 y_{it} + \Gamma \kappa_{it} + \rho_{it} + \epsilon_{it}, \]

where \( g_{it} \) denotes the five-year average annual growth rate of real GDP, real GDP per capita or real GDP per worker in country \( i \) at wave \( t \), \( A_{it} \) denotes the log level of agricultural productivity (defined as agricultural GDP per worker in the agricultural sector) in country \( i \) at wave \( t \), and \( y_{it} \) is the log value of per capita GDP in country \( i \) in the initial year of wave \( t \). Our theory predicts that \( \vartheta_1 > 0 \) and \( \vartheta_2 < 0 \). In other words, a higher level of

\[ \text{Figure 3: Agricultural productivity and R&D share} \]

Data source: FAO Statistical Yearbook for agricultural productivity and OECD Statistics for the R&D share of GDP. Agricultural productivity is in log and defined as agricultural GDP per worker in the agricultural sector. R&D share of GDP is defined as R&D expenditures as a share of GDP. The sample is from 1999-2001 and for OECD countries.

\[ \text{See also Chu, Fan and Wang (2020) who examine empirically and theoretically the interaction between status-seeking culture and income level.} \]
agricultural productivity generates a positive effect on economic growth at an early stage of economic development. As a country becomes more developed, this positive effect of agricultural productivity on economic growth becomes smaller.

$\pi_{it}$ is a vector of the average value of the following control variables in each wave: the log level of population, the log value of capital stock, a human capital index, the degree of openness (measured by the average ratio of export plus import to GDP), and the government consumption share of GDP. $g_t$ is the wave fixed effect, and $\epsilon_{it}$ is the error term. After merging data from the FAO Statistical Yearbook and the Penn World Table, we have a sample of up to 408 observations covering 149 countries. Table 1 provides the summary statistics of the variables.

Table 1: Summary statistics

<table>
<thead>
<tr>
<th>variable</th>
<th>obs</th>
<th>mean</th>
<th>std. dev.</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth of real GDP</td>
<td>408</td>
<td>0.046</td>
<td>0.054</td>
<td>-0.165</td>
<td>0.386</td>
</tr>
<tr>
<td>Growth of real GDP per capita</td>
<td>408</td>
<td>0.027</td>
<td>0.054</td>
<td>-0.202</td>
<td>0.354</td>
</tr>
<tr>
<td>Growth of real GDP per worker</td>
<td>402</td>
<td>0.024</td>
<td>0.053</td>
<td>-0.227</td>
<td>0.344</td>
</tr>
<tr>
<td>Log agricultural productivity</td>
<td>408</td>
<td>7.554</td>
<td>1.702</td>
<td>4.344</td>
<td>11.003</td>
</tr>
<tr>
<td>Log real GDP per capita</td>
<td>408</td>
<td>8.690</td>
<td>1.206</td>
<td>6.278</td>
<td>12.322</td>
</tr>
<tr>
<td>Log population (in 10 thousands)</td>
<td>408</td>
<td>6.613</td>
<td>1.900</td>
<td>1.426</td>
<td>11.793</td>
</tr>
<tr>
<td>Log capital stock</td>
<td>408</td>
<td>12.233</td>
<td>2.277</td>
<td>7.049</td>
<td>17.817</td>
</tr>
<tr>
<td>Human capital index</td>
<td>356</td>
<td>2.157</td>
<td>0.703</td>
<td>1.020</td>
<td>3.619</td>
</tr>
<tr>
<td>Openness</td>
<td>408</td>
<td>0.484</td>
<td>0.484</td>
<td>0.006</td>
<td>5.388</td>
</tr>
<tr>
<td>Government consumption share</td>
<td>408</td>
<td>0.192</td>
<td>0.093</td>
<td>0.007</td>
<td>0.664</td>
</tr>
</tbody>
</table>

Data source: FAO Statistical Yearbook for agricultural productivity and Penn World Table for others.

Table 2 reports the regression results. The dependent variable in columns (1)-(2) is the average annual growth rate of real GDP, whereas the dependent variable in columns (3)-(4) is the average annual growth rate of real GDP per capita. The dependent variable in columns (5)-(6) is the average annual growth rate of real GDP per worker. Odd columns present the baseline results without additional controls. In even columns, we further control for additional explanatory variables. As expected, in all the columns, the coefficient on agricultural productivity is significantly positive, whereas the interaction term between agricultural productivity and the income level is significantly negative.

For example, in column (4), the estimated coefficient on agricultural productivity is 0.054, which is statistically significant at the 1% level, whereas the estimated coefficient on the interaction term is -0.005, which is also statistically significant at the 1% level. These estimates imply that for a country with minimal GDP per capita, increasing agricultural productivity by 1% is associated with an increase in the growth rate by 2.26% (0.054-0.005*6.278)*1, which is statistically significant at the 1% level. For a country with average GDP per capita, increasing agricultural productivity by 1% is associated with an increase in the growth rate by 1.06% (0.054-0.005*8.690)*1, which is also statistically significant at the 1% level. For a country with maximal GDP per capita, increasing agricultural productivity by 1% is associated with a decrease in the growth rate by 0.76% (0.054-0.005*12.322)*1, which however

8Controlling country fixed effects yields the same signs but less significant coefficients given that we have only three waves of data. However, most of the regression coefficients still remain significant at least at the 5% level, except for column (2); see Table A1 in Appendix A.
is not statistically significant and has a \( p \)-value of 0.29. Therefore, the positive effect of agricultural productivity on economic growth becomes smaller and eventually disappears as the level of economic development increases.

Table 2: Effects of agricultural productivity on economic growth

<table>
<thead>
<tr>
<th></th>
<th>GDP growth</th>
<th>per capita GDP growth</th>
<th>per worker GDP growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>( A_{it} )</td>
<td>0.046***</td>
<td>0.041***</td>
<td>0.065***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>( A_{it} \times y_{it} )</td>
<td>-0.004***</td>
<td>-0.004**</td>
<td>-0.005***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>( y_{it} )</td>
<td>0.014</td>
<td>-0.005</td>
<td>0.023**</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Control variables</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Wave fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>408</td>
<td>356</td>
<td>408</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.159</td>
<td>0.205</td>
<td>0.207</td>
</tr>
</tbody>
</table>

Note: *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \). Standard errors are in parentheses. The dependent variable in columns (1)-(2) is the average annual growth rate of real GDP. The dependent variable in columns (3)-(4) is the average annual growth rate of real GDP per capita. The dependent variable in columns (5)-(6) is the average annual growth rate of real GDP per worker. Compared with odd columns, even columns add control variables including the log value of population, the log value of capital stock, a human capital index, the degree of openness (measured by the average ratio of export plus import to GDP), and the government consumption share of GDP.

3 A Schumpeterian model of endogenous takeoff

The model features both the improvement of existing intermediate goods (vertical innovation) and the creation of new intermediate goods (horizontal innovation). Incentives to undertake these activities depend on firm size. Consequently, whether the economy experiences the endogenous takeoff depends on the size of the market for intermediate goods. In the original version (Peretto, 2015) the size of this market is proportional to the size of the labor force. By incorporating an agricultural sector with subsistence consumption, we disentangle the size of the market for intermediate goods from the size of the labor force and obtain a structure where the size of the intermediate sector, and therefore the size of intermediate firms, depends on the reallocation of labor from agriculture to industry.

3.1 Household

There is a representative household with \( L_t = L_0 e^{\lambda t} \) identical members, where \( L_0 = 1 \) and \( \lambda > 0 \) is population growth rate. The household has Stone-Geary preferences

\[
U_0 = \int_0^\infty e^{-(\rho-\lambda)t} \left[ \ln c_t + \beta \ln (q_t - \eta) \right] dt, 
\] (1)
where \( c_t \) and \( q_t \) denote, respectively, consumption per capita of an industrial and of an agricultural good. The parameter \( \beta > 0 \) determines the importance of industrial consumption relative to agricultural consumption. The latter features a subsistence requirement \( \eta > 0 \). The parameter \( \rho > \lambda \) is the subjective discount rate.

The household maximizes utility subject to the asset-accumulation equation

\[
\dot{a}_t = (r_t - \lambda)a_t + w_t - c_t - p_t q_t, \tag{2}
\]

where \( a_t \) is wealth per capita and \( r_t \) is the real interest rate. Each member of the household supplies inelastically one unit of labor to earn the wage \( w_t \). Let the industrial good be our numeraire and \( p_t \) be the price of the agricultural good. The household sets:

\[
\begin{align*}
\frac{\dot{c}_t}{c_t} &= r_t - \rho; \quad \tag{3} \\
q_t &= \eta + \frac{\beta c_t}{p_t}. \quad \tag{4}
\end{align*}
\]

The first equation summarizes the intertemporal consumption-saving decision as the growth path of industrial consumption \( c_t \). The second summarizes the intratemporal allocation of expenditure across the two goods as the demand for agricultural consumption \( q_t \).

### 3.2 Agriculture

We follow Lagakos and Waugh (2013) and model agriculture as a competitive sector operating a linear technology

\[
Q_t = AL_{q,t}, \tag{5}
\]

where the parameter \( A > \eta \) is labor productivity and \( L_{q,t} \) is employment in agriculture. Profit maximization yields

\[
w_t = p_t A, \tag{6}
\]

which says that the wage rate in agriculture is equal to the marginal product of labor.

We omit land for simplicity. Including land produces the same qualitative results about endogenous takeoffs but the analysis is much more algebra-intensive. Vollrath (2011), among many others, studies the effects of land intensity and labor intensity in agriculture on industrialization. Our results are in line with the general insights produced by that work.

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9 This is a common feature of structural change models (see, e.g., Matsuyama (1992), Laitner (2000) and Kongsamut et al. (2001)), which study the implications of structural change for long-run (i.e., asymptotic) growth but not for endogenous takeoff. See Herrendorf et al. (2014) for an excellent survey of this literature and Herrendorf et al. (2020) for a recent contribution.
3.3 Industrial production

A representative competitive firm operates the assembly technology

\[ Y_t = \int_0^{N_t} X_t^\theta (i) \left[ Z_t^\alpha (i) Z_t^{1-\alpha} L_{y,t}/N_t^{1-\sigma} \right]^{1-\theta} di, \]  

(7)

where \( \{\theta, \alpha, \sigma\} \in (0,1) \). The key features are: (i) there is a continuum of non-durable differentiated intermediate goods \( i \in [0, N_t] \); (ii) \( X_t (i) \) is the quantity of intermediate good \( i \); (iii) the productivity of good \( i \) depends on its own quality \( Z_t (i) \) and on average quality \( Z_t \equiv \int_0^{N_t} Z_t (j) dj/N_t \); and (iv) overall productivity in assembly depends on product variety \( N_t \). Two parameters regulate technological spillovers: \( \alpha \) captures the private return to quality and hence \( 1 - \alpha \) determines vertical technological spillovers; and \( 1 - \sigma \) captures a congestion effect of product variety so that the social return to variety is \( \sigma \).

Let \( P_t (i) \) be the price of \( X_t (i) \). Profit maximization yields the conditional demands:

\[ L_{y,t} = (1 - \theta) \frac{Y_t}{w_t}; \]  

(8)

\[ X_t (i) = \left( \frac{\theta}{P_t (i)} \right)^{1/(1-\theta)} \frac{Z_t^\alpha (i) Z_t^{1-\alpha} L_{y,t}}{N_t^{1-\sigma}}. \]  

(9)

These expressions yield that the competitive industrial firm pays \( (1 - \theta) Y_t = w_t L_{y,t} \) for industrial labor and \( \theta Y_t = \int_0^{N_t} P_t (i) X_t (i) di \) for intermediate goods.

3.4 Intermediate goods and in-house R&D

A monopolistic firm produces differentiated intermediate good \( i \) with a linear technology that requires \( X_t (i) \) units of the industrial good to produce \( X_t (i) \) units of intermediate good \( i \) at quality \( Z_t (i) \), that is, the marginal cost of production is one. The firm also pays \( \phi Z_t^\alpha (i) Z_t^{1-\alpha} \) units of the industrial good as a fixed operating cost. To improve the quality of its product, the firm devotes \( I_t (i) \) units of the industrial good to in-house R&D. The innovation technology is

\[ \hat{Z}_t (i) = I_t (i). \]  

(10)

The firm’s gross profit (i.e., profit before-R&D) is

\[ \Pi_t (i) = [P_t (i) - 1] X_t (i) - \phi Z_t^\alpha (i) Z_t^{1-\alpha}. \]  

(11)

The value of the monopolistic firm is

\[ V_t (i) = \int_t^{\infty} \exp \left( - \int_t^s r_u du \right) \left[ \Pi_s (i) - I_s (i) \right] ds. \]  

(12)

The monopolistic firm maximizes (12) subject to (9) and (10).

We solve this dynamic optimization problem in Appendix A and find that the unconstrained profit-maximizing markup ratio is \( 1/\theta \). However, we assume that competitive fringe
firms can produce \( X_t(i) \) at quality \( Z_t(i) \) but at the higher marginal cost \( \mu \in (1, 1/\theta) \). The monopolistic firm then sets
\[
P_t(i) = \min \{ \mu, 1/\theta \} = \mu \tag{13}
\]
and prices fringe firms out of the market. The optimization problem also delivers the firm’s rate of return to innovation,
\[
r_t^q(i) = \frac{\alpha \Pi_t(i)}{Z_t(i)} = \alpha \left[ (\mu - 1) \frac{X_t(i)}{Z_t(i)} - \phi Z_t^{\alpha-1}(i) Z_t^{1-\alpha} \right],
\]
which is linear in quality-adjusted firm size \( X_t(i)/Z_t(i) \). This property is at the heart of the mechanism that we study: incentives to innovate depend on quality-adjusted firm size, which in turn depends on the size of the market. We now turn to this component of the logical chain.

In models of this class the equilibrium of the market for intermediate goods is symmetric, that is, intermediate firms start with the same initial quality \( Z_0(i) = Z_0 \) for \( i \in [0, N_t] \) and, facing a symmetric environment, make identical decisions. Consequently, they grow at the same rate and symmetry holds at any point in time. Using the limit price (13), quality-adjusted firm size is
\[
\frac{X_t(i)}{Z_t(i)} = \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{L_{y,t}}{N_t^{1-\sigma}} = \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}} \frac{L_{y,t}}{L_t}.
\]
We define the industrial employment share \( l_{y,t} \equiv L_{y,t}/L_t \) and the composite variable
\[
X_t \equiv \theta^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}.
\]
This variable compresses the two state variables \( L_t \) (population) and \( N_t \) (mass of firms) to the ratio \( L_t/N_t^{1-\sigma} \) and, therefore, makes the analysis of the model’s dynamics simple.

With this notation, quality-adjusted firm size becomes
\[
\frac{X_t}{Z_t} = \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{X_t}{\theta^{1/(1-\theta)} \frac{L_{y,t}}{L_t}} = \frac{X_t}{\theta^{1/(1-\theta)} \frac{L_{y,t}}{L_t}}.
\]
Accordingly, the rate of return to innovation is
\[
r_t^q = \frac{\alpha \Pi_t}{Z_t} = \alpha \left[ \frac{\mu - 1}{\mu^{1/(1-\theta)} \chi l_{y,t} - \phi} \right].
\]
To summarize, this structure captures two sides of the idea explored in this paper. First, agricultural employment implies \( l_{y,t} < 1 \) and thus reduces firm size in the intermediate sector and thereby depresses incentives to innovate. Second, the reallocation of labor from agriculture to industrial production is an essential component of the dynamics of takeoff and subsequent sustained growth: as \( l_{y,t} \) rises, the return to innovation rises faster than in the absence of structural change.

\[\text{Specifically, we allow for diffusion of knowledge from monopolistic firms to fringe firms that enables the latter to constrain the pricing behavior of the former. This structure disentangles markups from the technological parameter } \theta.\]
3.5 Entrants

Upon payment of a sunk cost of $\delta X_t$, $\delta > 0$, units of the industrial good, a new firm enters the market and offers a new differentiated good of average quality. This structure preserves the symmetry of the intermediate goods market equilibrium at all times. The asset-pricing equation governing the value of firms (old and new) is

$$r_t = \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t}.$$ (16)

Entry is positive when the free-entry condition holds, i.e., when

$$V_t = \delta X_t.$$ (17)

Substituting (9) and (13) into (11) and then using the resulting expression, (10), (16) and (17) yield the return to entry as

$$r^e_t = \frac{\mu^{1/(1-\theta)}}{\phi} \left( \frac{\mu - 1}{\mu^{1/(1-\theta)}} \frac{\phi + z_t}{\chi_{ly,t}} \right) + z_t + \frac{\dot{X}_t}{\chi_t} + \frac{\dot{y}_{iy,t}}{l_{y,t}},$$ (18)

where $z_t \equiv \dot{Z}_t/Z_t$ is the growth rate of average quality.

3.6 Aggregation

We define the general equilibrium in Appendix A. Substituting (9) and (13) into (7) yields the reduced-form representation of industrial production:

$$Y_t = \left( \frac{\theta}{\mu} \right)^{\theta/(1-\theta)} \dot{N}_t^\theta Z_t L_{ly,t}.$$ (19)

The associated growth rate of industrial output per capita, $y_t = Y_t/L_t$, is

$$g_t = \frac{\dot{y}_t}{y_t} = \sigma n_t + z_t + \frac{\dot{y}_{iy,t}}{l_{y,t}}.$$ (20)

This growth rate has three components: (i) the growth rate of the variety of intermediate goods, $n_t \equiv \dot{N}_t/N_t$; (ii) the growth rate of the average quality of intermediate goods, $z_t$; (iii) the growth rate of the industrial labor share $l_{y,t}$.

3.7 Labor allocation

The combination of labor demand from agriculture (6) and industry (8) yields

$$p_t = \frac{(1-\theta) Y_t}{AL_{ly,t}}.$$ (21)
Substituting the agricultural technology \((5)\) and the relative price \((21)\) in the demand function for \(q_t\) in \((4)\) yields the industrial labor share \(l_{y,t}\) as

\[
l_{y,t} = \left( 1 + \frac{\beta c_t}{1 - \theta y_t} \right)^{-1} \left( 1 - \frac{\eta}{A} \right). \tag{22}\]

This equation says that for given consumption-output ratio \(c_t/y_t\), the industrial labor share \(l_{y,t}\) is increasing in \(A\) (and conversely, the agricultural labor share \(l_{q,t} \equiv L_{q,t}/L_t\) is decreasing in \(A\) as in Figure 2) if and only if \(\eta > 0\). This property produces sectoral reallocation whereby an improvement in the agricultural technology releases labor from agriculture to the industrial sector.

### 4 Agriculture, takeoff and long-run growth

We now develop the main analytical insight of the paper. We first show that the economy begins in a pre-industrial era in which the growth rate of industrial output per capita is zero. It then enters the industrial era, which consists of two phases. In the first, only the development of new products marketed by new firms drives the growth rate of industrial output per capita. In the second, product-quality improvement by existing firms adds its contribution and produces an acceleration of the growth rate.\(^{11}\) The economy finally converges to constant growth of income per capita fueled by both vertical and horizontal innovation.

Next, we show that agriculture shapes this process of phase transitions and convergence: agricultural productivity determines the timing of the first phase transition, the endogenous takeoff of the economy, and of the second phase transition, the activation of vertical innovation. This timing effect has momentous consequences: although agricultural productivity does not affect steady-state growth due to the model’s sterilization of the scale effect, it has permanent and large effects on the economy’s time-profile of income. This property sheds new light on the debate about the role that agriculture plays in shaping the dynamics of cross-country income differences.

#### 4.1 Global dynamics

The equilibrium law of motion of the state variable \(\chi_t \equiv \theta^{1/(1-\theta)} L_t/N_t^{1-\sigma}\) defined in \((14)\) is

\[
\dot{\chi}_t = \left[ \lambda - (1 - \sigma)n_t \right] \chi_t, \tag{23}\]

where the variety growth rate \(n_t\) is either zero or an increasing function of \(\chi_t\) (see Appendix A). The dynamics of \(\chi_t\) in turn determines the dynamics of the economy, which converges to the balanced growth path if the following condition holds:

\[
\delta \phi > \frac{1}{\alpha} \left[ \mu - 1 - \delta \left( \rho + \frac{\sigma}{1 - \sigma} \lambda \right) \right] > \mu - 1. \tag{24}\]

\(^{11}\)We consider the realistic case in which product creation happens before quality improvement. See Peretto (2015) for details on this property of the baseline growth model.
In this case, given an initial $\chi_0$, the state variable $\chi_t$ increases over time and converges to

$$\chi^* = \mu^{1/(1-\theta)} \frac{(1 - \alpha)\phi - \left[\rho + \sigma\lambda/(1 - \sigma)\right]}{(1 - \alpha)(\mu - 1) - \delta \left[\rho + \sigma\lambda/(1 - \sigma)\right]} 1 + \beta \left(1 + \frac{\theta - \lambda}{\mu - 1 - \theta}\right)$$

as the variety growth rate converges to $n^* = \lambda/(1 - \sigma)$. Steady-state firm size and income per capita growth are (see Appendix A):

$$\chi^* I_y = \mu^{1/(1-\theta)} \frac{(1 - \alpha)\phi - \left[\rho + \sigma\lambda/(1 - \sigma)\right]}{(1 - \alpha)(\mu - 1) - \delta \left[\rho + \sigma\lambda/(1 - \sigma)\right]};$$

$$g^* = \alpha \left[\left(\mu - 1\right) \frac{(1 - \alpha)\phi - \left[\rho + \sigma\lambda/(1 - \sigma)\right]}{(1 - \alpha)(\mu - 1) - \delta \left[\rho + \sigma\lambda/(1 - \sigma)\right]} - \phi\right] - \rho > 0.$$  \hspace{1cm} (25)

This structure has two properties worth stressing:

First, the existence condition (24) consists of two inequalities that ensure that the steady state $\chi^*$ exists. To establish whether $\chi^*$ is the attractor of the model’s dynamics, we need to investigate the conditions for the occurrence of the two phase transitions discussed above. We do so in the remainder of this section, placing the role of agriculture at the center of the investigation. The exercise shows that the two inequalities also provide the condition for the occurrence of the second phase transition. The two conditions in (24) are then jointly sufficient for the full transition to the steady state $\chi^*$.

Second, (26) says that steady-state growth is independent of the sectoral allocation of labor due to the scale-invariance of the Schumpeterian growth model with endogenous market structure. This property is central to the paper’s insight. As we investigate the role of agriculture in driving the phase transitions, we find that because steady-state growth is invariant to $A$, cross-country differences in agricultural productivity produce a pattern of divergence-convergence, namely: (i) differences in $A$ generate differences in growth that are solely due to differences in the timing of takeoff; (ii) such differences are only temporary and eventually vanish so that all else equal, there is long-run growth equalization. It is worth stressing that differences in growth rates vanish, not differences in income levels. That is, differences in agricultural productivity imprint themselves on income levels and are amplified by the initial divergence in income dynamics caused by the different takeoff times. The amplification can be large since it leverages differences in growth rates that last several decades due to the model’s slow convergence to the steady state.

4.2 The pre-industrial era

In the pre-industrial era, there are two possible configurations of the intermediate-good sector. First, initially demand for each intermediate good is so small that a would-be monopolist operating the increasing-returns technology would earn negative profit (see Appendix A for details). Since the increasing-returns technology is not viable, the existing $N_0$ intermediate goods are produced by competitive firms that do not innovate and make zero profit at the equilibrium price $P_t(i) = \mu$. Anticipating this, entrepreneurs are not willing to pay the sunk entry cost and thus there is no variety innovation either. Initially, therefore, all technologies
in this economy exhibit constant returns to scale. Our variable $\chi_t$, the total output of the competitive firms producing each existing intermediate good, grows only because of exogenous population growth (i.e., $\frac{\dot{\chi}_t}{\chi_t} = \lambda$). In this era, more precisely, the initial number of intermediate goods $N_0$ is exogenous and predetermined while the market structure in each product line, i.e., the number of firms and the size of each firm, is indeterminate.

The second possible configuration occurs when the size of the market for intermediate goods grows sufficiently large that a would-be monopolist operating the increasing-returns technology could earn positive profit. We assume, however, that although the increasing-returns technology is now viable, agents do not deploy it yet because doing so requires payment of the sunk entry cost. The idea is that only innovation, in this case a process innovation, allows a new firm to monopolize an existing market. Hence, the pre-industrial era ends only when the present value of monopolistic firms is sufficiently large that the free-entry condition (17) holds.

As a result of the pre-industrial market structure outlined above, in the pre-industrial era the household’s industrial consumption is $c_t = w_t l_{y,t} = (1 - \theta) y_t$, which yields

$$\frac{c_t}{y_t} = 1 - \theta.$$  \hspace{1cm} (27)

Substituting (27) into (22) yields

$$l_y = \frac{1}{1 + \beta} \left( 1 - \frac{\eta}{A} \right).$$  \hspace{1cm} (28)

This says that the industrial labor share in the pre-industrial era is stationary and increasing in agricultural productivity $A$. From (20), the growth rate of industrial output per capita is

$$g_t = \sigma n_t + \left. z_t + \frac{i_{y,t}}{l_{y,t}} \right| = 0$$  \hspace{1cm} (29)

due to $n_t = z_t = i_{y,t}/l_{y,t} = 0$ in the pre-industrial era.

### 4.3 The industrial era: phase 1

Horizontal innovation (but not yet vertical innovation) activates when firm size $\chi_t l_{y,t}$ grows sufficiently large. In this phase, we have a positive variety growth rate $n_t > 0$ and a zero quality growth rate $z_t = 0$. When the free-entry condition holds, the consumption-output ratio $c_t/y_t$ and the industrial labor share $l_{y,t}$ jump to the steady-state values (derivation in Appendix A):

$$\left( \frac{c}{y} \right)^* = \left( \frac{\rho - \lambda}{\mu} \right) \delta \theta + 1 - \theta;$$  \hspace{1cm} (30)

$$l_y^* = \frac{1}{1 + \beta \left( 1 + \frac{\rho - \lambda}{\mu} \frac{\delta \theta}{1 - \eta} \right) \left( 1 - \frac{\eta}{A} \right)}.$$  \hspace{1cm} (31)

\[12^{12}\text{In Appendix B, we consider an extension of the model that does not rely on this assumption and show that the dynamics are less realistic.}\]
In the first phase of the industrial era, the growth rate of industrial output per capita becomes $g_t = \sigma n_t$ because $z_t = 0$. The growth rate of product variety $n_t$ can be derived as

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta} \left( \frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{\chi_t l^*_t} \right) + \lambda - \rho > 0,$$

which uses $\rho + g_t = \rho + \sigma n_t = r_t = r_t^i$ in (18). From (32), $n_t$ is positive if and only if

$$\chi_t > \frac{1 + \beta \left( 1 + \frac{\rho - \lambda}{\mu} \frac{\delta \theta}{1-\theta} \right)}{\mu - 1 - \delta (\rho - \lambda)} \frac{\mu^{1/(1-\theta)}\phi}{\left( 1 - \frac{\eta}{\lambda} \right)^{-1}} \equiv \chi_N. \quad (33)$$

Note that $n_t$ is increasing in agricultural technology $A$ via the industrial labor share $l^*_t$, which is increasing in $A$, and increasing in the state variable $\chi_t$ so that (23) describes a stable process.

The interpretation of this property in terms of the baseline growth model is that there exists a threshold of $\chi_t$ below which the economy operates under pre-industrial conditions and firm size grows only because of exogenous population growth. Eventually, the economy crosses the threshold $\chi_N$ but it takes

$$T_N = \frac{1}{\lambda} \log \left( \frac{\chi_N}{\chi_0} \right) \quad (34)$$

years to achieve such takeoff (derivation in Appendix A). Since $\chi_N$ is decreasing in $A$, the combination of (32) and (34) says that economies with higher agricultural productivity $A$ take off earlier and exhibit faster post-takeoff growth than economies with lower $A$.

An alternative interpretation is as follows. We write (33) as

$$A > \frac{\eta}{1 - \frac{\mu - 1 - \delta (\rho - \lambda)}{\mu - 1 - \delta (\rho - \lambda)} \left[ 1 + \beta \left( 1 + \frac{\rho - \lambda}{\mu} \frac{\delta \theta}{1-\theta} \right) \right] \frac{\mu^{1/(1-\theta)}\phi}{\chi_t}}. \quad (35)$$

This now says that, given $\chi_t$, when the agricultural technology $A$ is below this critical threshold the economy remains in the pre-industrial equilibrium. However, if $A$ rises above the threshold, the economy takes off immediately. In this sense, we have a condition determining when and how an Agricultural Revolution can trigger the Industrial Revolution. The two interpretations are complementary. The first holds $A$ constant and uses the model’s dynamics to compute the wait time to industrialization, i.e., how long it takes for $\chi_t$ to go from its initial value $\chi_0$ to the threshold value $\chi_N$. As shown, the wait time is lower the larger is $A$. The second interpretation fixes $\chi_t$ and asks how large an improvement in $A$ is needed to trigger immediately the activation of Schumpeterian innovation. Equation (35) says that economies with larger firms require smaller agricultural improvements to take off.

The important component of this mechanism is that when the agricultural technology improves, the economy reallocates labor from the agricultural sector to the industrial sector and that this reallocation alone can be sufficient to ignite industrialization. Figure 4 presents the time path of the growth rate $g_t$ when $A$ increases at time $t$ and causes the economy to escape the pre-industrial era and enter the first phase of the industrial era. The figure highlights the two complementary interpretations discussed above: (i) for a given $A$, the
model predicts a finite takeoff date with an associated wait time determined by the initial condition \( \chi_0 \) (equivalently, initial firm size \( \chi_0 l_y \)); and (ii) for a given firm size \( \chi t l_y^* \), the model identifies the minimum size of the improvement in \( A \)—an Agricultural Revolution—that triggers an immediate Industrial Revolution. The combination of (i) and (ii) says that low agricultural productivity delays industrialization and creates a temporary drag on post-industrialization growth. The drag is only temporary because our Schumpeterian growth model with endogenous market structure sterilizes the scale effect.

![Figure 4: Agricultural revolution and industrialization](image)

In Figure 4, agricultural productivity \( A \) increases at time \( t \), which leads to an immediate takeoff and an earlier activation of vertical innovation.

Finally, a related implication is that the R&D share of output (i.e., \( \dot{N}_t \delta X_t / Y_t \)) is increasing in the level of agricultural technology \( A \) for a given \( \chi_t \) (see Appendix A). This positive relationship between agricultural technology and the R&D share of output is consistent with the data in Figure 3.

### 4.4 The industrial era: phase 2

When firm size \( \chi t l_y^* \) is sufficiently large, horizontal innovation and vertical innovation occur simultaneously. In this case, we have a positive variety growth rate \( n_t > 0 \) and a positive quality growth rate \( z_t > 0 \). This is the second phase of the industrial era. Given active horizontal innovation, the consumption-output ratio and the industrial labor share remain at the steady-state values (30)-(31).

The growth rate of industrial output per capita can be derived from \( \rho + g_t = r_t = r_t^q \) in (15) as

\[
g_t = \alpha \left[ \frac{\mu - 1}{\mu l_y (1 - \gamma)} \chi t l_y^* - \phi \right] - \rho > 0, \tag{36}
\]
which is increasing in agricultural technology \(A\) via the industrial labor share \(l^*_y\) and increasing in firm size \(\chi_l l^*_y\). The growth rate of variety can be derived from \(\rho + g_t = \rho + \sigma n_t + z_t = r_t = r^*_t\) in (18) and is given by

\[
n_t = \frac{\mu^{1/(1-\theta)}}{\delta} \left( \frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{\chi_l l^*_y} \right) + \lambda - \rho > 0,
\]

where \(z_t\) can be derived from (36), (37) and \(g_t = \sigma n_t + z_t\) as

\[
z_t = \left(1 - \frac{\mu^{1/(1-\theta)}\sigma}{\delta \chi_l l^*_y}\right)^{-1} \left\{ \left[ \frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{\chi_l l^*_y} \right] \left[ \alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta \chi_l l^*_y} \right] - \rho + \sigma (\rho - \lambda) \right\}.
\]

The entry process in (37) determines the dynamics of \(\chi_t\) (derivation in Appendix A).

Given (24), this transition to phase 2 occurs when \(\chi_t\) rises above the following threshold:

\[
\chi_t > \left[ 1 + \beta \left(1 + \frac{\rho - \lambda}{\mu} \frac{\delta \theta}{1-\theta} \right) \right] \Omega \left(1 - \frac{\eta}{A}\right)^{-1} \equiv \chi_Z > \chi_N,
\]

where

\[
\Omega \equiv \text{arg solve} \left\{ \left[ \frac{\mu - 1}{\mu^{1/(1-\theta)}} - \phi \right] \left[ \alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta \omega} \right] = \rho - \sigma (\rho - \lambda) \right\}.
\]

As in the previous case, the standard interpretation of this condition is that for a given \(A\), there exists a threshold of firm size above which firms invest in-house and growth accelerates due to quality innovation.

The complementary interpretation of the threshold follows from rewriting (38) as

\[
A > \frac{\eta}{1 - \left[ 1 + \beta \left(1 + \frac{\rho - \lambda}{\mu} \frac{\delta \theta}{1-\theta} \right) \right] \Omega / \chi_t}.
\]

This says that for a given \(\chi_t\), a sufficiently large improvement in the level of agricultural technology \(A\) can cause the immediate activation of quality innovation if it causes the threshold \(\chi_Z\) to fall below \(\chi_t\). In this era, the R&D share of output (i.e., \((N_t l_t + \hat{N}_t \delta X_t)/Y_t\)) is also increasing in the level of agricultural technology \(A\) for a given \(\chi_t\) (see Appendix A). This positive relationship between agricultural technology and the R&D share of output is once again consistent with the data in Figure 3.

Finally, the economy converges to a constant growth rate of industrial output per capita fueled by both vertical and horizontal innovation. Firm size \(\chi^* l^*_y\) converges to its steady-state value in (25), while the growth rate \(g^*\) converges to its steady-state value in (26). Both \(\chi^* l^*_y\) and \(g^*\) are independent of agricultural technology \(A\).

### 4.5 Summary and discussion

We can summarize our main global dynamics result as follows.
Proposition 1 Given (24) and $y_0 < y_N < y_Z$, the economy begins in the pre-industrial era with no innovation of any kind. It then experiences the endogenous takeoff and enters the first phase of the industrial era where horizontal innovation alone fuels industrial growth. Finally, the economy enters the second phase of the industrial era with both vertical and horizontal innovation and converges to the balanced growth path. Agricultural productivity $A$ determines the timing of the two-phase transitions but does not affect the steady-state growth rate of the economy. Specifically, economies with higher agricultural productivity take off earlier and exhibit temporarily faster post-takeoff growth than economies with lower agricultural productivity, eventually converging to the scale-invariant growth rate $g^*$. 

Proof. See Appendix A. ■

These properties are important when looking at the data. As mentioned, economies with large populations (e.g., China and India) failed to industrialize for decades after smaller ones did (e.g., UK and USA). Growth theories based on increasing returns have obvious problems explaining this fact. Our analysis says that their allocation of labor to an unproductive agricultural sector played an important role in determining their industrialization lags both in terms of the timing of the takeoff and of the steepness of the post-takeoff income profile. The scale-invariance of steady-state growth implies that while agricultural productivity does not affect income growth asymptotically, it has permanent and large effects on the overall time-profile of income.

We see our results on the role of agriculture as part of a very broad agenda. Our emphasis is on analytical transparency and dynamics with phase-transitions. We see the latter as essential to understanding things like the timing of takeoff both in the time-series and the cross-sectional dimensions. The literature is rich in discussions of “timing” that, however, rarely provide details that allow one to see exactly how specific parameters determine it. Our contribution, in contrast, aims at identifying precisely the channels that run from fundamentals to timing, indeed to the overall shape of the transition path.

5 Quantitative analysis

In this section we complement our analytical work with quantitative exercises designed to illustrate some attractive properties of our framework. We begin with a simple counterfactual and then provide three applications. The first explores the role of intellectual property rights as an example of a potentially important policy instrument. The second explores the role of a general-purpose technology as an example of possible extensions of the theoretical framework that speak to important issues debated in the literature. The third explores the role of frictions in the reallocation of labor across agricultural and industrial sectors.

5.1 A simple counterfactual and model calibration

In the early 19th century, the agricultural share of the US workforce decreased from about 80% to 60%; see Baten (2016), Lebergott (1966) and Weiss (1986). We perform a counter-
factual analysis to assess how large an effect this reallocation of labor from agriculture to industry had on the takeoff of the US economy.

Recall that firm size, which determines the timing of the takeoff, is

$$\chi_t \delta_{y,t} = \chi_t (1 - \delta_{q,t}),$$

where $\delta_{q,t} \equiv L_{q,t}/L_t$ is the agricultural labor share. The takeoff occurs when $\chi_t$ reaches the threshold $\chi_N$. In terms of firm size we have

$$\chi_t \delta_{y,t} > \chi_N \delta_{y,t}^*. $$

A decrease in the agricultural labor share $\delta_{q,t}$ from 80% to 60% yields an increase in the industrial labor share $\delta_{y,t}$ from 20% to 40%. This expands firm size $\chi_t \delta_{y,t}$ by a factor of 2 for given $\chi_t$. In the pre-industrial era the state variable $\chi_t$ grows at rate $\lambda$. In the US, the long-run population growth rate is 1.8%. Therefore, without the increase in the industrial labor share, $\chi_t$ would take

$$t = \frac{\ln 2}{\lambda} = \frac{0.7}{1.8\%} = 39 \text{ years}$$

to increase by a factor of 2. In other words, without the reallocation of labor from agriculture to industry in the early 19th century, the takeoff of the US economy would have been delayed by about four decades. Furthermore, we can define $\varepsilon \equiv d\delta_{y,t}/\delta_{y,t}$, i.e., the percent change in $\delta_{y,t}$, and for $\varepsilon$ small obtain the approximation

$$t = \frac{\ln (1 + \varepsilon)}{\lambda} \approx \frac{\varepsilon}{\lambda} \text{ years.}$$

This says that, given a population growth rate $\lambda$ of 1.8%, a one-fifth increase in industrial employment reduces the wait time to takeoff by about a decade.

We now calibrate the rest of the model to the US economy to perform our quantitative analysis. In addition to the population growth rate $\lambda$, the model also features the following parameters: $\{\rho, \alpha, \sigma, \beta, \theta, \delta, \phi, \mu\}$. We set the discount rate $\rho$ to a conventional value of 0.05. We follow Iacopetta et al. (2019) to set the degree of technology spillovers $1 - \alpha$ to 0.833 and the social return of variety $\sigma$ to 0.25. Then, we calibrate $\beta$ using the current agricultural share of GDP in the US, which is about 1%. Furthermore, we calibrate $\{\theta, \delta, \phi\}$ by matching the following moments of the US economy: 60% for the labor income share of GDP, 62% for the consumption share of GDP, and 1% for the long-run growth rate. Finally, we calibrate the markup ratio $\mu$ by matching the average growth rates of the simulated path from our model and the historical path in the US. The calibrated parameter values are $\{\beta, \theta, \delta, \phi, \mu\} = \{0.016, 0.404, 2.547, 1.212, 1.630\}$. Table 3 summarizes the parameter values.

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13Here we are putting manufacturing and services together as the industrial sector that requires innovation; see e.g., United Nations (2011) for a review on the importance of innovation in the services sector. Kongsamut et al. (2001) show that manufacturing and services require the same technology growth rate in order for a balanced growth path to exist in their model.

14Data source: Maddison Project Database. The waiting time to takeoff is lower if the population growth rate is higher.

15There is also the subsistence ratio $\eta/A$, which we will calibrate using historical data.

16Here we assume that the subsistence requirement is no longer binding in modern days; i.e., $\eta/A \rightarrow 0$. 

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Table 3: Calibrated parameter values

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<td>0.404</td>
<td>2.547</td>
<td>1.212</td>
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To explore how well our model matches the historical path of the growth rate in the US, we first use historical data to calibrate a time path for the subsistence ratio $\eta/A$. Specifically, we calibrate the initial value of $\eta/A$ using an agricultural labor share of 80% at the beginning of the 19th century; see Baten (2016). Then, we use an agricultural labor share of 60% in 1840 and 53% in 1860 in Lebergott (1966) and Weiss (1986) and also an agricultural share of GDP of 30% in 1900, 20% in 1920-1930, 10% in 1950 and 2% in 1980 in Kongsamut et al. (2001) to compute a piecewise linear path of $\eta/A$. We model these changes as MIT shocks (i.e., a sequence of unanticipated, permanent changes). Based on this imputed path of $\eta/A$, Figure 5 simulates the path of the agricultural share of GDP, which decreases from about 70% in the early 19th century to 1% at the end of the 20th century as in the US data.

![Figure 5: Agricultural share of GDP](image1)

![Figure 6: Economic growth](image2)

Figure 5 presents the simulated path of agricultural expenditure as a share of GDP based on the calibrated path of agricultural productivity. Figure 6 presents (a) the simulated path of the growth rate of industrial output per worker based on the calibrated path of agricultural productivity, (b) the simulated path of the growth rate without agricultural improvement (i.e., agricultural productivity remains at its initial value), and (c) the HP-filter trend of the growth rate in the US.

Figure 6 presents the simulated path of the growth rate of industrial output per worker and the HP-filter trend of the US growth rate along with a simulated path of the growth rate without agricultural improvement (i.e., $\eta/A$ remains at its initial value). Unfortunately, we don’t have historical data on labor productivity growth in the US, so we use data on the growth rate of output per capita as a proxy. Here we pick an initial value $\chi_0$ such that the takeoff of the economy occurs before the mid-19th century. Following the onset of horizontal innovation, vertical innovation starts half a decade later. After that, the economy keeps growing and reaches a growth rate as high as 3% due to the expansion of the industrial sector, which helps to accelerate the rate of innovation. Around the time of the Great Depression in the 20th century, there is a pause in the reallocation of labor from agriculture to
the industrial sector, which translates into a temporary slow down in technological progress before a recovery. Before the end of the 20th century, the growth rate of the economy gradually falls towards the long-run growth rate due to the deceleration of sectoral reallocation. This simulated pattern replicates the data reasonably well with the average growth rate increasing from 1.08% in the 19th century to 2.24% in the 20th century before decreasing to 1.04% in the 21st century, whereas the corresponding data are 1.20%, 2.12% and 1.13% in the 19th, 20th and 21st centuries respectively. It is worth stressing that the simulated path of the growth rate cannot capture this inverted-U pattern in the data if we shut down agricultural improvement. Consequently, the novel ingredient of this paper — the agricultural sector with the associated labor reallocation mechanism — contributes significantly to the Schumpeterian model’s ability to match quantitatively the salient features of the secular growth path of the US economy.

5.2 Intellectual property rights

Our analysis above suggests that if we think about a group of countries, those with a larger manufacturing sector experience an earlier takeoff holding the other parameter values constant across the countries. Given the documented role of the timing of takeoff in driving persistent differences in income per capita across countries, this type of consideration tells us that identifying the factors that determine the takeoff is of first-order importance. As an example of such factors, consider North and Thomas (1973, p. 156), who argue that the industrial revolution happened in England because it "had developed an efficient set of property rights embedded in the common law [and...] begun to protect private property in knowledge with its patent law." Chu, Kou and Wang (2020) provide a qualitative analysis on the effects of patent protection on endogenous takeoff in the Schumpeterian growth model that we use here. Pursuing this line of thought, we now provide a quantitative comparison between the two alternative channels, patent protection and agricultural productivity, that determine the timing of takeoff and the shape of the subsequent path of economic growth.

Stronger patent protection allows monopolistic firms to charge a higher markup. Therefore, we consider an increase in \( \mu \) as a strengthening of patent protection. Equation (33) shows that an increase in \( \mu \in (1, 1/\theta) \) leads to a decrease in \( \chi_N \) and an earlier takeoff (as in the case of an increase in \( A \)). Equations (31), (32) and (36) show that an increase in \( \mu \in (1, 1/\theta) \) also leads to a higher growth rate \( g_t \) for a given \( \chi_t \) in the industrial era. As we have shown, an increase in \( A \) has the same qualitative effects. However, equation (26) shows that the steady-state growth rate \( g^* \) is decreasing in \( \mu \) because according to equation (25) firm size is decreasing in \( \mu \). Therefore, unlike agricultural technology \( A \), which does not affect steady-state growth, strengthening patent protection stifles steady-state growth despite generating an earlier takeoff of the economy.

We now perform a quantitative exercise. We assume that the industrial labor share starts at the same initial value \( l_{y,t} = 20\% \) as before and pick an initial value \( \chi_0 \) such that the takeoff of the economy occurs in 1840 given \( \mu = 1.630 \). Then we simulate the transition path of output growth rate \( g_t \) under different values of \( \mu \). Figure 7 presents the effects of \( \mu \) on the takeoff and economic growth. A larger \( \mu \) leads to an earlier takeoff but a lower steady-state growth rate. Table 4 shows that increasing \( \mu \) from 1.630 to 1.657 leads to an earlier takeoff
by only 1.15 years. However, the steady-state growth rate of quality drops to zero (i.e., \( z^* = 0 \)) and the steady-state growth rate of output becomes \( g^* = \sigma n^* = 0.60\% \). Therefore, the positive effect of a stronger patent protection on the timing of the takeoff is relatively small compared to its large negative effect on the steady-state growth rate. Therefore, an improvement in agricultural technology is a far more effective way to generate an earlier takeoff of the economy.

Figure 7: Dynamic effects of patent protection

Figure 7 presents the simulated paths of the growth rate based on different values of the markup.

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \Delta T_N )</th>
<th>( g^* )</th>
<th>( z^* )</th>
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<tr>
<td>1.635</td>
<td>-0.23</td>
<td>0.92%</td>
<td>0.32%</td>
</tr>
<tr>
<td>1.640</td>
<td>-0.44</td>
<td>0.85%</td>
<td>0.25%</td>
</tr>
<tr>
<td>1.645</td>
<td>-0.65</td>
<td>0.77%</td>
<td>0.17%</td>
</tr>
<tr>
<td>1.650</td>
<td>-0.86</td>
<td>0.70%</td>
<td>0.10%</td>
</tr>
<tr>
<td>1.655</td>
<td>-1.07</td>
<td>0.63%</td>
<td>0.03%</td>
</tr>
<tr>
<td>1.657</td>
<td>-1.15</td>
<td>0.60%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Table 4 presents the simulated values of (a) the change in the takeoff time, (b) the steady-state output growth rate \( g \) and (c) the steady-state quality growth rate \( z \), based on different values of the markup.

Before we close this section, we should emphasize that we have analyzed only one particular policy instrument within the patent system. Specifically, this patent policy instrument enables monopolistic firms to charge a higher markup and earn more monopolistic profit.
This is just one aspect of the patent system, and there are other policy instruments within the patent system. Moser (2013) provides an excellent review of empirical studies that examine the empirical relationship between patents and innovation using historical data. In summary, he finds that "patent policies, which grant strong intellectual property rights to early generations of inventors, may discourage innovation." Chu (2009) extends the Schumpeterian model in O’Donoghue and Zweimuller (2004) to provide a quantitative analysis on this negative effect of blocking patents. Therefore, it is important to note that different policy instruments within the patent system can have different implications on takeoff and long-run growth.

5.3 General-purpose technology

We now extend the model to allow for improvements in technology that affect both the agricultural sector and the industrial sector. This is a natural extension to consider since history suggests that many technological improvements contribute simultaneously to both sectors. The literature refers to such things as general-purpose technologies (GPTs). This extension is particularly relevant in the cross-country perspective highlighted in the previous subsection because differences in GPTs can serve as proxy for differences across countries in a broader set of fundamentals. The exercise, therefore, illustrates how one can use our model to shed light on the drivers of persistent differences in income per capita in a framework that accounts for the rich and non-linear properties of the transition path from stagnation to growth.

Formally, we modify the industrial technology (7) as follows:

\[ Y_t = \int_0^{N_t} X_t^\theta(i) \left[ A^\gamma Z_t^\alpha(i) Z_t^{1-\alpha} L_{y,t} / N_t^{1-\sigma} \right]^{1-\theta} di, \tag{40} \]

where \( \gamma \geq 0 \). When \( \gamma = 0 \), we are back to the benchmark model. When \( \gamma > 0 \), an increase in the level of GPT \( A \) has an additional positive effect in the industrial sector. The rest of the model is the same as before, except that \( \chi_t l_{y,t} \) is replaced by \( A^\gamma \chi_t l_{y,t} \). It is useful to note that due to the assumption of log utility in industrial and agricultural consumption, by itself the industrial technological improvement does not affect the allocation of labor across the two sectors. Thus the main channel in the model remains that the improvement in productivity in agriculture reallocates labor to industrial production.

We can show that the threshold \( \chi_N \) in (33) becomes

\[ \chi_N = \left[ 1 + \beta \left( 1 + \frac{\rho - \lambda}{\mu} \frac{\delta \gamma}{1-\theta} \right) \right] \mu^{1/(1-\theta)} \phi \left( 1 - \frac{\eta}{A} \right)^{-1} A^\gamma, \tag{41} \]

which is decreasing in \( A \) as before. However, there is now an additional negative effect via the term \( A^\gamma \). Therefore, an improvement in the GPT leads to an earlier takeoff than an

\[ \text{---See Chu (2022) for a survey on different patent instruments considered in the literature on patent policy and economic growth.} \]
improvement in agricultural technology alone. We can also show that the growth rate of variety in the first phase of the industrial era is

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta} \left( \frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{A^\gamma \chi_t l_y^*} \right) + \lambda - \rho > 0,$$

while the growth rate of output in the second phase of the industrial era is

$$g_t = \alpha \left[ \frac{\mu - 1}{\mu^{1/(1-\theta)}} A^\gamma \chi_t l_y^* - \phi \right] - \rho > 0,$$

where $l_y^*$ is increasing in $A$ as before. Notably, both the variety growth rate $n_t$ and the output growth rate $g_t$ are increasing in $A^\gamma$ for a given $\chi_t$. In other words, there is an additional positive effect on economic growth from an improvement in the GPT in both phases of the industrial era due to the simple fact that the GPT expands industrial production. However, due to the scale-invariance of the Schumpeterian growth model, this effect eventually vanishes and the steady-state growth rate $g^*$ is the same as in equation (26), which is independent of $A$ even when $\gamma > 0$.

Next, we assess quantitatively the effects of the GPT. We normalize the initial value of $A_0$ to 1, which implies that $\eta$ is 0.797 because the initial value of subsistence ratio $\eta/A$ in 1800 is 0.797. An increase in the industrial labor share $l_{y,t}$ from 20% to 40% in the early 19th century translates into a decrease of subsistence ratio $\eta/A$ from 0.797 to 0.593 and an increase in $A$ from 1 to 1.343. Then we simulate the effects of a permanent increase in $A$ from 1 to 1.343 in the pre-industrial era on takeoff and transition path of $g_t$. The effects of $A$ on the timing of the takeoff (i.e., the activation of variety-expanding innovation) and the activation of quality-improving innovation depend on $\gamma$. Specifically, a larger $\gamma$ leads to an earlier activation of both types of innovation and a higher transitional growth rate but does not affect the steady-state growth rate; see Figure 8.

Figure 8: Dynamic effects of general purpose technology

Figure 8 presents the simulated paths of the growth rate based on different values of GPT intensity.
It is worth stressing that in this exercise, and the one in the previous subsection, one can interpret each line as a specific country so that the figure speaks directly to the model’s ability to capture the cross-country variation over time of income paths. In this particular case, the model produces a great-divergence followed by great-convergence profile of growth rates due to two key properties: (i) the timing of takeoff depends on the level of the GPT; and (ii) the steady-state growth rate does not depend on the level of the GPT because the model sterilizes the strong scale effect. For example, Rostow (1956) documents the different timing of takeoff in a number of European countries, such as Britain (1783-1802), France (1830-1860), Belgium (1833-1860), Germany (1850-1873) and Sweden (1868-1890). Despite their different takeoff time by almost a century, these countries have a roughly similar income level in the modern era.

As mentioned above, industrial technological improvement does not affect the allocation of labor across sectors, due to the assumption of log utility in industrial consumption and agricultural consumption in our model. In the case of non-unitary elasticity of substitution between industrial and agricultural consumption, different rates of technological progress in the two sectors could lead to continuous reallocation of labor across sectors. In contrast, having the same growth rate in GPT $A$ under $\gamma = 1$ may accommodate a stationary allocation of labor across sectors.\[18\]

### 5.4 Skill acquisition

Our analysis so far assumes that workers are freely mobile between sectors. However, in reality, some frictions exist. For example, converting an agricultural worker into an industrial worker requires some training. In this section, we model this acquisition of skills in a simple and tractable way.

Specifically, we assume that each unit of industrial labor requires $\psi \in [0, 1)$ units of labor as training time. In this case, the asset-accumulation equation in (2) becomes

$$\dot{a}_t = (r_t - \lambda) a_t + w_{y,t} l_{y,t} + w_{q,t} l_{q,t} - c_t - p_t q_t,$$

where $w_{y,t}$ is the wage rate of industrial labor $l_{y,t}$ and $w_{q,t}$ is the wage rate of agricultural labor $l_{q,t}$. The time constraint of each household member is then

$$(1 + \psi) l_{y,t} + l_{q,t} = 1,$$

where $\psi$ captures the training time needed to work as industrial labor. In order for workers to be indifferent between working in the two sectors, the relative wage must be given by

$$\frac{w_{y,t}}{w_{q,t}} = 1 + \psi > 1,$$

where $\psi$ also determines the industrial wage premium. The rest of the model is the same as before, except that the industrial labor share $l_{y,t}$ becomes

$$l_{y,t} = \frac{1}{1 + \psi} \left( 1 + \frac{\beta}{1 - \theta} c_t \right) \left( 1 - \frac{\eta}{A} \right),$$

\[18\] This would also need the absence of the subsistence requirement in agricultural consumption (i.e., $\eta = 0)$.
which is smaller than before unless \( \psi = 0 \). As a result, the smaller firm size \( \chi_l l_{y,t} \) delays the takeoff by increasing the threshold \( \chi_N \) given by

\[
\chi_N(\psi) = \frac{(1 + \psi) \left[ 1 + \beta \left( 1 + \frac{\rho - \lambda}{\mu} \frac{\delta \theta}{1 - \theta} \right) \right]^{\mu^{-1}/(1-\theta)}}{\mu - 1 - \delta(\rho - \lambda)} \left( 1 - \frac{\eta}{A} \right)^{-1}.
\] 

(48)

In the rest of this section, we present the simulated paths of the growth rate under different values of \( \psi \). The industrial labor share \( l_{y,t} \) increased to 40% in the early 19th century in the US, which implies that the subsistence ratio \( \eta/A \) decreased to 0.593 given \( \psi = 0 \). We pick an initial value \( \chi_0 \) such that the takeoff of the economy occurs in around 1840 given \( \psi = 0 \). Then we simulate the transition path of output growth rate \( g_t \) under different values of \( \psi \) given \( \eta/A = 0.593 \). Figure 9 presents the effects of \( \psi \) on the takeoff and economic growth, and shows a larger \( \psi \) delays takeoff but does not affect the steady-state growth rate. The effects of \( \psi \) on the timing of the takeoff (i.e., the activation of variety-expanding innovation) and the activation of quality-improving innovation depend on \( \psi \). Specifically, a larger \( \psi \) leads to a later activation of both types of innovation and a lower transitional growth rate but does not affect the steady-state growth rate.

![Figure 9: Dynamic effects of skill acquisition](image_url)

Figure 9 presents the simulated paths of the growth rate based on different values of skill acquisition.

### 6 Conclusion

In this study, we have developed a Schumpeterian growth model with an agricultural sector in which the size of firms in the industrial sector determines the endogenous takeoff of the economy. The primary goal of the exercise is to shed new light on the important role of
agriculture in a dynamic process that historians describe narratively as follows (e.g., Nurkse 1953): at the heart of industrialization, large improvements in agricultural productivity liberate labor from food production and reallocate it to industrial production. The secondary goal is to shed new light on the role of agriculture in explaining why countries with large populations, such as China and India, did not experience an early industrial takeoff. Our explanation is that the vast majority of their population being in agriculture did not contribute to firm size in the industrial sector.

More broadly, the model delivers analytical insights on the mechanism through which an agricultural revolution determines the timing of the endogenous takeoff. A sectoral reallocation that expands firm size in the industrial sector produces an earlier transition from stagnation to growth. Our quantitative analysis indicates that the decline in the agricultural share of the US workforce in the early 19th century contributed to the takeoff of the US economy. Without the reallocation of labor from agriculture to industry, the takeoff of the US economy would have been delayed by four decades. Finally, although our model is designed to explore the takeoff of early industrialized countries in the 19th century, it is also relevant for the subsequent takeoff of emerging markets that need to rely on technologies for economic development. These countries may not have to reinvent the wheel, but the transfer of technologies from the global technology frontier also depends on incentives and the size of industrial firms in these countries.
References


[34] Lewis, A., 1954. Economic development with unlimited supplies of labor. The Manchester School, 22, 139-91.


### Appendix A

#### Table A1: Effects of agricultural productivity on economic growth

<table>
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<tr>
<th></th>
<th>GDP growth (1)</th>
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<td>0.147***</td>
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<td></td>
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<td>(0.055)</td>
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<td>0.487</td>
<td>0.513</td>
<td>0.543</td>
<td>0.493</td>
<td>0.518</td>
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Note: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors are in parentheses. The dependent variable in columns (1)-(2) is the average annual growth rate of real GDP. The dependent variable in columns (3)-(4) is the average annual growth rate of real GDP per capita. The dependent variable in columns (5)-(6) is the average annual growth rate of real GDP per worker. Compared with odd columns, even columns add control variables including the log value of population, the log value of capital stock, a human capital index, the degree of openness (measured by the average ratio of export plus import to GDP), and the government consumption share of GDP.

**Equilibrium.** The equilibrium is a time path of allocations $\{a_t, q_t, c_t, Y_t, X_t, I_t, L_{q,t}, L_{y,t}\}$ and prices $\{r_t, w_t, p_t, P_t, V_t\}$ such that:

- the household consumes $\{q_t, c_t\}$ to maximize utility taking $\{r_t, w_t, p_t\}$ as given;
- competitive firms produce $Q_t$ to maximize profits taking $\{w_t, p_t\}$ as given;
- competitive firms produce $Y_t$ to maximize profits taking $\{w_t, P_t\}$ as given;
- monopolistic intermediate-good firms choose $\{P_t, I_t\}$ to maximize $V_t$ taking $r_t$ as given;
- entrants make entry decisions taking $V_t$ as given;
- the aggregate value of monopolistic firms equals the household’s wealth, $a_t L_t = N_t V_t$;
- the labor market clears, $L_{q,t} + L_{y,t} = L_t$;
- the market for the agricultural good clears, $q_t L_t = A L_{q,t}$;
- the market-clearing condition of the final good holds:

$$Y_t = c_t L_t + \mu N_t X_t,$$

which applies to the pre-industrial era, and

$$Y_t = c_t L_t + N_t (X_t + \phi Z_t + I_t) + \tilde{N}_t \delta X_t,$$

which applies to the industrial era.
Dynamic optimization of monopolistic firms. The current-value Hamiltonian for monopolistic firm $i$ is
\[
H_t(i) = \Pi_t(i) - I_t(i) + \zeta_t(i) \dot{Z}_t(i) + \xi_t(i) [\mu - P_t(i)],
\]
where $\xi_t(i)$ is the multiplier on $P_t(i) \leq \mu$. We substitute (9)-(11) into (A1) and derive
\[
\frac{\partial H_t(i)}{\partial P_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial P_t(i)} = \xi_t(i),
\]
\[
\frac{\partial H_t(i)}{\partial I_t(i)} = 0 \Rightarrow \zeta_t(i) = 1,
\]
\[
\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha \left\{ [P_t(i) - 1] \left[ \frac{\theta}{P_t(i)} \right]^{1/(1-\theta)} \frac{L_{y,t}}{N_t^{1-\sigma}} - \phi \right\} Z_t^{\alpha-1}(i) Z_t^{1-\alpha} = r_t \zeta_t(i) - \dot{\zeta}_t(i).
\]
If $P_t(i) < \mu$, then $\xi_t(i) = 0$. In this case, $\partial \Pi_t(i) / \partial P_t(i) = 0$ yields $P_t(i) = 1/\theta$. If the constraint on $P_t(i)$ is binding, then $\zeta_t(i) > 0$. In this case, we have $P_t(i) = \mu$. Therefore, we have proven (13). Then, the assumption $\mu < 1/\theta$ implies $P_t(i) = \mu$. Substituting (A3), (14) and $P_t(i) = \mu$ into (A4) and imposing symmetry yield (15), where $l_{y,t} \equiv L_{y,t}/L_t$. ■

Monopolistic profit in the pre-industrial era. In the pre-industrial era, the firm size $\chi_i l_{y,t}$ is so small that monopolistic firms with increasing returns technology cannot earn a positive profit; i.e.,
\[
\chi_i l_{y,t} < \phi \mu^{1/(1-\theta)}/(\mu - 1) \Rightarrow \Pi_t < 0,
\]
where $l_y$ is given in (28). In this case, the existing intermediate goods $N_0$ are produced by competitive firms that make zero profit. When $\chi_i l_y$ reaches $\phi \mu^{1/(1-\theta)}/(\mu - 1)$, we assume that the increasing returns technology is not yet deployed until $\chi_t$ reaches $\chi_N$; see Appendix B for the case without this assumption. ■

Dynamics of the consumption-output ratio in the industrial era. The value of assets owned by each member of the household is
\[
a_t = V_t N_t / L_t.
\]
If $n_t > 0$, then $V_t = \delta X_t$ in (17) holds. Substituting (17) and $\mu X_t N_t = \theta Y_t$ into (A5) yields
\[
a_t = \delta X_t N_t / L_t = (\theta / \mu) \delta Y_t / L_t = (\theta / \mu) \delta y_t,
\]
which implies that $a_t / y_t$ is constant. Substituting (A6), (3) and (8) into (2) yields
\[
\frac{\dot{y}_t}{y_t} = \frac{\dot{a}_t}{a_t} = r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - c_t - p_t q_t}{a_t} \frac{\dot{a}_t}{a_t} = \frac{\dot{c}_t}{c_t} + \rho - \lambda + \frac{(1-\theta) \mu}{\delta \theta} - \frac{\mu c_t}{\delta \theta y_t},
\]
where we have also used $w_t L_{q,t} = p_t Q_t$. Equation (A7) can be rearranged as
\[
\frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = \frac{\mu c_t}{\delta \theta y_t} - \frac{(1-\theta) \mu}{\delta \theta} - (\rho - \lambda),
\]
35
which shows that the dynamics of $c_t/y_t$ is characterized by saddle-point stability such that $c_t/y_t$ jumps to its steady-state value in (30) whenever $n_t > 0$. Then, substituting (30) into (22) yields $l^*_y$ in (31).

**Proof of Proposition 1.** In the pre-industrial era, the firm size $\chi_t l^*_y$ is not sufficiently large for horizontal and vertical innovation to be viable such that the variety growth rate and the quality growth rate are both zero (i.e., $n_t = z_t = 0$). In this case, the industrial labor share $l_y$ is given by (28) and the state variable $\chi_t = \theta^{1/(1-\theta)} L_t/N_0^{1-\sigma}$ increases at the population growth rate $\lambda$. Therefore, in the pre-industrial era, the dynamics of $\chi_t$ is simply

$$\dot{\chi}_t = \lambda \chi_t > 0. \quad \text{(A9)}$$

In the first phase of the industrial era, the firm size $\chi_t l^*_y$ becomes sufficiently large for horizontal innovation (but not vertical innovation) to be viable such that $n_t > 0$ and $z_t = 0$. In this case, the variety growth rate $n_t$ is given by (32), which is positive if and only if

$$\chi_t > \frac{\mu^{1/(1-\theta)} \phi / l^*_y}{\mu - 1 - \delta (\rho - \lambda)} \equiv \chi_N > \chi_0, \quad \text{(A10)}$$

where $l^*_y$ is given by (31) and increasing in $\chi$. Given $\chi_0$, the state variable $\chi_t$ increases at rate $\lambda$ until it reaches $\chi_N$; therefore, the time this process takes is

$$T_N = \frac{1}{\lambda} \log \left( \frac{\chi_N}{\chi_0} \right).$$

After reaching $\chi_N$, the dynamics of $\chi_t$ in (23) becomes

$$\dot{\chi}_t = [\lambda - (1 - \sigma)n_t] \chi_t = \frac{1 - \sigma}{\delta} \left\{ \phi \mu^{1/(1-\theta)} l^*_y - \left[ \mu - 1 - \delta \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] \chi_t \right\} > 0, \quad \text{(A11)}$$

which uses (32) for $n_t$.

In the second phase of the industrial era, the firm size $\chi_t l^*_y$ becomes sufficiently large for both horizontal and vertical innovation to be viable such that $n_t > 0$ and $z_t > 0$. In this case, the quality growth rate $z_t$ is positive if and only if

$$\chi_t > \frac{\Omega}{l^*_y} \equiv \chi_Z > \chi_N, \quad \text{(A12)}$$

where $l^*_y$ is given by (31) and the composite parameter $\Omega$ is defined as before:

$$\Omega \equiv \arg \text{solve} \left\{ \left[ \frac{\mu - 1}{\mu^{1/(1-\theta)}} \omega - \phi \right] \left[ \alpha - \frac{\mu^{1/(1-\theta)} \sigma}{\delta \omega} \right] = \rho - \sigma (\rho - \lambda) \right\}.$$
where we have used $\sigma_\mu^{1/(1-\theta)}/\left(\chi_t l_y^*\right) \cong 0$. Then, we can use $n_t$ to derive $z_t = g_t - \sigma n_t$.

Given (24), the autonomous dynamics of $\chi_t$ is stable and captured by (A9), (A11) and (A13). Given an initial value $\chi_0$, the state variable $\chi_t$ increases according to (A9) until $\chi_t$ reaches the first threshold $\chi_N$, which is decreasing in $A$ via $l_y^*$. Then, $\chi_t$ increases according to (A11) until $\chi_t$ reaches the second threshold $\chi_Z$, which is also decreasing in $A$ via $l_y^*$. Finally, $\chi_t$ increases according to (A13) until $\chi_t$ converges to its steady state

$$\chi^* = \frac{\mu^{1/(1-\theta)}}{l_y^*} \left(1 - \alpha\right) \phi - \left[\rho + \sigma \lambda/(1 - \sigma)\right] = \frac{\delta \theta}{\mu} n_t,$$

where $l_y^*$ is given in (31). Substituting (A14) into (36) yields $g^*$ in (26).

**R&D share of output.** In the first phase of the industrial era, the R&D share of output is

$$\frac{\text{R&D}_t}{Y_t} = \frac{\tilde{N}_t \phi}{Y_t} = \frac{\tilde{N}_t \phi}{Y_t} = \frac{\delta \theta}{\mu} n_t$$

which uses $n_t$ in (32). $l_y^*$ is increasing in $A$ and $\text{R&D}_t/Y_t$ is increasing in $l_y^*$ for a given $\chi_t$ such that the R&D share of output is increasing in $A$ for a given $\chi_t$. In the second phase of the industrial era, the R&D share of output is

$$\frac{\text{R&D}_t}{Y_t} = \frac{N_t I_t + \tilde{N}_t \phi}{Y_t} = \frac{N_t \phi}{Y_t} = \frac{\delta \theta}{\mu} n_t + \frac{N_t Z_t}{\chi l_y^* \phi}$$

which uses $\chi_t$ in (14), $Y_t$ in (19) and $n_t$ in (37). $l_y^*$ is increasing in $A$ and $\text{R&D}_t/Y_t$ is increasing in $l_y^*$ for a given $\chi_t$ such that the R&D share of output is increasing in $A$ for a given $\chi_t$. In summary, in both phases of the industrial era, the R&D share of output is increasing in the level of agricultural technology $A$ for a given $\chi_t$. ■
Appendix B

In this appendix, we extend the baseline model to allow for the possibility that in the pre-industrial era (i.e., \( n_t = z_t = 0 \)), monopolistic profits become positive (i.e., \( \Pi_t > 0 \)) before the takeoff occurs. When \( n_t = 0 \), the entry condition in (17) does not hold. However, the asset-pricing equation in (16) still holds and becomes

\[
 r_t = \frac{\Pi_t}{V_t} + \dot{V}_t, \tag{B1}
\]

where \( I_t = z_t = 0 \). We use (A5) and \( n_t = 0 \) to derive \( \dot{a}_t/a_t = \dot{V}_t/V_t - \lambda \) and then substitute this equation into (2) to obtain

\[
 \frac{\dot{V}_t}{V_t} - \lambda = \frac{\dot{a}_t}{a_t} = r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - p_t q_t - c_t}{a_t}. \tag{B2}
\]

Substituting (B1) into (B2) yields

\[
 c_t = \frac{\Pi_t}{V_t} a_t + w_t l_{y,t} = \frac{N_t}{L_t} \Pi_t + (1 - \theta) y_t, \tag{B3}
\]

where we have used (A5), \( w_t l_{q,t} = p_t q_t \) and \( w_t l_{y,t} = (1 - \theta) y_t \). Then, substituting (11) and \( P_t = \mu \) into (B3) yields

\[
 c_t = \frac{N_t X_t (\mu - 1 - \phi Z_t / X_t)}{L_t} + (1 - \theta) y_t = \theta \mu^{\theta/(1 - \theta)} \left( \frac{\mu - 1}{\mu^{1/(1 - \theta)}} - \frac{\phi}{\chi l_{y,t}} \right) y_t + (1 - \theta) y_t, \tag{B4}
\]

where the second equality uses \( \theta Y_t = \mu N_t X_t \) and (14). The consumption-output ratio is

\[
 \frac{c_t}{y_t} = \theta \mu^{\theta/(1 - \theta)} \left( \frac{\mu - 1}{\mu^{1/(1 - \theta)}} - \frac{\phi}{\chi l_{y,t}} \right) + 1 - \theta, \tag{B5}
\]

which would increase from (27) to (30) if the firm size \( \chi l_{y,t} \) increases from \( \phi \mu^{1/(1 - \theta)}/(\mu - 1) \) to \( \phi \mu^{1/(1 - \theta)}/[\mu - 1 - \delta (\rho - \lambda)] \). Finally, we substitute (B5) into (22) and manipulate the equation to obtain the equilibrium firm size:

\[
 \chi l_{y,t} = \frac{\beta \phi}{1 - \beta} \mu^{\theta/(1 - \theta)} + \left( 1 - \frac{\phi}{\mu} \right) \chi_t \left( 1 + \frac{\theta}{\mu - 1} \right), \tag{B6}
\]

which continues to be increasing in the level of agricultural technology \( A \).

Given that the dynamics of \( \chi_t \) is still given by (A9) in the pre-industrial era, the firm size \( \chi l_{y,t} \) gradually increases towards the threshold in (A10) to trigger the takeoff as before. The only difference is that as \( \chi_t \) increases overtime, \( l_{y,t} \) in (B6) is gradually decreasing from \( l_y \) in (28) to \( l^*_y \) in (31) (instead of jumping from \( l_y \) to \( l^*_y \) at the time of the takeoff). This additional dynamics in \( l_{y,t} \) gives rise to negative growth in the industrial output per capita before the takeoff, which is less realistic than the dynamics in the baseline model.
Appendix C (not for publication)

In this appendix, we present a simple second-generation Schumpeterian growth model with a separate R&D sector. The household sector and the agricultural sector are the same as before.

**Industrial production**

A representative competitive firm operates the assembly technology

\[ Y_t = N_t \exp \left( \frac{1}{N_t} \int_0^{N_t} \ln X_t(i) di \right), \]  

(C1)

where \( N_t \) is the endogenous mass of differentiated intermediate goods. Profit maximization yields the following conditional demand function for \( X_t(i) \):

\[ X_t(i) = \frac{Y_t}{N_tP_t(i)}, \]  

(C2)

where \( P_t(i) \) is the price of \( X_t(i) \).

**Variety growth**

Following Howitt (2000), we specify the law of motion for \( N_t \) as

\[ \dot{N}_t = \xi L_t \Leftrightarrow g_{N,t} \equiv \frac{\dot{N}_t}{N_t} = \xi \frac{L_t}{N_t}, \]  

(C3)

where \( \xi > 0 \) is an exogenous parameter. The growth rate of variety \( g_{N,t} \) is determined by \( L_t/N_t \). A stationary \( \dot{N}_t/N_t \) on the balanced growth path implies a stationary ratio \( L_t/N_t \), which in turn implies that the long-run growth rate of \( N_t \) is also \( \lambda \). Therefore, \( N_t \) is proportional to \( L_t \) in the long run, such that \( (L_t/N_t)^* = \lambda/\xi \). We assume that \( L_0/N_0 < \lambda/\xi \) in which case \( L_t/N_t \) and also \( g_{N,t} \) rise towards their steady states.

**Intermediate goods**

There is a continuum of monopolistic industries producing differentiated intermediate goods. The production function of the industry leader in industry \( i \in [0,N_t] \) is

\[ X_t(i) = z^{n_t(i)}L_{x,t}(i), \]  

(C4)

where the parameter \( z > 1 \) is the quality step size, \( n_t(i) \) is the number of quality improvements that have occurred in industry \( i \) as of time \( t \), and \( L_{x,t}(i) \) is manufacturing labor employed in industry \( i \). Given the productivity level \( z^{n_t(i)} \), the marginal cost of the leader in industry \( i \) is \( w_t/z^{n_t(i)} \). The profit-maximizing monopolistic price is

\[ P_t(i) = \mu \frac{w_t}{z^{n_t(i)}}, \]  

(C5)

where the markup \( \mu \in (1,z] \) is a policy parameter determined by the government. The wage payment is

\[ w_tL_{x,t}(i) = \frac{1}{\mu} P_t(i) X_t(i) = \frac{Y_t}{\mu N_t}, \]  

(C6)
which implies that \( L_{x,t}(i) = L_{x,t}/N_t \) is the same across industries and \( w_t L_{x,t} = Y_t/\mu \). The monopolistic profit is

\[
\Pi_t(i) = P_t(i)X_t(i) - w_t L_{x,t}(i) = \frac{\mu - 1}{\mu} \frac{Y_t}{N_t},
\]

(C7)

**R&D**

Equation (C7) shows that \( \Pi_t(i) = \Pi_t \). Therefore, the value of inventions is the same across industries such that \( V_t(i) = V_t \). The no-arbitrage condition that determines \( V_t \) is

\[
r_t = \frac{\Pi_t + \dot{V}_t - \sigma_t V_t}{V_t},
\]

(C8)

which states that the rate of return on \( V_t \) is equal to \( r_t \). The return on \( V_t \) is the sum of monopolistic profit \( \Pi_t \), capital gain \( \dot{V}_t \) and expected capital loss \( \sigma_t V_t \), where \( \sigma_t \) is the arrival rate of innovation.

Competitive entrepreneurs maximize profit by devoting final good as R&D input to perform innovation. The arrival rate of innovation is

\[
\sigma_t = \varphi \frac{R_t}{Z_t},
\]

(C9)

where \( \varphi > 0 \) is a productivity parameter, \( Z_t \) denotes aggregate technology and \( R_t \) is the R&D input of final good in each industry. Because of symmetry, the R&D input is the same across industries \( R_t(i) = R_t \). The free-entry condition of R&D is

\[
\sigma_t V_t = R_t \iff \varphi \frac{V_t}{Z_t} = 1.
\]

(C10)

**Aggregation**

Aggregate technology \( Z_t \) is defined as

\[
Z_t \equiv \exp \left( \frac{1}{N_t} \int_0^{N_t} n_t(i) \ln z \right) = \exp \left( \int_0^t \sigma_\omega d\omega \ln z \right),
\]

(C11)

which uses the law of large numbers.\(^{19}\) Differentiating the log of \( Z_t \) with respect to time yields the growth rate of technology given by

\[
z_t \equiv \frac{\dot{Z}_t}{Z_t} = \sigma_t \ln z.
\]

(C12)

Substituting (C4) into (C1) yields the aggregate production function given by

\[
Y_t = N_t \exp \left( \frac{1}{N_t} \int_0^{N_t} n_t(i) \ln z + \frac{1}{N_t} \int_0^{N_t} \ln L_{x,t}(i)di \right) = Z_t L_{x,t},
\]

(C13)

\(^{19}\)Here we make the usual assumption that a new variety enters with the average level of quality in the economy.
and the growth rate of industrial output per capita \( y_t \equiv Y_t / L_t = Z_t l_{x,t} \) is

\[
g_t \equiv \frac{\dot{y}_t}{y_t} = z_t + \frac{\dot{l}_{x,t}}{l_{x,t}} = \sigma_t \ln z + \frac{\dot{l}_{x,t}}{l_{x,t}}, \tag{C14}\]

where \( l_{x,t} \equiv L_{x,t} / L_t \) is the share of industrial labor. We define \( l_{q,t} \equiv L_{q,t} / L_t = 1 - l_{x,t} \) as the share of agricultural labor. The growth rate of industrial output is \( g_{Y,t} = z_t + \dot{L}_{x,t} / L_{x,t} = z_t + l_{x,t} / l_{x,t} + \lambda = g_t + \lambda \).

The value of an invention is

\[
V_t = \frac{\Pi_t}{r_t - g_{c,t} + \sigma_t} = \frac{\mu - 1}{\mu N_t \rho + g_{c,t} - (g_{Y,t} - g_{N,t}) + \sigma_t} \frac{Y_t}{Z_t} = \frac{\mu - 1}{\mu \rho - \lambda + g_{c,t} - g_t + g_{N,t} + \sigma_t} \frac{N_t}{L_{x,t}} = \frac{\mu - 1}{\mu \xi \rho - \lambda + g_{c,t} - g_t + g_{N,t} + \sigma_t} Z_t, \tag{C15}\]

which uses \( r_t = \rho + \dot{c}_t / c_t \) in (3) and states that the value of an invention is increasing in the share of industrial labor \( l_{x,t} \) and the variety growth rate \( g_{N,t} \) for a given innovation arrival rate \( \sigma_t \). Substituting (C15) into (C10) yields

\[
\phi \frac{\mu - 1}{\mu \xi} \frac{g_{N,t} l_{x,t}}{\rho - \lambda + g_{c,t} - g_t + g_{N,t} + \sigma_t} = 1, \tag{C16}\]

in which we have re-expressed the free-entry condition of R&D in (C10).

**Agricultural productivity and takeoff**

In the pre-industrial era, the population size is so small that \( g_{N,t} \) (or \( L_t / N_t \)) is not big enough for the free-entry condition of R&D to hold and for innovation to take place. Therefore, the growth rate of industrial output per capita is zero (i.e., \( g_t = 0 \)). In this case, the population size \( L_t \) is so small that \( g_{N,t} \) (or \( L_t / N_t \)) is not large enough to ensure that the free-entry condition in (C10) holds such that entrepreneurs do not perform R&D. In this case, all industrial output is devoted to consumption (i.e., \( c_t L_t = Y_t \)), which yields

\[
\frac{c_t}{y_t} = 1. \tag{C17}\]

The combination of labor demand from agriculture in (6) and (C6) yields

\[
p_t = \frac{Y_t}{\mu AL_{x,t}}. \tag{C18}\]

Substituting the agricultural technology in (5) and the relative price in (C18) into the demand function for \( q_t \) in (4) yields the industrial labor share \( l_{x,t} \) as

\[
l_x = \frac{1}{1 + \beta \mu} \left( 1 - \frac{\eta}{A} \right), \tag{C19}\]
which also uses $c_t = y_t$ in (C17). This says that the industrial labor share in the pre-industrial era is stationary and increasing in the level of agricultural technology $A$. The associated growth rate of industrial output per capita is

$$g_t = z_t + \frac{\dot{i}_{x,t}}{i_{x,t}} = 0$$  \hspace{1cm} (C20)

because $z_t = \dot{i}_{x,t}/i_{x,t} = 0$.

Quality-improving innovation activates when the growth rate of variety $g_{N,t}$ (or $L_t/N_t$) grows sufficiently large. Substituting the agricultural technology in (5) and the relative price in (C18) into the demand function for $q_t$ in (4) yields the industrial labor share $l_{x,t}$ as

$$l_{x,t} = \left(1 + \beta \mu \frac{c_t}{y_t}\right)^{-1} \left(1 - \frac{\eta}{A}\right),$$  \hspace{1cm} (C21)

which is also increasing in $A$ for a given $c_t/y_t$. It can be shown that in the industrial era, the consumption-output ratio and the industrial labor share are given by

$$\frac{c_t}{y_t} = \frac{\varphi g_{N,t} (1 - \eta/A) - \xi \sigma_t}{\varphi g_{N,t} (1 - \eta/A) + \beta \mu \xi \sigma_t},$$  \hspace{1cm} (C22)

$$l_{x,t} = \frac{\beta \mu \xi \sigma_t}{(1 + \beta \mu) \varphi g_{N,t}} + \frac{1}{1 + \beta \mu} \left(1 - \frac{\eta}{A}\right).$$  \hspace{1cm} (C23)

To understand better the role of agricultural technology, we can derive a threshold of $g_{N,t}$, or equivalently $L_t/N_t$, below which the free-entry condition of R&D in (C10) does not hold such that $\varphi V_t/Z_t < 1$ and $\sigma_t = 0$. From (C16), this threshold is given by

$$\bar{g}_N = \left(\rho - \lambda\right)\frac{\mu \xi}{\varphi (\mu - 1) l_x - \mu \xi^3},$$  \hspace{1cm} (C24)

where $l_x$ is given by (C19). At the moment before innovation activates, we have $\sigma_t = 0$, $c_t/y_t = 1$ in (C22) and $l_{x,t} = (1 - \eta/A)/(1 + \beta \mu)$ in (C23). Equation (C24) says that innovation will not occur until $g_{N,t}$ (or $L_t/N_t$) crosses the threshold $\bar{g}_N$ due to population growth. An increase in agricultural technology $A$ leads to an increase in the industrial labor share $l_x$, which in turn reduces the threshold $\bar{g}_N$ and leads to an earlier takeoff.

In summary, given $L_0/N_0 < \bar{g}_N/\xi < \lambda/\xi$, the economy begins in the pre-industrial era. When $L_t/N_t$ rises above the threshold $\bar{g}_N/\xi$, the economy enters the industrial era, in which the population size (and also $g_{N,t}$) becomes large enough to trigger innovation and for the free-entry condition of R&D to hold. The growth rate of industrial output per capita $g_t$ increases overtime due to the accelerating growth rate of variety $g_{N,t}$ until it converges to its steady-state value $g^*_N = \lambda$. At this point, the growth rate of industrial output per capita also converges to a steady state value.

References