Agricultural Revolution and Industrialization

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Abstract

This study explores how agricultural technology affects the endogenous takeoff of an economy in the Schumpeterian growth model. Due to the subsistence requirement for agricultural consumption, an improvement in agricultural technology reallocates labor from agriculture to the industrial sector. Therefore, agricultural improvement expands firm size in the industrial sector, which determines innovation and triggers an endogenous transition from stagnation to growth. Calibrating the model to data, we find that without the reallocation of labor from agriculture to the industrial sector in the early 19th century, the takeoff of the US economy would have been delayed by about four decades.

JEL classification: O30, O40
Keywords: agricultural technology, endogenous takeoff, innovation, economic growth

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> The spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it. [...] The introduction of the turnip [...] made possible a change in crop rotation which [...] brought about a tremendous rise in agricultural productivity. As a result, more food could be grown with much less manpower. Manpower was released for capital construction. The growth of industry would not have been possible without the turnip and other improvements in agriculture. Nurkse (1953, p. 52-53)

1 Introduction

According to Nurkse (1953), among many others, improvements in agricultural technology that released labor from agriculture were crucial for the industrial revolution. The industrial revolution in turn sparked centuries of sustained economic growth. History thus suggests that improvements in agricultural technology propagate pervasively throughout the economy and have momentous consequences that far exceed what one can see by looking at the sector in isolation.

Modern growth economics has investigated extensively the forces driving the growth process, typically building on the theory of endogenous technological change (Romer 1990). Since at its core the theory has dynamic increasing returns, it identifies the size of the market in which firms operate as a, if not the, crucial factor determining incentives to innovate. A spectacular application of these ideas is the Unified Growth Theory of Galor and Weil (2000); see also Galor (2005, 2011). Models in this tradition produce an endogenous takeoff and a transition from stagnation to growth. Following these two influential branches of growth economics, and to place industry solidly at the forefront of the analysis, Peretto (2015) has developed an IO-based Schumpeterian growth model with endogenous takeoff in which firm size determines the incentives to innovate; see, e.g., Cohen and Klepper (1996a, b) and Laincz and Peretto (2006) for evidence on this channel. We use this model to formalize Nurkse's idea and then investigate the role that agriculture plays in shaping the growth path of the economy. This strikes us as a first-order question in light of studies like, among others, Voigtlander and Voth (2006), Vollrath (2011) and Lagakos and Waugh (2013) that document the important implications of productivity differences in agriculture for economic development across countries.[1]

In the baseline Schumpeterian model, firm size is increasing in population size and decreasing in the number of firms. All else equal, a larger population causes an earlier transition from stagnation to growth. However, countries with large population, such as China and India, did not experience an early industrial takeoff, arguably because the vast majority of their population was in agriculture and thus not contributing to firm size in industry. To capture this idea we introduce an agricultural sector and investigate how it affects the takeoff and the subsequent growth pattern. We preserve the analytical tractability of the original model and derive a closed-form solution for the equilibrium growth rate throughout the entire transition from stagnation to growth. We find that higher agricultural productivity causes

[1] Dalgaard *et al.* (2020) provide empirical evidence that fishery productivity also has a persistent positive effect on economic development since pre-industrial times and causes an early takeoff of the economy.

an earlier takeoff with faster post-takeoff growth and final convergence to scale-invariant steady-state growth.

At the heart of the mechanism driving this result is a subsistence requirement for agricultural consumption, which yields that when agricultural productivity improves, labor moves from agriculture to industry. This reallocation alone can be sufficient to ignite industrialization. More generally, we have that: (i) for given agricultural technology, the model predicts a finite takeoff date with an associated wait time that is co-determined by initial firm size and decreasing in agricultural productivity; and (ii) for given firm size, the model identifies the minimum size of the improvement in agricultural technology—an Agricultural Revolution—that triggers an immediate Industrial Revolution. The combination of (i) and (ii) says that low agricultural productivity *delays* industrialization and creates a *temporary drag* on post-industrialization growth. The drag is only temporary and not permanent because our Schumpeterian growth model with endogenous market structure sterilizes the strong scale effect.

These properties provide a new lens for interpreting the empirical evidence. As mentioned, economies with large populations (e.g., China and India) failed to industrialize for many decades after smaller ones did (e.g., UK and USA). Growth theories based on increasing returns have problems explaining this fact. The typical argument is that they had bad institutions (e.g., Acemoglu and Robinson, 2012). Our analysis develops the complementary hypothesis that the allocation of labor to agriculture played an important role in determining their industrialization lags. Moreover, the scale-invariance of steady-state growth implies that while agricultural productivity does not affect income growth asymptotically, it has permanent and large effects on the overall time-profile of income. This property sheds new light on the debate about the role that agriculture (more generally, the primary sector) plays in shaping the dynamics of cross-country income differences.

We calibrate the model to US data to perform an illustrative quantitative analysis. The agricultural share of the US workforce was about 80% in the early 19th century (see Baten 2016) and decreased to about 70% in 1830 and 60% in 1840 (see Lebergott 1966 and Weiss 1986). We find that this reallocation of labor from agriculture to industry was a powerful push toward the takeoff of the US economy. In line with our analytical result, absent this reallocation the takeoff of the US economy would have occurred four decades later. Finally, we derive a formula that shows that a one-fifth increase in industrial employment reduces the wait time to takeoff by about a decade.

To illustrate further the properties of our model, in particular in a cross-country perspective, we develop three applications. The first explores the role of intellectual property rights as an example of a potentially important policy instrument. The second explores the role of a general-purpose technology as an example of extensions of the theoretical framework that speak to important issues debated in the literature. In this example, the model produces a *great-divergence followed by great-convergence* profile of growth rates due to two key properties: (i) the timing of takeoff depends on the level of the general-purpose technology and (ii) the steady-state growth rate does not depend on the level of the general-purpose technology because the model sterilizes the strong scale effect. This result illustrates the model's ability to capture rich pattern of cross-country variation of income paths over time. The third application explores the role of frictions in the reallocation of labor across agricultural and industrial sectors.

This study relates to the literature on endogenous technological change. Romer (1990) develops the first R&D-based growth model driven by the invention of new products (horizontal innovation). Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) develop the creative-destruction Schumpeterian growth model driven by the improvement of the quality of products (vertical innovation). Peretto (1994, 1998, 1999), Smulders (1994) and Smulders and van de Klundert (1995) combine the two dimensions of innovation to develop the creative-accumulation Schumpeterian growth model with endogenous market structure.[2] Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008, 2010) and Ang and Madsen (2011) provide early evidence for this class of models. Garcia-Macia *et al.* (2019) provide the latest evidence that growth is driven by the in-house innovation activity of existing firms. We contribute to this literature by incorporating an agricultural sector in the creative-accumulation model.

This study also relates to the literature on endogenous takeoff. The seminal contribution in this literature is Galor and Weil (2000). They develop unified growth theory and show that the quality-quantity trade-off in child rearing and the accumulation of human capital enable an economy to escape the Malthusian trap and experience an endogenous transition from stagnation to growth; see also Galor and Moav (2002), Galor and Mountford (2008), Galor *et al.* (2009) and Ashraf and Galor (2011). Galor (2011) provides a comprehensive review of unified growth theory. A recent study by Madsen and Strulik (2020) introduces land-biased technological change driven by education to the unified growth model and explores how it affects the endogenous takeoff of the economy and also the evolution of income inequality. We focus, instead, on the role of Schumpeterian technological progress driven by innovation as a complementary channel for the endogenous takeoff of the economy. Hansen and Prescott (2002) is another early study on endogenous takeoff. Gollin *et al.* (2002) introduce an agricultural sector into the Hansen-Prescott model, which features exogenous technological progress, to explore how agricultural technology affects industrialization. Our Schumpeterian growth model features multiple dimensions of innovation, which complement these perspectives by exploring the endogenous activation of endogenous technological progress. More generally, and in line with the overall thrust of this literature, we formalize the idea of Nurkse (1953), and the related big push idea of Murphy *et al.* (1989), in a very tractable dynamic general equilibrium model.[3] Our model allows us to obtain analytical results and then quantify the effects of agricultural technology on the industrialization path of the economy—a path consisting of an endogenous takeoff followed by post-takeoff accelerating growth, with final convergence from below to scale-invariant innovation-led steady-state growth.

The rest of this paper is organized as follows. Section 2 presents some stylized facts. Section 3 describes the Schumpeterian growth model. Section 4 explores the effects of agricultural technology. Section 5 performs a quantitative analysis. Section 6 concludes.

[2] Howitt (1999) combines the two dimensions of technology to develop a creative-destruction version of the theory.

[3] Our model can be viewed as a modern version of the dual-sector model in Lewis (1954).

2 Stylized facts

In this section, we highlight some stylized facts from cross-country data. First, we look at available historical data in the 19th century for the following early industrialized countries: Belgium, Britain, France, Japan, Netherlands, Spain, Sweden, and the United States.[4] We compute the average values of GDP per capita and the agricultural share of GDP in each country for each decade in the 19th century. Figure 1 plots the relationship between these two variables and shows a clearly negative relationship. In other words, around the time of its takeoff, when a country had a large agricultural sector in the economy, it tended to have a low income level. Then, as the size of its agricultural sector decreased over time, its income level tended to rise.

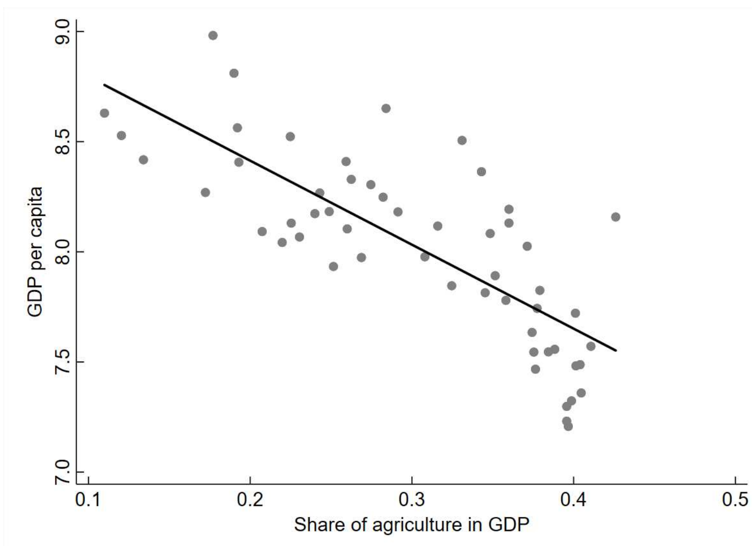

Figure 1: Agricultural share and GDP per capita

Data source: Roser (2013). GDP per capita is reported in log value. Agricultural share of GDP is defined as agricultural expenditures as a share of GDP. The data is from 1800 to 1899 based on the average value of each decade for the following early industrialized countries: Belgium, Britain, France, Japan, Netherlands, Spain, Sweden, and the United States.

A key component of our theoretical model is that the reduction in the size of the agricultural sector is driven by an increase in the level of agricultural productivity that leads to a reallocation of labor from agriculture to industry due to subsistence in agricultural consumption. Due to the lack of historical data on agricultural productivity, we look at recent data from developing countries, which only experienced industrialization recently or may even have yet to do so. Figure 2 plots the relationship between agricultural productivity and the agricultural share of labor in 1999-2001 and shows a clearly negative relationship

[4] According to Rostow (1956), most of these countries experienced their takeoff in the 19th century, with Britain being the first one in as early as 1783-1802 and Japan being relatively late in 1878-1900.

across countries.[5] In other words, a country that has a relatively high level of agricultural productivity tends to have a relatively low agricultural share of labor.

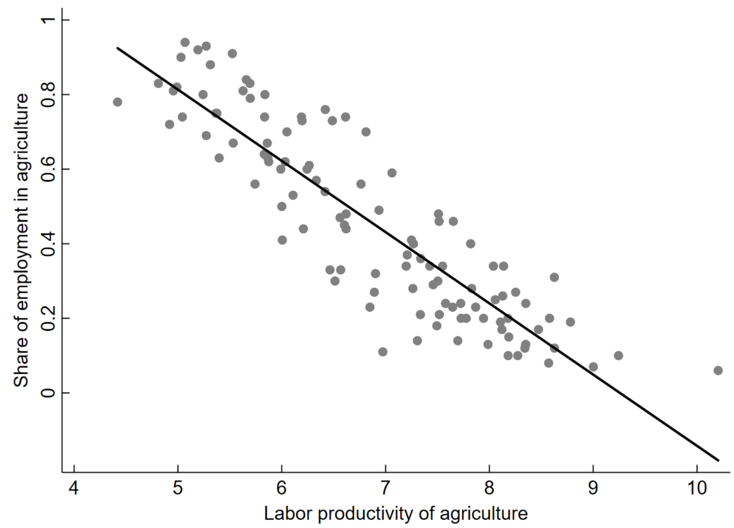

Figure 2: Agricultural productivity and labor share

Data source: FAO Statistical Yearbook by the Food and Agriculture Organization of the United Nations. Agricultural productivity is in log and defined as agricultural GDP per worker in the agricultural sector. Agricultural share of labor is defined as the share of workers in the agricultural sector. The sample is from 1999-2001 and for low-income and lower/upper middle-income countries as defined by the World Bank. The data that include high-income countries and from 1979-1981 and 1989-1991 show the same pattern.

Another important component of our theoretical model is that a higher level of agricultural productivity leads to a larger R&D share of output because the reallocation of labor from agriculture to industry increases industrial firm size, which in turn provides more incentives for R&D. Figure 3 plots the relationship between agricultural productivity and the R&D share of GDP in 1999-2001 and shows a clearly positive relationship for OECD countries.[6] In other words, a country that has a relatively high level of agricultural productivity tends to have a relatively large R&D share of GDP.

[5] See also Caselli (2005).

[6] Unfortunately, we only have R&D data for OECD countries.

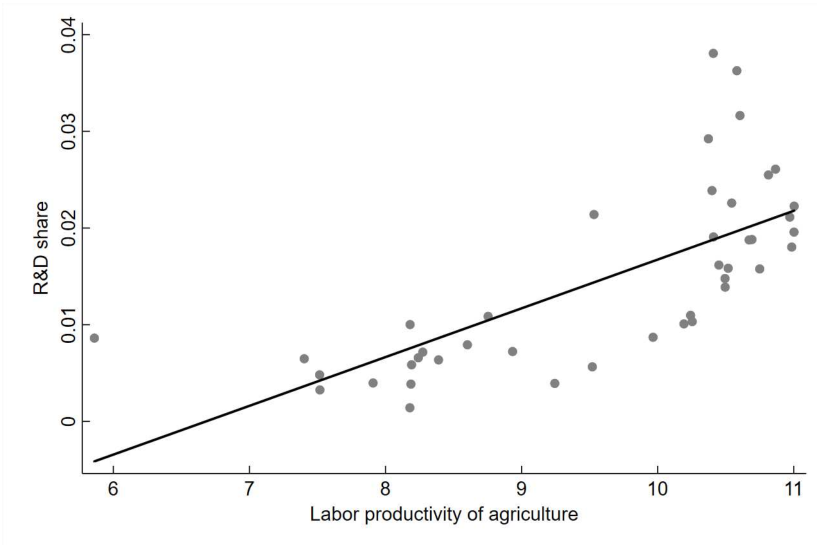

Figure 3: Agricultural productivity and R&D share

Data source: FAO Statistical Yearbook for agricultural productivity and OECD Statistics for the R&D share of GDP. Agricultural productivity is in log and defined as agricultural GDP per worker in the agricultural sector. R&D share of GDP is defined as R&D expenditures as a share of GDP. The sample is from 1999-2001 and for OECD countries.

These stylized facts support the mechanism in our theoretical model: higher agricultural productivity reallocates labor from agriculture to industry due to subsistence in agricultural consumption. The standard Schumpeterian mechanism then takes over: the reallocation of labor from agriculture to industry increases industrial firm size, which in turn provides more incentives for R&D and innovation. As stated in the introduction, we exploit this structure to investigate how agricultural productivity shapes the whole transition path of the economy from stagnation to growth.

Our theoretical model also predicts that a rise in agricultural productivity has a positive effect on economic growth at an early stage of economic development. Then, as the country develops over time, this positive effect on economic growth becomes smaller and eventually disappears. Therefore, we use the following empirical specification to examine our theory:[7]

$$g_{it} = \vartheta_1 A_{it} + \vartheta_2 A_{it} \times y_{it} + \vartheta_3 y_{it} + \Gamma \varkappa_{it} + \varrho_t + \epsilon_{it},$$

where g_{it} denotes the five-year average annual growth rate of real GDP, real GDP per capita or real GDP per worker in country i at wave t, A_{it} denotes the log level of agricultural productivity (defined as agricultural GDP per worker in the agricultural sector) in country i at wave t, and y_{it} is the log value of per capita GDP in country i in the initial year of wave t. Our theory predicts that $\vartheta_1 > 0$ and $\vartheta_2 < 0$. In other words, a higher level of

[7] See also Chu, Fan and Wang (2020) who examine empirically and theoretically the interaction between status-seeking culture and income level.

agricultural productivity generates a positive effect on economic growth at an early stage of economic development. As a country becomes more developed, this positive effect of agricultural productivity on economic growth becomes smaller.

$\varkappa_{it}$ is a vector of the average value of the following control variables in each wave: the log level of population, the log value of capital stock, a human capital index, the degree of openness (measured by the average ratio of export plus import to GDP), and the government consumption share of GDP. ϱ_t is the wave fixed effect, and ϵ_{it} is the error term. After merging data from the FAO Statistical Yearbook and the Penn World Table, we have a sample of up to 408 observations covering 149 countries. Table 1 provides the summary statistics of the variables.

Table 1: Summary statistics

variable	obs	mean	std. dev.	min	max
Growth of real GDP	408	0.046	0.054	-0.165	0.386
Growth of real GDP per capita	408	0.027	0.054	-0.202	0.351
Growth of real GDP per worker	402	0.024	0.053	-0.227	0.344
Log agricultural productivity	408	7.554	1.702	4.344	11.003
Log real GDP per capita	408	8.690	1.206	6.278	12.322
Log population (in 10 thousands)	408	6.613	1.900	1.426	11.793
Log capital stock	408	12.233	2.277	7.049	17.817
Human capital index	356	2.157	0.703	1.020	3.619
Openness	408	0.484	0.484	0.006	5.388
Government consumption share	408	0.192	0.093	0.007	0.664

Data source: FAO Statistical Yearbook for agricultural productivity and Penn World Table for others.

Table 2 reports the regression results.[8] The dependent variable in columns (1)-(2) is the average annual growth rate of real GDP, whereas the dependent variable in columns (3)-(4) is the average annual growth rate of real GDP per capita. The dependent variable in columns (5)-(6) is the average annual growth rate of real GDP per worker. Odd columns present the baseline results without additional controls. In even columns, we further control for additional explanatory variables. As expected, in all the columns, the coefficient on agricultural productivity is significantly positive, whereas the interaction term between agricultural productivity and the income level is significantly negative.

For example, in column (4), the estimated coefficient on agricultural productivity is 0.054, which is statistically significant at the 1% level, whereas the estimated coefficient on the interaction term is -0.005, which is also statistically significant at the 1% level. These estimates imply that for a country with minimal GDP per capita, increasing agricultural productivity by 1% is associated with an increase in the growth rate by 2.26% ((0.054-0.005*6.278)*1), which is statistically significant at the 1% level. For a country with average GDP per capita, increasing agricultural productivity by 1% is associated with an increase in the growth rate by 1.06% ((0.054-0.005*8.690)*1), which is also statistically significant at the 1% level. For a country with maximal GDP per capita, increasing agricultural productivity by 1% is associated with a decrease in the growth rate by 0.76% ((0.054-0.005*12.322)*1), which however

[8] Controlling country fixed effects yields the same signs but less significant coefficients given that we have only three waves of data. However, most of the regression coefficients still remain significant at least at the 5% level, except for column (2); see Table A1 in Appendix A.

is not statistically significant and has a p-value of 0.29. Therefore, the positive effect of agricultural productivity on economic growth becomes smaller and eventually disappears as the level of economic development increases.

Table 2: Effects of agricultural productivity on economic growth

	GDP growth		per capita GDP growth		per worker GDP growth	
	(1)	(2)	(3)	(4)	(5)	(6)
A_{it}	0.046***	0.041***	0.065***	0.054***	0.056***	0.045***
	(0.013)	(0.014)	(0.012)	(0.014)	(0.012)	(0.013)
$A_{it} \times y_{it}$	-0.004***	-0.004**	-0.006***	-0.005***	-0.005***	-0.004***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
y_{it}	0.014	-0.005	0.023**	0.000	0.014	-0.009
	(0.011)	(0.013)	(0.010)	(0.013)	(0.011)	(0.013)
Control variables	No	Yes	No	Yes	No	Yes
Wave fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	408	356	408	356	402	356
R-squared	0.159	0.205	0.207	0.277	0.179	0.258

Note: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors are in parentheses. The dependent variable in columns (1)-(2) is the average annual growth rate of real GDP. The dependent variable in columns (3)-(4) is the average annual growth rate of real GDP per capita. The dependent variable in columns (5)-(6) is the average annual growth rate of real GDP per worker. Compared with odd columns, even columns add control variables including the log value of population, the log value of capital stock, a human capital index, the degree of openness (measured by the average ratio of export plus import to GDP), and the government consumption share of GDP.

3 A Schumpeterian model of endogenous takeoff

The model features both the improvement of existing intermediate goods (vertical innovation) and the creation of new intermediate goods (horizontal innovation). Incentives to undertake these activities depend on firm size. Consequently, whether the economy experiences the endogenous takeoff depends on the size of the market for intermediate goods. In the original version (Peretto, 2015) the size of this market is proportional to the size of the labor force. By incorporating an agricultural sector with subsistence consumption, we disentangle the size of the market for intermediate goods from the size of the labor force and obtain a structure where the size of the intermediate sector, and therefore the size of intermediate firms, depends on the reallocation of labor from agriculture to industry.

3.1 Household

There is a representative household with $L_t = L_0 e^{\lambda t}$ identical members, where $L_0 = 1$ and $\lambda > 0$ is population growth rate. The household has Stone-Geary preferences

$$U_0 = \int_0^{\infty} e^{-(\rho-\lambda)t} \left[\ln c_t + \beta \ln(q_t - \eta)\right] dt, \tag{1}$$

where c_t and q_t denote, respectively, consumption per capita of an industrial and of an agricultural good. The parameter $\beta > 0$ determines the importance of industrial consumption relative to agricultural consumption. The latter features a subsistence requirement $\eta > 0$.[9] The parameter $\rho > \lambda$ is the subjective discount rate.

The household maximizes utility subject to the asset-accumulation equation

$$\dot{a}_t = (r_t - \lambda)a_t + w_t - c_t - p_t q_t, \tag{2}$$

where a_t is wealth per capita and r_t is the real interest rate. Each member of the household supplies inelastically one unit of labor to earn the wage w_t. Let the industrial good be our numeraire and p_t be the price of the agricultural good. The household sets:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho; \tag{3}$$

$$q_t = \eta + \frac{\beta c_t}{p_t}. \tag{4}$$

The first equation summarizes the intertemporal consumption-saving decision as the growth path of industrial consumption c_t. The second summarizes the intratemporal allocation of expenditure across the two goods as the demand for agricultural consumption q_t.

3.2 Agriculture

We follow Lagakos and Waugh (2013) and model agriculture as a competitive sector operating a linear technology

$$Q_t = AL_{q,t}, \tag{5}$$

where the parameter $A > \eta$ is labor productivity and $L_{q,t}$ is employment in agriculture. Profit maximization yields

$$w_t = p_t A, \tag{6}$$

which says that the wage rate in agriculture is equal to the marginal product of labor.

We omit land for simplicity. Including land produces the same qualitative results about endogenous takeoffs but the analysis is much more algebra-intensive. Vollrath (2011), among many others, studies the effects of land intensity and labor intensity in agriculture on industrialization. Our results are in line with the general insights produced by that work.

[9] This is a common feature of structural change models (see, e.g., Matsuyama (1992), Laitner (2000) and Kongsamut *et al.* (2001)), which study the implications of structural change for long-run (i.e., asymptotic) growth but not for endogenous takeoff. See Herrendorf *et al.* (2014) for an excellent survey of this literature and Herrendorf *et al.* (2020) for a recent contribution.

3.3 Industrial production

A representative competitive firm operates the assembly technology

$$Y_t = \int_0^{N_t} X_t^{\theta}(i) \left[Z_t^{\alpha}(i) Z_t^{1-\alpha} L_{y,t} / N_t^{1-\sigma} \right]^{1-\theta} di, \tag{7}$$

where $\{\theta, \alpha, \sigma\} \in (0,1)$. The key features are: (i) there is a continuum of non-durable differentiated intermediate goods $i \in [0, N_t]$; (ii) $X_t(i)$ is the quantity of intermediate good i; (iii) the productivity of good i depends on its own quality $Z_t(i)$ and on average quality $Z_t \equiv \int_0^{N_t} Z_t(j)\, dj / N_t$; and (iv) overall productivity in assembly depends on product variety N_t. Two parameters regulate technological spillovers: α captures the private return to quality and hence $1-\alpha$ determines vertical technological spillovers; and $1-\sigma$ captures a congestion effect of product variety so that the social return to variety is σ.

Let $P_t(i)$ be the price of $X_t(i)$. Profit maximization yields the conditional demands:

$$L_{y,t} = (1-\theta) \frac{Y_t}{w_t}; \tag{8}$$

$$X_t(i) = \left(\frac{\theta}{P_t(i)} \right)^{1/(1-\theta)} \frac{Z_t^{\alpha}(i) Z_t^{1-\alpha} L_{y,t}}{N_t^{1-\sigma}}. \tag{9}$$

These expressions yield that the competitive industrial firm pays $(1-\theta) Y_t = w_t L_{y,t}$ for industrial labor and $\theta Y_t = \int_0^{N_t} P_t(i) X_t(i)\, di$ for intermediate goods.

3.4 Intermediate goods and in-house R&D

A monopolistic firm produces differentiated intermediate good i with a linear technology that requires $X_t(i)$ units of the industrial good to produce $X_t(i)$ units of intermediate good i at quality $Z_t(i)$, that is, the marginal cost of production is one. The firm also pays $\phi Z_t^{\alpha}(i) Z_t^{1-\alpha}$ units of the industrial good as a fixed operating cost. To improve the quality of its product, the firm devotes $I_t(i)$ units of the industrial good to in-house R&D. The innovation technology is

$$\dot{Z}_t(i) = I_t(i). \tag{10}$$

The firm's gross profit (i.e., profit before-R&D) is

$$\Pi_t(i) = [P_t(i) - 1] X_t(i) - \phi Z_t^{\alpha}(i) Z_t^{1-\alpha}. \tag{11}$$

The value of the monopolistic firm is

$$V_t(i) = \int_t^{\infty} \exp\left(-\int_t^s r_u du \right) [\Pi_s(i) - I_s(i)]\, ds. \tag{12}$$

The monopolistic firm maximizes (12) subject to (9) and (10).

We solve this dynamic optimization problem in Appendix A and find that the unconstrained profit-maximizing markup ratio is $1/\theta$. However, we assume that competitive fringe firms can produce $X_t(i)$ at quality $Z_t(i)$ but at the higher marginal cost $\mu \in (1, 1/\theta)$.[10] The monopolistic firm then sets

$$P_t(i) = \min\{\mu, 1/\theta\} = \mu \tag{13}$$

and prices fringe firms out of the market. The optimization problem also delivers the firm's rate of return to innovation,

$$r_t^q(i) = \alpha \frac{\Pi_t(i)}{Z_t(i)} = \alpha \left[(\mu - 1) \frac{X_t(i)}{Z_t(i)} - \phi Z_t^{\alpha-1}(i) Z_t^{1-\alpha} \right],$$

which is linear in quality-adjusted firm size $X_t(i)/Z_t(i)$. This property is at the heart of the mechanism that we study: incentives to innovate depend on quality-adjusted firm size, which in turn depends on the size of the market. We now turn to this component of the logical chain.

In models of this class the equilibrium of the market for intermediate goods is symmetric, that is, intermediate firms start with the same initial quality $Z_0(i) = Z_0$ for $i \in [0, N_t]$ and, facing a symmetric environment, make identical decisions. Consequently, they grow at the same rate and symmetry holds at any point in time. Using the limit price (13), quality-adjusted firm size is

$$\frac{X_t(i)}{Z_t(i)} = \frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_{y,t}}{N_t^{1-\sigma}} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}} \frac{L_{y,t}}{L_t}.$$

We define the industrial employment share $l_{y,t} \equiv L_{y,t}/L_t$ and the composite variable

$$\chi_t \equiv \theta^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}. \tag{14}$$

This variable compresses the two state variables L_t (population) and N_t (mass of firms) to the ratio $L_t/N_t^{1-\sigma}$ and, therefore, makes the analysis of the model's dynamics simple.

With this notation, quality-adjusted firm size becomes

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{\chi_t}{\theta^{1/(1-\theta)}} \frac{L_{y,t}}{L_t} = \frac{\chi_t l_{y,t}}{\mu^{1/(1-\theta)}}.$$

Accordingly, the rate of return to innovation is

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} \chi_t l_{y,t} - \phi \right]. \tag{15}$$

To summarize, this structure captures two sides of the idea explored in this paper. First, agricultural employment implies $l_{y,t} < 1$ and thus reduces firm size in the intermediate sector and thereby depresses incentives to innovate. Second, the reallocation of labor from agriculture to industrial production is an essential component of the dynamics of takeoff and subsequent sustained growth: as $l_{y,t}$ rises, the return to innovation rises faster than in the absence of structural change.

[10] Specifically, we allow for diffusion of knowledge from monopolistic firms to fringe firms that enables the latter to constrain the pricing behavior of the former. This structure disentangles markups from the technological parameter θ.

3.5 Entrants

Upon payment of a sunk cost of δX_t, $\delta > 0$, units of the industrial good, a new firm enters the market and offers a new differentiated good of average quality. This structure preserves the symmetry of the intermediate goods market equilibrium at all times. The asset-pricing equation governing the value of firms (old and new) is

$$r_t = \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{16}$$

Entry is positive when the free-entry condition holds, i.e., when

$$V_t = \delta X_t. \tag{17}$$

Substituting (9) and (13) into (11) and then using the resulting expression, (10), (16) and (17) yield the return to entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\delta} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{\chi_t l_{y,t}} \right) + z_t + \frac{\dot{\chi}_t}{\chi_t} + \frac{\dot{l}_{y,t}}{l_{y,t}}, \tag{18}$$

where $z_t \equiv \dot{Z}_t / Z_t$ is the growth rate of average quality.

3.6 Aggregation

We define the general equilibrium in Appendix A. Substituting (9) and (13) into (7) yields the reduced-form representation of industrial production:

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{\sigma} Z_t L_{y,t}. \tag{19}$$

The associated growth rate of industrial output per capita, $y_t = Y_t / L_t$, is

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \sigma n_t + z_t + \frac{\dot{l}_{y,t}}{l_{y,t}}. \tag{20}$$

This growth rate has three components: (i) the growth rate of the variety of intermediate goods, $n_t \equiv \dot{N}_t / N_t$; (ii) the growth rate of the average quality of intermediate goods, z_t; (iii) the growth rate of the industrial labor share $l_{y,t}$.

3.7 Labor allocation

The combination of labor demand from agriculture (6) and industry (8) yields

$$p_t = \frac{(1-\theta) Y_t}{A L_{y,t}}. \tag{21}$$

Substituting the agricultural technology (5) and the relative price (21) in the demand function for q_t in (4) yields the industrial labor share $l_{y,t}$ as

$$l_{y,t} = \left(1 + \frac{\beta}{1-\theta}\frac{c_t}{y_t}\right)^{-1}\left(1 - \frac{\eta}{A}\right). \tag{22}$$

This equation says that for given consumption-output ratio c_t/y_t, the industrial labor share $l_{y,t}$ is *increasing* in A (and conversely, the agricultural labor share $l_{q,t} \equiv L_{q,t}/L_t$ is *decreasing* in A as in Figure 2) if and only if $\eta > 0$. This property produces sectoral reallocation whereby an improvement in the agricultural technology releases labor from agriculture to the industrial sector.

4 Agriculture, takeoff and long-run growth

We now develop the main analytical insight of the paper. We first show that the economy begins in a pre-industrial era in which the growth rate of industrial output per capita is zero. It then enters the industrial era, which consists of two phases. In the first, only the development of new products marketed by new firms drives the growth rate of industrial output per capita. In the second, product-quality improvement by existing firms adds its contribution and produces an acceleration of the growth rate.[11] The economy finally converges to constant growth of income per capita fueled by both vertical and horizontal innovation.

Next, we show that agriculture shapes this process of phase transitions and convergence: agricultural productivity determines the timing of the first phase transition, the endogenous takeoff of the economy, and of the second phase transition, the activation of vertical innovation. This *timing effect* has momentous consequences: although agricultural productivity does not affect steady-state growth due to the model's sterilization of the scale effect, it has permanent and large effects on the economy's time-profile of income. This property sheds new light on the debate about the role that agriculture plays in shaping the dynamics of cross-country income differences.

4.1 Global dynamics

The equilibrium law of motion of the state variable $\chi_t \equiv \theta^{1/(1-\theta)} L_t / N_t^{1-\sigma}$ defined in (14) is

$$\dot{\chi}_t = \left[\lambda - (1-\sigma)n_t\right]\chi_t, \tag{23}$$

where the variety growth rate n_t is either zero or an increasing function of χ_t (see Appendix A). The dynamics of χ_t in turn determines the dynamics of the economy, which converges to the balanced growth path if the following condition holds:

$$\delta\phi > \frac{1}{\alpha}\left[\mu - 1 - \delta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right] > \mu - 1. \tag{24}$$

[11] We consider the realistic case in which product creation happens before quality improvement. See Peretto (2015) for details on this property of the baseline growth model.

In this case, given an initial χ_0, the state variable χ_t increases over time and converges to

$$\chi^* = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \sigma\lambda/(1-\sigma)\right]} \frac{1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)}{\left(1 - \frac{\eta}{A}\right)}$$

as the variety growth rate converges to $n^* = \lambda/(1-\sigma)$. Steady-state firm size and income per capita growth are (see Appendix A):

$$\chi^* l_y^* = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \sigma\lambda/(1-\sigma)\right]}; \tag{25}$$

$$g^* = \alpha\left[(\mu-1)\frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \sigma\lambda/(1-\sigma)\right]} - \phi\right] - \rho > 0. \tag{26}$$

This structure has two properties worth stressing.

First, the existence condition (24) consists of two inequalities that ensure that the steady state χ^* exists. To establish whether χ^* is the attractor of the model's dynamics, we need to investigate the conditions for the occurrence of the two phase transitions discussed above. We do so in the remainder of this section, placing the role of agriculture at the center of the investigation. The exercise shows that the two inequalities also provide the condition for the occurrence of the second phase transition. The two conditions in (24) are then jointly sufficient for the full transition to the steady state χ^*.

Second, (26) says that steady-state growth is independent of the sectoral allocation of labor due to the scale-invariance of the Schumpeterian growth model with endogenous market structure. This property is central to the paper's insight. As we investigate the role of agriculture in driving the phase transitions, we find that because steady-state growth is invariant to A, cross-country differences in agricultural productivity produce a pattern of *divergence-convergence*, namely: (i) differences in A generate differences in growth that are solely due to differences in the timing of takeoff; (ii) such differences are only temporary and eventually vanish so that all else equal, there is long-run growth equalization. It is worth stressing that differences in growth rates vanish, not differences in income levels. That is, differences in agricultural productivity imprint themselves on income levels and are amplified by the initial divergence in income dynamics caused by the different takeoff times. The amplification can be large since it leverages differences in growth rates that last several decades due to the model's slow convergence to the steady state.

4.2 The pre-industrial era

In the pre-industrial era, there are two possible configurations of the intermediate-good sector. First, initially demand for each intermediate good is so small that a would-be monopolist operating the increasing-returns technology would earn negative profit (see Appendix A for details). Since the increasing-returns technology is not viable, the existing N_0 intermediate goods are produced by competitive firms that do not innovate and make zero profit at the equilibrium price $P_t(i) = \mu$. Anticipating this, entrepreneurs are not willing to pay the sunk entry cost and thus there is no variety innovation either. Initially, therefore, all technologies in this economy exhibit constant returns to scale. Our variable χ_t, the total output of the competitive firms producing each existing intermediate good, grows only because of exogenous population growth (i.e., $\dot{\chi}_t/\chi_t = \lambda$). In this era, more precisely, the initial number of intermediate goods N_0 is exogenous and predetermined while the market structure in each product line, i.e., the number of firms and the size of each firm, is indeterminate.

The second possible configuration occurs when the size of the market for intermediate goods grows sufficiently large that a would-be monopolist operating the increasing-returns technology could earn positive profit. We assume, however, that although the increasing-returns technology is now viable, agents do not deploy it yet because doing so requires payment of the sunk entry cost.[12] The idea is that only innovation, in this case a process innovation, allows a new firm to monopolize an existing market. Hence, the pre-industrial era ends only when the present value of monopolistic firms is sufficiently large that the free-entry condition (17) holds.

As a result of the pre-industrial market structure outlined above, in the pre-industrial era the household's industrial consumption is $c_t = w_t l_{y,t} = (1-\theta)y_t$, which yields

$$\frac{c_t}{y_t} = 1 - \theta. \tag{27}$$

Substituting (27) into (22) yields

$$l_y = \frac{1}{1+\beta}\left(1 - \frac{\eta}{A}\right). \tag{28}$$

This says that the industrial labor share in the pre-industrial era is stationary and increasing in agricultural productivity A. From (20), the growth rate of industrial output per capita is

$$g_t = \sigma n_t + z_t + \frac{\dot{l}_{y,t}}{l_{y,t}} = 0 \tag{29}$$

because $n_t = z_t = \dot{l}_{y,t}/l_{y,t} = 0$ in the pre-industrial era.

4.3 The industrial era: phase 1

Horizontal innovation (but not yet vertical innovation) activates when firm size $\chi_t l_{y,t}$ grows sufficiently large. In this phase, we have a positive variety growth rate $n_t > 0$ and a zero quality growth rate $z_t = 0$. When the free-entry condition holds, the consumption-output ratio c_t/y_t and the industrial labor share $l_{y,t}$ jump to the steady-state values (derivation in Appendix A):

$$\left(\frac{c}{y}\right)^* = \frac{(\rho-\lambda)\delta\theta}{\mu} + 1 - \theta; \tag{30}$$

$$l_y^* = \frac{1}{1+\beta\left(1+\frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)}\left(1 - \frac{\eta}{A}\right). \tag{31}$$

[12] In Appendix B, we consider an extension of the model that does not rely on this assumption and show that the dynamics are less realistic.

In the first phase of the industrial era, the growth rate of industrial output per capita becomes $g_t = \sigma n_t$ because $z_t = 0$. The growth rate of product variety n_t can be derived as

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta}\left(\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi}{\chi_t l_y^*}\right) + \lambda - \rho > 0, \tag{32}$$

which uses $\rho + g_t = \rho + \sigma n_t = r_t = r_t^e$ in (18). From (32), n_t is positive if and only if

$$\chi_t > \frac{\left[1+\beta\left(1+\frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\mu^{1/(1-\theta)}\phi}{\mu-1-\delta(\rho-\lambda)}\left(1-\frac{\eta}{A}\right)^{-1} \equiv \chi_N. \tag{33}$$

Note that n_t is increasing in agricultural technology A via the industrial labor share l_y^*, which is increasing in A, and increasing in the state variable χ_t so that (23) describes a stable process.

The interpretation of this property in terms of the baseline growth model is that there exists a threshold of χ_t below which the economy operates under pre-industrial conditions and firm size grows only because of exogenous population growth. Eventually, the economy crosses the threshold χ_N but it takes

$$T_N = \frac{1}{\lambda}\log\left(\frac{\chi_N}{\chi_0}\right) \tag{34}$$

years to achieve such takeoff (derivation in Appendix A). Since χ_N is decreasing in A, the combination of (32) and (34) says that economies with higher agricultural productivity A take off earlier and exhibit faster post-takeoff growth than economies with lower A.

An alternative interpretation is as follows. We write (33) as

$$A > \frac{\eta}{1-\frac{1}{\mu-1-\delta(\rho-\lambda)}\left[1+\beta\left(1+\frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\mu^{1/(1-\theta)}\phi/\chi_t}. \tag{35}$$

This now says that, given χ_t, when the agricultural technology A is below this critical threshold the economy remains in the pre-industrial equilibrium. However, if A rises above the threshold, the economy takes off immediately. In this sense, we have a condition determining when and how an Agricultural Revolution can trigger the Industrial Revolution. The two interpretations are complementary. The first holds A constant and uses the model's dynamics to compute the wait time to industrialization, i.e., how long it takes for χ_t to go from its initial value χ_0 to the threshold value χ_N. As shown, the wait time is lower the larger is A. The second interpretation fixes χ_t and asks how large an improvement in A is needed to trigger immediately the activation of Schumpeterian innovation. Equation (35) says that economies with larger firms require smaller agricultural improvements to take off.

The important component of this mechanism is that when the agricultural technology improves, the economy reallocates labor from the agricultural sector to the industrial sector and that *this reallocation alone can be sufficient to ignite industrialization.* Figure 4 presents the time path of the growth rate g_t when A increases at time t and causes the economy to escape the pre-industrial era and enter the first phase of the industrial era. The figure highlights the two complementary interpretations discussed above: (i) for a given A, the model predicts a finite takeoff date with an associated wait time determined by the initial condition χ_0 (equivalently, initial firm size $\chi_0 l_y$); and (ii) for a given firm size $\chi_t l_y^*$, the model identifies the minimum size of the improvement in A—an Agricultural Revolution—that triggers an immediate Industrial Revolution. The combination of (i) and (ii) says that low agricultural productivity *delays* industrialization and creates a *temporary drag* on post-industrialization growth. The drag is only temporary because our Schumpeterian growth model with endogenous market structure sterilizes the scale effect.

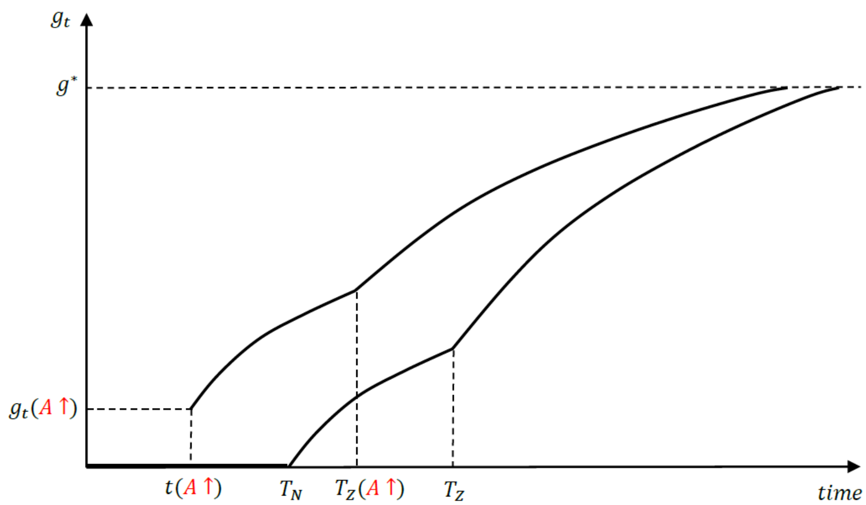

Figure 4: Agricultural revolution and industrialization

In Figure 4, agricultural productivity A increases at time t, which leads to an immediate takeoff and an earlier activation of vertical innovation.

Finally, a related implication is that the R&D share of output (i.e., $\dot{N}_t \delta X_t / Y_t$) is increasing in the level of agricultural technology A for a given χ_t (see Appendix A). This positive relationship between agricultural technology and the R&D share of output is consistent with the data in Figure 3.

4.4 The industrial era: phase 2

When firm size $\chi_t l_y^*$ is sufficiently large, horizontal innovation and vertical innovation occur simultaneously. In this case, we have a positive variety growth rate $n_t > 0$ and a positive quality growth rate $z_t > 0$. This is the second phase of the industrial era. Given active horizontal innovation, the consumption-output ratio and the industrial labor share remain at the steady-state values (30)-(31).

The growth rate of industrial output per capita can be derived from $\rho + g_t = r_t = r_t^q$ in (15) as

$$g_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} \chi_t l_y^* - \phi \right] - \rho > 0, \tag{36}$$

which is increasing in agricultural technology A via the industrial labor share l_y^* and increasing in firm size $\chi_t l_y^*$. The growth rate of variety can be derived from $\rho + g_t = \rho + \sigma n_t + z_t = r_t = r_t^e$ in (18) and is given by

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta}\left(\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{\chi_t l_y^*}\right) + \lambda - \rho > 0, \tag{37}$$

where z_t can be derived from (36), (37) and $g_t = \sigma n_t + z_t$ as

$$z_t = \left(1 - \frac{\mu^{1/(1-\theta)}\sigma}{\delta \chi_t l_y^*}\right)^{-1}\left\{\left[\frac{\mu-1}{\mu^{1/(1-\theta)}}\chi_t l_y^* - \phi\right]\left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta\chi_t l_y^*}\right] - \rho + \sigma\left(\rho - \lambda\right)\right\}.$$

The entry process in (37) determines the dynamics of χ_t (derivation in Appendix A).

Given (24), this transition to phase 2 occurs when χ_t rises above the following threshold:

$$\chi_t > \left[1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\Omega\left(1 - \frac{\eta}{A}\right)^{-1} \equiv \chi_Z > \chi_N, \tag{38}$$

where

$$\Omega \equiv \underset{\omega}{\arg\,\text{solve}}\left\{\left[\frac{\mu-1}{\mu^{1/(1-\theta)}}\omega - \phi\right]\left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta\omega}\right] = \rho - \sigma\left(\rho - \lambda\right)\right\}.$$

As in the previous case, the standard interpretation of this condition is that for a given A, there exists a threshold of firm size above which firms invest in-house and growth accelerates due to quality innovation.

The complementary interpretation of the threshold follows from rewriting (38) as

$$A > \frac{\eta}{1 - \left[1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\Omega/\chi_t}. \tag{39}$$

This says that for a given χ_t, a sufficiently large improvement in the level of agricultural technology A can cause the immediate activation of quality innovation if it causes the threshold χ_Z to fall below χ_t. In this era, the R&D share of output (i.e., $(N_t I_t + \dot{N}_t \delta X_t)/Y_t$) is also increasing in the level of agricultural technology A for a given χ_t (see Appendix A). This positive relationship between agricultural technology and the R&D share of output is once again consistent with the data in Figure 3.

Finally, the economy converges to a constant growth rate of industrial output per capita fueled by both vertical and horizontal innovation. Firm size $\chi^* l_y^*$ converges to its steady-state value in (25), while the growth rate g^* converges to its steady-state value in (26). Both $\chi^* l_y^*$ and g^* are independent of agricultural technology A.

4.5 Summary and discussion

We can summarize our main global dynamics result as follows.

Proposition 1 *Given (24) and $\chi_0 < \chi_N < \chi_Z$, the economy begins in the pre-industrial era with no innovation of any kind. It then experiences the endogenous takeoff and enters the first phase of the industrial era where horizontal innovation alone fuels industrial growth. Finally, the economy enters the second phase of the industrial era with both vertical and horizontal innovation and converges to the balanced growth path. Agricultural productivity A determines the timing of the two-phase transitions but does not affect the steady-state growth rate of the economy. Specifically, economies with higher agricultural productivity take off earlier and exhibit temporarily faster post-takeoff growth than economies with lower agricultural productivity, eventually converging to the scale-invariant growth rate g^*.*

Proof. See Appendix A. ■

These properties are important when looking at the data. As mentioned, economies with large populations (e.g., China and India) failed to industrialize for decades after smaller ones did (e.g., UK and USA). Growth theories based on increasing returns have obvious problems explaining this fact. Our analysis says that their allocation of labor to an unproductive agricultural sector played an important role in determining their industrialization lags both in terms of the timing of the takeoff and of the steepness of the post-takeoff income profile. The scale-invariance of steady-state growth implies that while agricultural productivity does not affect income growth asymptotically, it has permanent and large effects on the overall time-profile of income.

We see our results on the role of agriculture as part of a very broad agenda. Our emphasis is on analytical transparency and dynamics with phase-transitions. We see the latter as essential to understanding things like the timing of takeoff both in the time-series and the cross-sectional dimensions. The literature is rich in discussions of "timing" that, however, rarely provide details that allow one to see exactly how specific parameters determine it. Our contribution, in contrast, aims at identifying precisely the channels that run from fundamentals to timing, indeed to the overall shape of the transition path.

5 Quantitative analysis

In this section we complement our analytical work with quantitative exercises designed to illustrate some attractive properties of our framework. We begin with a simple counterfactual and then provide three applications. The first explores the role of intellectual property rights as an example of a potentially important policy instrument. The second explores the role of a general-purpose technology as an example of possible extensions of the theoretical framework that speak to important issues debated in the literature. The third explores the role of frictions in the reallocation of labor across agricultural and industrial sectors.

5.1 A simple counterfactual and model calibration

In the early 19th century, the agricultural share of the US workforce decreased from about 80% to 60%; see Baten (2016), Lebergott (1966) and Weiss (1986). We perform a counterfactual analysis to assess how large an effect this reallocation of labor from agriculture to industry had on the takeoff of the US economy.

Recall that firm size, which determines the timing of the takeoff, is

$$\chi_t l_{y,t} = \chi_t (1 - l_{q,t}),$$

where $l_{q,t} \equiv L_{q,t}/L_t$ is the agricultural labor share. The takeoff occurs when χ_t reaches the threshold χ_N. In terms of firm size we have

$$\chi_t l_{y,t} > \chi_N l_y^*.$$

A decrease in the agricultural labor share $l_{q,t}$ from 80% to 60% yields an increase in the industrial labor share $l_{y,t}$ from 20% to 40%.[13] This expands firm size $\chi_t l_{y,t}$ by a factor of 2 for given χ_t. In the pre-industrial era the state variable χ_t grows at rate λ. In the US, the long-run population growth rate is 1.8%.[14] Therefore, without the increase in the industrial labor share, χ_t would take

$$t = \frac{\ln 2}{\lambda} = \frac{0.7}{1.8\%} = 39 \text{ years}$$

to increase by a factor of 2. In other words, without the reallocation of labor from agriculture to industry in the early 19th century, the takeoff of the US economy would have been delayed by about four decades. Furthermore, we can define $\varkappa \equiv dl_{y,t}/l_{y,t}$, i.e., the percent change in $l_{y,t}$, and for $\varkappa$ small obtain the approximation

$$t = \frac{\ln(1+\varkappa)}{\lambda} \approx \frac{\varkappa}{\lambda} \text{ years}.$$

This says that, given a population growth rate λ of 1.8%, a one-fifth increase in industrial employment reduces the wait time to takeoff by about a decade.

We now calibrate the rest of the model to the US economy to perform our quantitative analysis. In addition to the population growth rate λ, the model also features the following parameters: $\{\rho, \alpha, \sigma, \beta, \theta, \delta, \phi, \mu\}$.[15] We set the discount rate ρ to a conventional value of 0.05. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers $1 - \alpha$ to 0.833 and the social return of variety σ to 0.25. Then, we calibrate β using the current agricultural share of GDP in the US, which is about 1%.[16] Furthermore, we calibrate $\{\theta, \delta, \phi\}$ by matching the following moments of the US economy: 60% for the labor income share of GDP, 62% for the consumption share of GDP, and 1% for the long-run growth rate. Finally, we calibrate the markup ratio μ by matching the average growth rates of the simulated path from our model and the historical path in the US. The calibrated parameter values are $\{\beta, \theta, \delta, \phi, \mu\} = \{0.016, 0.404, 2.547, 1.212, 1.630\}$. Table 3 summarizes the parameter values.

[13] Here we are putting manufacturing and services together as the industrial sector that requires innovation; see e.g., United Nations (2011) for a review on the importance of innovation in the services sector. Kongsamut *et al.* (2001) show that manufacturing and services require the same technology growth rate in order for a balanced growth path to exist in their model.

[14] Data source: Maddison Project Database. The waiting time to takeoff is lower if the population growth rate is higher.

[15] There is also the subsistence ratio η/A, which we will calibrate using historical data.

[16] Here we assume that the subsistence requirement is no longer binding in modern days; i.e., $\eta/A \to 0$.

Table 3: Calibrated parameter values

λ	ρ	α	σ	β	θ	δ	ϕ	μ
0.018	0.050	0.167	0.250	0.016	0.404	2.547	1.212	1.630

To explore how well our model matches the historical path of the growth rate in the US, we first use historical data to calibrate a time path for the subsistence ratio η/A. Specifically, we calibrate the initial value of η/A using an agricultural labor share of 80% at the beginning of the 19th century; see Baten (2016). Then, we use an agricultural labor share of 60% in 1840 and 53% in 1860 in Lebergott (1966) and Weiss (1986) and also an agricultural share of GDP of 30% in 1900, 20% in 1920-1930, 10% in 1950 and 2% in 1980 in Kongsamut *et al.* (2001) to compute a piecewise linear path of η/A. We model these changes as MIT shocks (i.e., a sequence of unanticipated, permanent changes). Based on this imputed path of η/A, Figure 5 simulates the path of the agricultural share of GDP, which decreases from about 70% in the early 19th century to 1% at the end of the 20th century as in the US data.

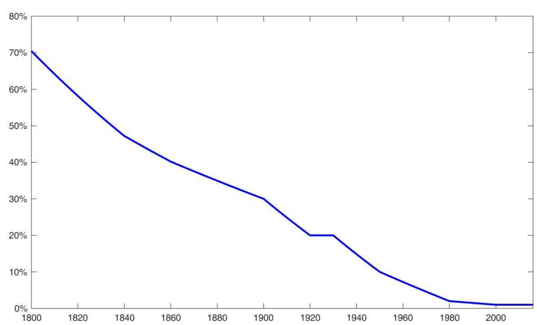

Figure 5: Agricultural share of GDP

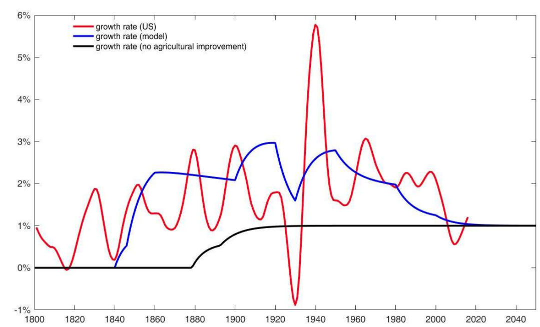

Figure 6: Economic growth

Figure 5 presents the simulated path of agricultural expenditure as a share of GDP based on the calibrated path of agricultural productivity. Figure 6 presents (a) the simulated path of the growth rate of industrial output per worker based on the calibrated path of agricultural productivity, (b) the simulated path of the growth rate without agricultural improvement (i.e., agricultural productivity remains at its initial value), and (c) the HP-filter trend of the growth rate in the US.

Figure 6 presents the simulated path of the growth rate of industrial output per worker and the HP-filter trend of the US growth rate along with a simulated path of the growth rate without agricultural improvement (i.e., η/A remains at its initial value). Unfortunately, we don't have historical data on labor productivity growth in the US, so we use data on the growth rate of output per capita as a proxy. Here we pick an initial value χ_0 such that the takeoff of the economy occurs before the mid-19th century. Following the onset of horizontal innovation, vertical innovation starts half a decade later. After that, the economy keeps growing and reaches a growth rate as high as 3% due to the expansion of the industrial sector, which helps to accelerate the rate of innovation. Around the time of the Great Depression in the 20th century, there is a pause in the reallocation of labor from agriculture to the industrial sector, which translates into a temporary slow down in technological progress before a recovery. Before the end of the 20th century, the growth rate of the economy gradually falls towards the long-run growth rate due to the deceleration of sectoral reallocation. This simulated pattern replicates the data reasonably well with the average growth rate increasing from 1.08% in the 19th century to 2.24% in the 20th century before decreasing to 1.04% in the 21st century, whereas the corresponding data are 1.20%, 2.12% and 1.13% in the 19th, 20th and 21st centuries respectively. It is worth stressing that the simulated path of the growth rate cannot capture this inverted-U pattern in the data if we shut down agricultural improvement. Consequently, the novel ingredient of this paper — the agricultural sector with the associated labor reallocation mechanism — contributes significantly to the Schumpeterian model's ability to match quantitatively the salient features of the secular growth path of the US economy.

5.2 Intellectual property rights

Our analysis above suggests that if we think about a group of countries, those with a larger manufacturing sector experience an earlier takeoff holding the other parameter values constant across the countries. Given the documented role of the timing of takeoff in driving persistent differences in income per capita across countries, this type of consideration tells us that identifying the factors that determine the takeoff is of first-order importance. As an example of such factors, consider North and Thomas (1973, p. 156), who argue that the industrial revolution happened in England because it "had developed an efficient set of property rights embedded in the common law [and...] begun to protect private property in knowledge with its patent law." Chu, Kou and Wang (2020) provide a qualitative analysis on the effects of patent protection on endogenous takeoff in the Schumpeterian growth model that we use here. Pursuing this line of thought, we now provide a quantitative comparison between the two alternative channels, patent protection and agricultural productivity, that determine the timing of takeoff and the shape of the subsequent path of economic growth.

Stronger patent protection allows monopolistic firms to charge a higher markup. Therefore, we consider an increase in μ as a strengthening of patent protection. Equation (33) shows that an increase in $\mu \in (1, 1/\theta)$ leads to a decrease in χ_N and an earlier takeoff (as in the case of an increase in A). Equations (31), (32) and (36) show that an increase in $\mu \in (1, 1/\theta)$ also leads to a higher growth rate g_t for a given χ_t in the industrial era. As we have shown, an increase in A has the same qualitative effects. However, equation (26) shows that the steady-state growth rate g^* is decreasing in μ because according to equation (25) firm size is decreasing in μ. Therefore, unlike agricultural technology A, which does not affect steady-state growth, strengthening patent protection stifles steady-state growth despite generating an earlier takeoff of the economy.

We now perform a quantitative exercise. We assume that the industrial labor share starts at the same initial value $l_{y,t} = 20\%$ as before and pick an initial value χ_0 such that the takeoff of the economy occurs in 1840 given $\mu = 1.630$. Then we simulate the transition path of output growth rate g_t under different values of μ. Figure 7 presents the effects of μ on the takeoff and economic growth. A larger μ leads to an earlier takeoff but a lower steady-state growth rate. Table 4 shows that increasing μ from 1.630 to 1.657 leads to an earlier takeoff by only 1.15 years. However, the steady-state growth rate of quality drops to zero (i.e., $z^* = 0$) and the steady-state growth rate of output becomes $g^* = \sigma n^* = 0.60\%$. Therefore, the positive effect of a stronger patent protection on the timing of the takeoff is relatively small compared to its large negative effect on the steady-state growth rate. Therefore, an improvement in agricultural technology is a far more effective way to generate an earlier takeoff of the economy.

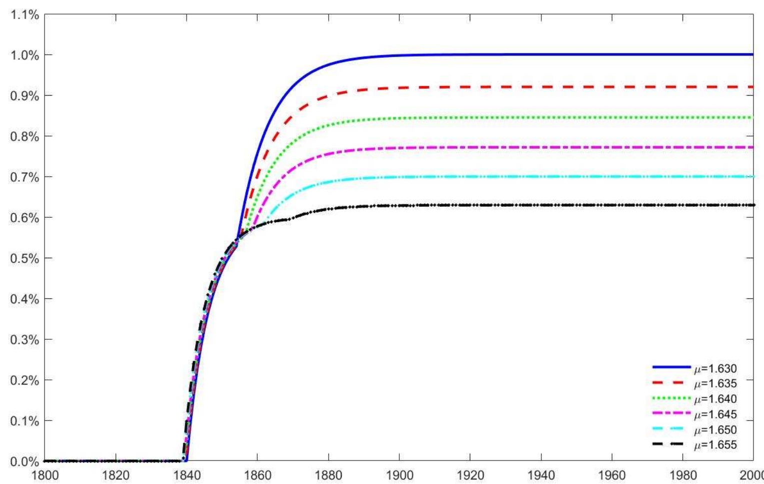

Figure 7: Dynamic effects of patent protection

Figure 7 presents the simulated paths of the growth rate based on different values of the markup.

Table 4: Effects of patent protection on takeoff and growth			
μ	ΔT_N	g^*	z^*
1.635	-0.23	0.92%	0.32%
1.640	-0.44	0.85%	0.25%
1.645	-0.65	0.77%	0.17%
1.650	-0.86	0.70%	0.10%
1.655	-1.07	0.63%	0.03%
1.657	-1.15	0.60%	0.00%

Table 4 presents the simulated values of (a) the change in the takeoff time, (b) the steady-state output growth rate g and (c) the steady-state quality growth rate z, based on different values of the markup.

Before we close this section, we should emphasize that we have analyzed only one particular policy instrument within the patent system. Specifically, this patent policy instrument enables monopolistic firms to charge a higher markup and earn more monopolistic profit.

This is just one aspect of the patent system, and there are other policy instruments within the patent system.[17] Moser (2013) provides an excellent review of empirical studies that examine the empirical relationship between patents and innovation using historical data. In summary, he finds that "patent policies, which grant strong intellectual property rights to early generations of inventors, may discourage innovation." Chu (2009) extends the Schumpeterian model in O'Donoghue and Zweimuller (2004) to provide a quantitative analysis on this negative effect of blocking patents. Therefore, it is important to note that different policy instruments within the patent system can have different implications on takeoff and long-run growth.

5.3 General-purpose technology

We now extend the model to allow for improvements in technology that affect both the agricultural sector and the industrial sector. This is a natural extension to consider since history suggests that many technological improvements contribute simultaneously to both sectors. The literature refers to such things as *general-purpose* technologies (GPTs). This extension is particularly relevant in the cross-country perspective highlighted in the previous subsection because differences in GPTs can serve as proxy for differences across countries in a broader set of fundamentals. The exercise, therefore, illustrates how one can use our model to shed light on the drivers of persistent differences in income per capita in a framework that accounts for the rich and non-linear properties of the transition path from stagnation to growth.

Formally, we modify the industrial technology (7) as follows:

$$Y_t = \int_0^{N_t} X_t^{\theta}(i) \left[A^{\gamma} Z_t^{\alpha}(i) Z_t^{1-\alpha} L_{y,t} / N_t^{1-\sigma} \right]^{1-\theta} di, \tag{40}$$

where $\gamma \geq 0$. When $\gamma = 0$, we are back to the benchmark model. When $\gamma > 0$, an increase in the level of GPT A has an additional positive effect in the industrial sector. The rest of the model is the same as before, except that $\chi_t l_{y,t}$ is replaced by $A^{\gamma}\chi_t l_{y,t}$. It is useful to note that due to the assumption of log utility in industrial and agricultural consumption, by itself the industrial technological improvement does not affect the allocation of labor across the two sectors. Thus the main channel in the model remains that the improvement in productivity in agriculture reallocates labor to industrial production.

We can show that the threshold χ_N in (33) becomes

$$\chi_N \equiv \frac{\left[1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right] \mu^{1/(1-\theta)}\phi}{\left[\mu - 1 - \delta(\rho-\lambda)\right] A^{\gamma}} \left(1 - \frac{\eta}{A}\right)^{-1}, \tag{41}$$

which is decreasing in A as before. However, there is now an additional negative effect via the term A^{γ}. Therefore, an improvement in the GPT leads to an earlier takeoff than an

[17] See Chu (2022) for a survey on different patent instruments considered in the literature on patent policy and economic growth.

improvement in agricultural technology alone. We can also show that the growth rate of variety in the first phase of the industrial era is

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta}\left(\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi}{A^{\gamma}\chi_t l_y^*}\right) + \lambda - \rho > 0, \tag{42}$$

while the growth rate of output in the second phase of the industrial era is

$$g_t = \alpha\left[\frac{\mu-1}{\mu^{1/(1-\theta)}} A^{\gamma}\chi_t l_y^* - \phi\right] - \rho > 0, \tag{43}$$

where l_y^* is increasing in A as before. Notably, both the variety growth rate n_t and the output growth rate g_t are increasing in A^{γ} for a given χ_t. In other words, there is an additional positive effect on economic growth from an improvement in the GPT in both phases of the industrial era due to the simple fact that the GPT expands industrial production. However, due to the scale-invariance of the Schumpeterian growth model, this effect eventually vanishes and the steady-state growth rate g^* is the same as in equation (26), which is independent of A even when $\gamma > 0$.

Next, we assess quantitatively the effects of the GPT. We normalize the initial value of A_0 to 1, which implies that η is 0.797 because the initial value of subsistence ratio η/A in 1800 is 0.797. An increase in the industrial labor share $l_{y,t}$ from 20% to 40% in the early 19th century translates into a decrease of subsistence ratio η/A from 0.797 to 0.593 and an increase in A from 1 to 1.343. Then we simulate the effects of a permanent increase in A from 1 to 1.343 in the pre-industrial era on takeoff and transition path of g_t. The effects of A on the timing of the takeoff (i.e., the activation of variety-expanding innovation) and the activation of quality-improving innovation depend on γ. Specifically, a larger γ leads to an earlier activation of both types of innovation and a higher transitional growth rate but does not affect the steady-state growth rate; see Figure 8.

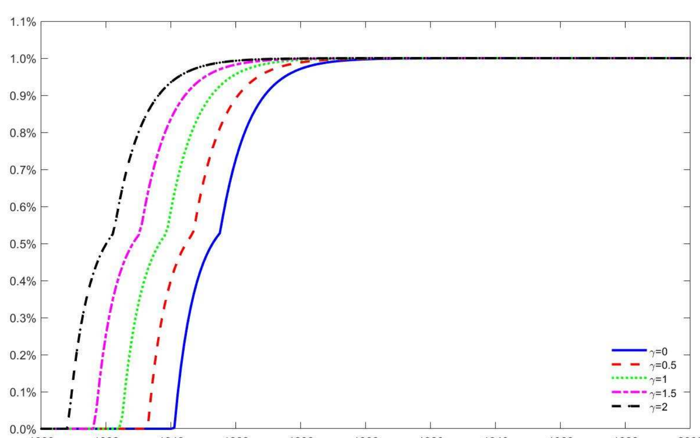

Figure 8: Dynamic effects of general purpose technology

Figure 8 presents the simulated paths of the growth rate based on different values of GPT intensity.

It is worth stressing that in this exercise, and the one in the previous subsection, one can interpret each line as a specific country so that the figure speaks directly to the model's ability to capture the cross-country variation over time of income paths. In this particular case, the model produces a *great-divergence followed by great-convergence* profile of growth rates due to two key properties: (i) the timing of takeoff depends on the level of the GPT; and (ii) the steady-state growth rate does not depend on the level of the GPT because the model sterilizes the strong scale effect. For example, Rostow (1956) documents the different timing of takeoff in a number of European countries, such as Britain (1783-1802), France (1830-1860), Belgium (1833-1860), Germany (1850-1873) and Sweden (1868-1890). Despite their different takeoff time by almost a century, these countries have a roughly similar income level in the modern era.

As mentioned above, industrial technological improvement does not affect the allocation of labor across sectors, due to the assumption of log utility in industrial consumption and agricultural consumption in our model. In the case of non-unitary elasticity of substitution between industrial and agricultural consumption, different rates of technological progress in the two sectors could lead to continuous reallocation of labor across sectors. In contrast, having the same growth rate in GPT A under $\gamma = 1$ may accommodate a stationary allocation of labor across sectors.[18]

5.4 Skill acquisition

Our analysis so far assumes that workers are freely mobile between sectors. However, in reality, some frictions exist. For example, converting an agricultural worker into an industrial worker requires some training. In this section, we model this acquisition of skills in a simple and tractable way.

Specifically, we assume that each unit of industrial labor requires $\psi \in [0, 1)$ units of labor as training time. In this case, the asset-accumulation equation in (2) becomes

$$\dot{a}_t = (r_t - \lambda)a_t + w_{y,t}l_{y,t} + w_{q,t}l_{q,t} - c_t - p_t q_t, \tag{44}$$

where $w_{y,t}$ is the wage rate of industrial labor $l_{y,t}$ and $w_{q,t}$ is the wage rate of agricultural labor $l_{q,t}$. The time constraint of each household member is then

$$(1 + \psi)l_{y,t} + l_{q,t} = 1, \tag{45}$$

where ψ captures the training time needed to work as industrial labor. In order for workers to be indifferent between working in the two sectors, the relative wage must be given by

$$\frac{w_{y,t}}{w_{q,t}} = 1 + \psi > 1, \tag{46}$$

where ψ also determines the industrial wage premium. The rest of the model is the same as before, except that the industrial labor share $l_{y,t}$ becomes

$$l_{y,t} = \frac{1}{1+\psi}\left(1 + \frac{\beta}{1-\theta}\frac{c_t}{y_t}\right)^{-1}\left(1 - \frac{\eta}{A}\right), \tag{47}$$

[18] This would also need the absence of the subsistence requirement in agricultural consumption (i.e., $\eta = 0$).

which is smaller than before unless $\psi = 0$. As a result, the smaller firm size $\chi_t l_{y,t}$ delays the takeoff by increasing the threshold χ_N given by

$$\underset{+}{\chi_N(\psi)} = \frac{(1+\psi)\left[1+\beta\left(1+\frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\mu^{1/(1-\theta)}\phi}{\mu-1-\delta(\rho-\lambda)}\left(1-\frac{\eta}{A}\right)^{-1}. \tag{48}$$

In the rest of this section, we present the simulated paths of the growth rate under different values of ψ. The industrial labor share $l_{y,t}$ increased to 40% in the early 19th century in the US, which implies that the subsistence ratio η/A decreased to 0.593 given $\psi = 0$. We pick an initial value χ_0 such that the takeoff of the economy occurs in around 1840 given $\psi = 0$. Then we simulate the transition path of output growth rate g_t under different values of ψ given $\eta/A = 0.593$. Figure 9 presents the effects of ψ on the takeoff and economic growth, and shows a larger ψ delays takeoff but does not affect the steady-state growth rate. The effects of ψ on the timing of the takeoff (i.e., the activation of variety-expanding innovation) and the activation of quality-improving innovation depend on ψ. Specifically, a larger ψ leads to a later activation of both types of innovation and a lower transitional growth rate but does not affect the steady-state growth rate.

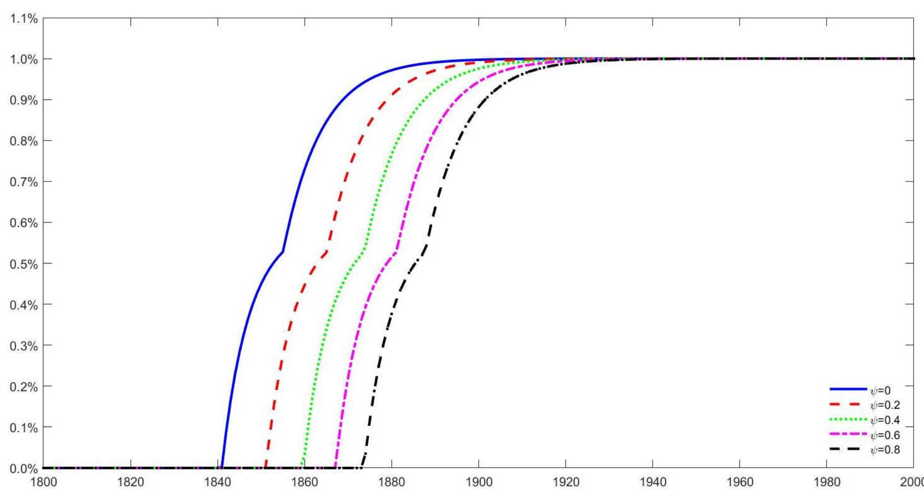

Figure 9: Dynamic effects of skill acquisition

Figure 9 presents the simulated paths of the growth rate based on different values of skill acquisition.

6 Conclusion

In this study, we have developed a Schumpeterian growth model with an agricultural sector in which the size of firms in the industrial sector determines the endogenous takeoff of the economy. The primary goal of the exercise is to shed new light on the important role of agriculture in a dynamic process that historians describe narratively as follows (e.g., Nurkse 1953): at the heart of industrialization, large improvements in agricultural productivity liberate labor from food production and reallocate it to industrial production. The secondary goal is to shed new light on the role of agriculture in explaining why countries with large populations, such as China and India, did not experience an early industrial takeoff. Our explanation is that the vast majority of their population being in agriculture did not contribute to firm size in the industrial sector.

More broadly, the model delivers analytical insights on the mechanism through which an agricultural revolution determines the timing of the endogenous takeoff. A sectoral reallocation that expands firm size in the industrial sector produces an earlier transition from stagnation to growth. Our quantitative analysis indicates that the decline in the agricultural share of the US workforce in the early 19th century contributed to the takeoff of the US economy. Without the reallocation of labor from agriculture to industry, the takeoff of the US economy would have been delayed by four decades. Finally, although our model is designed to explore the takeoff of early industrialized countries in the 19th century, it is also relevant for the subsequent takeoff of emerging markets that need to rely on technologies for economic development. These countries may not have to reinvent the wheel, but the transfer of technologies from the global technology frontier also depends on incentives and the size of industrial firms in these countries.

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Appendix A

Table A1: Effects of agricultural productivity on economic growth

	GDP growth		per capita GDP growth		per worker GDP growth	
	(1)	(2)	(3)	(4)	(5)	(6)
A_{it}	0.133***	0.094	0.176***	0.147***	0.174***	0.143***
	(0.048)	(0.058)	(0.046)	(0.055)	(0.046)	(0.055)
$A_{it} \times y_{it}$	-0.010**	-0.005	-0.014***	-0.011**	-0.015***	-0.011**
	(0.005)	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)
y_{it}	-0.001	-0.052	0.029	-0.015	0.034	-0.011
	(0.034)	(0.046)	(0.033)	(0.044)	(0.033)	(0.044)
Control variables	No	Yes	No	Yes	No	Yes
Wave fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	408	356	408	356	402	356
R-squared	0.463	0.487	0.513	0.543	0.493	0.518

Note: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors are in parentheses. The dependent variable in columns (1)-(2) is the average annual growth rate of real GDP. The dependent variable in columns (3)-(4) is the average annual growth rate of real GDP per capita. The dependent variable in columns (5)-(6) is the average annual growth rate of real GDP per worker. Compared with odd columns, even columns add control variables including the log value of population, the log value of capital stock, a human capital index, the degree of openness (measured by the average ratio of export plus import to GDP), and the government consumption share of GDP.

Equilibrium. The equilibrium is a time path of allocations $\{a_t, q_t, c_t, Y_t, X_t, I_t, L_{y,t}, L_{q,t}\}$ and prices $\{r_t, w_t, p_t, P_t, V_t\}$ such that:

- the household consumes $\{q_t, c_t\}$ to maximize utility taking $\{r_t, w_t, p_t\}$ as given;
- competitive firms produce Q_t to maximize profits taking $\{w_t, p_t\}$ as given;
- competitive firms produce Y_t to maximize profits taking $\{w_t, P_t\}$ as given;
- monopolistic intermediate-good firms choose $\{P_t, I_t\}$ to maximize V_t taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the aggregate value of monopolistic firms equals the household's wealth, $a_t L_t = N_t V_t$;
- the labor market clears, $L_{q,t} + L_{y,t} = L_t$;
- the market for the agricultural good clears, $q_t L_t = A L_{q,t}$;
- the market-clearing condition of the final good holds:

$$Y_t = c_t L_t + \mu N_t X_t,$$

which applies to the pre-industrial era, and

$$Y_t = c_t L_t + N_t \left(X_t + \phi Z_t + I_t \right) + \dot{N}_t \delta X_t,$$

which applies to the industrial era.

Dynamic optimization of monopolistic firms. The current-value Hamiltonian for monopolistic firm i is

$$H_t(i) = \Pi_t(i) - I_t(i) + \zeta_t(i)\dot{Z}_t(i) + \xi_t(i)[\mu - P_t(i)], \tag{A1}$$

where $\xi_t(i)$ is the multiplier on $P_t(i) \leq \mu$. We substitute (9)-(11) into (A1) and derive

$$\frac{\partial H_t(i)}{\partial P_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial P_t(i)} = \xi_t(i), \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial I_t(i)} = 0 \Rightarrow \zeta_t(i) = 1, \tag{A3}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha\left\{[P_t(i) - 1]\left[\frac{\theta}{P_t(i)}\right]^{1/(1-\theta)}\frac{L_{y,t}}{N_t^{1-\sigma}} - \phi\right\} Z_t^{\alpha-1}(i) Z_t^{1-\alpha} = r_t\zeta_t(i) - \dot{\zeta}_t(i). \tag{A4}$$

If $P_t(i) < \mu$, then $\xi_t(i) = 0$. In this case, $\partial\Pi_t(i)/\partial P_t(i) = 0$ yields $P_t(i) = 1/\theta$. If the constraint on $P_t(i)$ is binding, then $\xi_t(i) > 0$. In this case, we have $P_t(i) = \mu$. Therefore, we have proven (13). Then, the assumption $\mu < 1/\theta$ implies $P_t(i) = \mu$. Substituting (A3), (14) and $P_t(i) = \mu$ into (A4) and imposing symmetry yield (15), where $l_{y,t} \equiv L_{y,t}/L_t$. ■

Monopolistic profit in the pre-industrial era. In the pre-industrial era, the firm size $\chi_t l_{y,t}$ is so small that monopolistic firms with increasing returns technology cannot earn a positive profit; i.e.,

$$\chi_t l_{y,t} < \phi\mu^{1/(1-\theta)}/(\mu - 1) \Leftrightarrow \Pi_t < 0,$$

where l_y is given in (28). In this case, the existing intermediate goods N_0 are produced by competitive firms that make zero profit. When $\chi_t l_y$ reaches $\phi\mu^{1/(1-\theta)}/(\mu - 1)$, we assume that the increasing returns technology is not yet deployed until χ_t reaches χ_N; see Appendix B for the case without this assumption. ■

Dynamics of the consumption-output ratio in the industrial era. The value of assets owned by each member of the household is

$$a_t = V_t N_t / L_t. \tag{A5}$$

If $n_t > 0$, then $V_t = \delta X_t$ in (17) holds. Substituting (17) and $\mu X_t N_t = \theta Y_t$ into (A5) yields

$$a_t = \delta X_t N_t / L_t = (\theta/\mu)\delta Y_t / L_t = (\theta/\mu)\delta y_t, \tag{A6}$$

which implies that a_t/y_t is constant. Substituting (A6), (3) and (8) into (2) yields

$$\begin{aligned}\frac{\dot{y}_t}{y_t} = \frac{\dot{a}_t}{a_t} &= r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - c_t - p_t q_t}{a_t} \\ &= \frac{\dot{c}_t}{c_t} + \rho - \lambda + \frac{(1-\theta)\mu}{\delta\theta} - \frac{\mu}{\delta\theta}\frac{c_t}{y_t},\end{aligned} \tag{A7}$$

where we have also used $w_t L_{q,t} = p_t Q_t$. Equation (A7) can be rearranged as

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = \frac{\mu}{\delta\theta}\frac{c_t}{y_t} - \frac{(1-\theta)\mu}{\delta\theta} - (\rho - \lambda), \tag{A8}$$

which shows that the dynamics of c_t/y_t is characterized by saddle-point stability such that c_t/y_t jumps to its steady-state value in (30) whenever $n_t > 0$. Then, substituting (30) into (22) yields l_y^* in (31). ■

Proof of Proposition 1. In the pre-industrial era, the firm size $\chi_t l_y$ is not sufficiently large for horizontal and vertical innovation to be viable such that the variety growth rate and the quality growth rate are both zero (i.e., $n_t = z_t = 0$). In this case, the industrial labor share l_y is given by (28) and the state variable $\chi_t = \theta^{1/(1-\theta)} L_t / N_0^{1-\sigma}$ increases at the population growth rate λ. Therefore, in the pre-industrial era, the dynamics of χ_t is simply

$$\dot{\chi}_t = \lambda \chi_t > 0. \tag{A9}$$

In the first phase of the industrial era, the firm size $\chi_t l_y^*$ becomes sufficiently large for horizontal innovation (but not vertical innovation) to be viable such that $n_t > 0$ and $z_t = 0$. In this case, the variety growth rate n_t is given by (32), which is positive if and only if

$$\chi_t > \frac{\mu^{1/(1-\theta)} \phi / l_y^*}{\mu - 1 - \delta(\rho - \lambda)} \equiv \chi_N > \chi_0, \tag{A10}$$

where l_y^* is given by (31) and increasing in A. Given χ_0, the state variable χ_t increases at rate λ until it reaches χ_N; therefore, the time this process takes is

$$T_N = \frac{1}{\lambda} \log \left(\frac{\chi_N}{\chi_0} \right).$$

After reaching χ_N, the dynamics of χ_t in (23) becomes

$$\dot{\chi}_t = \left[\lambda - (1-\sigma) n_t \right] \chi_t = \frac{1-\sigma}{\delta} \left\{ \frac{\phi \mu^{1/(1-\theta)}}{l_y^*} - \left[\mu - 1 - \delta \left(\rho + \frac{\sigma \lambda}{1-\sigma} \right) \right] \chi_t \right\} > 0, \tag{A11}$$

which uses (32) for n_t.

In the second phase of the industrial era, the firm size $\chi_t l_y^*$ becomes sufficiently large for both horizontal and vertical innovation to be viable such that $n_t > 0$ and $z_t > 0$. In this case, the quality growth rate z_t is positive if and only if

$$\chi_t > \frac{\Omega}{l_y^*} \equiv \chi_Z > \chi_N, \tag{A12}$$

where l_y^* is given by (31) and the composite parameter Ω is defined as before:

$$\Omega \equiv \underset{\omega}{\arg\text{solve}} \left\{ \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} \omega - \phi \right] \left[\alpha - \frac{\mu^{1/(1-\theta)} \sigma}{\delta \omega} \right] = \rho - \sigma (\rho - \lambda) \right\}.$$

In this regime, the equilibrium growth rate in (36) is derived from $g_t = r_t^q - \rho$, where r_t^q is given in (15). Then, we use (36), (37) and $z_t = g_t - \sigma n_t$ to derive n_t and the linearized dynamics of χ_t as

$$\dot{\chi}_t = \frac{1-\sigma}{\delta} \left\{ \left[(1-\alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma} \right) \right] \frac{\mu^{1/(1-\theta)}}{l_y^*} - \left[(1-\alpha)(\mu - 1) - \delta \left(\rho + \frac{\sigma\lambda}{1-\sigma} \right) \right] \chi_t \right\} \geq 0, \tag{A13}$$

where we have used $\sigma\mu^{1/(1-\theta)}/\left(\chi_t l_y^*\right) \cong 0$. Then, we can use n_t to derive $z_t = g_t - \sigma n_t$.

Given (24), the autonomous dynamics of χ_t is stable and captured by (A9), (A11) and (A13). Given an initial value χ_0, the state variable χ_t increases according to (A9) until χ_t reaches the first threshold χ_N, which is decreasing in A via l_y^*. Then, χ_t increases according to (A11) until χ_t reaches the second threshold χ_Z, which is also decreasing in A via l_y^*. Finally, χ_t increases according to (A13) until χ_t converges to its steady state

$$\chi^* = \frac{\mu^{1/(1-\theta)}}{l_y^*} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \sigma\lambda/(1-\sigma)\right]}, \tag{A14}$$

where l_y^* is given in (31). Substituting (A14) into (36) yields g^* in (26). ■

R&D share of output. In the first phase of the industrial era, the R&D share of output is

$$\begin{aligned} \frac{R\&D_t}{Y_t} &= \frac{\dot{N}_t \delta X_t}{Y_t} = \frac{\dot{N}_t}{N_t} \frac{\delta X_t N_t}{Y_t} = \frac{\delta\theta}{\mu} n_t \\ &= \frac{\theta}{\mu}\left[\mu - 1 - \mu^{1/(1-\theta)} \frac{\phi}{\chi_t l_y^*} + \delta\left(\lambda - \rho\right)\right] \end{aligned} \tag{A15}$$

which uses n_t in (32). l_y^* is increasing in A and $R\&D_t/Y_t$ is increasing in l_y^* for a given χ_t such that the R&D share of output is increasing in A for a given χ_t. In the second phase of the industrial era, the R&D share of output is

$$\begin{aligned} \frac{R\&D_t}{Y_t} &= \frac{N_t I_t + \dot{N}_t \delta X_t}{Y_t} = \frac{N_t \dot{Z}_t}{Y_t} + \frac{\delta\theta}{\mu} n_t = \frac{N_t Z_t}{\left(\frac{\theta}{\mu}\right)^{\theta/(1-\theta)} N_t^{\sigma} Z_t L_{y,t}} z_t + \frac{\delta\theta}{\mu} n_t \\ &= \frac{\theta}{\mu}\left[\mu^{1/(1-\theta)} \frac{z_t}{\chi_t l_y^*} + \delta n_t\right] = \frac{\theta}{\mu}\left[\mu - 1 - \mu^{1/(1-\theta)} \frac{\phi}{\chi_t l_y^*} + \delta\left(\lambda - \rho\right)\right] \end{aligned} \tag{A16}$$

which uses χ_t in (14), Y_t in (19) and n_t in (37). l_y^* is increasing in A and $R\&D_t/Y_t$ is increasing in l_y^* for a given χ_t such that the R&D share of output is increasing in A for a given χ_t. In summary, in both phases of the industrial era, the R&D share of output is increasing in the level of agricultural technology A for a given χ_t. ■

Appendix B

In this appendix, we extend the baseline model to allow for the possibility that in the pre-industrial era (i.e., $n_t = z_t = 0$), monopolistic profits become positive (i.e., $\Pi_t > 0$) before the takeoff occurs. When $n_t = 0$, the entry condition in (17) does not hold. However, the asset-pricing equation in (16) still holds and becomes

$$r_t = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t}, \tag{B1}$$

where $I_t = z_t = 0$. We use (A5) and $n_t = 0$ to derive $\dot{a}_t/a_t = \dot{V}_t/V_t - \lambda$ and then substitute this equation into (2) to obtain

$$\frac{\dot{V}_t}{V_t} - \lambda = \frac{\dot{a}_t}{a_t} = r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - p_t q_t - c_t}{a_t}. \tag{B2}$$

Substituting (B1) into (B2) yields

$$c_t = \frac{\Pi_t}{V_t} a_t + w_t l_{y,t} = \frac{N_t}{L_t}\Pi_t + (1-\theta)\, y_t, \tag{B3}$$

where we have used (A5), $w_t l_{q,t} = p_t q_t$ and $w_t l_{y,t} = (1-\theta)\, y_t$. Then, substituting (11) and $P_t = \mu$ into (B3) yields

$$c_t = \frac{N_t X_t \left(\mu - 1 - \phi Z_t/X_t\right)}{L_t} + (1-\theta)\, y_t = \theta \mu^{\theta/(1-\theta)} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{\chi_t l_{y,t}}\right) y_t + (1-\theta)\, y_t, \tag{B4}$$

where the second equality uses $\theta Y_t = \mu N_t X_t$ and (14). The consumption-output ratio is

$$\frac{c_t}{y_t} = \theta \mu^{\theta/(1-\theta)} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{\chi_t l_{y,t}}\right) + 1 - \theta, \tag{B5}$$

which would increase from (27) to (30) if the firm size $\chi_t l_{y,t}$ increases from $\phi\mu^{1/(1-\theta)}/(\mu - 1)$ to $\phi\mu^{1/(1-\theta)}/[\mu - 1 - \delta(\rho - \lambda)]$. Finally, we substitute (B5) into (22) and manipulate the equation to obtain the equilibrium firm size:

$$\chi_t l_{y,t} = \frac{\frac{\beta\theta\phi}{1-\theta}\mu^{\theta/(1-\theta)} + \left(1 - \frac{\eta}{A}\right)\chi_t}{1 + \beta\left(1 + \frac{\theta}{1-\theta}\frac{\mu - 1}{\mu}\right)}, \tag{B6}$$

which continues to be increasing in the level of agricultural technology A.

Given that the dynamics of χ_t is still given by (A9) in the pre-industrial era, the firm size $\chi_t l_{y,t}$ gradually increases towards the threshold in (A10) to trigger the takeoff as before. The only difference is that as χ_t increases overtime, $l_{y,t}$ in (B6) is gradually decreasing from l_y in (28) to l_y^* in (31) (instead of jumping from l_y to l_y^* at the time of the takeoff). This additional dynamics in $l_{y,t}$ gives rise to negative growth in the industrial output per capita before the takeoff, which is less realistic than the dynamics in the baseline model.

Appendix C (not for publication)

In this appendix, we present a simple second-generation Schumpeterian growth model with a separate R&D sector. The household sector and the agricultural sector are the same as before.

Industrial production
A representative competitive firm operates the assembly technology

$$Y_t = N_t \exp\left(\frac{1}{N_t}\int_0^{N_t} \ln X_t(i)di\right), \tag{C1}$$

where N_t is the endogenous mass of differentiated intermediate goods. Profit maximization yields the following conditional demand function for $X_t(i)$:

$$X_t(i) = \frac{Y_t}{N_t P_t(i)}, \tag{C2}$$

where $P_t(i)$ is the price of $X_t(i)$.

Variety growth
Following Howitt (2000), we specify the law of motion for N_t as

$$\dot{N}_t = \xi L_t \Leftrightarrow g_{N,t} \equiv \frac{\dot{N}_t}{N_t} = \xi \frac{L_t}{N_t}, \tag{C3}$$

where $\xi > 0$ is an exogenous parameter. The growth rate of variety $g_{N,t}$ is determined by L_t/N_t. A stationary $\dot{N}_t/N_t$ on the balanced growth path implies a stationary ratio L_t/N_t, which in turn implies that the long-run growth rate of N_t is also λ. Therefore, N_t is proportional to L_t in the long run, such that $(L_t/N_t)^* = \lambda/\xi$. We assume that $L_0/N_0 < \lambda/\xi$ in which case L_t/N_t and also $g_{N,t}$ rise towards their steady states.

Intermediate goods
There is a continuum of monopolistic industries producing differentiated intermediate goods. The production function of the industry leader in industry $i \in [0, N_t]$ is

$$X_t(i) = z^{n_t(i)} L_{x,t}(i), \tag{C4}$$

where the parameter $z > 1$ is the quality step size, $n_t(i)$ is the number of quality improvements that have occurred in industry i as of time t, and $L_{x,t}(i)$ is manufacturing labor employed in industry i. Given the productivity level $z^{n_t(i)}$, the marginal cost of the leader in industry i is $w_t/z^{n_t(i)}$. The profit-maximizing monopolistic price is

$$P_t(i) = \mu \frac{w_t}{z^{n_t(i)}}, \tag{C5}$$

where the markup $\mu \in (1, z]$ is a policy parameter determined by the government. The wage payment is

$$w_t L_{x,t}(i) = \frac{1}{\mu} P_t(i) X_t(i) = \frac{Y_t}{\mu N_t}, \tag{C6}$$

which implies that $L_{x,t}(i) = L_{x,t}/N_t$ is the same across industries and $w_t L_{x,t} = Y_t/\mu$. The monopolistic profit is

$$\Pi_t(i) = P_t(i)X_t(i) - w_t L_{x,t}(i) = \frac{\mu - 1}{\mu}\frac{Y_t}{N_t}. \tag{C7}$$

R&D

Equation (C7) shows that $\Pi_t(i) = \Pi_t$. Therefore, the value of inventions is the same across industries such that $V_t(i) = V_t$. The no-arbitrage condition that determines V_t is

$$r_t = \frac{\Pi_t + \dot{V}_t - \sigma_t V_t}{V_t}, \tag{C8}$$

which states that the rate of return on V_t is equal to r_t. The return on V_t is the sum of monopolistic profit Π_t, capital gain $\dot{V}_t$ and expected capital loss $\sigma_t V_t$, where σ_t is the arrival rate of innovation.

Competitive entrepreneurs maximize profit by devoting final good as R&D input to perform innovation. The arrival rate of innovation is

$$\sigma_t = \varphi \frac{R_t}{Z_t}, \tag{C9}$$

where $\varphi > 0$ is a productivity parameter, Z_t denotes aggregate technology and R_t is the R&D input of final good in each industry. Because of symmetry, the R&D input is the same across industries $R_t(i) = R_t$. The free-entry condition of R&D is

$$\sigma_t V_t = R_t \Leftrightarrow \varphi \frac{V_t}{Z_t} = 1. \tag{C10}$$

Aggregation

Aggregate technology Z_t is defined as

$$Z_t \equiv \exp\left(\frac{1}{N_t}\int_0^{N_t} n_t(i)di \ln z\right) = \exp\left(\int_0^t \sigma_\omega d\omega \ln z\right), \tag{C11}$$

which uses the law of large numbers.[19] Differentiating the log of Z_t with respect to time yields the growth rate of technology given by

$$z_t \equiv \frac{\dot{Z}_t}{Z_t} = \sigma_t \ln z. \tag{C12}$$

Substituting (C4) into (C1) yields the aggregate production function given by

$$Y_t = N_t \exp\left(\frac{1}{N_t}\int_0^{N_t} n_t(i)di \ln z + \frac{1}{N_t}\int_0^{N_t} \ln L_{x,t}(i)di\right) = Z_t L_{x,t}, \tag{C13}$$

[19] Here we make the usual assumption that a new variety enters with the average level of quality in the economy.

and the growth rate of industrial output per capita $y_t \equiv Y_t/L_t = Z_t l_{x,t}$ is

$$g_t \equiv \frac{\dot{y}_t}{y_t} = z_t + \frac{\dot{l}_{x,t}}{l_{x,t}} = \sigma_t \ln z + \frac{\dot{l}_{x,t}}{l_{x,t}}, \tag{C14}$$

where $l_{x,t} \equiv L_{x,t}/L_t$ is the share of industrial labor. We define $l_{q,t} \equiv L_{q,t}/L_t = 1 - l_{x,t}$ as the share of agricultural labor. The growth rate of industrial output is $g_{Y,t} = z_t + \dot{L}_{x,t}/L_{x,t} = z_t + \dot{l}_{x,t}/l_{x,t} + \lambda = g_t + \lambda$.

The value of an invention is

$$\begin{aligned} V_t &= \frac{\Pi_t}{r_t - g_\Pi + \sigma_t} = \frac{\mu - 1}{\mu N_t} \frac{Y_t}{\rho + g_{c,t} - (g_{Y,t} - g_{N,t}) + \sigma_t} \\ &= \frac{\mu - 1}{\mu} \frac{Z_t}{\rho - \lambda + g_{c,t} - g_t + g_{N,t} + \sigma_t} \frac{L_{x,t}}{N_t} \\ &= \frac{\mu - 1}{\mu \xi} \frac{g_{N,t} l_{x,t}}{\rho - \lambda + g_{c,t} - g_t + g_{N,t} + \sigma_t} Z_t, \end{aligned} \tag{C15}$$

which uses $r_t = \rho + \dot{c}_t/c_t$ in (3) and states that the value of an invention is increasing in the share of industrial labor $l_{x,t}$ and the variety growth rate $g_{N,t}$ for a given innovation arrival rate σ_t. Substituting (C15) into (C10) yields

$$\varphi \frac{\mu - 1}{\mu \xi} \frac{g_{N,t} l_{x,t}}{\rho - \lambda + g_{c,t} - g_t + g_{N,t} + \sigma_t} = 1, \tag{C16}$$

in which we have re-expressed the free-entry condition of R&D in (C10).

Agricultural productivity and takeoff

In the pre-industrial era, the population size is so small that $g_{N,t}$ (or L_t/N_t) is not big enough for the free-entry condition of R&D to hold and for innovation to take place. Therefore, the growth rate of industrial output per capita is zero (i.e., $g_t = 0$). In this case, the population size L_t is so small that $g_{N,t}$ (or L_t/N_t) is not large enough to ensure that the free-entry condition in (C10) holds such that entrepreneurs do not perform R&D. In this case, all industrial output is devoted to consumption (i.e., $c_t L_t = Y_t$), which yields

$$\frac{c_t}{y_t} = 1. \tag{C17}$$

The combination of labor demand from agriculture in (6) and (C6) yields

$$p_t = \frac{Y_t}{\mu A L_{x,t}}. \tag{C18}$$

Substituting the agricultural technology in (5) and the relative price in (C18) into the demand function for q_t in (4) yields the industrial labor share $l_{x,t}$ as

$$l_x = \frac{1}{1 + \beta\mu} \left(1 - \frac{\eta}{A}\right), \tag{C19}$$

which also uses $c_t = y_t$ in (C17). This says that the industrial labor share in the pre-industrial era is stationary and increasing in the level of agricultural technology A. The associated growth rate of industrial output per capita is

$$g_t = z_t + \frac{\dot{l}_{x,t}}{l_{x,t}} = 0 \tag{C20}$$

because $z_t = \dot{l}_{x,t}/l_{x,t} = 0$.

Quality-improving innovation activates when the growth rate of variety $g_{N,t}$ (or L_t/N_t) grows sufficiently large. Substituting the agricultural technology in (5) and the relative price in (C18) into the demand function for q_t in (4) yields the industrial labor share $l_{x,t}$ as

$$l_{x,t} = \left(1 + \beta\mu\frac{c_t}{y_t}\right)^{-1}\left(1 - \frac{\eta}{A}\right), \tag{C21}$$

which is also increasing in A for a given c_t/y_t. It can be shown that in the industrial era, the consumption-output ratio and the industrial labor share are given by

$$\frac{c_t}{y_t} = \frac{\varphi g_{N,t}\left(1 - \eta/A\right) - \xi\sigma_t}{\varphi g_{N,t}\left(1 - \eta/A\right) + \beta\mu\xi\sigma_t}; \tag{C22}$$

$$l_{x,t} = \frac{\beta\mu\xi\sigma_t}{(1+\beta\mu)\varphi g_{N,t}} + \frac{1}{1+\beta\mu}\left(1 - \frac{\eta}{A}\right). \tag{C23}$$

To understand better the role of agricultural technology, we can derive a threshold of $g_{N,t}$, or equivalently L_t/N_t, below which the free-entry condition of R&D in (C10) does not hold such that $\varphi V_t/Z_t < 1$ and $\sigma_t = 0$. From (C16), this threshold is given by

$$\bar{g}_N = (\rho - \lambda)\frac{\mu\xi}{\varphi(\mu - 1)l_x - \mu\xi}, \tag{C24}$$

where l_x is given by (C19). At the moment before innovation activates, we have $\sigma_t = 0$, $c_t/y_t = 1$ in (C22) and $l_{x,t} = \left(1 - \eta/A\right)/\left(1 + \beta\mu\right)$ in (C23). Equation (C24) says that innovation will not occur until $g_{N,t}$ (or L_t/N_t) crosses the threshold $\bar{g}_N$ due to population growth. An increase in agricultural technology A leads to an increase in the industrial labor share l_x, which in turn reduces the threshold $\bar{g}_N$ and leads to an earlier takeoff.

In summary, given $L_0/N_0 < \bar{g}_N/\xi < \lambda/\xi$, the economy begins in the pre-industrial era. When L_t/N_t rises above the threshold $\bar{g}_N/\xi$, the economy enters the industrial era, in which the population size (and also $g_{N,t}$) becomes large enough to trigger innovation and for the free-entry condition of R&D to hold. The growth rate of industrial output per capita g_t increases overtime due to the accelerating growth rate of variety $g_{N,t}$ until it converges to its steady-state value $g_N^* = \lambda$. At this point, the growth rate of industrial output per capita also converges to a steady state value.

References

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