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Abstract

This study explores class struggle between workers and capitalists in a Schumpeterian economy. We consider the limit on the market power of monopolistic firms as a government policy instrument and derive its optimal levels for workers and capitalists, respectively. Capitalists prefer more powerful monopolistic firms than workers. However, even workers prefer monopolistic firms to have some market power because profit provides incentives for innovation. Workers' utility-maximizing degree of monopoly power is decreasing in their discount rate and increasing in innovation productivity and the quality step size. Capitalists' utility-maximizing degree of monopoly power is increasing in the quality step size. We use the difference in the utility-maximizing degrees of monopoly power for workers and capitalists to measure the severity of their conflict, which becomes less severe when workers' discount rate falls or innovation productivity rises. Finally, at a small (large) quality step size, enlarging the step size mitigates (worsens) their conflict.

JEL classification: O30, O40, E11
Keywords: economic growth, workers, capitalists, class struggle

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"What are the common wages of labour, depends everywhere upon the contract usually made between those two parties [workers and capitalists], whose interests are by no means the same." Adam Smith (1776, p. 81)

1 Introduction

The Chinese government's action in limiting the market power of its tech giants is referred to as "de-tycoonification" by *The Economist*.[1] A purpose of this de-tycoonification is to maintain social stability by softening the conflict of interests between workers and capitalists. This study uses a Schumpeterian growth model to explore this issue. According to Dutt (1990), the degree of monopoly power can capture the rate of capitalists' exploitation on workers. Therefore, we consider the limit on the market power of monopolistic firms as a policy instrument and derive its optimal levels for workers and capitalists, respectively. Our results are summarized as follows.

Due to their ownership of monopolistic firms, capitalists prefer a higher degree of monopoly power than workers. Interestingly, even workers may prefer monopolistic firms to have some market power because monopolistic profit provides incentives for innovation. However, an increase in monopolistic profit reduces the labor share of income, so workers prefer a lower degree of monopoly power than capitalists, who receive monopolistic profit. When the government puts more emphasis on workers relative to capitalists, it limits the market power of monopolistic firms, at the expense of innovation and economic growth as some analysts on China are anticipating.[2] Workers' utility-maximizing degree of monopoly power is decreasing in their discount rate and increasing in innovation productivity and the quality step size, whereas capitalists' utility-maximizing degree of monopoly power is increasing in the quality step size.

We use the difference in the utility-maximizing degrees of monopoly power for workers and capitalists to measure the severity of their conflict of interests. We find that their conflict becomes less severe when the discount rate falls or innovation productivity rises. Intuitively, the benefit of monopolistic profit for workers comes solely from innovation, so a fall in their discount rate or a rise in innovation productivity enables workers to benefit more from economic growth. As for the quality step size, its effect on the severity of their conflict is U-shaped. Specifically, at a small (large) quality step size, enlarging the size of quality improvement mitigates (worsens) their conflict of interests. We discuss the intuition of this result in the main text.

Harris (1978), Marglin (1984) and Dutt (1990) are early studies in the literature on Marxian growth theory; see Dutt and Veneziani (2019, 2020) for recent studies. Studies in this literature follow the tradition of Solow (1956) by considering physical/human capital accumulation as the growth engine. We differ from studies in this literature by exploring the conflict of interests between workers and capitalists, which is commonly referred to as class struggle, in a Schumpeterian growth model in which the economy is characterized

[1] https://www.economist.com/business/2021/04/08/chinas-rulers-want-more-control-of-big-tech

[2] https://www.cnbc.com/2021/06/30/chinas-crackdown-on-tech-firms-will-hurt-economic-growth-says-analyst.html

by monopolistic competition and features market-driven innovation as the growth engine. Kalecki (1971) emphasized the importance of imperfect competition in the analysis of class struggle and wrote that "only by [...] penetrating the world of imperfect competition [...] are we able to draw any reasonable conclusion on the impact of bargaining for wages on the distribution of income."

This study relates most closely to the literature on innovation and economic growth. The seminal study in this literature is Romer (1990), who also emphasizes the importance of imperfect competition and develops the first R&D-based growth model in which economic growth is due to the development of new products. Then, Aghion and Howitt (1992) develop the Schumpeterian growth model in which economic growth is driven by the quality improvement of products.[3] Subsequent studies apply the Schumpeterian model to explore various policy instruments, including patent breadth that also determines monopoly power. Li (2001) explores the effects of patent breadth on innovation, whereas Goh and Olivier (2002), Chu (2011), Zeng *et al.* (2014), Saito (2017), Iwaisako (2020) and Yang (2021) derive optimal patent breadth for a representative household. Chu (2010), Chu and Cozzi (2018) and Chu *et al.* (2021) analyze the effects of patent breadth on income inequality of heterogeneous households. Chu (2008) incorporates special interest politics into the Schumpeterian model to analyze how campaign contributions and political lobbying affect the level of patent protection. This study contributes to this literature by exploring the political economics behind the market power of monopolistic firms and comparing the different degrees of monopoly power preferred by workers and capitalists.

2 A Schumpeterian growth model with class struggle

In the Schumpeterian growth model developed by Aghion and Howitt (1992),[4] innovation is driven by quality improvement. Given that the Schumpeterian model has been studied extensively, we omit some details. The key modification is that we replace the representative household by two distinct classes: workers and capitalists.[5]

2.1 Capitalists and workers

Capitalists and workers, indexed by $i \in \{c, w\}$ respectively, have lifetime utility:

$$U^i = \int_0^\infty e^{-\rho t} \ln c_t^i dt, \tag{1}$$

[3]See also Segerstrom *et al.* (1990) and Grossman and Helpman (1991) for other early studies and Aghion *et al.* (2014) for a survey.

[4]Here we follow the treatment in Grossman and Helpman (1991).

[5]We assume homogeneous workers and homogeneous capitalists in the main text. However, our results are robust to heterogeneous workers and heterogeneous capitalists; see Appendix B for the derivations.

where $\rho > 0$ is the discount rate. c_t^c denotes consumption of capitalists at time t whereas c_t^w denotes consumption of workers. Workers supply one unit of labor to earn wage w_t. They simply consume their wage income, such that $c_t^w = w_t$.

Capitalists own assets and do not work. The asset-accumulation equation is

$$\dot{a}_t = r_t a_t - c_t^c, \tag{2}$$

where a_t is the value of assets (i.e., the share of monopolistic firms) and r_t is the interest rate. Dynamic optimization yields the consumption path of capitalists as

$$\frac{\dot{c}_t^c}{c_t^c} = r_t - \rho. \tag{3}$$

2.2 Final good

Competitive firms use the following Cobb-Douglas aggregator to produce final good y_t:

$$y_t = \exp\left(\int_0^1 \ln x_t(j) dj\right), \tag{4}$$

in which $x_t(j)$ for $j \in [0,1]$ denotes differentiated intermediate goods. Maximizing profit yields the conditional demand function:

$$x_t(j) = \frac{y_t}{p_t(j)}, \tag{5}$$

where $p_t(j)$ denotes the price of $x_t(j)$.

2.3 Intermediate goods

The economy features a unit continuum of monopolistic industries that produce intermediate goods. Each industry is dominated by a temporary industry leader (who owns the latest innovation) until the arrival of the next innovation. The production function of the leader in industry $j \in [0,1]$ is

$$x_t(j) = z^{q_t(j)} l_t(j), \tag{6}$$

where $z > 1$ is the quality step size, $q_t(j)$ is the number of quality improvements that have occurred in industry j as of time t, and $l_t(j)$ is production labor.

Given the productivity level $z^{q_t(j)}$, the marginal cost of the leader in industry j is $w_t/z^{q_t(j)}$. From the Bertrand competition between the current leader and the previous leader, the profit-maximizing price for the current leader is

$$p_t(j) = \mu \frac{w_t}{z^{q_t(j)}}, \tag{7}$$

where $\mu \in (1, z]$ is the markup ratio. Grossman and Helpman (1991) and Aghion and Howitt (1992) assume that the markup μ is equal to the quality step size z. Here we consider $\mu \leq z$ as a policy instrument of the government, which uses its authority to limit the market power of monopolistic firms as in China recently.[6]

The wage payment in industry j is

$$w_t l_t(j) = \frac{1}{\mu} p_t(j) x_t(j) = \frac{1}{\mu} y_t, \tag{8}$$

whereas the monopolistic profit is

$$\pi_t(j) = p_t(j) x_t(j) - w_t l_t(j) = \frac{\mu - 1}{\mu} y_t. \tag{9}$$

Equation (8) shows that $w_t l_t / y_t$ is decreasing in the markup μ, which is interpreted as capitalists' exploitation on workers in Marxian economics.

2.4 R&D

Equation (9) shows that $\pi_t(j) = \pi_t$. The value of inventions is symmetric across industries such that $v_t(j) = v_t$ for $j \in [0, 1]$.[7] The no-arbitrage condition that determines v_t is

$$r_t = \frac{\pi_t + \dot{v}_t - \lambda_t v_t}{v_t}. \tag{10}$$

Intuitively, the no-arbitrage condition equates the interest rate r_t to the rate of return on v_t given by the sum of monopolistic profit π_t, capital gain $\dot{v}_t$ and expected capital loss $\lambda_t v_t$, where λ_t is the arrival rate of innovation. When the next innovation occurs, the previous technology becomes obsolete.[8]

Competitive entrepreneurs devote R_t units of final good to perform innovation in each industry. We specify the arrival rate of innovation as

$$\lambda_t = \frac{\varphi R_t}{Z_t}, \tag{11}$$

where $\varphi > 0$ is a productivity parameter and Z_t denotes aggregate technology, which captures an increasing-difficulty effect of R&D. The free-entry condition for R&D is

$$\lambda_t v_t = R_t \Leftrightarrow \frac{\varphi v_t}{Z_t} = 1, \tag{12}$$

where the second equality uses (11).

[6] Li (2001) interprets $\mu < z$ as incomplete patent breadth.

[7] See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium.

[8] See Cozzi (2007) for a discussion on the Arrow replacement effect.

2.5 Economic growth

Aggregate technology Z_t is defined as

$$Z_t \equiv \exp\left(\int_0^1 q_t(j)dj \ln z\right) = \exp\left(\int_0^t \lambda_\omega d\omega \ln z\right), \tag{13}$$

which uses the law of large numbers and equates the average number of quality improvements $\int_0^1 q_t(j)dj$ that have occurred to the average number of innovation arrivals $\int_0^t \lambda_\omega d\omega$ as of time t. Differentiating the log of Z_t with respect to time yields

$$g_t \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z. \tag{14}$$

Substituting (6) into (4) yields the aggregate production function:

$$y_t = \exp\left(\int_0^1 q_t(j)dj \ln z + \int_0^1 \ln l_t(j)dj\right) = Z_t, \tag{15}$$

where we have used the symmetry condition and the resource constraint: $l_t(j) = l_t = 1$. Therefore, the growth rate of final good y_t is also g_t, which is determined by λ_t as in (14).

Using $\dot{c}_t^c/c_t^c = g_t$ and (3) in (10), we derive the balanced-growth value of v_t as

$$v_t = \frac{\pi_t}{\rho+\lambda} = \frac{\mu-1}{\mu}\frac{Z_t}{\rho+\lambda}, \tag{16}$$

which uses (9) and (15). Equation (16) shows that v_t is increasing in the markup μ. Substituting (16) into (12) yields

$$\lambda^* = \frac{\mu-1}{\mu}\varphi - \rho, \tag{17}$$

which is the steady-state arrival rate of innovation and increasing in the markup μ. The steady-state growth rate is

$$g^* = \lambda^* \ln z = \left(\frac{\mu-1}{\mu}\varphi - \rho\right)\ln z, \tag{18}$$

which is also increasing in the markup μ.[9]

3 Conflict between workers and capitalists

We now derive the utility-maximizing degrees of monopoly power for capitalists and workers, respectively. Given that the economy is always on the balanced growth path,[10] we can rewrite (1) as

$$U^i = \frac{1}{\rho}\left(\ln c_0^i + \frac{g^*}{\rho}\right) \tag{19}$$

[9]This result originates from Li (2001), who analyzes patent breadth in the Schumpeterian model.

[10]Appendix A shows that the economy always jumps to the unique balanced growth path.

for $i \in \{c, w\}$. The resource constraint on final good is

$$y_t = c_t^c + c_t^w + R_t. \tag{20}$$

Using (8) and (15), we derive the consumption of workers as

$$c_t^w = w_t l_t = \frac{y_t}{\mu} = \frac{Z_t}{\mu}, \tag{21}$$

which is decreasing in the markup μ. Using (11) and (17), we derive the level of R&D as

$$R_t = \frac{\lambda_t Z_t}{\varphi} = \left(\frac{\mu - 1}{\mu}\varphi - \rho\right)\frac{Z_t}{\varphi}. \tag{22}$$

Substituting (15), (21) and (22) into (20) yields

$$c_t^c = y_t - c_t^w - R_t = \frac{\rho}{\varphi} Z_t, \tag{23}$$

which is independent of the markup μ. It is useful to note that $c_t^c = \pi_t - R_t$ is independent of μ because both π_t and R_t are increasing in μ.

Substituting (18) and (23) into (19) yields the welfare function of capitalists as

$$U^c = \frac{1}{\rho}\left[\ln\left(\frac{\rho Z_0}{\varphi}\right) + \left(\frac{\mu - 1}{\mu}\varphi - \rho\right)\frac{\ln z}{\rho}\right], \tag{24}$$

where initial technology Z_0 is exogenous. U^c is increasing in μ due to its positive effect on economic growth. Therefore, the capitalists prefer the maximum markup:

$$\mu^c = z, \tag{25}$$

which is increasing in the quality step size.[11] Substituting (18) and (21) into (19) yields the welfare function of workers as

$$U^w = \frac{1}{\rho}\left[\ln\left(\frac{Z_0}{\mu}\right) + \left(\frac{\mu - 1}{\mu}\varphi - \rho\right)\frac{\ln z}{\rho}\right]. \tag{26}$$

The level of markup that maximizes U^w is

$$\mu^w = \max\left\{\frac{\varphi \ln z}{\rho}, 1\right\}. \tag{27}$$

The intuition for μ^w can be explained as follows. Monopoly power provides incentives for innovation, so even workers may prefer monopolistic firms to have some market power. This is the case when innovation productivity is sufficiently high (i.e., $\varphi > \rho / \ln z$). Given

[11] This upper bound on the markup arises from the constraint due to the Bertrand competition. If current industry leaders can consolidate market power with previous industry leaders, then they would choose an even higher markup, which however would still be proportional the quality step size z; see O'Donoghue and Zweimuller (2004) for such an analysis.

that the benefit of monopoly power for workers comes solely from innovation, a fall in their discount rate or a rise in innovation productivity or a larger quality step size enables workers to benefit more from economic growth. Therefore, μ^w is increasing in R&D productivity φ and the quality step size z but decreasing in the discount rate ρ. We impose the following parameter restriction:[12]

$$\frac{\varphi \ln z}{\rho} < z, \tag{28}$$

which ensures that $\mu^w < \mu^c$. Workers prefer less powerful monopolistic firms than capitalists because a larger markup reduces the labor share of income given by $w_t l_t / y_t = 1/\mu$.

Proposition 1 *Given (28), workers prefer a lower markup than capitalists, who prefer the maximum markup $\mu^c = z$. If $\varphi \leq \rho / \ln z$, workers prefer a zero markup (i.e., $\mu^w = 1$). If $\rho / \ln z < \varphi < z\rho / \ln z$, workers prefer a positive markup (i.e., $\mu^w > 1$), which is rising in R&D productivity φ and the quality step size z but decreasing in the discount rate ρ.*

Proof. Compare (25) and (27). Then, use (27) to show that μ^w is increasing in φ and z but decreasing in ρ. ■

Suppose both workers and capitalists try to influence the government. We follow Grossman and Helpman (2001) to specify the government's objective function as

$$\widetilde{U} \equiv \theta U^w + (1-\theta) U^c = \frac{1}{\rho}\left[\theta \ln\left(\frac{Z_0}{\mu}\right) + (1-\theta)\ln\left(\frac{\rho Z_0}{\varphi}\right) + \left(\frac{\mu - 1}{\mu}\varphi - \rho\right)\frac{\ln z}{\rho}\right], \tag{29}$$

where $\theta \in (0,1)$ is the weight that the government places on workers relative to capitalists and captures the power of workers in their class struggle against capitalists. Maximizing (29), the government chooses

$$\widetilde{\mu} = \min\left\{\frac{\varphi \ln z}{\theta \rho}, z\right\} \in (\mu^w, \mu^c], \tag{30}$$

which is decreasing in θ. For example, as the Chinese government puts more emphasis on workers, it reduces the market power of monopolistic firms at the expense of economic growth because monopolistic profit serves as incentive for innovation, which is a core element in R&D-based growth theory. As Jones (2019) nicely summarizes, "imperfect competition provides the profits that incentivize entrepreneurs to innovate."

Proposition 2 *A larger weight θ on workers in the government's objective function leads to a lower market power of monopolistic firms, which reduces innovation and growth.*

Proof. Use (30) to show that $\widetilde{\mu}$ is decreasing in θ. Use (17) and (18) to show that λ^* and g^* are increasing in μ. ■

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[12] If this inequality does not hold, then even workers would prefer the maximum level of markup such that $\mu^w = z$, which is neither realistic nor interesting.

The government chooses $\widetilde{\mu}$ to balance the conflict of interests between workers and capitalists but cannot satisfy both groups unless they prefer the same degree of monopoly power. Therefore, we use the difference in the utility-maximizing degrees of monopoly power for workers and capitalists to measure the severity of their conflict. Formally,

$$\sigma \equiv \mu^c - \mu^w = z - \frac{\varphi \ln z}{\rho}, \tag{31}$$

which is increasing in the discount rate ρ and decreasing in R&D productivity φ. Intuitively, a fall in the workers' discount rate or a rise in innovation productivity enables them to benefit more from economic growth and increases their utility-maximizing degree of monopoly power towards that of the capitalists. As a result, the tension between workers and capitalists falls.

As for the quality step size z, its effect on σ is U-shaped. Specifically, at a small (large) quality step size, raising the step size z reduces (raises) σ. A larger quality step size z increases the utility-maximizing degrees of monopoly power for both workers and capitalists. For workers, a larger quality step size affects their utility via its positive effect on economic growth, captured by the term $\ln z$. For capitalists, a larger quality step size affects their utility via its positive effect on monopolistic profit, captured by the term $\mu = z$. The *growth* effect is relatively strong at a small z, whereas the *profit* effect is relatively strong at a large z. Therefore, at a small (large) z, enlarging the quality step size closes (widens) the gap between the different utility-maximizing degrees of monopoly power for workers and capitalists.

Proposition 3 *The severity σ of the conflict of interests between workers and capitalists is decreasing in R&D productivity φ, increasing in the discount rate ρ and U-shaped in the quality step size z.*

Proof. Use (31) to show that σ is decreasing in φ, increasing in ρ and U-shaped in z. ■

4 Discussion

In this study, we have explored the determinants of the class struggle between workers and capitalists but not its destructive consequences on the society. One can reasonably specify a process in which the probability of social unrest is increasing in σ. Our analysis implies that the Chinese government could invest in education to enhance the innovation capacity φ of its workforce in order to reduce σ and avoid social unrest. This implication is also consistent with Galor's (2022, p. 74) observation that "technological transformation of the production process in fact made human capital an increasingly critical element in the boosting of industrial productivity. Instead of a communist revolution, therefore, industrialisation triggered a revolution in mass education. Capitalists' profit margins stopped shrinking and workers' wages started rising, and ultimately the threat of class conflict - the beating heart of Marxism - began to fade."

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Online Appendix A: Dynamics

In this appendix, we derive the dynamics of the economy and show that it jumps to a unique and stable balanced growth path. The free-entry condition for R&D in (12) shows that the value of an invention is $v_t = Z_t/\varphi = y_t/\varphi$, where the second equality holds because $y_t = Z_t$ in (15). The aggregate value of assets owned by capitalists is $a_t = v_t = y_t/\varphi$ because of symmetry $v_t(j) = v_t$ and a unit continuum of industries $j \in [0,1]$. Therefore, we can rewrite the capitalists' asset-accumulation equation in (2) as

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{a}_t}{a_t} = r_t - \frac{c_t^c}{a_t} = r_t - \varphi\frac{c_t^c}{y_t}. \tag{A1}$$

Substituting the Euler equation in (3) into (A1) yields

$$\frac{\dot{c}_t^c}{c_t^c} - \frac{\dot{y}_t}{y_t} = \varphi\frac{c_t^c}{y_t} - \rho, \tag{A2}$$

which implies that c_t^c/y_t must jump to its unique steady-state value $c_t^c/y_t = \rho/\varphi$ such that $g_t \equiv \dot{y}_t/y_t = \dot{c}_t^c/c_t^c = r_t - \rho$ at all t. Substituting (9), $r_t = \rho + g_t$ and $v_t = y_t/\varphi$ into the no-arbitrage condition in (10) yields

$$\rho + g_t = r_t = \varphi\frac{\mu - 1}{\mu} + g_t - \lambda_t, \tag{A3}$$

which also uses $\dot{v}_t/v_t = \dot{y}_t/y_t$ and shows that the arrival rate of innovation is

$$\lambda_t = \lambda^* = \frac{\mu - 1}{\mu}\varphi - \rho. \tag{A4}$$

Therefore, the economy jumps to a unique and stable balanced growth path along which the growth rates of y_t, Z_t, c_t^c, c_t^w, w_t, a_t and v_t jump to the same steady-state value $g^* = \lambda^* \ln z$ and the real interest rate jumps to its steady-state value $r^* = \rho + g^*$.

Online Appendix B: Heterogeneity

It may seem that our analysis relies on the assumption of homogeneous workers and homogeneous capitalists. In this appendix, we show that all our results are robust to heterogeneous workers and heterogeneous capitalists. Suppose there is a unit continuum of workers indexed by $h \in [0,1]$. Worker h is exogenously endowed with $l(h)$ units of labor, which follows a general distribution with a mean of unity such that

$$\int_0^1 l(h)dh = 1. \tag{B1}$$

Worker h's consumption is given by

$$c_t^w(h) = w_t l(h). \tag{B2}$$

Using (8) and (15), we can derive $c_t^w(h)$ as

$$c_t^w(h) = \frac{y_t}{\mu} l(h) = \frac{Z_t}{\mu} l(h). \tag{B3}$$

Substituting (B3) into the welfare function of worker h yields

$$U^w(h) = \frac{1}{\rho}\left[\ln c_0^w(h) + \frac{g^*}{\rho}\right] = \frac{1}{\rho}\left[\ln l(h) + \ln\left(\frac{Z_0}{\mu}\right) + \frac{g^*}{\rho}\right], \tag{B4}$$

in which $\ln l(h)$ affects the utility of worker h but is independent of the markup μ whereas g^* is given by (17) and (18) as before. Therefore, the utility-maximizing level of markup for all workers $h \in [0,1]$ is given by μ^w in (27) as before.

Suppose there is a unit continuum of capitalists indexed by $k \in [0,1]$. At time 0, capitalist k is exogenously endowed with $a_0(k)$ units of assets, where $\int_0^1 a_0(k)dk = a_0 = v_0$. Her asset-accumulation equation is

$$\dot{a}_t(k) = r_t a_t(k) - c_t^c(k). \tag{B5}$$

Dynamic optimization yields the consumption path of capitalist k as

$$\frac{\dot{c}_t^c(k)}{c_t^c(k)} = r_t - \rho, \tag{B6}$$

which implies that the growth rate of $c_t^c = \int_0^1 c_t^c(k)dk$ is also given by $\dot{c}_t^c/c_t^c = r_t - \rho$. Therefore, the distribution of consumption share $c_t^c(k)/c_t^c$ among capitalists is stationary. Combining (B5) and (B6) yields

$$\frac{\dot{c}_t^c(k)}{c_t^c(k)} - \frac{\dot{a}_t(k)}{a_t(k)} = \frac{c_t^c(k)}{a_t(k)} - \rho, \tag{B7}$$

which shows that the consumption-asset ratio $c_t^c(k)/a_t(k)$ of capitalist k jumps to ρ. In other words, we have

$$c_t^c(k) = \rho a_t(k), \tag{B8}$$

which implies $c_t^c = \rho a_t$ and $c_t^c(k)/c_t^c = a_t(k)/a_t$. Therefore, the stationary distribution of consumption share $c_t^c(k)/c_t^c$ implies that the distribution of asset share $a_t(k)/a_t$ is also stationary. Let's denote the initial share as $s(k) \equiv a_0(k)/a_0$, which is exogenously given at time 0 and remains stationary. Then, capitalist k's consumption is given by

$$c_t^c(k) = \rho s(k) a_t = s(k) \frac{\rho Z_t}{\varphi}, \tag{B9}$$

where the second equality uses $a_t = v_t$ and (12). Substituting (B9) into the welfare function of capitalist k yields

$$U^c(k) = \frac{1}{\rho}\left[\ln c_0^c(k) + \frac{g^*}{\rho}\right] = \frac{1}{\rho}\left[\ln s(k) + \ln\left(\frac{\rho Z_0}{\varphi}\right) + \frac{g^*}{\rho}\right], \tag{B10}$$

in which $\ln s(k)$ affects the utility of capitalist k but is independent of the markup μ whereas g^* is given by (17) and (18) as before. Therefore, the utility-maximizing level of markup for all capitalists $k \in [0, 1]$ is given by μ^c in (25) as before. As a result, all three propositions in the main text continue to hold.