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Abstract

In a mechanism, a designer may reveal some information to influence agents' private types in order to obtain more payoffs. In the literature, the information is usually represented as random variables, the value of which are realized by the nature. However, this representation of information may not be proper in some practical cases. In this paper, we propose a type-adjustable mechanism where the information sent by the designer is modeled as a solution of her optimization problem. From the designer's perspective, the probability distributions of agents' private types may be optimally controlled. By constructing a type-adjustable first-price sealed-bid auction, we show that the seller may obtain more expected payoffs than what she could obtain at most in the traditional optimal auction model. Interestingly, to the satisfaction of all, each agent's *ex-ante* expected payoffs may be increased too. In the end, we compare the type-adjustable mechanism with other relevant models.

Key words: Mechanism design; Optimal auction; Bayesian implementation.

1 Introduction

In mechanism design theory [1–3], there are one designer and some agents. [1] The designer would like to implement a desired social choice function which specifies an outcome for each possible profile of agents' private types. In order to implement the social choice function, the designer constructs a mechanism which specifies each agent's feasible strategy set (*i.e.*, the allowed actions of

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[1] In this paper, the designer is denoted as "She", and the agent is denoted as "He".

each agent) and an outcome function (*i.e.*, a rule for how agents' actions get turned into a social choice).

Note that the designer's role is changed during the period of a mechanism. Before announcing a mechanism, the designer acts like a dominator, since she can decide all agents' feasible strategy sets and the outcome function that yields the final result. However, after announcing a mechanism, the designer acts like a passive receiver, since she should accept any outcome yielded by the mechanism, even if some outcome may correspond to low payoffs from her perspective. The reason for this embarrassment is that the outcome is not only dependent on the outcome function, but also dependent on agents' private types, which are unknown to the designer.

There have been several possible ways to improve the designer's situations. For example, the designer may hold a charity auction. Engers and McManus [4] proposed that agents' bids in a first-price charity auction are greater than those in a standard (non-charity) auction [5] because of the charitable benefit that winners receive from their own payments.

Additionally, agents' private types may not be fixed. Maschler *et al* ([6], Section 2.9.5) claimed that agents' types may change with changing circumstances. Hence, in order to improve payoffs, the designer may reveal some information to influence agents' informational environments, and then induce agents to change their private types. For example, Bergemann and Välimäki [7] proposed that for many practical cases, agents' types cannot be considered to be independent of the mechanism, and the seller may have control over pieces of evidence that determine the bidders' private valuations for the object on sale. In auctions with interdependent values, the seller may have some information that would affect the valuations of the bidders if known by them [8], [9]. Kamenica and Gentzkow [10] proposed a model of Bayesian persuasion in which the Sender can strategically control the the Receiver's information to influence her beliefs, and thus affect the actions that she takes.

So far, in the literature, the information revealed by the designer is usually represented as random variables, the value of which are realized randomly by the nature. It should be noted that this representation of information may not be proper in some practical cases. For example, let us consider a symmetric first-price sealed-bid (FPSB) auction, where the probability distribution of each bidder i's initial private valuation is identical to each other. Before each bidder i submits his bid b_i, the seller sends a signal [2] with a positive cost c as

[2] The signal is just an abstract notion, and practically it can denote different things, such as an advertisement, or a training course to improve bidders' knowledge to understand the sold object. The reason why the signal is assumed to be costly is that this is common in practical cases, and more importantly, a costly signal is more credible than a costless signal from any rational agent's perspective.

public information to change bidders' information environments. We assume that the seller knows the probability density function of each bidder's initial private valuation, and knows how each probability density function changes with the signal cost.[3] After learning about the signal, each agent adjusts his private valuation θ_i to the sold object, and then submits his bid b_i.

Without loss of generality, we assume that: 1) Each agent i's private valuation θ_i and bid b_i both increase concavely with the signal cost c. 2) If c is small, then a tiny increment δ_c of cost c will yield a big increment δ_{bi} of each bid b_i, *i.e.*, $\delta_c/\delta_{bi} < 1$ for a small c. Since the seller's payoffs is the winner's bid minus the seller's signal cost, then it will always be beneficial for the seller to increase the signal cost c as long as $\delta_c/\delta_{bi} < 1$ holds. Obviously, the seller's payoffs will reach the maximum when the signal cost c reaches a critical value such that $\delta_c = \delta_{bi}$ for each bidder i.

In the above-mentioned example, the information (*i.e.*, the costly signal) sent by the seller is modeled as a solution of an optimization problem, which aims to find a critical signal cost. Obviously, the information is deterministic. Hence, this circumstance should not be analyzed by using traditional mechanisms, where the information sent by the designer is represented as random variables.

This paper aims to propose a generalized mechanism to model such case. In Section 2, a series of notions are defined, such as type-adjustable mechanism, type adjustment function, optimal adjustment cost and type-adjustable Bayesian implementation. In Section 3, an example is constructed to show that by sending an optimal signal, the designer may obtain more expected payoffs than what she could obtain at most in the traditional optimal auction model. Interestingly, to the satisfaction of all, each agent's expected payoffs may be increased too. In Section 4, we compare the type-adjustable mechanism with other related models in the literature. Section 5 draws conclusions.

2 Model

Following Section 23.B of MWG's book [1], we consider a setting with one designer and I agents ($i = 1, \cdots, I$) with private types. The set of each agent i's possible type θ_i is denoted as Θ_i, and $\Theta = \Theta_1 \times \cdots \times \Theta_I$. Each agent knows his type, but not necessarily the types of the others. For any outcome $x \in X$, the utility function of each agent i with type θ_i is denoted as $u_i(x, \theta_i) : X \times \Theta_i \to \mathbb{R}$, the designer's utility is denoted as $u_d(x) \in \mathbb{R}$. For a set X

[3] A similar assumption can be seen in Section 2.1 of Ref [7], where Bergemann and Välimäki assumed that the initial type and transition function of each agent are common knowledge.

of possible outcomes, a social choice function (SCF) $f : \Theta \to X$ specifies an outcome for each profile of agents' types $\theta \in \Theta$. For each agent $i = 1, \cdots, I$, his *initial type* [4] is denoted as $\theta_i^0 \in \Theta_i$. Let $\phi^0(\theta^0) = (\phi_1^0(\theta_1^0), \cdots, \phi_I^0(\theta_I^0))$ denote the profile of probability density functions of agents' initial types $\theta^0 \in \Theta$.

Assumption 1: The designer knows the profile of probability density functions of agents' initial types $\phi^0(\theta^0)$.

Definition 1 (Type-adjustable mechanism):
Given a set of outcomes X and an SCF $f : \Theta_1 \times \cdots \times \Theta_I \to X$, a *type-adjustable* mechanism is denoted as $\Gamma^c = (S_1, \cdots, S_I, g, c)$, and is constructed by the following steps:
1) The designer performs a costly action a which is observable to all agents. The cost $c \in \mathbb{R}_+$ of the action a is assumed to be known by all agents. [5]
2) The designer specifies a feasible strategy set S_i for each agent i, and an outcome function $g : S_1 \times \cdots \times S_I \to X$.
3) After observing the action a and knowing the outcome function g, each agent i adjusts his initial type $\theta_i^0 \in \Theta_i$ by using an type adjustment function, which is defined in the following Definition 2.
4) Each agent i chooses a strategy $s_i \in S_i$ to perform.
5) The mechanism yields the outcome $g(s_1, \cdots, s_I) \in X$.

Definition 2 (Type adjustment function):
For each agent $i = 1, \cdots, I$ with private type θ_i, given an action with cost c, his *type adjustment function* is defined as $\mu_i(\theta_i, c) : \Theta_i \times \mathbb{R}_+ \to \Theta_i$ [6]. Let $\mu(\theta, c) = (\mu_1(\theta_1, c), \cdots, \mu_I(\theta_I, c))$ denote the profile of agents' type adjustment functions, in which $\theta = (\theta_1, \cdots, \theta_I)$ is a profile of all agents' types. Consider two following cases of cost c:
• $c = 0$: It is reasonable to assume that no agent will adjust his initial type after observing the action, *i.e.*, $\mu(\theta^0, 0) = \theta^0$ for any $\theta^0 \in \Theta$.
• $c > 0$: Each agent will adjust his initial type after observing the action. Let θ_i^c denote the result of agent i's adjusted type, *i.e.*, $\mu_i(\theta_i^0, c) = \theta_i^c$. Let

$$\theta^c = (\theta_1^c, \cdots, \theta_I^c) \in \Theta, \quad \theta_{-i}^c = (\theta_1^c, \cdots, \theta_{i-1}^c, \theta_{i+1}^c, \cdots, \theta_I^c), \quad \mu(\theta^0, c) = \theta^c.$$

[4] Here, the term "initial type" denotes each agent's private type *before* the designer constructs any mechanism.
[5] Since the action a acts as an open signal to all agents, it is natural to require the action a be costly. Otherwise if the designer can send any signal without any cost, it is unreasonable to assume that any rational agent is willing to believe such costless signal.
[6] In Ref [5] (Page 60, Line 12), Myerson proposed: "if there are quality uncertainties, then bidder i might tend to revise his valuation of the object after learning about other bidders' value estimates." Here, different from Myerson's proposition, in Definition 2 we propose that each agent revises his type after learning about the signal cost c. An example will be given in Section 3.

Assumption 2: The designer knows each agent i's type adjustment function $\mu_i(\theta_i, c)$.

Let $\phi^c(\theta^c) = (\phi_1^c(\theta_1^c), \cdots, \phi_I^c(\theta_I^c))$ denote the profile of probability density functions of agents' adjusted types $\theta^c \in \Theta$. For each $i = 1, \cdots, I$, let

$$\phi_{-i}^0(\theta_{-i}^0) = (\phi_1^0(\theta_1^0), \cdots, \phi_{i-1}^0(\theta_{i-1}^0), \phi_{i+1}^0(\theta_{i+1}^0), \cdots, \phi_I^0(\theta_I^0)),$$
$$\phi_{-i}^c(\theta_{-i}^c) = (\phi_1^c(\theta_1^c), \cdots, \phi_{i-1}^c(\theta_{i-1}^c), \phi_{i+1}^c(\theta_{i+1}^c), \cdots, \phi_I^c(\theta_I^c)).$$

Definition 3 (The designer's expected payoffs):
Given an *SCF* $f : \Theta \to X$ and a profile $\phi^0(\theta^0)$, *the designer's initial expected payoffs* is denoted as

$$\bar{u}_d(0) = E_{\theta^0} u_d(f(\theta^0)) = \int_{\theta^0 \in \Theta} u_d(f(\theta^0)) \phi^0(\theta^0) d\theta^0.$$

By choosing an action with cost c, *the designer's expected payoffs with type adjustment* is denoted as

$$\bar{u}_d(c) = E_{\theta^c} u_d(f(\theta^c)) - c = \int_{\theta^0 \in \Theta} u_d(f(\mu(\theta^0, c))) \phi^0(\theta^0) d\theta^0 - c.$$

Assumption 3: For any outcome $f(\theta^c)$, the designer's utility function $u_d(f(\theta^c))$ is assumed to be concave with respect to the cost c,[7] *i.e.*,

$$\frac{\partial u_d(f(\theta^c))}{\partial c} > 0, \quad \frac{\partial^2 u_d(f(\theta^c))}{\partial c^2} < 0, \quad \text{for any } c > 0.$$

Definition 4 (Optimal adjustment cost):
If there exists a cost $c^* > 0$ such that

$$\left. \frac{\partial \bar{u}_d(c)}{\partial c} \right|_{c=c^*} = 0,$$

then the designer's expected payoffs with type adjustment $\bar{u}_d(c)$ will reach its maximum at $c = c^*$, and c^* is denoted as the *optimal adjustment cost.*

Definition 5 (Type-adjustable Bayesian implementation):
Given an *SCF* f and a profile $\phi^0(\theta^0)$, f is called *type-adjustable Bayesian implementable* if the following conditions are satisfied:
1) There exists a positive optimal adjustment cost, *i.e.*, $c^* > 0$.
2) There exists a type-adjustable mechanism $\Gamma^{c^*} = (S_1, \cdots, S_I, g(\cdot), c^*)$ that implements f in Bayesian equilibrium. That is, there exists a strategy profile

[7] The concave function means that with the increasement of the cost c, the outcome $f(\theta^c)$ will become better from the viewpoint of the designer, however the marginal increasement of utility $u_d(f(\theta^c))$ decreases with the increasement of c.

$s^*(\cdot) = (s_1^*(\cdot), \cdots, s_I^*(\cdot))$ such that:
(i) For all agent i, all $\theta_i^{c^*} \in \Theta_i$, and all $\hat{s}_i \in S_i$, [8]

$$E_{\theta_{-i}^{c^*}}[u_i(g(s_i^*(\theta_i^{c^*}), s_{-i}^*(\theta_{-i}^{c^*})), \theta_i^{c^*})|\theta_i^{c^*}] \geq E_{\theta_{-i}^{c^*}}[u_i(g(\hat{s}_i, s_{-i}^*(\theta_{-i}^{c^*})), \theta_i^{c^*})|\theta_i^{c^*}]. \quad (1)$$

(ii) $g(s^*(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

Proposition 1: Given an *SCF* f and a profile $\phi^0(\theta^0)$, if f is type-adjustable Bayesian implementable, then the designer will obtain more expected payoffs than her initial expected payoffs $\bar{u}_d(0)$.
Proof: Since f is type-adjustable Bayesian implementable, then according to Definition 5, there exists an optimal adjustment cost $c^* > 0$ such that $\bar{u}_d(c^*)$ is the designer's maximum expected payoffs with type adjustment, and $\bar{u}_d(c^*) > \bar{u}_d(0)$. □

Proposition 2: If $\bar{u}_d(c)$ satisfies the following condition,

$$\left.\frac{\partial \bar{u}_d(c)}{\partial c}\right|_{c=0} \leq 0, \quad (2)$$

then $\bar{u}_d(0)$ is the designer's maximum expected payoffs, and it is worthless for the designer to adjust agents' types.
Proof: The proof is straightforward and omitted.

3 Example

Following the first-price sealed-bid (FPSB) auction given in MWG's book (Ref [1], Page 865), in this section we will construct a revised FPSB auction, and point out that it is type-adjustable Bayesian implementable. By analyzing the seller and each bidder's expected payoffs, we point out that all members will benefit from the type-adjustable mechanism.

3.1 A revised first-price sealed-bid auction

Suppose there are one seller and two bidders, each bidder i's initial valuation (*i.e.*, his initial type) θ_i^0 is drawn independently from the uniform distribution on $[0, 1]$. This distribution is known by the seller but the exact value of each θ_i^0 is bidder i's private information.

[8] In formula (1), the probability density functions of agents' types (except i) are the adjusted functions $\phi_{-i}^{c^*}(\theta_{-i}^{c^*})$. As a comparison, in the traditional notion of Bayesian equilibrium ([1], Page 883, Definition 23.D.1), the probability density functions of agents' types (except i) are just initial functions $\phi_{-i}^0(\theta_{-i}^0)$.

Different from the traditional FPSB auction, our revision is that before the auction the seller sends a signal with cost $c \geq 0$, which is observable to two bidders. Let $\beta > 0$ be a coefficient. [9] After observing the signal, each bidder i is assumed to adjust his private valuation to the object according to the following concave function,

$$\theta_i^c = (1 + \beta\sqrt{c})\theta_i^0. \tag{3}$$

Then each bidder i submits a sealed bid $b_i \geq 0$ to the seller. The bidder with the higher bid wins the object, and must pay money equal to his bid to the seller. Let $\theta = (\theta_1, \theta_2)$, consider the following social choice function

$$f(\theta) = (y_1(\theta), y_2(\theta), y_d(\theta), t_1(\theta), t_2(\theta), t_d(\theta)), \tag{4}$$

in which

$$\begin{aligned}
y_1(\theta) &= 1, \quad \text{if } \theta_1 \geq \theta_2; \quad = 0 \text{ if } \theta_1 < \theta_2 \\
y_2(\theta) &= 1, \quad \text{if } \theta_1 < \theta_2; \quad = 0 \text{ if } \theta_1 \geq \theta_2 \\
y_d(\theta) &= 0, \quad \text{for all } \theta \in \Theta \\
t_1(\theta) &= -\theta_1 y_1(\theta)/2 \\
t_2(\theta) &= -\theta_2 y_2(\theta)/2 \\
t_d(\theta) &= [\theta_1 y_1(\theta) + \theta_2 y_2(\theta)]/2.
\end{aligned}$$

The subscript "d" stands for the seller, and the subscript "1", "2" stands for the bidder 1 and bidder 2 respectively. $y_i = 1$ means that bidder i gets the object, t_i denotes bidder i's payment to the seller, t_d denotes the sum of two bidders' payment to the seller.

3.2 The SCF f is Bayesian implementable

Let us investigate whether the social choice function $f(\theta)$ is Bayesian implementable. We will look for a Bayesian equilibrium in which each bidder i's strategy $b_i(\cdot)$ takes the form $b_i(\theta_i^c) = \alpha_i\theta_i^c = \alpha_i(1+\beta\sqrt{c})\theta_i^0$ for $\alpha_i \in [0, 1]$. Suppose that bidder 2's strategy has this form, and consider bidder 1's problem. For each possible θ_1^c, bidder 1 wants to solve the following problem:

$$\max_{b_1 \geq 0} (\theta_1^c - b_1)\text{Prob}(b_2(\theta_2^c) \leq b_1). \tag{5}$$

Because bidder 2's highest possible bid is $\alpha_2(1 + \beta\sqrt{c})$ when $\theta_2^0 = 1$, it is evident that bidder 1's bid b_1 should not be greater than $\alpha_2(1 + \beta\sqrt{c})$. Note

[9] The greater the value β is, the more significantly the signal increases bidders' valuations to the sold object.

that θ_2^0 is uniformly distributed on $[0,1]$, and $b_2(\theta_2^c) = \alpha_2(1+\beta\sqrt{c})\theta_2^0 \leq b_1$ means that $\theta_2^0 \leq b_1/[\alpha_2(1+\beta\sqrt{c})]$. Thus,

$$\text{Prob}(b_2(\theta_2^c) \leq b_1) = \text{Prob}(\theta_2^0 \leq b_1/[\alpha_2(1+\beta\sqrt{c})]) = \frac{b_1}{\alpha_2(1+\beta\sqrt{c})}.$$

Now we can rewrite bidder 1's problem (formula 5) as:

$$\max_{0\leq b_1\leq\alpha_2(1+\beta\sqrt{c})} \frac{(\theta_1^c - b_1)b_1}{\alpha_2(1+\beta\sqrt{c})}$$

The solution to this maximum problem is

$$b_1^*(\theta_1^c) = \begin{cases} \theta_1^c/2, & \text{if } \theta_1^0/2 \leq \alpha_2 \\ \alpha_2(1+\beta\sqrt{c}), & \text{if } \theta_1^0/2 > \alpha_2 \end{cases}.$$

Similarly,

$$b_2^*(\theta_2^c) = \begin{cases} \theta_2^c/2, & \text{if } \theta_2^0/2 \leq \alpha_1 \\ \alpha_1(1+\beta\sqrt{c}), & \text{if } \theta_2^0/2 > \alpha_1 \end{cases}.$$

Let $\alpha_1 = \alpha_2 = 1/2$, we see that the strategies $b_i^*(\theta_i^c) = \theta_i^c/2 = (1+\beta\sqrt{c})\theta_i^0/2$ for $i = 1, 2$ constitute a Bayesian equilibrium for this revised FPSB auction.

Thus, the social choice function f is implemented in Bayesian equilibrium by the revised FPSB auction. Hence, f is Bayesian implementable.

3.3 The SCF f is type-adjustable Bayesian implementable

Let us consider the seller's expected payoffs with type adjustment:

$$\bar{u}_d(c) = (1+\beta\sqrt{c})E[\theta_1^0 y_1(\theta^0) + \theta_2^0 y_2(\theta^0)]/2 - c.$$

The seller's problem is to maximize $\bar{u}_d(c)$, *i.e.*,

$$\max_{c\geq 0}(1+\beta\sqrt{c})E[\theta_1^0 y_1(\theta^0) + \theta_2^0 y_2(\theta^0)]/2 - c.$$

By appendix, the seller's initial expected payoffs is $\bar{u}_d(0) = E[\theta_1^0 y_1(\theta^0) + \theta_2^0 y_2(\theta^0)]/2 = 1/3$. Thus, the seller's optimization problem is reformulated as:

$$\max_{c\geq 0}(1+\beta\sqrt{c})/3 - c.$$

It can be easily seen that Assumption 3 holds and the optimal adjustment cost is $c^* = \beta^2/36 > 0$. According to Section 3.2, the strategies $b_i^*(\theta_i^{c^*}) = \theta_i^{c^*}/2 = (1+\beta\sqrt{c^*})\theta_i^0/2$ for $i = 1, 2$ constitute a Bayesian equilibrium for the revised FPSB auction. By Definition 5, the social choice function f is type-adjustable Bayesian implementable.

3.4 The seller's expected payoffs with type adjustment

The seller's maximum expected payoffs with type adjustment is:

$$\bar{u}_d(c^*) = (1 + \beta\sqrt{c^*})/3 - c^* = \frac{1}{3}(1 + \frac{\beta^2}{12}).$$

Hence, if $\beta > \sqrt{3}$, then $\bar{u}_d(c^*) > 5/12$. Note that the seller's maximum expected payoffs in the traditional optimal auction with two bidders is 5/12 (Ref [8], Page 23, the ninth line from the bottom). Therefore, if $\beta > \sqrt{3}$, then by choosing the optimal adjustment cost $c^* = \beta^2/36$, *the seller can obtain more expected payoffs than what she could obtain at most in the traditional optimal auction model.*

3.5 Each bidder's ex ante expected payoffs

Now we consider each bidder's *ex ante* expected payoffs when the seller chooses the optimal adjustment cost $c^* = \beta^2/36$. By appendix, for the case of two bidders, the winner bidder's expected payoffs is denoted as follows:

$$\begin{aligned} E[\theta^{c^*}_{winner} - b^*_{winner}(\theta^{c^*}_{winner})] &= E[\theta^{c^*}_{winner}/2] = (1 + \beta\sqrt{c^*})E[\theta^0_{winner}]/2 \\ &= (1 + \beta\sqrt{c^*})E[\theta^0_1 y_1(\theta^0) + \theta^0_2 y_2(\theta^0)]/2 \\ &= \frac{1}{3} + \frac{\beta^2}{18}. \end{aligned}$$

Note that the loser bidder's expected payoffs is zero. Since the two bidders are symmetric, then each of them has the same probability 1/2 to be the winner bidder. Therefore, each bidder's *ex ante* expected payoffs is half of the winner's expected payoffs, *i.e.*, $1/6 + \beta^2/36$.

4 Comparison with related models

4.1 Optimal auction model

Let us recall the traditional first-price auction model with reserve price (Ref [8], Page 21). There is one object for sale, and N potential buyers are bidding for the object. Let $r > 0$ be the reserve price, and $[r, \omega]$ be the interval of each bidder i's valuation which is independently and identically distributed according to an increasing distribution function F. Fix a bidder, G denotes the distribution function of the highest valuation among the rest remaining bidders. According to Krishna's book (Ref [8], Page 22, Line 13), the *ex ante* expected payment of a bidder is

$$r(1-F(r))G(r)+\int_r^{\omega} y(1-F(y))g(y)dy. \tag{6}$$

For the case of two bidders with valuation range $[r,1]$ and uniform distribution,

$$\begin{aligned}&F(r)=r,\ G(r)=r,\ \omega=1,\\&F(y)=y,\ g(y)=1, \text{for any } y\in[r,1].\end{aligned}$$

By Ref [8] (Page 23), when each of two bidder's valuation to the object is uniformly distributed on interval $[0,1]$, the optimal reserve price $r^*=1/2$. Therefore, each bidder's *ex ante* expected payment in formula (6) is

$$\begin{aligned}&r^*(1-r^*)r^*+\int_{r^*}^1 y(1-y)dy\\&=\frac{1}{8}+\int_{\frac{1}{2}}^1 y(1-y)dy=\frac{5}{24}.\end{aligned}$$

Since the optimal reserve price is $1/2$, then each bidder's valuation to the object is uniformly distributed on interval $[1/2,1]$. Hence, each bidder's expected valuation is the middle point of interval $[1/2,1]$, *i.e.*, $3/4$.

Consequently, in the traditional optimal auction, each bidder's *ex ante* expected payoffs is his expected valuation $3/4$ minus his *ex ante* expected payment $5/24$, *i.e.*,

$$\frac{3}{4}-\frac{5}{24}=\frac{13}{24}. \tag{7}$$

As specified in Section 3.5, in the type-adjustable first-price sealed-bid auction, when the seller chooses the optimal adjustment cost $c^*=\beta^2/36$, each bidder's *ex ante* expected payoffs will be $1/6+\beta^2/36$. Obviously, if $\beta>\sqrt{27/2}$, then each bidder's *ex ante* expected payoffs will be greater than the corresponding value $13/24$ occurred in the traditional optimal auction.

According to Section 3.4, if $\beta>\sqrt{3}$, then the seller can obtain more expected payoffs than what she could obtain at most in the traditional optimal auction model.

To sum up, if $\beta>\sqrt{27/2}$, then for the social choice function specified by formula (4) in Section 3.1, not only the seller but also each bidder can obtain more payoffs from the type-adjustable mechanism than what they could obtain at most in traditional optimal auctions.

4.2 Auctions with interdependent values

The main distinctions between our model and the auctions with interdependent values [8] are as follows:
1) In auctions with interdependent values, the public information is represented as a random variable, and each bidder's initial private valuation is dependent to each other.
2) In our model, the public information sent by the designer is modeled as a solution of an optimization problem, and each bidder's initial private valuation is independent to each other. After learning about the public information, each bidder's private valuation is relevant to his own initial private valuation and the public information.

4.3 Dynamic mechanism design

Bergemann and Välimäki [11] proposed that agents' types may change in a nontrivial manner across periods of a dynamic mechanism. Consider a discounted discrete-time model with a finite or infinite ending date T. In each period $t \leq T$, each agent $i \in \{1, \cdots, I\}$ receives a payoff that depends on the current physical allocation $x_t \in X_t$, the current monetary payment $p_{i,t} \in \mathbb{R}$. The Bernoulli utility function u_i of agent i takes the form:

$$u_i(x_t, p_t, \theta_t) = v_i(x_t, \theta_{i,t}) - p_{i,t}$$

It is assumed that the type $\theta_{i,t}$ of agent i follows a *controlled Markov process* on his state space Θ_i. The utility function u_i and the Markov transition function of each agent i are assumed to be common knowledge at $t = 0$. There are three possible interpretations of controllable types $\theta_{i,t}$ (Ref [11], Section 2.2): 1) All agents are present in all periods of the game, and their types evolve according to an exogenous stochastic process on Θ_i. 2) All agents are present in all periods, but their future types depend endogenously on current allocations. 3) Not all agents are present in all periods.

Note: In Bergemann and Välimäki's model, each agent's private type is controllable, and the type transition function of each agent is common knowledge. Hence, it looks similar with our model. The main distinctions between this model and our model are clear: 1) The mechanism defined in our model is a one-stage game, not a multistage game as specified in the literature of dynamic mechanisms [11] [12]. 2) The type adjustment function in our model is a deterministic function, whereas the type transition function of each agent i in dynamic mechanism is a stochastic function following a Markov process.

4.4 Bayesian persuasion

In 2011, Kamenica and Gentzkow [10] proposed a model of Bayesian persuasion. The model consists of a player called Sender and a player called Receiver. Each of them has a utility function depending on the Receiver's action $a \in A$ and the state of the world $\omega \in \Omega$. The Sender and Receiver share a common prior μ_0 on Ω. Let S be a sufficiently large set of signal realizations. A signal $\pi : \Omega \to \Delta(S)$ is a map from the state to the distribution over signal realizations. The working steps are as follows: 1) Sender chooses a signal π. 2) Receiver observes which signal was chosen. 3) Nature chooses ω according to μ_0. 4) Nature chooses a realized signal realization s according to $\pi(\omega)$. 5) Receiver observes s. 6) Receiver takes action a.

Given the knowledge of π, Receiver uses Bayes' rule to update his belief from the prior to the posterior, and then chooses an action that maximizes his expected utility. Given this behavior by Receiver, Sender solves an optimization problem to maximize her expected utility.

Note: The main distinctions between this model and our model are as follows: in our model, there is no state of the world, and the signal chosen by the designer is a solution of the designer's optimization problem, but not a map from a state to the distribution over signal realizations.

4.5 Information design

According to Kamenica's descriptions [13], information design is similar to Bayesian persuasion. However, the former is used more when the designer is a social planner and there are multiple interacting receivers, and the latter is used more when the designer is one of the players in the game and there is a single receiver.

Following the model proposed by Bergemann and Morris [14], there are I players. Let Θ be the set of payoff states of the world, and θ be a typical element of Θ. An information structure S consists of (1) for each player i, a finite set of types $t_i \in T_i$; and (2) a type distribution $\pi : \Theta \to \Delta(T)$, where $T = T_1 \times \cdots \times T_I$. Thus, $S = ((T_i)_{i=1}^I, \pi)$.

In general, the information designer could follow any rule for generating messages. A "revelation principle" implies that the information designer sends only action recommendations that are obeyed. Given this restriction, the information designer chooses among decision rules $\sigma : T \times \Theta \to \Delta(A)$. The decision rule encodes the information that the players receive about the realized state of the world, the types and actions of the other players. The conditional dependence of the recommended action a on state of the world θ and type profile t represents the information conveyed to the players.

Note: In the model of information design, the information sent to the players is modeled as a rule which obeys the probability distribution $\sigma : T \times \Theta \to \Delta(A)$. As a comparison, in our model, there is no such decision rule and the information is a solution of the designer's optimization problem. These are the main distinctions between two models.

4.6 Persuasion with communication costs

Hedlund [15] studied strategic transmission of verifiable information with reporting costs that continuously increase in the precision of the report. In his model, there are two players, a Sender and a Receiver. The game has two stages: in the first stage, nature reveals the value of a parameter $t \in T = [0, 1]$ to the Sender, which is referred to as the Sender's type, and then the Sender chooses a costly report which he delivers to the Receiver. The report takes the form of a closed interval contained in type space T. Precision is the amount of relevant information in a report, and the cost of producing a report depends only on its precision. In the second stage, the Receiver observes the report, forms a posterior belief with respect to the Sender's type, chooses an action and then the game ends.

Note: In our model, the designer (*i.e.*, the Sender) has no type, and the information sent by the designer does not represent any precision of report. Hence, Hedlund's model is very different from our model.

4.7 Signaling games

Since the seller performs a costly action as an open signal to bidders, some one may consider our model to be similar with the signaling game model. However, the two models are different:

1) In the signaling games (Ref [16], Section 8.2.1, Page 324), there are one leader and one follower. The leader has private information about his type, and the follower has no private information. Before the game begins, it is common knowledge that the follower has prior beliefs about the leader's private type. The leader moves first, and the follower observes the leader's action (*i.e.*, signal), then updates his beliefs about the leader's type and chooses his own action. The equilibrium of the signaling game is the perfect Bayesian equilibrium (PBE), which is simply a set of strategies and beliefs such that, at

any stage of the game, strategies are optimal given the beliefs, and beliefs are obtained from equilibrium strategies and observed actions using Bayes' rule.

2) In our model, the leader (*i.e.*, the designer of a mechanism, or the seller of an auction) has no private information, and the followers (*i.e.*, the agents of an mechanism, or the bidders of an auction) have private information (*i.e.* private types, or private valuations to the sold object). The leader moves first (*i.e.*, sends a costly signal), then the followers observe the leader's action and choose their own actions (*i.e.*, perform their strategies, or submits their bids). At last, the mechanism yields the outcome according to the outcome function. The equilibrium of our model is the type-adjustable Bayesian equilibrium defined by formula (1).

4.8 Inducing agents to invest efforts strategically

Kleinberg and Raghavan [18] investigated how the designer induces agents to invest efforts strategically. There are two kinds of players: an evaluator creates a decision rule for assessing an agent in terms of a set of features, and this leads the agent to make choices about how to invest private effort across their actions to improve these observable features. Kleinberg and Raghavan developed a model (hereafter KR model) for this process of incentivizing private efforts, when actions can only be measured through intermediate features. The evaluator's design task is to create an evaluation rule that takes the feature values as input, and produces a numerical score as output. The agent's goal is to achieve a high score, and to do this, they will optimize how they allocate their effort across actions.

Note: The distinctions between KR model and our model are as follows:
1) The KR model considers the interaction between the designer (*i.e.*, the evaluator) and agents, but does not consider the interactions among agents. As a comparison, our model considers the strategic interactions among agents.

2) In the KR model, what the agent observes from the designer (*i.e.*, the evaluator) is a decision rule. The agents strategically choose private efforts, generate their observable features, and the evaluator rewards them in some way based on those features. What the evaluator wants to do is to incentivize agents to choose certain effort profile. As a comparison, in our model, the agents observes a costly signal from the designer. Given agents' initial profile of probability distributions of private types, what the designer wants to do is to choose an optimal signal in order to induce agents to adjust distributions of private types to optimal profile.

4.9 Auctions with information release

Szech [19] investigated optimal disclosure of costly information packages in auctions. Consider a seller to sell one indivisible object to n risk-neutral bidders with independent valuations via a second-price auction. The precise timing of the model is as follows: 1) The seller announces individual entry fees to each bidder, and commits to giving out an information structure (describing which bidder will get how much information), excluding all bidders that refuse to pay. 2) The bidders decide if they want to pay their fee. 3) The bidders who have paid get their information. 4) All bidders who have paid participate in the second-price auction.

Note: The distinctions between Szech's model [19] and our model are that:
1) In our model, the public information is a costly message sent by the seller, and is free from each bidder's perspective. As a comparison, in Szech's model each bidder need to pay fees for the information, which is represented as some package which can be allocated among the bidders.
2) In our model, the probability density function of each bidder's private valuation is controlled by the seller through choosing a costly signal. As a comparison, there is no such control in Szech's model.

4.10 Information structures in optimal auctions

Bergemann and Pesendorfer [20] proposed a model of information structures in optimal auctions, where the seller may control the bidders' information structures which generate the bidders' private information. Each bidder's information structure $\mathcal{S}_i$ is represented as a pair containing a space of signal realization and a joint probability distribution over the space of valuations and signals. The seller may assign an information structure that informs a bidder perfectly, or an information structure that gives the bidder only a rough guess about her true value for the object. The seller's choice of information structure is made prior to the auction and does not involve transfer payments from the bidders. At the interim stage, every bidder observes privately a signal s_i rather than his true valuation v_i of the object. Given the signal s_i and the information structure $\mathcal{S}_i$, each bidder forms an estimate about his true valuation of the object, then report their value estimate to a revelation mechanism which determines the probability of winning the object and a transfer payment for every bidder.

Note: In Bergemann and Pesendorfer's model, the signal is represented as a random variable, which is different from our model. Another distinction is that in our model, each bidder has an initial private valuation of the object before the designer sends a costly signal to each bidder.

4.11 Optimal auctions with information disclosure

Gershkov [21] analyzed the properties of an optimal selling mechanism when the bidders' information about the quality of the object to be auctioned is under control of the seller. In Gershkov's model, bidders' valuations for the object consist of two components: a private and a common value component. While each bidder observes his private value component individually before the auction, the information that all bidders observe about the common value component depends on the disclosure policy of the seller. The seller may adopt a disclosure policy that reveals all information to all bidders, or, alternatively, to conceal it from the bidders. After the choice of disclosure policy, all bidders observe their information about the common value component as specified by the chosen policy. Finally, at the last stage of the game, an auction takes place. The main result of Gershkov's model is that: *in the optimal mechanism the seller reveals all information to all bidders and therefore implements a second price or English auction with a reservation price at the final stage.*

Note: In Gershkov's model, the information about common value component is assumed to have been existed before the seller chooses disclosure policy, and is represented as a random variable. What the seller can control is to choose whether to disclose the information or to conceal it. Therefore, Gershkov's model is different from our model.

4.12 Optimal advertising of auctions

Szech [22] analyzed a symmetric independent private values auction model: a revenue-maximizing seller faces a cost c_n of attracting n bidders. These costs can be thought of as advertising costs, or as costs of making bidders familiar with the object to be auctioned. Szech's question is that: How many bidders does the seller choose to attract compared to the socially optimal number? The main result is the following: If the distribution of valuations has an increasing failure rate (IFR), then the seller overadvertises the auction. Conversely, with a decreasing failure rate (DFR), the seller underadvertises.

Fang and Li [23] examine a model where a seller holds an object for sale through a second-price auction to n risk-neutral potential bidders, all of whom are initially unaware of the auction and must be solicited. The seller is allowed to set a reserve price and employs an advertising policy consisting of the potential bidders' entering probabilities. After receiving the advertisement, potential bidders know their valuations of the object for sale and participate in the auction according to independent Bernoulli trials.

Note: The two papers both consider the advertisement as a tool to solicit potential bidders. As a comparison, in our model there is no such advertisement for soliciting bidders, and the signal sent by the seller is used to induce each bidder to adjust his private type. Hence, the model settings of advertising are different from our model.

5 Conclusions

In this paper we propose a type-adjustable mechanism in which the public information sent by the designer is not represented as a random variable, but is modeled as a solution of an optimization problem. Therefore, from the designer's perspective, the probability distribution of each agent's type may be *optimally* controlled as she wishes. In this sense, each agent's private type is not his own endogenous property, but is decided by each agent himself and the designer together. The main results of this paper are as follows:
1) As proved in Proposition 1, for a type-adjustable Bayesian implementable social choice function, the seller may obtain more expected payoffs than her initial expected payoffs.
2) As shown in Section 3, the seller may breakthrough the limit of expected payoffs which she could obtain at most in the traditional optimal auction model:
• If $\beta > \sqrt{3}$, then by choosing the optimal adjustment cost $c^* = \beta^2/36$, the seller can obtain more expected payoffs than the maximum expected payoffs 5/12 yielded by the traditional optimal auction.
• If $\beta > \sqrt{27/2}$, then each bidder's *ex ante* expected payoffs $1/6 + \beta^2/36$ will be greater than the corresponding value 13/24 in the traditional optimal auction. Put differently, every member in the type-adjustable auction may benefit from the seller's optimal signal, and hence this is Pareto-efficient.

Appendix

As specified in Section 3, θ_1^0 and θ_2^0 are drawn independently from the uniform distribution on $[0, 1]$. Let Z be a random variable $Z = \theta_1^0 y_1(\theta^0) + \theta_2^0 y_2(\theta^0)$.

$$f_{\theta_1^0}(z) = \begin{cases} 0, & z < 0 \\ 1, & z \in [0,1] \\ 0, & z > 1 \end{cases}.$$

$$F_{\theta_1^0}(z) = Prob\{\theta_1^0 \leq z\} = \begin{cases} 0, & z < 0 \\ z, & z \in [0,1] \\ 1, & z > 1 \end{cases}.$$

$$F_Z(z) = [F_{\theta_1^0}(z)]^2 = \begin{cases} 0, & z < 0 \\ z^2, & z \in [0,1] \\ 1, & z > 1 \end{cases}.$$

Therefore,

$$f_Z(z) = \begin{cases} 0, & z < 0 \\ 2z, & z \in [0,1] \\ 0, & z > 1 \end{cases}.$$

As a result,

$$E(Z) = \int_0^1 z \cdot 2zdz = \int_0^1 2z^2 dz = 2/3.$$

Therefore, $E[\theta_1^0 y_1(\theta^0) + \theta_2^0 y_2(\theta^0)]/2 = 1/3$.

By formula (4), the seller's initial expected payoffs is the sum of two bidders' payment to the seller when the cost is zero, $\bar{u}_d(0) = E[\theta_1^0 y_1(\theta^0) + \theta_2^0 y_2(\theta^0)]/2 = 1/3$.

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