Implicit Trade in Risk and Risk Aversion

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Abstract

Using a simple duopolistic trade model with demand uncertainty and an identical traded product, we show that we can view trade in goods as implicit exports/imports of risk and risk aversion. Specifically, we show that a relatively "risk-aversion abundant" country is more likely to be a net importer of the product - hence an importer of low risk-aversion. Similarly, a "relatively high-risk abundant" country is more likely to be a net exporter of the product - hence importer of low risk.

We also show that risk and risk aversion differences, and the presence of market correlation, are sources of implicit risk-sharing and diversification gains from trade. Consequently, the relatively high-risk or high-risk-aversion country always gains from trade, whereas the other country will most likely gain unless markets are highly, positively correlated. Furthermore, we show that world gains from trade are always strictly positive, and in general, are likely to decrease with risk and correlation. Comparing gains from trade with and without uncertainty, we find that, sometimes, both world and country gains may be higher with uncertainty than without it. Finally, to get a sense of the importance of risk, risk aversion and correlation, we calculate local measures of world gains from trade elasticities. We find that world gains from trade are most responsive to changes in market correlation, highlighting the importance of diversification.

KEYWORDS: Patterns of Trade, Gains from Trade, Risk, Risk Aversion, Exports.

JEL Classification: F12, F13, F15, D81.

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1 Introduction

Traditional trade theories explain trade patterns and gains from trade (GFT) by differences among countries. For example, Ricardian models focus on technology differences, whereas the Heckscher-Ohlin-Samuelson model focuses on differences in endowments. Relatively more modern trade theories focus on non-competitive markets, strategic behaviour, economies of scale and other imperfections, allowing them to explain also other phenomena, such as trade in similar commodities among similar countries, win-win outcomes, Etc.

Although these strands of international trade theory have generally been addressed under certainty, the importance of uncertainty has long been recognized in the literature. For example, Batra (1975) examined the validity of the Heckscher-Ohlin-Samuelson theorem in the context of uncertainty; Batra and Russell (1974) examined the GFT under uncertainty, and the effects of uncertainty within the context of the Ricardian model were studied by Ruffin (1974) and Turnovsky (1974). Other examples include Helpman and Razin (1978), who provided a comprehensive analysis of the impact of uncertainty on trade in the presence of stock markets, and Appelbaum and Kohli (1997), who examined the effects of uncertainty on income distribution. Extensive literature also examines the condition under which the HO model holds, or does not hold, under uncertainty. For example, Hoff (1994) investigates when and why the Heckscher-Ohlin-Samuelson model does not hold under uncertainty; Anderson (1981) shows the conditions under which the Heckscher-Ohlin and Travis-Vanek theorems will hold under uncertainty, and Dumas (1980) extends trade theorems to a broader class of uncertainty models.[1]

More recently, the effects of uncertainty have been studied in the context of trade agreements (Limão and Maggi (2015), Appelbaum and Melatos (2016, 2020)); trade policy (Handley (2014), Feng et al. (2017)); sourcing and export decisions ((Gervais (2018), Lewis (2014); trade generation in a general equilibrium framework (Baley et al. (2020)) and the effects of uncertainty on trade flows and income distribution (Novi and Taylor (2020)).

This paper aims to examine the role of risk, risk aversion, and market correlation (RRAC) in determining trade patterns, gains from trade, and hence, motives for trade. First, using a simple partial equilibrium, duopolistic trade model with an identical product (similar to Brander and Krugman (1983)), we show that differences in risk and risk aversion (RRA) provide (in themselves) an explanation of trade patterns. In the presence of RRA differences, in turn, market correlation (which is a non-difference-based characteristic) can exacerbate or mitigate the effects of RRA. More importantly, in the spirit of the "content of trade" models,[2] this paper shows that, indeed, the pattern of trade in goods also reflects the implicit RRA "contents of trade." Therefore, the flow of goods can be interpreted as a "flow" of risk and risk aversion.

[1] Cheng (1987) examines the conditions under which self-sufficiency is optimal under uncertainty.

[2] See Vanek (1968), Helpman (1984), Davis and Weinstein (2002).

Specifically, we show that, in general, if countries differ only in their attitudes toward risk, the less risk-averse country will be a net exporter of the product. We can interpret this result as saying that if a country is relatively "abundant in low risk-aversion," it will be a net exporter of low risk-aversion. We can think of this as the "risk aversion content of trade." We also show that if countries differ only in their risks, the country with lower risk will be a net importer of the product. Thus, we can interpret this result as saying that if a country is relatively "abundant in low risk," it will be an exporter of low risk.

We then consider a more general case, where both risk and risk aversion are different. We show that a country is more likely to be an importer of the product (an importer of low risk-aversion) when its measure of risk aversion is high. However, it is more likely to be an exporter of the product (an importer of low risk) when its risk is high. We find that the role of correlation is a bit more complicated: the likelihood that a country will become an importer decreases with correlation when the correlation is "sufficiently high" but increases with correlation when the correlation is "sufficiently low."[3]

Second, within the same model framework, we show that RRAC introduce GFT due to implicit risk-sharing and diversification benefits that reduce the "cost of uncertainty." These difference-based and non-difference-based sources of GFT are in addition to the standard pro-competitive effect of trade (which is present with and without uncertainty). Differences in risk aversion generate GFT because they introduce indirect (non-cooperative) risk-sharing benefits, viewed as implicit partial insurance. To demonstrate this implicit insurance effect, consider the following. As is well known, bargaining or collusive equilibria always result in Pareto efficient risk-sharing, i.e., efficient insurance. We do not get this result, however, with non-cooperative interactions as in our duopoly model. Nevertheless, although there is no direct risk-sharing in our duopoly model, implicit risk-sharing, albeit not Pareto efficient, still occurs.[4] Moreover, the extent to which it occurs depends on the RRAC parameters.

As to the diversification benefits, the mere existence of a risky foreign market introduces diversification possibilities. Market correlation can give rise to further diversification benefits. The intuition behind these sources of GFT is simple. Essentially, for two countries under autarky, trade is akin to introducing "new risky assets" to choose from when selecting an "optimal portfolio." Clearly, a larger feasible assets-set expands an investor's (portfolio choice) efficiency frontier in the absence of strategic behaviour. An expanded efficiency frontier allows for better risk-management/diversification by taking into account the new assets' (in our case, the foreign country's) risks and market correlation. It is not just the mere introduction of a new asset that is beneficial;

[3] Throughout the paper, when we talk about high and low correlation, we refer to the non-absolute values.

[4] For example, within a cooperative/bargaining framework, a risk-neutral agent always provides full insurance to a risk-averse agent, giving rise to Pareto efficient risk-sharing. However, in a non-cooperative duopoly model, the agents/firms do not collude or engage in cooperative bargaining. Therefore, although the resultant risk-sharing will not be Pareto efficient, it will not be entirely absent. See footnote 15 below.

it is the new risk-management possibilities that the new asset brings. However, the picture in our model is more complicated because, unlike in portfolio choice theory, the countries behave strategically. Consequently, each country's ultimate portfolio value depends on its rival's actions and is determined by the game's Nash equilibrium. Nevertheless, the diversification benefits of an expanded choice set are still present.

We show that the country whose risk or risk-aversion is relatively high always gains from trade. The other country will, generally, also gain unless markets are highly, positively correlated. However, the world always gains from trade, regardless of whether the other country gains. We also show that, in general, world gains from trade are likely to decrease with risk and the correlation, but the impact of risk aversion may be positive or negative (depending on whether the risk is low or high). We compare gains from trade with and without uncertainty and show that both world and country gains may, sometimes, be higher with uncertainty than without it. Finally, to assess the responsiveness of world gains from trade, we calculate local measures of the RRAC elasticities and find that world gains from trade are most responsive to changes in market correlation.

2 The Model

Consider two countries, each with one firm. To isolate the effects of uncertainty, we assume that the two firms (countries) are identical in all respects, except for those that pertain to risk or risk aversion. Furthermore, we make the common assumption that the markets in the two countries are physically distinct. Hence, we assume that the firms produce an identical product, x, using the same technology. Denote the amount of x that the firm in Country i (Firm i) sells in Country j as x_{ij}, $i, j = 1, 2$. The firms' cost functions are linear and given by,

$$C_i = c_i(x_{i1} + x_{i2}),\ i = 1, 2$$

where C_i is total cost in Country $i = 1, 2$, and c_i is the fixed marginal (and average) cost. To focus on the impact of uncertainty, we ignore transportation (and fixed) costs.[5]

We assume that both countries' firms face uncertain output prices, $p_j,\ j = 1, 2$. Prices are uncertain because some demand function parameter is uncertain. Specifically, we take the two demand functions as:

$$p_j = a_j - (x_{1j} + x_{2j}),\ j = 1, 2, \tag{1}$$

where a_1 and a_2 are random variables with a joint distribution function $f(a_1, a_2)$, whose means and covariance matrix are given by $\mu = (\mu_1,\ \mu_2)$ and Σ (whose elements are σ_{ij}, where $\sigma_{ii} > 0$), respectively.[6] Demand curves are, therefore, uncertain because the intercepts are random.

[5] See Brander and Krugman (1983).

[6] Since we are not interested in the other demand functions' parameters, we set the slope parameters to one.

The firms' profits are given by:

$$\pi_1 = \sum_{j=1}^{2}([a_j - (x_{1j} + x_{2j})] - c_1)x_{1j}, \quad \pi_2 = \sum_{j=1}^{2}([a_j - (x_{2j} + x_{1j})] - c_2)x_{2j} \tag{2}$$

If we define the utility of the random variable π_i as $U_i[\pi_i]$ and if U_i has the expected utility form, then $U_i[\pi_i] = E[u_i(\pi_i)]$, where $u_i(\pi_i)$ is the utility of the realizations π_i (corresponding to the realization of a_1 and a_2). We assume that u_i is strictly monotonically increasing and (at least weakly) concave. In such a case, both firms maximize their expected utility of profits so that we can write their problems as:

$$\max_{x_{11},\ x_{12}} E[u_1(\pi_1)], \qquad \max_{x_{21},\ x_{22}} E[u_2(\pi_2)] \tag{3}$$

The first thing that is clear from these expected utility maximization problems is that while the two markets are physically distinct, they are no longer distinct for all nonlinear utility functions as far as the firms' decisions are concerned. For example, Firm 1 cannot choose its output in each market separately: x_{11} and x_{12} must be jointly chosen. We will pursue this point further later.

Unfortunately, using general (increasing and concave) utility functions substantially complicates the analysis. For example, even for a single decision-maker, properties of high order derivatives of the utility function, as well as moments of an order higher than two, may be required. Furthermore, in this model, we need to find the Nash Equilibrium of a game with four best-reply functions. Since this paper aims to explain trade patterns and gains from trade, we choose the simplest possible framework that enables us to do so. Thus, defining the mean and variance of π_i as $E(\pi_i)$ and $Var(\pi_i)$, we use the standard approximation of the expected utility, given by:

$$E\{u_i[\pi_i]\} \approx u_i\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\} \equiv u_i\{E(\pi_i) - \theta_i\},\ i = 1, 2, \tag{4}$$

where R_i is the measure of absolute risk aversion in Country i, which is assumed to be constant[7], and θ_i Country $i's$ risk premium, defined as:

$$\theta_i \equiv \frac{1}{2}R_i Var(\pi_i). \tag{5}$$

Since the utility functions, u_i, are strictly monotonically increasing, the maximization of $u_i\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\}$ is equivalent to the maximization of $E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)$ - in the sense that they yield the same solutions for the $x'_{ij}s$. Thus, the two countries' maximization problems can be written as:

$$\max_{x_{11},\ x_{12}}\{E(\pi_1) - \frac{1}{2}R_1 Var(\pi_1)\}, \max_{x_{21},\ x_{22}}\{E(\pi_2) - \frac{1}{2}R_2 Var(\pi_2)\}, \tag{6}$$

[7] In fact, it can be shown that if a_1 and a_2 are jointly elliptically distributed, expected utility is completely characterized by its mean and variance. Moreover, if we have a constant absolute risk aversion utility function, the expected utility of π_i is linear in the mean and variance of π_i.

where

$$E(\pi_1) = \sum_{j=1}^{2} ([\mu_j - (x_{1j} + x_{2j})] - c_1)x_{1j}, \quad E(\pi_2) = \sum_{j=1}^{2} ([\mu_j - (x_{2j} + x_{1j})] - c_2)x_{2j} \tag{7}$$

and,

$$Var(\pi_1) = x_{11}^2 v_1 + x_{12}^2 v_2 + 2x_{11}x_{12}\sigma_{12} = x_{11}^2 v_1 + x_{12}^2 v_2 + 2x_{11}x_{12}\rho\sqrt{v_1}\sqrt{v_2} \tag{8}$$

$$Var(\pi_2) = x_{21}^2 v_1 + x_{22}^2 v_2 + 2x_{21}x_{22}\sigma_{21} = x_{21}^2 v_1 + x_{22}^2 v_2 + 2x_{21}x_{22}\rho\sqrt{v_1}\sqrt{v_2},$$

where ρ, v_1 and v_2 are the correlation coefficient and variances, respectively.

From equations (6) and the variances in (8), it is clear that, in general, the firms cannot treat the two markets as distinct, even though they are physically distinct. Specifically, we have the following proposition:

Proposition 1 *If demand functions are uncertain, the two markets can be treated as distinct if and only if* $R_i\rho = 0$.

Proof. Let $\lambda_{ij}^i \equiv \frac{\partial^2\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\}}{\partial x_{ii}\partial x_{ij}}$, $i \neq j$. The two markets can be treated as distinct if and only if $\lambda_{ij}^i = 0$. But, $\frac{\partial^2\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\}}{\partial x_{ii}\partial x_{ij}} = -R_i\rho\sigma_i v_j$. Thus, the two markets can be treated as distinct, if and only if $R_i\rho = 0$. In other words, we need either risk neutrality ($R_i = 0$), or no correlation ($\rho = 0$).[8] ■

Therefore, although the markets are physically distinct, they will not be treated as distinct unless firms are risk neutral or demand functions are uncorrelated. Consequently, each country must choose its outputs in the two markets simultaneously.

Now, defining the firms' output vectors as $x_1 = (x_{11}, x_{12})$, $x_2 = (x_{21}, x_{22})$ and denoting the two countries' parameter vectors $\gamma_1 \equiv (\mu_1, \mu_2, c_1, v_1, v_2, \rho, R_1)$ and $\gamma_2 \equiv (\mu_1, \mu_2, c_2, v_1, v_2, \rho, R_2)$, we can write the two objective functions as:

$$F^1(x_1, x_2; \gamma_1) \equiv E(\pi_1) - \frac{1}{2}R_1 Var(\pi_1) \tag{9}$$

$$F^2(x_1, x_2; \gamma_2) \equiv E(\pi_2) - \frac{1}{2}R_2 Var(\pi_2) \tag{10}$$

It is easy to verify that the functions $F^1(x_1, x_2; \gamma_1)$ and $F^2(x_1, x_2; \gamma_2)$ are strictly concave in x_1 and x_2, respectively.[9]

Defining the vector of all parameters as $\gamma \equiv (\gamma_1, \gamma_2) = (\mu_1, \mu_2, c_1, c_2, v_1, v_2, \rho, R_1, R_2)$, the Nash equilibrium of this two-firm game is given by (the vectors) $x_1^*(\gamma) = \{x_{11}^*(\gamma), x_{12}^*(\gamma)\}$, and $x_2^*(\gamma) = \{x_{21}^*(\gamma), x_{22}^*(\gamma)\}$, such that:

$$\begin{aligned} F^1[x_1^*, x_2^*; \gamma_1] &\geq F^1[x_1, x_2^*; \gamma_1] \\ F^2[x_1^*, x_2^*; \gamma_2] &\geq F^2(x_1^*, x_2; \gamma_2) \end{aligned}$$

[8] Or both.

[9] For example, in equation (9), the mean is concave in x_1 and, since the variance is a convex function, $-\theta_i$ is concave. The same is true for (10). Moreover, we can easily verify that the determinant of the corresponding Hessian matrix is strictly positive.

In other words, the pair (x_1^*, x_2^*) is the simultaneous solution to the two problems:

$$\max_{x_1}\{\,F^1(x_1, x_2; \gamma_1)\}$$

$$\max_{x_2}\{\,F^2(x_1, x_2; \gamma_2)\}$$

The two countries' first-order conditions are given by the following equations, respectively,

$$\begin{aligned}\frac{\partial F^1(x_1, x_2; \gamma_1)}{\partial x_{11}} &= 0, \quad \frac{\partial F^1(x_1, x_2; \gamma_1)}{\partial x_{12}} = 0 \\ \frac{\partial F^2(x_1, x_2; \gamma_2)}{\partial x_{21}} &= 0, \quad \frac{\partial F^2(x_1, x_2; \gamma_2)}{\partial x_{22}} = 0\end{aligned}$$

These conditions define a pair of best reply functions for each country, given by:[10]

$$\begin{aligned} x_{11} &= \frac{1-(2R_1\rho\sqrt{v_1}\sqrt{v_2}x_{12}+2x_{21})}{2(R_1v_1+2)}, \; x_{12} = \frac{1-(2R_1\rho\sqrt{v_1}\sqrt{v_2}x_{11}+2x_{22})}{2(R_1v_1+2)} \\ x_{21} &= \frac{1-(2R_2\rho\sqrt{v_1}\sqrt{v_2}x_{22}+2x_{11})}{2(R_2v_1+2)}, \; x_{22} = \frac{1-(2R_2\rho\sqrt{v_1}\sqrt{v_2}x_{21}+2x_{12})}{2(R_2v_1+2)} \end{aligned} \tag{11}$$

As can be seen from these best reply functions, the pairs (x_{11}, x_{12}), (x_{11}, x_{21}), (x_{22}, x_{12}) and (x_{22}, x_{21}) are strategic substitutes, whereas the pairs (x_{11}, x_{22}) and (x_{12}, x_{21}) are strategic neutrals.

The solution to these four equations can be solved explicitly to obtain the Nash equilibrium solution.[11] Let this solution be denoted as:

$$x_1^*(\gamma) = \{x_{11}^*(\gamma), x_{12}^*(\gamma)\}, \quad x_2^*(\gamma) = \{x_{21}^*(\gamma), x_{22}^*(\gamma)\} \tag{12}$$

and let the corresponding maximum functions be defined as[12]:

$$F^{*i}(\gamma) \equiv F^i[x_1^*(\gamma), x_2^*(\gamma); \gamma_i]. \tag{13}$$

3 Trade Patterns

Given the Nash equilibrium values of $x_1^*(\gamma)$, and $x_2^*(\gamma)$, as defined in equations (12), we can now examine the effects of uncertainty on equilibrium trade patterns. Define net exports of countries 1 and 2 as:

$$z_1^*(\gamma) = x_{12}^*(\gamma) - x_{21}^*(\gamma), \quad z_2^*(\gamma) = x_{21}^*(\gamma) - x_{12}^*(\gamma) = -z_1^*(\gamma) \tag{14}$$

[10] Given that the $F^1(x_1, x_2; \gamma)$ and $F^1(x_1, x_2; \gamma)$ functions are strictly concave, the best reply functions are continuous.

[11] The expressions for $x_1^*(\gamma)$ and $x_2^*(\gamma)$ are given in Appendix 6.1.

[12] The expressions are rather long, so they are not included in the paper but are available upon request.

First, note that there are eleven parameters in the model (given by the vector γ). However, we are interested in two identical countries in all respects, except those related to risk and risk aversion. Thus, to isolate the impact of uncertainty, we assign specific, equal values to all parameters that are not directly related to uncertainty. We set the values of those "unrelated" parameters as follows: (i) marginal costs: $c_1 = c_2 = .5$, (ii) means of demand intercepts: $\mu_1 = \mu_2 = 1$.[13] Specifying these values leaves us with five "uncertainty-related" parameters: four are country-specific: $R_1,\ R_2,\ v_1,\ v_2$ and one is the common ρ.

From equations (11), it is clear that obtaining the effects of parameter changes on the patterns of trade or the level of net exports may not always be easy to obtain. First, all best reply functions are highly nonlinear in the parameters, so results may very well be "local" rather than "global.". Second, and more importantly, changes in $v_1,\ v_2$ and ρ shift *all* four best reply functions, and a change in each country's risk aversion shifts its *pair* of best reply functions. Consequently, changes in $v_1,\ v_2$ and ρ, directly and indirectly, affect the equilibrium levels of net exports. Such changes directly shift all best reply functions, hence changing each x_{ij}, for any given values of the other outputs. But, because all other best reply functions also shift, the values of other outputs also change. Therefore, the equilibrium values of $x^*_{12}(v_1, R_1, \rho)$ and $x^*_{21}(v_1, R_1, \rho)$ change although x_{12} and x_{21} are strategic neutrals. A change in a country's measure of risk aversion directly shifts its two best reply functions, but it does not directly affect the other country's two best reply functions. However, a shift in a Country 1's best reply functions, for example, indirectly affects the values of all other outputs, thus changing the equilibrium values of $x^*_{12}(v_1, R_1, \rho)$ and $x^*_{21}(v_1, R_1, \rho)$, even though they are strategic neutrals. For the remainder of this section, we look at the effects of parameter changes on (Country 1's) net export, the difference between $x^*_{12}(v_1, R_1, \rho)$ and $x^*_{21}(v_1, R_1, \rho)$ rather than $x^*_{12}(v_1, R_1, \rho)$ and $x^*_{21}(v_1, R_1, \rho)$ individually.

As a first step, to separate the risk and risk aversion effects, we consider two cases. In the first case, we assume that risks (variances) are the same, but the measures of risk aversion differ, whereas in the second case, we assume that the measures of risk aversion are the same, but risks differ. Since ■ is a common parameter, we do not examine its effects separately but, instead, we examine the extent to which it amplifies, or mitigates, the effects of differences in risk and risk aversion.

3.1 Differences in Measures of Risk Aversion

Setting $v_1 = v_2 = v$ and using the Nash equilibrium values of the firms' outputs, we calculate the countries' net export. These are given by:

$$z_1^* = -\frac{1}{2}\frac{(1+\rho)(R_1 - R_2)v}{(1+\rho)^2 R_1 R_2 v^2 \ + 2v(1+\rho)(R_1 + R_2) + 3}, \quad z_2^* = -z_1^* \tag{15}$$

[13] And, of course, the demand functions' slopes were already set to 1.

Thus, when $v_1 = v_2 = v > 0$, we have:

Proposition 2: For all values of $v > 0$,

(i) If $R_2 < R_1$ and $\rho > -1$, County 2 is a net exporter of x ($z_2^* > 0$), and Country 1 is a net importer of x.

(ii) If $R_2 < R_1$ but $\rho = -1$, neither country is a net importer ($z_1^* = z_2^* = 0$).[14]

Alternatively, Proposition 2 can be restated as:

Proposition 2a: For all values of $v > 0$, if Country 2 is relatively low-risk-aversion abundant ($R_2 < R_1$) and $\rho > -1$, then Country 2 is a net exporter of low risk-aversion, whereas Country 1 is a net importer of low risk-aversion.

Thus, although risk aversion is country-specific and immobile, effectively, it is imported/exported via net exports of the product. This result is reminiscent of the standard notion of the "factor content" of trade. In our context, we can think of it as the "risk aversion content of trade" (although its measurability is not clear). If $R_2 < R_1$, we can think of Country 2 as having a risk-aversion-driven uncertainty-cost comparative advantage. As a result, Country 2 is a net exporter of x. By importing x, Country 1 effectively imports lower risk aversion, mitigating its risk-aversion-driven uncertainty-cost comparative disadvantage. Alternatively, we can interpret the result as reflecting implicit risk-sharing.[15]

3.2 Differences in Risk

Setting $R_1 = R_2 = R > 0$ and using the Nash equilibrium values of the firms' outputs we calculate the countries' net export as:

$$z_1^* = \frac{1}{2} \frac{(v_1 - v_2)R}{(1-\rho^2)v_1 v_2 R^2 \ + 3R(v_1 + v_2) + 9} \tag{16}$$

Thus, when $R_1 = R_2 = R > 0$, we have:

Proposition 3: For all values of ρ and R, if $v_1 > v_2$, Country 1 is a net exporter of x (i.e., $z_1^* > 0$), and Country 2 is a net importer of x.[16]

Alternatively, Proposition 3 can be restated as:

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[14] Proposition 2 follows immediately from equation (15) and the fact that if $\rho > -1$, both numerator and denominator are strictly positive, but when $\rho = -1$, the numerator is zero.

[15] It can be easily verified that with full collusion, with $R_1 = 0 < R_2$, the solution will always be $x_{11}^* = x_{12}^* = 0$, $x_{21}^* > 0$, $x_{22}^* > 0$. Namely, full insurance. On the other hand, without collusion, we have $x_{11}^* > 0$, $x_{12}^* > 0$. The two countries' total (utility of) profits will be higher with full insurance than without it, and thus, assuming the firms share that total (with Country 1's share being fixed), they will each be better off.

[16] Since in equation (16) $v_1 > v_2$ and $(1 - \rho^2) \geq 0$.

Proposition 3a: If Country 1 is relatively high-risk abundant ($v_1 > v_2$), then, for all values of ρ and R, it is a net importer of low risk (through net exports of x), whereas Country 2 is a net exporter of low risk.

As in the case of differences in risk aversion, we can think of it as the "risk content of trade." Even though a country's risk is immobile, it is imported/exported via net exports/imports of the product. When $v_2 < v_1$, we can think of Country 2 as having a low-risk-driven uncertainty-cost comparative advantage. But, Country 1 can "import" Country 2's low (risk-driven) uncertainty cost technology by diverting sales to Country 2 through exports. Doing so mitigates Country 1's uncertainty-cost comparative disadvantage. In part, this reflects the benefits of diversification.[17]

3.3 General Trade Patterns

In the two cases above, we compared the two countries by focusing on "one difference at a time." In the following, we carry out the comparison by allowing for differences in risk and risk aversion.

Since the countries' indexing is arbitrary, we assume that Country 1 is more risk-averse: $R_1 > R_2$. Moreover, since we are interested in the relative abundance of low risk-aversion and low risk, we first fix "benchmark values" for Country 2's risk and risk aversion parameters. We then measure Country 1's corresponding parameters relative to those benchmark values. Specifically, we take $R_2 = 1 < R_1$ and $v_2 = 1$, but we do not make assumptions regarding the ranking of v_1 and v_2, and neither do we make assumptions regarding the common parameter, ρ. Thus, this leaves us with three parameters: v_1, R_1 and ρ. Furthermore, changes in v_1 and R_1 can now be viewed as changes relative to the benchmark values of v_2 and R_2.

Country 1's Nash equilibrium level of net exports is now given by:[18]

$$z_1^* = x_{12}^*(v_1, R_1, \rho) - x_{21}^*(v_1, R_1, \rho) \tag{17}$$

Note that to determine the effects of parameter changes on the *level* of net exports, we need to know the signs of the derivatives of $z_1^*(v_1, R_1, \rho)$. But, to determine *trade patterns,* it suffices to know the *sign* of $z_1^*(v_1, R_1)$. Although these two questions are complementary and closely related, they require separate analyses. For example, if we find that over the whole parameter space, we have $\partial z_1^*(v_1, R_1, \rho)/\partial v_1 > 0$, it is likely that "*eventually*" we would have $z_1^*(v_1, R_1, \rho) > 0$. Locally, however, a marginal increase of v_1 may increase $z_1^*(v_1, R_1, \rho)$, but this may not be enough to make $z_1^*(v_1, R_1)$ strictly positive.

[17] Which, by the way, would exist even with identical risks.
[18] The precise solution is given in Appendix 6.2.

3.3.1 The Determinants of Trade Patterns

For any level of net exports by Country 1, given by k, define Country 1's net exports iso-value curve (IVC), denoted by S_k^1, as:

$$S_k^1 \equiv \{(v_1, R_1, \rho) \ : z_1^*(v_1, R_1, \rho) = k\} \tag{18}$$

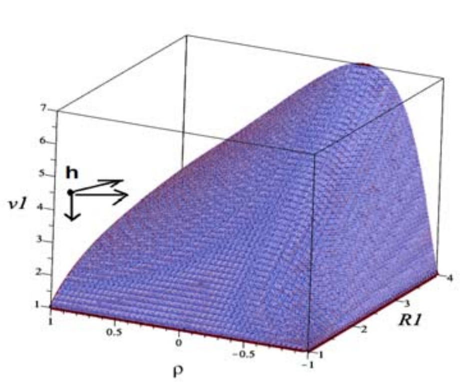
For example, S_0^1 is the zero net exports IVC, taking $k = 0$. The (strict) net import and net export sets (NIS, and NES, respectively) are then defined by:

$$NIS \equiv \{(v_1, R_1, \rho) \ : z_1^*(v_1, R_1, \rho) < 0\} \tag{19}$$

$$NES \equiv \{(v_1, R_1, \rho) \ : z_1^*(v_1, R_1, \rho) > 0\} \tag{20}$$

In other words, NIS and NES are the sets of the parameters (v_1, R_1, ρ) that, respectively, lie on "opposite sides" of the iso-value curve S_0^1.

The S_0^1 IVC with the corresponding NIS and NES (in three-dimensional (v_1, R_1, ρ) space) are shown in Figure 1. The skewed "tent-like" blue surface shows the iso-value curve S_0^1. The corresponding NIS and NES are the areas below and above that blue surface, respectively.[19] As Figure 1 shows, the S_0^1 curve is continuous[20] and "well-behaved." For example, Figure 1a shows the contours of S_0^1 for different values of v_1 (higher values are farther away from the origin). As Figure 1a shows, NIS is a convex set, so that S_0^1 is quasi-concave relative to the (R_1, ρ) base.[21]



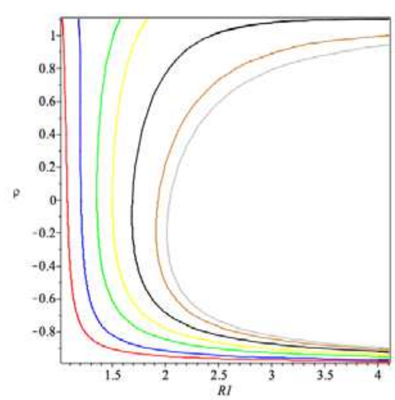


Figure 1: Zero Net Exports Iso-Surface. Figure 1a: Top View of Iso-Value Curves.

A single parameter change corresponds to cross-sectional movements in the three-dimensional (v_1, R_1, ρ) space. The effects of such parameter changes on the patterns of trade are shown in Figure 1. We do the following to

[19] Notice that for all $0 < v_1 < 1$, Country 1 is always an importer of x.
[20] Its continuity can be verified from the explicit expression for equation (17).
[21] But NES is not convex.

determine whether a parameter change makes it more likely for Country 1 to be an importer of x. We pick any initial point strictly above S_0^1 (strictly inside the NES). For example, take a point h in Figure 1 that lies (strictly) above S_0^1. Since point h is above S_0^1, we know that, at h, Country 1 is a net exporter. Now, from point h, there are three possible cross-sectional movements: up/down (a change in v_1), forward/backward (a change in R_1) and left/right (a change in ρ). These three possible cross-sectional movements represent a change in one of the three parameters. For each of these cross-sectional movements, the direction we need to follow to "hit" the iso-value curve S_0^1 is the direction that increases the likelihood that Country 1 will become a net importer. On the other hand, if the initial point h is already (strictly) below S_0^1 (inside NIS), any cross-sectional movement toward the surface makes it less likely that Country 1 remains an importer of x. As mentioned above, given the nonlinearity of S_0^1, the result may depend on the location of point h.

Thus, Figure 1 shows that:

Proposition 4: Other things being equal, (i) *Country 1 is more likely to be an importer of* x (an importer of low risk-aversion) *when its measure of risk aversion is high* (ii) *Country 1 is more likely to be an exporter of* x (hence an importer of low risk) *when its risk is high.*

Proposition 4 and the intuition behind it are similar to and consistent with propositions 2 and 3 above.

Unlike the global results in Propositions 4, the effect of a change in correlation is local. As Figure 1 shows, starting from an initial position that lies (strictly) above S_0^1 (say, point h) where correlation is positive and high, Country 1 is more likely to be an importer as ρ decreases and moves to the right toward the "ridge/spine" of S_0^1. Beyond that "ridge," any further decrease in ρ makes Country 1 less likely to be an importer. Specifically, define the point where the effect of a decrease in ρ changes its sign as $\widetilde{\rho}(R_1)$.[22] Then, we have:

Proposition 5: *For all $\rho > \widetilde{\rho}$ (and given values of* v_1 and R_1), *Country 1 is more likely to become an importer as ρ decreases. But, for all $\rho < \widetilde{\rho}$, Country 1 is less likely to become an importer as ρ decreases.*

The effect of correlation is not global because, in general, diversification benefits are not monotonic in ρ over its whole domain. This result is related to the usual result in portfolio choice theory, where, generally, corner solutions for asset holdings are not optimal. For example, Country 1's "cost of uncertainty," as captured by its risk premium (and primarily determined by the variance of profits), is not monotonic in ρ for all $\rho \in [-1..1]$.[23]

[22] The value for $\widetilde{\rho}$ can be obtained as follows. Solve the equation $z_1^*(v_1, R_1, \rho) = 0$ for v_1 to obtain $v_1 = v_1(R_1, \rho)$. Then solve the problem: $\max_\rho\{v_1(R_1, \rho)\}$. The solution to this problem is $\widetilde{\rho}(R_1)$. In other words, for any R_1, $\widetilde{\rho}(R_1)$ is the value of correlation that maximizes v_1, giving us the top of the ridge. It is easy to show that (as Figure 1 shows) $\widetilde{\rho}(R_1)$ decreases with R_1.

[23] Neither "corner" represents minimum profits variance, which is similar to what standard mean-variance portfolio choice theory suggests.

3.4 Comparative Statics

The effects of the three parameters on the Nash equilibrium level of net exports are given by:

$$\frac{\partial z_1^*(v_1, R_1, \rho)}{\partial n}, \; n = v_1, R_1, \rho$$

To obtain the signs of these effects, we define the zero-derivative iso-value curves $D^k(v_1, R_1, \rho)$ as,

$$D^n(v_1, R_1, \rho) \equiv \{(v_1, R_1, \rho) \; : \frac{\partial z_1^*(v_1, R_1, \rho)}{\partial n} = 0\}, \quad n = v_1, R_1, \rho$$

The iso-value curves $D^{R_1}(v_1, R_1, \rho)$ and $D^{\rho}(v_1, R_1, \rho)$ are shown in Figures 2a and 2b by red and blue surfaces, respectively. They enable us to determine the effects of a marginal increase in R_1 and ρ on Country 1's net exports. Within the area to the right of the red iso-value curve in Figure 2a, we have $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} < 0$ (and to the left of the red surface, we have $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} > 0$). Therefore, as Figure 2a shows, the parameter set for which $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} > 0$ is negligible.[24] In other words, in general, an increase in R_1 reduces Country 1's net export level, effectively increasing the incentive to import low risk-aversion from Country 2.[25] The exception to this negative relationship between R_1 and z_1^* is not very likely and can only happen when the correlation is close to -1,[26] and even then, primarily, for low measures of risk aversion.

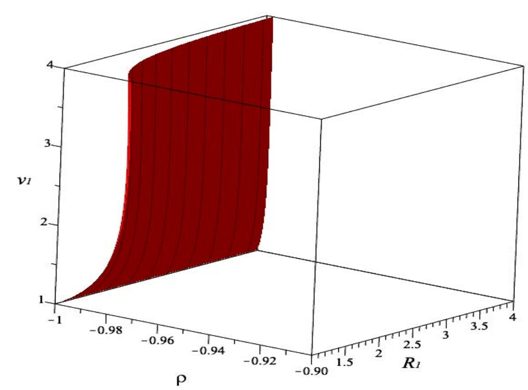


Figure 2a: The Marginal Effects of R_1.

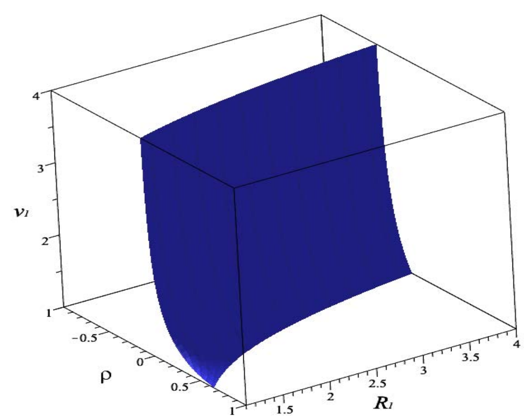


Figure 2b: The Marginal Effects of ρ.

[24] To better identify the impact, the correlation range in Figure 2a is reduced to $\rho \in [-.90, \; -1]$.

[25] Diagram 2a is shown for values of v_1 that are higher than 1. The reason is that for all $0 < v_1 < 1$, we always have: $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} < 0$.

[26] It must be somewhere within the interval $[-.972, \; -1]$, depending on the values of v_1 and R_1. The precise value can be obtained as follows. Solve the equation $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} = 0$ for ρ. Define the solution as $\rho(v_1, R_1)$. In other words, $\frac{\partial z_1^*(v_1,R_1,\rho(v_1,R_1))}{\partial R_1} = 0$. Then, for all $\rho > \rho(v_1, R_1)$, we have $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} < 0$ and for all $\rho < \rho(v_1, R_1)$ we have $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} > 0$. It can be shown that $\overline{\rho} = -.972 \geq \rho(v_1, R_1) \geq -1$, where $\overline{\rho} = -.972$ is the lowest possible correlation that still *guarantees* that $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} < 0$ for all values of v_1, R_1. It is the solution to the problem $\min_{v_1,R_1} \{\rho(v_1, R_1)\}$. Thus, a sufficient (but not a necessary) condition for the requirement that $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} < 0$ for all values of v_1, R_1 is: $\rho > -.972$. A necessary and sufficient condition is that $\rho > \rho(v_1, R_1)$, which is less restrictive; thus, it is less likely to have $\rho < \rho(v_1, R_1)$ (i.e., $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial R_1} > 0$).

Hence, we have:

Proposition 6: For any correlation such that $\rho > -.972$, *an increase in Country 1's measure of risk aversion reduces its net exports of* x*, hence increasing its imports of low risk-aversion from Country 2.*

Within the area in front of the blue curve in Figure 2b, we have $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial \rho} > 0$. Thus, as Figure 2b shows, for any given $v_1 > 0$ and $R_1 > 1$, for sufficiently low (non-absolute) values of ρ (and for almost all $\rho < 0$), Country 1's net exports decrease with correlation (i.e., its net import increase). But, as correlation becomes higher (in non-absolute terms), beyond some point (which "comes sooner" for high v_1 and R_1), when ρ is sufficiently high (for the same v_1 and R_1), Country 1's exports increase with correlation. This result is consistent with the non-monotonicity in ρ seen in Figure 1 (reflecting the non-monotonicity of diversification benefits).

The effect of an increase in v_1 is not shown because $\frac{\partial z_1^*(v_1,R_1,\rho)}{\partial v_1}$ is always positive, which means that Country 1's net exports increase with its risk for all values of (v_1, R_1, ρ). Thus we have:

Proposition 7: *An increase in Country 1's risk increases its net exports of* x*, thereby increasing its imports of low risk from Country 2, for all* values of (v_1, R_1, ρ).

These comparative statics results are consistent with the findings regarding trade patterns. There is, however, a difference. When we examined trade patterns, we considered the sign of $z_1^*(v_1, R_1, \rho)$, whereas here, we look at the marginal effect of parameter changes. But, for example, a marginal effect of a parameter change may take us in "the direction" of $z_1^*(v_1, R_1, \rho) < 0$ but may not get us there "yet." It may do so eventually, but not necessarily with a marginal change.

3.5 Special Case: Country 2 is Risk-Neutral

In this section, we consider the special case when Country 2 is risk-neutral. Specifically, we take $R_1 > R_2 = 0$ and $v_2 = 1$. Using the Nash equilibrium values of the firms' outputs, we calculate Country 1's corresponding net exports as:

$$z_1^* = -\frac{R_1}{2} \frac{2 + v_1 + 2R_1 v_1(1-\rho^2) + 3\rho\sqrt{v_1}}{8(1-\rho^2)v_1 R_1^2 + 12R_1(v_1+v_2) + 18} \tag{21}$$

Since ρ may be positive or negative, the sign of the numerator is ambiguous. Thus, we do not know if Country 1 is a net importer or net exporter in general. However, equation (21) implies that if $\rho \geq 0$, Country 1 is a net importer ($z_1^* < 0$). Moreover, Country 1 can only be a net exporter when the correlation is negative and high in absolute terms. But how likely is it for this to happen? To answer this question, note that Country 1's IVC, S_0^1, in this case, is given by:

$$S_0^1 \equiv \{(v_1, R_1, \rho) \ : \frac{R_1}{2}\frac{v_1 + 2 + 2R_1 v_1(1-\rho^2) + 3\rho\sqrt{v_1}}{8(1-\rho^2)v_1 R_1^2 \ + 12R_1(v_1+v_2) + 18} = 0 \tag{22}$$

The S_0^1 IVC is shown in Figure 3 below. For all parameter combinations "outside" (left) of the red surface, we have $z_1^* > 0$, and for all parameter combinations "inside" the red surface, we have $z_1^* < 0$.[27]

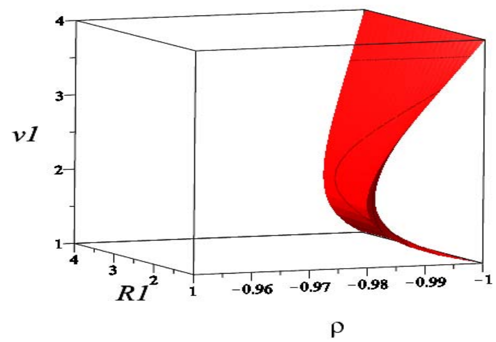


Figure 3: S_0^1 IVC When Country 2 is Risk Neutral.

As is clear from Figure 3, when Country 2 is risk-neutral but Country 1 is risk-averse, it is highly unlikely for Country 1 to be a net importer: the area to the right of the red IVC is "almost negligible," for all values of v_1 and R_1. For example, a simple calculation shows that even when $R_1 = 10^{-5}$, to be within the NIS, we must have a close to perfect negative correlation. Thus, when facing a risk-neutral country, we conclude that a risk-averse country is not likely to be a net importer, regardless of the values of risk and risk aversion parameters, and even if the correlation is near -1. We can view this result as reflecting the importance of implicit risk-sharing when Country 2 is risk-neutral.

3.6 Prices

Before we conclude this section, let us examine the effects of RRAC on prices in the two countries. Using the Nash equilibrium values of the firms' outputs (as defined in equation (12)), total amounts sold in Countries 1 and 2 are $(x_{11}^* + x_{21}^*)$ and $(x_{22}^* + x_{12}^*)$, respectively, and the corresponding Nash equilibrium prices are defined as $p_1^* = p(x_{11}^* + x_{21}^*),\ p_2^* = p(x_{22}^* + x_{12}^*)$. Then, with the normalization $R_2 = 1,\ v_2 = 1$, we obtain the following two propositions:

Proposition 8: For all values of $R \geq 1,\ v_1 > 1$ and for all values of ρ, we have $x_{12}^* - x_{11}^* > 0$ and $x_{22}^* - x_{21}^* > 0$.

[27] Again, to better identify the zone for which $z_1^* < 0$, the correlation range in Figure 2a is reduced to $\rho \in [-.90, \ -1]$.

Proof. It is easy to show that for Countries 1 and 2, the sets of parameters $B^1 \equiv \{(v_1, R_1, \rho)\colon\ x^*_{12} - x^*_{11} \leq 0\}$ and $B^2 \equiv \{(v_1, R_1, \rho)\colon\ x^*_{22} - x^*_{21} \leq 0\}$ are empty for all $R \geq 1,\ v_1 > 1$ and $-1 \leq \rho \leq 1$. Thus, both countries always sell more in Country 2 than in Country 1. ∎

Proposition 8a: For all values of $R \geq 1,\ v_1 > 1$ and $-1 \leq \rho \leq 1$, we have $p^*_2 < p^*_1$.

Proof. From Proposition 8, it follows that $x^*_{12} + x^*_{22} > x^*_{11} + x^*_{21}$, which, in turn, implies that $p(x^*_{11} + x^*_{21}) >\ p^*_2 = p(x^*_{22} + x^*_{12})$. ∎

Proposition 8a implies that consumer surplus will be higher in Country 2 than in Country 1. We will use this result in the next section when we compare the countries' gains from trade.

4 The Gains from Trade

4.1 Autarky

Under autarky, each country has a single monopolist. If we set $x_{ij} = 0$ for $i \neq j$, then the mean and variance of profits of monopolist $i's$ profits are given by equations (7) and (8). Specifically, under autarky, we have:

$$E(\pi_i) = (\mu_i - x_{ii} - c_i)x_{ii}, \quad i = 1, 2 \tag{23}$$

$$Var(\pi_i) = x^2_{ii}v_i, \quad i = 1, 2 \tag{24}$$

The corresponding maximization problem is, then, given by (equation (6) with $x_{ij} = 0$ for $i \neq j$):

$$\max_{x_{ii}}\{(\mu_i - x_{ii} - c_i)x_{ii} - \frac{1}{2}R_i x^2_{ii}v_i\},\ i = 1, 2 \tag{25}$$

Or, using the notation in equations (9) and (10) above, this can be written as:

$$\max_{x_{ii}}\{F^{ai}(x_{ii}; \gamma^a_i)\}$$

where $\gamma^a_i = (\mu_i, v_i, R_i, c_i)$ is the vector of parameters in Country i and $F^{ai}(x_{ii}; \gamma^a_i)$ is the objective function of Country i under autarky.

Define the solution for this problem as: $x^{*a}_{ii}(\gamma^a_i),\ i = 1, 2$, and let the corresponding maximum function be defined as: $F^{*ai}(\gamma^a_i) \equiv F^{ai}(x^{a*}_{ii}(\gamma^a_i); \gamma^a_i)$. The explicit solution for $F^{*ai}(\gamma^a_i)$ is easily obtained as:

$$F^{*ai}(\gamma^a_i) = \frac{1}{2}\frac{(\mu_i - c_i)^2}{R_i v_i + 2} \tag{26}$$

We set the values of the "uncertainty-unrelated" parameters in line with the normalizations applied in earlier sections: (i) marginal costs: $c_1 = c_2 = .5$, (ii) means of demand intercepts: $\mu_1 = \mu_2 = 1$. As a result, we are left with two parameters in each country, so $\gamma_i^a = (v_i, R_i)$, and the solution becomes:

$$F^{*ai}(R_i, v_i) = \frac{1}{8}\frac{1}{R_i v_i + 2} \tag{27}$$

where the Nash equilibrium value of x_{ii} is:

$$x_{ii}^{*a}(R_i, v_i) = \frac{1}{2}\frac{1}{R_i v_i + 2} \tag{28}$$

4.2 The Countries' Gains from Trade

We take the sum of consumer and producer surplus to measure welfare in both the autarky and trade cases. Since we use linear demand functions, the calculation of consumer surplus is quite simple. Specifically, using Country $i's$ Nash equilibrium solutions, with and without trade ($x_i^*(\gamma)$ as defined in equations (12) and $x_{ii}^{*a}(\gamma_i^a)$, as defined in equations (28)), consumer surplus in Country i, with and without trade, denoted as, cs^i and cs^{ai}, respectively, is given by:

$$cs^i = \frac{1}{2}[x_{ii}^*(\gamma) + x_{ji}^*(\gamma)]^2, \quad cs^{ai} = \frac{1}{2}[x_{ii}^{*a}(\gamma_i^a)]^2 \tag{29}$$

Country $i's$ welfare, with and without trade is, therefore, given by:

$$w^i(\gamma) = F^{*i}(\gamma) + cs^i(\gamma), \quad w^{ai}(\gamma_i^a) = F^{*ai}(\gamma_i^a) + cs^{ai}(\gamma_i^a) \tag{30}$$

Given our normalizations of Country 2's RRA parameters ($v_2 = 1,\ R_2 = 1$), the only remaining (three) parameters are $v_1,\ R_1$ and ρ. Thus, we can write the countries' gains from trade as,

$$G^1(v_1, R_1, \rho) \equiv w^1(v_1, R_1, \rho) - w^{a1}(R_1, v_1), \quad G^2(v_1, R_1, \rho) \equiv w^2(v_1, R_1, \rho) - w^{a2}(1, 1) \tag{31}$$

Two points are worth mentioning before we consider the effects of uncertainty on the countries' GFT. First, it is important to remember that, as is well known, one source of GFT is the pro-competitive effect of trade, which results in lower markups, thus reducing prices. This pro-competitive effect is, of course, present with and without uncertainty. Therefore, in addition to measuring the GFT (comparing autarky with trade), it is also important to compare GFT with and without uncertainty. Such a comparison will provide what can be viewed as the net gain from trade (NGFT), disentangling gains due to the pro-competitive effect from uncertainty-related ones. Since all parameters that are not directly related to uncertainty were taken as constants, the gains from trade with no uncertainty are also constant and equal.[28] We can calculate them by taking $R_i = 0$ (thus making the risk premia in the two countries equal to zero) and define them as $G_{nu}^1 = G_{nu}^1 = G_{nu} > 0$.[29] Then, Country $i's$ NGFT is given by $G^i(\gamma) - G_{nu}$.

[28] Since the countries only differ in terms of RRA, with no uncertainty, they become identical, so their GFT are the same.
[29] Where $G_{nu} = 5/288$.

Second, given our normalizations, it follows immediately from (27) and (28) that for all $R_1 > 1$ and $v_1 > 1$, Country 2's welfare, under autarky, is higher than Country 1's: $w^{a2} > w^{a1}$.

Now, for any GFT level given by q_i, define Country $i's$ corresponding gains from trade iso-value curve, B_q^i, as:

$$B^i_{q_i} \equiv \{(v_1, R_1, \rho)\ : G^i(v_1, R_1, \rho) = q_i\}, \quad i = 1, 2 \tag{32}$$

It is easy to verify that for Country 1, for all values of the parameters, we have $G^1(v_1, R_1, \rho) > 0$. In other words, the set of parameters B^1_0 is empty, so Country 1 always gains from trade.[30] However, although Country 1's GFT are always positive, it is unclear whether its gains without uncertainty are lower or higher than with uncertainty. The reason is that although uncertainty is costly with and without trade, its adverse impact will, in general, not be the same in the two cases. Thus, if uncertainty's impact on Country 1 is more severe under autarky than with trade, Country 1's GFT may be higher with uncertainty.

To compare the gains with and without uncertainty, we need to consider Country 1's IVC that corresponds to $q_1 = G^1_{nu}$, the level of Country $1's$ GFT without uncertainty. This IVC, defined by,

$$B^1_{G^1_{nu}} \equiv \{(v_1, R_1, \rho)\ : G^1(v_1, R_1, \rho) = G^1_{nu}\}, \tag{33}$$

is shown by the red curve in Figure 4a. For all combinations of parameters on the red surface, Country 1's GFT will have the same (positive value) with and without uncertainty. All points to the left of the red surface represent strictly positive NGFT. As is clear from the figure, country 1's NGFT will be strictly positive for a wide range of parameter values. How likely is such a case to occur? Since G^i_{nu} is constant, all we need to know is which parameter combinations are likely to place us to the left of the red surface in Figure 4a. As Figure 4a shows, this is more likely to happen when RRAC are low.[31]

For Country 2, we define two iso-value curves: one for zero GFT (G^2_0) and one for the autarky value of GFT when there is no uncertainty (G^2_{nu}). These are shown in Figure 4b by the blue and green curves, respectively. Country 2's GTF are positive to the left of the blue G^2_0 curve. Its NGFT are positive to the left of the green curve. Between the green and blue curves, Country 2's GFT are positive but lower than under autarky with no uncertainty. As Figure 4b shows, Country 2's GFT are positive for a very wide range of parameter values (except when the correlation is positive and very high).[32] Furthermore, Country 2's NGFT will also be strictly positive

[30] If we breakup Country 1's GFT into their two components, we can show that, as a result of trade, consumers surplus always increases and, generally (but not always), producer surplus also increases (especially when v_1 is high, and the non-absolute value of ρ is low - but for $\rho < 0$ it always increases). The sum of these two components is, however, always positive.

[31] The non-absolute values in the case of correlation.

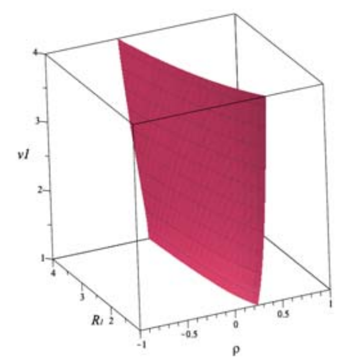
[32] As to the two components of Country 2's GFT, its change in producer surplus (going from autarky to trade) is positive over a smaller parameter set compared to country 1 (it increases when the non-absolute value of the correlation is low, R_1 is high and v_1 is low). Usually, the change in its consumer surplus is positive, but not always. For example, its consumer surplus decreases when the correlation is positive and very high (with a high R_1 and low v_1).

for a wide range of parameter values. Specifically, its NGFT are likely to be positive when ρ and v_1 are low and when R_1 is high.[33]

To easily compare the countries' GFT, we place the red and green curves from Figures 4a and 4b side by side in Figure 4c. As Figure 4c shows, Country 1's GFT are likely to be higher than Country 2's when v_1 is low, and R_1 is high. Moreover, Figure 4c also shows that Country 1's NGFT will be strictly positive over a "larger" set of parameter values than Country 2. Thus, even though Country 2's risk and risk aversion are lower than Country 1's, and its consumer and producer surpluses (before and after the trade) are higher than Country 1's, its GFT may not always be positive, whereas Country 1's GFT is always positive.

Hence, we have:

Proposition 8: (i) Country 1's gains from trade are always positive, and Country 2's gains from trade are positive unless the correlation is positive and very high. (ii) For a wide range of parameters, both countries' NGFT can be strictly positive. (iii) Country 1's NGFT are more likely to be strictly positive than Country 2's.



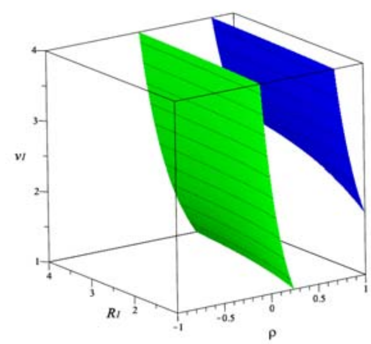


Figure 4a: Country 1's $G^1 = G^1_{nu}$ IVS Figure 4b: $G^2 = G^2_0$ (Blue) and $G^2 = G^2_{nu}$ (Green) IVCs.

[33] Moreover, its GFT decreases with ρ (unless ρ is nearly -1), decreases with v_1 (when ρ is sufficiently high) and increases with R_1 (over the whole domain shown in the diagram.

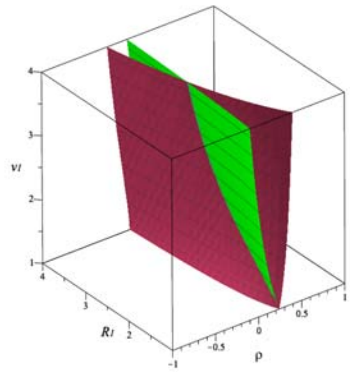


Figure 4c: $G^1 = G^1_{nu}$ (Red) and $G^2 = G^2_{nu}$ (Green) Iso-Value Curves.

Proposition 8 is based on the intuition behind the diversification and implicit insurance sources of GFT. It disentangles the pro-competitive effects from uncertainty-related ones and shows the RRAC parameters' role in determining the GFT.

4.3 World Gain from Trade

Having considered the two countries' GFT, we now examine world gain from trade (WGFT). As we showed, Country 2's gains may be negative. Thus, whether or not WGFT is positive becomes a particularly interesting and important question. To answer this question, first, we define the world gains from trade, G^w, as,

$$G^w(v_1, R_1, \rho) = G^1(v_1, R_1, \rho) + G^2(v_1, R_1, \rho)$$

Then, for any level WGFT given by q, define the iso-value curve, B^w_q, as:

$$B^w_q(v_1, R_1, \rho) \equiv \{(v_1, R_1, \rho) \ : G^w(v_1, R_1, \rho) = q\} \tag{34}$$

It is easy to verify that for all values of the parameter values, we have $G^w(v_1, R_1, \rho) > 0$.[34] In other words, the world always gains from trade. But, how will the world's gain compare to its gain with no uncertainty? As we did above, to compare the gains with and without uncertainty, we need to consider the world's IVC that corresponds to $q = G^1_{nu} + G^2_{nu} = 2G_{nu} \equiv G^w_{nu}$: the level of WGFT without uncertainty. The green surface in Figure 5 shows the IVC given by,

$$B^w_{G^w_{nu}} \equiv \{(v_1, R_1, \rho) \ : G^w(v_1, R_1, \rho) = G^w_{nu}\}.$$

WGFT will have the same (positive) value with and without uncertainty for all combinations of parameters along the green curve. All points to the left of the green surface represent positive world net gains from trade (WNGFT),

[34] Therefore, we cannot provide a diagram for the B^w_0.

i.e., positive gains that are greater with than without uncertainty (note that $G^w(v_1, R_1, \rho) = 0$ IVC is not shown in Figure 5 because we always have $G^w(v_1, R_1, \rho) > 0$). On the other hand, all points to the green curve's right represent positive WGFT but negative WNGFT (world gains are still strictly positive but lower than they would be with no uncertainty).

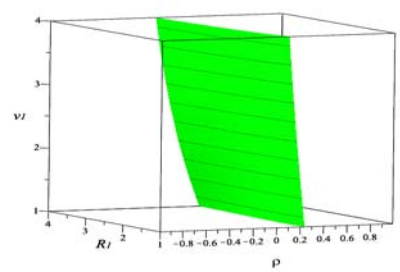


Figure 5: The World's $G^w = G^w_{nu}$ IVC. $G^w > G^w_{nu}$ to the Left of the Green Curve.

There are three possible cross-sectional movements for every initial point on the green curve, representing a change in one of the three parameters. The direction of a parameter change required to move to the left of the green curve is the change that increases WGFT, hence making WNGFT strictly positive. Since the green curve is nonlinear, the result may depend on the initial point's location on the green curve. Nevertheless, Figure 5 indicates that WNGFT will be strictly positive for a wide range of parameter values. Moreover, it is more likely to happen, primarily when v_1 and ρ are low. The impact of R_1 is more likely to depend on the initial position's location on the green curve: when risk is high, a higher R_1 moves us to the right of the green curve (reducing WGFT), whereas for low risk, a higher R_1 move us to the left of the green curve (increasing WGFT).

We can summarize these results with the following proposition:

Proposition 9: World gains from trade are: (i) always positive, and the world net gains from trade are often also positive. (ii) likely to decrease with risk and the correlation, but the impact of risk aversion depends on the risk level.

Proposition 9 implies that if transfer/compensation payments are possible, trade is likely to emerge.[35]

[35] When examining the figures above, it may be useful to consider the likely domain for the parameters v_1 and R_1. Specifically, how high can Country 1's risk and risk aversion be relative to Country 2? It may not be unreasonable to assume that the countries are not "radically" different. In such a case, we may wish to restrict the parameter space by taking, for example, $v_1 \leq 2v_2 = 2$, $R_1 \leq 2R_2 = 2$.

4.4 Elasticities

Finally, to get a sense of the importance of the qualitative comparative statics results, we calculate the RRAC elasticities of WGFT. We define the elasticity of $G^w(v_1, R_1, \rho)$ with respect to variable k as,

$$e_k(v_1, R_1, \rho) = \frac{\partial \ln[G^w(v_1, R_1, \rho)]}{\partial \ln(k)}, \quad k = v_1, R_1, \rho$$

First, note that these elasticities are all local measures. Second, since $G^w(v_1, R_1, \rho)$ itself is a rather complicated expression, clearly, so are the elasticities' expressions. Thus, to simplify matters, we provide diagrams of the elasticities for two fixed values of the correlation: $\rho = -.8$ and $\rho = .8$. The elasticity of G^w with respect to R_1 (red curve) and v_1 (green curve), with $\rho = -.8$ and $\rho = .8$, respectively, are shown in Figures 6a and 6b. Several points should be noted in these two figures. First, the absolute values of both $e_{R_1}(v_1, R_1, \rho)$ and $e_{v_1}(v_1, R_1, \rho)$ are smaller than 1, and this is true for both (fixed) correlation values. In other words, G^w is inelastic with respect to changes in risk and risk aversion. Second, in general, for a given correlation, the elasticity of WGFT with respect to R_1 (red curve) is greater than the elasticity of G^w with respect to v_1 (green curve). Third, elasticity with respect to risk is always higher (in absolute terms) when the correlation is positive.[36] However, the elasticity with respect to risk aversion is only higher (in absolute terms) when the correlation is positive if v_1 is high and R_1 is low.[37] Figure 6c shows the elasticity of WGFT with respect to the correlation, again with $\rho = -.8$ (red curve) and $\rho = .8$ (green curve). As Figure 6c shows, the elasticity of G^w with respect ρ is higher than $e_{R_1}(v_1, R_1, \rho)$ and $e_{v_1}(v_1, R_1, \rho)$, for the same values of v_1, R_1 and ρ. Moreover, for a wide range of high R_1 and high v_1 values, world gains from trade are elastic with respect to ρ when ρ is low (when $\rho = -.8$), and for all values of v_1 and R_1 when ρ is high (when $\rho = .8$). In conclusion, Figures 6a-6c suggest that correlation plays an important role in determining WGFT, highlighting the diversification effect's importance. Furthermore, WGFT are often more responsive to risk aversion changes, presumably, because risk aversion captures and affects the countries' risk costs both at home and abroad.

[36] The green curve in Figure 6b (where $\rho = .8$) is lower (thus, higher in absolute terms) than the green curve in Figures 6a (where $\rho = -.8$).

[37] The red curve in Figure 6b (where $\rho = .8$) is not always higher than the red curve in Figures 6a (where $\rho = -.8$).

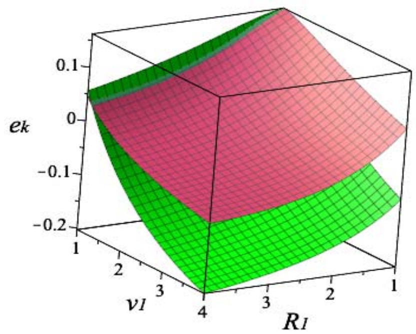


Figure 6a: e_{R_1} (red) and e_{v_1} (green), with $\rho = -.8$.

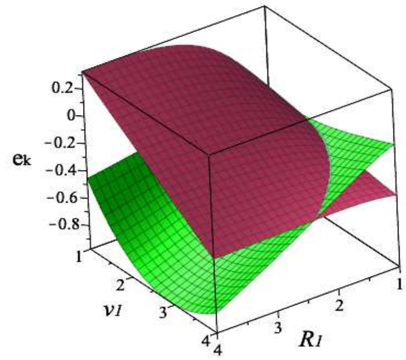


Figure 6b: e_{R_1} (red) and e_{v_1} (green), with with $\rho = .8$.

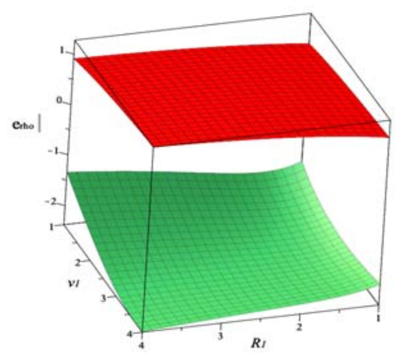


Figure 6c: e_{R_ρ}, with $\rho = -.8$. ρ (red) and $\rho = .8$ (green).

5 Conclusion

We show that market correlation and RAR differences can explain trade patterns by using a simple partial equilibrium duopolistic trade model with an identical product and demand uncertainty. We demonstrate that we can view export/import patterns as reflecting trade's implicit risk and risk aversion content based on their relative abundance. Specifically, a relatively "risk-aversion abundant" country is likely to be a net importer of the product - hence an importer of low risk-aversion; and a relatively high-risk abundant country is more likely to be a net exporter of the product - hence an importer of low risk.

We also show that RAR differences and market correlation are sources of GFT due to implicit insurance and diversification benefits. The GFT are generally positive for both countries, and consequently, world gains from trade are always positive. Furthermore, GFT may, sometimes, be higher with uncertainty than without it. To get a sense of the importance of RRAC, we calculate local measures of the WGFT's elasticities. We find that world gains from trade are most responsive to changes in market correlation.

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