Private Overborrowing under Sovereign Risk

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Abstract

This paper proposes a quantitative theory of the interaction between private and public debt in an open economy. Excessive private debt increases the frequency of financial crises. During such crises the government provides fiscal bailouts financed with risky public debt. This response may cause a sovereign debt crisis, which is characterized by a higher probability of a sovereign default. The model is quantitatively consistent with the evolution of private debt, public debt, and sovereign spreads in Spain from 1999 to 2015, and provides an estimate of the degree of overborrowing, its effect on the spreads, and the optimal macroprudential policy.

Keywords: Bailouts, credit frictions, financial crises, macroprudential policy, sovereign default

JEL Classifications: E32, E44, F41, G01, G28

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1 Introduction

A feature of the 2010–2015 European Debt Crisis is that governments that had previously pursued fiscally frugal policies saw significant increases in their borrowing costs. One of those countries was Spain. From the introduction of the euro in 1999 up to the global financial crisis in 2008, Spain was the largest economy in the Eurozone in uninterrupted compliance with the budgetary and public debt limits set by the Stability and Growth Pact.[1] During this same period, however, Spain accumulated a large stock of international private debt, primarily in its banking sector.[2] As the financial turmoil accelerated, the government responded with multiple rounds of bailouts to highly indebted financial institutions. These interventions led to an abrupt increase in Spain's public debt and its interest rate spreads. These events have raised questions about how private crises can lead to public debt crises and how a sovereign with defaultable debt should respond to systemic vulnerabilities in international private credit.[3] This paper is among the first few to provide a joint analysis of the interplay of private debt and sovereign risk is necessary in order to provide adequate policy prescriptions.

This paper provides quantitative answers to the following three questions. First, was the Spanish private sector excessively indebted in the lead-up to the crisis and, if so, by how much? Second, what was the effect of excessive private debt on the severity of the sovereign debt crisis that followed? Third, how do the optimal macroprudential policy prescriptions change when one takes sovereign risk into account?

To answer these questions, I build a small open economy model with both financial crises caused by collateral debt constraints on private debt and sovereign default crises caused by long-term defaultable public debt. First, the model is quantitatively consistent with the Spanish data, and yields a measure of excessive private debt stock, 5% of gross domestic product (GDP) on average. Second, the model also matches the dynamics of private debt, public debt, and sovereign spread during the 2008-2012 crisis, and allows me to construct counterfactual dynamics under optimal macroprudential policies. Third, I show that the optimal macroprudential tax increases by 0.7 percentage points (p.p.) on average because of the interaction between private debt and defaultable public debt.

Private debt is modeled as in Mendoza (2002) and Bianchi (2011), and the sovereign debt structure follows the tradition of Eaton and Gersovitz (1981) with long-term bonds as in Hatchondo and

[1] Morris et al. (2006) discuss the reform of the pact in 2005 and distinguish Spain for its compliance. Schuknecht et al. (2011) describe the evolution of deficits and sovereign debt in the post-reform period and document Spanish compliance up to the 2008 recession.

[2] Lane (2013) and Chen et al. (2013) discuss the current account imbalances of periphery European countries. Hale and Obstfeld (2016) and Hobza and Zeugner (2014) analyze capital flows within the Eurozone and document the flow in the form of debt instruments from "core" countries toward financial institutions in the periphery. In't Veld et al. (2014) and Ratto and Roegera (2015) link the increase in capital flows to Spanish banks financing a boom in the construction sector.

[3] This is not the first time that private credit booms have been linked to subsequent sovereign debt crises. An earlier literature analyzing the 1997 currency crises in Thailand, Korea, and Indonesia stresses this link. Burnside et al. (2001) argue that implicit bailout guarantees lead to private credit booms and raise expectations of large fiscal deficits in the future. Schneider and Tornell (2004) show that systemic bailout guarantees cause both credit cycles and self-fulfilling crises.

Martinez (2009).[4] I solve two versions of the model. In the baseline version, a continuum of identical households makes the private borrowing decisions and a benevolent government makes the taxes, default, and public borrowing decisions. In the normative version, a benevolent social planner (SP) makes aggregate borrowing decisions about both private and public assets and then transfers the proceeds to the households that make all consumption choices. Thus, the planner and the competitive households are subject to the same market clearing conditions, as well as credit constraints. Nevertheless, the planner's choice of allocations may be different from that of the competitive equilibrium because the planner internalizes the general equilibrium effects of the aggregate choices that are made. I show that the planner's allocations can be decentralized by extending the baseline framework to allow the government to impose state-dependent taxes on private borrowing. I also find that the socially planned version features a lower level of private debt, a lower level of public debt, and a lower interest rate spread. While the first result is known in the sudden-stops literature, the other two are new results from this paper. These differences allow the planner to achieve a higher level of welfare.

In the quantitative section I first calibrate the baseline version of the model to the Spanish data from 1999 to 2011, before the peak of the crisis. The calibrated model matches the Spanish environment before the crisis – namely, low public debt, high private debt, and near-zero interest rate spreads. I then use the calibrated parameters to solve the socially planned version of the model. Comparing the socially planned economy and the baseline model at their respective ergodic distributions provides a measure of excessive private debt stock, 5% of GDP on average.

I then use the 2008-2015 Spanish data to simulate the crisis in the model. I feed into the model the exogenous shocks from the data. To infer shocks unobserved in the data, I use the particle filter approach proposed in Bocola and Dovis (2019). As in the data, the government in the baseline model finds it optimal to provide large transfers to the private sector, which are financed with external public debt. This response in turn leads to a sudden decrease in private debt and a rise in the public interest rate spread commensurate with the increase observed in Spain. Facing the same shocks, the social planner completely avoids an increase in the interest rate on public debt through a combination of low private and public debt.

Lastly, I compute the optimal macroprudential policies that implement the allocations chosen by the social planner. I compare these taxes to those that implement efficiency in an economy without public debt and sovereign risk. I find that in the presence of defaultable sovereign debt macroprudential policies are tighter. Specifically, relative to Bianchi (2011) economy, optimal macroprudential policies in my model are 0.7 p.p. higher on average.

The key mechanism of the paper is understating why the baseline and socially planned allocations differ. The allocations differ because of two general equilibrium effects that the social planner incor-

[4] See also Kehoe and Levine (1993) for earlier implementations of collateral debt constraints in a general equilibrium context and Arellano (2008) and Aguiar and Gopinath (2006) for early quantitative adaptations of the Eaton and Gersovitz (1981), model.

porates in their decision-making and that the competitive households in the baseline version do not. The first one is common in the literature, and the second one is novel to this paper.

The first mechanism is what Mendoza (2002) and Bianchi (2011) named Irving Fisher's classic debt-deflation effect and is present in most models with a collateral constraint that depends on market-determined current prices. The planner, but not the households, internalizes that higher current private borrowing lowers the future price of nontradables. As a result, the baseline version exhibits a higher level of private debt, more frequent periods when the credit constraint binds, and sharper contractions in consumption during these periods.

The second and novel mechanism of the paper is the interaction between Fisherian deflation and the government's borrowing and default decisions. Each period, the government evaluates the benefits of providing households positive transfers (bailouts) financed with external public debt against the expected costs of either higher taxes or a sovereign default in the future. When the households are unconstrained, these transfers are offset by reductions in private borrowing due to standard Ricardian equivalence effects. In contrast, when the credit constraint binds, the marginal benefit of current consumption exceeds the marginal cost of lower future consumption. In these instances, a positive fiscal transfer leads to higher individual consumption. At the aggregate level, the increase in consumption raises the relative price of nontradables and with it the value of collateral. The increasing valuation of collateral allows for a higher level of private debt, which in turn translates into an additional increase in consumption. This effect makes bailouts desirable when the collateral constraint binds. Since the constraint binds more frequently in the baseline version than in socially planned version, bailouts are more frequent and public debt is higher. Consequently, in the baseline version sovereign risk will be higher and the government will face a worse schedule of prices for its debt.

Macroprudential policies, equated in this paper to taxes on private borrowing, allow the government to decentralize the socially efficient level of private borrowing. The benefits of restoring the socially efficient level of private debt in this context are twofold. First, by decreasing the level of private borrowing, the planner decreases the severity and frequency of private financial crises. Second, fewer crises reduce the need for government bailouts. Fewer bailouts then translate into lower public debt and a lower probability of a sovereign default. The combination of these two factors implies lower interest rate spreads on public debt.

Related Literature: Following the theoretical framework of sovereign defaultable debt introduced in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008) developed quantitative models of sovereign debt and business cycles. A growing literature has emerged extending their framework. Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016) highlight the importance of long-term debt in generating dynamics of the interest rate spread that are consistent with the data. The model presented here incorporates these findings by assuming a long-term structure for public debt while keeping, for simplicity's sake, the short-term maturity in private debt.[5] The paper is closely related to the branch of the sovereign debt literature that focuses on the link between sovereign debt and the private economy. In contrast to Mendoza and Yue (2009) and Arellano et al. (2017), the analysis presented here assumes that private agents have access to international credit markets even during sovereign default episodes. The paper shares this feature with Kaas et al. (2020). The main difference with this recent work is that private debt in my model is inefficiently high from a social perspective, and this inefficiency increases the incidence and magnitude of crises. As a result, the frequency of public bailouts, in response to reductions in the borrowing capacity in the private sector, is an endogenous outcome of the model.

The paper is also related to the literature that studies the trade-offs between centralized international public debt and decentralized international private debt. With complete markets, Jeske (2006) and Wright (2006) find that a centralized environment, where only the government can issue international debt and default on it, is preferable to a decentralized environment where individual households make the borrowing and default choices. With incomplete markets, Kim and Zhang (2012) find underborrowing in an environment where decentralized households make the borrowing choices and a centralized government makes the default choice for all agents. My paper assumes incomplete markets and two distinct assets: private and public bonds. Only public debt enjoys sovereign immunity, and the government cannot force private agents to default. In my environment, the decentralization of the private bonds leads to overborrowing in both assets, in contrast to a centralized environment where a planner chooses the optimal portfolio.

Furthermore, the paper contributes to the literature on credit frictions, financial crises, and macroprudential policies. In particular, it belongs to the branch on systemic credit risk (see Lorenzoni (2008), Bianchi (2011), and Dávila and Korinek (2018)) and its management with taxes on private borrowing (see Bianchi and Mendoza (2018), Farhi and Werning (2016), and Jeanne and Korinek (2019)).[6] This paper also shows that government bailouts financed with external defaultable debt are not a substitute for optimal macroprudential policies. The role of bailouts in the model is similar to the one found in Bianchi (2016), Keister (2016), and Chari and Kehoe (2016). In contrast to those papers, I distinctly assume here that the bailouts are paid for with long-term strategically defaultable debt. This feature allows the model to create a path from financial crises to sovereign debt crises– a relationship observed in the data.[7]

By analyzing how private credit affects the sovereign spread, I also contribute to a growing literature on the feedback loop between sovereigns and the domestic financial sector referred to as "doom

[5]The presence of multiple maturities links the paper to literature studying the role of the optimal debt maturity structure, such as Arellano and Ramanarayanan (2012) and Sanchez et al. (2018). This paper differentiates itself from this literature by assuming that the government will not be able to fully control the issuances of short-term private debt.

[6]Bianchi (2011) and Arce et al. (2021) propose alternative implementations of optimal allocations.

[7]The literature on bailouts also deals extensively with the issue of moral hazard that the expectation of government bailouts induces. This concern is not addressed in this paper because households take as given that government policies are functions of aggregate states and not their individual actions.

loops." Theoretical models of this issue are presented in Korinek (2012), Brunnermeier et al. (2016), and Farhi and Tirole (2018).[8] Another strand of the literature, exemplified by Perez (2015), Bocola (2016), and Sosa-Padilla (2018), has focused instead on developing quantitative models that capture only a part of this loop, the transmission of sovereign risk to private risk, usually through the balance sheet of domestic banks. This paper complements the existing quantitative literature by focusing on the other part the loop, where a financial crisis in the private sector precipitates a sovereign debt crisis. In the model, excessive private credit will endogenously generate financial crises and increase the incentives for government interventions that increase default risk and spreads.[9]

Finally, methodologically the paper applies recent techniques in dynamic discrete choice methods to solve a sovereign debt model drawing from the contributions of Dvorkin et al. (Forthcoming).[10] Additionally, to simulate the Spanish debt crisis, I use the nonlinear particle filter method proposed by Kitagawa (1996). This technique uses likelihood functions to construct a numerical approximation of an unobserved stochastic shock and was first applied to quantitative business cycle models in Bocola (2016) and Bocola and Dovis (2019).

Layout: The paper is organized as follows. Section 2 outlines the motivating empirical facts in the Spanish data. Section 3 presents the model and the main theoretical results. Section 4 details the calibration. Section 5 provides the quantitative results of the paper. Finally, Section 6 summarizes the conclusions and is followed by an extensive appendix.

2 Motivation: The path of debt and spreads in Spain, 1999-2015

This section documents the evolution of international private and public debt in Spain from the creation of the Eurozone in 1999 to the end of the Spanish sovereign debt crisis in 2015. The pattern consists of a period of large accumulation of private debt, with low levels of public debt and low spreads, followed by financial and sovereign debt crises. Figure 1 shows this pattern for Spain; however, as noted by Reinhart and Rogoff (2011), Lane (2013), and Gennaioli et al. (2018), similar patterns have been seen in other countries and periods.

Figure 1 plots the evolution of the Spanish debt crisis. The left axis plots the evolution of the international investment position as a percentage of GDP on an inverted scale; that is, positive numbers represent net liabilities.[11] All types of assets are accounted for in this aggregate. Nevertheless, throughout the paper I refer to this measure of net international liabilities as debt. The right axis plots

[8] For empirical evidence on this issue, see Acharya et al. (2014).

[9] A recent paper featuring a closed economy quantitative model of doom loops is Hur et al. (2021). In contrast, this paper proposes a model where both private and public agents have access to international credit markets.

[10] Other models using this technique include Mihalache (2020). A review of the method and an alternative can be found in Gordon (2019).

[11] Annualized data are from the Bank of Spain; more details can be found in Appendix C.

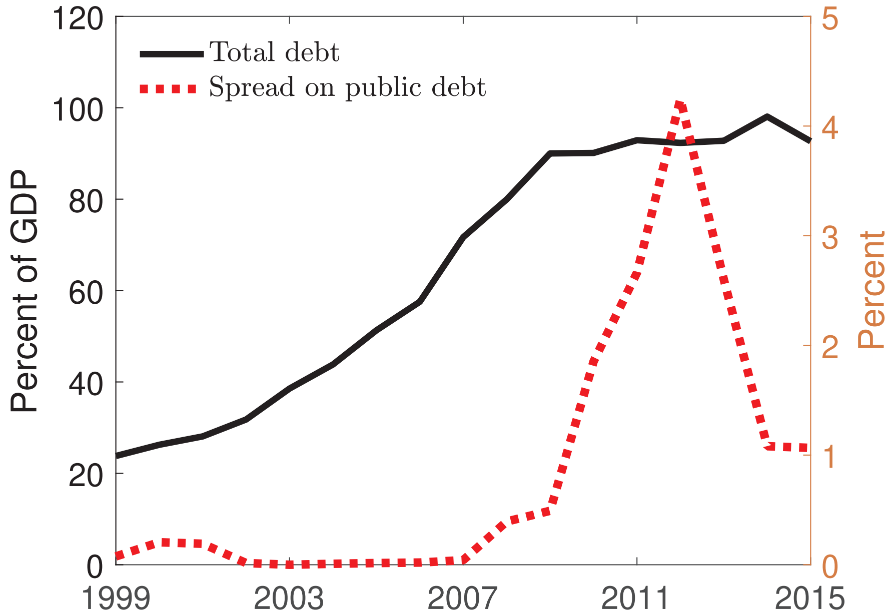

Figure 1: Total international debt and sovereign spread

Note: Total debt corresponds to the inverse of the international investment positions. The spreads correspond to the average difference between the interest rate on a Spanish six-year treasury bill and the interest rate on the German equivalent. The data source for debt is the Bank of Spain, and the interest rate data are from Bloomberg. More details can be found in Appendix C.

the sovereign spread (dotted line), calculated as the difference between a six-year treasury bond issued by Spain and its German counterpart.[12] The figure shows a first period of accumulation of external debt between 1999 and 2008, followed by a period where total debt remained constant at around 92% of GDP. Interest rate spreads remain close to zero up to 2009 and then spiked up in 2012. Some observers, such as Banco de España (2017), find it hard to reconcile rational financial markets with a period of rapidly increasing debt but low spreads (1999–2008) and a period of significant movement in the spread but steady total debt (2009–2015). In this paper, I will argue that these two things are not incompatible.

Next, I summarize in Figure 2 the evolution of the private international liabilities during this time period. The left axis corresponds to the debt position of the private sector as a percentage of GDP (solid line), and the right axis corresponds to nonperforming loans as a percentage of gross loans (dashed line).[13] The evolution of private debt also reveals two distinct periods. Net liabilities in the private sector grew from 20% of GDP in 1999 to 70% of GDP in 2009. Contemporary observers of this trend, such as the International Monetary Fund (2007), classified the growth in private credit

[12] This maturity is chosen because it corresponds to the average maturity of public debt in Spain during this period. For more details, see Section 4 and Appendix C.

[13] To compute the position of the private sector, I subtract from total debt the assets held by the public administration and the Bank of Spain. See details in Appendix C.

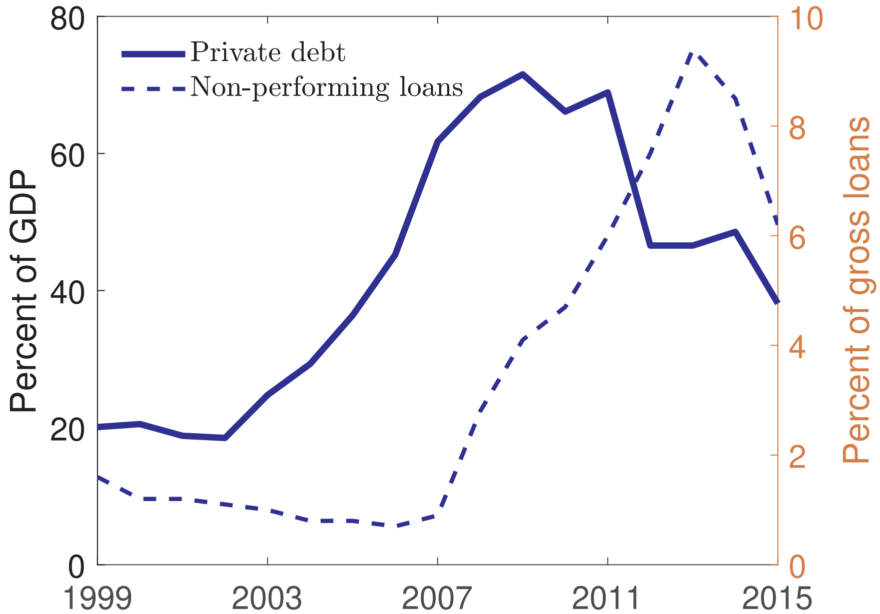

Figure 2: Private debt and nonperforming loans
Note: Private debt corresponds to the inverse of the international investment positions of the financial and nonfinancial private sector. Nonperforming loans are computed as a share of total gross loans. The data source for debt is the Bank of Spain, and the loans data are from Bloomberg. More details can be found in Appendix C.

as the main risk to Spanish growth, but predicted that the imbalances would gradually disappear.[14] After declining slightly for two years, private debt dropped by 22% of GDP in 2012. As noted by International Monetary Fund (2012), International Monetary Fund (2014), and Martin et al. (2019), among others, the buildup of external private debt was primarily driven by a banking sector that was financing a construction boom. When housing prices fell and mortgages started going unpaid, private debt became increasingly more difficult to roll over abroad. For this reason, I use the percentage of nonperforming loans as a proxy measure of aggregate default risk in the private sector. Figure 2 shows that the rapid increase in private debt stopped roughly at the same time as the share of nonperforming loans started increasing. Moreover, the abrupt drop in 2012 coincided with a high mark of the share of private default. On average, 7.5% of gross loans were nonperforming between 2011 and 2015.

Finally Figure 3 complements the analysis by showing the joint evolution of public and private debt. Combined, these two series add up to the total debt presented in Figure 1. The symmetry between these two aggregates highlights the importance of the decomposition presented in this section. From 1999 to 2007, public external debt in Spain was below 20% of GDP. In contrast, from 2008 to 2015, public external debt increased from 11% to 55% of GDP. More importantly, the largest yearly increase was also in 2012, when public liabilities increased by 22% of GDP, exactly mirroring the drop in private

[14]The empirical literature finds that strong link between international private credit growth and financial crises. See for instance, Schularick and Taylor (2012) and Davis et al. (2016).

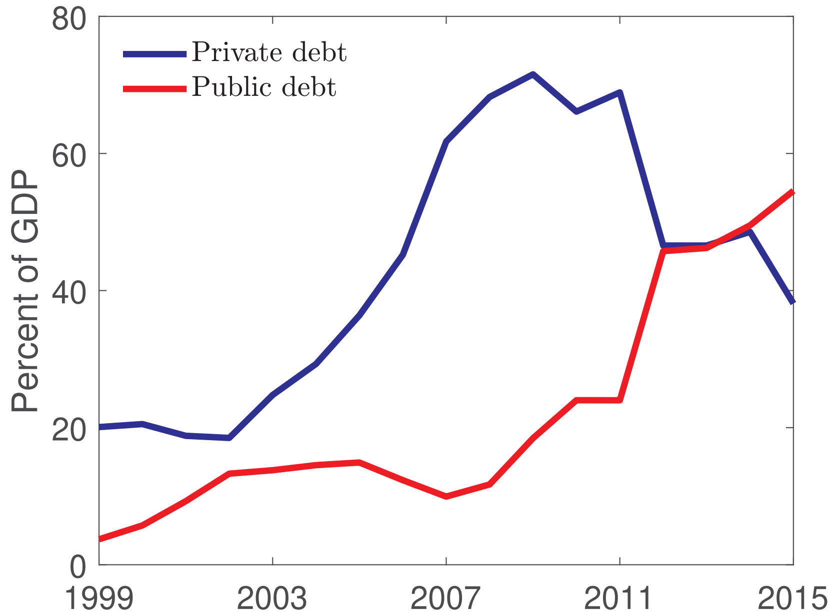

Figure 3: Private and public debt
Note: Private debt corresponds to the inverse of the international investment positions of the financial, and nonfinancial private sector. Public debt corresponds to the inverse of the international investment position of the Bank of Spain and other public administrations. The data source is the Bank of Spain. More details can be found in Appendix C.

debt. As noted in Banco de España (2017), this symmetry is not a coincidence. Between 2008 and 2012, the Spanish government funneled financial assistance to its lending institution primarily in the forms of bailouts and transfers of toxic assets. Total direct aid to the Spanish banking sector amounted to 70 € billion or around 7% of GDP, with most of these funds being transferred by the newly created Fund for the Orderly Restructuring of the Banking Sector (FROB).[15]

To summarize, in the pre-crisis years of 1999–2007, large buildups of private debt coexist with low public debt and public spreads close to zero. This period was followed by a private financial crisis, corresponding in the data to the years 2008 to 2011. The financial crisis is characterized by an increase in nonperforming loans in the private sector and a moderate private deleveraging. Throughout this period, public debt and spreads increased but stayed relatively low. The final period, from 2012 to 2015, corresponds to the sovereign debt crisis. These years are characterized by large public bailouts that reduce net liabilities in the private sector but are financed with issuances of public debt. The symmetric evolution of debt positions coincides with significant increases in the interest rate spread on public debt. The next section will propose a theory that generates dynamics consistent with the facts presented in this section.

[15]Beyond direct transfers, private debt declined following liquidation of private assets while public debt increased to finance unemployment benefits and economic stimulus programs to mitigate the financial crisis. A full overview of the restructuring of the Spanish financial sector is beyond the scope of this paper. More details can be found in International Monetary Fund (2010) and Banco de España (2017).

3 A model of financial and sovereign debt crises

This section presents a dynamic small open-economy model with one-period international private bonds subject to an occasionally binding borrowing constraint, as in Bianchi (2011), and long-term, strategically defaultable international public bonds, as in Hatchondo and Martinez (2009). The first subsection presents the economy's environment and technologies. The second subsection defines and characterizes the baseline unregulated, competitive equilibrium where the government only has access to public debt and lump-sum transfers. The third shows the optimal policy problem of a social planner who makes all borrowing decisions for both assets. The fourth subsection demonstrates that the SP's allocations are equivalent to those of a competitive equilibrium where the government gains access to state-contingent taxes on private debt. The last subsection explains the main mechanism of the model.

3.1 Environment

Time is discrete and indexed by $t \in \{0, 1, ..., \infty\}$. The economy is composed of a continuum of identical households of unit measure, a benevolent domestic government, and a continuum of risk-neutral, competitive foreign creditors who lend to both domestic agents via two different assets. The focus is on real values as opposed to nominal ones because most Spanish debt was denominated in euros, whose supply is controlled by the European Central Bank.[16]

3.1.1 Households

Preferences: The representative household has an infinite life horizon and preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + D_t], \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is a discount factor; and $u(\cdot)$ is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada condition. The term D_t is an additive preference shifter that depends on government decisions and that the households take as given. The consumption basket c is an Armington-type constant elasticity of substitution (CES) aggregator with an elasticity of substitution $1/(\eta+1)$ between tradable goods c^T and nontradable goods c^N, given by

$$c = \left[\omega \left(c^T\right)^{-\eta} + (1-\omega)\left(c^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0,1).$$

[16]The interaction of sovereign default and the inability to inflate away the debt in the context of the European Debt Crisis is studied in Aguiar et al. (2014) and Aguiar et al. (2015). For the specific case of Spain, Bianchi and Mondragon (2018) explore this issue in an environment with nominal rigidities.

Endowments: Each period the economy receives a stochastic endowment of tradable goods $y^T \in \mathbb{R}^+$ and nontradable goods $y^N \in \mathbb{R}^+$. Both endowments are drawn from first-order Markov processes independently of each other and of all other stochastic shocks in the model. The numeraire is the tradable good.

Private Debt: Households can borrow using a one-period non-state-contingent debt denominated in units of tradables. Following the standard convention, b denotes the individual level of private debt and B denotes the aggregate level. Each period a stochastic fraction π_t of these bonds are defaulted on. Including these private default shocks allows the model to capture the dynamics of nonperforming loans in Spain, but has otherwise no major implications in the model. Like the endowment shocks, the fraction of defaulted private bonds is drawn from a first-order Markov process independently from all the other stochastic shocks in the model. Private debt is issued in international competitive credit markets at price q_t.

In addition, private bonds issuances are subject to a collateral credit constraint, as follows:

$$q_t b_{t+1} \leq \kappa_t \left(y_t^T + p_t^N y^N \right), \tag{2}$$

where p_t^N is the equilibrium price of nontradable goods in units of tradables. The market value of private debt issuances $q_t b_{t+1}$ is capped at a fraction $\kappa_t \geq 0$ of the market value of current income.

This credit constraint captures in a parsimonious way the empirical fact that income is critical in determining credit market access.[17] Theoretically, the constraint can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ_t of the value of the endowment owned by a defaulting household.[18] Nontradable goods enter the collateral constraint because even though foreign creditors do not value them, I assume they can be seized in the event of default and sold in exchange for tradable goods in the domestic market.[19] Collateral constraints are commonly used in mortgage lending. Consequently, this assumption is particularly suitable in the Spanish context where mortgage loans played an important role in the buildup of private credit. Note that while private debt is explicitly modeled here as issued internationally by the households, the same constraint arises under a broader set of assumptions. In particular, I could assume instead that credit is provided to households by a competitive domestic financial system with unrestricted access to global capital markets but subject to the same enforcement friction. As noted in Section 2, this interpretation is more in line with the events that unfolded in Spain. Commercial and savings banks borrowed internationally and then channeled these

[17] See Jappelli (1990).

[18] In this context, the punishment is only triggered by private default above the exogenous fraction drawn in each period.

[19] The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks. See Bianchi and Mendoza (2018) for a derivation of a similar constraint.

funds to households and construction firms. The assumption of short-term maturity is consistent with the empirical literature documenting a reduction in the maturity of private bonds issued in advanced economies during this period.[20]

The fraction of market income required as collateral κ_t is stochastic and drawn from a first-order Markov process. Throughout the paper, I refer to this shock as the financial shock. Stochastic changes in collateral requirements can be viewed as shocks to the creditors' risk assessment of the borrowers. Financial shocks of this form have been shown to be capable of accounting for the dynamics of private financial crises in advanced economies (see Jermann and Quadrini (2012) and Boz and Mendoza (2014)), as well as balance of payment crises in emerging economies (see Mendoza (2002)). From a modeling perspective, these shocks generate fluctuations in private borrowing that are not caused by fluctuations in other domestic fundamentals. This is consistent with recent empirical work by Forbes and Warnock (2020). They document that shocks in international volatility, monetary policy, or sudden-stop crises in similar and/or neighboring countries can cause fluctuations in the lenders' perceptions about the private sector's solvency. In the context of interest, these shocks allow the model to account for a change in investors' behavior toward Eurozone banks in the wake of the Greek sovereign debt crisis.

Finally, note that neither the existence of the financial amplification mechanism nor the government's best responses presented later rely on κ_t or π_t being stochastic.[21] Nevertheless, these shocks will generate fluctuations in private borrowing independently from income fluctuations and as such will have a different impact on government policies.

Households' budget constraint: Each period, individual households face a budget constraint of the form

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t b_{t+1}, + y_t^T + p_t^N y^N + T_t, \tag{3}$$

where T_t is a lump-sum transfer from the government. A positive transfer indicates a bailout, while a negative one denotes a lump-sum tax. This transfer is the primary link between the households and the government and will be present in all versions of the model. Access to this instrument allows the government to directly modify the household's cash in hand without introducing additional distortions. As a result, the interactions that will arise between private and public debt in this paper are not a consequence of a restrictive set of tax instruments. The last subsection will consider the implications of giving the government an additional tax instrument, a linear tax on private borrowing, τ_t, used for macroprudential purposes.

[20]See, for instance, Gorton et al. (2020) and Chen et al. (2019).

[21]Models with a constant κ and no private default, such as Mendoza (2010), also generate private crisis dynamics with realistic business cycle features.

3.1.2 Government

Public debt: The government borrows by issuing without commitment a long-term bond ($L \geq 0$) on international capital markets *à la* Eaton and Gersovitz (1981). Each period the sovereign chooses to either default ($d \in \{0, 1\}$) or keep its credit market access by paying its obligations and reissuing new ones. As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), I assume that a bond issued in period t promises in case of repayment a deterministic infinite stream of coupons that decreases at an exogenous constant rate δ. As such, one unit issued in the current period promises to pay a fraction $(1 - \delta)$ of all remaining debt each following period. An advantage of this payment structure is that it condenses all future payment obligations into a one-dimensional state variable proportional to the quantity of long-term coupon obligations that mature in the current period. Hence, the debt dynamics can be summarized by

$$L_{t+1} = (1 - \delta)L_t + i_t, \tag{4}$$

where L_t is the number of public bonds due at the beginning of period t and where i_t is the bond issuances at t. As is in common in the literature, I assume that sovereign debt only takes values in a finite and bounded support with $\mathcal{J}$ points.[22] The grid of potential long-term debt positions can be summarized by a vector Λ, where L_j is the j^{th} element; consequently,

$$\Lambda = \Big[L_1, L_2, ..L_{\mathcal{J}}\Big]^T.$$

Default: Default brings immediate financial autarky and an additive utility cost that is an increasing function of tradable output $\phi(y_t^T)$.[23] For simplicity's sake, I assume that the government returns to international credit markets with zero debt after one period of exclusion from the markets.[24] Note that sovereign default does not imply default on private debt nor an exclusion of private agents from financial markets. This is in contrast to other papers with both public and private international debt, such as Mendoza and Yue (2009). I make this assumption because empirically, Kalemli-Ozcan et al. (2018), Gennaioli et al. (2018), and Bottero et al. (2020) find that although private borrowing declines during a sovereign default crisis, it is still quantitatively significant.

[22] The assumption of a discrete and bounded support is usual in the sovereign default literature with long-term debt; see Chatterjee and Eyigungor (2012).

[23] Utility losses from default in sovereign debt models are also used in Aguiar and Amador (2013), Bianchi and Sosa-Padilla (2020), and Roch and Uhlig (2018), among others. A common alternative is output costs of default. If the utility function is log over the composite consumption and if output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications.

[24] Assuming an exogenous probability of reentry into financial markets, as in Arellano (2008), would not change the results but would require the model to keep track of an additional state.

Government's preferences: The sovereign is benevolent and therefore has the same utility and discount factors as the households. Furthermore, for computational tractability, I follow Dvorkin et al. (Forthcoming) and assume that each period the government draws a random vector $\boldsymbol{\epsilon}$ of size $\mathcal{J}+1$ of additive taste shocks. One element of the vector is associated with the choice of default, while the remaining $\mathcal{J}$ elements are associated with each debt choice on Λ in case of repayment. The elements of the vector are labeled

$$\epsilon(L_j) = \epsilon_j,$$
$$\epsilon^{Def} = \epsilon_{\mathcal{J}+1}.$$

The taste shock $\boldsymbol{\epsilon}$ is independent and identically distributed (i.i.d.) over time and within Λ. Furthermore, I assume that its distribution is a multivariate generalized extreme value with mean m and variance $v > 0$.[25] Combining all this, the government's flow utility at time t is

$$u(C_t) + d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1 - d_t)\epsilon_t(L_{t+1}),$$

where d_t is the government default decision, C_t is private consumption, $\phi(y_t)$ is the utility cost of default, and ϵ_t is the additive taste shock. This equation provides an explicit formulation of the additive preference term in the household preferences (1), namely,

$$D_t = d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1 - d_t)\epsilon_t(L_{t+1}).$$

Government's budget constraint: Each period the government's budget constraint is given by its default decision d_t, the public debt dynamics (4), and the lump-sum transfers T_t.[26] The budget constraint is

$$T_t = (1 - d_t)\Big[Q_t\big[L_{t+1} - (1 - \delta)L_t\big] - \delta L_t\Big], \tag{5}$$

where L_t is the long-term public debt at the beginning of period t and where L_{t+1} is the long-term debt at the end. Finally, Q_t is the price at which lenders purchase these bonds, which in equilibrium depends on the government's and household's portfolio decisions and the exogenous shocks.

[25] For additional details regarding the distribution of taste shocks, see Appendix A. Preference shocks affecting the default decisions are now common in the literature; see, for instance, Arellano et al. (2017), Aguiar et al. (2019), and Aguiar et al. (2020). They are considered an alternative to the i.i.d. income shocks also encountered in the literature (e.g., Chatterjee and Eyigungor (2012)). In this model, the shocks allow the government to break ties between similar portfolio positions. An interpretation of these shocks is that they capture additional costs or benefits of default, such as the perceptions of policymakers of the costs of default. At the same time, as noted by Dvorkin et al. (Forthcoming), provided that the variance of the shocks is small enough, they will have small quantitative consequences in aggregate moments.

[26] Subsection 3.4 will relax this assumption by granting the government access to taxes on private debt.

3.1.3 International lenders

Private and sovereign bonds are traded with a continuum of risk-neutral, competitive foreign lenders. Lenders have access to a one-period risk-free security paying a net interest rate r. The equilibrium price of private bonds is given by the no-arbitrage condition

$$q_t = \frac{\mathbb{E}_t[1-\pi_{t+1}]}{1+r}.$$

In equilibrium, investors must be indifferent between purchasing a risk-free security and buying a private bond at price q_t. Since private debt is only held for one period, lenders use the exogenous probability of default one period ahead to price it. Similarly, bond prices for sovereign debt in case of repayment are

$$Q_t = \frac{\mathbb{E}_t}{1+r}\left[(1-d_{t+1})\big(\delta + (1-\delta)Q_{t+1}\big)\right].$$

As before, the no-arbitrage condition implies that investors will purchase government bonds at a price Q_t that compensates them for the risk of default they bear. In case of default, no public debt is recovered. In case of repayment, the payoff is given by the coupon δ plus the market value Q_{t+1} of the nonmaturing fraction of the bonds next period.

3.1.4 Resource constraints

Since both types of debt are denominated in tradables, the market clearing conditions are

$$c_t^N = y_t^N, \tag{6}$$

$$c_t^T + (1-\pi_t)b_t = y_t^T + q_t b_{t+1} + T_t. \tag{7}$$

3.2 Baseline unregulated competitive equilibrium

This subsection defines and characterizes the baseline problem in recursive form. I first discuss the equilibrium concept and the timing of the events and introduce the notation used throughout the paper. I then present, in order, the problems of the government, the households, and the lenders. I conclude with the formal definition of a competitive equilibrium for this baseline version of the model.

Equilibrium concept: This paper focuses on a Markov perfect equilibrium. Consequently, the current period decisions of all agents will be functions of payoff-relevant state variables and will take all future policy rules as given. The focus on a Markov perfect equilibrium is important. An environment with strategically defaultable long-term bonds with a government that cannot commit to future debt issuances induces a time-inconsistency problem known as debt dilution. The solutions to the recursive, time-consistent problem do not coincide with the solutions to the sequential problem with commitment. Throughout the paper, the focus is on the time-consistent policies.[27] Additionally, government default, borrowing and transfer strategies each period will only depend on current period payoff-relevant states.

One could interpret this environment as a game where the government makes current period decisions while taking as given the best response functions of the other players, households, and foreign lenders, as well as the strategies of future governments that decide policies later on. Thus, the government considers the general equilibrium effects of its policies on the aggregate choices of the private sector, consumption, and private borrowing, as well as all prices, nontradables, and bonds; however, the government cannot choose those functions.

Recursive notation and timing: In all cases, I denote with a prime symbol the end-of-period levels of private and public debt. The timing of events within the period is as follows:

- The economy enters the period with private debt B and public debt L.
- All shocks are realized. The exogenous state is $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$.
- The state space is now $S = \{s, L, B\}$.
- The government acts first. Facing S, the government makes default d and public debt L' choices.
- The aggregate state of the economy incorporating the government's policies is $S_G = \{S, d, L'\}$.
- Households act second. Facing S_G, households choose consumption and private debt, which determine the aggregate consumption C^T and C^N and the aggregate private debt B'.
- The lenders act last. They choose bond schedules Q and q using only the payoff-relevant states.

Policy decisions and best responses: The government's policy decisions are $\boldsymbol{d}(S)$ and $\mathcal{L}'(S)$. The private sector's aggregate best responses are $\mathcal{C}^T(S_G)$, $\mathcal{C}^N(S_G)$, and $\mathcal{B}'(S_G)$. The foreign lenders' best responses are the schedules for public bond $Q(s, L', \mathcal{B}'(S_G))$ and for private bond $q(s)$.

Government: Given the best responses of the private sector and foreign lenders, the government chooses $\boldsymbol{d}(S)$ and $\mathcal{L}'(S)$ that maximizes the household's welfare subject to the period budget constraint (5) and the resource constraints ((6) and (7)). In detail, the government's problem is

$$W(S) = \max_{d \in \{0,1\}} [1 - d] W^R(S) + d W^D(S), \tag{8}$$

[27] For a discussion of policies that remedy debt dilution, see Hatchondo et al. (2016) and Aguiar et al. (2019).

where $d = 1$ if the government defaults and $d = 0$ otherwise. If the government repays, $S_G = (S, 0, L')$, and the value of repayment is

$$W^R(S) = \max_{L' \in \Lambda} \; u\big(C^T(S_G), C^N(S_G)\big) + \epsilon(L') + \beta \mathbb{E}_s[W(s', L', \mathcal{B}'(S_G))] \tag{9}$$

$$\text{subject to}$$
$$T(S_G) = Q(s, L', \mathcal{B}'(S_G))[L' - (1-\delta)L] - \delta L,$$
$$C^T(S_G) + (1-\pi)B = y^T + q(s)\mathcal{B}'(S_G) + T(S_G),$$
$$C^N(S_G) = y^N.$$

Note that in repayment states, the government's public debt decision will affect the value of the transfer directly through issuances and indirectly through the bond schedule. The choice of public debt will then affect the households' decisions on consumption of tradables and private debt via the transfer. The government internalizes that its borrowing decision affects the choices of the households and the price that the lenders will charge for public debt.

In the case of default, $S_G = (S, 1, 0)$, and the government's value is

$$W^D(S) = u\Big(C^T(S_G), C^N(S_G)\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s\Big[W\big(s', 0, B'(S_G)\big)\Big] \tag{10}$$

$$\text{subject to}$$
$$T = 0,$$
$$C^T(S_G) + (1-\pi)B = y^T + q(s)\mathcal{B}'(S_G),$$
$$C^N(S_G) = y^N.$$

While in default, the government loses access to public borrowing. Thus, the transfer is zero. Nevertheless, households still maintain access to financial markets and are still liable for their obligations. Consequently, a sovereign default can still leave the economy highly leveraged, albeit in private bonds.[28]

The solution to the government's problem yields decision rules for default $\boldsymbol{d}(S)$ and public debt $\mathcal{L}'(S)$, which in turn determine the transfers $T(S_G)$ and the preference shift $D(S_G)$ as follows:

$$T(S_G) = (1 - \boldsymbol{d}(S)) \times \Big(Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))[\mathcal{L}'(S) - (1-\delta)L] - \delta L\Big), \tag{11}$$

$$D(S_G) = \big(1 - \boldsymbol{d}(S)\big)\epsilon\big(\mathcal{L}'(S)\big) + \boldsymbol{d}(S)\big(\epsilon^{def} - \phi(y^t)\big). \tag{12}$$

Households: The households make decisions based on their current level of individual debt b and the aggregate state of the economy when they act S_G. The aggregate state comprises the exogenous

[28]See Mendoza and Yue (2009) for a case where public default also triggers private default.

shocks s, the initial level of government debt L, the current level of aggregate private debt B, and the decisions made by the government in the current period regarding default d and public debt L'. Households are competitive, and as such they take all prices and aggregate laws of motion as given: the price of nontradables $p^N(S_G)$, the equilibrium price of private bonds $q(s)$, the government's current and all future borrowing decisions $\mathcal{L}'$ and default decisions $\boldsymbol{d}$,[29] transfers T, and the preference shock D. Under rational expectations, households predict future states using the perceived law of motion of aggregate private debt $\mathcal{B}'$. The households' optimization problem in recursive form is

$$V(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s[V(S'_G, b')] \tag{13}$$

subject to

$$\begin{aligned}
c^T + p^N(S_G)c^N + (1-\pi)b &= y^T + p^N(S_G)y^N + q(s)b' + T, \\
q(s)b' &\leq \kappa[p^N(S_G)y^N + y^T], \\
T &= T(S_G), \\
D &= D(S_G), \\
B' &= \mathcal{B}'(S_G), \\
L' &= \mathcal{L}'(S), \\
S'_G &= (s', L', B', \boldsymbol{d}(s', L', B'), \mathcal{L}'(s', L', B')).
\end{aligned}$$

In equilibrium, $p^N(S_G)$ is the price of nontradables, and $q(s)$ is the price of private bonds. The solution to the household problem yields decision rules for individual bond holdings $\hat{b}'(S_G, b)$, tradable consumption $\hat{c}^T(S_G, b)$, and nontradable consumption $\hat{c}^N(S_G, b)$. The household optimization problem induces a mapping from the perceived law of motion for aggregate bond holdings, $\mathcal{B}'(S_G)$, to an actual law of motion, given the representative agent's choice $\hat{b}'(S_G, B)$. In a rational expectations equilibrium, these two functions must coincide. The same is true for the laws of motion of aggregate consumption in the economy $\{C^i(s, L, B)\}_{i=T,N}$.

The solutions to the households' problem solve the optimality conditions that include the budget constraint (3), the credit constraint (2), and the first-order conditions. In particular, the households' intratemporal optimality condition pins down the equilibrium price of nontradables:

$$p^N(S_G) = \frac{1-\omega}{\omega}\left(\frac{C^T(S_G}{y^N}\right)^{\eta+1}. \tag{14}$$

Condition (14) is a static optimality condition equating the marginal rate of substitution between tradable and nontradable goods to their relative price. The equation implies that the price of nontrad-

[29] For concision's sake, I equate in the discussion the solutions to the current government policy functions with the strategies of future governments. This equality holds in a Markov perfect equilibrium. Alternatively, one could impose this equality as an equilibrium condition, as in Bianchi and Mendoza (2018).

ables is an increasing function of c^T.

A pecuniary externality arises in this problem because this equilibrium price affects the value of collateral (2) and therefore the level of borrowing in some states. Consequently, a reduction in c^T causes in equilibrium a reduction in the collateral value (2). In states where the credit constraint binds, this reduction triggers the financial amplification mechanism, whereby a drop in consumption induces a contraction in private borrowing, which in turn drives consumption further down. Because of standard consumption-smoothing effects, consumption increases with the cash in hand of the households. Since the government can increase the cash in hand of the households via the fiscal transfer, mitigating the amplification mechanism is an important incentive for government bailouts.

Lenders: The risk-neutral, competitive foreign lenders use the decision rules of current and future governments and households to price the bonds. The solution to the problem of the lenders yields the bond price schedule for private debt,

$$q(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{15}$$

and the bond price schedule for public debt,

$$Q(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\Big[\Big[1 - d'\Big] \times \Big[\delta + (1-\delta)Q\Big(s', L'', B''\Big)\Big]\Big], \tag{16}$$

where

$$\begin{aligned} B'' &= \mathcal{B}'(s', L', B'), \\ L'' &= \mathcal{L}'(s', L', B'), \\ d' &= \boldsymbol{d}(s', L', B'). \end{aligned}$$

The lenders price the debt contracts based on their expectations of future defaults and new issuances of public debt. As a result, when pricing private debt, the only payoff-relevant state is the exogenous shock s. In contrast, when pricing public debt, the payoff-relevant states for the lenders also include the end-of-period levels of private B' and public debt L'. Note that both the levels and composition of debt are important because they affect the future governments' default and public debt issuance decisions.

Definition of equilibrium: The competitive Markov equilibrium combines the problems of the government, households, and lenders, as well as the resource constraints of the economy. Moreover, it also has rational expectations conditions guaranteeing that in equilibrium the households' borrowing and consumption decisions are consistent with the perceived law of motion that all agents are using in their decisions.

Definition 1. *A Markov unregulated competitive equilibrium is a set of value functions* $\{V, W, W^R, W^D\}$, *policy functions for the private sector* $\{\hat{b}, \hat{c}^T, \hat{c}^N\}$*, policy functions for the public sector* $\{\boldsymbol{d}, \mathcal{L}'\}$*, a pricing function for nontradable goods* p^N*, pricing functions for public debt* Q *and private debt* q*, and perceived laws of motion* $\{\mathcal{B}', C^T, C^N, Q\}$ *such that*

1. *Given prices* $\{p^N, q\}$*, government policies* $\{\boldsymbol{d}, \mathcal{L}'\}$*, and perceived law of motion* $\mathcal{B}'$*, the private policy functions* $\{\hat{b}', \hat{c}^T, \hat{c}^N\}$ *and value function* V *solve the household's problem (13).*
2. *Given bond prices* $\{Q, q\}$ *and aggregate laws of motion* $\{\mathcal{B}', C^T, C^N\}$*, the public policy functions* $\{\boldsymbol{d}, \mathcal{L}'\}$ *and value functions* W*,* W^R*, and* W^D *solve the Bellman equations (8)–(9).*
3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion* $\{\mathcal{B}'(S_G) = \hat{b}'(S_G, B), C^T(S_G) = \hat{c}^T(S_G, B), C^N(S_G) = \hat{c}^N(S_G, B)\}$.
4. *The private bond price function* $q(s)$ *satisfies (15).*
5. *Given public* $\{\boldsymbol{d}, \mathcal{L}'\}$ *and private* $\{\mathcal{B}'\}$ *policies, the public bond price* $Q(s, L', B')$ *satisfies (16).*
6. *Goods market clear:*

$$
\begin{aligned}
C^N(S_G) =& y^N, \\
C^T(S_G) + (1-\pi)B =& y^T + q(s)\mathcal{B}'(S_G) + \\
& \Big\{1 - \boldsymbol{d}(S)\Big\}\Big\{Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))\Big[\mathcal{L}'(S) - (1-\delta)L\Big] - \delta L\Big\}.
\end{aligned}
$$

3.3 Recursive social planner's problem

This subsection formulates the problem of a social planner in the same environment. The formulation is similar to the "primal approach" to optimal policy analysis. The planner chooses aggregate allocations subject to resource, implementability, and collateral constraints. Note that the planner does not set prices and instead takes the pricing functions that solve the lenders' problem as given. However, the planner internalizes how their consumption and borrowing decisions affect all general equilibrium prices. As such, the planner behaves like a strategic player and not competitively as the households do in the previous subsection. Therefore, the equilibrium price of nontradable goods (p^N) and bonds (q, Q) will enter the SP problem as implementability constraints.[30] As before, the focus is on the Markov perfect stationary equilibrium. I assume that the planner cannot commit to future policy rules, including future defaulting and borrowing decisions. Consequently, the planner chooses current period allocations, taking as given the strategies of future planners. Equilibrium is characterized by a fixed point of these policy rules.

[30] This formulation is equivalent to letting the planner make all borrowing decisions and transfer the proceeds to competitive households that make all consumption decisions, taking prices as given.

The social planner's optimization problem consists of maximizing the utility of the households (1) subject to the credit constraint (2), the resource constraints ((6) and (7)), and equilibrium prices ((14), (15), and (16)).[31] Denote $\{\mathcal{L}^{SP\prime}$ and $\mathcal{B}'^{SP\prime}\}$ as the public and private borrowing decisions, respectively. Let $\boldsymbol{d}^{SP}$ be the default decisions of future planners that the current SP takes as given. The planning problem is[32]

$$W^{SP}(s,L,B) = \max_{d\in\{0,1\}} [1-d]W^{SP,R}(s,L,B) + dW^{SP,D}(s,B), \tag{17}$$

where the default value of the planner $W^{SP,D}(s,B)$ is

$$\begin{aligned}
W^{SP,D}(s,B) &= \max_{c^T,B'} u(c^T,y^N) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\Big[W^{SP}(s',0,B')\Big],\\
c^T + B(1-\pi) &= y^T + q^{SP}(s)B',\\
q^{SP}(s)B' &\leq \kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T\right),\\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}.
\end{aligned} \tag{18}$$

And the value of the planner under repayment $W^{SP,R}(s,L,B)$ is

$$\begin{aligned}
W^{SP,R}(s,L,B) &= \max_{c^T,B',L'\in\Lambda} u(c^T,y^N) + \epsilon(L') + \beta\mathbb{E}_s[W^{SP}(s',L',B')],\\
c^T + B(1-\pi) + \delta L &= y^T + q^{SP}(s)B + Q^{SP}(s,L',B')[L'-(1-\delta)L],\\
q^{SP}(s)B' &\leq \kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T\right),\\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r},\\
Q^{SP}(s,L',B') &= \frac{\mathbb{E}_s\Big[\Big[1-\boldsymbol{d}^{SP}(s',L',B')\Big]\times\Big[\delta+(1-\delta)Q^{SP}\Big(s',\mathcal{L}^{SP\prime}(s',L',B'),\mathcal{B}^{SP\prime}(s',L',B')\Big)\Big]\Big]}{1+r}.
\end{aligned}$$

Like the government, the planner chooses aggregate private debt L'. In contrast to the government in the baseline version, the planner also directly controls the level of aggregate private borrowing B'. The planner's decisions take into account the effect of these assets on: the price of nontradables (14). the value of collateral (2), and the price of public debt (16).

Definition 2. *A Markov stationary socially planned equilibrium is a set of value functions* $\{W^{SP}, W^{SP,R}, W^{SP,D}\}$*, policy functions for allocations* $\{C^{SP,T}, C^{SP,N}, \mathcal{L}^{SP\prime}, \mathcal{B}^{SP\prime}\}$*, defaulting* $\boldsymbol{d}^{SP}$*, and pricing functions*

[31] The household budget constraint is automatically satisfied by Walras's law.

[32] For concision, the equilibrium price of nontradables (14) and the resource constraint of nontradables (6) are already incorporated in this formulation. The price of public bonds Q^{SP} is equated with the equilibrium best response of risk-neutral, competitive lenders.

for public Q^{SP} and private q^{SP} debt that solve (17) given conjectured future policies $\{C^{SP,T}, C^{SP,N}, \mathcal{L}^{SP\prime}, \boldsymbol{d}^{SP}\}$

3.4 Decentralization with macroprudential policies

In this subsection, I consider another version of the model where the government gains access to state-contingent linear taxes on private borrowing. I show that the Markov competitive equilibrium allocation solves the planner's problem presented in the previous subsection. The households' budget constraint (3) becomes

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t(1-\tau_t)b_{t+1}, + y_t^T + p_t^N y^N + T_t,, \tag{19}$$

where τ_t is the tax rate on private borrowing. The introduction of taxes does not modify the credit constraint (2). As with all other government policies, taxes on private debt are taken as given by households. At the same time, the government can still tax the households using lump-sum transfers. The budget constraint (5) is now

$$T_t = (1-d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big] + \tau_t q_t B_{t+1}. \tag{20}$$

Note that the government can still tax private debt and use lump-sum transfers while in default. Appendix A provides a complete recursive formulation and characterization of the decentralized equilibrium with taxes.

Proposition 1. *The socially planned equilibrium allocation can be decentralized with a state-contingent tax on debt that satisfies*

$$\begin{aligned} 1-\boldsymbol{\tau}(s,L,B) =& \frac{1}{q^{SP}(s)u_T(C^{SP,T}(s,L,B), y^N)} \times \Bigg(\mu^{SP}(s,L,B)q^{SP}(s) + \\ &+ \beta\mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(s',L',B'), C^{SP,N}(s',L',B'))\Big)\Big]\Bigg), \end{aligned} \tag{21}$$

where μ^{SP} corresponds to the Lagrange multiplier associated with the credit constraint in the planner problem (17).

Proof: See Appendix B.

The proof is done in two steps. First, I show that the planning problem is equivalent to a relaxed version of the competitive equilibrium with taxes. Second, I show that solutions to the planning problem are sufficient to construct policies that satisfy the additional constraints of the competitive equilibrium problem with taxes.

3.5 Difference between the baseline and planned economies

This subsection explains the intuition behind the main difference between the two versions of the model. For this purpose, I compare the intertemporal optimality conditions of the baseline and planner problems presented before. Consider the intertemporal optimality conditions of the households in the baseline problem (13),[33]

$$q(s)u_T(C^T(S_G)) = \beta\mathbb{E}_s[(1-\pi')u'_T(C^{T\prime}(S_G)] + \mu(S_G)q(s), \tag{22}$$

$$0 \leq \kappa(p^N(S_G)y^N + y^T) - q(s)\mathcal{B}'(S_G) \quad \text{with equality if } \mu(S_G) > 0, \tag{23}$$

where $u_T(.)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$, the marginal utility of the consumption of tradables, and where μ is the Lagrange multiplier on the credit constraint. Condition (22) is the household's Euler equation for private debt, and (23) is the complementary slackness condition. If $\mu > 0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit of private debt and repaying next period. The main difference between the baseline problem and the planner's problem is in the private borrowing decision. Consequently, I compare the Euler equation of private bonds for each problem.[34] Using the same notation as before, the planner policies (SP) are:[35]

$$q^{SP} + Q^{SP}_{B'}(\mathcal{L}^{SP\prime} - (1-\delta)L) = \frac{\beta\mathbb{E}_s\left[(1-\pi')\left(u^{SP}_T(C^{SP,T\prime}) + \mu^{SP\prime}\psi^{SP\prime}\right)\right] + \mu^{SP}q^{SP}}{u^{SP}_T(C^{SP,T}) + \mu^{SP}\psi^{SP}}. \tag{24}$$

The prime notation denotes future values of the marginal utility of consumption (u^{SP}_T) and of the Lagrange multiplier (μ^{SP}). In contrast to the baseline's condition (22), the planner's Euler equation includes the marginal effect on the collateral value of an additional unit of tradable consumption $\psi^{SP} = \kappa(1+\eta)\frac{(1-\omega)}{\omega}(\frac{C^{SP,T}}{y^N})^{\eta}$, public borrowing policies $\mathcal{L}^{SP\prime}$, and the marginal effect on the price of public bonds of an additional unit of tradable consumption $Q^{SP}_{B'}$. These terms capture the additional general equilibrium effects that the planner considers when deciding the level of private borrowing. While the first term is common in the Fisherian debt deflation literature, the other two are encountered in the sovereign debt maturity management literature. I now briefly discuss the effect of each of them.

The term ψ^{SP} appears in Bianchi (2011). It captures that relative to the households in the baseline model, the planner considers the marginal benefit of an extra unit of private borrowing on the current and future real exchange rate. First, additional borrowing increases the consumption of tradables and

[33]These expressions are obtained by assuming that the policy and value functions are differentiable and then applying the standard envelope theorem to the first-order conditions of the household problem while assuming that rational expectations hold.

[34]The complete characterization of the optimality conditions of the planning problem is discussed in Appendix A.

[35]As before, these first-order conditions are obtained by assuming differentiability of policy and value functions and that the standard envelope conditions. I also assume that the equilibrium price of bonds is differentiable.

therefore the price of nontradables, which in turn relaxes the credit constraint ($\mu^{SP}\psi^{SP}$). Quantitatively, this effect is generally small, given that numerically I find that $\psi^{SP} < 1$. [36] Second, additional private borrowing decreases expected cash in hand next period, depressing the expected future price of nontradables ($\mu^{SP\prime}\psi^{SP\prime}$). Thus, additional borrowing increases the probability of facing a binding constraint next period. The planner internalizes this cost; the competitive households in the baseline model do not.[37] Consequently, the planner borrows less. This effect is quantitatively significant and the source of private overborrowing in the baseline model.

The terms $\mathcal{L}^{SP\prime}$ and $Q^{SP}_{B'}$ are seen in models where the government has access to public bonds of different maturities, as those in Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016). The private bond discussed here has a short-term maturity but differs from the short-term asset discussed in those papers in two ways. First, it is not directly controlled by the government in the baseline model; instead, it is controlled by the households. Second, it is not strategically defaultable and is instead subject to the collateral constraint. Nevertheless, some of the trade-offs described in those models apply here. Private borrowing increases the probability of default and also increases the expected issuances of public debt in case of repayment. Keeping all other things equal, an extra unit of private bonds decreases expected wealth next period. Mechanically, this increases the probability of sovereign default. Moreover, even in states of repayment, higher private debt increases the probability of a debt-financed bailout. As a result, in some states an extra unit of private debt is also associated with an expected increase in future public debt. As a consequence of these two effects, increasing private debt increases the premium paid on public debt. The planner, who optimally manages the issuances of both assets, chooses a lower level of private debt to lower the interest on public debt. Lenders internalize that the government in the baseline problem cannot guarantee this optimal portfolio in either the current or future periods. Consequently, lenders offer a worse price schedule to the government than to the counterfactual social planner. This bond schedule combined with more frequent use of public bailouts will quantitatively explain the difference in average spreads between the baseline and socially planned equilibria.

4 Quantitative analysis

In this section, I solve numerically the two versions of the model presented in the previous section. The baseline is solved using time iteration for the private equilibrium and value function iteration for the government problem. The socially planned economy can be solved by value function iteration. More details regarding the numerical solution methods are described in Appendices D and E.

[36] If $\psi^{SP} > 1$, this can instead lead to underborrowing and/or multiple equilibria. In all quantitative specifications considered in the paper, this case is never encountered. For specifications where this is violated, see Schmitt-Grohé and Uribe (2019). For other models of Fisherian deflation with underborrowing, see Benigno et al. (2013).

[37] Note that the decision to ignore this effect is rational from the individual household perspective. Each household is small and does not control aggregate borrowing. As a result, its borrowing choices do not affect aggregate prices.

4.1 Calibration

The baseline version of the model is calibrated using Spanish macroeconomic data from 1999 to 2011. One period in the model corresponds to one year in the data. I assume that Spain was at the ergodic distribution of the baseline version of the model during this period. The calibration consists of selecting a set of parameters so that the ergodic distribution averages coincide with the relevant macroeconomic moments in the data.

The starting year is chosen to coincide with the creation of the Eurozone. Before this, most Spanish public debt was in domestic currency, and therefore its nominal value was subject to government choices. The end year of 2011 is chosen to keep out of sample the significant European policies introduced in 2012. Some of these policies conflict with some of the fundamental assumptions underlying the baseline version of the model. Although Spain had implemented countercyclical prudential policies for its domestic banking sector in 1999, up until 2011 there were no systematic controls on private international borrowing within the European Union.[38] This changed in June of 2012, when European heads of state proposed the creation of the Single Supervisory Mechanism (SSM) to supervise bank debt within the union. By 2014, the Bank of Spain had transferred a substantial portion of its supervisory powers to the SSM. In addition, in June of 2012, European leaders also agreed to allow the European Stability Mechanism to offer direct help to Spanish banks. Finally, one month later, in July 2012, then-president of the European Central Bank (ECB) Mario Draghi famously signaled the commitment of the institution to do "whatever it takes to preserve the euro." That statement was interpreted at the time as a commitment from the ECB to buy Eurozone public bonds from distressed countries.[39]

Given that the baseline version of the model assumes no restrictions in international private debt and that the last two mechanisms of supranational bailouts are not explicitly modeled, I restrict the sample to the year prior to their introduction. As a consequence of this assumption, in the next section, I will use the comparison between the model and the data responses to the large financial shock as an out-of-sample validation.

Functional forms: The utility function is of the constant relative risk aversion (CRRA) form on the composite CES good:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \quad \text{with } \sigma > 0.$$

The default utility cost is parameterized as follows:

[38] Saurina and Trucharte (2017) provide a detailed account of the history of banking regulation in Spain and how it adapted to the adoption of international accounting standards during this period. For an overview of the current provisions, see Mencia and Saurina Salas (2016).

[39] For a discussion of how beliefs can be crucial for sovereign default incentives, see Cole and Kehoe (2000), and Aguiar et al. (2020).

$$\phi(y^T) = \max\{0, \phi_0 + \phi_1 \ln y^T\}.$$

As Arellano (2008) and Chatterjee and Eyigungor (2012) discuss, a nonlinear specification of the default costs allows the model to reproduce the mean and standard deviation of spreads in the data. In particular, I follow Bianchi et al. (2018) in specifying the default cost function in terms of utility to avoid introducing a direct interaction between sovereign default decisions and private borrowing capacity.

Table 1: Parameters estimated outside of the model

Description	Parameter	Value
Risk aversion	σ	2.0
Elasticity of substitution	$1/(1+\eta)$	.83
Share of tradables	ω	.39
Persistence of tradables	ρ^y	.75
Volatility of tradables	σ^y	.010
Mean private default rate	$\bar{\pi}$	.021
Persistence private default rate	ρ^π	.82
Volatility private default rates	σ^π	.33
Risk free interest rate	r	.027
Duration of long-term bonds	δ	.14

Note: The risk aversion and elasticity of substitution between tradables and nontradables are standard in the literature. The share of tradables is the average share of value added of agriculture, manufacturing, and tradable services of GDP. The risk-free rate is the average yield of one-year German treasury bonds. The duration parameter is chosen to match the average bond duration of six years of Spanish bonds. The tradable income and private default shock parameters are estimated by fitting a first-order autoregressive process on the logs of the tradable share of GDP and share of nonperforming gross loans, respectively. All public bond and yield data are from 1999 to 2011, and the processes for tradable income and nonperforming loans are estimated using the longest available series. The data source for bond yields and nonperforming loans is Bloomberg, and the sectoral GDP series are taken from Eurostat. For details, see data Appendix C.

Estimated parameters: Table 1 shows the set of parameters that are estimated outside of the model. The risk aversion σ and elasticity of substitution between tradables and nontradables $1/(1+\eta)$ are set at values frequently encountered in the literature.[40] To reduce the state space, I awr the endowment of nontradables y^N to a constant normalized to one. I assume that the endowment of tradables is drawn from a first-order log-normal autoregressive (AR(1)) process. I estimate this process using the cyclical component of linearly detrended tradable GDP for Spain. Since the focus is on fluctuations around the business cycle, I use the cyclical component of the linearly detrended share of tradable output.[41] The estimated values for persistence and volatility are $\rho^y = .75$ and $\sigma^y = .01$, respectively.

[40]See for instance, Garcia-Cicco et al. (2010), and Bengui and Bianchi (2018).

[41]Details and sources in Appendix C.

The recursive specification is

$$\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y \quad \text{with } \varepsilon_t^y \sim N(0, \sigma^y).$$

The value of ω is chosen to replicate the share of nontradable GDP in the data, which is 60%.[42] To compute the model counterpart of this object at the ergodic distribution, I use the mean value of external private liabilities $\bar{b}$ and external public liabilities $\bar{L}$ at their targeted values.[43] The value of ω is then set so that $\frac{\bar{p}^N y^N}{\bar{p}^N y^N + y^T} = 0.60$, where $\bar{p}^N = \frac{1-\omega}{\omega} \frac{y^T - r\bar{b} - \delta r \bar{L}}{y^N}$. Since the average tradable and nontradable endowments are one, this yields $\omega = 0.39$.

Similarly, I assume that the exogenous share of private bonds defaulted on each period follows a log-normal AR(1) process. The parameters of this process are estimated using the gross share of nonperforming loans as a percentage of total loans.[44] The estimation yields an average private default rate $\bar{\pi} = 2.1\%$, a persistence parameter $\rho^\pi = .82$, and a volatility $\sigma^\pi = .33$. The recursive specification of the process is

$$\ln \pi_t = (1 - \rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon_t^\pi \quad \text{with } \varepsilon_t^\pi \sim N(0, \sigma^\pi).$$

Two parameters affecting interest rates, r and δ, are estimated outside of the model. The risk-free interest rate is set to the average yield of the one-year German treasury bill over the calibration period, $r = 2.7\%$. One-year bonds are chosen as a benchmark to reproduce the maturity of the short-term private bond in the model. The duration parameter δ is chosen so that average duration in the model corresponds to the average maturity of Spanish bonds in the data. Using Bank of Spain data, I find an average maturity of public debt of six years during the period of interest. This calculation is in line with previous estimates of Spanish bond maturity, as those from Hatchondo et al. (2016) and Bianchi and Mondragon (2018). The Macaulay definition of duration of a bond given the coupon structure of the model is

$$M = \frac{1 + \bar{i_L}}{\delta + \bar{i_L}},$$

where $\bar{i_L}$ is the constant per-period yield delivered by a long-term bond held to maturity (forever) with no default.[45] The implied duration is then $\delta = .14$.

Calibrated parameters: Six parameters are calibrated to match six aggregate moments from the Spanish data. The calibrated parameters are the two constants in the default cost function ϕ_0 and ϕ_1, the discount factor β, the standard deviation of the taste shocks σ^ϵ, and the constants determining

[42] Tradable GDP is computed using the value-added shares of agriculture, manufacturing, and tradable services. More details can be found in Appendix C.

[43] In the baseline calibration described below, $\bar{b} = 0.42$ and $\frac{\delta}{1+\frac{1-\delta}{1+r}}\bar{L} = .14$

[44] Details and sources are in Appendix C.

[45] In the baseline calibration, it corresponds to the targeted spread plus the risk-free rate, $\bar{i_L} = 3.1\%$.

Table 2: Calibrated parameters

Description	Parameter	Value	Moment	Target	Model
Discount factor	β	.92	Avg. total debt	.56	.56
Vol. taste shock	σ^{ϵ}	.020	Vol. total debt	.048	.050
Avg. financial shock	$\bar{\kappa}$	.45	Avg. private debt	.42	.42
Vol. financial shock	σ^{κ}	.020	Vol. private debt	.071	.058
Default cost	ϕ_0	.31	Avg. spread	.0045	.0045
Default cost	ϕ_1	1.9	Vol. spread	.0061	.0061

Note: Total debt and private debt are computed using the international investment position presented in Section 2. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. All moments are computed using data from 1999 to 2011. For additional details, see Appendix C.

the process of the financial shocks $\bar{\kappa}$ and σ^{κ}.[46] Table 2 shows a summary of all the targets and their model counterparts.

The parameters associated with the default costs ϕ_0 and ϕ_1 are measured in the data using the difference in returns between the average Spanish six-year bond and the average German bond of the same maturity. The targeted moments are the average and the standard deviation of this spread, and their model counterparts are the average and standard deviation of the spread of the long-term bond L_t. To compute the sovereign spread in the model that is implicit in a bond price Q, I use the definition of the constant per-period yield. Given the coupon structure, the yield satisfies

$$Q = \sum_{j=1}^{\infty} \delta \frac{(1-\delta)^{j-1}}{(1+\bar{i_L})^j}.$$

The average targeted spread is 0.45% with a standard deviation of 0.47%, which implies values for the default cost parameters of $\phi_0 = .3$ and $\phi_1 = 1.9$. The targets are low when compared to the related literature because they are computed using 1999-2011 data. Other quantitative analyses of the sovereign debt crisis in Spain, such as those of Hatchondo et al. (2016) and Bianchi and Mondragon (2018), focus on spreads only from a later period (2011-2015) and, consequently, target a higher spread. This paper deviates from that by including in the calibration the years 1999-2007, when the interest rate spread of Spanish government debt was very close to zero. Since the aim of the paper is to study the link between the buildup of private debt during those years and the subsequent sovereign debt crisis, it is important for the model to simultaneously match both the years with zero spreads and the large spikes observed during the crisis.

The discount factor β and the volatility of the taste shocks σ^{ϵ} are selected to match the average and standard deviation of the total debt. To compute the model counterparts of these measures, I first

[46]The mean of the taste shocks is irrelevant for their quantitative properties and is selected to achieve numerical tractability. More details can be found in Appendix D.

calculate the international positions of the public and private sectors. The stock of public debt as a percentage of output at time t in the model is calculated for our coupon structure as the present value of future payment obligations discounted at the risk-free rate, that is, $\frac{\delta}{1+\left(\frac{1-\delta}{1+r}\right)} \times \frac{L_t}{(p_t^N y_t^N + y_t^T)}$. By contrast, the international position of the private sector as a percentage of output at time t is simply $\frac{B_t}{(p_t^N y_t^N + y_t^T)}$. At the calibrated values, $\beta = .92$ and $\sigma^{\epsilon} = .02$.

Finally, since the buildup in private debt in the years leading up to the crisis is a motivating fact of the model, the last two targeted aggregated moments are the average and standard deviations of the private debt. Note that because of the symmetry in the evolution of private and public stocks, the volatility of the private and public positions is higher than the volatility of the total debt. It is therefore important that the model matches not only the aggregate positions but also some of their decomposition. I calibrate the process of financial shocks κ_t to match this. As with the other exogenous shocks in the model, I assume that the financial shock follows a first-order normal AR(1) process of the form

$$\kappa_{t+1} = (1-\rho^{\kappa})\bar{\kappa} + \rho^{\kappa}\kappa_t + \varepsilon_t^{\kappa} \quad \text{with } \varepsilon_t^{\kappa} \sim N(0, \sigma^{\kappa}).$$

For simplicity's sake, I assume that the persistence parameter coincides with the persistence of tradable income $\rho^{\kappa} = \rho$ while the mean ($\bar{\kappa}$) and volatility parameters (σ^{κ}) are estimated within the model. The model successfully replicates the average debt of the private sector and a higher volatility for the private position relative to the aggregate. However, it fits less well the large standard deviation seen in the data. At the baseline calibration, $\bar{\kappa} = .45$ and $\sigma^{\kappa} = .02$.

The policy functions of debt accumulation for the baseline and planned economies at the calibrated values can be found on Appendix F. The policy functions of private debt have similar characteristics as those found in Bianchi (2011). Similarly, the policy functions of public debt are analogous to those found Hatchondo and Martinez (2009).

5 Results: Quantitative implications of private overborrowing

This section details the main quantitative findings of the paper. The first subsection shows the results obtained by comparing the baseline and regulated economies at their ergodic distributions. The second subsection simulates the model dynamics during the Spanish debt crisis and the counterfactual in a socially planned economy. The final subsection discusses the policy implications of the paper.

5.1 Social planner and baseline economies at the ergodic distribution

Table 3 presents the first set of quantitative results of the paper. The table shows the values first and second moments in the data and at the ergodic distributions of the baseline and the socially planned economies. The baseline version of the model is calibrated to match the moments from the data; the socially planned economy is not. Instead, I use the calibrated parameters of the baseline to compute the ergodic distribution of the planned problem. The average private debt at the ergodic distribution for the social planner is 36% of output, whereas it is 41% in the baseline case. This difference of 5% of output is the estimate of the total amount of excessive private debt in Spain in the lead-up to the crisis. The table shows that the increase in private debt in the baseline economy relative to the socially planned economy is insufficient to explain the increase in overall indebtedness. That is, the baseline economy accumulates on average more public debt, around 2% of output.

The explanation for why there is more public debt in the baseline can be seen in the bottom half of the table. In this part, I compute four measures of aggregate well-being for the baseline and planned economies, namely, the probability of a binding credit constraint, the probability of a financial crisis, the probability of a sovereign default, and a measure of welfare gains. The credit constraint binds more frequently under the baseline. As explained in the previous section, optimal government borrowing is higher when the constraint binds. As a result, average public debt is higher under the baseline because the government must respond more often to crises. I define a financial crisis as an episode with a binding constraint and a contraction of more than one standard deviation below the mean of the current account of the private sector.[47] Under this definition, I find that excessive private borrowing increases the incidence of financial crises by 2.40 p.p. on average.

Furthermore, Table 3 provides the interest rate spreads on public debt for both the baseline and socially planned economies. In the planned economy, spreads are on average an order of magnitude below their baseline counterparts. The reduction in the spread occurs both because the planner borrows less in general and because it faces a binding constraint less often. The result is also consistent with the smaller average probability of sovereign default in the regulated economy relative to the baseline.

Finally, Table 3 shows the welfare gains of moving from the baseline to the planned economy. The welfare gains are calculated as the proportional increase in consumption for all possible future states that would make the households indifferent between staying in the baseline and moving to the centralized equilibrium. This measure explicitly incorporates the cost of lower consumption in the transition to the ergodic distribution of the planned economy. Given the homoscedasticity of the utility function, the expected welfare gains in state (s_0, L_0, B_0) are:

$$\boldsymbol{\theta}(s_0, L_0, B_0) = \left(\frac{\boldsymbol{W}^{SP}(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1}{\boldsymbol{W}(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1} \right)^{\frac{1}{1-\sigma}} - 1. \tag{25}$$

On average at the ergodic state, households would need to receive a permanent increase of 0.36% in consumption to be indifferent between the two economies. These welfare gains are larger than the ones encountered in the literature. In Bianchi (2011), the welfare gains from correcting the overbor-

[47] Similar definitions are encountered in the related literature; see, for instance, Bianchi (2011) and Bengui and Bianchi (2018).

Table 3: Baseline and social planner aggregate moments at the ergodic distribution

Moment	Data	Baseline	Social planner
Total debt	.56	.56	.49
Private debt	.42	.42	.37
Mean spread	.0045	.0045	.00034
Volatility debt	.048	.050	.027
Volatility private debt	.071	.058	.071
Volatility spread	.0061	.0061	.00030
Probability of a binding constraint	-	.099	.024
Probability of a financial crisis	-	.025	.0010
Probability of default	-	.0046	.00030
Welfare gains	-	-	.0041

Note: All calibrated parameters are kept constant in the computation of the socially planned economy. A financial crisis is defined as a episode in which the credit constraint binds and the current account of the private sector contracts by more than one standard deviation below the mean. Welfare gains are calculated as the proportional increase in permanent consumption under the baseline. Debt levels in the data are calculated using the international investment positions. More details are explained in Appendix C.

rowing externality are around 0.13%. The welfare gains are larger in my model because optimal private debt management also decreases the probability of experiencing the deadweight losses of sovereign default.

In addition to the targeted moments presented in Table 3, the quantitative performance of the model for untargeted business cycle moments is presented in Appendix G. Using simulated data, one can show that the baseline model successfully approximates the volatility of consumption, the current account, and the trade balance, but overestimates the volatility of output. Moreover, the baseline model correctly predicts the sign of the correlations between output and consumption, output and the current account, output and the spread on public debt, and the public debt level and the spread on public debt.

5.2 Simulating the 2012 debt crisis

This subsection uses the data and the calibrated models to provide a model simulation of the events that unfolded in Spain between 2008 and 2015. To shed light on what optimal policies could have achieved, I also plot, alongside the baseline model and the data, the counterfactual dynamics of the socially planned economy. The idea is to feed into the model the exogenous shocks that affected Spain during this period and then contrast the endogenous responses in terms of debt and spreads of the baseline model and socially planned model with their data counterparts. The three fundamental exogenous shocks I feed into the model are: the income shock, the private default shock, and the financial shock. Public and private debt as well as the spread on public bonds are then allowed to respond endogenously to these shocks.

The exogenous income shock, y_t, is taken directly from the Spanish tradable GDP data. Similarly, the share of private bonds defaulted on, π_t, matches exactly the data on gross nonperforming loans during this period. The taste shocks, ϵ_t, are all set to zero. The financial shock, κ_t, is unobserved in the data. To circumvent this problem, I apply the particle filter method proposed by Bocola and Dovis (2019) to my model. Additional details about the particle filter method can be found in appendix H; here I present a summary of the methodology.

The baseline model defines a nonlinear state-space system:

$$\boldsymbol{Y}_t = g(\boldsymbol{S}_t) + e_t,$$
$$\boldsymbol{S}_t = f(\boldsymbol{S}_{t-1}, \varepsilon_t),$$

where $\boldsymbol{S}_t = [L_t, B_t, y_{t-1}^T, \pi_{t-1}, \kappa_{t-1}]$ is the state vector and where ε_t is the vector collecting all the innovations in the three structural exogenous shocks. The vector of observables, $\boldsymbol{Y}_t$, includes average private and public debt as a share of GDP, detrended tradable output, the share of nonperforming loans, and the interest rate spreads on public bonds.[48] The vector e_t represents uncorrelated Gaussian measurement errors and is equal to the difference between the data aggregates $\boldsymbol{Y}_t$ and their model counterparts $g(\boldsymbol{S}_t)$. The functions $g(\cdot)$ and $f(\cdot)$ come from the calibrated numerical solutions of the baseline model. The realizations of the state vector are estimated by applying the particle filter to this system of equations and data from 2008 to 2015. The process yields a path of financial shocks and a set of initial endogenous states. I then feed these shocks into the social planner policy functions $f^{SP}(\cdot)$ to generate the allocations of debts and spreads that would have emerged under counterfactual optimal policies. Note that the social planner functions are not used to estimate the system and are only used ex post to generate counterfactuals. Finally, I also construct the implied tax on borrowing that implements the planner allocations in a competitive equilibrium.

5.2.1 Counterfactuals

I assume that only tradable output and nonperforming private loans are observed with no error. This leaves three observable variables not perfectly fitted in $\boldsymbol{Y}_t$: public debt, private debt, and spreads. To match them, there are three stochastic variables in $\boldsymbol{S}_t$, namely, B_t, L_t, and κ_t. By setting the variance of all measurement errors to 1% of their sample variance, I compute the filtered path of these three

[48] As in the calibration, I use the linearly detrended cyclical component of tradable output. Public debt is initialized at zero, and initial private debt is adjusted to match the composition of total debt in the data.

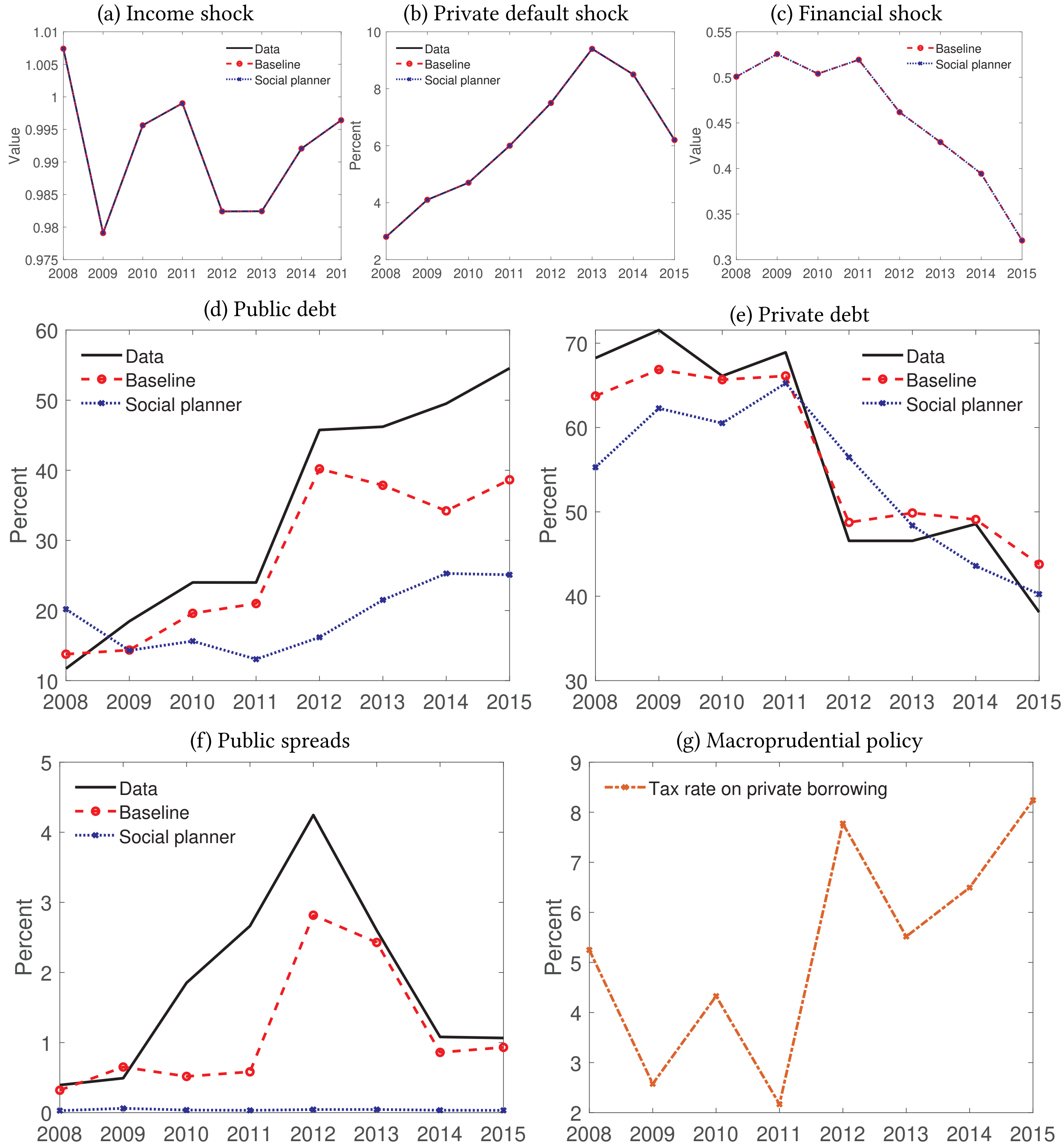

Figure 4: Evolution of debt, taxes, spreads, and exogenous shock, 2008–2015: data and models

Note: Model simulations are obtained by feeding into the model observed income shocks, nonperforming loans, and the most likely series of financial shocks from the particle filter. Public debt, private debt, and spreads are the particle-filtered weighted averages. Both debt series are expressed as a percentage of output, and nonperforming loans are expressed as a percentage of gross loans. Taxes and interest rate spreads are expressed in percentages. Data sources can be found in Appendix C, and details on the particle filter can be found in Appendix H.

stochastic variables that is consistent with the data. Figure 4 summarizes the results of this exercise.

Positive counterfactual: The baseline model, whose responses are plotted as dashed red lines, captures the main events of the crisis. In particular, the magnitude of the 2012 public bailout, around 12% of GDP, is financed by an equivalent increase in public debt. This leads to an increase in the interest rate spread on public bonds of around 3 p.p., equivalent to 80% of the increase observed in the data. The baseline model is less successful at tracking the evolution of public debt after 2012; it predicts a lower indebtedness than what is observed in the data. Similarly, the interest rate spread increase in the model before 2012 is below its data counterpart. Two observations could partially explain these discrepancies. First, while the model captures some of the fluctuations in the external conditions for borrowing via the financial shock, it may be the case that this shock is not enough to fully replicate the uncertainty around government bonds of Eurozone countries during the worst years of the Greek debt crisis. Second, there is no model counterpart to the Mario Draghi speech of 2012 that can replicate its effect on interest rate spreads. Accordingly, the model expects less public debt than the data to replicate the drop in spreads observed in the 2013-2015 period. All things considered, the baseline model predicts a pattern of public debt, private debt, and spreads that is consistent with the data and validates the approach of the paper.

Normative counterfactual: Having validated the positive model, I now turn to the normative counterfactual. In contrast to the baseline case, the socially planned economy is predicting a smooth transition from private liabilities to public debt. Instead of a large bailout in 2012, the planner deleverages in the private bond in three years. The dynamics allow the planner to maintain the interest rate spread near zero throughout the period and halves the size of the 2012 bailout to around 10% of GDP. Note that with the exception of 2012, private debt is lower in the planned economy in all years.

The government could have implemented this with a macroprudential tax on private borrowing that is on average 5% during this period. Similarly, public debt in the socially planned economy is significantly below the levels observed in the data for most of the period, and importantly, even after the bailouts take place.

This exercise shows that the 2012 spike in the interest rate spread could have been avoided if a planner had managed public and private borrowing optimally. In Appendix I, I take advantage of the probabilistic framework induced by the taste shocks to conduct another counterfactual exercise. I restrict the issuances of public debt to the levels observed in the data and compute the evolution of the spread in the socially planned economy. Even in this case, the spike on interest rate spreads in 2012 is 3.8 p.p. below the level observed in the data.

5.3 Policy implication

This subsection will present the implications of sovereign risk for macroprudential policies. Using the calibrated parameters, I compute the state-dependent tax on private debt that decentralizes the allocations that solve the socially planned problem (see Proposition 1). I simulate 10,000 observations to approximate the ergodic distribution. These simulations allow me to compute the entire density of taxes at the ergodic distribution. This density is plotted as a solid red line in Figure 5.

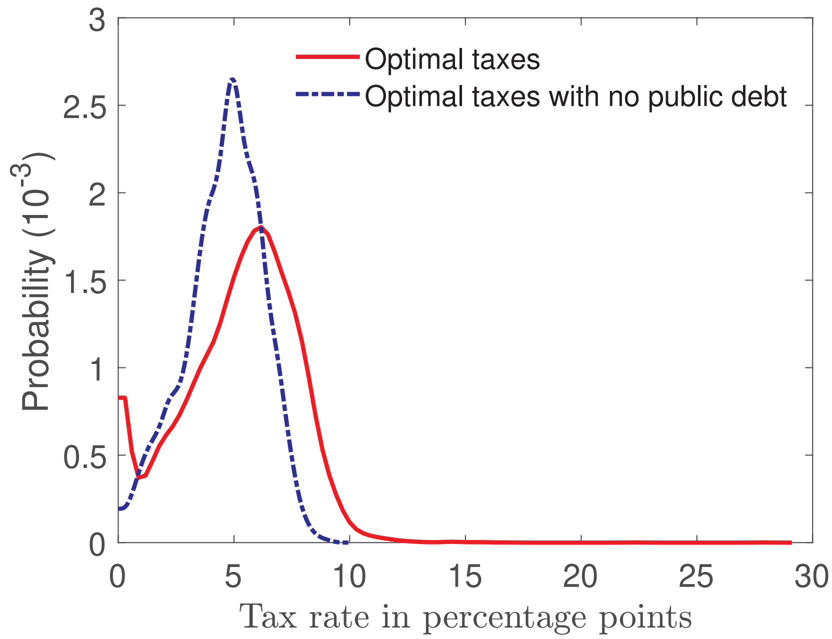

Figure 5: Optimal taxes on private debt issuances at the ergodic distribution
Note: Density functions of the optimal tax on private debt at the ergodic distribution. This distribution is constructed by simulating 10, 000 observations of the calibrated model presented in section 4 (solid red line) and using the same parameters to compute optimal taxes in the Bianchi (2011) model without sovereign risk (dotted blue line).

To compare this optimal tax to a relevant benchmark, I solve a version of the model without public debt using the calibrated parameters presented in Table 2. This version coincides exactly with the canonical sudden-stop model developed in Bianchi (2011). As in this paper, the canonical model also calls for a tax on international private debt to decentralize the allocations that solve the social planner's problem. The density distribution of the optimal tax for this model is plotted as a dotted blue line in Figure 5. The policy functions of the optimal tax have the same monotonic properties in both models. In particular, a tax rate of zero implements the planner's allocations when the collateral constraint binds; thus, the density distributions exhibit a mass at zero.

The main takeaway from Figure 5 is that the average tax rate in an economy with sovereign risk is higher. The average tax rate that decentralizes the planner's allocations is 5.3% as opposed to 4.6% in the economy with no public debt. Since a private financial crisis increases the risk of a sovereign default, more restrictive prudential policies are called for.[49]

[49] Additional comparisons of aggregate moments of the model to those of nested variants of the model (with no public debt and no private debt) can be found in Appendix J.

6 Conclusions

This paper develops a theory that is quantitatively consistent with the evolution of debt and spreads in Spain that culminated in the 2012 sovereign debt crisis. The theory presented here is also consistent with the business cycle statistics observed in the data during this time period.

The model focuses on the interaction between systemic externalities in private credit and sovereign default. The combination of competitive private households whose borrowing is constrained to a fraction of the market value of their current income and a benevolent government capable of assisting them with public funds creates a pathway from financial crises to sovereign debt crises. The process begins with a buildup of private debt when financial conditions are favorable. During this time, public debt remains low and the government faces low spreads. As the private sector accumulates more debt, a financial crisis becomes more likely. Eventually an adverse shock materializes, and the households face a tight borrowing limit. In the model, I allow for a crisis to be triggered by the following exogenous factors: slowdowns in output, increases in private default, and shocks to international financial markets. Confronted with an imminent and painful private deleveraging, the government responds with fiscal transfers financed by new issuances of public debt. Bailouts have a multiplicative positive effect in this context. A positive transfer causes an appreciation in the value of collateral and increases the borrowing capacity of the private sector. As a result, bailouts allow credit-constrained households to accrue more private debt and further increase consumption. Unfortunately, these gains come at the expense of raising the specter of a sovereign default. In all cases, the interest rate spread on government debt increase, and in some particularly adverse circumstances, default materializes.

The paper quantifies the level of excessive private borrowing and its impact. I estimate that in the lead-up to the crisis, excessive private debt in Spain was equivalent to 5% of GDP. As a result, the annual probability of experiencing a financial crisis was 2.4 p.p. above the socially desirable level. Simulating the 2012 crisis, I show the increase in spreads would not have materialized under optimal policies. Finally, I show that optimal borrowing policies could have been implemented by pairing public debt management with state-dependent taxes on private borrowing. I estimate an average tax rate of 5.3% for Spain. This estimate is 0.7 p.p. above what a version of the model without sovereign risk would have called for.

Several interesting avenues for future research remain open. It could be fruitful to investigate the quantitative consequences of introducing moral hazard into the motivations for private overborrowing. Alternatively, one could explore how budgetary covenants or other fiscal limits could deal simultaneously with the incentives for bailouts and with public debt dilution, as in Hatchondo et al. (2016) and Aguiar and Amador (2018). A final extension would be to investigate how a monetary response to private overborrowing would interact with the fiscal response presented here.

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Appendices

A Recursive competitive problem with taxes

For the representative household, the aggregate state of the economy includes the exogenous aggregate shocks denoted by $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$, the initial level of government debt L, the initial level of aggregate private debt B, and the initial level of its own debt b. Following the same notation than in the body of the paper I denote $S = (s, L, B)$ the state space of the economy before government actions. Similarly, let $S_G = (S, d, L', \tau)$ denote the state space after government actions. Note that now that state includes the choice of taxes.

As before, households take as given the price of non-tradables $p^{N\tau}(S_G)$, the equilibrium price of price bonds $q^{\tau}(s)$, and government's current and future decisions regarding default $\boldsymbol{d}^{tau}$, public debt $\mathcal{L}^{\tau}$, and taxes $\boldsymbol{\tau}$. They also know the functions associates with these choices, the lump-sum transfer $\mathcal{T}^{\tau}$ and the preference shock $\mathcal{D}^{\tau}$. Finally, they also have a perceived law of motion of aggregate private debt $\mathcal{B}^{\tau\prime}$. The household's optimization problem in recursive form is:

$$V^{\tau}(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s[V^{\tau}(S'_G, b')] \tag{26}$$

$$\begin{aligned}
&\text{subject to} \\
c^T + p^{N,\tau}(S_G)c^N + (1-\pi)b &= y^T + p^{N,\tau}(S_G)y^N + q^{\tau}(s)(1-\tau)b' + T, \\
q^{\tau}(s)b' &\leq \kappa[p^{N,\tau}(S_G)y^N + y^T], \\
T &= \mathcal{T}^{\tau}(S_G), \\
D &= \mathcal{D}^{\tau}(S_G), \\
B' &= \mathcal{B}^{\tau\prime}(S_G), \\
L' &= \mathcal{L}^{\tau\prime}(S_G) \\
\tau &= \boldsymbol{\tau}(S_G), \\
\text{And } S'_G &= (s', L', B', \boldsymbol{d}^{\tau}(s', L', B'), \mathcal{L}^{\tau\prime}(s', L', B'), \boldsymbol{\tau}(s', L', B')).
\end{aligned}$$

Using the same notation than in the baseline case for the aggregate laws of motion of the private sector are $\mathcal{B}^{\tau\prime}(S_G)$, and $\{C^{i,\tau}(S_G)\}_{i=T,N}$, and public bond pricing $\boldsymbol{Q}^{\tau}(s, L', B')$ function. The government's problem is:

$$W^{\tau}(S) = \max_{d \in \{0,1\}} [1-d]W^{R.\tau}(S) + dW^{D,\tau}(S) \tag{27}$$

In case of default, $S_G = (S, 1, 0, \tau)$ and $W^{D,\tau}(S)$ is given by:

$$W^{D,\tau}(S) = \max_{\tau} u\Big(C^T, C^N\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s\Big[W^\tau\big(s', 0, B'(S_G)\big)\Big] \tag{28}$$

$$\text{subject to}$$

$$\begin{aligned} C^{T,\tau}(S_G) + (1-\pi)B &= y^T + q^\tau(s)(1-\tau)B' + T \\ C^{\mathcal{N},\tau}(S_G) &= y^N \\ T &= q^\tau(s)\tau B' \\ D &= \epsilon^{Def} - \phi(y^T) \\ B' &= \mathcal{B}^{\tau\prime}(S_G) \end{aligned}$$

Note that transfers can still be strictly positive in default since the government transfers the proceeds to of the private debt tax to the households. In case of repayment, $S_G = (S, 0, L', \tau)$ and the value is:

$$W^{R,\tau}(S) = \max_{\tau, L' \in \Lambda} u\big(C^{T,\tau}, C^{N,\tau}\big) + \epsilon(L') + \beta \mathbb{E}_s[W^\tau(s', L', B')] \tag{29}$$

$$\text{subject to}$$

$$\begin{aligned} C^{\mathcal{T},\tau}(S_G) + (1-\pi)B &= y^T + q^\tau(s)(1-\tau)B' + T, \\ C^{\mathcal{N},\tau}(S_G) &= y^N, \\ T &= Q^\tau(s, L', \tau, B')[L' - (1-\delta)L] - \delta L + q^\tau(s)\tau B', \\ D &= \epsilon(L'), \\ B' &= \mathcal{B}^{\tau\prime}(S_G) \end{aligned}$$

The solution to the government's problem yields decision rules for default $\boldsymbol{d}^\tau(S)$, public borrowing $\mathcal{L}^{\tau\prime}(S)$, and taxes $\boldsymbol{\tau}(S)$. The transfers $\mathcal{T}^\tau(S_G)$ and preference shifter $D^\tau(S_G)$ are also pinned down by these decisions. The solution to the problem of competitive risk neutral foreign lenders yields the bond price schedule for private debt:

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{30}$$

and for public debt:

$$Q^{\tau}(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s \left[\left[1 - d' \right] \times \left[\delta + (1-\delta) Q^{\tau}\left(s', L'', B'' \right) \right] \right], \tag{31}$$

$$\begin{aligned} &\text{Where:} \\ B'' &= \mathcal{B}^{\tau\prime}(s', L', B'), \\ L'' &= \mathcal{L}^{\tau\prime}(s', L', B'), \\ d' &= \boldsymbol{d}^{\tau}(s', L', B') \end{aligned}$$

Definition 3. *A Markov regulated competitive equilibrium with taxes is defined by, a set of value functions $\{V^{\tau}, W^{\tau}, W^{R,\tau}, W^{D,\tau}\}$, policy functions for the private sector $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$, policy functions for the public sector $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, a pricing function for nontradable goods $p^{N,\tau}$, pricing functions for public debt Q^{τ} and private debt q^{τ}, and perceived laws of motion $\{\mathcal{B}^{\tau\prime}, C^{T,\tau}, C^{N,\tau}\}$ such that*

1. *Given prices $\{p^{N,\tau}, q^{\tau}\}$, government policies $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, and perceived law of motion $\mathcal{B}^{\tau\prime}$, the private policy functions $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$ and value function V solve the household's problem (26)*

2. *Given bond prices $\{Q^{\tau}, q\}$ and aggregate laws of motion $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}\}$, the public policy functions $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$ and value functions W^{τ}, $W^{R,\tau}$, and $W^{D,\tau}$, solve the Bellman equations (27)–(29)*

3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion $\{\mathcal{B}'(S_G) = \hat{b}^{\tau\prime}(S_G, B), C^{T,\tau}(S) = \hat{c}^{T,\tau}(S_G, B), C^{N,\tau}(S_G) = \hat{c}^{N,\tau} S_G, B)\}$*

4. *The private bond price function $q^{\tau}(s)$ satisfies (30)*

5. *Given public $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, and private $\{\mathcal{B}^{\tau\prime}\}$, policies the public bond price $Q^{\tau}(s, \mathcal{L}^{\tau}(S)', \mathcal{B}^{\tau}(S_G)')$ satisfies (31)*

6. *Goods market clear:*

$$\begin{aligned} C^{N,\tau}(S_G) &= y^N \\ C^{T,\tau}(S_G) + (1-\pi)B &= y^T + q^{\tau}(s)\mathcal{B}^{\tau\prime}(S_G) + \left\{ 1 - \boldsymbol{d}^{\tau}(S) \right\} \times \\ &\quad \left\{ Q^{\tau}(s, \mathcal{L}^{\tau}(S)', \mathcal{B}^{\tau}(S_G)') \left[\mathcal{L}^{\tau\prime}(S) - (1-\delta)L \right] - \delta L \right\} \end{aligned} \tag{32}$$

Similarly to the baseline model the optimality conditions of the households problem are:

$$q^{\tau}(s)(1 - \boldsymbol{\tau}(S) u_T(C^{T,\tau}(S_G)) = \beta \mathbb{E}_s[(1-\pi') u_T(C^{T,\tau\prime}(S_G))] + \mu^{\tau}(S_G) q^{\tau}(s),$$

$$p^{N,\tau}(S_G) = \frac{1-\omega}{\omega} \left(\frac{C^{T,\tau}(S_G)}{y^N} \right)^{\eta+1},$$

$$0 \leq \kappa(p^{N,\tau}(S_G)y^N + y^T) - q^\tau(s)\mathcal{B}^{\tau\prime}(S_G) \quad \text{with equality if } \mu^\tau(S_G) > 0,$$

where μ^τ is the Lagrange multiplier associated with the credit constraint.

B Proof of proposition 1

This is a proof by construction. We will show that the recursive equilibrium with taxes can be written as a government problem that coincides with the planning problem (17). Start from the recursive competitive equilibrium problem with taxes described in Appendix B.

The problem with taxes is equivalent to the recursive problem of a government given that chooses allocations for the current period while taking future policies and prices as given. Denote these policies $\{\boldsymbol{d}^\tau(S), \boldsymbol{\mathcal{L}}^{\tau\prime}(S), \boldsymbol{\tau}(S), \boldsymbol{C}^{T,\tau}(S_G), \boldsymbol{C}^{N,\tau}(S_G), \mathcal{B}^{\tau\prime}(S_G)\}$. This government maximizes utility considering the optimal responses of households and lenders. This is equivalent to let the government choose all policies using the Kuhn-Tucker conditions of households and lenders as constraints. The problem is therefore:

$$W^\tau(S) = \max_{d\in\{0,1\}} [1-d]W^{R,\tau}(S) + dW^{D,\tau}(S),$$

Let $S' = (S', B', L')$ the default value $W^{D,\tau}(S)$ is:

$$\begin{aligned}
W^{D,\tau}(S) &= \max_{c^T,c^N,B',\tau,\mu} u(c^T, c^N) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\Big[W^\tau(S')\Big] \\
&\text{subject to} \\
c^T + B(1-\pi) &= y^T + q^\tau(s)B', \\
c^N &= y^N, \\
q^\tau(s)B' &\leq \kappa\Bigg(p^{N,\tau}c^N + y^T\Bigg), \\
q^\tau(s)(1-\tau)u_T(c^T, c^N) &= \beta E_s[(1-\pi')u_T(\boldsymbol{C}^{T,\tau}, \boldsymbol{C}^{N,\tau}(S', \boldsymbol{d}^\tau(S'), \boldsymbol{\mathcal{L}}^{\tau\prime}(S'), \boldsymbol{\tau}(S'))] + \mu q^\tau(s) \\
p^{N,\tau} &= \frac{1-\omega}{\omega}\Big(\frac{c^T}{c^N}\Big)^{1+\eta} \\
(\kappa(p^{N,\tau}c^N + y^T) - q^\tau(s)B')\mu &= 0 \\
\mu &\geq 0 \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned}$$

The value under repayment $W^{R,\tau}(S)$ is:

$$W^{R,\tau}(S) = \max_{c^T,c^N,B',\tau,\mu,L'\in\Lambda} u\big(c^T,c^N\big) + \epsilon(L') + \beta\mathbb{E}_s[W^\tau(S')]$$

$$\text{subject to}$$

$$c^T + B(1-\pi) + \delta L = y^T + q^\tau(s)B + Q^\tau(s,L',B')[L' - (1-\delta)L],$$

$$q^\tau(s)B' \leq \kappa\left(p^{N,\tau}c^N + y^T\right),$$

$$q^\tau(s)(1-\tau)u_T(c^T,c^N) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau},C^{N,\tau}(S',\boldsymbol{d}^\tau(S'),\mathcal{L}^{\tau\prime}(S'),\boldsymbol{\tau}(S'))] + \mu q^\tau(s)$$

$$p^{N,\tau} = \frac{1-\omega}{\omega}\big(\frac{c^T}{c^N}\big)^{1+\eta}$$

$$(\kappa(p^{N,\tau}c^N + y^T) - q^\tau(s)B')\mu = 0$$

$$\mu \geq 0$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}$$

$$Q^\tau(s,L',B') = \frac{1}{1+r}\times\mathbb{E}_s\left[\left[1-\boldsymbol{d}^\tau(S')\right]\times\left[\delta + (1-\delta)Q^\tau\Big(s',\mathcal{L}^{\tau\prime}(S'),\mathcal{B}^{\tau\prime}(S',\boldsymbol{d}^\tau(S'),\mathcal{L}^{\tau\prime}(S'),\boldsymbol{\tau}(S'))\Big)\right]\right]$$

Substituting in the resource constraint for non tradables, and the intratemporal conditions that problem can be simplified to:

$$W^\tau(S) = \max_{d\in\{0,1\}}[1-d]W^{R,\tau}(S) + dW^{D,\tau}(S), \tag{33}$$

where default value $W^{D,\tau}(S)$ is:

$$W^{D,\tau}\big(S\big) = \max_{c^T,B',\tau,\mu} u\big(c^T,y^N\big) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\Big[W^\tau\big(S'\big)\Big]$$

$$c^T + B(1-\pi) = y^T + q^\tau(s)B',$$

$$q^\tau(s)B' \leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big)$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}$$

$$q^\tau(s)(1-\tau)u_T(c^T,y^N) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau},C^{N,\tau})] + \mu q^\tau(s)$$

$$0 = \left[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^\tau(s)B'\right]\mu$$

$$\mu \geq 0$$

and value under repayment $W^{R,\tau}(S')$ is:

$$W^{R,\tau}(S') = \max_{c^T,B',\tau,\mu,L'\in\Lambda} u\big(c^T, y^N\big) + \epsilon(L') + \beta\mathbb{E}_s[W^\tau(S')]$$

$$c^T + B(1-\pi) + \delta L = y^T + q^\tau(s)B + Q^\tau(s,L',B')[L' - (1-\delta)L] \tag{34}$$

$$q^\tau(s)B' \leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) \tag{35}$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r} \tag{36}$$

$$Q^\tau(s,L',B') = \frac{1}{1+r}\times\mathbb{E}_s\bigg[\Big[1-\boldsymbol{d}^\tau\Big]\times\Big[\delta + (1-\delta)Q^\tau\Big(s',\mathcal{L}^{\tau\prime},\mathcal{B}^{\tau\prime}\Big)\Big]\bigg] \tag{37}$$

$$q^\tau(s)(1-\tau)u_T(c^T,y^N) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau},C^{N,\tau})] + \mu q^\tau(s) \tag{38}$$

$$0 = \Bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^\tau(s)B'\Bigg]\mu \tag{39}$$

$$\mu \geq 0 \tag{40}$$

In this formulation it is apparent that the social planner problem (17) is a relaxed version of problem (33). In problem (33) the government must satisfy three additional constraints (38)–(40) and has access to two additional instruments μ and τ. Crucially, both μ and τ only appear in problem (33) in constraints (38)–(40). As such, problem (17) will be equivalent to problem (33) if we can use the solutions of (17) to construct two functions $\mu(s,L,B)$ and $\tau(s,L,B)$ that satisfy (38)–(40).

Let $\{C^{SP,T}(s,L,B), C^{SP,N}(s,L,B), \mathcal{L}^{SP\prime}(s,L,B), \mathcal{B}^{SP\prime}(s,L,B), \boldsymbol{d}^{SP}(s,L,B), Q^{SP}, q^{SP}(s)\}$ be a solution of problem (17). Additionally let $\mu^{SP}(s,L,B) \geq 0$ be the multiplier on the collateral constraint of the planner problem (17). μ^{SP} corresponds to the shadow value of relaxing the collateral constraint from the planner's perspective. This multiplier is different from μ which corresponds to the shadow value of relaxing the collateral constraint for individual households, and is a variable chosen by the government in (33). The complementary slackness condition of the social planner problem (17) is:

$$0 = \Bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{C^{SP,T}(s,L,B)}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^{SP}(s)\mathcal{B}^{SP\prime}(s,L,B),'\Bigg]\mu^{SP}(s,L,B). \tag{41}$$

As such by setting:

$$\mu(s,B,L) = \mu^{SP}(s,L,B)$$

$$1-\boldsymbol{\tau}(s,L,B) = \frac{\beta\mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(S'),C^{SP,N}(S'))\Big)\Big] + \mu^{SP}(s,L,B)q^{SP}(s)}{q^{SP}(s)u_T(C^{SP,T}(s,L,B),y^N)},$$

We can see that (38)–(40) are satisfied and therefore the two problems are equivalent.

C Data Appendix

Gross Domestic Product (GDP): *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64,-*. Corresponds to Total gross value added in all NACE activities. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Non-tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64.* Corresponds to the share of total value added produced in the following industries: public administration, wholesale and retail, construction, and real state. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64.* Corresponds to the complement of nontradable valued added as a share of total value added. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Private debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector".* Corresponds to the inverse of the net international investment position of Spanish monetary financial institutions (excluding the Bank of Spain) and other resident sectors. The data series used are 3273771 and 3273777. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Public debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector".* Corresponds to the inverse of the net international investment position of the Bank of Spain and all public administrations. The data series used are 2386960 and 3273774. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Total debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector".* Corresponds to the inverse of the net international investment position of Spain and is calculated as the consolidation of private and public positions. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011.

Risk free rate: *Bloomberg ticker GTDEM1Y Govt*, Corresponds to the average interest rate spread paid on 1 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2011.

Spread on public bonds: *Bloomberg tickers GTESP6YR Govt and GTDEM6Y Govt*, Corresponds to the difference between average interest rate paid on 6 year Spanish treasury bonds and 6 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2015. In the calibration we use data only from 1999 to 2011.

Average Maturity: *Table 5 from the Bank of Spain's economic bulletin Alloza et al. (2019), of March 2019*, Average maturity of the stock of public debt for Spain in years. Annual data from 1999 to 2011.

Nonperforming loans: *Bloomberg ticker BLTLWESP Index*, Nonperforming loans as a share of total gross loans. Annual data from 1999 to 2015.

Consumption: *Eurostat , GDP and main components (output, expenditure and income) nama_10_gdp*. Corresponds to final consumption expenditure. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2017.

Current Account: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to current account as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

Trade Balance: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to the balance of trade on goods and services as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

D Solution Method: The Government's ex-ante problem

Following the approach of Dvorkin et al. (Forthcoming), I can re-write the government's Bellman equations before the ϵ shocks are realized. From an ex-ante point of view, the shocks ϵ make the default and borrowing decisions stochastic. By taking expectations over these shocks, the decisions can be viewed as probabilistic. If we view the previously defined equilibrium as a game between the private and public sector each period, the ϵ shocks allow the government to play mixed strategies. This makes the computation of this problem using value function iteration possible. We follow this approach to write (8) from a an ex-ante perspective. That is when all the aggregate states have realized except the ϵ. For this we summarize all other exogenous state variables in $z = (y^T, y^N, \kappa, \pi)$. As mentioned in the main text we assume that L' is a finite and bounded grid with $\mathcal{J}$ elements. Denote by $F(\epsilon)$ the joint cumulative density function of the taste shocks and by $f(\epsilon)$ its joint density function. To simplify notation in what follows, the following operator to denotes the expectation of any function

$Z(\epsilon)$ with respect to all the elements of ,

$$\boldsymbol{Z} = \mathbb{E}_{\epsilon} Z(\boldsymbol{\epsilon}) = \int_{\epsilon_1} \int_{\epsilon_2} ... \int_{\epsilon_{\mathcal{J}+1}} Z(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) f(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) d\epsilon_1, ..d\epsilon_{\mathcal{J}+1} \tag{42}$$

Given this notation we have that:

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}[W(s, L, B)]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\big\{W^R(s, L, B); W^D(s, B)\big\}\Big]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\big\{\max_{L' \in \Lambda}\{u(C(s, L, B)) + \epsilon(L') + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', L', \mathcal{B}'(s, L, B))\};$$

$$u(C(s, 0, B)) - \phi(y^T) + \epsilon^{Def} + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))\big\}\Big]$$

Subject to the resource constraints:

$$C^T(s, L, B) = y^T + q(s)\mathcal{B}'(s, L, B) - (1 - \pi)B + Q(s, L', B')[L' - (1 - \delta)\mathcal{B}'(s, L, B)] - \delta \mathcal{B}'(s, L, B)$$

$$C^N(s, L, B) = y^N$$

Furthermore, if its convenient to define the following expected utility objects:

$$\Upsilon_{L,L'}(z, B) = u(C(s, L, B)) + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', L, \mathcal{B}'(s, L, B))$$

$$\Upsilon_{def}(z, B) = u(C(s, 0, B)) - \phi(y^T) + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))$$

Lemma 2. *Suppose that the $\boldsymbol{\epsilon}$ shocks follow a multivariate generalized extreme value distribution with parameters $\{m, v, p\}$ and are i.i,d over time. Where v is the scale parameter and p is the shape parameter and is set to 1. m corresponds to the location parameter and is set to $-v\gamma$ where γ is the Euler constant. Suppose that public debt L is on a grid with $\mathcal{J}$ points. Then the ex-ante value function of the government's recursive problem can be re-written as*

$$\boldsymbol{W}(z, L, B) = \Upsilon_{def} + v \log\left[1 + \left(\sum_{L' \in \Lambda} \exp\left(-\frac{\Upsilon_{def} - \Upsilon_{L,L'}}{pv}\right)\right)^p\right] \tag{43}$$

Additionally given this distributional assumptions there are closed form solutions for the ex-ante probability of default and borrowing policy functions conditional on repayment.

Proof. Given our distributional assumptions

$$F(\boldsymbol{\epsilon}) = \exp\left[-\left(\sum_{j=1}^{\mathcal{J}} \exp\left(-\frac{\epsilon_j - m}{v}\right)\right) - \exp\left(-\frac{\epsilon_{\mathcal{J}+1} - m}{v}\right)\right] \tag{44}$$

For $j \in [\![0, \mathcal{J}+1]\!]$ we denote by $F_j(\boldsymbol{\epsilon}) = \frac{\partial F(\boldsymbol{\epsilon})}{\partial \epsilon_j}$, the marginal with respect to element j^{th} element of $\boldsymbol{\epsilon}$.

$$F_j(\boldsymbol{\epsilon}) = \begin{cases} \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] \quad \exp(-\frac{\epsilon_j - m}{v}) & for\ j = 1..\mathcal{J} \\ \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] \quad \exp(-\frac{\epsilon^{def} - m}{v}) & for\ j = \mathcal{J}+1 \end{cases}$$

Using this notation ant the dropping the states (z, B) from the previously defined $\Upsilon_{L,L'}(z, B)$ functions we can compute the ex-ante policy functions of the government in close form solutions. Let the probability of default be $\boldsymbol{d}(z, L, B) = \mathbb{E}_{\boldsymbol{\epsilon}} \boldsymbol{d}(z, L, B, \boldsymbol{\epsilon})$. Note that:

$$\begin{aligned} \boldsymbol{d}(z, L, B) &= \int_{-\infty}^{\infty} F_{\mathcal{J}+1}(\Upsilon_{def} + \epsilon^{def} - \Upsilon_1, ..., \Upsilon_{def} + \epsilon^{def} - \Upsilon_{def}) d\epsilon^{def} \qquad (45) \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} + \epsilon^{def} - \Upsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \right) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \end{aligned}$$

Define $\exp(\phi_{def}) = 1 + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_h}{v})$. We can use this to rewrite (45) as:

$$\begin{aligned} \boldsymbol{d}(z, L, B) &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \exp(\phi_{def}) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\ &= \frac{1}{v \exp(\phi_{def})} \underbrace{\int_{-\infty}^{\infty} \exp\left[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \right] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def}}_{=v} \\ &= \frac{1}{1 + \left(\sum_{L' \in \Lambda} \exp\left(- \frac{\Upsilon_{def} - \Upsilon_{L,L'}}{v} \right) \right)} \qquad (46) \end{aligned}$$

Where the last equivalence uses the fact that the PDF of the generalized extreme distribution integrates to 1. Similarly, conditional on repayment, the random component $\boldsymbol{\epsilon}$ make the public borrowing decisions random from an ex-ante perspective. Given a set of current aggregate states relevant for the government, it is useful to introduce the probability of choosing an amount of public debt L' *conditional on not defaulting* as:

$$\boldsymbol{G}_{z,L,B}(L') = \mathbb{P}_{\boldsymbol{\epsilon}}\big(L' | d(z, L, B, \boldsymbol{\epsilon}) = 0\big)$$

Using the same notation as before we have that for the L' that is the j^{th} element of Λ:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{1-\boldsymbol{d}(z,L,B)} \int_{-\infty}^{\infty} F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B))v} \times \\
& \int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m}{v}) \left(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \right) \right] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j
\end{aligned}
$$

Defining $\exp(\phi_j) = \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v})$, we can simplify:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{(1-\boldsymbol{d}(z,L,B))v} \int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m}{v}) \exp(\phi_j) \right] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B))v \exp(\phi_j)} \underbrace{\int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \right] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j}_{=v} \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B)) \exp(\phi_j)}
\end{aligned}
$$

Finally this can be further simplified to:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{(1-\boldsymbol{d}(z,L,B))} \times \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})}{\sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{1}{\sum_{H\in\Lambda} \exp\left(\frac{\Upsilon_{L,H} - \Upsilon_{L,L'}}{v} \right)}
\end{aligned} \tag{47}
$$

Finally the value $\boldsymbol{W}(z, L, B)$ is given by:

$$\boldsymbol{W}(z,L,B) = \sum_{j=1}^{\mathcal{J}+1} \int_{-\infty}^{\infty} (\Upsilon_j + \epsilon_j) F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j$$

$$= \sum_{j=1}^{\mathcal{J}} \int_{-\infty}^{\infty} \frac{\Upsilon_j + \epsilon_j}{v} \times$$

$$\exp\Bigg[- \exp(-\frac{\epsilon^j - m}{v}) \Bigg(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \Bigg) \Bigg] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j$$

$$+ \int_{-\infty}^{\infty} \frac{\Upsilon_{def} + \epsilon_{def}}{v} \times$$

$$\exp\Bigg[- \exp(-\frac{\epsilon^{def} - m}{v}) \Bigg(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \Bigg) \Bigg] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def}$$

$$= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j) \times$$

$$\Bigg[\Upsilon_j + m + v\phi_j + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon_j - m - v\phi_j}{v}) \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j}_{=v\gamma} \Bigg]$$

$$+ \exp(-\phi_{def}) \times$$

$$\Bigg[\Upsilon_{def} + m + v\phi_{def} + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \exp\big[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \big] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def}}_{=v\gamma} \Bigg]$$

Where in the last equivalence we have used the fact that for all j:

$$\Upsilon_j + m + v\phi_j = \frac{(\Upsilon_j + m + v\phi_j) \int_{-\infty}^{\infty} \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Big]}{v}$$

The last step (underscored in the above equations) uses one of the integral properties of the Euler constant. We now use the fact we assumed the distribution of shocks to be mean zero, that is $m = -\gamma v$. Using the definition of ϕ_{def} one can see that:

$$\exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] = \frac{\Upsilon_{def} + v \log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})}$$

The value of the government is then given by:

$$
\begin{aligned}
\boldsymbol{W}(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j)[\Upsilon_j + v\phi_j] + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j + v\log(\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v})} + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j - \frac{v\Upsilon_j}{v} + v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j}{v})(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))} + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \frac{v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j}{v}) + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \left[\frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})}\right]\left[\sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + 1\right] \\
\boldsymbol{W}(z,L,B) &= \Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})) \qquad (48)
\end{aligned}
$$

To sum up the distributional assumptions allow us to obtain closed form solutions for the ex-ante value function (48), the policy functions for default (46), the public borrowing conditional on repayment (47), ∎

Note that the functions $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z,L,B)$ are sufficient to express all government decisions. Using the fact that the shocks are i.i.d over time, and assuming a guess $\boldsymbol{Q}$ of next price schedule functions, we can use $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z,L,B)$ to write the pricing equation of public bonds (16):

$$
Q(z,L',B') = q(z)\mathbb{E}_{z'|z}\left[\left[1 - \boldsymbol{d}(z',L',B')\right]\left[\delta + (1-\delta)\sum_{L''\in\Lambda} \boldsymbol{Q}\big(z',L'',\mathcal{B}'(z',L',B')\big)\boldsymbol{G}_{z',L',B'}(L'')\right]\right] \qquad (49)
$$

In the quantitative section we assume that the shocks are mean zero ($m = -\gamma v$). We also assume that the shape parameter p is one, therefore taste shocks are independent from each other within the period as well. The scale parameter v is calibrated to match the variance of public debt in the data.

E Numerical Solution

In this section we provide more detail about the solution methods we use to solve both the baseline and planner version of the model described in the main text. For both solutions methods we use the closed form ex-ante solutions of the government's problem described in detail in Appendix D.

Baseline. This version is solved in three steps. The first step solves the households problem while taking government policies and bond prices as given using time iteration method. The second step uses the implied policy functions of the private sector from the first step and the assumed bond schedules, and computes the closed form solutions that solve the government's ex-ante problem. Finally using private and public policy functions the schedule of private bonds is updated. Iterate until convergence in private en public policies.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (15).
- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}(z, L, B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}(z, L', B')$.
- Construct the implied transfer function $\boldsymbol{T}(z, B, L, L')$ using the government budget constraint (5).
- Taking all these functions as given find the optimal private borrowing $B'(z, L, B, L')$ and consumption decisions $C'(z, L, B, L')$ using the private sector Euler equation (**??**) to find the binding and non binding states.
- Given households optimal policies $B'(z, L, B, L')$, and $C'(z, L, B, L')$, and the guess schedule of public bonds $\boldsymbol{Q}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the government using (46) and (47). Update the government policy functions.
- Compute the government ex-ante value function $W(z, L, B)$ using (48).
- Update the schedule of public bonds $\boldsymbol{Q}(z, L', B')$ using (49).
- Repeat until convergence in $W(z, L, B)$,$B'(z, L, B, L')$, and $C'(z, L, B, L')$, and $\boldsymbol{Q}(z, L', B')$ is achieved.

Social planner. This version is solved in three steps. The first step finds optimal private borrowing on a grid (*grid search method*) given an initial guess of public for each potential default and public borrowing decisions. The second step uses this optimal private borrowing policy and the assumed bond schedules to computes the closed form solutions for public borrowing and default and the value function. Finally using private and public borrowing policy functions the schedule of private bonds is updated. Iterate until convergence in private borrowing policies and the value function is achieved.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (15).
- Construct a grid of potential private borrowing choices B'.
- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}^{SP}(z, L, B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}^{SP}(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$.
- Taking all these functions as given find the optimal private borrowing $B^{SP\prime}(z, L, B, L')$ in the finite grid discarding all choices that violate the credit constraint (**??**) for each potential public borrowing and default decision.
- Given optimal private borrowig policy $B^{SP\prime}(z, L, B, L')$ and the guess schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the planner using (46) and (47). Update the planner public borrowing and default policy functions.
- Compute the ex-ante value function $W^{SP}(z, L, B)$ using (48).
- Update the schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$ using (49).
- Repeat until convergence in $W^{SP}(z, L, B)$,$B^{SP\prime}(z, L, B, L')$, and $\boldsymbol{Q}^{SP}(z, L', B')$ is achieved.

F Policy functions of private and public debt

To shed light on the workings of the model, this section shows an analysis of the policy functions for public and private debt accumulation. Both variables are functions of the exogenous shocks of the model and of the initial portfolio composition. To fix ideas, this section will first show how the accumulation of private and public debt varies with respect to the two main exogenous shocks, income and financial shocks. Then, I will show how both types of debt issuances vary with the endogenous states, the initial level of total debt and end-of-period public debt. Since the government acts first, the end-of-period private debt is a function of both the beginning of period debt of the country and the newly issued public debt. Considering the best response from the households, the government chooses the issuance of public debt optimally. For simplicity, the initial level of public debt has been set to zero in all the policy function plots, making all initial debt private. Nevertheless, all the implications follow through with a strictly positive level of initial public debt. Unless otherwise specified all debt levels are expressed as a share of mean output at the ergodic distribution.

Policy functions of private debt: Figure 6 depicts the optimal private debt accumulation as a function of the income and financial shocks. Panel (a) shows end-of-period private debt as a function of the endowment of tradable shocks, for the mean value of κ and π_t and for two possible values of initial debt. Panel (b) plots end-of-period private debt as a function of the financial shock, for the mean value of y^T, again for two possible values of initial debt.

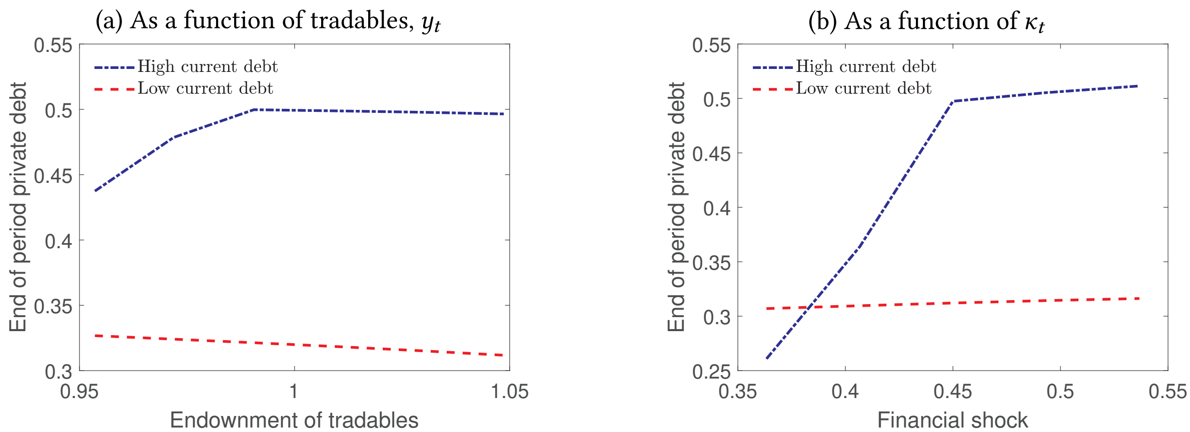

Figure 6: Policy function for private debt relative to the exogenous states

The figure shows that households' borrowing choices are most sensitive to the exogenous shocks when the households are facing a binding credit constraint. If the initial level of debt is low, represented by the dashed line in the plot, end-of-period private debt increases only slightly when income is low or the borrowing capacity is larger (smaller y^T or higher κ). However, if the current debt is high enough, households borrow up to their credit constraint. As a result, increases in the endowment of tradables or the value of the financial shock (higher y^T or higher κ) are met with equivalent increases in private borrowing.

Focusing now on the endogenous states, Figure 7 plots the law of motion of end-of-period private debt as a function of the initial level of debt, panel (a), and to end-of-period public debt, panel (b). To help visualize the importance of the credit constraint, the total borrowing capacity of the private sector (debt limit) is plotted alongside the policy functions. In both panels, the exogenous shocks are kept constant. In the first panel, the level of end-of-period public debt is set at zero, and in the second panel, the starting level of debt is one standard deviation above the mean.

Panel (a) shows that for low levels of initial debt, the credit constraint does not bind, and end-of-period private debt increases with current total debt. The change in the sign of the slope of the policy function indicates the point at which the credit constraint is satisfied with equality. Beyond this point, higher levels of initial debt imply a lower level of tradable consumption. This in turn lowers the price of nontradables p^N and further restricts the borrowing capacity of the economy. This is therefore an illustration of the Fisherian debt deflation mechanism discussed in the previous section. As a result, similar policy functions can be seen Bianchi (2011) and Bianchi and Mendoza (2018).

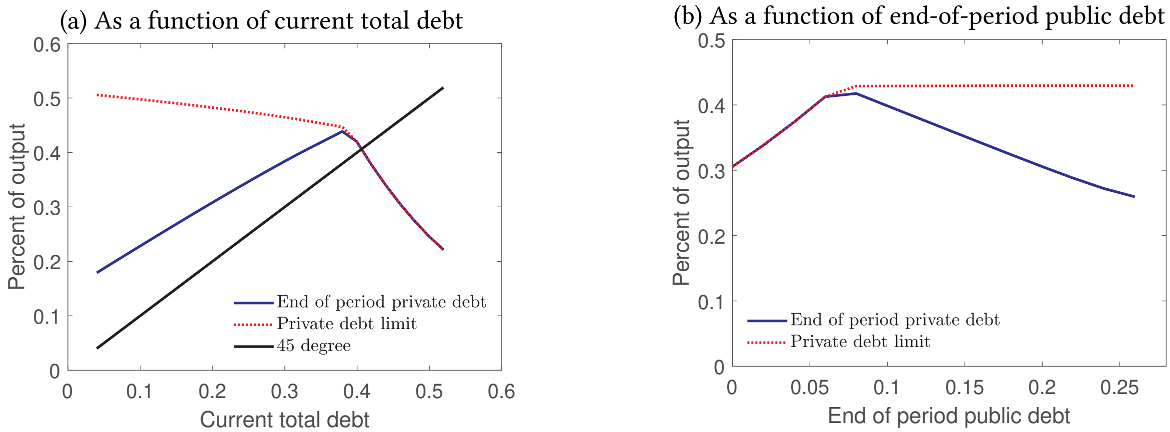

Figure 7: Policy function of private debt relative to the endogenous states

In contrast, panel (b) depicts the private sector response to the government's end-of-period debt and is novel to this paper. Low levels of end-of-period public debt imply a reduction in the fiscal transfer received by the household. At the plotted values, without substantial government assistance (above 8% of output), private borrowing will be constrained. Given the financial amplification mechanism described before, in this constrained area, higher government borrowing increases the consumption of tradables, the price of nontradables, the borrowing limit of the private sector, and private borrowing. This process comes to a halt once government assistance is large enough to ensure that the households will not borrow up to their limit. Further government borrowing continues to increase the transfer received by the households, but they now respond by borrowing less. For these states, private and public debt are substitutes.

Figure 8 shows the optimal public debt accumulation policy as a function of the income (panel (a)) and financial shocks (panel (b)). When initial debt is low, or when the endowment and the financial capacity κ are high, the optimal end-of-period debt remains mostly constant around a positive value. As in other models with multiple maturity assets, such as Arellano and Ramanarayanan (2012), long-term bonds provide rollover benefits relative to the short-term bonds. Long-term bonds provide more insurance against income fluctuation, which facilitates consumption smoothing. As a result, the government finds it optimal to always have a strictly positive level of public debt, even when the households are unconstrained. Since private and public debt are substitutes in these states, the government can issue debt at low spreads as long as total public debt remains low.

Policy functions of public debt: The government considers the household's best responses when choosing the level of public borrowing. Since the choice of public debt is also affected by the taste shock drawn, I now plot the expected level of end-of-period public debt conditional on repayment. All values are plotted as a share of output. I start by showing public debt as a function of the income and financial shocks and then show how it changes with initial debt.

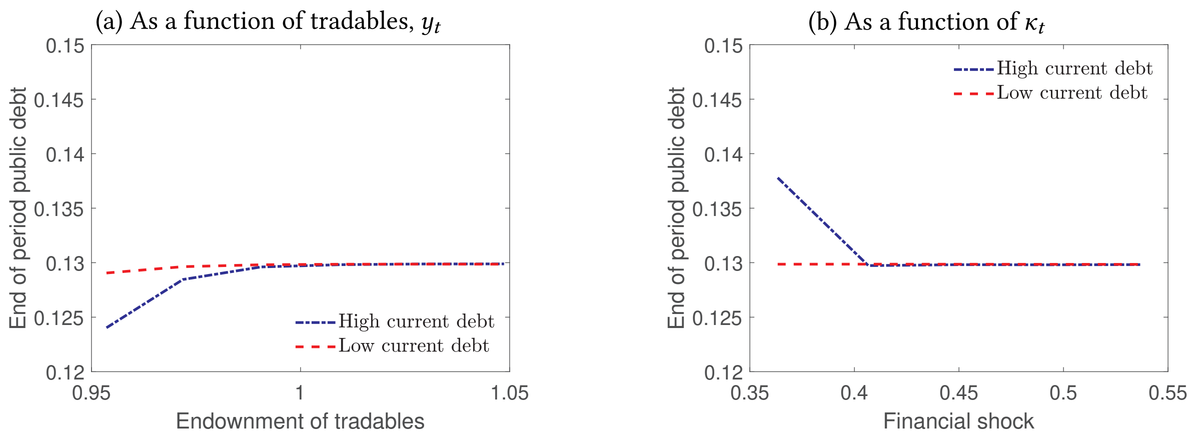

Figure 8: Policy function of public debt relative to the exogenous states

In contrast, when total debt is high, end-of period public debt varies differently with each type of exogenous shock. A low tradable endowment implies higher default risk and higher spreads, and therefore public borrowing decreases. Instead, an adverse financial shock (low κ) means that private borrowing is more likely constrained. Public debt in these cases has the twofold beneficial effect detailed in the previous section. Public debt allows for higher consumption when the households are constrained. This relaxes the credit constraint by depreciating the real exchange rate and allows for higher private borrowing. Thus, higher end-of-period public debt is desirable.

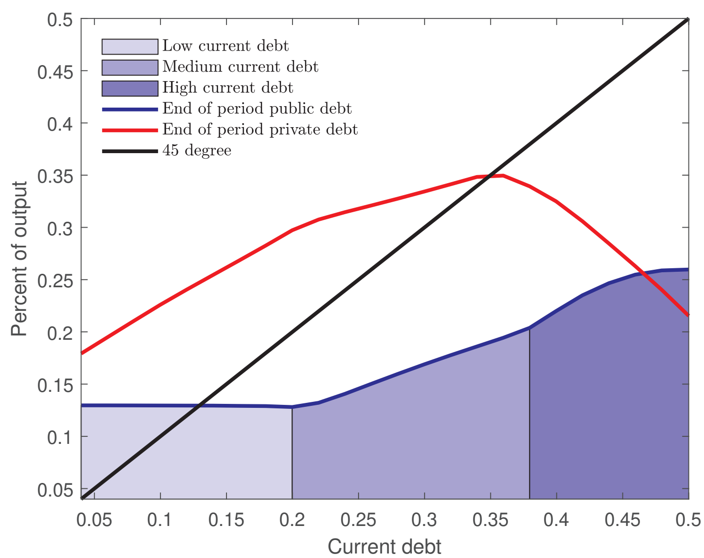

Figure 9: Expected end-of-period public and private debt as function of initial debt

Finally, Figure 9 shows the expected level of end-of-period public debt as a function of the current level of debt (blue line). To help visualize the situation of the households, the figure also shows the expected end-of-period private debt. All values are plotted as a share of output, and all exogenous shocks and the initial level of public debt are kept at constant values. Depending on the initial level of debt, three types of responses in terms of public debt are possible.

When the initial level of debt is low, issuances of public debt are kept relatively constant and low. Public debt is issued here because of its rollover benefits. Long-term debt allows the government to partially insure the households against transitory fluctuations in all exogenous shocks. Private debt is increasing in initial debt while public debt is almost constant. If the initial debt is large enough, however, the constraint for the private sector will bind if the government end-of-period debt is zero. At these medium levels of initial debt, households are not expected to face a credit constraint on average. The government is expected to transfer enough resources to the household so that the constraint will not bind. Consequently, private and public debt levels are increasing in the initial level of debt. The slope of private debt accumulation is smaller than in the previous case because households will be constrained in some states. Finally, if the initial level of debt is very high, it is never optimal to provide a large enough bailout that would prevent the households from facing a binding constraint. In these cases, issuances of public debt are at their highest. This is because in these states, public debt has a significant positive impact on the private borrowing capacity. The higher the initial level of debt, the more constrained the households are expected to end up, even after receiving transfers, and therefore the lower the level of end-of-period private debt.

Comparison with the socially planned economy: A social planner who controls the issuance of both types of assets would have similar policy functions. In this subsection, we compare those policies to those presented in the baseline model discussed above.

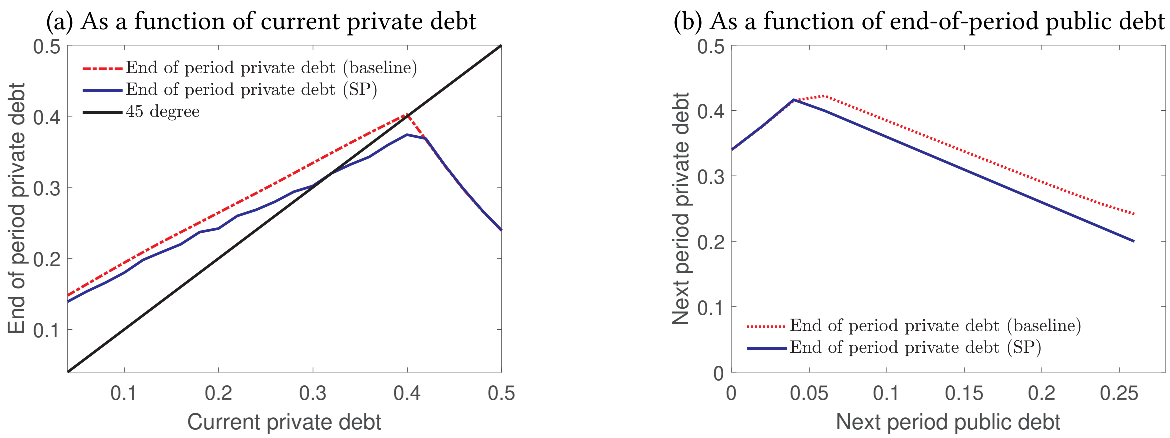

Figure 10: Policy function of private debt, baseline and SP

Figure 10 compares the evolution of end-of-period private debt in the baseline and socially planned economy as a function of the initial stock of private debt (panel (a)) and end-of-period public debt (panel (b)). In both panels, overborrowing in the baseline economy is present only when the constraint does not bind. When the constraint binds, private borrowing is pinned down by the resource constraints, and therefore there is no room for disagreement between the models. The sources of private overborrowing in both panels, however, are different. In the first panel, households overborrow for low levels of initial private debt because they do not internalize the marginal effect of their debt on the probability of facing a binding constraint next period. This figure is common to models of private overborrowing with a credit constraint that is increasing in the price of nontradables, such as Bianchi (2011) and Bianchi and Mendoza (2018). In contrast, the second panel is novel to this paper. Overborrowing is now caused by a smaller private borrowing response to government issuances of public debt. Unlike the planner, the households do not internalize that higher private debt increases the probability of sovereign default next period. Thus, individual households substitute less private debt for the same increase in public debt relative to the planner.

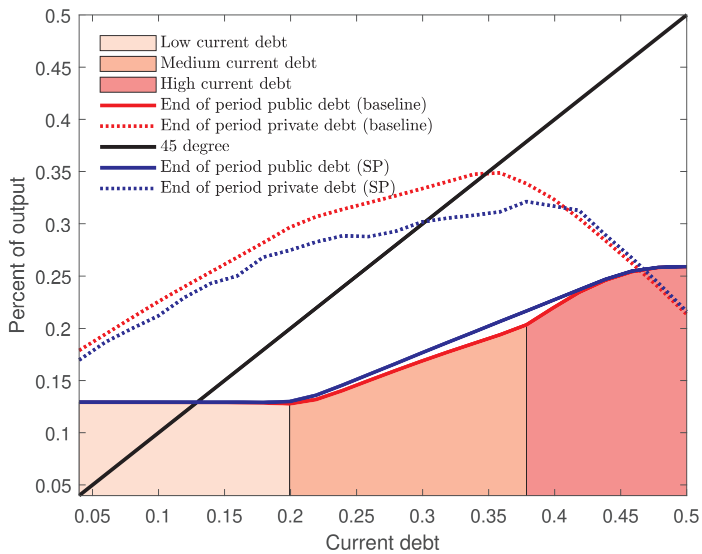

Figure 11: Expected end-of-period public and private debt as function of initial debt

Figure 11 compares the expected optimal level of public borrowing, conditional on repayment, in the baseline and socially planned economies as a function of the initial debt. As before, the households' private debt responses are plotted alongside the planners'. The figure also shows private overborrowing in the baseline model when the constraint does not bind. Public borrowing is higher in the planned economy when initial debt is small or medium. In these areas, the planner internalizes that it is approaching its borrowing capacity on the private bond and substitutes some of that borrowing with the public bond. The government in the decentralized economy would like to implement the same policy but does not control the issuances of the private bond. Correctly predicting that the household will not reduce private borrowing at the same rate as a planner would, the government decides to issue less public debt. The differences in public borrowing are, however, quantitatively smaller than the differences in private borrowing. As shown in the next section, when we compare the ergodic distributions, the small differences in public borrowing will not compensate for the fact that the baseline economy hits the credit constraint more often than the planned one. Consequently, the government must more frequently relieve the households by issuing public debt. When the constraint is expected to bind, the two economies mostly coincide.[50]

I also compare the evolution of the expected interest rate spreads paid on public debt in both economies conditional on repayment. Figure 12 plots the spreads as a function of the initial debt. The figure is computed at the same states as in Figure 11. The spreads peak when the debt enters the high debt zone. The shape of this plot shows that the interest rate spreads are mostly driven by the evolution of total end-of-period debt. Default is more likely in a more indebted economy. Up until the moment the constraint binds, both private and public debt are increasing with initial debt. Beyond this point, however, the private sector deleverages at a rate that outpaces the increase in public borrowing. As a result, total indebtedness decreases. This reduces the probability of default and the spread. In all cases, the spreads are higher in the baseline economy. This is the case even though Figure 12 shows that for medium or high levels of debt, the planner is expected to issue more public debt. The gap in interest rates exists because total debt is higher in the baseline economy as a result of household overborrowing. Anticipating this, foreign lenders demand a higher spread from the government.

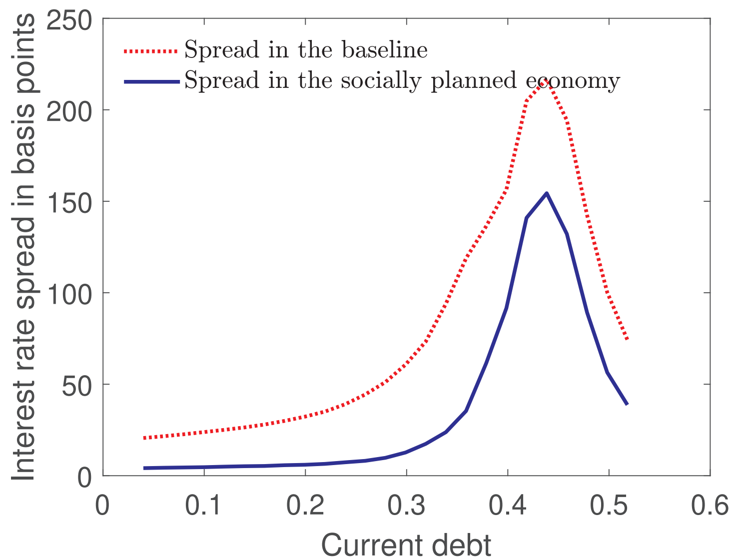

Figure 12: Expected spreads on public debt as function of initial debt

G Untargeted business cycle properties

This subsection evaluates the model's quantitative performance by comparing untargeted moments from the data with moments from the model at the ergodic distribution. I compute the model's moments by simulating the exogenous processes for 10,000 periods and eliminating the first 500 observations. The moments from the data are computed with annual data for the sample period 1999-2017.

[50]The small amount of underborrowing in the baseline economy in this context is caused by fact that the planner faces a more favorable price schedule and therefore can relax the constraint a little bit more.

The longer sample period is chosen to avoid small sample bias. Similar results are obtained when restricting the sample to the period 1999-2011. In Table 4, real GDP is equated with output, and consumption corresponds to total final consumption expenditure and is measured in real terms. GDP and consumption data are detrended. The current account and trade balance are computed as a percentage of GDP. All data are from Eurostat, and additional descriptions of the sources can be found in Appendix C.

Table 4 compares the unconditional second moments in the Spanish data with their baseline model counterparts at the ergodic distribution. The model successfully captures the volatility of consumption, of the current account and of the trade balance, and overestimates the volatility of output. Nevertheless, the model correctly predicts that the volatility of output will exceed the volatility of consumption. This contrasts with traditional sovereign default models where the opposite is true.[51] This suggests that explicitly modelling international private debt is important to simultaneously achieve a volatility of consumption and net capital flows consistent with the Spanish data. Table 4 also computes correlations between output and the other business cycle statistics. The model correctly predicts the sign of all the correlations.

Table 4: Untargeted business cycle statistics

Statistic	Data	Calibration
Volatility		
Output	.032	.062
Consumption	.031	.037
Current account	.041	.046
Trade balance	.034	.040
Correlations		
Output - Consumption	.97	.99
Output - Current account	-.59	-.91
Output - Trade balance	-.54	-.94
Output - Spread on public debt	-.46	-.10
Public debt - Spread on public debt	.53	.28

Note: Output corresponds to real gross domestic product and consumption to real final consumption expenditure, and both series are detrended. Current account and trade balance are measured as a percentage of output. Public debt corresponds to the international investment position of the public sector. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. For additional details, see Appendix C.

[51] Neumeyer and Perri (2005) find that consumption is more volatile than output in emerging economies whereas the opposite is true in advanced economies. Spain is listed by the International Monetary Fund (IMF) as an advanced economy.

H Particle filter method

This appendix details the particle filter method used to conduct the counterfactual exercises of section 5. It follows closely the approach presented in Bocola and Dovis (2019). As noted in the main text, the state space representation of the model is:

$$\boldsymbol{Y}_t = g(\boldsymbol{S}_t) + e_t \tag{50}$$

$$\boldsymbol{S}_t = f(\boldsymbol{S}_{t-1}, \varepsilon_t). \tag{51}$$

In this formulation, the first equation captures the measurement error e_t, a vector of i.i.d. normally distributed errors with mean zero and a diagonal variance-covariance matrix Σ. The vector of observable, $\boldsymbol{Y}_t$, includes average private and public debt as share of GDP, detrended tradable output, the share of nonperforming loans, and interest rate spreads on public bonds. The second equation describes the law of motion of the baseline model state variables $\boldsymbol{S}_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$. The vector ε_t corresponds to the innovations in the AR 1 process of the three structural shocks $[y^T_t, \pi_t, \kappa_t]$.

$$\begin{aligned}
y^T_t &= \exp(\rho^y \ln y^T_{t-1} + \varepsilon^y_t) \\
\pi^T_t &= \exp((1-\rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon^\pi_t) \\
\kappa_t &= (1-\rho^\kappa)\bar{\kappa} + \rho^\kappa \kappa_t + \varepsilon^\kappa_t
\end{aligned}$$

Since we did not observe any defaults in the time periods considered we use the repayment policy functions to compute the transitions. Using the notation of section 3 the evolution of private and public debt in the first exercise is then:

$$\begin{aligned}
L_{t+1} &= \mathcal{L}'(s_t, L_t, B_t) = \mathcal{L}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t) \\
B_{t+1} &= \mathcal{B}'(s_t, L_t, B_t) = \mathcal{B}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t)
\end{aligned}$$

In the first exercise all taste shocks are set to to zero. In the second exercise, we still focus on repayment but this time we select the taste shocks to match public debt exactly to it's data counter part and let private debt the respond endogenously:

$$\begin{aligned}
L_{t+1} &== L^{data}_{t+1} \\
B_{t+1} &= \tilde{B}'(y^T_t, \pi_t, \kappa_t, L_t, B_t, 0, L^{data}_{t+1}, \tilde{T}(s_t, L_t, L^{data}_{t+1}))
\end{aligned}$$

These transitions are summarized in function $f(\cdot)$ for each exercise. Similarly we can generate numerical solutions to compute the model counterparts to debt to output ratios and the public spreads and summarize them in $g(\cdot)$.

Let $\boldsymbol{Y}^t = [\boldsymbol{Y}_1, ..\boldsymbol{Y}_t]$, and denote by $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$ the conditional distribution of the state vector given a history of observations up to period t. In general there is no analytical solution for the density function $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$. The particle filter method approaches this density by using the fact that the conditional density of $\boldsymbol{Y}_t$ given $\boldsymbol{S}_t$ is Gaussian. It consists of finding a set of pairs of states and weights $\{S_t^i, \tilde{w}_t^i\}_{i=1}^N$ such that for all function $h(\cdot)$:

$$\frac{1}{N}\sum_{i=1}^{N} h(\boldsymbol{S}_t^i)\tilde{w}_t^i \underset{a.s}{\longrightarrow} \mathbb{E}[h(\boldsymbol{S}_t)|\boldsymbol{Y}^t].$$

This approximation can then be used to obtain the weighted average path of the state vector over the sample. The states selected S_t^i are called particles and $\tilde{w}_t^i$ corresponds to their weight. To construct this set we follow the algorithm proposed by Kitagawa (1996).

Step 1: Initialization Set $t = 1$ and $\forall i$ $\tilde{w}_0^i = 1$, draw S_0^i from the ergodic distribution of the baseline model.

Step 2: Transition For each $i = 1..N$ compute the state vector $\boldsymbol{S}_{t|t-1}^i$ given vector $\boldsymbol{S}_{t-1}^i$ by drawing innovations for the fundamental shocks from the calibrated distributions and using the policy functions summa zed in $f(\cdot)$.

Step 3: Filter Assign to each particle $\boldsymbol{S}_{t|t-1}^i$ the weight

$$w_t^i = p(\boldsymbol{Y}|\boldsymbol{S}_{t|t-1}^i)\tilde{w}_{t-1}^i$$

where $p(\boldsymbol{Y}|\boldsymbol{S}_{t|t-1}^i)$ is a multivariate Normal density.

Step 4: Rescale & Resample Rescale the weights $\{w_t^i\}$ so that they add up to one, and denote these new weights $\{\tilde{w}_t^i\}$. Sample with replacement N values of the state vector from the set $\{\boldsymbol{S}_{t|t-1}^i\}$ using $\{\tilde{w}_t^i\}$ as sample weights. Denote this draws $\{\boldsymbol{S}_t^i\}$. Set $\tilde{w}_t^i = 1$ $\forall i$. If $t < T$ set $t = t + 1$ and go to Step 2. Otherwise stop.

In both exercises, it is assumed that measurement error associated with y_t^T and π_t is zero, as such the variance of the measurement error is set to zero for these variables in the measurement equation and the innovations ε_t^y and ε_t^π are set to match the empirical counterparts exactly. Since κ_t has no empirical counterpart, the algorithm help us find the most likely path using its effects on debt aggregates and the spreads. As in Bocola and Dovis (2019) the filter is tuned with $N = 100,000$.

Equipped with a set of particles and weights $\{S_t^i, \tilde{w}_t^i\}_{i=1}^N$ and the policy functions summarized in $g(\cdot)$ one can approximate the model predictions plotted in figures 4 and 13. As an example for all

$t = [2008, ..., 2015]$ the predicted interest rate spread, $spr_t^{Baseline}$ at time t is:

$$spr_t^{Baseline} = \sum_i^N \tilde{w}_t^i [\frac{\delta - \delta \boldsymbol{Q}(S_t^i)}{\boldsymbol{Q}(S_t^i)} - r]$$

Similar weighted averages are computed for the debt to output ratio and the exogenous shocks. When computing objects for the social planner the function $g^{SP}(\cdot)$ is used instead.

I Second counterfactual

In section 5.2, I compare the responses of the baseline and socially planned economies to the shocks that affected Spain during the 2012 crisis. I show that the 2012 spike in the spread of public debt would have been completely avoided if a planner had managed public and private borrowing optimally. The reduction in spreads is, however, the result of less issuances of both public and private debt. To disentangle how much of the difference is caused by lower public borrowing and how much is caused by excessive private debt, in this appendix I conduct a second counterfactual exercise. Taking advantage of the probabilistic framework of the model, I can select the taste shocks ϵ_t such that the path of public debt coincides exactly with the one observed in the data in both the baseline and planned economies. With the path of public debt restricted to the data values, the policy functions are used to compute the other endogenous series. The particle filter is then applied to back out the implied financial shock and the filtered endogenous evolution of private debt and the sovereign spread. As before, I then feed this sequence of exogenous shocks into the planner policy functions to compute the counterfactual private debt, and spreads. Finally, I use the planner's policies to compute the optimal taxes on borrowing that could have decentralized these dynamics. The results of the second exercise are presented in Figure 13.

Positive counterfactual: The model once again predicts a drop in private debt of 20% of GDP, close in magnitude to the one observed in the data. Overall, private debt is around 5% below what is observed in the data for most of the period. The spread on public debt increases from close to zero in 2008, peaks in 2012, and then falls from 2013 onward. The magnitude of the increase between 2008 and 2012 is not the same in the baseline and the data, however, the model experiences a larger rise in 2012. The small mismatch in private debt and the larger spread are both consequences of the requirement to fit public debt exactly in this exercise. Nevertheless, the baseline model can still replicate the patterns of interest.

Normative counterfactual: Finally, I compare the evolution of the data and the socially planned economy. Private indebtedness in the planned economy is still lower than in the baseline and the data. In this exercise, the data on the evolution of public debt impose that the main bailout takes place

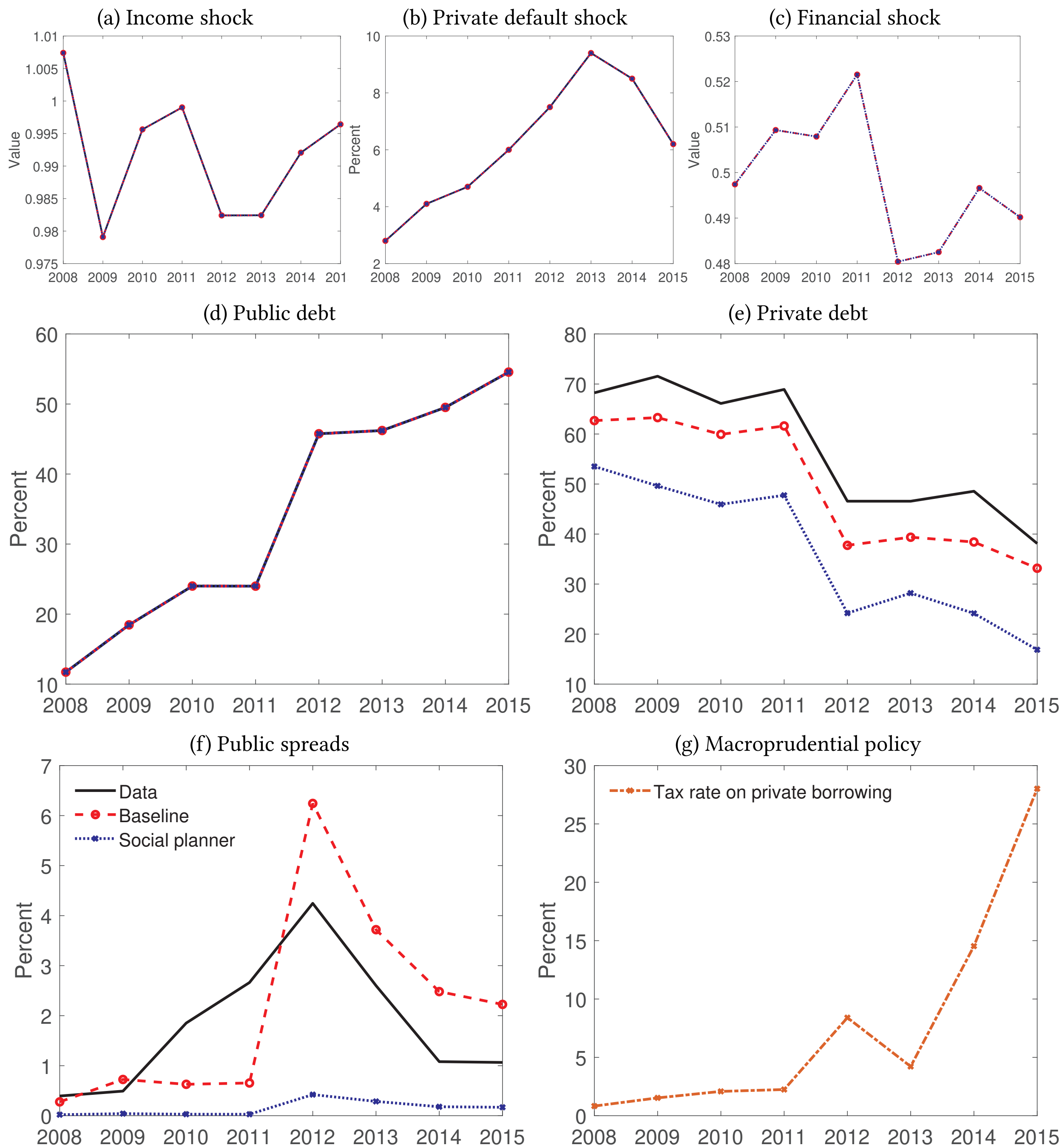

Figure 13: Evolution of debt, taxes, spreads, and exogenous shocks, 2008–2015: data and models

Note: Model simulations are obtained by feeding into the model observed income shocks, nonperforming loans, and taste shocks to match exactly the evolution of public debt. The most likely path of financial shocks is computed using the particle filter. Private debt and spreads are filtered weighted averages. Both debt series are expressed as a percentage of output, while nonperforming loans are expressed as a percentage of gross loans. Taxes and interest rate spreads are expressed in percentages. Data sources can be found in Appendix C, while details on the particle filter can be found in Appendix H.

in 2012. As a result, the public spread in the planned economy also peaks in 2012. The peak value is .4%, or 3.8 percentage points below the spread observed in the Spanish data. This is the lower bound estimate of the increase in the severity of the sovereign debt crisis caused by excessive private debt. It should be restated here that this estimate is obtained while keeping the paths of public debt at their data values. The reduction in the spread is therefore not a consequence of lower public borrowing but of the only other endogenous factor, private debt. In the planned economy, the lenders internalize that the regulator will pair the increase in high public debt with high taxes on private debt, which is 8% on average during the period. This leads to a reduction in private debt and thus reduces the probability of a sovereign default in the future.

J Comparison to nested models

This appendix compares the model to the two existing models of international borrowing that are nested within it. This comparison is useful to illustrate the role that private and public debt play for the quantitative properties of the model. The results of this comparison are presented in table 5. Throughout the comparison, I use the calibrated parameters presented in section 4 to solve all models. The welfare gains are computed as in section 5 in terms of equivalent consumption.

Table 5: Comparison relative to nested models

Related model			Bianchi (2011)	Bianchi (2011)	Hatchondo and Martinez (2009)	Arellano (2008)
Average	Baseline	Planner	Laisse-faire	Efficient	$\delta = .14$	$\delta = 1$
Private debt as a % of output	42	37	44	43	-	-
Public debt as a % of output	15	12	-	-	13	15
Spread in percent	.45	.034	-	-	0.08	.28
Probability of a financial crisis	2.5	.10	6.4	1.8	-	-
Probability of sovereign default	.46	.030	-	-	0.04	.35
Welfare gain relative to Baseline	-	.41	-6.4	-6.2	-2.4	-2.7

Note: Simulated moments computed at the calibrated parameters for different versions of the model. The first two columns correspond to the baseline and socially planned version calibrated in section 4. The third and fourth column correspond to a version of the model with no public debt that coincides with the model presented in Bianchi (2011). The third column correspond to the decentralized case where competitive household choose their individual level of borrowing. The fourth column corresponds to the case where a benevolent social planner makes the aggregate borrowing decision. The fifth and sixth column correspond to a version of the model with no private debt. In the fifth column the public debt is long term and has the same maturity as in this paper. In the last column, the government only has access to one period debt.

As in the last part of section 5, I compute a version of the model without sovereign debt. In other words, a version of the model with only international private debt subject to a collateral constraint. This corresponds exactly with the model presented in Bianchi (2011) and its properties are on the third and fourth columns of table 5. Due to the pecuniary externality in private debt, two versions of this model exist, a decentralized and a constrained efficient. Table 5, shows that both versions of the model exhibit a higher level of private debt than even the baseline version of the model. Nevertheless, the international debt position is significantly improved due to the absence of public debt. One can also see, that in the absence of the public debt instrument financial crises are significantly more frequent. This is consistent with the fact that we know that the government will use public debt to move the households away from the constraint to avoid crises. Losing access to the long-term debt instrument, also increases the exposure to rollover risk. These two reason, the higher frequency of crises and the increase exposure to roll-over risk, explain the welfare loses showed in the last raw.

Similarly, I also compute a version of the model without private debt. This corresponds to a two goods version of the standard Eaton and Gersovitz (1981) framework. In the fifth column, I maintain the long-term maturity of the public bond as in Hatchondo and Martinez (2009). The effect of this is that the level of public debt is between the baseline and socially planned versions of the model. The same is true for the interest rate spread and the probability of default. Consequently, we can see that it is only when private debt is issued by competitive agents, as in the baseline, that the economy is exposed to higher spreads and more frequent defaults. Nonetheless, losing access to the risk free bond also leads to a significant reduction in overall indebtedness which is costly for impatient households. The overall effect on welfare ends up being negative. Finally, in the sixth column I give the government access to only a one period bond as in Arellano (2008). The results are similar to those of the previous column, except that debt levels and spreads are higher, and defaults are more frequent. Consequently, the welfare loses are higher.