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In Search of A Giffen Input:
A Comprehensive Analysis of The Wold-Juréen (1953) Production Function

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Abstract: The Wold-Juréen (1953) functional form has proven to be invaluable to the development of consumer theory, and to the general search for utility functions which have the potential to yield a Giffen good. Because of this success, it seems only natural to ask about the value of Wold-Juréen (1953) functional form to the development of producer theory, and to the general quest for production functions which have the potential to yield a Giffen factor input. To date, this set of research questions remains unanswered, aside from one unenlightening comment by Weber (2001, page 622). The present paper answers these questions by offering a comprehensive analysis of the Wold-Juréen (1953) production function in the ongoing search for production functions which have the potential to yield a Giffen factor.

Keywords: Slutsky decomposition, Giffen factor, Wold-Juréen (1953) production function, pedagogy.

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1. Introduction

The Wold-Juréen (1953) functional form has proven to be invaluable to the development of consumer theory, and to the general search for utility functions which have the potential to yield a Giffen good. See, for example, Weber (1997), Sproule (2020), and Sproule and Karras (2022).

Because of this success, it seems only natural to ask about the value of Wold-Juréen (1953) functional form to the development of producer theory, and to the general quest for production functions which have the potential to yield a Giffen factor. [2, 3].To date, no research has answered this question in full. However a solitary paper by Christian Weber (2001) made mention of the potential value of the Wold-Juréen (1953) functional form to the study of Giffenity when the decision maker (hereafter DM) is a producer (rather than a consumer). The key particulars of this paper are these: (a) Weber (2001) began by assessing the capability of the Gil Epstein and Uriel Spiegel (2000) production function to generate a Giffen factor, and then (b) Weber (2001) finished by giving (no more than) a nod to the potential value of the Wold-Juréen (1953) functional form in the ongoing search for Giffen behavior by the producer.

The present paper answered the question about the value and viability of the Wold-Juréen (1953) production function to generate a Giffen factor, and this it does within the context of three separate analytical or problem types. The core question posed, and the three problem types, are as follows:

[2] The conceptual backdrop for this paper should be seen as the intersection set of three overlapping research domains within microeconomics: production economics, production functions, and cost functions. For details, the reader is directed to one or more of the following eleven monographs: Beattie et al. (2009), Chambers (1988), Coto-Millán (2003), Debertin (1986), Färe (1988), Førsund (1984), Fuss and McFadden (1978), Hackman (2008), Heathfield (1971), Rasmussen (2013), and Shephard (1981).
[3] To fasten down the terminology used in this paper, please note: by the phrase "Factor 1 is a Giffen factor", we mean that the market price of Factor 1 and quantity demanded of Factor 1 move in the same direction [Jensen and Miller (2008)].

- **Question 1:** Can the constrained-output-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?
- **Question 2:** Can the constrained-cost-minimization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?
- **Question 3:** Can the unconstrained-profit-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?

The answers to these three questions are useful in that they would facilitate the direct comparison between: (a) the properties of the producer model predicated on the Wold-Juréen (1953) production function versus (b) the properties of the producer model predicated on the much used "well-behaved arbitrary production function". One prime example of the usefulness of this comparison is our assessment of the analysis articulated by Keizo Nagatani (1978), who uses the well-behaved arbitrary production function, vis-à-vis the Wold-Juréen (1953) production function.

This paper is organized as follows. In Section 2, we define the Wold-Juréen (1953) production function, and its key properties. In Section 3, we address Questions 1 and 2. There we show that: (a) the answer to Question 1 [viz., can the constrained-output-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?] is "yes", and (b) the answer to Question 2 [viz., can the constrained-cost-minimization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?] is "no",

In Section 4, we address Question 3 [viz., can the unconstrained-profit-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function? .There, we find that the short answer is "no". The long answer to Question 3 is this: our findings both agree, and are at odds, with the research by Nagatani (1978). In particular: (a) Like Nagatani (1978), who uses an arbitrary production function, rather than the Wold-Juréen (1953) production function, we answer "no, the unconstrained-profit-maximization problem cannot yield a Giffen factor if the production function is the Wold-Juréen (1953) production function". (b) But unlike Nagatani

(1978), our negative answer flows from the fact that the related unconditional factor demand function does not exist. Our final remarks are offered in Section 5.

2. The Wold-Juréen (1953) Production Function, and Its Core Properties

Suppose that our DM's production function is the Wold-Juréen (1953) production function. That is, suppose that our DM's production function conforms to Definition 1:

Definition 1: After Wold and Juréen (1953), Weber (1997), Sproule (2020), and Sproule and Karras (2022), suppose that our DM's production function is defined as $Q = (x_1 - 1)(x_2 - 2)^{-2}$ where Q denotes the level of output, where x_1 denotes the related quantity of Factor 1, where x_2 denotes the related quantity of Factor 2, and where $x_1 > 1$ and where $0 < x_2 < 2$

Several properties of the Wold-Juréen (1953) production function flow from Definition 1 Some of these are reported in Definitions 2 to 5 and in Proposition 1. All of these follow next.

Definition 2: Because $x_1 > 1$ and where $0 < x_2 < 2$, it follows that

$$Q_{x_1} = (x_2 - 2)^{-2} > 0$$

$$Q_{x_2} = -2(x_1 - 1)(x_2 - 2)^{-3} > 0$$

$$Q_{x_1 x_1} = 0$$

$$Q_{x_1 x_2} = -2(x_2 - 2)^{-3} > 0$$

$$Q_{x_2 x_2} = 6(x_1 - 1)(x_2 - 2)^{-4} > 0$$

Definition 3: Definitions 1 and 2 aid in defining and in signing a particular Hessian determinant of subsequent interest. This is:

$$|H_2| = \begin{vmatrix} Q_{x_1x_1} & Q_{x_1x_2} \\ Q_{x_1x_2} & Q_{x_2x_2} \end{vmatrix} = \begin{vmatrix} 0 & -2(x_2-2)^{-3} \\ -2(x_2-2)^{-3} & 6(x_1-1)(x_2-2)^{-4} \end{vmatrix}$$

$$= -\left(-2(x_2-2)^{-3}\right)^2 < 0$$

Definition 4 [Weber (1997, page 39)]: Definitions 1 and 2 also aid in defining and in signing one of two particular bordered Hessian determinants of subsequent interest. The first one is,

$$|\bar{B}_1| = \begin{vmatrix} 0 & Q_{x_1} & Q_{x_2} \\ Q_{x_1} & Q_{x_1x_1} & Q_{x_1x_2} \\ Q_{x_2} & Q_{x_1x_2} & Q_{x_2x_2} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & (x_2-2)^{-2} & -2(x_1-1)(x_2-2)^{-3} \\ (x_2-2)^{-2} & 0 & -2(x_2-2)^{-3} \\ -2(x_1-1)(x_2-2)^{-3} & -2(x_2-2)^{-3} & 6(x_1-1)(x_2-2)^{-4} \end{vmatrix}$$

$$= 2(x_1-1)(x_2-2)^{-8} > 0$$

since $x_1 > 1$.

Definition 5: Likewise, Definitions 1 and 2 aid in defining and in signing the second of two particular bordered Hessian determinants of subsequent interest. Under the assumption that there exists a scalar, $\lambda' > 0$, the second one is:

$$|\bar{B}_2| = \begin{vmatrix} 0 & Q_{x_1} & Q_{x_2} \\ Q_{x_1} & -\lambda' Q_{x_1x_1} & -\lambda' Q_{x_1x_2} \\ Q_{x_2} & -\lambda' Q_{x_1x_2} & -\lambda' Q_{x_2x_2} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & (x_2-2)^{-2} & -2(x_1-1)(x_2-2)^{-3} \\ (x_2-2)^{-2} & 0 & \lambda' 2(x_2-2)^{-3} \\ -2(x_1-1)(x_2-2)^{-3} & \lambda' 2(x_2-2)^{-3} & -\lambda' 6(x_1-1)(x_2-2)^{-4} \end{vmatrix}$$

$$= -\lambda'\left(2(x_1-1)(x_2-2)^{-8}\right) < 0$$

since $x_1 > 1$.and $\lambda' > 0$,

Finally, it is worth noting that the relationship between the two particular bordered Hessian determinants reported in Definitions 4 and 5 above can be reconciled as follows.

Proposition 1: $|\bar{B}_2| = -\lambda'|\bar{B}_1| < 0$ since $\lambda' > 0$ and $|\bar{B}_1| > 0.$

3. In Search of A Giffen Input Under the Wold-Juréen (1953) Production Function -- The Producer's Twin Constrained-Optimization Problems

In this section, we explore the prospect of Giffenity within the primal and dual of the producer's twin constrained-optimization problems. The first problem to be considered will be the primal problem, which we define as the constrained-output-maximization problem under the Wold-Juréen (1953) production function. The second problem to be considered will be the dual problem, which we define as the constrained-cost-minimization problem under the Wold-Juréen (1953) production function.

3.1. The Search for A Giffen Input Within the Primal Problem: Since the analytical problem to be inspected here is the constrained-output-maximization problem, we begin with the following Lagrange function:

$$\text{maximize}_{x_1, x_2, \lambda}\ L = Q(x_1, x_2) + \lambda\left(p_1 x_1 + p_2 x_2 - \bar{C}\right) \tag{1}$$

where

$Q(x_1, x_2) = (x_1 - 1)(x_2 - 2)^{-2}$ such that $x_1 > 1$ and $0 < x_2 < 2$,
p_1 denotes the unit price of Factor 1,
p_2 denotes the unit price of Factor 2, and
$\bar{C}$ denotes constant costs, viz., $\bar{C} = p_1 x_1^{C1}(p_1, p_2, C) + p_2 x_2^{C1}(p_1, p_2, C)$.

Regarding Equation (1), it follows that if the first-order conditions (hereafter FOCs) and the second-order condition (hereafter SOC) for an interior maximum hold, then there exists the first of two types of conditional factor demand functions. These factor demand functions have the form:

$$x_1^{C1} = x_1^{C1}(p_1, p_2, C) = 2 + \frac{2p_2 - C}{p_1} \tag{2}$$

$$x_2^{C1} = x_2^{C1}(p_1, p_2, C) = 2\left(\frac{C - p_1}{p_2} - 1\right) \tag{3}$$

In an effort to render these factor demand functions distinct from the other two types (which will be defined and discussed below), we employ the superscript "C1".

Remark 1 [Weber (1997, page 39), and Chiang and Wainwright (2005, page 376)]: It should be noted: (a) that the bordered Hessian determinant contained in Definition 4 is the SOC for Equation (1). and (b) in view of its sign, we conclude that the SOC for Equation (1) holds.

We turn next to the parallel analyses of Sproule (2020) and Sproule and Karras (2022) regarding the presence of a Giffen factor in the constrained-output-maximization problem when the DM's production function is the Wold-Juréen (1953) production function,

Sproule's (2020) analysis begins with these three lemmas.

Lemma 1 [Sproule (2020)]: If the DM's production function is the Wold-Juréen (1953) production function, then $C < p_1 + 2p_2$.

Lemma 2 [Sproule (2020)]: If the DM's production function is the Wold-Juréen (1953) production function, then $p_1 + p_2 < C$

Lemma 3 [Sproule (2020)]: If the DM's production function is the Wold-Juréen (1953) production function, then (by Lemmas 1 and 2) $p_1 - p_2 < C - 2p_2 < p_1$.

Proposition 2 [Sproule (2020)]: If the DM's production function is the Wold-Juréen (1953) production function, and if $p_1 \geq p_2$, then Factor 1 is a Giffen factor.

Proof: If the DM's production function is the Wold-Juréen (1953) production function, and if $p_1 \geq p_2$, then $C - 2p_2 > 0$ [Lemma 3] and $\dfrac{\partial x_1^{C1}}{\partial p_1} = \dfrac{C - 2p_2}{(p_1)^2} > 0$ •

There exists an alternative to Proposition 2. That is, unlike Proposition 2 which requires the precondition $p_1 \geq p_2$, the alternative does not require any precondition. Why is that? Proposition 2 makes use of only Equation (2), whereas its alternative makes use of both Equation (2) and Equation (3). That alternative to Proposition 2 is as follows:

Proposition 3 [Sproule and Karras (2022)]: If the DM's production function is the Wold-Juréen (1953) production function, then: (a) Factor 2 is a complement to Factor 1, and therefore (b) Factor 1 is a Giffen factor.

Proof: If the DM's production function is the Wold-Juréen (1953) production function, then $\dfrac{\partial x_2^{C1}}{\partial p_1} < 0$ [Equation (3)], and because $\bar{C} = p_1 x_1^{C1}(p_1, p_2, C) + p_2 x_2^{C1}(p_1, p_2, C)$, it follows that $\text{sign}\left(\dfrac{\partial x_1^{C1}}{\partial p_1}\right) = -\,\text{sign}\left(\dfrac{\partial x_2^{C1}}{\partial p_1}\right)$ and that $\dfrac{\partial x_1^{C1}}{\partial p_1} > 0$ •

3.2. The Search for A Giffen Input Within the Dual Problem: Since the analytical problem to be inspected here is the constrained-cost-minimization, we begin with the following Lagrange function:

$$\text{minimize}_{\,x_1, x_2, \lambda'}\ L' = p_1 x_1 + p_2 x_2 - \lambda' \left(Q(x_1, x_2) - \bar{Q} \right) \tag{4}$$

where once again $Q(x_1, x_2) = (x_1 - 1)(x_2 - 2)^{-2}$ such that $x_1 > 1$ and $0 < x_2 < 2$.

Regarding Equation (4), it follows that if the FOC and SOC for an interior minimum hold, then there exists the second of two types of conditional factor demand functions. These factor demand functions have the form:

$$x_1^{C2} = x_1^{C2}(p_1, p_2, Q) = 1 + \frac{1}{4Q}\left(\frac{p_2}{p_1}\right)^2 \tag{5}$$

$$x_2^{C2} = x_2^{C2}(p_1, p_2, Q) = 2 - \frac{1}{2Q}\left(\frac{p_2}{p_1}\right) \tag{6}$$

In an effort to render these factor demand functions distinct from the other two types discussed in this paper, we employ the superscript "C2".

Remark 2 [Chiang and Wainwright (2005, pages 376 and 392), and Proposition 1]: It should be noted: (a) that the bordered Hessian determinant contained in Definition 5 is the SOC for Equation (4), and (b) in view of its sign, we conclude that the SOC for Equation (4) holds.

Proposition 4: If the optimization problem is the constrained-cost-minimization problem, if our DM's production function is the Wold-Juréen (1953) production function, then neither Factor 1 nor Factor can be a Giffen factor.

Proof: From Equation (5), $\frac{\partial x_1^{C2}}{\partial p_1} = -\frac{1}{2Q}\frac{(p_2)^2}{(p_1)^3} < 0$ And from Equation (6),

$$\frac{\partial x_2^{C2}}{\partial p_2} = -\frac{1}{2Q}\left(\frac{1}{p_1}\right) < 0 \bullet$$

4. In Search of A Giffen Input Under the Wold-Juréen (1953) Production Function -- The Producer's Unconstrained-Profit-Maximization Problem

In this section, we explore the prospect of Giffenity in the producer's unconstrained-optimization problem under two scenarios: (a) the case in which the producer's production function is an arbitrary production function, and (b) the case in which the producer's production function is the Wold-Juréen (1953) production function. The exploration of the first scenario does not require the development of new theory, We merely have to draw upon the findings of Keizo Nagatani (1978), which we review next:

4.1. A Review of Nagatani (1978): The paper by Nagatani (1978) is predicated on the producer's unconstrained-optimization problem and on an arbitrary production function.[4]

[4] Takayama (1993, pages 188-192) provides a detailed overview both to Nagatani (1978) and to the literature which predates Nagatani's (1978) contribution.

Hence the analytical platform for Nagatani's (1978) paper must be viewed as the following objective function:

.

$$\max_{x_1,x_2} \pi \;=\; P\,Q(x_1, x_2) \;-\; \left(p_1 x_1 \;+\; p_2 x_2\right) \tag{7}$$

where $Q = Q(x_1, x_2)$ denotes an arbitrary production function. Now if the FOCs and the SOCs associated with Equation (7) hold, then there exists a pair of unconditional factor demand functions of the form:

$$x_j^U \;=\; x_j^U(p_1, p_2, P) \tag{8}$$

where i=1,2 and where the superscript "U" refers to an "unconditional factor demand function", and this superscript is employed to render the related factor demand functions distinct from the other two types of factor demand functions discussed in the above.

It is important to note that the SOCs for Equation (7) are:

$$\pi_{x_1x_1} \;<\; 0 \text{ and } |H_1| \;=\; \begin{vmatrix} \pi_{x_1x_1} & \pi_{x_1x_2} \\ \pi_{x_1x_2} & \pi_{x_2x_2} \end{vmatrix} \;>\; 0 \tag{9}$$

[Chiang and Wainwright (2005, page 332)]. Likewise, it is important to also note that these properties are tied to the properties of the underlying arbitrary production function, $Q \;=\; Q\left(x_1, x_2\right)$. That is,

(a) Recall that the properties of the production function (provided that it has a "bliss point" or an interior maximum) include

$$Q_{x_1x_1} \;<\; 0 \text{ and } |H_2| \;=\; \begin{vmatrix} Q_{x_1x_1} & Q_{x_1x_2} \\ Q_{x_1x_2} & Q_{x_2x_2} \end{vmatrix} \;>\; 0. \tag{10}$$

(b) From the properties contained in Equation (10), there flows the SOCs in Equation (9), in that:

$$\pi_{x_1x_1} \;=\; P\,Q_{x_1x_1} \;<\; 0 \text{ and } |H_2| \;=\; \begin{vmatrix} \pi_{x_1x_1} & \pi_{x_1x_2} \\ \pi_{x_1x_2} & \pi_{x_2x_2} \end{vmatrix} \;=\; P|H_1| \;>\; 0 \tag{11}$$

The above lays the initial groundwork for this discussion. We turn next to outlining the contribution of Nagatani (1978), which is "there can be no Giffen input" [Nagatani (1978, page 521)].

To begin, we note that, from the duality properties in producer theory in the long run, we may write:

$$x_j^U(p_1, p_2, P) \;=\; x_j^{C2}(p_1, p_2, Q) \;=\; x_j^{C2}(p_1, p_2, Q(p_1, p_2, P)) \qquad (12)$$

Next, we note that the partial differentiation of Equation (12) with respect to p_i yields:

$$\frac{\partial x_j^U(p_1, p_2, P)}{\partial p_j} = \frac{\partial x_j^{C2}(p_1, p_2, Q(p_1, p_2, P))}{\partial p_j} + \frac{\partial x_j^{C2}(p_1, p_2, Q(p_1, p_2, P))}{\partial Q}\ \frac{\partial Q(p_1, p_2, P)}{\partial p_j}$$

where we note that:

$\dfrac{\partial x_j^U(p_1, p_2, P)}{\partial p_j}$ is termed the "total effect";

$\dfrac{\partial x_j^{C2}(p_1, p_2, Q(p_1, p_2, P))}{\partial p_j}$ is termed the "substitution effect"; and

$\dfrac{\partial x_j^{C2}(p_1, p_2, Q(p_1, p_2, P))}{\partial Q}\ \dfrac{\partial Q(p_1, p_2, P)}{\partial p_j}$ is termed the "expansion effect".

What Nagatani (1978) demonstrated is that the expansion effect is always negative. And because of this, and because of the fact that the substitution effect is also always negative, Nagatani (1978) was able to infer or asset that the total effect must also be negative. All of the above considerations led Nagatani (1978, page 521) to conclude that "there can be no Giffen input" [Nagatani (1978, page 521)].

One issue likely remains in the mind of the reader, and that is: how did Nagatani demonstrate that the expansion effect is always negative? He did so by showing that the two component parts of the expansion effect are of opposite sign. That is, Nagatani showed that

$$\text{sign}\left(\frac{\partial x_j^{C2}(p_1, p_2, Q(p_1, p_2, P))}{\partial Q}\right) = -\ \text{sign}\left(\frac{\partial Q(p_1, p_2, P)}{\partial p_j}\right) \tag{13}$$

and then (from this) Nagatani inferred that the expansion effect is always negative, viz.,

$$\frac{\partial x_j^{C2}(p_1, p_2, Q(p_1, p_2, P))}{\partial Q}\ \frac{\partial Q(p_1, p_2, P)}{\partial p_j} < 0$$

In summary, Proposition 5:

Proposition 5 [Nagatani (1978)]: If the optimization problem is the unconstrained-profit-maximization problem, if our DM's production function is the arbitrary production function, then neither Factor 1 nor Factor 2 can be a Giffen factor.

4.2. Does the Nagatani (1978) Result Carry Over From The Arbitrary Production Function To The Wold-Juréen (1953) Production Function? The simple answer is no, and here is why.

- Nagatani's (1978) result is predicated (in part or in whole) on the SOCs when the production function is the arbitrary production function. And these SOCs are tied to the properties of the underlying arbitrary production function (which has a "bliss point"), viz.,

$$Q_{x_1x_1} < 0 \text{ and } |H_2| = \begin{vmatrix} Q_{x_1x_1} & Q_{x_1x_2} \\ Q_{x_1x_2} & Q_{x_2x_2} \end{vmatrix} > 0. \tag{14}$$

- The SOCs in Equation (9) do not hold when the arbitrary production function is replaced by the Wold-Juréen (1953) production function, We can demonstrate this by comparing Equation (14) to Definitions 1 and 2, which state:

$$Q_{x_1x_1} = 0 \text{ and } |H_2| = \begin{vmatrix} 0 & Q_{x_1x_2} \\ Q_{x_1x_2} & Q_{x_2x_2} \end{vmatrix} < 0. \quad (15)$$

- Because Equation (15) contradicts Equation (14), we conclude that when the production function is the Wold-Juréen (1953) production function: (a) the unconditional factor demand functions do not exist, and (b) Nagatani's (1978) claim ("there can be no Giffen input") does not apply.

Proposition 6: If the optimization problem is the unconstrained-profit-maximization problem, if our DM's production function is the Wold-Juréen (1953) production function, then the Giffen behavior of Factor 1 and Factor 2 is indeterminate.

5. Conclusion

The present paper explored the value of Wold-Juréen (1953) functional form in the search for Giffen behavior within the context of producer theory.

To date, the research literature is silent about the value and viability of the Wold-Juréen (1953) production function in generating a Giffen factor within each of three analytical or problem types. In particular, the core question posed, within each of three problem types, are these:

- Question 1: Can the constrained-output-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?
- Question 2: Can the constrained-cost-minimization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?
- Question 3: Can the unconstrained-profit-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?

We determined that Giffenity only occurs in the constrained-output-maximization problem (when the production function is the Wold-Juréen (1953) production function).

This paper is organized as follows. In Section 2, we defined the Wold-Juréen (1953) production function, and its key properties. In Section 3, we addressed Questions 1 and 2. There we showed that: (a) the answer to Question 1 [viz., Can the constrained-output-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?] is "yes", and (b) the answer to Question 2 [viz., Can the constrained-cost-minimization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?] is "no", In Section 4, we addressed Question 3 [viz., Can the unconstrained-profit-maximization problem yield a Giffen factor if the production function is the Wold-Juréen (1953) production function?].There, we provided both a short answer, which is "no", and a long answer. Our long answer is also "no", but it has two nuanced comments: (a) Like Nagatani (1978), who uses an arbitrary production function, rather than the Wold-Juréen (1953) production function, we answer "no, the unconstrained-profit-maximization problem cannot yield a Giffen factor if the production function is the Wold-Juréen (1953) production function". (b) But unlike Nagatani (1978), our negative answer flows from the fact that the related unconditional factor demand function does not exist. Our final remarks are offered in Section 5.

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