The North-South Trade Featuring the Leontief Trade Improves Poverty and Inequality

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ABSTRACT - This study derives the general trade equilibrium with factor price non-equalizations and demonstrates that the Leontief trade (the trade pattern of the Leontief paradox) is a regular trade pattern theoretically. It also shows that the doctrine and the sign predictions of the effective endowments (seeTrefler, 1993) and the virtual endowments (see Fisher and Marshal 2008) adopted the Leontief trade already. The intersection of goods price diversification cones (Fisher, 2011) can additionally demonstrate it. There are a hundred more pieces of literature reporting the evidence of the Leontief paradox. All of them are about the trades between North countries and South countries. The study illustrates that the essential features of the North-south trade are the Leontief trade. A unique phenomenon observed in the North-South trades is wages increasing both in the south and in the north. The Leontief trade can explain it well.

Keywords: Localized factor prices, factor price equalization, factor price non-equalization, General equilibrium of trade, Leontief Paradox, Leontief trade

JEL Classification Code: F10, F15

1. Introduction

1.1 The study purpose and approaches

International trades and investments integrate the world economy. Free Trade as a substitution of factor mobility plays important role in the world's economic and social developments. International investments change the world production distributions across

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geographic areas and improve directly and indirectly world trade structures. International trade changes the consumption distributions of world products.

To understand the social consequences of international trade, we need to know the economic consequence of international trade first. It is a gap in international economics, which addressed the trade consequence by the Heckscher-Ohlin-Samuelson theory with the assumption of the same technologies across countries. The study shows that both the effective endowments and the virtual endowments demonstrated a doctrine that *A country exports its effective (virtual) abundant factor rather than its actual abundant factor.* The trade direction of commodities by the doctrine is that *a country will export a commodity that is produced by using its effective (or virtual) abundant factor, rather than its actual abundant factor, intensively.* The Leontief trade occurs when a country's actual abundant factor is not its effective abundant factor[1].

This study finds that under the assumption that countries have different productivities, there are three trade patterns rather than one trade pattern of the Heckscher-Ohlin trade. Another two trade patterns are the mutual Leontief trade and the FIR Leontief trade. The FIR Leontief trade is caused by the factor intensity reversal, in which one country does Leontief trade, another does the Heckscher-Ohlin trade. The mutual Leontief trade occurs when the actual factor abundances conflict with their effective factor abundances in both countries. It happens in the absence of an FIR. The study presents the exact conditions when and how Leontief trades occur, and how the trade equilibrium is achieved for the "paradox" trade patterns. All three trade patterns satisfy the logic that a country exports the services of its effective (or virtual) abundant factor.

More than 150 pieces of literature reported evidence of the Leontief trade. All are about the trade between north countries and south countries. This study shows the essential feature of the north-south trade is the Leontief trade. The most recent empirical studies in this century reported the observations of skill intensity reversals and showed wage increases both in North countries and South countries (see Kurokawa, 2011, Reshef, 2007, Sampson, 2016). This study explains this phenomenon and illustrates that it is another distinct property of the Leontief trade.

[1] International economics igornas it for thee decades.

The paper provides three different approaches to demonstrate the Leontief trade. The first one is the price-trade equilibrium with the factor price localizations. It proves that the Leontief trade is a regular trade consequence under factor price localization.

The second way is by using the doctrine of the effective endowments and the virtual endowments. Trefler's (1993) effective endowments and Fisher and Marshall (2008) explored an important logic that a country will export its commodity that is produced by using its effective abundant factor rather than its actual abundant factor, intensively. The study shows that both the Leontief trade and the Heckscher-Ohlin trade follow this logic and that countries gain from trade by the two trade pattens, under the law of comparative advantage.

The last approach uses the idea of Fisher (2011)'s intersection of goods price diversification cones. Any commodity price, which falls within the intersection of goods price diversification cones, can be used to illustrate the trade direction of commodities. Given a price, there is a share of GNP for a country. We can obtain a trade direction by the price. All the prices falling within the intersection cone illustrate the same trade directions. The paper also provides geometrical diagrams to illustrate the Leontief trade.

1.2 Literature related

The three trade patterns proposed in this study roots in the classical trade theories and follow the law of comparative advantages.

Samuelson (1948) presented the famous theorem of factor price equalization (FPE). Dixit and Norman (1980, chapter 4) proposed the Integrated World Equilibrium (IWE) to illustrate the factor price equalization, which perfectly fulfilled the factor mobility analysis. They proved that the world prices remain the same when the allocation of factor endowments changes within the factor price equalization (FPE) set in the IWE diagram.

McKenzie (1955) proposed the diversification cone of factor endowments, which is critical to understanding factor price equalization (FPE) and trade balance from production constraints. Vanek (1968) proposed the HOV model that presented factor contents of trade. The share of GNP in the HOV model engaged prices with trade and consumption. It resulted in the theoretical and application issue on how to convert the assumption of homothetic taste into consumption balance. Fisher (2011) proposed "the goods price diversification cone," which is the counterpart of the factor diversification cone. He also

offered another insight into the intersection of goods price cones to specify the set of the factor price non-equalization when countries have different technologies.

The Leontief test (Leontief, 1953) showed that the US, as a capital-abundant country, exported its labor-intensive commodities. It counters the common sense of international economics then. The Leontief paradox impelled the HOV studies aimed to supply alternative approaches to explain it. Leontief (1953) proposed the productivity-equivalent factor (workers) to explain his test results. Trefler (1993) implemented Leontief's idea with factor-argument parameters as effective (equivalent) endowments. The model is also instrumental for theoretical analyses to reach factor price non-equalizations[2]. Fisher and Marshall (2008) proposed another excellent approach to involving different technologies using the virtual endowments and the conversion matrix.

Deardorff (1986) presented the diversification cones of the FIRs. He showed the double factor intensity reversals. He suggested a way to turn any model with the FIRs into a model without it, and vice versa, by simply redefining goods.

Chipman (1969), Trefler (1993), Krugman (2000), Fisher (2011), Leamer (2000), Rassekh and Thompson (1993), and many other studies had argued the need for factor price non-equalization when considering different technologies across countries.

Helpman and Krugman (1985, pp. 24) proposed an insight idea of trade volume that is defined with domestic factor endowments in the IWE diagram. They abstracted a unique principle as "the differences in factor composition are the sole basis of trade." Guo (2015) used their idea to obtain the price-trade equilibrium in the integrated world economy[3]. This study extends it to the equilibrium of factor price non-equalization under different productivities. The equilibrium supplies a vehicle to understand trade patterns.

The author organizes the study into six sections. Section 2 derives the general trade equilibrium of factor price localizations. Section 3 illustrates that conceptually there are three trade patterns: the Heckscher-Ohlin trade, the FIR Leontief trade, and the mutual Leontief trade when countries have different productivities. Section 4 presents trade patterns by localized factor prices and discusses the trade effects. Section 5 analyzes the

[2] This paper uses factor price localizations and factor price non-equalizations alternatively for the phenomena that local factors are rewarded differently under the common world commodity prices when countries have different productivities.

[3] Guo (2019) published his equilibrium result briefly.

trade patterns by virtual factor endowments. It concludes that the trade patterns under the virtual endowments will be as same as trade patterns under the effective endowments. Section 6 reviews the empirical studies that reported the Leontief paradox in different countries. It shows skill intensity reversals in recent empirical studies, are just the localized factor prices by the FIR Leontief trade. The last section is the concluding remarks.

2. The Price-Trade Equilibrium When Countries Have Different Productivities

Trefler (1993) proposed the first HOV model to incorporate different productivities across countries within the Heckscher-Ohlin framework magnificently. We use it to illustrate the factor price localizations. We use all assumptions in the Trefler model such as free trade, same taste for consumption, constant return of scale, no cost for trade, and productivities different across countries. The illustration is by $2 \times 2 \times 2$ model.

2.1 Review of the Trefler Model

The central assumption in the Trefler model is to express productivity differences by factor input requirements as

$$A^H = \begin{bmatrix} a_{K1}^H & a_{K2}^H \\ a_{L1}^H & a_{L2}^H \end{bmatrix} = \Pi A^F = \begin{bmatrix} \pi_K & 0 \\ 0 & \pi_L \end{bmatrix} A^F \tag{2-1}$$

where Π is a 2×2 diagonal matrix, its element π_k is the factor productivity-argument parameter, $k = K, L$, K for capital, L for Labor. A^h is the 2×2 technology matrix of country h, its element $a_{ik}^h(w/r)$ is the input requirement of factor k needed to produce one unit of output i, $i=1,2$, $k=L, K$.

The production constraint function and the unit cost function for country H are

$$A^H X^H = V^H \tag{2-2}$$

$$(A^H)' W^H = P^H \tag{2-3}$$

For country F, they are

$$\Pi^{-1} A^H X^F = V^F \tag{2-4}$$

$$(\Pi^{-1} A^H)' W^F = P^F \tag{2-5}$$

where V^h is the 2×1 vector of factor endowments with elements K as capital and L as labor; X^h is the 2×1 vector of commodity output; W^h is the 2×1 vector of factor prices with elements r as rental and w as wage; P^h is the 2×1 vector of commodity prices with elements p_1^h and p_2^h; $h = H, F$ for countries.

The Trefler model is with a single cone of goods price diversifications[4]. Its factor cost ratio ranks, which show the rays of the cone of goods prices in algebra, are

$$\frac{a_{K1}^H}{a_{K2}^H} = \left(\frac{a_{K1}^F}{a_{K2}^F} = \frac{a_{K1}^H/\pi_K}{a_{K2}^H/\pi_K}\right) \quad > \quad \frac{P_1^*}{P_2^*} \quad > \quad \frac{a_{L1}^H}{a_{L2}^H} = \left(\frac{a_{L1}^F}{a_{L2}^F} = \frac{a_{L1}^H/\pi_L}{a_{L2}^H/\pi_L}\right) \tag{2-6}$$

where we assume both countries are capital intensive on sector 1. The single cone of goods price diversifications reduces the difficulties of analyses of the price-trade equilibrium. The Trefler model does have two cones of the factor diversifications, which show different productivities across countries. (2-6) also implies the absence of FIRs.

Bernhofen (2011, p104) emphasized the way to calculate factor content, "A country's factor content is defined using the country's domestic technology matrix[5]." This idea is a critical point in analyses of trade equilibrium when countries with different productivities.

The world's effective endowments by referring to country H's productivities are

$$K^{hW} = K^H + \pi_K K^F \tag{2-7}$$

$$L^{hW} = L^H + \pi_L L^F \tag{2-8}$$

The world's effective endowments by country F's productivities are

$$K^{fW} = K^F + K^H/\pi_K \tag{2-9}$$

$$L^{fW} = L^F + L^H/\pi_L \tag{2-10}$$

We use the lowercase character *h* to indict the country referred to as its productivities.

2.3 Factor Price Localizations at Equilibrium

Trefler (1993) described that the factor price equalization hypothesis and the HOV theorem hold in his equivalent-productivities system[6]. When the effective system is built (or mapped) by the referring to country H's productivities, the equalized factor price is the localized factor price in country H. Similarly, when the effective system is built (or mapped) by referring to country F's productivities, the equalized factor price in the system is country F's prices.

Let's express an effective productivity system formally by referring to country H's productivity. Equations (2-2) and (2-3) for country H are still the same. Rewrite (2-4) as

$$A^H X^F = V^{hF} \tag{2-11}$$

[4] See Fisher (2011) for the cone of goods price diversification.

[5] The sign predictions both by effective endowments and by virtual endowments say this also.

[6] Fisher (2011) also mentioned that factor price equalization and Hechscher-Ohlin theorem hold in the virtual endowment system.

where

$$V^{hF}=\Pi V^{F} \tag{2-12}$$

Trefler specified the following

$$W^{F}=\Pi W^{H} \tag{2-13}$$

Substituting it into (2-5) yields

$$(A^{H})'W^{H}=P^{F} \tag{2-14}$$

Equations (2-2), (2-3), (2-11), and (2-14) compose the effective system, or mapped system by matrix A^{H}. Mathematically it is a Heckscher-Ohlin model exactly. It is just the model that Feenstra (2004, pp.) described the equalized factor price in the effective endowments as "Let A now denote the amounts of effective factors needed per unit of output in each industry. We continue to assume that factor price equalization holds in terms of effective factor prices, so with identical technologies, the matrix A is the same across countries". All the theorems and equilibrium result in the Heckscher-Ohlin model can apply to it. Guo (2015, 2019) proposed the price-trade equilibrium of the Heckscher-Ohlin model[7]. It can be applied to the effective system above directly as

$$s^{h}=\frac{1}{2}\left(\frac{K^{h}}{K^{hW}}+\frac{L^{h}}{L^{hW}}\right) \qquad h=(H,F) \tag{2-15}$$

$$W^{*H}=\begin{bmatrix} L^{HW} \\ K^{HW} \end{bmatrix} \tag{2-16}$$

$$P^{*}=(A^{H})'\,W^{*H} \tag{2-17}$$

$$W^{*F}=\Pi W^{*H} \tag{2-18}$$

$$F_{K}^{h}=s^{h}K^{hW}-K^{h}\ =-\frac{1}{2}\frac{K^{h}L^{hW}-K^{hW}L^{h}}{L^{hW}} \qquad h=(H,F) \tag{2-19}$$

$$F_{L}^{h}=s^{h}L^{hW}-L^{h}\ =\frac{1}{2}\frac{K^{h}L^{hW}-K^{hW}L^{h}}{K^{hW}} \qquad h=(H,F) \tag{2-20}$$

$$T_{1}^{h}=s^{h}\,x_{1}^{W}-x_{1}^{h} \qquad (h=H,F) \tag{2-21}$$

$$T_{2}^{h}=s^{h}x_{2}^{W}-x_{2}^{h} \qquad (h=H,F) \tag{2-22}$$

We assume $w^{*H}=K^{hW}$ to drop one market clear condition by Walras equilibrium law. w^{*H} serves as a "benchmark" price to be referred to by the other three factors' prices and two world commodity prices. Equation (2-18) is by the assumption of the Trefler model.

[7] Appendix A is the price-trade equilibrium proposed by Guo (2019), for the Heckscher-Ohlin model.

The key relationships for localized factor prices are

$$\frac{w^{*H}}{r^{*H}} = \frac{K^{hW}}{L^{hW}} \tag{2-23}$$

$$\frac{w^{*F}}{r^{*F}} = \frac{K^{fW}}{L^{fW}} \tag{2-24}$$

The numerators in (2-19) and (2-20) show if

$$\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}} \tag{2-25}$$

then $F_K^H < 0$ and $F_L^H > 0$. It just says that a country exports the services of its effective abundant factor (Recall that we use negative sign express export). It shows the logic that *a country exports the services of a factor that is its effective abundant factor, compared to another factor.* It is also the doctrine and the result of the sign predictions in the effective endowments and the virtual endowments. Equations (2-19) through (2-22) imply that a country being effective-capital abundance will export its capital-intensive goods and import its labor-intensive goods. Section 3.5 is proof of the Leontief trade.

3. Trade Patterns Specified by Effective Endowments

3.1 the logic of trade direction when countries have different productivities

The Heckscher-Ohlin theorem guides the trade direction under the same technologies. The HOV studies did accumulate some understanding of trade patterns when countries have different productivities also. The logic of the effective endowments can be abstracted as

Trade direction of factor content (Trefler 1993) - a country exports the services of a factor that is its effective abundant factor, compared to another factor.

This logic is widely accepted, when countries have different productivities, in the HOV studies. The sign predictions for effective endowments and virtual endowments both use this logic. It can be extended into the following also,

Trade direction of commodities - A country will export a commodity that is produced by using its effective abundant factor rather than using its actual abundant factor intensively.

It is a useful extension. It will lead to recognizing the Leontief trades as a regular trade pattern immediately[8]. When the actual factor abundance in a country conflicts with its effective factor abundance, the Leontief trade occurs. The general trade equilibrium in the last section proves[9] it analytically (see section 3.5). This section will show if one country's actual factor abundance conflicts with its effective factor abundance, it is the FIR Leontief trade. If both countries' actual factor abundance conflict with their effective factor abundance, it is the mutual Leontief trade.

3.2 Factor Intensity and Factor Intensity Reversals

The Heckscher-Ohlin theory defines the factor intensities between two industries as[10]

$$\frac{a_{K1}}{a_{L1}} > \frac{a_{K1}}{a_{L2}} \tag{3-1}$$

Most theorems of the Heckscher-Ohlin model require the absence of factor intensity reversals (FIRs), which are defined as the elasticities and substitutions in a production function[11]. When two countries have different production functions by different parameters, it has more chances to present the factor intensity reversals. The system functions (2-2) through (2-5) are a snapshot of the production productions, in a linear system, by giving a group of factor endowments. The system functions should be with the capacity to express factor intensity reversals.

For the Trefler model by (2-2) through (2-5), the productivity differences can be presented both by the factor diversifications cone (see Mckenzie, 1955) and by cones of goods price diversifications (see Fisher, 2011) in the HOV studies. These two types of cones can reflect factor intensities and factor intensity reversals in production technology structures.

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[8] No literiture goes this direction yet.

[9] There are two approaches to prove the trade direction of commodity output by the direction of factor content of trade. One is by Leamer (1984,p.9-10). Another is by Helpman and Krugman (1985, p17). Both can be extended to analyze the commodity trade direction under effective endowments or virtual endowments.

[10] It uses the approach by Leamer (1984, p.9-10).

[11] Constant-elasticity-of-substitution (CES) production can specify the factor intensity reversals. When both goods have Cobb-Douglas production function, the factor intensity reversal are impossible (see Bhagwati, Panagariya, and Srinivasan (1998, p.61). However, when assume technology differences and using it in production function even by Cobb-Douglas functions, the factor intensity reversals will be more significant.

The Heckscher-Ohlin model only analyzes factor intensity between industries, since the two countries' technologies are the same. When countries have different productivities (or technologies), the following cones of factor diversifications may occur,

$$\frac{a_{K1}^{H}}{a_{L1}^{H}} > \frac{a_{K1}^{H}}{a_{L2}^{H}} \tag{3-2}$$

$$\frac{a_{K1}^{F}}{a_{L1}^{F}} < \frac{a_{K1}^{F}}{a_{L2}^{F}} \tag{3-3}$$

It implies that country H is a capital intensity to product commodity 1, and country F is capital intensive to product commodity 2. This is a factor intensity reversal across countries. It is an essential term related to trade direction when countries have different productivities.

The goods price diversification cone is an idea that makes sure factor prices are positive when the vector of commodity price falls within it. The intersection of goods price diversification cones will make sure that both countries' factor prices will be positive when the vector of world commodity price falls within it. The intersection of goods price diversification cones can be used to illustrate factor intensity well.

The following cost requirement ratio ranks are typical for normal factor intensity across countries,

$$\frac{a_{K1}^{H}}{a_{K2}^{H}} > \frac{a_{K1}^{F}}{a_{K2}^{F}} > \frac{a_{L1}^{H}}{a_{L2}^{H}} > \frac{a_{L1}^{F}}{a_{L2}^{F}} \tag{3-4}$$

They are also the rays of the two goods price cones of the two countries. The intersection cone is

$$\frac{a_{K1}^{F}}{a_{K2}^{F}} > \frac{a_{L1}^{H}}{a_{L2}^{H}} \tag{3-5}$$

Goods prices by free trade must fall in this cone[12]. Equation (3-4) shows that both countries are factor intensity in industry 1. It is the case of the absence of factor intensity reversal. The equations (3-2) and (3-3) can be characterized by

$$\left|A^{H}\right|\left|A^{F}\right| > 0 \tag{3-6}$$

[12] The relative commodity price must fall within the intersection cone as $\frac{a_{L1}^{F}}{a_{L2}^{F}} > \frac{p_1^*}{p_1^2} > \frac{a_{L1}^{H}}{a_{L2}^{H}}$, see Fisher (2011).

where $\left|A^h\right|$ is the determinant of the technology matrix A^h of country h, $h = H, F$. $\left|A^h\right| > 0$ means that country h is capital-intensive in industry 1. $\left|A^h\right| < 0$ means that country h is labor-intensive in industry 1.

Look at another cost requirement ratio ranks

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{L1}^H}{a_{L2}^H} > \frac{a_{K1}^F}{a_{K2}^F} \tag{3-7}$$

It implies (3-2) and (3-3). We call it factor intensity reversal across countries (Briefly, we still call it FIR as it is named in production function analyses). The intersection cone of two goods price diversification cones is

$$\frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{L1}^H}{a_{L2}^H} \tag{3-8}$$

The equations (3-7) can be characterized by

$$\left|A^H\right|\left|A^F\right| < 0 \tag{3-9}$$

We do not introduce any new definition for factor intensity reversals but identify them in the existing price-production system equations to help to see their trade consequence.

3.3 The Heckscher-Ohlin trade

There are three trade patterns conceptually when countries have different productivities. The first one is the Heckscher-Ohlin trade, which is well-known when assuming that countries have the same technologies. It says that a country abundant in the endowment of a factor will export the commodity that uses this factor intensively. It is also a trade pattern when countries have different productivities. The following conditions make the Heckscher-Ohlin trade occur, assuming the absence of FIR,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-11}$$

$$\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}} \tag{3-12}$$

$$\frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{3-13}$$

Equation (3-11) is about the actual factor abundance[13]. Equations (3-12) and (3-13) uses effective endowments. The feature of this trade pattern is that both countries' actual factor

[13] Feenstra and Taylor (2012, p102) first used term "actual factor endowment" to different from "effective factor endowment".

abundances are consistent with their effective factor abundance. If both countries are capital intensive in product 1 as (3-4), county H will export product 1; and country F will export product 2.

3.3 Mutual Leontief Trade

Assume the absence of factor intensity reversal in the production technology structures of the two countries. The following conditions will lead to the mutual Leontief trade,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-14}$$

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{3-15}$$

$$\frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}} \tag{3-16}$$

They show that both countries' actual factor abundances conflict with their effective factor abundances. If both countries are capital intensive in producing 1 as described as (3-4), County H will export product 2, and country F will export product 1. It is just different from the Heckscher-Ohlin trade.

We illustrate how it happens. Assuming that country H be actual factor abundant as (3-14) if the following is true,

$$\frac{K^H L^F}{L^H K^F} < \frac{\pi_K}{\pi_L} \tag{3-17}$$

equations (3-15) and (3-16) will occur. Equation (3-17) can be rewritten as the following separately

$$\frac{K^H}{L^H} < \frac{\pi_K K^F}{\pi_L L^F} = \frac{K^{hF}}{L^{hF}} \tag{3-18}$$

$$\frac{K^F}{L^F} > \frac{K^H/\pi_K}{L^H/\pi_L} = \frac{K^{fH}}{L^{fH}} \tag{3-19}$$

Equation (3-18) implies[14] (3-15). And equation (3-19) implies (3-16). As we illustrated in 3.1 that *A country will export a commodity that is produced by using its effective abundant factor intensively,* county H will export product 2, and country F will export product 1.

[14] $\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} < \frac{K^{hF}}{L^{hF}}$ is always true.

The mutual Leontief trade may occur within the Trefler model in the absence of the FIR. It is a new trade pattern that we get to notice in this study. The scope of the presence of the Leontief trade is much larger than what we expected before.

3.4 The FIR Leontief Trade - Factor Conversion Trade

The factor intensity reversals source the FIR Leontief trade. We specify the Trefler model a little bit differently by assuming that the technological matrices of the two countries are implemented as

$$A^H = \psi A^F = \begin{bmatrix} 0 & \theta_K \\ \theta_L & 0 \end{bmatrix} A^F \tag{3-20}$$

where ψ is a 2×2 anti-diagonal matrix, its element θ_k is the productivity-across-factor-argument parameter, $k = K, L$. Denote

$$A^H = \begin{bmatrix} a_{K1}^H & a_{K2}^H \\ a_{L1}^H & a_{L2}^H \end{bmatrix} \tag{3-21}$$

The technology matrix in country F will be

$$A^F = \psi^{-1} A^H = \begin{bmatrix} \frac{1}{\theta_L} a_{L1}^H & \frac{1}{\theta_L} a_{L2}^H \\ \frac{1}{\theta_K} a_{K1}^H & \frac{1}{\theta_K} a_{K2}^H \end{bmatrix} \tag{3-22}$$

Those two matrices compose a model with the FIRs as

$$A^H X^H = V^H \tag{3-23}$$

$$(A^H)' W^H = P^H \tag{3-24}$$

$$\psi^{-1} A^H X^F = V^F \tag{3-25}$$

$$(\psi^{-1} A^H)' W^F = P^F \tag{3-26}$$

Rewrite (3-25) as

$$A^H X^F = \psi V^F = \begin{bmatrix} \theta_L L^F \\ \theta_K K^F \end{bmatrix} \tag{3-27}$$

It shows that the effective endowments[15] measured by country H's productivities to produce commodity X^F. The world's effective endowments by referring to country H's productivities are

$$K^{hW} = K^H + \theta_L L^F \ , \qquad L^{hW} = L^H + \theta_K K^F \tag{3-28}$$

Similarly, the world's effective endowments by referring to country F's productivities are

[15] It can be expressed also as $\begin{bmatrix} K^{hF} \\ L^{hF} \end{bmatrix} = \left(A^H A^{F-1}\right) \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \frac{\theta_L L^F}{\theta_K K^F}$.

$$K^{fW} = K^F + L^F/\theta_L, \qquad L^{fW} = L^F + K^F/\theta_K \tag{3-29}$$

The cost requirement ratios, which write down the rays of goods price diversification cones in algebra, are

$$\frac{a_{K1}^H}{a_{K2}^H} = \left(\frac{a_{L1}^F}{a_{L2}^F} = \frac{a_{K1}^H/\theta_L}{a_{K2}^H/\theta_L}\right), \quad \frac{a_{L1}^H}{a_{L2}^H} = \left(\frac{a_{K1}^F}{a_{K2}^F} = \frac{a_{L1}^H/\theta_K}{a_{L2}^H/\theta_K}\right) \tag{3-30}$$

It is also the case of the single cone of goods prices diversification, in which the two cones intersected fully, but in reversal. If country H is capital-intensive to produce commodity 1,

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{L1}^H}{a_{L2}^H} \tag{3-31}$$

by (3-30), country F will be capital-intensive to produce commodity 2,

$$\frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{K1}^F}{a_{K2}^F} \tag{3-32}$$

The FIR Leontief trade will occur if a trade model presents the FIR. The following conditions judge the FIR Leontief trade,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-33}$$

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{3-34}$$

$$\frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{3-35}$$

By inequality (3-34), country H is labor effective abundance. It will export commodity 2 since it is labor intensity to product commodity 2 by (3-31). And country F will export commodity 1, by (3-35) that it is effective labor abundant and by (3-32) that it is labor-intensive to produce community 2.

The commodity trades equilibrate in the normal way[16] as

$$T^H = -T^F \tag{3-36}$$

Both countries are effective labor abundant by (3-34) and (3-35). Both countries will export labor services and import capital services. We call it the reversals of factor content of trade. Both countries export commodities that are produced by using the same factor (labor) intensively, but they export different commodities since they are labor-intensive in different products.

[16] Some studies explained the commodity trade direction as that both countries export two commodities and imports same two commodities. The explanation built another paradox.

Let's see how (3-34) and (3-35) occur. Equation (3-34) implies[17]

$$\frac{K^H}{L^H} < \frac{K^{hF}}{L^{hF}} = \frac{\theta_L L^F}{\theta_K K^F} \tag{3-37}$$

It means that country H is effective labor abundant. Equation (3-37) can be rewritten as

$$\frac{K^F}{L^F} < \frac{L^H/\theta_K}{K^H/\theta_L} = \frac{K^{fH}}{L^{fH}} \tag{3-38}$$

It implies that country F also is effective labor abundant.

Under comparative advantage law, a country exports its product with a relative advantage to produce. The net exported factor will be rewarded with a higher price than its price in autarky. Both countries' labor will be rewarded better. It is another distinct characteristic of the FIR Leontief trade. Section 4 shows that like the Heckscher-Ohlin trade, the Leontief trade can make sure of gains from trade for both countries.

The Trefler FIR model is a Trefler model mathematically. The result of general trade equilibrium (2-15) through (2-22) can be applied to the Trefler FIR model.

With factor content of trade reversals, both countries will consume more on their effective scarce factor, embodied in the trade flows[18]. It is a new type of comparative advantage to use the global resource more efficiently.

For a particular case when country F produces commodity 1 by using technologies of industry 2 in country H and produces commodity 2 by using the technologies of industry 1 in country H as

$$\psi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{3-39}$$

the localized factor prices will be[19]

$$\frac{w^{*H}}{r^{*H}} = \frac{r^{*F}}{w^{*F}} \tag{3-40}$$

---

[17] The following always holds, $\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}} > \frac{K^{hF}}{L^{hF}} = \frac{\theta_L L^F}{\theta_K K^F}$, if $\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}}$.

[18] Free trade transforms the global effective abundant factor into global effective scarce factor, embodied in the commodity trade flows. The FIR Leontief trade phenomenon is a little bit like the "black hole" in astronomy (Black hole is defined as that a region of space having a gravitational field so intense that no matter or radiation can escape). Free trade traps or absorbers the global effective abundant factor, which cannot "escape" from the market. At the same time, free trade is also like the "white hole" (the white hole is a hypothetical region of spacetime, which cannot be entered from the outside, although matter and light can escape from it. In this sense, it is the reverse of a black hole, which can only be entered from the outside and from which matter and light cannot escape). Free trade releases or transforms the global effective scarce factor to both countries.

[19] The equilibrium result in section 2 can present it exactly.

It reflects the switch order of goods in the analyses by production function in the Heckscher-Ohlin model, which lead to the presence of the FIR.

Deardorff (2006, page 102) defined the factor intensity reversal as "A property of technology (page 268) for two industries such as that their ordering of relative factor intensities is different at different factor prices. For example, one industry may be relatively capital intensive compared to the other at high relative wages and labor intensive at low relative wage". This study describes the same issue reversely. We use factor intensity reversals to illustrate factor price reversal. The results and conclusion are the same about localized factor prices. Appendix A is a numerical example of the FIR Leontief trade.

By the law of comparative advantage, each country exports its product which is produced with relative advantage. The factor used intensively in the exported good is a relatively effective abundant and relatively cheaper factor. Equations (3-34) and (3-35) show that labor is relatively cheaper in both countries by factor intensity reversals, so both countries export labor. International trade will improve the payment of labor in both countries[20].

3.4 The Proof of the Commodity Trade Direction for the FIR Leontief trade

Assume the model is in the presence of FIRs, in which $|A^H| > 0$ and $|A^F| < 0$. Assume also that country H is actual-capital abundance; country H is effective-labor abundance; country F is effective-labor abundance also, as specified in equations (3-34) through (3-35).

The vector of the output exports in the home country is the difference between production output X^H and consumption[21] C^H:

$$T^H = C^H - H^H = (A^H)^{-1}(s^H V^{hW} - V^H) \tag{3-41}$$

which is $(A^H)^{-1}$ times the vector of excess effective factor supplies,

$$F^H = s^H V^{hW} - V^H = \begin{bmatrix} s^H K^{hW} - K^H \\ s^H L^{hW} - L^H \end{bmatrix} = \begin{bmatrix} K^{hW}(s^H - K^H/K^{hW}) \\ L^{hW}(s^H - L^H/L^{hW}) \end{bmatrix} \tag{3-42}$$

---

[20] If reassuming (3-33) through(3-36) as

$$\frac{K^H}{L^H} < \frac{K^F}{L^F}, \qquad \frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}}, \qquad \frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}}$$

Both countries will export capital services. The payment of capital will be increase in both the countries. This is another theoritical possible result. However, no expirimental reported this case.

[21] For this paper, export is expressed by negative value.

If the home country will is effective-labor abundant, The vector of factor content of trade has a sign pattern as

$$F^H = \begin{bmatrix} + \\ - \end{bmatrix} \tag{3-43}$$

The signs pattern[22] of trade flow of the home country will be

$$T^H = (A^H)^{-1}F^H = \begin{bmatrix} + & - \\ - & + \end{bmatrix}\begin{bmatrix} + \\ - \end{bmatrix}=\begin{bmatrix} + \\ - \end{bmatrix} \tag{3-44}$$

This is due to the home country being capital intensive in output 1 by $|A^H| > 0$. Country F's trade flow will be

$$T^F = (A^F)^{-1}F^F = \begin{bmatrix} - & + \\ + & - \end{bmatrix}\begin{bmatrix} + \\ - \end{bmatrix}=\begin{bmatrix} - \\ + \end{bmatrix} \tag{3-45}$$

This is due to the foreign country being capital intensive in output 2 by $|A^F| < 0$

The trade flows of the two countries are under normal trade relationships as

$$T^H = -T^F \tag{3-46}$$

It shows the logic that *A country will export a commodity that is produced by using its effective abundant factor intensively.*

Similarly, we can show the logic goes with the Heckscher-Ohlin trade and the mutual Leontief trade also.

4. Factor Price Definitions of the Trade Patterns

4.1 Factor Price Premium

To compare relative factor prices across countries after free trade, this section introduces the terms of the factor productivity incentives and the factor reward incentives.

We say that labor in country H is on productivity incentive if the localized factor prices of the two countries satisfied the following relationship,

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} \tag{4-1}$$

It also implies that capital in country F is on productivity incentive. This is an international comparison.

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[22] Leamer (1984, page 9) first used this method to present the commodity trade direction for the Heckscher-Ohlin model. He shows $A^{-1} = \begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix}^{-1} = \begin{bmatrix} a_{L2} & -a_{K2} \\ -a_{L1} & a_{K1} \end{bmatrix}/|A|$
where $|A| = a_{L1}a_{L2}\left({}^{a_{K1}}/_{a_{L1}} - {}^{a_{K2}}/_{a_{L2}}\right) > 0$

We say that labor in country h is on reward incentive if the localized factor prices and the autarky price in a country satisfied the following relationship.

$$\frac{w^{*h}}{r^{*h}} > \frac{w^{ah}}{r^{ah}} \qquad (h = H, F) \tag{4-2}$$

where w^{ah} is autarky wage in country h; and r^{ah} is autarky rental in country h, $h = H, F$. This is a domestic comparison.

4.2 Factor Price Definition of the trade patterns

There are two alternative ways of defining factor abundance[23]. Country H is said to be capital abundant by either of the followings,

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \qquad "phsical\ defination" \tag{4-3}$$

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} \qquad "price\ defination" \tag{4-4}$$

The price definition is not as popular as the physical definition since autarky prices are not available. Guo (2019) proved the logic of autarky prices as

$$\frac{w^{ah}}{r^{ah}} = \frac{K^h}{L^h} \qquad (h = H, F) \tag{4-5}$$

It implies that (4-3) and (4-4) are the same things. It is a useful condition to define trade patterns by factor prices.

The localized wage-rental ratio for a country in (2-29) and (2-30) is

$$\frac{w^{*h}}{r^{*h}} = \frac{K^{hW}}{L^{hW}} \qquad (h = H, F) \tag{4-6}$$

The Heckscher-Ohlin trade is specified by the physical factor abundances in the last section as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}\ , \qquad \frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}}\ , \qquad \frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{4-7}$$

Substituting (4-5) and (4-6) into the inequalities above yields,

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}\ , \qquad \frac{w^{aH}}{r^{aH}} > \frac{w^{*H}}{r^{*H}}\ , \qquad \frac{w^{aF}}{r^{aF}} < \frac{w^{*F}}{r^{*F}} \tag{4-8}$$

It is the factor price definition of the Heckscher-Ohlin trade when countries have different productivities. Inequalities (4-8) are theoretical analyses to help in reasoning and

[23] see Bhagwati, Panagariya, and Srinivasan (1998, p.63).

understanding[24]. It illustrates that the trade will benefit capital services in country H and labor in country F. Free trades benefit the effective-abundant factors, which are actual abundant factors also for the Heckscher-Ohlin trade.

From (4-8), two possible ranks of wage-rental ratios are

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{aF}}{r^{aF}} \tag{4-9}$$

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{aF}}{r^{aF}} \tag{4-10}$$

(4-9) implies

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} \tag{4-11}$$

(4-10) implies

$$\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} \tag{4-12}$$

Inequality (4-11) tells that country H is labor reward incentive, and that country F is capital reward incentive. Inequality (4-12) reverses (4-11). The Heckscher-Ohlin trade can generate both, depending on if $\frac{K^{hW}}{L^{hW}} > \frac{K^{fW}}{L^{fW}}$ or not.

The mutual Leontief trade by physical factor abundance is expressed as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}, \qquad \frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}}, \qquad \frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}} \tag{4-13}$$

Substituting (4-5) and (4-6) into them yields,

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}, \qquad \frac{w^{aH}}{r^{aH}} < \frac{w^{*H}}{r^{*H}}, \qquad \frac{w^{aF}}{r^{aF}} > \frac{w^{*F}}{r^{*F}} \tag{4-14}$$

It shows that the trade will benefit labor in country H and capital in country F. They are actual-scarce factors in each country. However, they are effective-abundant factors in each country. Rewrite (4-14) as

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} > \frac{w^{*F}}{r^{*F}} \tag{4-15}$$

It is with only one productivity incentive relationship as

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} \tag{4-16}$$

[24] It assumes that two countries engaged in free trade immediately form a trade pattern in (4-7). It may be not realistic in the real world. It shows relationships among variables within the price-trade system.

The FIR Leontief trade is specified as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}, \qquad \frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}}, \qquad \frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{4-17}$$

Substituting (4-3) and (4-4) into them yields

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}, \qquad \frac{w^{aH}}{r^{aH}} < \frac{w^{*H}}{r^{*H}}, \qquad \frac{w^{aF}}{r^{aF}} < \frac{w^{*F}}{r^{*F}} \tag{4-18}$$

The last two inequalities above depict that the trade will benefit laborers in both countries, which are effective-abundant factors in each country. This is a unique feature of the FIR Leontief trade that the effective abundant factors are the same factor for both countries. Meanwhile, trade compensates more consumption on effective scarce factors for both countries, as if trade converts the part of the effective abundant factor into an effective scarce factor in consumption equilibrium. Free trade makes the usage of resources more efficient. The technology differences, indicated by $\left|A^H \right| \left|A^F\right| < 0$, endow factor intensity reversals that source factor abundance reversals, the reversals factor content of trade, and factor price reversals. It just explains the phenomime of skill intensity reversal reported in many empirical studies on trades between North countries and South countries[25]. It shows a new trade mechanism of consumption compensation and comparative advantage.

From (4-18), two possible wage-rental ratio ranks are

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{aF}}{r^{aF}} \tag{4-19}$$

$$\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} \tag{4-20}$$

Those two inequality chains can present in (4-12) and (4-13) also. It implies both the Heckscher-Ohlin trade and the FIR Leontief trade can generate different factor reward incentives.

The structures of localized factor prices are as complex as trade patterns. Some of the patterns look strange in the first glace. However, all the price patterns above make sure of gains from trades. The factor price definitions of trade patterns show a new way to view practices of international trade. Appendix B illustrates the gains from trade by localized factor prices. Whether the model is the same technical model or not, gains from trade are

[25] Section 6 will discuss it in detail.

the basic requirements of an equilibrium solution for models under the Heckscher-Ohlin frameworks[26].

5. Analyses of Trade Patterns by the Virtual Endowments

The idea of the virtual endowments presented the full technologies difference across countries in the Heckscher-Ohlin framework (see Fisher and Marshall, 2008). It is more complex in model structure and equilibrium analyses, although its mathematical expression is still concise. Fisher (2011) proposed the interception of goods diversification cones, which explored the most challenging part of the model's general trade equilibrium with virtual endowments[27].

Fisher (2011) also mentioned that under the virtual endowment assumptions, the classical Heckscher-Ohlin theory holds when technologies and factor prices are identical to those of the reference country. The behaviors and the trade patterns of the virtual endowment model are identical to the Trefler model's behaviors.

The $2 \times 2 \times 2$ model with virtual endowments[28] can be expressed as

$$A^h X^h = V^h \qquad (h = H, F) \tag{5-1}$$

$$(A^h)' W^h = P^h \qquad (h = H, F) \tag{5-2}$$

where $A^H \neq A^F$ in general. The world virtual endowments referring to the home country's technology can be expressed with the conversion matrix as

$$V^{hW} = V^H + (A^H)^{-1} A^F V^F \tag{5-3}$$

The world virtual endowments referring to the foreign country's technology can be expressed with the conversion matrix as

$$V^{fW} = V^F + (A^F)^{-1} A^H V^H \tag{5-4}$$

---

[26] The localized factor prices also satisfied with Helpman (1984) restrictions between factor price differences and factor content of trade

$$(w^j - w^i)' F^{ij} > 0$$
$$(w^j - w^i)' \left(F^{ij} - F^{ji}\right) > 0$$

where w^j is the vector of payment in country j and F^{ij} is the vector of factor content of trade exported from country *j* to country *i*, *i=1,2*, and *j=1,2*. This can be displayed numerically for the three trade patterns.

[27] Appendix C presents a geometrical expression of the interception of goods diversification cones.

[28] In their original notation, they consider the indirect primary factors by intermediate input in their empirical analysis, such as $A = B(I - \tilde{A})$, where B is input-output matrix, $\tilde{A}$ is directe factor requirement matrix.

where V^{hW} is the vector of is the factor services needed to produce world commodity x^w using a reference to the technology matrix of country h as A^h , $h = h, f$. We use the same notations as those used in the effective endowments.

Unlike the Trefler model, the model of virtual endowments is with two goods price diversification cones. which will bring some difficulty to analyze its equilibrium solution and trade consequence. The solution of price-trade equilibrium by referring to country H's technology is slightly different from the solution by using country F's technology, from a strict view of analytical analyses on the result of share GNP of a country. It needs more studies about it for a theoretical result of price-trade equilibrium.

Illustrating trade pattern by the intersection cone of commodity price numerically

Any (numerical) value in the intersection of goods price diversification cones can be used to illustrate trade directions either commodity or factor content of trade.

Formula (3-4) shows the rays of the two goods price cones of the two countries under normal factor intensity in the absence of FIR, across countries. A relative commodity price must be within the intersection of cones as

$$\frac{a_{K1}^F}{a_{K2}^F} > \frac{p_1^c}{p_2^c} > \frac{a_{L1}^H}{a_{L2}^H} \tag{5-5}$$

Giving a value to $\frac{p_1^c}{p_2^c}$, numerically, there is a share of GNP of a country as s^h. Using equations (B-3) and (B-4), we can obtain the trade directions[29].

Similarly, for the case of factor intensity reversal (3-7), a commodity price must fall within the intersection of cones (3-8) as

$$\frac{a_{L1}^F}{a_{L2}^F} > \frac{p_1^c}{p_2^c} > \frac{a_{L1}^H}{a_{L2}^H} \tag{5-6}$$

Any relative commodity price that is within the range by (5-6) can illustrate a Leontief trade.

6. Empirical studies on the Leontief trade and the North-South trade

6.1 Empirical Studies Showed Co-existences of the Heckscher-Ohlin trade and the Leontief Trade

[29] It can show that the trade directions are same for any prices fall within the intersection, numerically.

Kwok and Yu (2005) investigated the 52 countries' data using differentiated factor intensity techniques and concluded that the Leontief paradox "is found to be either disappeared or eased."

The first wave of literature about the Leontief paradox was published between the 1960s and the 1990s. Half of them concluded that the paradox persists, and half were consistent with the Heckscher-Ohlin theory. The half to half results confused economists then. Nevertheless, all the tests are still meaningful from the view of the trade patterns of this study.

The empirical studies in this period mostly used sign prediction based on the same technology assumption,

$$(V_k^i - s^i V_k^W) F_k^i > 0 \qquad (6\text{-}1)$$

The reality is that countries are with different productivities. If (6-1) is failed in a study with its data, it implies actual factor abundance is a conflict with effective factor abundance. Therefore, the failure implies the Leontief trade. Half of the tests at this period reported the Leontief trade[30], which is denied by lacking an adequate conceptual foundation. This paper does supply the conceptual description of the Leontief trade. Based on the half of test results at this period, we may see the co-existence of the Heckscher-Ohlin trade and Leontief trades.

6.2 Skill Intensity Reversal (Factor Reward Intensity Reversal) and North-South Trade

Some studies in this century show evidence of the Leontief trade by factor intensity reversals. Kurokawa (2011) showed "clear-cut evidence for the existence of the skill intensity reversal" in his empirical study of the USA-Mexico economy. Sampson (2016) interpreted his assignment reversals of the skilled workforce between North and South by factor intensity reversal. Takahashi (2004) studied the postwar Japanese economy. He interpreted Japan's economic growth as a capital-intensity reversal.

Reshef (2007) claimed "One of the most prevalent economic phenomena in the last two decades of the 20th century has been the increase in skill premia in many countries around

[30] We cite fewer of test with Leontief trade ant the period. Keesing (1966) inspected the factor contents of trade in some OECD countries and reported that US exports have higher skill input than their imports[30]. Heller (1976) studied the Japanese economy and documented the changes in trade factor contents. Roskamp (1963) noted that in 1954 West German experts were more labor-intensive than imports. Baldwin (1971) showed that U.S. imports were 27% more capital-intensive than U.S. exports in the 1962 trade data, using a measure like Leontief's.

the globe skilled workers have been receiving a higher share of income and higher wages relative to their less-skilled fellow workers. The magnitude of this increase varies considerably across countries but is economically large almost everywhere". He cited other five studies which presented the same results[31]. Kozo and Yoshinori (2017) found the existence of factor intensity reversals in their study as well. They wrote, "Using newly developed region-level data; however, we argue that the abandonment of factor intensity reversals in the empirical analysis has been premature. Specifically, we find that the degree of the factor intensity reversals is higher than that found in previous studies on average".

Sampson (2016) specially mentioned in his study, "Therefore, assignment reversals offer a new explanation for why trade liberalization has led to increased wage inequality not only in the relative skill abundant North but also in the relative skill scarce South". Equations (4-16) through (4-18) present the reward increasing on the same factor in both countries. It is another typical character of the Leontief FIR trade.

6.3 The sign predictions by the effective endowments and virtual endowments favor both Leontief trade and Heckscher-Ohlin Trade

In empirical studies, the sign prediction for the effective endowments can be written as

$$(V_k^i - s^i \sum_j \pi_k^i V_k^j) F_k^i > 0 \tag{6-2}$$

where V_k^j is the element of vector V^j which is defined as $V^j = \Pi_j^{-1} A^0 y^j$. And V_k^j is the factor services needed to product country j's commodity y^j using a reference to productivity in country i as A^0. F_k^i is the factor services exported by country i.

The sign prediction for virtual endowments is

$$(V_k^{vi} - s^i \sum_j V_k^{vj}) F_k^{vi} > 0 \tag{6-3}$$

where V_k^{vj} is the element of vector V^{vj} which is defined as $V^{vj} = A^0 y^j$. V_k^{vj} is the factor services needed to product country j's commodity y^j using a reference to technology matrix in country i as A^0. F_k^{vi} is the factor services exported by country i.

Both the Heckscher-Ohlin trade and the Leontief trades are under the logic (6-2) and (6-3). They are derived from the logic of those signs. Therefore, the signs above favor

[31] Acemoglu (2003), Behrman et al. (2003), Gorg, and Strobl (2002), and Hoekman and Winters (2005). All of them is about the phenomenon that skill intensive reversal has indeed been global, as both developed, and less-developed countries have experienced it.

both the Heckscher-Ohlin trade and the Leontief trades. It is not sufficient to use those signs and their test results to clear the issue of the Leontief paradox simply.

Conclusion

This paper uses three different approaches to demonstrate that there are three trade patterns when considering countries having different productivities. They are the Heckscher-Ohlin trade, the mutaul Leontief trade, the FIR Leontief trade.

The price-trade equilibrium with factor price localization shows the Leontief trades are trade consequences.

The study demonstrates that the ideas and the sign predictions both of the effective endowments and the virtual endowments favor either the Heckscher-Ohlin trade or the Leontief trade. It implies that the Leontief trade conceptually is correct when considering different productivities across countries[32].

The numerical simulation by the intersection cone of goods price diversification cones can illustrate the three trade patterns[33] more straightforwardly.

More than half of the empirical studies reported evidence of the Leontief paradox. These results cannot be ignored conceptually. All of the works of literature are about the trade between north countries and source countries. This is the first theoretical result in international economics that free trade will be beneficial to workers both in the south countries and in the north countries. It implies that trade will be an important activity to improve poverty and inequality and reduce labor migration.

Appendix A – Numerical Example of the FIR Leontief Trade

The technological matrix for the home country, in this example, is

$$A^H = \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The matrix for the foreign country is

[32] Theoretically, it is true also that Leontief trade conceptually has no background when considering two countries have the same technologies.

[33] It is a rare case that a researcher provide three different ways to demonsatrate a economic logic or theory.

$$A^F = \begin{bmatrix} 0.0 & 1/0.9 \\ 1/0.8 & 1.0 \end{bmatrix} \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The factor intensities of the two countries are

$$a_{K1}^H / a_{L1}^H = 2.0 > \ a_{K2}^H / a_{L2}^H = 0.5$$

$$a_{K1}^F / a_{L1}^F = \ 0.562 < a_{K2}^F / a_{L2}^F = 2.25$$

The home country is capital intensive in product 1, and the foreign country is capital intensive in industry 2. The system is in the presence of the FIRs. We take the factor endowments for the two countries as

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 4200 \\ 3000 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 3187.5 \\ 2666.6 \end{bmatrix}$$

The outputs of the two countries are

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 1200.0 \\ 600.0 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 500.0 \\ 900.0 \end{bmatrix}$$

The home country is actual capital abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \ \frac{K^F}{L^F} = \frac{3187.5}{2666.6} = 1.19$$

The home country is effective capital abundant also as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \ \frac{K^{hF}}{L^{hF}} = \frac{2400}{2550} = 0.94$$

Therefore, the home country exports capital services and exports commodity 1 since commodity 1 uses the capital intensively.

The foreign country is effective capital abundant also as

$$\frac{K^F}{L^F} = \frac{318.75}{2666.6} = 1.19 > \ \frac{K^{fH}}{L^{fH}} = \frac{3750}{4666} = 0.80$$

Therefore, the foreign country exports capital services too. It will export commodity 2 since commodity 2 used the capital intensively. The home country is with the Heckscher-Ohlin trade and the foreign country is with the Leontief trade.

The share of GNP is, $s^H = 0.5884$. The trade flows and the factor contents of trades by the share of GNP are:

$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} -199.6 \\ 282.6 \end{bmatrix}, \ \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = \begin{bmatrix} 199.6 \\ -282.6 \end{bmatrix}, \ \begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} -316.2 \\ 265.9 \end{bmatrix}, \quad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} -332.8 \\ 351.3 \end{bmatrix}$$

We see that both countries export capital services and import labor services. The trade converts the globally effective abundant factor into the globally scarce factor. However, the trade flows are normal, country H exports product 1, and country F exports product 2.

At the equilibrium, the world prices and the localized factor prices are

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 4.0227 \\ 2.8409 \end{bmatrix}, \quad \begin{bmatrix} r^{*H} \\ w^{*H} \end{bmatrix} = \begin{bmatrix} 0.8409 \\ 1.0000 \end{bmatrix}, \quad \begin{bmatrix} r^{*F} \\ w^{*F} \end{bmatrix} = \begin{bmatrix} 0.8000 \\ 0.7568 \end{bmatrix}$$

Here, we assume $w^{*H} = 1$. The autarky prices for the two countries are

$$\begin{bmatrix} r^{aH} \\ w^{aH} \end{bmatrix} = \begin{bmatrix} 0.7142 \\ 1.0 \end{bmatrix}, \begin{bmatrix} r^{aF} \\ w^{aF} \end{bmatrix} = \begin{bmatrix} 0.8366 \\ 1.0 \end{bmatrix}$$

Here, we assume $w^{aH} = 1$, and $w^{aF} = 1$. The gains from trade are

$$g^H = W^{aH} \cdot F^H = 40.04, \quad g^H = W^{aH} \cdot F^F = 73.27$$

Appendix B – Gains from trade by localized factor prices

We express the gains from trade for country H as

$$W^{aH} \cdot F^H > 0 \tag{B-1}$$

where W^{aH} is autarky factor prices in country H. It can be expressed as[34]

$$W^{aH} = \begin{bmatrix} K^H \\ L^H \end{bmatrix} \tag{B-2}$$

The factor content of trade of country H by (2-19) and (2-20) is

$$F^H = \begin{bmatrix} -\frac{1}{2}\frac{K^H L^{hW} - L^{hW} L^H}{L^{hW}} \\ \frac{1}{2}\frac{K^H L^{hW} - K^{hW} L^H}{K^{hW}} \end{bmatrix} \tag{B-3}$$

Substituting (B-2) through (B-3) into (B-1) yields

$$[L^H \quad K^H] \begin{bmatrix} -\frac{1}{2}\frac{K^H L^{hW} - L^{hW} L^H}{L^{hW}} \\ \frac{1}{2}\frac{K^H L^{hW} - K^{hW} L^H}{K^{hW}} \end{bmatrix} > 0 \tag{B-4}$$

Reduced it to

$$\frac{(K^H L^{hW} - K^{hW} L^H)^2}{2K^{hW} L^{hW}} > 0 \tag{B-5}$$

Similarly, we can obtain the gain from trade for country F.

Reference

[34] see Guo (2019)

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