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Kanninen, Ohto and Böckerman, Petri and Suoniemi, Ilpo

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Ohto Kanninen*, Petri Böckerman**, and Ilpo Suoniemi***

* Labour Institute for Economic Research LABORE. E-mail: ohto.kanninen@labour.fi

** University of Jyväskylä, Labour Institute for Economic Research LABORE and IZA Institute of Labor Economics. E-mail: petri.bockerman@labour.fi

*** Labour Institute for Economic Research LABORE. E-mail: ilpo.suoniemi@gmail.com

Abstract

We propose a method of studying the value of insurance. For this purpose, we analyze well-being of the same individuals, comparing sick and healthy years in German panel survey data on life satisfaction. To impose structure on the income–well-being gradient, we fit a flexible utility function to the data, focusing on the differences in marginal utility in the sick and the healthy state, by allowing for a "fixed cost of sickness". We find that marginal utility of income is higher in the sick state. We use our estimates to gauge the value of sickness insurance for Baily-Chetty type optimal policy calculations. We also show that the income–well-being gradient has steepened over time in Germany and we use the fitted model to characterize this change.

Keywords: life satisfaction, state dependence, risk aversion, social insurance, optimal benefits, sickness absence

JEL classification: C13, H55, I13

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1. Introduction

Social insurance that protects workers against unemployment, sickness, and old-age accounts for a significant share of government expenditures. The potential negative moral hazard effects of these programs on workers' labor supply have been the topic of an ever-growing literature (see e.g., the reviews in Chetty and Finkelstein, 2013; Schmieder and Von Wachter, 2016).

The optimal social insurance – characterized by the Baily-Chetty formula – balances the moral hazard cost of providing insurance with its value to the insured. The value of insurance is captured by the marginal rate of substitution (MRS) between in-work and out-of-work consumption. However, there is less evidence on the value of social insurance relative to its costs. The main reason for this is that social insurance programs, like sickness insurance (SI), are mandated, not leaving opportunities to study the benefits through choices.

The traditional approach in the literature, implemented by Gruber (1997) in the context of unemployment (UI), is to focus on consumption smoothing. The estimated drop in consumption in response to an adverse event can be scaled by workers' risk aversion to get an estimate of the value of providing additional insurance against that event, assuming state independent utility.

Landais and Spinnewijn (2020) introduce an alternative method of estimating the value of unemployment insurance (UI) using consumption data. They consider the difference in marginal propensity to consume (MPC) when unemployed vs. employed, which allows them to identify the difference in shadow prices to smooth consumption in the respective states.

In this paper, we explore a third approach by using panel data on subjective well-being as a proxy for utility, allowing for state-dependence in marginal utility. We use German panel data on life satisfaction and income. We first study the raw income–well-being gradient in the sick and the healthy states, and then impose structure on the data by fitting a flexible utility function. The benefit of insurance follows from the difference in marginal utility in the employed and sick states, which we identify by focusing on individuals who switch between states (i.e., sickness and health). We compare our results to those of Finkelstein et al. (2013) and use the results in Baily-Chetty type calculations to characterize the optimal policy rule. We focus on the sickness insurance, an important policy priority, with sickness causing a loss of nearly 10% of annual working days in some OECD countries (DICE Database, 2017) and functioning as a important pathway to disability pension.

In other words, one can infer risk preferences and state dependence empirically by studying individuals' revealed preferences (e.g., Cohen and Einav, 2007) or by analyzing subjective well-being in different states (Finkelstein et al., 2009; Mata et al., 2018). We take the latter approach.

In our empirical utility function fits, we allow for a horizontal shift parameter corresponding to a "fixed cost of sickness", which affects the utility gain of consumption smoothing, and could in principle be of either sign. If high, it can potentially by a major determinant of the shift in relative risk aversion and marginal utility in the sick state compared to the employed state. We also allow for level shifts in utility between states. The level shift does not affect marginal utility and thus policy. Our method leads to a domain-focused view on risk. A mapping of net income on life satisfaction or the income–well-being gradient for individuals who alternate between sickness states, as shown in Figure 1, immediately reveals two qualitative and systematic patterns concerning the sickness absence state. First, life satisfaction in the sick state is lower conditional on income. Second, assuming that the relationship is causal, marginal utility in the sick state is higher (i.e., a positive state dependence). Consequently, there is also a convergence in life satisfaction between the two states at higher levels of income.

[Figure 1 here]

Our analysis assumes that life satisfaction approximates utility. The fitted parametric form utilizes minimal restrictive assumptions and allows us to identify the relevant parameters of the utility function, including the fixed cost of sickness term. The estimates show that the fixed cost is a parsimonious way to model state dependence in marginal utility and to capture the salient features in Figure 1, except at very low incomes. We employ cross-sectional and panel variation when estimating the utility function and the estimates are biased by omitted variables such as education or reverse causality from well-being to income. However, the estimated cost of sickness is identified using within-individual variation in the sickness state. For policy purposes, we are interested in how sickness is associated with shifts in the utility function, compared to the utility function of the same individuals in full health. Even if sickness is caused by e.g., other shocks in life that also directly affect marginal utility, a sickness insurance policy should be optimized for the new marginal utility, independent of the mechanisms causing the change.

The empirical findings support our method of incorporating the standard utility theory and insights from recent behavioral economics research. The estimate of the utility function under the healthy state conforms to the standard utility function, again, with possible endogeneity bias. However, allowing for and empirically finding a significant difference in the utility curve for those who are on sick leave emphasizes the importance of state dependence in risk. A limitation of our analysis is the fact that we can only identify sickness spells at the annual level and are not aware whether life satisfaction is reported during the sickness spell or not. Such a limitation leads to measurement error, whose importance we study with alternative specifications.

Economic theory argues that the optimal policy rule in insurance schemes constitutes a trade-off between benefits (i.e., the consumption smoothing effect) and costs (i.e., due to hidden actions, the moral hazard effect). The canonical Baily-Chetty formula is based on a state-independent utility function (Baily, 1978; Chetty, 2006, 2008). We relax this restriction and show that the standard measure of relative risk aversion is empirically lacking and that the fixed cost of sickness explains a substantial part of the effect of sickness on marginal utility. The last point also implies that social insurance schemes must be calibrated according to our best empirical understanding of utility and risk in each policy domain.

Our results are relevant for the core features of sickness absence policies. Three points stand out. First, we contribute to the literature on state dependence by showing that marginal utility is higher in the sick state, conditional on income. Traditionally, the focus has been on consumption smoothing because consumption and the curvature of the utility function are the sufficient statistics in the received view to characterize the benefit of the insurance. But a more subtle understanding of marginal utility of the sick versus employed states is required. Second, we establish that contrary to conventional wisdom, relative risk aversion in the employed state increases with income. This stems from the negative estimated horizontal shift or "institutions" parameter. Third, the effect of the fixed cost of sickness is the main reason to insure against sickness for lower income individuals due to the baseline protection given by the institutions.

Concerning sickness, the assumption of state-independence of the utility function has been challenged by previous contributions. The empirical literature provides conflicting results concerning whether the marginal utility is higher or lower for the sick population (see Finkelstein et al., 2009, p. 117; Viscusi and Evans, 1990; Viscusi, 2019). The empirical literature on insurance choice in economics along with the psychological literature has found that risk taking is highly domain-specific (Einav et al., 2012; Weber et al., 2002), which is consistent with state-dependent utility functions.

Using US panel data, Finkelstein et al. (2013) estimate that a one standard deviation increase in the incidence of chronic disease leads to a 10%–25% drop in the marginal utility of consumption relative to the healthy population. Their result is thus opposite to ours. There are several possible reasons for such a deviation. First, their main specification is a log-linear mapping from income to well-being, i.e., Finkelstein et al. (2013) assume a relative risk aversion of 1, although they find that their result is robust to some relaxation of this assumption. Conversely, we focus on a non-linear specification in which we estimate relative risk aversion (which is non-constant by income), the level effect of sickness and the fixed cost of sickness. Their functional form is thus a special case of the one we utilize. However, implementing the panel specification in Finkelstein et al. (2013, p. 236) with our data by replacing the number of diseases with a sickness dummy, we obtain a statistically significant and opposite result to theirs (not reported). Second, their response variable is a binary happiness indicator, as opposed to our cardinal measure of life satisfaction. Third, the two studies focus on contrasting populations. Finkelstein et al. (2013) study those aged above 50 who are not in the labor force. We study working-age employed population. Also, they study the effect of chronic conditions, whereas we examine any sickness absence of above 6-week length.

The paper is structured as follows. Section 2 discusses the key theoretical aspects of an optimal sickness insurance system. Section 3 describes the data and characterizes the utility function and empirical estimation methods. Section 4 reports the estimation results. The last section concludes.

2. Optimal sickness insurance

The goal of social insurance is to transfer resources from states of the world with low marginal utility to states of the world with high marginal utility. We apply and develop the Baily-Chetty approach to sickness insurance (Baily, 1978; Chetty, 2006; Chetty and Finkelstein, 2013).[1] The theoretical model describes a welfare-maximizing social planner's optimal choice of sickness benefits and taxes given the costs and benefits of a higher sickness allowance for a utility-maximizing representative agent who chooses the length of the sickness absence spell. Following the literature, the agent cannot borrow. Chetty (2008) studies the implications of liquidity-constraints on optimal unemployment insurance. The costs of higher replacement rates consist of unobservable hidden actions, the effect of longer sickness spells at the intensive margin and more sickness spells at the extensive margin.

We make two departures from the standard theoretical model, as outlined by Chetty (2006). On the cost side, we examine the relative contribution of the extensive and intensive margins by allowing the probability of becoming sick to vary in the model as a function of effort, which is unobservable to the social planner.[2] On the benefit side, we allow for a fixed cost of sickness (θ in the model), i.e., we depart from the standard assumption of the state-independence of marginal utility.[3]

Given a concave utility function, the fixed cost of sickness, which fundamentally affects the utility gain of consumption smoothing, could in principle be of either sign. The fixed cost of sickness being positive (negative) implies a positive (negative) state dependence, meaning

[1] See Palme and Persson (2019) for the description of sickness insurance systems in Europe.

[2] Jaeger, Schoefer and Zweimuller (2018) and Kolsrud et al. (2018) among others study the extensive margin in the context of unemployment insurance.

[3] See Chetty and Finkelstein (2013, pp. 155-156) for an alternative way to incorporate state dependence.

that the marginal utility is higher (lower) in the sick state. The importance of state dependence in optimal sickness insurance has been acknowledged since at least Zeckhauser (1970) and Arrow (1974). The state dependence implied by the presence of a fixed cost of sickness underlines the importance of interstate consumption smoothing of social insurance. The prior evidence (see Finkelstein et al., 2009, for a review), however, focuses on the relationship between health (e.g., Finkelstein et al., 2013, study chronic disease) and marginal utility. Our focus is on the relationship between sickness absence and marginal utility. In what follows, we characterize and estimate θ.

The model yields an implicit equation for the optimal benefit, *b,* which is based on the sufficient statistics approach (an augmented Baily-Chetty formula; see Appendix 2 for the detailed derivation of the model):

$$\epsilon_{r,b} + \epsilon_{D,b} = \frac{u'(c_s,1) - u'(c_e,0)}{u'(c_e,0)} \approx \gamma \frac{\Delta c + \theta}{c_e}\left[1 + \frac{1}{2}\rho \frac{\Delta c + \theta}{c_e}\right], \tag{1}$$

where $\epsilon_{r,b} = \frac{d\log(\frac{p}{1-p})}{d\log(b)}$ is the elasticity of the odds ratio ($r = \frac{p}{1-p}$) of sickness leave with respect to the sickness benefit, i.e., the extensive margin; and $\epsilon_{D,b} = \frac{d\log(D)}{d\log(b)}$ is the elasticity of the duration (*D*) of sick leave with respect to the sickness benefit, i.e., the intensive margin; $u(c_e, 0)$ and $u(c_s, 1) = u(c_s - \theta, 0)$ are the utility functions and c_e and c_s are consumption in the employment ($S = 0$) states and sickness leave ($S = 1$), respectively; $\frac{\Delta c}{c_e} = \frac{c_e - c_s}{c_e}$ is the proportional drop in consumption when on sick leave; $\gamma = -\frac{c_e u''(c_e)}{u'(c_e)}$ is the coefficient of relative risk aversion; $\rho = -\frac{c_e u'''(c_e)}{u''(c_e)}$ is the coefficient of relative prudence. The envelope theorem guarantees that all other behavioral responses can be ignored when setting the optimal benefit level, except for the elasticity parameters ($\epsilon_{D,b}$ and $\epsilon_{r,b}$) that enter the government budget constraint directly (Appendix 2).

The model has an intuitive interpretation. The left-hand side of the equality in equation (1) disentangles the extensive ($\epsilon_{r,b}$) and intensive ($\epsilon_{D,b}$) margins of the effect due to hidden actions.

The right-hand side of equation (1) defines the value of the insurance, i.e., the change in relative marginal utility, and the fixed cost of sickness under sick leave. The reduction in consumption is a function of the replacement rate. We estimate the utility function parameters, including the fixed cost of sickness, allowing the value of insurance to vary as a function of income (for more details, see Section 3).

The Baily-Chetty formula is based on simplifying assumptions. The model does not account for possible preference for vertical redistribution across individuals, general equilibrium effects on wages, the marginal cost of public funds, externalities on government budget or other externalities (Pichler and Ziebarth, 2017). In addition, reference dependence might play a role in the utility function in this context (Kahneman & Tversky, 1979). However, any reference dependence not captured by θ is not considered.

3. Empirical approach

3.1. Data

Our main dataset is the Socio-Economic Panel (SOEP), which is a large survey on German households on a wide range of variables including incomes and subjective well-being. We restrict the data to years 1993–2018 such that all our relevant variables are found for all the years.[4]

[4] See https://www.diw.de/en/diw_01.c.678568.en/research_data_center_soep.html. The data are public and available to other researchers, and our method is replicable for sickness insurance and other policy domains.

The level of life satisfaction is measured on an eleven-point scale from 0 to 10, where 0 is 'not at all satisfied' and 10 is 'completely satisfied'. We give the life satisfaction variable a cardinal (not ordinal) interpretation to accomplish our analyses, following, e.g., Layard et al. (2008), which requires comparability of utility across individuals.[5] Our focus is on the policy, which is conditional on sickness absence. The measure of subjective well-being is life satisfaction. It is the best available survey measure of decision utility (Benjamin et al., 2012; 2014a-2014b; for discussion of decision versus experienced utility, see Kahneman et al., 2007). The income variable we use for all our analyses is net income.

A year with sickness is defined by the question asking whether the individual spent more than 6 weeks in sickness absence in the previous year, which we then assign to the relevant year. Note that our variable does not allow us to separate the effect of sickness from that of sickness absence. We use both terms interchangeably. Also, the individual might not be sick during the interview. Our estimates should be interpreted a wider and long-term effect of sickness on life satisfaction. In case the interview was held before the onset of the sickness, there is measurement error and the resulting attenuation bias.

We restrict our sample to year-person observations in which the individual worked more than 1,000 hours at ages 19 to 64. We construct our sample by selecting those who switch between the sickness and healthy states during the sample period. This focus on "switchers" ensures that the estimated utility curve for the sick and the healthy is estimated using the same individuals. We show our results for differently constructed samples as well. Descriptive statistics are presented in Table 1. Figures A1–A2 display the histograms of the incomes and life satisfaction of the two subsamples (sick leave vs. healthy), respectively. The number of

[5] See Bond and Lang (2019), Chen et al. (2019) and Kaiser and Vendrik (2019) for critique and discussion of happiness scales.

year-individual observations in the healty state is larger (~56,000) than the subsample for those in the sick state (~11,000), since the "switchers" have more healthy years than sick years. There are 6,994 different individuals with on average 8.97 periods of which 1.48 in sickness. Mean life satisfaction is ~0.3 points lower in the sick state.

[Table 1 here]

3.2. A utility function compliant with data

We estimate utility functions in the healthy and sick states using life satisfaction as a measure of subjective well-being, allowing us to infer both the degree of risk aversion and state dependence. There is no consensus on the sign of state dependence, possibly due to varying contexts (see Finkelstein et al., 2009; Finkelstein et al., 2013; Figure A3).

For a non-parametric analysis of the relationship between income and life satisfaction in the data, we fit a spline. A visual inspection of the spline fit of the income–well-being gradient as shown in Figure 1 reveals that the utility on sick leave is lower with a higher slope, compared with those who are employed. To account for such a relationship, we utilize a parametric functional form in the family of HARA (Hyperbolic Absolute Risk Aversion) utility functions. It offers a flexible and tractable functional form that encompasses the most commonly used functions in macroeconomics and finance and emerges from economic reasoning (Perets and Yashiv, 2016). HARA utility functions can account for an increasing, decreasing, or constant relative risk aversion (see Merton, 1971; Meyer and Meyer, 2005, for a review):

$$u(c(y), S) = \frac{\gamma}{1-\gamma}\left(\frac{\alpha_H y - \omega - \theta S}{\gamma}\right)^{1-\gamma}, \tag{2}$$

where S is an indicator for sickness leave, and ω is a horizontal shift parameter, the fixed cost of sickness in the economic model presented in Section 2. Similar shift parameters have been used in dynamic analyses incorporating stock effects, such as habit formation; see Phlips

(1978). As in the economic model presented in Section 2, θ is the fixed cost of sickness. For simplicity, we set the income scale parameter $\alpha_H = 1$ (see Section 3.3. for more details).

Unlike in the standard CRRA (Constant Relative Risk Aversion) model, the relative risk aversion (RRA) and relative prudence (RP) are functions of y and ω. Assuming that agents also know their utility function in the sick state, their relative risk aversion and relative prudence are also functions of θ:

$$RRA_S(y,1) = -y\frac{u''(y,1)}{u'(y,1)} = \frac{\gamma y}{y-\omega-\theta}, if\ c > \omega + \theta \tag{3}$$

$$RP_S(y,1) = -y\frac{u'''(y,1)}{u''(y,1)} = \frac{(\gamma+1)y}{y-\omega-\theta}, if\ c > \omega + \theta. \tag{4}$$

Using the functional form from equation (2) and assuming that equivalized disposable income is a near equivalent to consumption (but see equation (13)), we can explicitly solve equation (1) for the optimal replacement rate (RR):

$$RR = 1 - \frac{\Delta y}{y_e} = \left(\frac{\omega}{y_e} + \frac{\theta}{y_e}\right) + \left(1 - \frac{\omega}{y_e}\right)\left(1 + \epsilon_{r,b} + \epsilon_{D,b}\right)^{-\frac{1}{\gamma}}, \tag{5}$$

where $\Delta y = y_e - y_s$. Equation (5) yields the optimal benefit schedule in terms of replacement rates conditional on the income level. Note that the presence of θ on the right-hand side of equation (5) implies that pecuniary and non-pecuniary costs must be considered when characterizing the total benefit of insurance. If $\theta > 0$, the state dependence is positive and *vice versa.*

3.3. Estimation

For the empirical specification of the utility function, we estimate a non-linear least squares fit of the following form:

$$SWB(c(y_i), S_i) = \alpha + \frac{\beta}{1-\gamma}\left(\frac{y_i - \omega - \theta S_i}{\gamma}\right)^{1-\gamma} + \delta S_i + \varepsilon_{S_i,i}, \tag{6}$$

which is equation (2) augmented with additional parameters (the constant α and the level of sickness absence δ) to increase flexibility. S_i is the sickness leave indicator, y_i is equivalized disposable income, and SWB_i is life satisfaction. The sick state ($S_i = 1$) is measured as the state of having sickness absences for more than six weeks during the year. The healthy state ($S_i = 0$) is measured as spending below six weeks in sickness absence (see Section 3.1. for more details). The parameters α (constant) and β (scale parameter) whose values are not our focus, are included to account for the scale used to measure life satisfaction. Note that parameters α, β and δ do not affect the relative marginal utility in equation (1), nor the value of relative risk-aversion. The parameter ω gives the horizontal shift of the curve. This parameter is crucial in fitting the level of well-being for low-income individuals. In essence, assuming all utility is a function of consumption, it plays a major role in defining the level of consumption at zero income. At zero income, savings can be consumed, but also publicly provided goods and services. We consequently call this parameter the "institutions parameter", although we recognize that much more goes into this parameter than only publicly provided goods and services, such as access to credit and functioning of financial markets. To understand better its nature, we implement a multi-country analysis using EU-SILC data. We find that the highest country correlation with the "institutions parameter" is with trust in institutions ($\rho = 0.82$) and interpersonal trust ($\rho = 0.80$). For a more direct measure of the magnitude of the welfare state, social spending in euros, yield a lower correlation of 0.49 with the institutions parameter. See Appendix 4 for EU-SILC aggregate analysis and Appendix 5 for EU-SILC country-level analysis, which also includes the above-mentioned correlations.

We apply the R package "minpack-lm", which is based on a modified Levenberg-Marquardt-type algorithm to obtain our fit. The maximizing problem is non-smooth across the boundary of having the value one in the γ parameter. We choose the fit with the lowest sum of squared errors across the regions, which is obtained with an above-one initial γ parameter value.

If $\gamma > 1$ equation (6) yields a bounded utility function. Since the data have a natural upper bound of 10, we estimate the model assuming that $\alpha = 10$, which implies that when income tends to infinity, life satisfaction tends to 10. Given the state-dependent shift and level parameter, δ, the model can flexibly fit both positive and negative state dependence according to data. For example, a relationship shown in Panel B of Figure A3 would be produced by $\theta < 0$ and $\delta < 0$.

The functional form (6) corresponds to HARA $\left(U_{HARA}(c(y)) = \frac{\gamma}{1-\gamma}\left(\frac{\alpha_H y+\beta_H}{\gamma}\right)^{1-\gamma}\right)$, with the simplifying restriction that $\frac{\alpha_H}{\gamma} = \frac{1}{1000}$, i.e., we measure income in thousands of annual euros. Scaling income to a similar order of magnitude as life satisfaction slightly increases estimation robustness, due to particulars of the numeric estimation algorithm. The numerical values of the parameters of interest $\{\theta, \omega\}$ remain stable but are scaled by $\frac{1}{1000}$, and the estimation of the crucial parameter γ is not qualitatively affected by the scale.

Empirically, at any point in time, we assume there is a difference in the marginal utility of the healthy ($S = 0$) and sick ($S = 1$) populations that is captured by the fixed cost of sickness (θ). The parameter θ can be decomposed into two parts, $\theta = \theta_s + \theta_b$, where θ_s is the effect of sickness absence across states and θ_b is the difference in the utility across individuals. The latter component is the selection or estimation bias. Since we estimate θ using the same individuals in the two states, θ_b is likely to be small. For the policy to be a pure insurance, the social planner will only consider θ_s. To maximize utility across states and across individuals, the social planner will consider both components.

3.4. Replacement rate: income vs. consumption

We use equivalized disposable income to approximate consumption as closely as possible with an income measure to explicitly solve for optimal replacement rates; see equation (3). Not using actual consumption levels induces a potential bias (see Gruber, 1997; Kolsrud et al., 2018).[6] A recent work by Meyer and Mok (2019) analyzes the effect of disability on consumption (food and housing) using the PSID and presents calculations for optimal policy rules. Our data do not allow us to capture actual consumption. However, assuming that our equation (6) uncovers marginal utilities conditional on income levels in the sick and employed states, we can write:

$$u'(y,S) = u'(c(y),S)c'(y,S) = SWB'(c(y),S)c'(y,S) = \beta(y-\omega-\theta S)^{-\gamma}. \qquad (12)$$

Note that equation (12) is a function of y, not of c.

More generally, one can calibrate the optimum (equation 1) in terms of our observables using $u'(y,S) = u'(c(y),S)c'(y,S)$ to obtain in a general case:

$$\epsilon_{r,b} + \epsilon_{D,b} = \frac{u'(c_s,1) - u'(c_e,0)}{u'(c_e,0)} = \frac{(c'(y_e,0)/c'(y_s,1))u'(y_s,1) - u'(y_e,0)}{u'(y_e,0)}$$

$$= \frac{u'(y_s,0) - u'(y_e,0)}{u'(y_e,0)} + \frac{u'(y_s,1) - u'(y_s,0)}{u'(y_e,0)}$$

$$- \frac{((c'(y_e,0) - c'(y_s,1))/c'(y_s,1))u'(y_s,1)}{u'(y_e,0)}. \qquad (13)$$

[6] The bias is due to additional hidden effects, which affect the marginal propensity to consume, i.e., the increase in benefits crowding out savings (Engen and Gruber, 2001) or spousal labor supply (Cullen and Gruber, 2000). However, in the case of sickness insurance, these effects are negligible in comparison with old age insurance and smaller than in the case of unemployment insurance. For an analysis of old age insurance, see Feldstein (1974).

Using our modelling assumption $u'(c(y),S) = u'\big(c(y-\theta S)\big)$, with $S \in \{0,1\}$, and a Taylor expansion we obtain the approximation for the RHS of (13):

$$\approx \gamma \frac{\Delta y + \theta}{y_e}\left[1 + \frac{1}{2}\rho \frac{\Delta y + \theta}{y_e}\right] - \frac{c'(y_e,0) - c'(y_s,1)}{c'(y_s,1)}\left[1 + \gamma \frac{\Delta y + \theta}{y_e}\right], \qquad (14)$$

The first term on the RHS of (13) is the well-known, state-independent Baily-Chetty expression in terms of income. The next two terms describe the effects of consumption smoothing, i.e. how marginal changes in income are transformed into utility under a state change. Here, the second term accounts for the state-dependent change in marginal utilities and the last term accounts for the possible change in marginal propensities to consume in different states. In the special case of linear consumption functions that allow the constant term to be state-dependent, $c(y,S) = c_0(S) + c_1 y$, the last term on the RHS of (13) would be zero.

However, in the context of unemployment Kolsrud et al. (2018) utilize register-based data on consumption to draw attention to both state-specificity in marginal propensities to consume and on its substantial change during the duration of unemployment spells with important implications to the shape of benefit function as a function of unemployment spell. Since we have no such consumption data, we ignore the last term and rely on our compound function (6) in capturing the effects of state-dependence in the marginal propensities to consume.

Note that equation (6) estimates γ and ρ as risk parameters relative to income, not to consumption. In addition, our local recommendations concerning policy are based on the first-order conditions (Appendix 2). The estimated optimal policy rule should be considered with this in mind. Since parameters can vary with policy rule, they might not apply globally.

4. Results

4.1. Estimation results

To model the empirical relationship observed in Figure 1 and to obtain the numerical estimates of the parameters of the utility function, including the fixed cost of sickness, we fit equation (6). The estimated parameter values are documented in Table 2. The main specification result is presented in column 1 of Table 2. The parameters with policy significance, $\{\gamma, \theta, \omega\}$, are all statistically highly significant. The fixed cost of sickness is estimated to be approximately seven thousand euros per year. There is also a statistically significant level drop in the estimated utility curve of 0.18 life satisfaction points for those with sickness.

The resultant utility curves, overlaid on the spline fit, are presented in Figure 3. The estimates confirm the visual observation that the association between life satisfaction and income is steeper, conditional on income in the sickness absence state. Using our functional form, the positive state dependence stems from the positive and significant fixed cost of the sickness parameter, θ. Visually, the fitted curve describes the spline fit rather well, except at low incomes of below 5,000 to 10,000 euros. Net incomes below 10,000 euros represent only 0.8% of the sample. The fitted utility curves are weighted by sample weights, unlike the spline curves.

Column 2 of Table 2 presents the result when the sick state is defined as having at least 180 days of sickness during the calendar year. Since there are far fewer individuals with such long sickness absences and the sample is limited to “switchers”, the sample size is smaller in this regression. The benefit of focusing on this population, however, is that each individual has at least a 50% probability of being on sick leave when the interview is conducted. The point estimate for fixed cost of sickness is higher for this sample than in column 1. Longer-term sickness absences apparently leads to slightly larger estimated costs in utility. However, the difference in estimated values is not statistically significant from the main specification.

[Table 2 and Figure 2 here]

In columns 3 and 4 we split the sample at year 2005, comparing the period 1992–2004 to 2005–2018. Here, the main difference between the estimates is the large drop in the estimated institutions parameter. A drop in the institutions parameter, with no change in other parameter values, would suggest a drop in life satisfaction at low incomes. However, the interpretation of such a change depends on the changes in the other parameters, which do seem rather similar across the two fits. For a graphical exposition, we show the spline fits for the two periods and the corresponding parametric fits in Figure 4. The spline shown in light blue and pink illustrates the relationship between income and life satisfaction has steepened in time due to both a lower reported life satisfaction at low incomes and a higher life satisfaction at higher incomes. Also, the curvature of the life satisfaction curve apparently decreases over time, which is reflected in a slightly lower relative risk aversion estimate.

The steeper form, and the corresponding drop in the estimated institutions parameter imply that there is now a larger difference in well-being by income level. The first possible explanation to this phenomenon is that there might be a composition change. Individuals with higher well-being might have moved up the income ladder and there is no real societal change. On the other hand, the German labor market has seen major institutional changes, especially the Hartz reforms (see e.g., Krebs & Scheffel, 2013). The reforms' effect might reflect in the steeper form of the empirical life satisfaction curve as well. Lastly, it is possible that deflating by CPI might not fully reflect the true changes in the purchasing power of money over such a long time period (1992–2004 vs 2005–2018), which could leave a mark on the estimates.

In our main specification, we differentiate sick years from healthy years by a measure of at least 6 weeks of sickness absences from work. Such a measure is far from perfect. One issue might be lingering effects of sickness after return to work. To minimize such measurement error, we check robustness of our results to excluding the adjacent years when a switch is made from health to sickness or *vice versa.* We report the result in the fifth column of Table 2. We find that the estimated level effect of sickness increases substantially and significantly compared to the main specification, while the fixed cost of sickness is unchanged. The level effect of sickness more than triples. Other parameters remain nearly unchanged. This result suggests that our main specification does underestimate the level effect of sickness due to inaccurate measurement of sickness. However, dropping the switching years can also produce bias, since now life satisfaction in sickness and health is measured further apart in time, which might amplify any endogeneity between incomes, life satisfaction and health. We also report the reverse robustness check, which only includes the adjacent years when the switch between states took place (column 6). Consistently, the estimated costs of sickness now fall to a point where the fixed cost of sickness no longer is statistically significant.

Across all samples, the estimated relative risk aversion parameter (γ) is highly stable, varying between 1.26 and 1.33 in point estimate, making the institutions parameter crucial in defining the form of the curve.

We replicate all our results with the EU SILC cross-sectional data for European countries. The results are presented in Appendices 4 and 5.

4.2. Policy implications

To assess the significance of our estimates to policy, we take the main specification results (Table 2, column 1) and exploit the optimal policy that the Bailly-Chetty model implies (Equation 1). First, we calculate the relative risk aversion (RRA) and relative prudence parameters using equations (3) and (4). The former is shown in Figure A3. The estimated RRA increases with income. This result stems from the $-\omega$ and $-\theta$ terms in the denominator of equation (3). As long as $-\omega - \theta > 0$, RRA will be increasing and tend towards γ as incomes increase. In essence, the pattern of increasing RRA follows from having a negative horizontal shift parameter or institutions parameter ω. Intuitively, risks for lower income individuals are smaller, since they are relatively more protected by the "institutions". This result challenges the conventional wisdom of decreasing relative risk aversion (see Meyer and Meyer, 2005). In our application, the emphasis is on the estimate for the sick state, in which relative risk aversion is higher.

We apply equation (5) to characterize the potential implications for optimal policy of the estimated and assumed parameter values. We are also interested in the role of θ in determining the optimal policy curve (Figures 5–6). Since the utility curve itself is estimated with different individuals across the income spectrum, we focus on the effect of the estimated fixed cost of sickness on optimal policy. We further assume that $\epsilon_{r,b} + \epsilon_{D,b} = 1.5$, i.e., the combined effect of the extensive and intensive margins sums to 1.5.[7]

[Figures 5–6 here]

We find that the optimal replacement rate curve is non-linear and increases slightly with income. In Figure 5, we show that the fixed cost of sickness reshapes the replacement rate curve at the lower end of the income scale. For lower income individuals, who have some safety given by the horizontal shift parameter (ω), the effect of the fixed cost of sickness, or the state-dependence of relative risk-aversion, is the main reason to insure against sickness. A key feature is that relative risk aversion is higher in the sick state (Figure A3).

[7] Echoing the estimate that we use in the calculations, Ziebarth and Karlsson (2014) argue that the consensus estimate of the literature is ~1. Böckerman et al. (2018) also find that $\epsilon_{D,b} \approx 1$.

Figure 6 shows the optimal policy curve. Our estimated replacement rates for most income earners fall close to the current French policy of a linear 0.5 replacement rate. Note, however, that the optimal curve is based on net income, whereas the current policy schemes are based on earnings. Since our estimate for the utility curve itself is biased by the fact that it is estimated for different individuals across the income spectrum, the derived level of optimal policy is biased. However, the core observation arising from our estimates is that in the utility function itself is different in the sick state due to a fixed cost of sickness. This state dependence has a significant effect on optimal policy as shown in Figure 6.

5. Conclusions

We study the income–well-being gradient in the sickness and health to understand its state-dependent nature using German panel data on income and life satisfaction. We use subjective well-being data to measure utility and characterize risk in a state-dependent manner. The representative survey data cover 30 countries in Europe, allowing us to address institutional variation. We impose structure in the data in the form of a least-squares fit of a flexible utility function to extract the magnitudes of the costs of sickness in monetary terms.

We obtain three main results. First, the marginal utility is higher in the sick state, conditional on income (i.e., positive state dependence). We model this feature with a fixed cost of sickness. Second, relative risk aversion increases with income, stemming from the baseline well-being provided by "institutions" at lower incomes. Third, for lower income individuals, who are protected by the institutions, the effect of the fixed cost of sickness is the main reason to insure against sickness. In addition, we find that the income–well-being gradient has steepened over time in Germany. This steepening has come from both a drop in well-being at low incomes and an increase at higher incomes. In the language of the parametric model, the shift or "institutions" parameter has decreased, which implies that the insurance value of the German social system has decreased markedly from the period 1992–2004 to 2005–2018.

The institutions parameter, which has a notable influence on the shape of income–well-being gradient, captures the effects of predetermined stock variables, such as the value of institutions. We present country-specific estimates of the institutions parameter and report unconditional correlations lending credence to the value of institutions across countries in Europe, such as interpersonal and institutional trust.

Our findings challenge the standard view of risk aversion. Based on our analysis, the relative marginal utility is state-dependent and can be captured by a fixed cost of sickness. The role of optimal public policy is to mitigate this welfare cost, which is more pronounced at low levels of income.

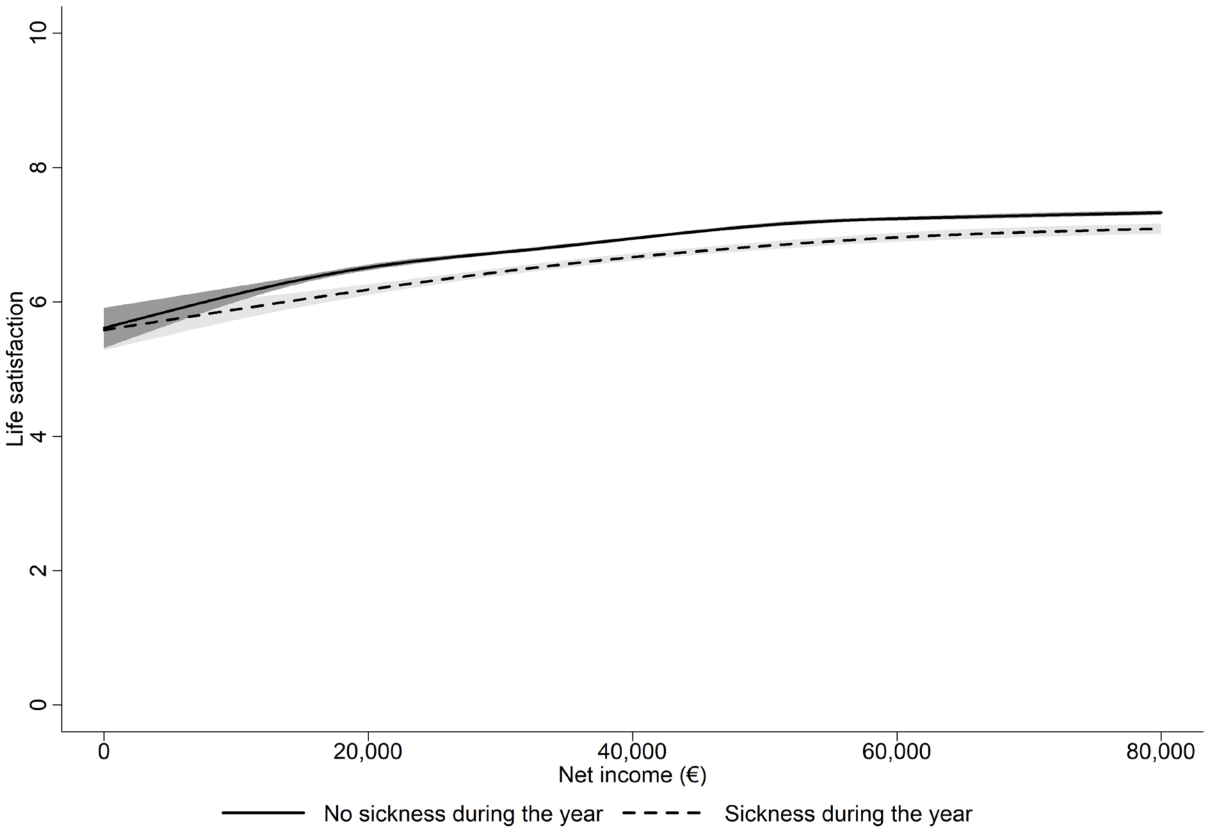
We propose the following procedure when assessing state-dependent risk. First, estimate the standard measure of relative risk aversion in the state in which risk has been realized. Second, evaluate the state dependence of marginal utility.

To identify utility functions, we assume that life satisfaction is a sufficiently satisfactory measure of utility, and our fit of equation (6) guarantees a sufficiently good fit. We compare same individuals in the two states. However, the baseline form of the utility curve is biased by comparing different individuals across the income distribution. Consequently, optimal levels of sickness insurance cannot be inferred from our work alone.

The state dependence of the utility function implies that each relevant policy domain must be studied separately for optimal policy design. Future research should consider other risks, such as unemployment and old age.

Figures and Tables

Figure 1. Spline fit of income–well-being gradient in Germany split by sickness states



Notes. The estimates are cubic splines with six knots estimated with the R package "bigsplines" for a cross-section using the following parameter values; unrounded, no manual tuning, unique knots assuming Gaussian standard errors. The gray area around the curves represents the 95% confidence interval. Sample size: 56,268 year-person observations with no sickness during the year, 10,961 year-person observations with sickness during the year.

Table 1. Descriptive statistics

	Healthy		Sick	
	Mean	SD	Mean	SD
Life satisfaction	6.97	1.65	6.65	1.85
Net income	47426.65	28599.94	45656.40	36880.43
N	56,268	-	10,961	-

Notes. Healthy state is defined as not having had a sickness absence of at least 6 weeks during the year. The sick state is defined as having had a sickness absence of at least 6 weeks during the year. There are 6994 different individuals with on average 8.97 periods of which 1.48 in sickness.

Figure 2. Theoretical patterns of marginal utilities

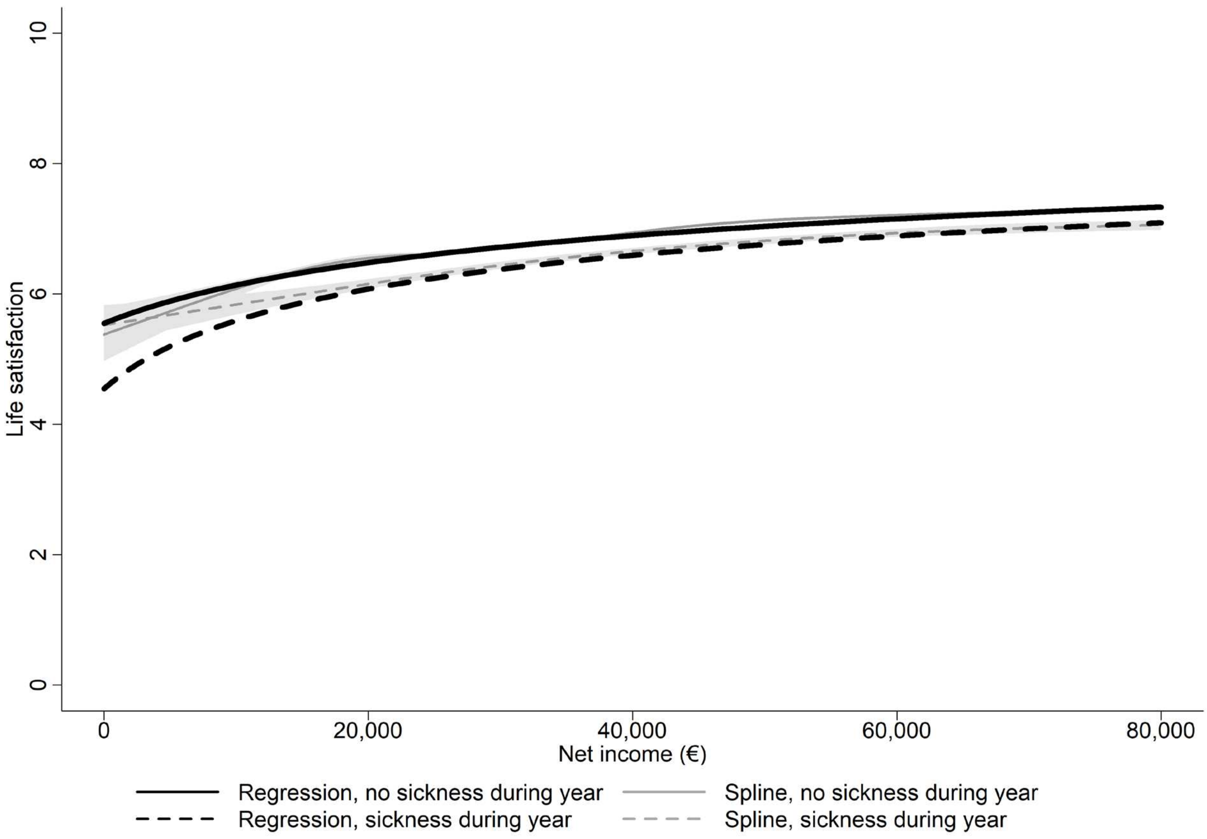
Notes. Adapted from Finkelstein et al. *(2013).* ***Panel A.*** *The panel presents a utility function with positive state dependence,* i.e., *a utility function with higher marginal utility at each consumption level when on sick leave.* ***Panel B.*** *The panel presents a utility function with negative state dependence,* i.e., *a utility function with lower marginal utility at each consumption level when on sick leave.*

Table 2. Estimates

	Main specification	180 days of sickness	Years 1992 to 2004	Years 2005 to 2018	Excl. adjacent switching years	Only adjacent switching years
	(1)	(2)	(3)	(4)	(5)	(6)
Scale parameter (β)	2.64*** (0.52)	2.92*** (0.67)	4.20* (2.34)	2.15*** (0.38)	2.27*** (0.58)	2.84*** (0.89)
Relative risk aversion parameter (γ)	1.28*** (0.02)	1.29*** (0.03)	1.33*** (0.07)	1.26*** (0.02)	1.26 *** (0.03)	1.28*** (0.04)
Institutions parameter (ω)	-15.11*** (4.13)	-4.84 (3.69)	-30.9** (13.83)	-7.85** (3.13)	-13.64** (5.38)	-13.28** (5.96)
Fixed cost of sickness (θ)	6.88*** (1.89)	8.55*** (2.8)	12.2*** (4.71)	3.76** (1.80)	6.42* (3.66)	3.2 (2.49)
Level effect of sickness (δ)	-0.18*** (0.04)	-0.14 (0.11)	-0.10 (0.09)	-0.24*** (0.04)	-0.64*** (0.09)	-0.06 (0.06)
N	67,229	5,448	28,686	38,543	44,958	22,271

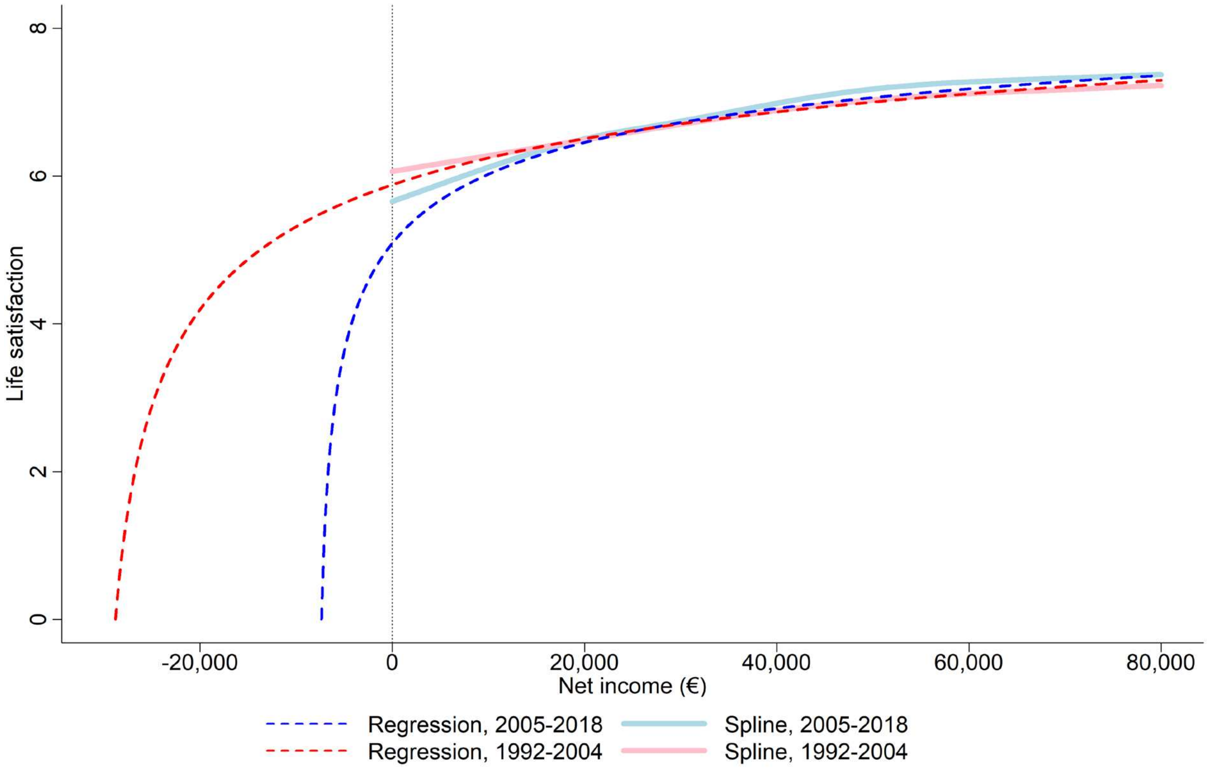
*Notes. Statistical significance: * p<0.1; ** p<0.05; *** p<0.01. The non-linear regression is the fit with a modified Levenberg-Marquardt-type algorithm with sampling weights. The standard errors are in parentheses. All models are estimated with equation (6) with α set at 10. The starting values, where applicable, are:* $\{\beta = 0,\ \gamma = 1.4,\ \omega = -15,\ \theta = 15,\ \delta = 0\}$. *For the estimation, the income variable is in thousands of annual euros. The SOEP population is limited to 'switchers', i.e., those who had one or more years with sick leave and one or more years without sick leave. To be defined as having sick leave, in columns 1 and 2 requires 6 weeks of absences and for column 3, 180 days. In columns 3 and 4 we replicate the main specification for years before and after the start of 2005, respectively. In column 5, we exclude years when an individual switches between sickness states. In column 6, we include only those years, when an individual switches between sickness states.*

Figure 3. Spline and non-linear fit of life satisfaction and net income split by sickness states



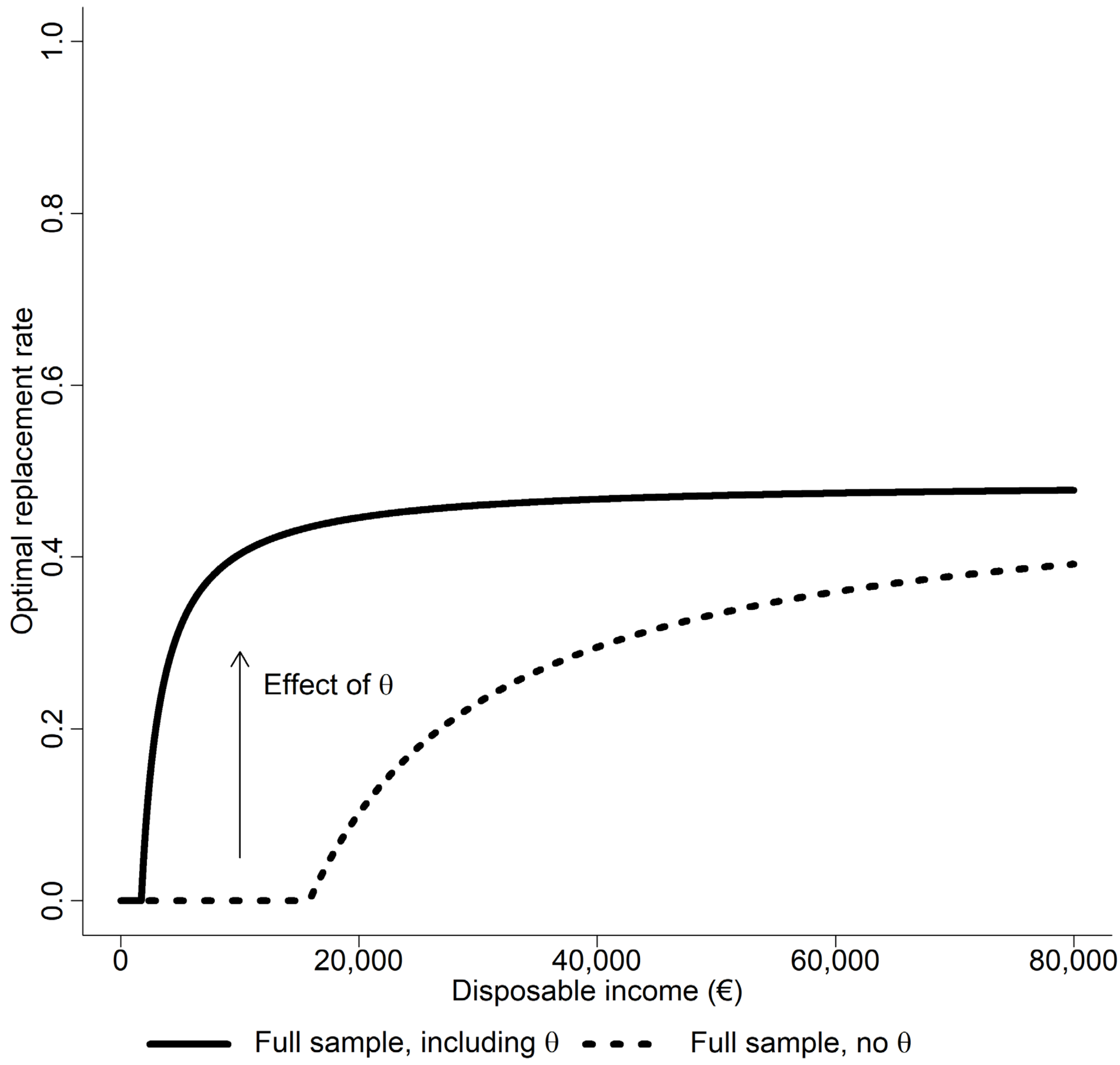
Notes. The non-parametric estimate is a spline fit. The x-axis in the figure is truncated at 80,000 euros. The gray area around the curves represents the 95% confidence interval. The non-linear regression in black fit is equation (6) with α set at 10, parameter values in Table 2, model 1. Sample size: 56,268 year-person observations with no sickness during the year, 10,961 year-person observations with sickness during the year.

Figure 4. Spline and non-linear fit of life satisfaction and income



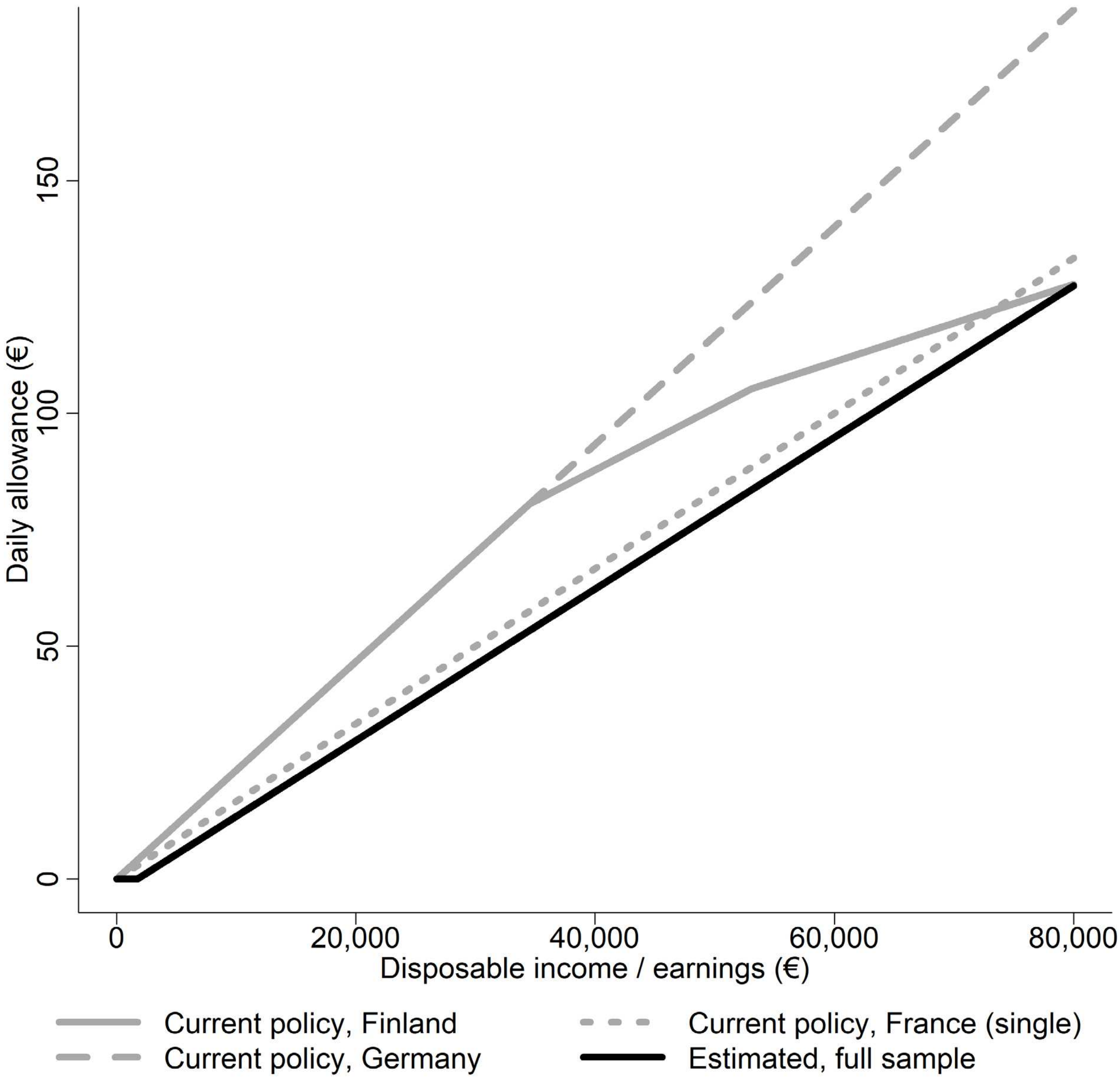
Notes. The non-parametric estimate is a spline fit shown in pink (1992 to 2004) and light blue (2005 to 2018). The x-axis in the figure is truncated at 80,000 euros. Negative incomes are not observed in the data, but extrapolated from the fitted utility function. Confidence intervals are not shown for maximum clarity. The non-linear regression in black fit is equation (6) with α set at 10, parameter values in Table 2, model 1. Sample size: 56,268 year-person observations with no sickness during the year, 10,961 year-person observations with sickness during the year.

Figure 5. Optimal replacement rates



Notes. The optimal replacement rates are calculated with the augmented Baily-Chetty formula (equation 1). The relative risk aversion values are from a generalized CRRA utility function with the parameter values of $\{\gamma,\ \omega,\ \theta\} = \{1.28, -15.11, 6.88\}$ *at different levels of net income, shown in Table 2, model 1, obtained from estimating equation (6).* θ *is the fixed cost of sickness, which affects the optimal replacement rate through relative risk aversion (RRA) and the augmented Baily-Chetty formula. Additionally, we assume that* $\epsilon_{r,b} + \epsilon_{D,b} = 1.5$.

Figure 6. Prevailing universal sickness insurance policy and estimated optimal curves



Notes. The optimal replacement rates are calculated with the augmented Baily-Chetty formula (equation 1). The relative risk aversion values are from a generalized CRRA utility function with the parameter values of $\{\gamma,\ \omega,\ \theta\} = \{1.28, -15.11, 6.88\}$ *for the solid line at different levels of net income, shown in Table 2, model 1, obtained from estimating equation (6).* θ *is the fixed cost of sickness, which affects the optimal replacement rate through relative risk aversion (RRA) and the augmented Baily-Chetty formula. Additionally, we assume that* $\epsilon_{r,b} + \epsilon_{D,b} = 1.5$. *"Single" refers to a one-member household. The optimal curve is based on equivalized disposable income, whereas the current policy curves are based on earnings.*

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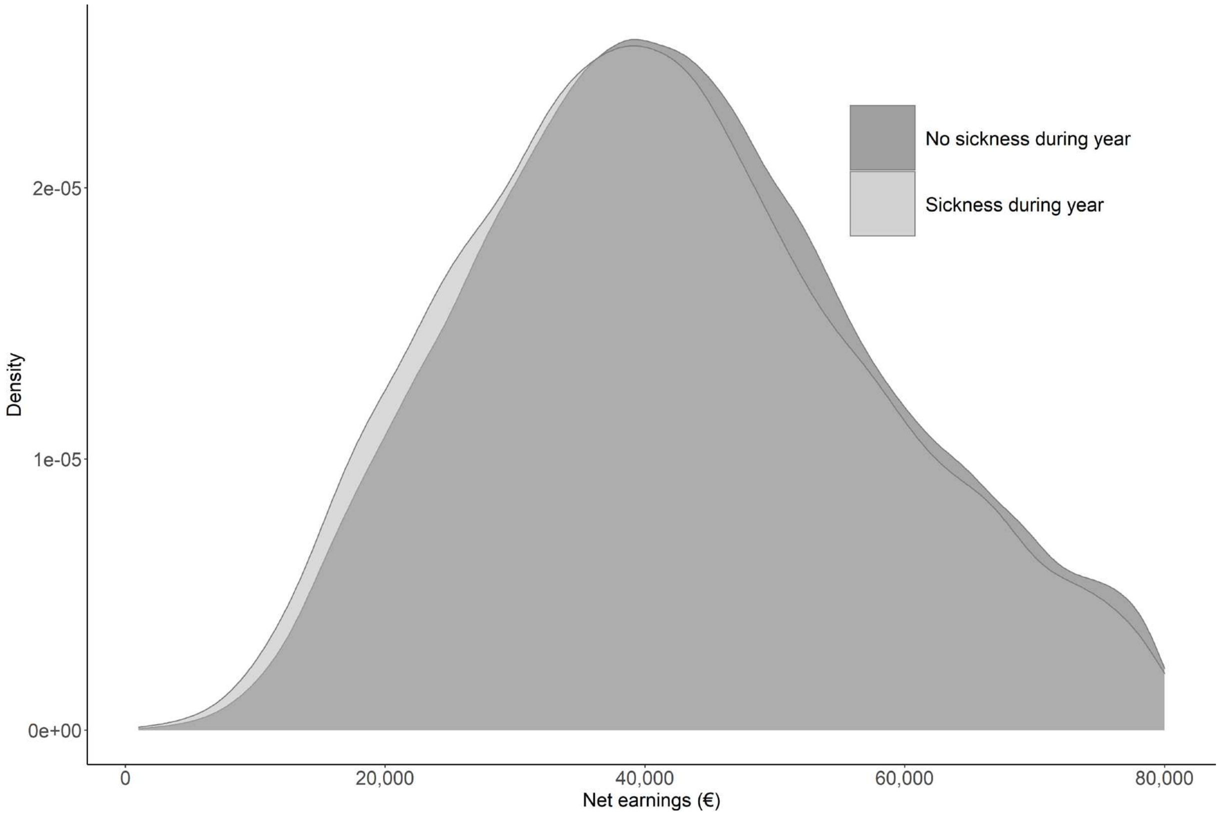
ONLINE SUPPLEMENTARY APPENDIX

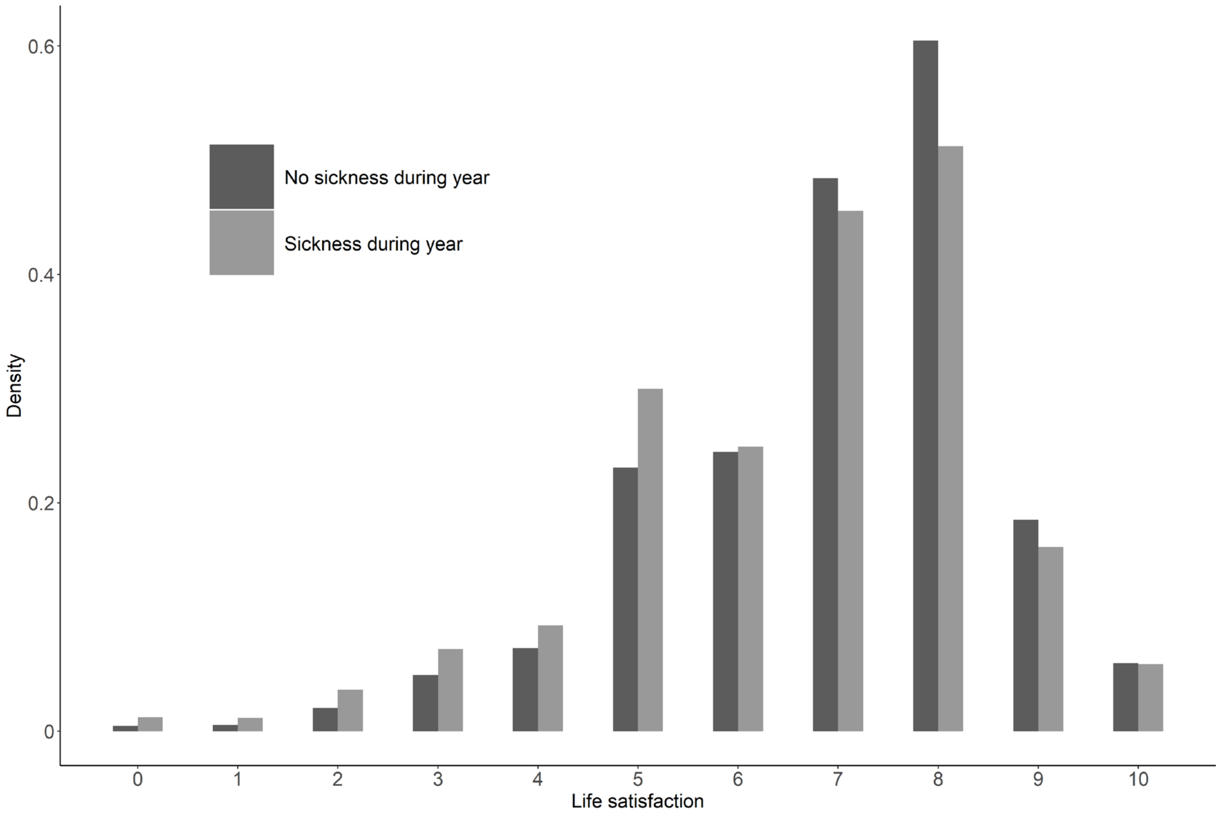
Appendix 1: Additional figures and tables

Figure A1. Density of incomes by sickness status



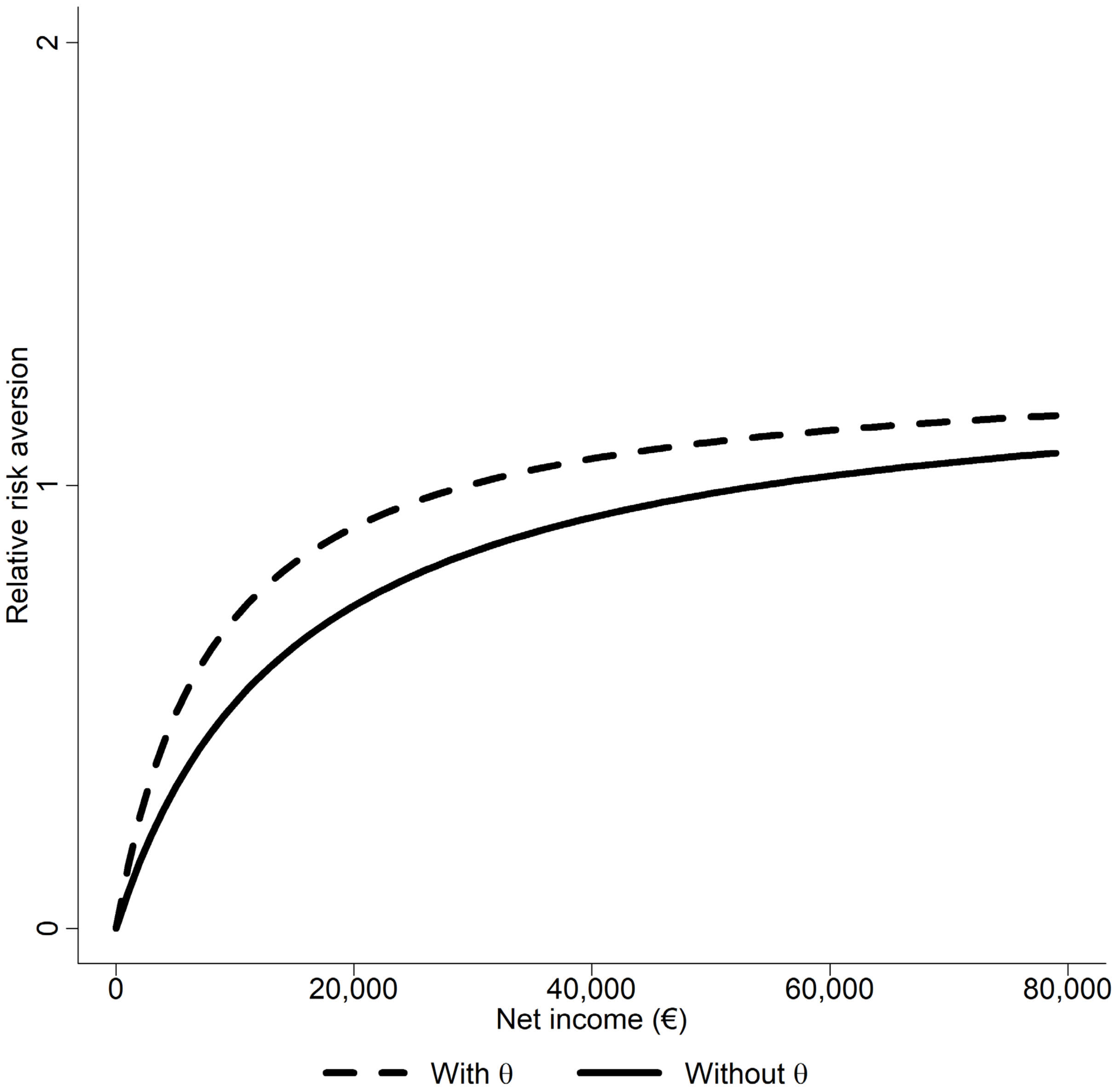
Notes. Sample size: 56,268 year-person observations with no sickness during the year, 10,961 year-person observations with sickness during the year.

Figure A2. Histogram of life satisfaction by sickness status



Notes. Sample size: 56,268 year-person observations with no sickness during the year, 10,961 year-person observations with sickness during the year.

Figure A3. Estimated relative risk aversion



Notes. The relative risk aversion values are from a generalized CRRA utility function with parameter values of $\{\gamma,\ \omega,\ \theta\} = \{1.28, -15.11, 6.88\}$ *at different levels of disposable equivalized income which by assumption equals consumption, shown in Table 2, model 2, obtained from estimating equation (6).*

Appendix 2: Augmented Baily-Chetty model

We adapt the canonical Baily-Chetty model of unemployment insurance to sickness insurance (Baily, 1978; Chetty, 2006; Chetty and Finkelstein, 2013). Consider a representative worker who has an initial level of assets A_0 and wage w. Assume that the agent is injured or becomes ill at work with probability $p(E)$, usually denoted p. $p(E)$ is a decreasing function of E, his chosen sickness-avoidance effort level, with convex effort cost $\psi_E(E)$. If the agent is injured or becomes ill, he takes sick leave. In the sick state, there is no risk of repeated sickness or unemployment, and the agent makes no labor supply choices. In the sick state, the agent must be rehabilitated to return to work.

In the sick state, the agent receives a benefit, b, for the duration of the sickness benefit and subsequently returns to work. The sickness duration, $D,$ is assumed to be a choice variable. Non-pecuniary costs and benefits of sickness duration and effort are captured by concave increasing functions $\psi_D(D)$ and $\psi_E(E)$. Let $k \in \{e, s\}$ and $U_k(c_k)$ be strictly concave utility over consumption, where subscripts *e* and *s* stand for being at work and on sick leave, respectively. The utility is assumed to be state-dependent, specifically with a fixed cost of sickness, $u(c, 1) = u(c - \theta) + \delta$, with $\delta < 0$ to allow for $u(c, 1) < u(c, 0)$. The agent chooses c_e, c_s, *E* and *D* at time 0 to solve,

$$max\ (1 - p(E))u(c_e, 0) + p(E)\big(u(c_s, 1) + \psi_D(D)\big) - \psi_E(E)$$

$$s.t. A_0 + (w - \tau) - c_e \geq 0$$

$$A_0 + bD + w(1 - D) - c_s \geq 0.$$

while taking $(b,\ \tau)$ as fixed. This assumption is critical. The social planner chooses the benefits, *b*, that maximize the agent's indirect utility under the condition that taxes collected

(τ) equal benefits paid. The taxes here are modeled to be lump sum, so they do not affect labor supply choices under no sickness.[8] The social planner's problem, with p*(E),* written as p, is as follows:

$$\max_{\tau,b,E} V(b,\tau,E)$$

$$s.t.(1-p)\tau \geq pbD.$$

At the optimum, the optimal benefit rate, $b*$, must satisfy the following:

$$\frac{dV(b,\tau,E)}{d(b^*)} = 0,$$

where τ and E are functions of *b*.

$$V(b) = \max_{c_e,c_s,D,E,\lambda_e,\lambda_s} (1-p)u(c_e,0) + p\big(u(c_s,1) + \psi_D(D)\big) - \psi_E(E) + \lambda_e[A_0 + w - \tau - c_e]$$
$$+ \lambda_u[A_0 + bD + (w-\tau)(1-D) - c_s].$$

The function is optimized over $\{c_e, c_s, D, E, \lambda_e, \lambda_s\}$. We assume that the value function $V(b)$ is differentiable such that the envelope theorem applies. Thus, following the envelope theorem, changes in the functions have no first-order effect. Specifically, $\frac{dE}{db} = 0$, giving the interior optimum as follows:

$$\frac{dV(b,\tau,E)}{db(b^*)} = -\lambda_e \frac{d\tau}{db} + \lambda_s D = 0. \quad \text{(A1)}$$

From the agent optimization, we know that

$$\lambda_e = (1-p)u(c_e,0) \text{ and } \lambda_s = pu(c_s,1). \quad \text{(A2)}$$

From the social planner budget constraint, on which the change in effort does have a first-order effect:

[8] If modeled, they would add a component on the cost side.

$$\frac{d\tau}{db} = \frac{\mathrm{p}}{1-\mathrm{p}}\left(\mathrm{D} + \frac{\mathrm{bdD}}{\mathrm{db}}\right) + \frac{\mathrm{Db}}{(1-p)}\frac{\mathrm{d}\log(\frac{p}{1-p})}{\mathrm{db}} \tag{A3}$$

Substituting (A3) and (A2) into (A1) yields an implicit equation for the optimal policy (an augmented Baily-Chetty formula):

$$\epsilon_{D,b} + \epsilon_{r,b} = \frac{u'(c_s,1) - u'(c_e,0)}{u'(c_e,0)} = \frac{u'(c_s-\theta,1) - u'(c_e,0)}{u'(c_e,0)} \approx \gamma\frac{\Delta c+\theta}{c_e}\left[1 + \frac{1}{2}\rho\frac{\Delta c+\theta}{c_e}\right], \tag{A4}$$

where $\epsilon_{r,b} = \frac{d\log(\frac{p}{1-p})}{d\log(b)}$ is the elasticity of the odds ratio ($r = \frac{p}{1-p}$) of sickness leave with respect to the sickness benefit, i.e., the extensive margin; and $\epsilon_{D,b} = \frac{d\log(D)}{d\log(b)}$ is the elasticity of the duration of sick leave with respect to the sickness benefit, i.e., the intensive margin; $\frac{\Delta c}{c_e}$ is the proportional drop in consumption while on sick leave; $\gamma = -\frac{C_e u''(C_e,0)}{u'(C_e,0)}$ is the coefficient of relative risk aversion; $\rho = -\frac{C_e u'''(C_e,0)}{u''(C_e,0)}$ is the coefficient of relative prudence. The right-hand side of the formula approximates the increase in relative marginal utility given the drop in consumption under sick leave and yields an implicit equation for the optimal benefit, *b,* which is based on the sufficient statistics approach, $\left(\epsilon_b, \frac{\Delta c+\theta}{c_e}, \gamma\right), \epsilon_b = \epsilon_{D,b} + \epsilon_{r,b}$.

The welfare change can be written in terms of relative marginal utilities of consumption in the two states. If individuals' behaviors were not distorted by the provision of insurance, the social planner would achieve the first best by setting b to perfectly smooth utilities, $u'_s(c_s, 1) = u'_e(c_e, 0)$. Note that equation (A4) is an implicit one. However, the envelope theorem guarantees that one need not fully characterize all of the margins to which individuals can respond to calculate the net welfare gain of social insurance. In particular, all other behavioral responses can be ignored when setting the optimal benefit level except for the elasticity parameters ($\epsilon_{D,b}$ and $\epsilon_{r,b}$) that enter the government budget constraint directly. However, the social planner cannot directly choose observed consumption levels or Δc

(hidden savings); rather, it determines the benefit level, which influences income replacement rates, which are observable. Kolsrud et al. (2018) find that the consumption drop increases with the duration of an unemployment spell and that savings and credit play a limited role in smoothing consumption. Equating consumption with income, we can directly solve for the optimal $\frac{\Delta y}{y_e}$ using equation (2):

$$RR = 1 - \frac{\Delta y}{y_e} = \left(\frac{\omega}{y_e} + \frac{\theta}{y_e}\right) + \left(1 - \frac{\omega}{y_e}\right)\left(1 + \epsilon_{r,b} + \epsilon_{D,b}\right)^{-\frac{1}{\gamma}}, \qquad \text{(A5)}$$

We employ the form $u(y, S) = u(c(y) - \theta S)$ as a simple parametrization of the state-dependent utility of the qualitative type we have observed in Figure 1. The social planner now must consider θ in addition to the standard Baily-Chetty parameters $\{\epsilon_b, \gamma, \rho\}$ for optimal policy. The relationship observed by Finkelstein et al. (2013) would require an alternative functional form.

The envelope theorem plays a critical role in generalizing (A4) with minor modifications to more realistic dynamic models with endogenous savings and borrowing constraints (Chetty and Finkelstein, 2013). One could also complement the model following Kolsrud et al. (2018), who model the effect of duration-dependent benefit rates in unemployment.

In the standard Baily-Chetty formula, it is possible that a non-linear benefit rule is optimal if risk aversion or the incentive effect varies significantly according to the income level. Additionally, if the aim of the insurance scheme is to contribute to the redistribution of income from rich to poor households, a non-linear benefit rule might be motivated well.

Appendix 3: Sickness insurance in Europe

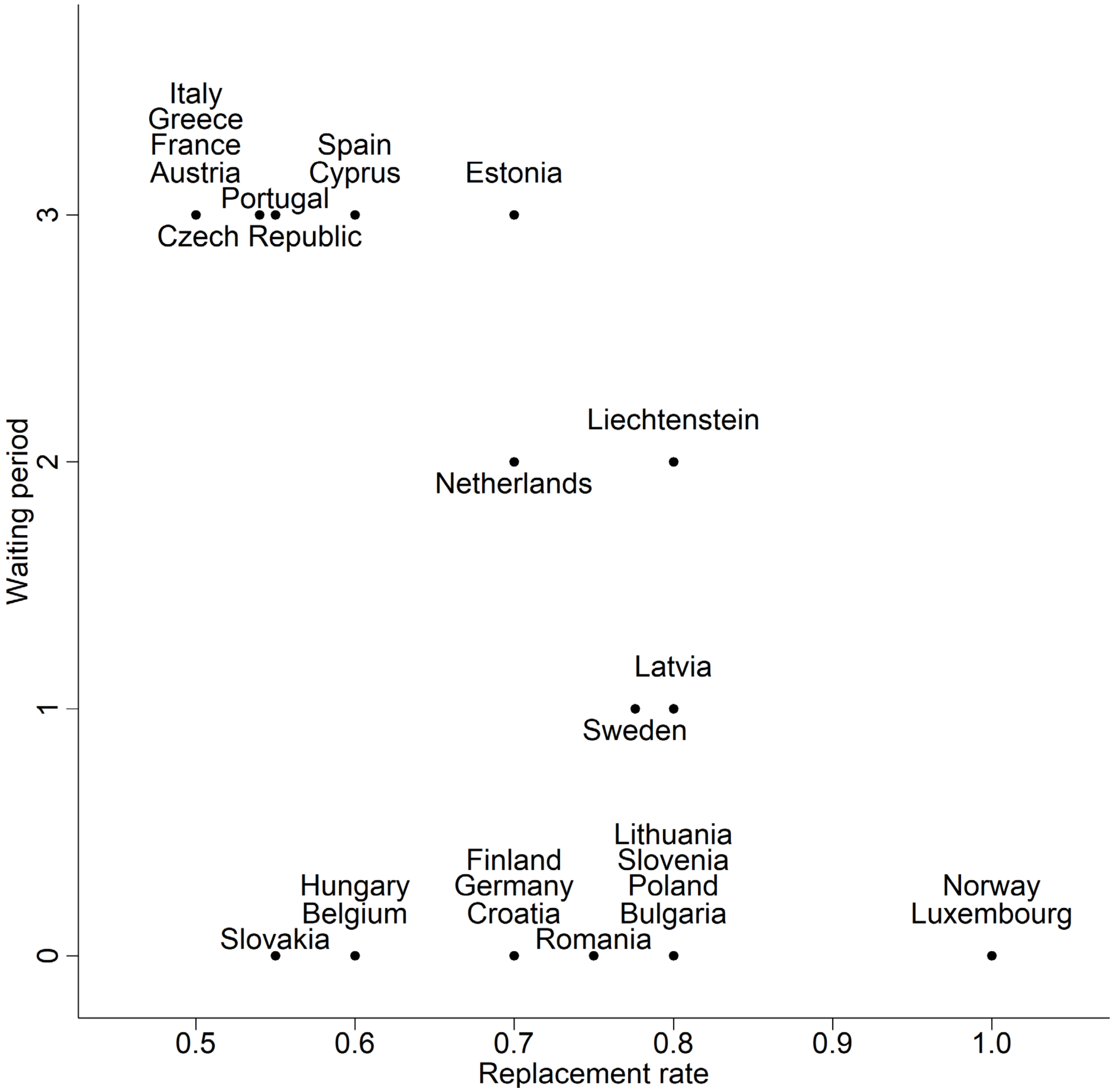
MISSOC (2017) comparative tables describe the European sickness insurance schemes (cf. Frick and Malo, 2008). The tables distinguish at least five dimensions, in which the schemes differ. Two of the key dimensions are depicted in Figure 3. The crucial aspect in any social insurance system is the replacement rate, i.e., the rate at which pre-sickness income is covered by sickness insurance. The replacement rates vary in Europe from 50% (Italy, Greece, France and Austria) to 100% (Luxembourg and Norway). However, some European countries (Iceland, Ireland, Malta and the UK) have a lump-sum benefit. Lump-sum benefits imply highly regressive replacement rates and are therefore not shown in Figure A2.1.

The other important dimension presented in Figure A2.1 is the waiting period. A waiting period is the amount of time the person must pass on sick leave before being eligible for the benefit. The waiting periods vary between 0 and 3 days in the countries with proportional replacement rates. Three-day waiting periods are found in Southern Europe, the Czech Republic and Estonia. Northern European countries tend to have no waiting periods at all. The waiting period plays a large role in short sickness spells.

The other three dimensions in which European sickness insurance schemes differ are coverage, maximum duration and qualifying period. Coverage is broad for full-time employees in all countries in Europe and varies primarily in terms of how the self-employed are treated. Maximum durations vary slightly between countries such that the mode is at one year. The qualifying periods, that is, the time required at the job before eligibility, vary from none to 6 months.

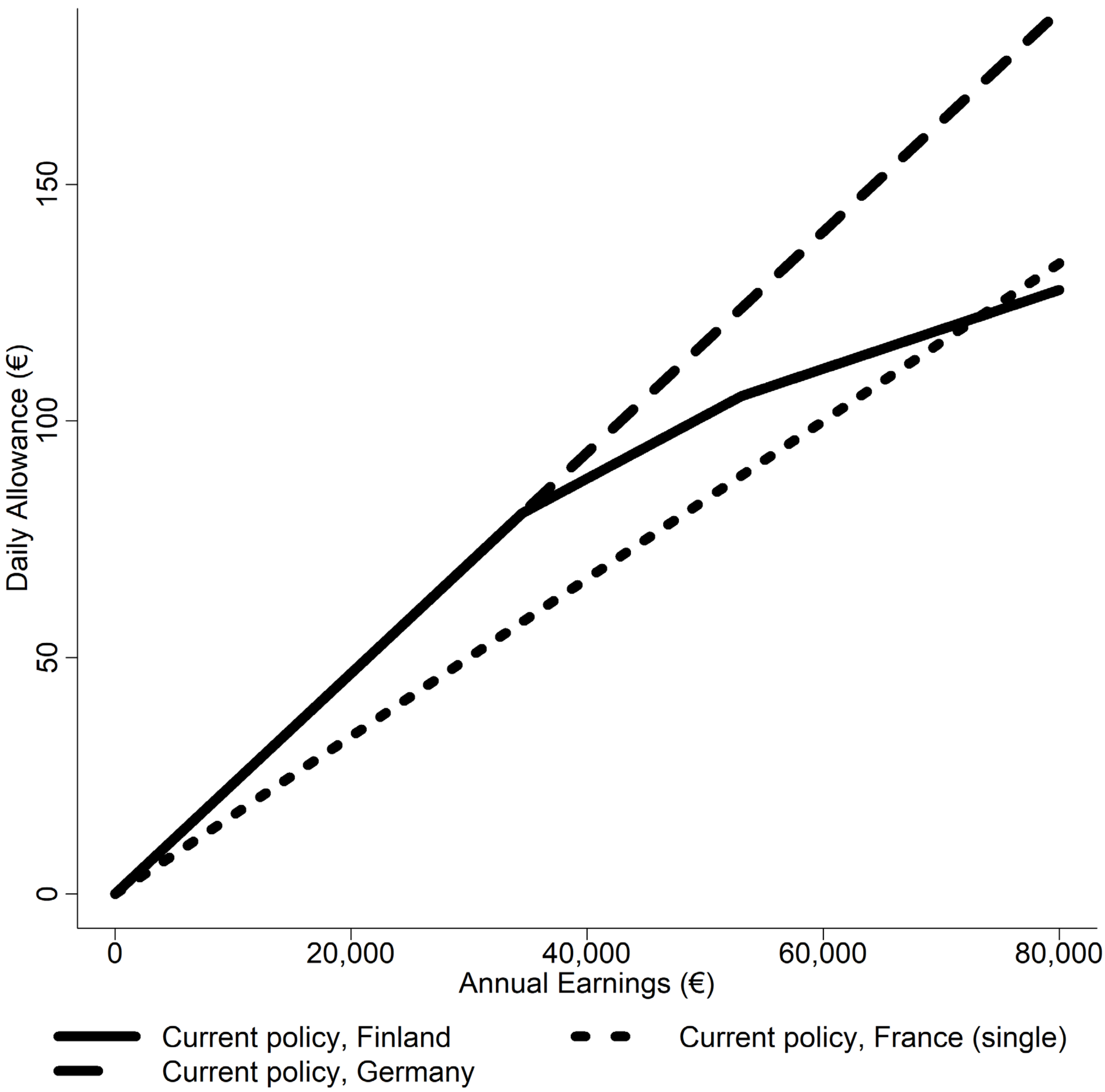
To capture within-country heterogeneity in the replacement rates, Figure A2.1 is insufficient. Some countries, such as Finland, have notably non-linear benefit rules. The benefit curves for Germany, France and Finland are depicted in Figure A2.2.

Figure A2.1. Characteristics of sickness insurance schemes in Europe



Notes. Single, lowest income bracket, initial benefit level, and the general case for long-term employment. Denmark: not defined in the table. Iceland, Ireland, Malta and the UK: lump sum benefit. Switzerland: varies by individual contract. Source: MISSOC comparative table, 2017/07/01.

Figure A2.2. Generosity of sickness insurance schemes in three European countries



Appendix 4. Replication of main results using EU-SILC cross-sectional data

We replicate the main results using the EU Statistics on Income and Living Conditions (EU-SILC) data. The EU-SILC is a harmonized dataset on income, social inclusion and living conditions that covers the material and subjective aspects of well-being.[9] The EU-SILC data are based on a combination of survey and register-based information, depending upon the source country. We use the 2013 data for all 27 countries that were members of the European Union in 2013 (see Appendix 3 for a description of sickness insurance institutions in Europe). In addition, we use data on Iceland, Norway and Switzerland, for a total of 30 countries.

Descriptive statistics are presented in Table 4.1. The subsample that is employed is large (~125,000), whereas the subsample for those on sick leave is substantially smaller (~1,200). Mean life satisfaction is ~0.8 points lower for those on sick leave.

[9] See http://ec.europa.eu/eurostat/web/income-and-living-conditions/overview. The data are public and available to other researchers subject to following the EU procedures, and our method is replicable for sickness insurance and other policy domains.

For the EU-SILC, we use four variables to construct our estimates. To define the working population, we restrict the employed sample to those aged 18 to 64 who usually work more than 30 hours per week (the variable PL060 in EU-SILC). We define the sick leave population as those who usually work less than 30 hours per week due to "disability or illness" (PL120). Our measure of sickness absence thus captures more longer-term sickness that causes the largest financial burden to the health care system. Note that our variable does not allow us to separate the effect of sickness from that of sickness absence. We thus use both terms interchangeably. Our focus is on the policy, which is conditional on sickness absence. The measure of subjective well-being is life satisfaction. It is the best available survey measure of decision utility (Benjamin et al., 2012; 2014a-2014b; for discussion of decision versus experienced utility, see Kahneman et al., 2007). We use the standard life satisfaction question (PW010): "Overall, how satisfied are you with your life nowadays?" For income, we use PPP-adjusted equivalized disposable household income per consumption unit (HX090; see Section 3.5 for a discussion of consumption vs. income).

Using EU-SILC data, Tables 4.1 and 4.2 and Figures 4.1 to 4.4. replicate the main Tables 1 and 2 and Figures 1 to 4 done with the SOEP panel data and discussed in the main text. The results qualitatively concur with the panel estimate with SOEP data. However, since the population is all of Europe and the sickness state parameters are estimated with a different population in the cross-section, we observe that the fixed cost of sickness at around 11 thousand euros is higher than the main estimate we obtain with SOEP data. The difference could partly stem from the fact that in the EU-SILC cross-section the sick individuals are not the same as the those in the healthy state, unlike in the SOEP main estimations. We repeat the analysis at the country level and study correlation of the institutions parameter estimates with those of institutional variables in Appendix 5.

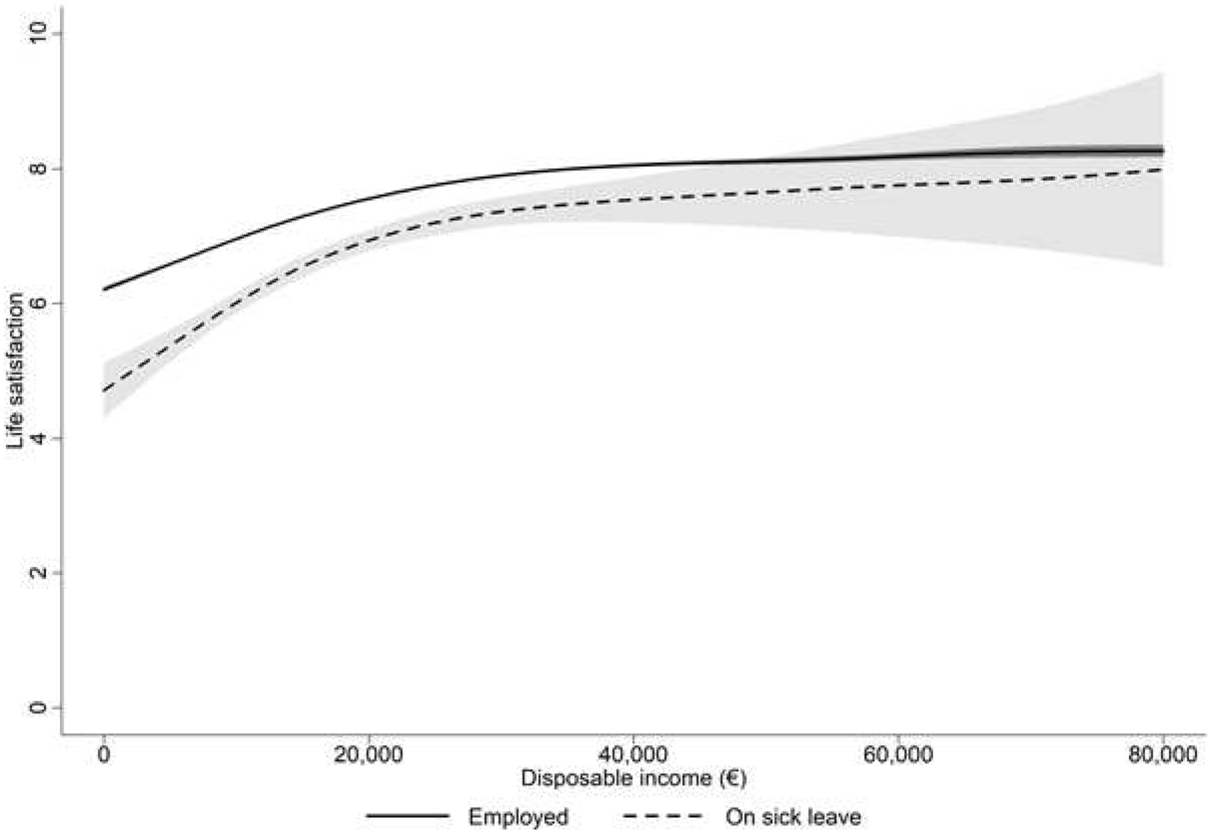
Figures and Tables

Table 4.1. Descriptive statistics

	Employed		On sick leave	
	Mean	SD	Mean	SD
Life satisfaction	7.38	1.79	6.56	2.20
Equivalized disposable income (thousands €)	19.94	15.07	18.39	11.59
Age	43.77	10.77	51.18	8.95
Female	0.45	0.50	0.70	0.46
Tertiary education	0.69	0.46	0.42	0.51
N	125,166	-	1,236	-

Notes. All variable means differ statistically significantly at the 5% level.

Figure 4.1. Spline fit of life satisfaction and income in Europe and UK, employed vs. sick



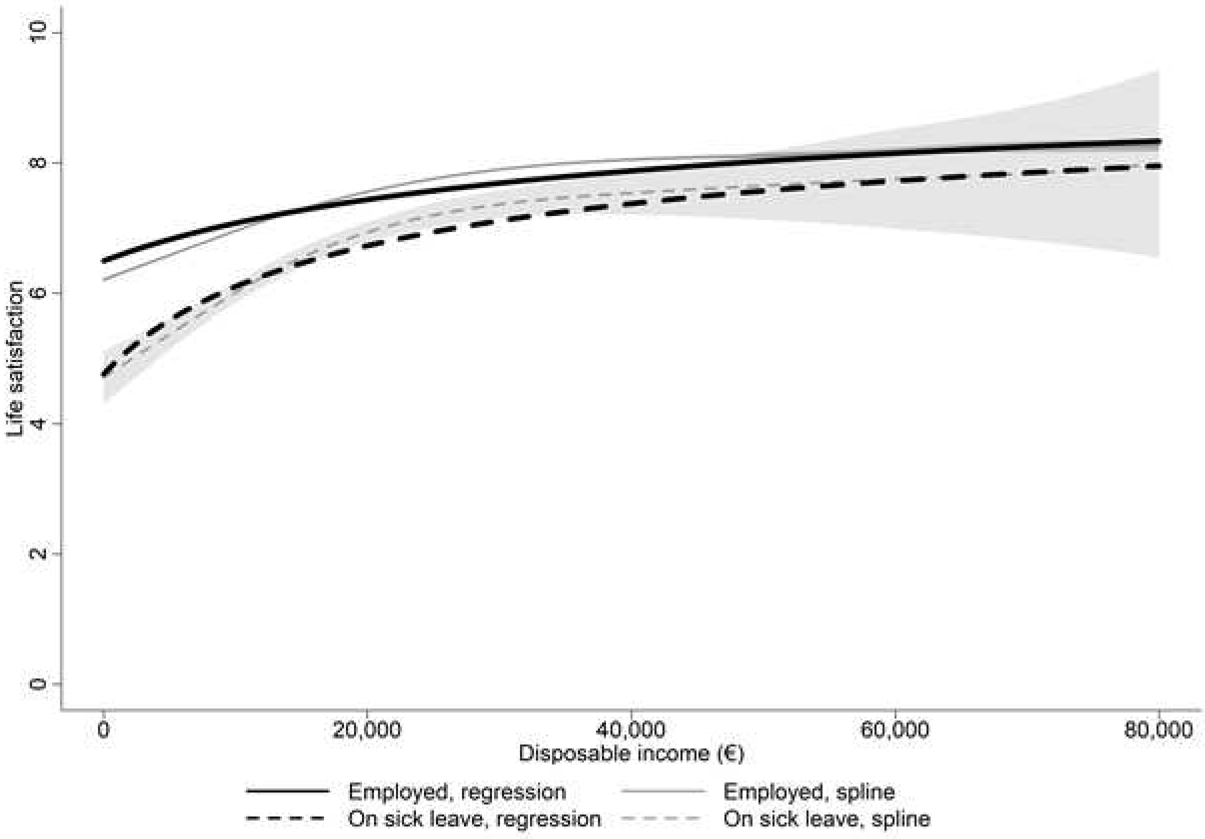
Notes. The estimates are cubic splines with six knots estimated with the R package "bigsplines" for a cross-section using the following parameter values; unrounded, no manual tuning, unique knots assuming Gaussian standard errors. The gray area around the curves represents the 95% confidence interval. Sample size: 125,166 in employment, 1,236 on sick leave.

Table 4.2. Estimation.

Variable	Estimate
Scale parameter (β)	2.85*** (2.10)
Relative risk aversion parameter (γ)	1.49*** (0.04)
Institutions parameter (ω)	-22.37*** (3.18)
Fixed cost of sickness (θ)	11.37*** (1.93)
Level effect of sickness (δ)	-0.28** (0.12)
N	126,402

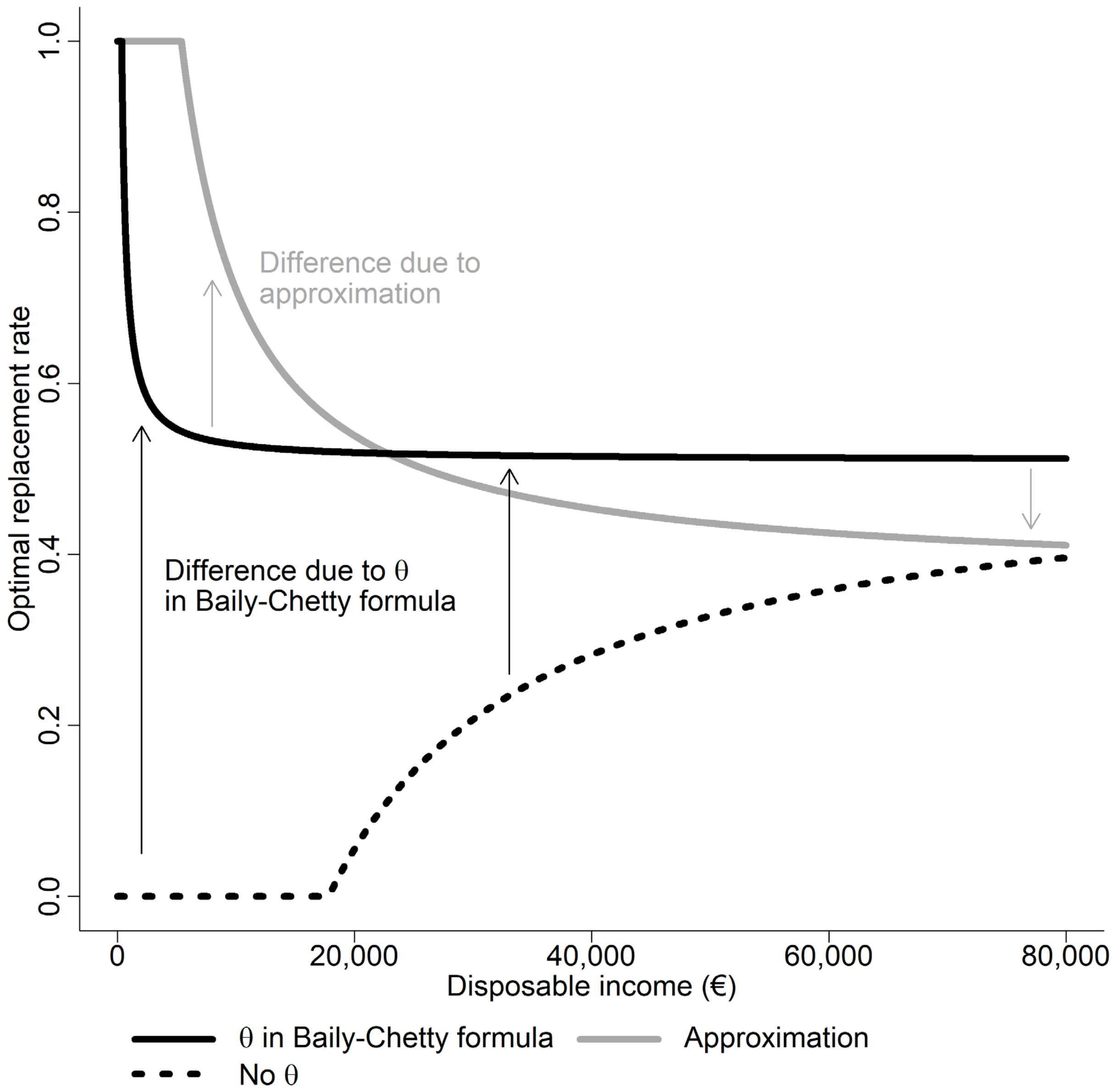
*Notes. Statistical significance: * p<0.1; ** p<0.05; *** p<0.01. The non-linear regression is the fit with a modified Levenberg-Marquardt-type algorithm with sampling weights. The standard errors are in parentheses. All models are estimated with equation (6) with α set at 10. The starting values, where applicable, are:* $\{\beta = 0,\ \gamma = 1.4,\ \omega = -15,\ \theta = 15,\ \delta = 0\}$. *For the estimation, the income variable is in thousands of annual euros.*

Figure 4.2. Spline and non-linear fit of life satisfaction and income in Europe, employed vs. sick



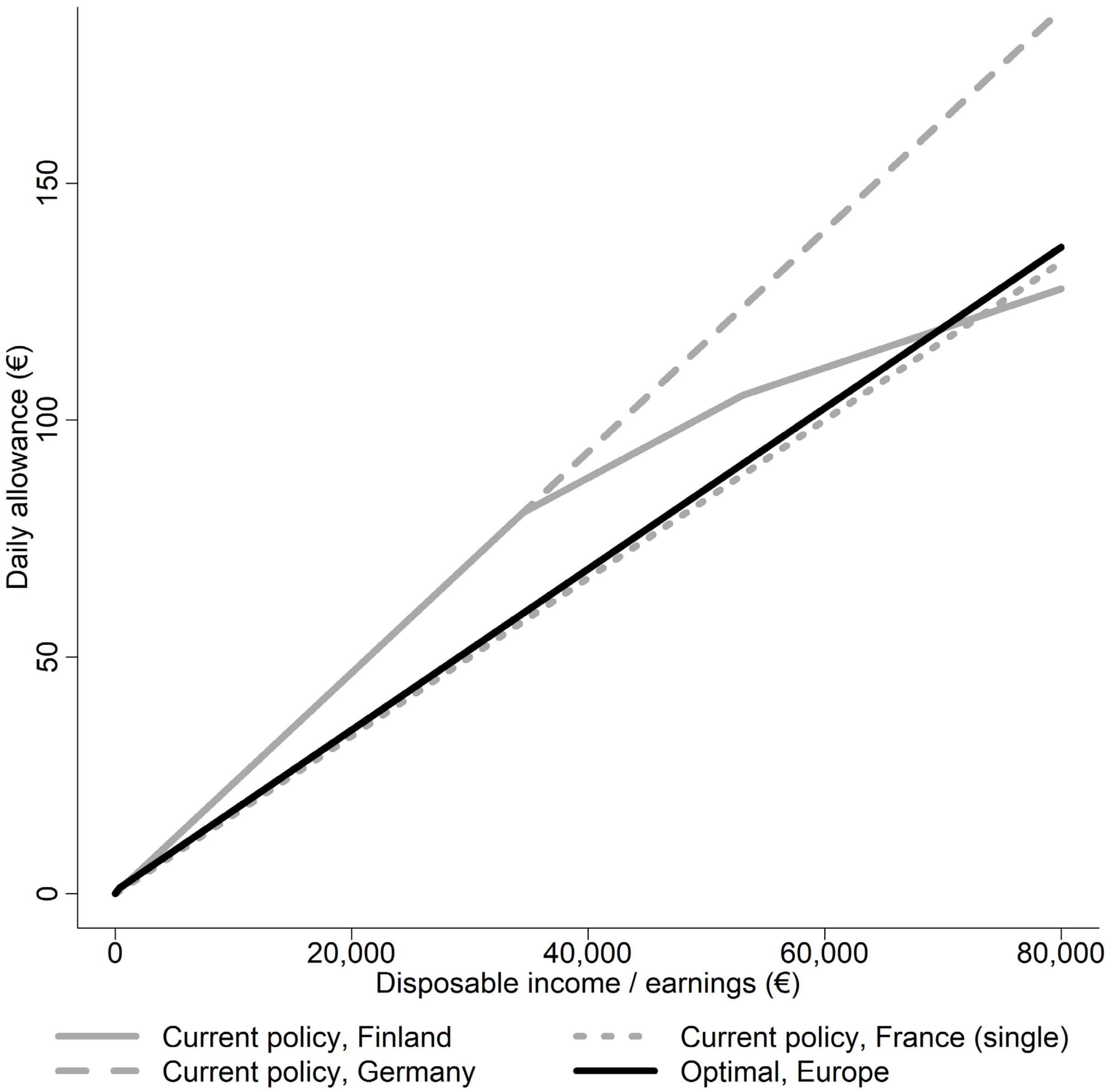
Notes. The non-parametric estimate is a spline fit. The x-axis in the figure is truncated at 80,000 euros. The non-linear regression fit is equation (6) with α set at 10, parameter values in Table 2, model 1, estimated using the whole income distribution. Sample size: in employment 125,166, on sick leave 1,236.

Figure 4.3. Optimal replacement rates



Notes. The optimal replacement rates are calculated with the augmented Baily-Chetty formula (equation 1). The relative risk aversion values are from a generalized CRRA utility function with the parameter values of $\{\gamma,\ \omega,\ \theta\} = \{1.36, -18.5, 9.3\}$ *at different levels of disposable equivalized income which by assumption equals consumption, shown in Table 2, model 2, obtained from estimating equation (7).* θ *is the fixed cost of sickness, which affects the optimal replacement rate through relative risk aversion (RRA) and the augmented Baily-Chetty formula. Additionally, we assume that* $\epsilon_{r,b} + \epsilon_{D,b} = 1.5$. *The approximation is performed using equations (1), (3), (4) and (6).*

Figure 4.4. Prevailing universal sickness insurance policy and estimated optimal curves



Notes. The optimal replacement rates are calculated with the augmented Baily-Chetty formula (equation 1). The relative risk aversion values are from a generalized CRRA utility function with the parameter values of $\{\gamma,\ \omega,\ \theta\} = \{1.36, -18.5, 9.3\}$ *at different levels of disposable equivalized income which by assumption equals consumption, shown in Table 2, model 2, obtained from estimating equation (7).* θ *is the fixed cost of sickness, which affects the optimal replacement rate through relative risk aversion (RRA) and the augmented Baily-Chetty formula. Additionally, we assume that* $\epsilon_{r,b} + \epsilon_{D,b} = 1.5$. *"Single" refers to a one-member household. The optimal curve is based on equivalized disposable income, whereas the current policy curves are based on earnings.*

Appendix 5. Country-specific estimates of ω (i.e., the value of institutions) with EU-SILC cross-sectional data

We replicate the aggregate analysis at the country level. Figure A5.1 presents country-level profiles for the relationship between disposable income and life satisfaction as a spline fit using EU-SILC cross-sectional data. The figures also show the fit of equation (6), where only ω and θ are allowed to vary and all other parameters are held constant at values presented in Table 2, column 2. Table A5.1. shows the sample size by each country.

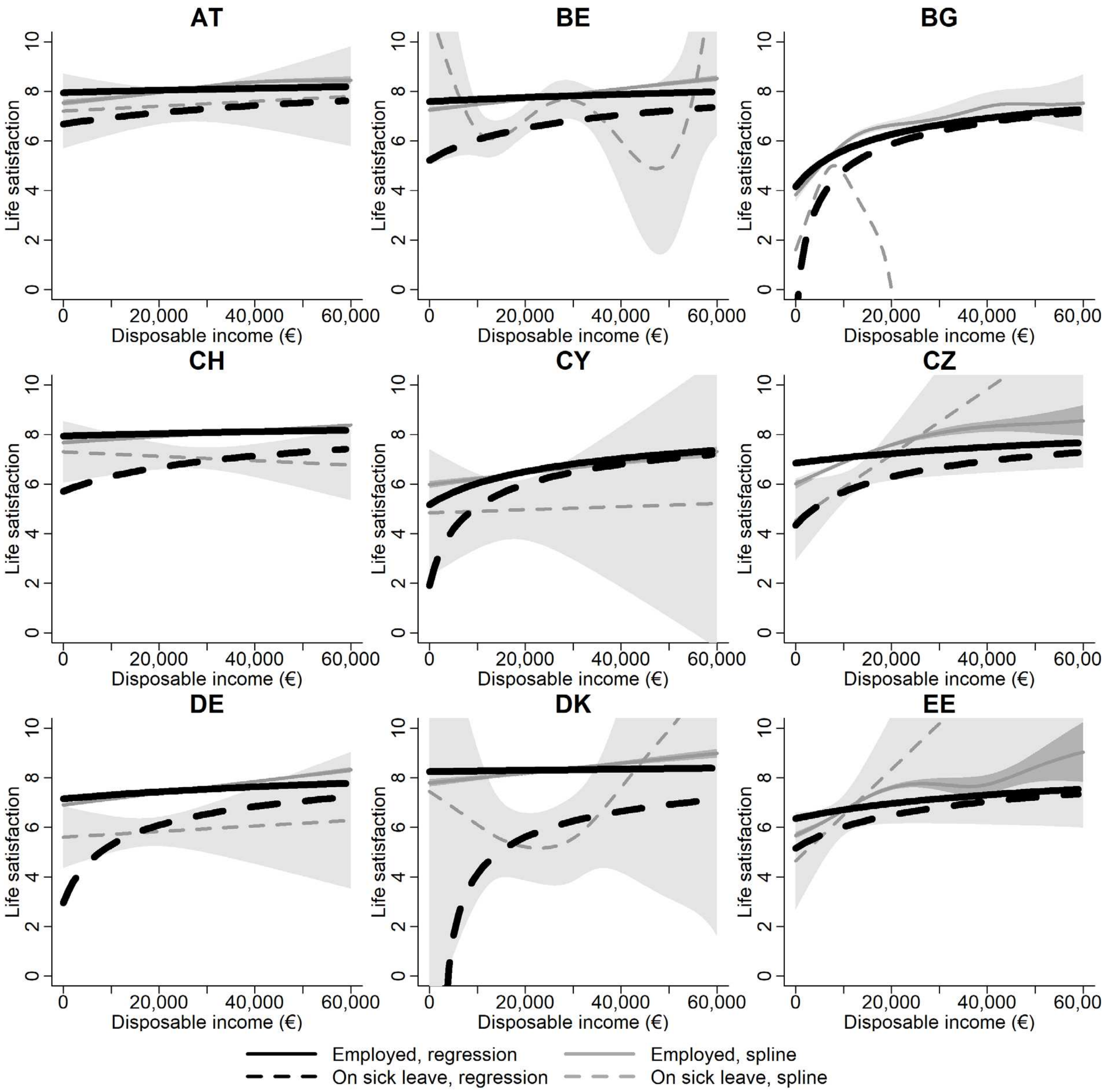
The functional fit follows the pattern of the aggregate fit remarkably well given the smaller sample size for each country. The estimation of ω is robust to the exclusion of low-income individuals in the data because the whole range of incomes is used for the estimation. However, the non-parametric spline is inaccurate for most countries due to small sample size.

We extract the ω parameter point estimates from the country-level fits and correlate them with the measures of institutions (Figure A5.2). The ω parameter is equivalent to giving each citizen an equal increase in income, increasing utility at all income levels. The ω parameter, which measures this shared increase in utility, captures the value of *all* of the characteristics of a country, including for example its institutions, social norms, culture, and geographical features. For brevity, we call ω the institutions parameter. In contrast to Jones and Klenow (2016), the ω parameter abstracts from consumption levels.

We find that the Nordic welfare states have a high ω. By contrast, high-income Southern European countries have a low ω. The high correlation coefficient between the institutions parameter and trust is notable, at 0.80 for interpersonal trust and 0.82 for the mean trust in the police, the legal system and the political system. Additionally, the Gini coefficient of equivalized disposable income has a highly significant correlation with the institutions parameter, at -0.59. The high correlations suggest that the institutions parameter captures something that is of real-world significance. However, one should interpret numeric values with caution, even when the rankings and relative values appear to matter.

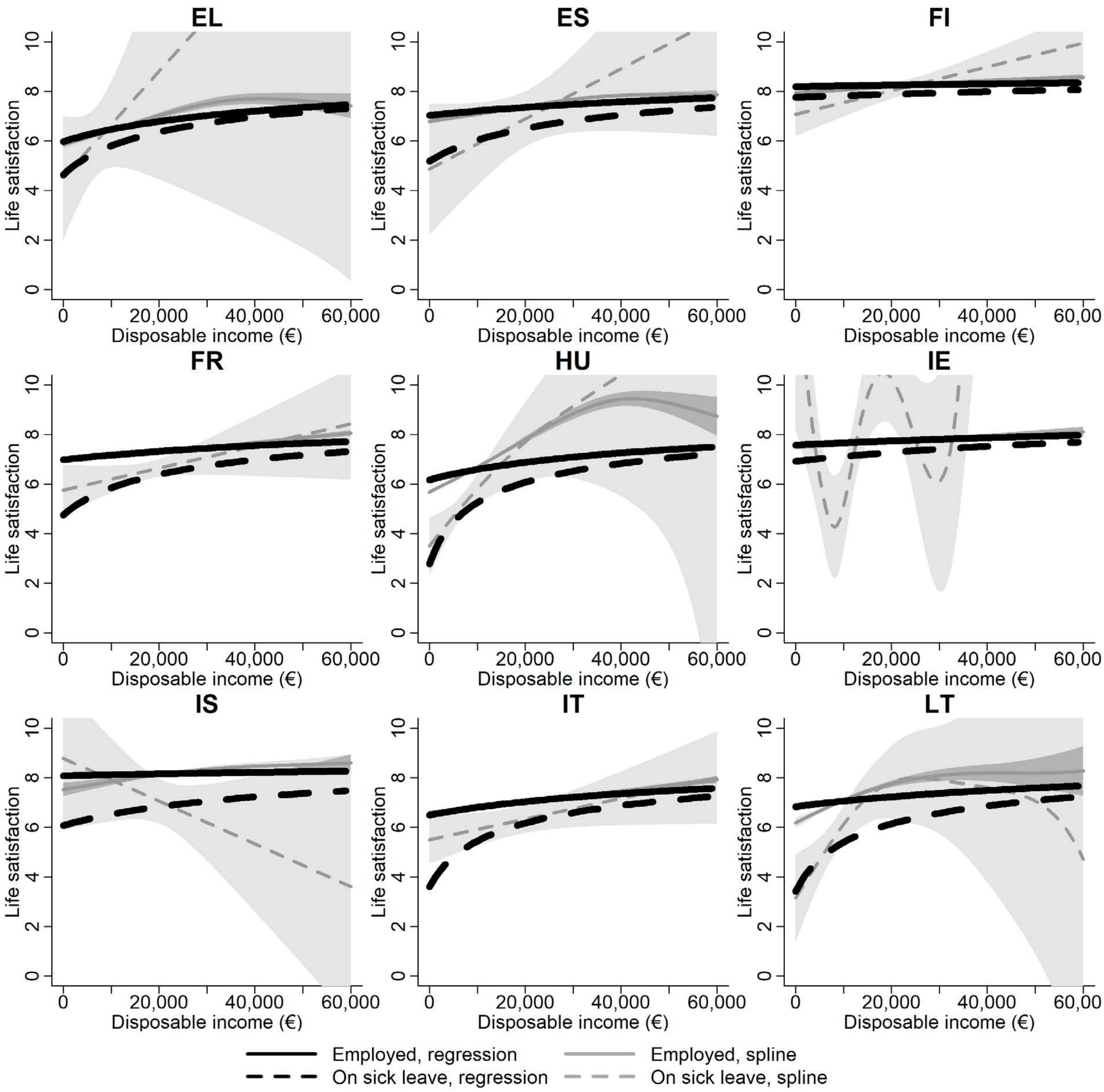
The sickness benefit can affect the value of institutions and the functional form between income and life satisfaction across countries. However, the correlation coefficient between the replacement rate and the estimated value of institutions is low at 0.20 and not statistically significant. Table A5.2. reports the estimated contribution of institutions and equivalized income to the mean utility by country.

Figure A5.1. Spline and non-linear regression fit of life satisfaction and income by country, employed vs. sick



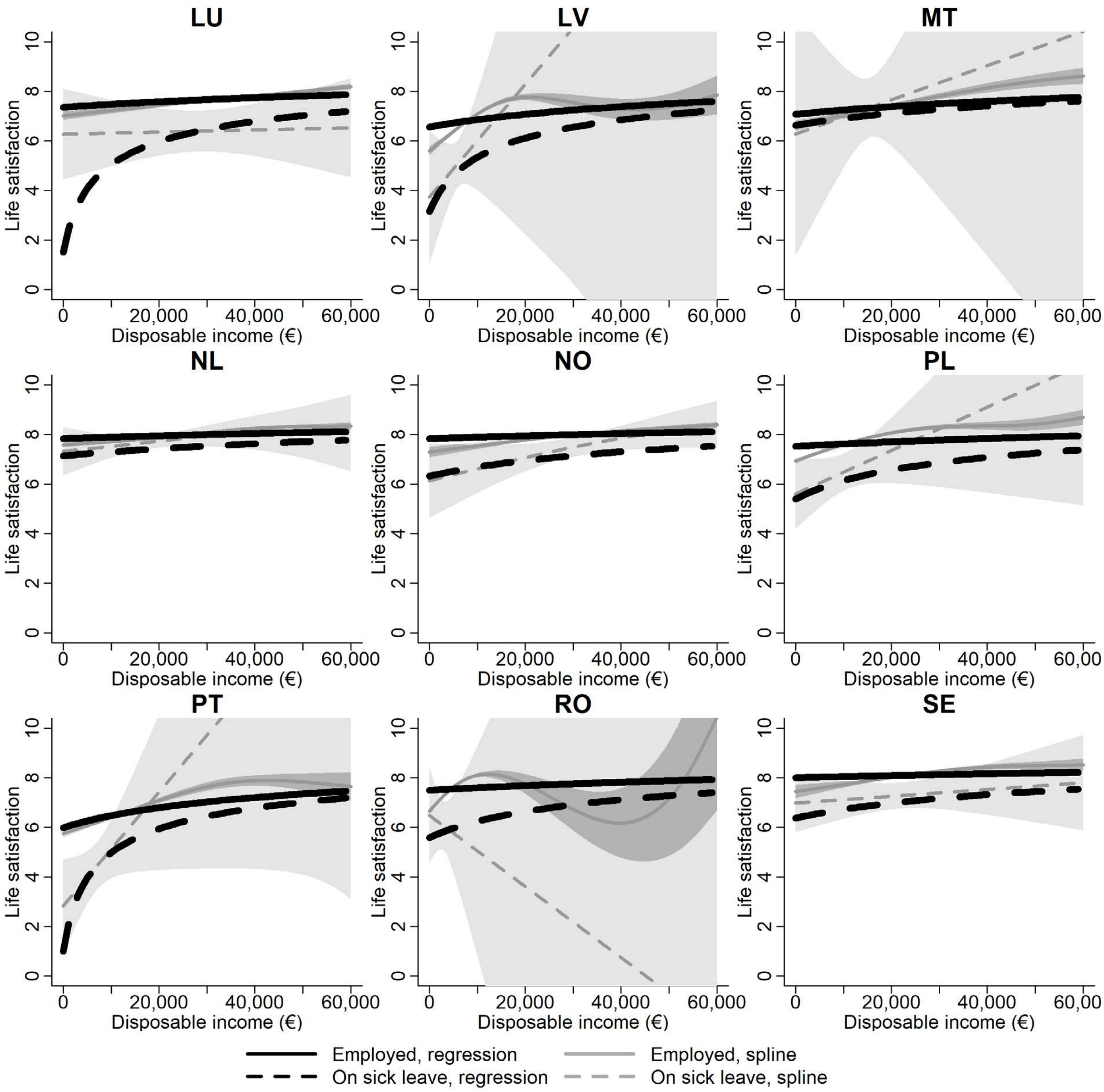
Notes. The estimate is a spline fit. The fit is performed using the whole income distribution, although the x-axis in the figure is truncated at 60,000 euros. Country codes: AT=Austria, BE=Belgium, BG=Bulgaria, CH=Switzerland, CY=Cyprus, CZ=Czech Republic, DE=Germany, DK=Denmark, EE=Estonia.

Figure A5.1 (cont.). Spline and non-linear regression fit of life satisfaction and income by country, employed vs. sick



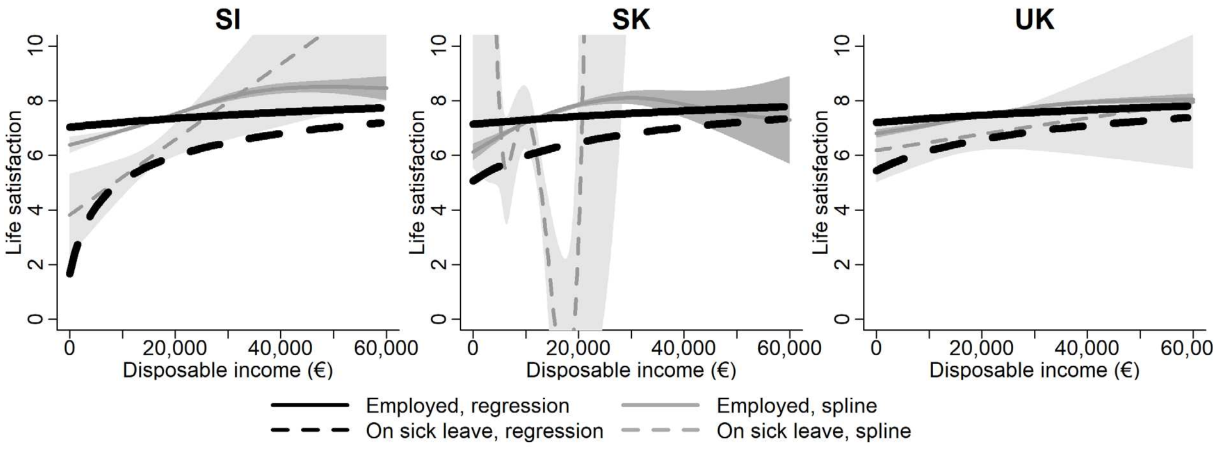
Notes. The estimate is a spline fit. The fit is performed using the whole income distribution, although the x-axis in the figure is truncated at 60,000 euros. Country codes: EL=Greece, ES=Spain, FI=Finland, FR=France, HU=Hungary, IE=Ireland, IS=Iceland, IT=Italy, LT=Lithuania.

Figure A5.1 (cont.). Spline and non-linear regression fit of life satisfaction and income by country, employed vs. sick



Notes. The estimate is a spline fit. The fit is performed using the whole income distribution, although the x-axis in the figure is truncated at 60,000 euros. Country codes: LU=Luxembourg, LV=Latvia, MT=Malta, NL=Netherlands, NO=Norway, PL=Poland, PT=Portugal, RO=Romania, SE=Sweden.

Figure A5.1 (cont.). Spline and non-linear regression fit of life satisfaction and income by country, employed vs. sick

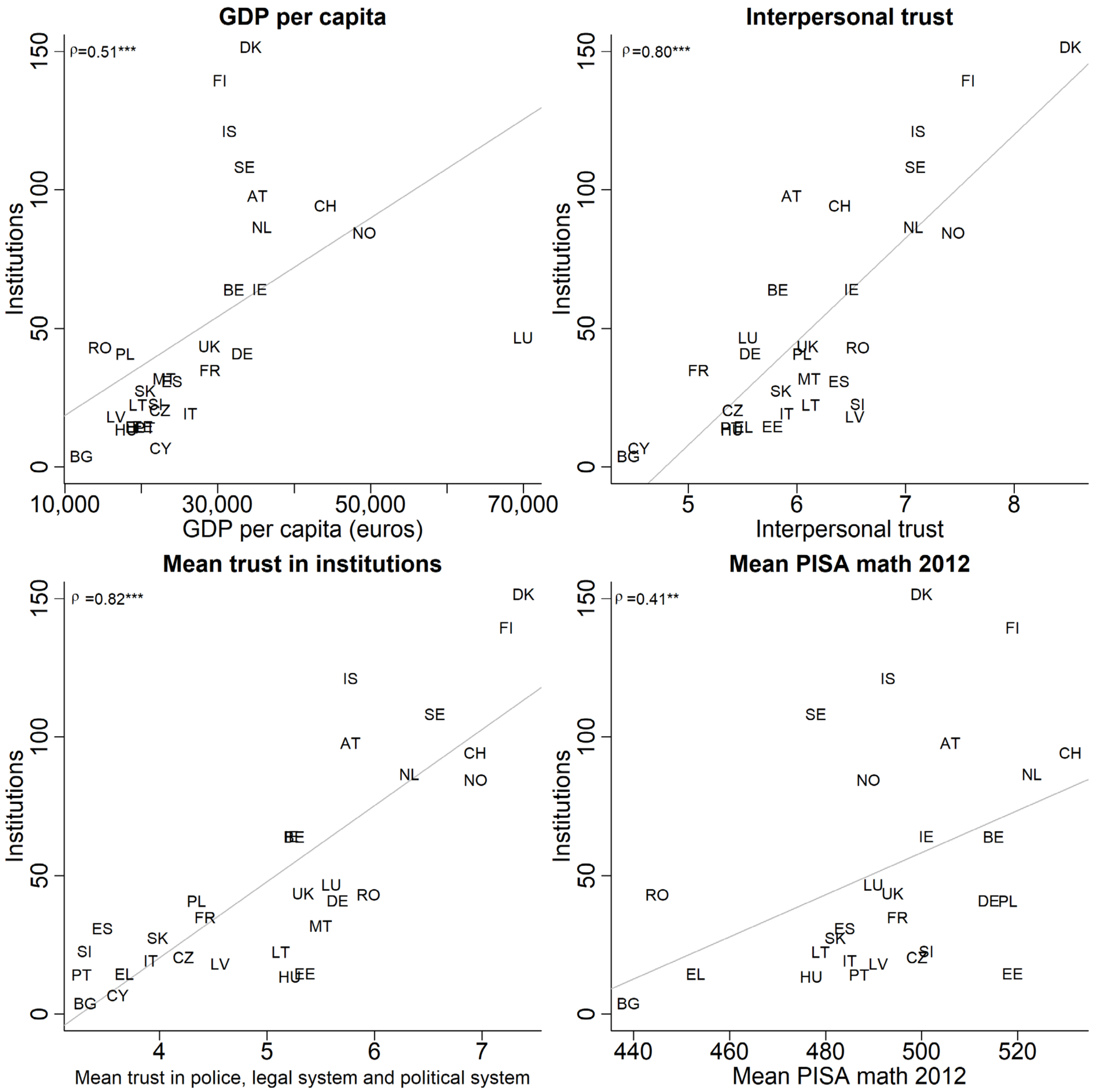


Notes. The estimate is a spline fit. The fit is performed using the whole income distribution, although the x-axis in the figure is truncated at 60,000 euros. Country codes: SI=Slovenia, SK=Slovakia, UK=United Kingdom.

Table A5.1. Sample size by country and subset

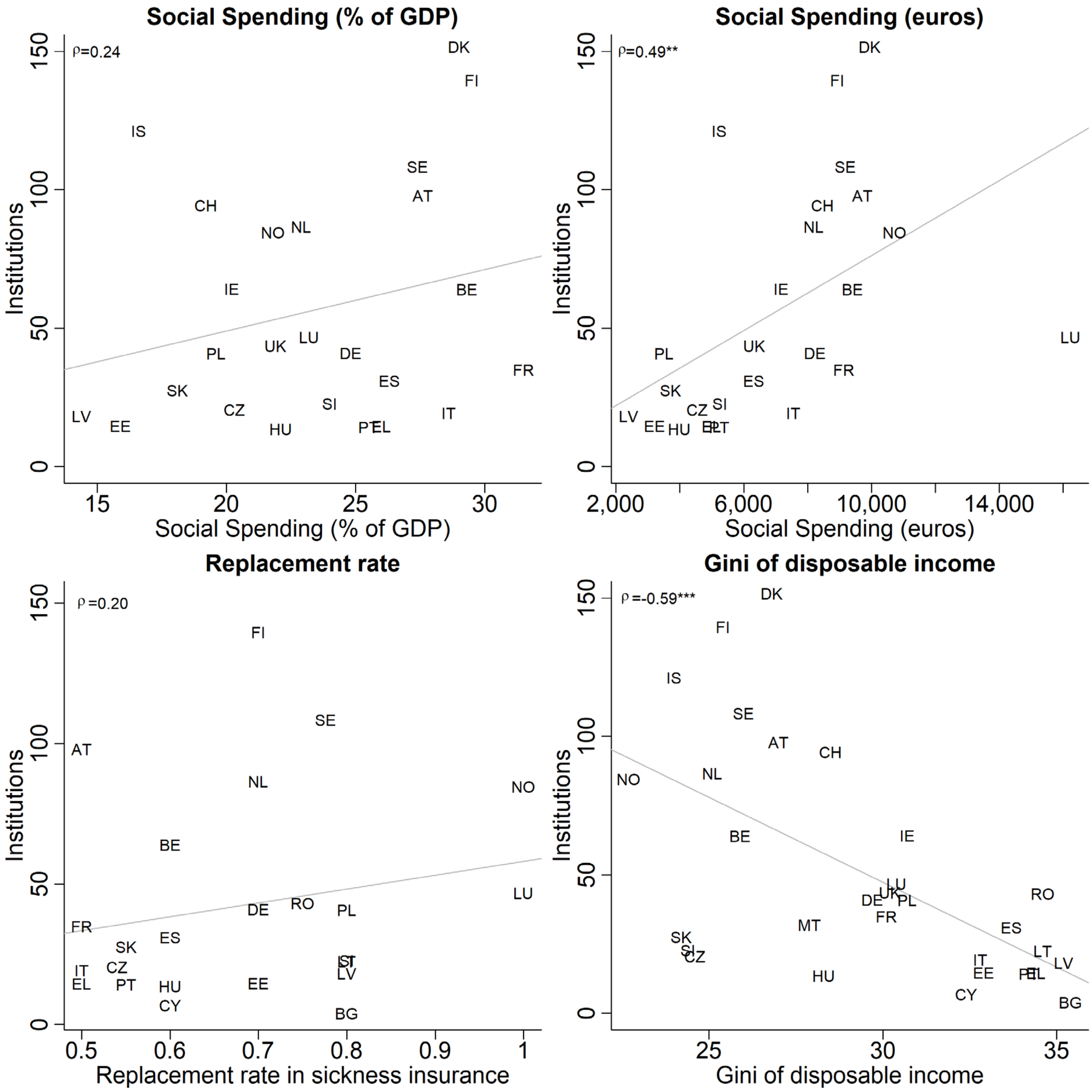
	Employed	On sick leave
Austria	3,805	36
Belgium	3,521	39
Bulgaria	2,757	3
Switzerland	5,143	95
Cyprus	3,944	18
Czech Republic	4,357	45
Germany	6,857	55
Denmark	2,428	14
Estonia	4,077	33
Greece	3,782	16
Spain	7,757	17
Finland	4,782	40
France	5,605	89
Hungary	6,416	94
Ireland	1,637	11
Iceland	1,429	15
Italy	7,984	39
Lithuania	2,839	38
Luxembourg	1,946	31
Latvia	3,257	16
Malta	2,179	6
The Netherlands	3,740	99
Norway	3,046	98
Poland	7,609	31
Portugal	3,649	21
Romania	4,767	14
Sweden	2,606	55
Slovenia	3,722	81
Slovakia	4,729	16
United Kingdom	4,796	71

Figure A5.2. Country-level scatter plots



*Notes. Statistical significance of correlation: * p<0.1; ** p<0.05; *** p<0.01. All figures show a scatter plot and correlations for 30 countries except the top left panel, which is for 25 countries, and the bottom right panel, which is for 28 countries. Source. Income: Eurostat ppp GDP per capita. PISA math: PISA. All other sources: own calculations using EU-SILC.*

Figure A5.2 (cont.). Country-level scatter plots



*Notes. Statistical significance of correlation: * p<0.1; ** p<0.05; *** p<0.01. All figures show a scatter plot and correlations for 25 countries in the top panels and 24 and 30 countries in the bottom left and right panels, respectively. Source. Social spending: OECD, GDP and Gini: Eurostat. Replacement rate: MISSOC (2017). All other sources: own calculations using EU-SILC.*

Table A5.2. Estimated relative contribution of the institutions parameter (ω) and equivalized disposable income at mean income by country

Country	Mean life satisfaction (EU-SILC)	Estimated utility at mean GDP	Estimated utility at zero income	Contribution of income to utility	Relative contribution of institutions to utility (%)
Austria	8.1	7.9	7.6	0.2	96.8
Belgium	7.8	7.6	7.2	0.4	95.0
Bulgaria	5.5	5.4	2.5	3.0	45.1
Switzerland	8.1	7.9	7.6	0.3	96.0
Cyprus	6.5	6.3	3.8	2.5	59.8
Czech Republic	7.2	6.8	5.8	1.0	85.6
Germany	7.5	7.4	6.7	0.6	91.4
Denmark	8.3	8.1	8.0	0.1	98.2
Estonia	6.8	6.5	5.3	1.3	80.6
Greece	6.6	6.5	5.2	1.2	80.8
Spain	7.4	7.1	6.4	0.7	90.4
Finland	8.3	8.0	7.9	0.1	98.2
France	7.4	7.2	6.5	0.7	90.6
Hungary	6.6	6.4	5.1	1.3	80.1
Ireland	7.8	7.6	7.2	0.4	94.6
Iceland	8.2	8.0	7.8	0.2	97.8
Italy	7.1	6.9	5.7	1.1	83.3
Lithuania	7.1	6.8	5.9	0.8	87.9
Luxembourg	7.7	7.8	6.9	0.9	88.7
Latvia	6.9	6.5	5.6	0.9	86.0
Malta	7.4	7.1	6.4	0.6	91.0
Netherlands	8.0	7.8	7.5	0.3	96.2
Norway	8.0	7.9	7.5	0.4	95.1
Poland	7.6	7.1	6.7	0.4	94.4
Portugal	6.6	6.5	5.2	1.3	79.9
Romania	7.6	7.1	6.8	0.3	95.5
Sweden	8.1	7.9	7.7	0.2	97.3
Slovenia	7.3	6.8	6.0	0.9	87.2
Slovakia	7.3	6.9	6.2	0.7	90.1
United Kingdom	7.5	7.3	6.8	0.5	92.7

Notes. The values presented in columns 3–6 are based on a fit of equation (7), in which only ω and θ are allowed to vary and all other parameters are held constant at values presented in Table 2, column 2.