Value-at-Risk (VAR) Estimation Methods: Empirical Analysis based on BRICS Markets

Ben Salem, Ameni and Safer, Imene and Khefacha, Islem

FSEG Mahdia, University of Monastir

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Value-at-Risk (VAR) Estimation Methods:  
Empirical Analysis based on BRICS Markets

Ameni BEN SALEM[^4], University of Sousse, FSEG Sousse, Tunisia.  
E-Mail: amanibensalem06@gmail.com

Imene SAFER, University of Monastir, FSEG Mahdia, BESTMOD, ISG of Tunis, University of Tunis. E-Mail: Imen.Safer@fsegma.u-monastir.tn

Islem KHEFACHA, University of Monastir, FSEG Mahdia, LaREMFiQ, IHEC of Sousse, University of Sousse, Tunisia. E-Mail: Islem.Khefacha@fsegma.u-monastir.tn  
[https://orcid.org/0000-0002-5636-0692](https://orcid.org/0000-0002-5636-0692)

**Abstract**

The purpose of this paper is to investigate some statistical methods to estimate the value-at-Risk (VaR) for stock returns in the BRICS countries for the period between 2011 to 2018. Four different risk methods are used to estimate VaR: Historical Simulation (HS), Riskmetrics, Historical Method and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Process. By applying the Backtesting technique, we try to test the effectiveness of this different methods by comparing the calculated VaR with the real realized losses (or gain) of the portfolio or the index.

The results show that for the all-BRICS countries and at different confidence level; the Historical Method and the Historical Simulation are the appropriate methods. While the GARCH model failed to predict precisely the VaR for all BRICS countries.

**Keywords:**  
Value-at-Risk, BRICS, Riskmetrics, Historical Simulation, GARCH, Historical Method, Backtesting, Confidence level.

**JEL Code:** C01 ; C58 ; D84 ; G00 ; G17

1. **Introduction**

The quantification, forecasting and management of market risks are major concerns for financial institutions. This is because exposure to extreme price fluctuations in financial markets can lead to sudden and significant losses. Therefore, managers and researchers are
responsible for ensuring financial stability. So, they should rely on a large database and metrics tables to properly identify potential risks.

In recent years, many concepts of risk measurement have been developed. The main risk management methodology is the Value-at-Risk VaR method which is combined with other risk minimization techniques in order to achieve optimal results. VaR is the largest portfolio loss we can expect over a given period, and in a certain level of confidence. This value is a simple and easily understandable number which presents the risk to which the institution is exposed in the financial market. Despite its simple implementation, VaR has been the subject of several criticisms (Artzner et al., 1999, Yamai et Yoshiba, 2002, 2005, Sobreira et al. 2020).

In this research, we will compare the performance of different VaR estimation techniques for the BRICS countries (Brazil, Russia, India, China, and South Africa) under the period from 2011 to 2018. We underline that this Research compares VaR based on the stock returns of the market indexes. In fact, choosing an appropriate measure of VaR that gives an accurate estimate is an important but difficult task.

In this study, VaR is estimated using four different risk methods: Historical Simulation (HS), RiskMetrics, Historical Method and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process.

Our objective through this research is to improve the existing literature that deals with risk management by measuring VaR. Indeed, in frequent are the researches that have studied the performance of these different methods, particularly in the context of BRICS countries. The goal is to test the reliability of the different methods in order to retain the best methods which estimate the VaR. VaR’s obtained results will be evaluated with a backtesting and compared using a loss function approach.

On the bases of the objectives previously mentioned the problem that can outlined is : What is the most reliable method for estimating VaR and to what extent changes in data and confidence level have an effect on performance and reliability value-at-risk (VaR) measures in BRICS countries?

2. VaR Estimation Method

VaR is a measure of the risk of loss of investments. It estimates how much a set of investments could lose (with a given probability), under some market conditions, over a defined period of
time such as a day. VaR is typically used by businesses and financial sector regulators to assess the amount of assets needed to cover potential losses (Bonga-Bonga and Nleya, 2016). This definition accepted by all financial investors is as follows: “VaR is the maximum potential loss that a portfolio can suffer, for a given time horizon and a given level of probability, assuming that this portfolio remains unchanged for the specified horizon.” Manganelli and al. (2001)

Bayer (2018) argues that although it is difficult, it is important to choose between alternative modeling and value-at-risk (VaR) forecasting strategies. An improperly selected risk model can have dramatic effects on portfolios and the market as a whole, as evidenced by the stock market crash of 2015 when many standard approaches predict insufficiently low levels of risk. Choosing an appropriate VaR estimation method is an important but difficult task. Indeed, Hendricks (1996) suggested that further research aimed at comparing and combining the best features of the approaches examined might be useful. For this, it seems necessary to us to compare the different estimation methods of VaR, namely RiskMetrics, Historical Simulation, Historical Method and Variance-Covariance Method as under the GARCH name.

2.1 Historical Simulation

Some researchers such as Jawwad and Palgrave (2014), explain that the Historical Simulation (HS) is the most popular and efficient method. The characteristics of the HS method:

- Relatively simple to set up
- Does not assume any form of distribution.
- Depends on the quality and availability of data.

According to Gajadharsingh (2013): “The empirical quantile method (or Historical Simulation) is a quite simple method of estimating risk measures. It is based on the empirical distribution of historical data on the returns of a financial portfolio. Formally, VaR is estimated simply by directly reading the empirical fractiles of past returns.”

Wiener (1999) asserts that historical simulation belongs to the nonparametric method of calculating VaR. What is common to all nonparametric approaches is the use of the empirical distribution, obtained from the observed data, as opposed to the parametric approach (where assumptions about the theoretical distributions of return are used). The main feature of historical simulation is its ease of implementation.
The Historical Simulation allows us to estimate the VaR of a portfolio by considering the amount invested in the portfolio in general and in each of its securities in particular.

2.2 Historical Method

After identifying the significant risk factors for a financial market, we use the historical data collected in order to deduce the amount of loss. According to Didier (2014): "The historical method requires knowing the price history for an index in order to calculate the change in its value over time. This method is very inexpensive in terms of calculation and technique. In addition, no prior assumption on the form of the distribution is required."

This method is able to determine the daily Profits & Losses (P&L) of a market index which is then ranked in ascending order. Depending on the number of P&L calculated and the desired confidence interval, the historical VaR is equal to the corresponding P&L value.

This simplicity of implementation generates many limits. While among its drawbacks, this method is not suitable for derivative products (options, warrants, futures contracts, etc.). In addition, historical data must be sufficiently and widely large compared to the horizon of the VaR and its confidence level but not too much, to ensure that the law of probability has not changed too much over the given period.

2.3 Riskmetrics

RiskMetrics was introduced in 1994. It contains datasets and techniques used to calculate the value at risk (VaR) of a portfolio of stocks or a market index. Morgan and Reuters collaborated in 1996 to develop the methodology and make the data widely available to practitioners, managers, and researchers. The objective is to improve and promote the transparency of market risks and subsequently to create a benchmark for risk measurement by providing advice to clients on the management of market risks.

Morgan calculates the VaR as the conditional variance as a weighted average shifted by one period and squared logarithms at period t-1 (Sobreira and Louro 2020):

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2$$

$\sigma_t^2$: the conditional variance; $r_t^2$: Square yield

Usually, $\lambda = 0.94$ for the forecast of daily volatility.
2.4 Generalized autoregressive conditional heteroskedasticity process (GARCH)

In 1982, Engle presented in his famous research paper entitled "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation" the family of conditional autoregressive heteroskedastic (ARCH) models. Since then, other research has focused on the modeling of conditional volatility, such as the work of Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Diebold and Lopez (1995). Other papers have compared different specific models for predicting conditional volatility such as West and Cho (1994) and Heynen and Kat (1993).

The steps for calculating the VaR by the GARCH method are schematized as follows:

1. First develop a Best Fit ARIMA Process
2. Test the ARCH effects in Residuals
   - If ARCH exist in residuals then
     - Develop a best GARCH (p, q)
     - Test the Value-at-Risk (VaR)
   - If ARCH does not exist in residuals then
     - There should be no autocorrelation heteroscedasticity and ARCH effect.
     - However, normality in residuals of GARCH process is generally not required
3. End the process

According to Angelidisa et al. (2004) the GARCH (p, q) model successfully captures several features of financial time series, such as thick-tailed returns and volatility clustering, as noted by Mandelbrot (1963) “… big changes tend to be followed by big changes in one or the other of the signs, and small changes tend to be followed by small changes…”. On the other hand, the GARCH structure presents some drawbacks of implementation, since the variance depends only on the magnitude and not on the sign of $\varepsilon_t$, which is in contradiction with the empirical behavior of stock prices where a leverage effect may be present. This term, introduced by Black (1976) refers to the tendency of changes in stock returns to be negatively correlated with changes in return volatility, so that volatility tends to increase in response to bad news, ($\varepsilon_t < 0$) and decrease in response to good news ($\varepsilon_t > 0$). Additionally, Brooks and Persand (2003) state that a VaR model that ignores asymmetries in the volatility specification is most likely to generate inaccurate predictions.

3. Backtesting

3.1 Definition

Considering the existence and the great diversity of methods for providing the VaR’s estimation, many studies propose that different models applied for the same research generally led to vastly different estimates of VaR, and therefore risk, for the same portfolio or the same market index.

Risk Managers need to assess VaR forecasts outside the regulatory standards imposed of Basel II by setting up Backtesting procedures (Silver and al. 2020).

Backtesting is a set of statistical procedures used in financial institutions to designate the testing of a strategy of a predictive model from existing historical data in order to verify that the actual losses observed are in line with the expected losses. This involves systematically comparing the historical VaR forecasts with the observed returns of the portfolio (Jorion 2007). This kind of simulation makes it possible to refine a model and verify hypotheses. Backtesting is requires real historical data.

According to Niepolla (2009): “The results of the Backtests provide an indication of potential problems within the system. A severe underestimation of risk is discovered, especially for stocks and stock options. However, the turbulent market environment poses challenges in evaluating backtesting results, as VaR models are only known to be accurate under normal market conditions.”
However, such a Backtest involves some verification risks. First, we know that data from the past is not necessarily a guide to future performance. It is therefore desirable to keep realistic and simple assumptions. Over-optimizing a backtest would not lead to optimizing a strategy, but to optimizing the past so that the strategy is always the best.

Backtesting must therefore make it possible to determine the most appropriate method (or methods) (Historical Simulation, Historical Method, RiskMetrics, GARCH) to predict the Var. We must distinguish between the forecast validation test and the comparison test of forecast such as the Kupiec TUFF test, the Kupiec POF test or the Christoffersen independence test. Only large institutions and professional fund managers use Backtesting because of the expense of obtaining and using detailed data sets. However, backtrading is used on a large basis and independent backtesting platforms. Although the technique is widely used, it has weaknesses.

### 3.2 Backtesting Value at Risk Forecast: Kupiec Pof-Test

The POF (proportion of failure) test examines whether the number of exceptions meets the given confidence level. The null hypothesis of failure is expressed as follows:

$$H_0 : p = \hat{p} = \frac{x}{T} \times 100$$

Where:
- $p$: percentage of failure
- $\hat{p}$: The observed failure rate
- $x$: Number of exceptions
- $T$: Number of total observations

Once the one-day VaR and the number of exceptions for each confidence level are known; the likelihood ratio test must be calculated.

In the event that the calculated LR exceeds the critical value, the null hypothesis and the accuracy of the model must be rejected for a certain level of confidence.

The “LR” likelihood ratio test is expressed according to the following expression:

$$LR_{POF} = -2 \ln \left( \frac{(1 - p)^{T-x} * p^x)}{\left[ 1 - \left( \frac{x}{T} \right) \right]^{T-x} * \left( \frac{x}{T} \right)^x} \right)$$

Where:
- $p$: confidence level
- $T$: Total number of observations
- $x$: Number of exceptions
4. Empirical analysis

When we seek to invest in the stock market, we tend to focus on the developed markets of the European Union or the United States and we forget the emerging countries, namely the countries of the BRICS group, which are distinguished by their vast growing economies. Indeed, the BRICS countries attract a large part of capital inflows and represent a destination of choice for the investments of many global portfolio managers. The main problem with stock markets in developing countries is the access to markets and financial information. Unless investors know emerging markets like the back of their hand, they are therefore discouraged from investing in BRICS markets individually. They should therefore ask funds and apply risk measurement methods based on historical data in order to build a complete idea of the market.

4.1 Empirical results

4.1.1 Historical Simulation

The data used for the statistical calculations come from a secondary source, specifically, the share prices of 20 companies with the largest market capitalizations (see Annex 1) for a period of 2085 days. The data was collected via the “Datastream” financial and macroeconomic data platform. First, we assumed we have $2000 to invest in a portfolio at the rate of $100 for each company. Thus, daily returns are calculated for each company, then the daily return of the portfolio is calculated so that the daily returns of twenty companies are added up.

The third step is to calculate the overnight VaR for the portfolio at the confidence levels of 95% and 99% respectively using the formula (percentile) on excel:

\[ \text{Var} (99\%) = \text{CENTILE} (n_1 : n_t ; 99\%) \]

The daily losses are then considered in order to compare these values with the estimated calculation of the VaR. If the value of the portfolio loss is greater than the predicted overnight VaR value, then the exception exists. This comparison is necessary to see how many exceptions occur at the 95% and 99% confidence level. (Annex 2)
<table>
<thead>
<tr>
<th>Countries</th>
<th>confidence level</th>
<th>VaR%</th>
<th>Number of exceptions</th>
<th>Number of total observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>95% (α=5%)</td>
<td>-46,5514749</td>
<td>103</td>
<td>2085</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-74,7337198</td>
<td>19</td>
<td>2085</td>
</tr>
<tr>
<td>Russia</td>
<td>95% (α=5%)</td>
<td>-38,10863374</td>
<td>109</td>
<td>2085</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-61,97126617</td>
<td>32</td>
<td>2085</td>
</tr>
<tr>
<td>India</td>
<td>95% (α=5%)</td>
<td>-32,8803826</td>
<td>94</td>
<td>2085</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-49,2278663</td>
<td>23</td>
<td>2085</td>
</tr>
<tr>
<td>China</td>
<td>95% (α=5%)</td>
<td>-44,29199755</td>
<td>95</td>
<td>2085</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-74,22332947</td>
<td>29</td>
<td>2085</td>
</tr>
<tr>
<td>South Africa</td>
<td>95% (α=5%)</td>
<td>-35,38409499</td>
<td>122</td>
<td>2085</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-61,00736923</td>
<td>20</td>
<td>2085</td>
</tr>
</tbody>
</table>

The table shows the estimated VaR for the BRICS group at the 99% and 95% thresholds, as well as the number of exceptions (losses that have exceeded the VaR) and the number of total observations.

The highest number of exceptions is recorded in South Africa at the 95%. This means that following the estimation of the VaR by the HS, 122 values exceeded the worst expected loss in Brazil at the threshold by 95%. While there are only 20 losses that have exceeded the VaR at the 99% threshold.

Generally, and depending on the results obtained, the VaR estimated at the 99% is lower than that at the 95% since the confidence level will be more limited (there is only a 1% chance that the losses will exceed the Value at risk). And even for the number of exceptions (losses that exceeded VaR at the 99% are therefore less than that at the 95%).

### 4.1.2. The Historical Method

We carry out our analysis on the basis of the repatriation of the daily closing values over the last 8 years (from 2011 to 2018) of the BRICS group market indices (BOVESPA, RTS, SENSEX, SSE, JSE). The data was extracted from the financial data platform "factset". We thus calculate the daily earnings which are sorted by increasing value. The confidence level is then calculated from the number of observations (number of the day) according to the following expressions:
VaR reference = (N° line / Total number of lines)

Var at x% = (100 - Ref VaR)

The risk value is then obtained at the 99% and 95% levels by calculating the sorted gains. (Annex 3)
The daily losses are then considered in order to compare these values with the estimated calculation of the VaR. If the negative return (loss) of the index is greater than the expected overnight VaR value, the exception exists. This comparison is necessary to see how many exceptions occur at the 95% and 99% confidence level.

Table 2: VaR estimation by the Historical Method

<table>
<thead>
<tr>
<th>Countries</th>
<th>confidence level</th>
<th>VaR</th>
<th>VaR in % =</th>
<th>Number of exceptions</th>
<th>Number of total observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>95% (α=5%)</td>
<td>-1317</td>
<td>1.62</td>
<td>98</td>
<td>1977</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-2017</td>
<td>2.25%</td>
<td>19</td>
<td>1977</td>
</tr>
<tr>
<td>Russia</td>
<td>95% (α=5%)</td>
<td>-32,93</td>
<td>3.11%</td>
<td>99</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-62,87</td>
<td>5.44%</td>
<td>19</td>
<td>2000</td>
</tr>
<tr>
<td>India</td>
<td>95% (α=5%)</td>
<td>-351,28</td>
<td>0.94%</td>
<td>98</td>
<td>1972</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-590,05</td>
<td>1.62%</td>
<td>19</td>
<td>1972</td>
</tr>
<tr>
<td>China</td>
<td>95% (α=5%)</td>
<td>-58,167</td>
<td>2.11%</td>
<td>96</td>
<td>1945</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-175,56</td>
<td>6.58%</td>
<td>18</td>
<td>1945</td>
</tr>
<tr>
<td>South Africa</td>
<td>95% (α=5%)</td>
<td>-680,96</td>
<td>1.32%</td>
<td>103</td>
<td>2085</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>-1161,52</td>
<td>2.52%</td>
<td>20</td>
<td>2085</td>
</tr>
</tbody>
</table>

This table presents the estimated VaR for each country of the BRICS group at the 99% and 95% thresholds. The number of exceptions (losses that exceed the VaR) and the number of total observations are different from each country due to national holidays and missing data for some indices.
The VaR for Brazil at the 95% is equal to -1317 and -2017 at the 99% meaning that there is a 5% chance that the loss will exceed -1317 and 1% chance that the loss will exceed -2017.
The number of exceptions in the five countries is almost equal. On average, 19 Return (loss) values exceed the VaR at the 99% for all countries. One hundred stocks were the exception at the level 95% threshold. This means that the measure of VaR by the Historical Method is robust and gives the same estimates for all countries (it is no longer affected by the database).

4.1.3. Riskmetrics :

As the "Historical Method" technique, we use the same database to calculate the daily returns of the market index for each country according to the following expression:

\[ R_t = \ln \left( \frac{P_{ct}}{P_{c(t-1)}} \right) \]

Where:

- \( R_t \): daily returns
- \( P_{ct} \): The closing price at time \( t \).
- \( P_{c(t-1)} \): The closing price at time \( t-1 \).

We then calculate the variance and the standard deviation in order to estimate the VaR at levels 95% and 99% by the following expression (See Annex 4):

\[ \text{VaR} (1-\alpha) = \sigma_t \ast \text{NORMAL.STANDARD.INVERSE.LAW.N}(\alpha) \]

Table 3: VaR estimation by RiskMetrics

<table>
<thead>
<tr>
<th>Countries</th>
<th>Confidence level</th>
<th>VaR</th>
<th>Number of exceptions</th>
<th>Number of total observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>95% (( \alpha = 5% ))</td>
<td>-2.388%</td>
<td>92</td>
<td>1977</td>
</tr>
<tr>
<td></td>
<td>99% (( \alpha = 1% ))</td>
<td>-3.378%</td>
<td>20</td>
<td>1977</td>
</tr>
<tr>
<td>Russia</td>
<td>95% (( \alpha = 5% ))</td>
<td>-2.956%</td>
<td>83</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td>99% (( \alpha = 1% ))</td>
<td>-4.181%</td>
<td>32</td>
<td>2000</td>
</tr>
<tr>
<td>India</td>
<td>95% (( \alpha = 5% ))</td>
<td>-1.576%</td>
<td>98</td>
<td>1972</td>
</tr>
<tr>
<td></td>
<td>99% (( \alpha = 1% ))</td>
<td>-2.229%</td>
<td>30</td>
<td>1972</td>
</tr>
<tr>
<td>China</td>
<td>95% (( \alpha = 5% ))</td>
<td>-2.268%</td>
<td>82</td>
<td>1945</td>
</tr>
<tr>
<td></td>
<td>99% (( \alpha = 1% ))</td>
<td>-3.208%</td>
<td>40</td>
<td>1945</td>
</tr>
<tr>
<td>South Africa</td>
<td>95% (( \alpha = 5% ))</td>
<td>-1.680%</td>
<td>103</td>
<td>2085</td>
</tr>
<tr>
<td></td>
<td>99% (( \alpha = 1% ))</td>
<td>-2.376%</td>
<td>36</td>
<td>2085</td>
</tr>
</tbody>
</table>
Table 2.3 shows the estimated VaRs for each country of the BRICS group at the levels 99% and 95%, as well as the number of exceptions. The worst loss recorded by the RiskMetrics method is that of the Russian (-4.181%) at the 99% threshold. Among the 2000 observations 32 performance values (losses) exceed the VaR.

4.1.4 GARCH

We describe in the following the different steps of the application of the GARCH method in order to estimate the VaR.

**Step 1:** download the data.

We download the adjusted closing prices of market indices from January 1, 2011 to December 31, 2018 using Yahoo Finance. Since we have missing data, we use the na.omit command. This function removes all incomplete cases from the data (See Annex 5).

**Step 2:** Obtain data returns

Based on the daily returns, we can conclude that high volatility days are followed by high volatility days and low volatility days by low volatility days (See Annex 6).

**Step 3:** Find the best model using ARIMA

In this step it is a question of finding the best ARIMA model (p, d, q) based on the Bayesian information criteria. Note that the returns of a financial series is always stationary and therefore integrated of order 0 I (0). Therefore, ARIMA is in fact only an ARMA (p, q) process (See Annex 7).

**Step 4:** Testing the ARCH effect

In order to validate the ARIMA-type modelling, it is necessary to test the absence of heteroskedasticity through the ARCH effect. To do this, it is a question of applying the Ljung-Box test on the first 12 shifts of the squared residuals of the best ARIMA model under the null hypothesis of no ARCH effect. (See Annex 7)

If the value of p of the Ljung-Box test is less than 5% of significance, the ARCH effect is indeed present and the modulization of the GARCH type is then essential (See Annex 7).

**Step 5:** Development of a GARCH model

For the GARCH theory, we specify the object called res_garch_spec in which we want to develop a GARCH (p, q) on ARIMA (p, 0, q).
Step 6: Backtesting the risk model

Once the GARCH model has been estimated, we verify the performance of the model by performing a historical backtest. To do this, we can compare the estimated VaR (value at risk) with the actual return over the period. If the return is more negative than the VaR, we have exceeded the VaR. In our case, exceeding the VaR should only occur 1% of the time if we have specified a confidence level of 99%, and 5% of the cases if we have specified a confidence level of 95%. The 1% and 5% VaR show the 1% and 5% probability of its extreme loss. (See Annex8)

Table 4: estimation VaR by the GARCH model

<table>
<thead>
<tr>
<th>Pays</th>
<th>Confidence level</th>
<th>Number of exceptions</th>
<th>Number of total observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>95% (α=5%)</td>
<td>115</td>
<td>1854</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>32</td>
<td>1854</td>
</tr>
<tr>
<td>Russia</td>
<td>95% (α=5%)</td>
<td>84</td>
<td>1310</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>20</td>
<td>1310</td>
</tr>
<tr>
<td>India</td>
<td>95% (α=5%)</td>
<td>149</td>
<td>1840</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>72</td>
<td>1840</td>
</tr>
<tr>
<td>China</td>
<td>95% (α=5%)</td>
<td>101</td>
<td>1824</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>49</td>
<td>1824</td>
</tr>
<tr>
<td>South Africa</td>
<td>95% (α=5%)</td>
<td>140</td>
<td>1965</td>
</tr>
<tr>
<td></td>
<td>99% (α=1%)</td>
<td>50</td>
<td>1965</td>
</tr>
</tbody>
</table>

This table shows the number of exceptions and the total number of observations, which differ between countries due to missing data.

The GARCH method records extremely high numbers of exceptions for the 5 countries, a sign of an inaccuracy in the estimate of the VaR by this method.

4.2 Backtesting Value at Risk Forecast: Kupiec Pof-Test

The percentages indicated in the table reflect the percentages of rejections of the null hypothesis. For the more liberal level of coverage (α = 5%), GARCH has the worst performance for most countries, while HS fails in both Russia and South Africa. In addition, RiskMetrics dominates the three other methods. When a more conservative level of coverage is considered (α = 1%), the historical method has shown the best overall performance, more clearly outperforming GARCH and RiskMetrics. In general, we get a higher percentage of VaR method for more liberal coverage levels.

Once the one-day VaR and the number of exceptions for each confidence level are known; the likelihood ratio test must be calculated.
In the event that the calculated LR exceeds the critical value, the null hypothesis and the model accuracy must be rejected for a certain level of confidence.

**Table 5: Percentages of exceptions : A comparative results**

<table>
<thead>
<tr>
<th></th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HS</td>
<td>Historical Method</td>
</tr>
<tr>
<td>Brazil</td>
<td>4.94%</td>
<td>4,957%</td>
</tr>
<tr>
<td>Russia</td>
<td>5.32%</td>
<td>4,950%</td>
</tr>
<tr>
<td>India</td>
<td>4.51%</td>
<td>4,970%</td>
</tr>
<tr>
<td>China</td>
<td>4.56%</td>
<td>4,936%</td>
</tr>
<tr>
<td>South Africa</td>
<td>5.85%</td>
<td>4,938%</td>
</tr>
</tbody>
</table>

The null hypothesis indicates that the observed failure percentage equals the failure rate, which is suggested by the confidence interval. Moreover, the purpose of accepting the null hypothesis is to prove that the model is accurate. In the case where the quantity of likelihood ratio is greater than the critical value of $\chi^2$, the conclusion on the rejection of the null hypothesis and the inaccuracy of the model would be made.

The “LR” likelihood ratio test is expressed according to the following formula:

$$LR POF = -2 \ln \left( \frac{((1 - p)^{T-x} \cdot p^x)}{[1 - (\frac{x}{T})]^{T-x} \cdot (\frac{x}{T})^x} \right)$$

Where:

- $p$: Confidence level
- $T$: Total number of observations
- $x$: Number of exceptions

According to Jorion (2001), “the likelihood ratio is a statistical test that calculates the ratio between the maximum probabilities of a result under two alternative hypotheses. The maximum probability of the result observed under the null hypothesis is defined in the numerator and the maximum probability of the result observed under the alternative hypothesis is defined in the denominator. The decision is then based on the value of this ratio. The smaller the ratio, the larger the LR statistic will be. If the value becomes too large and greater than the critical value
of the chi-square distribution, the null hypothesis is rejected. According to statistical decision theory, the likelihood ratio test is the most powerful test in its class.”

As we have already specified for the POF test, the calculation of the likelihood test is necessary. Thus, it can be calculated by plugging the appropriate data from the table (1, 2, 3 and 4) into the likelihood ratio formula. This means that robust evidence is needed to reject the null hypothesis and the accuracy of the model. In order to draw a valid conclusion about the validity of the model, the critical value at the two levels 5% and 1% are determined from the chi-square table, the two values are 3.84 at level 5% and 6.63 at level 1%. (Annex 9)

### Table 6: Kupiec-POF test results

<table>
<thead>
<tr>
<th></th>
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<th>99%</th>
</tr>
</thead>
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<tr>
<td></td>
<td>HS</td>
<td>Riskmetrics</td>
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<td>Accepted</td>
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<td>Russia</td>
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<td>Accepted</td>
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<td>India</td>
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<tr>
<td>China</td>
<td>Accepted</td>
<td>Accepted</td>
</tr>
<tr>
<td>South Africa</td>
<td>Accepted</td>
<td>Accepted</td>
</tr>
</tbody>
</table>

The test used for the Backtesting of the amount of VaR expected in this research is a so-called failure proportion test. This test only considers the number of exceptions, not when the particular exception occurs. Therefore, the number of exceptions is critical information necessary for the model to be accurate or not (whether the null hypothesis is rejected or accepted).

- If we refer to the historical simulation method and to the historical method, the two methods are reliable for all countries at levels 95% and 99% thresholds. While the difference lies in the Riskmetricks method which underestimates the risk at levels 99% level for China and South Africa. Whereas the GARCH method gives a poor estimate for both thresholds and for all countries, except at level 95% for China and at 99% for Russia.
- If we seek to estimate the risk in Brazil and India, we must apply either the historical method or historical simulation or Riskmetrics. These methods gave us a satisfactory estimate at levels 99% and 95%.
The only method that should not be applied for risk measurement in Russia is GARCH.

The risk estimate for China at level 99% can be applied by the four methods of measuring VaR. While at level 95%, the GARCH method can no longer be used because, according to the POF test, this method is no longer reliable.

The results obtained from these four methods and after the application of the validation test, the VaR at level 95% must be estimated by the historical method, Riskmetrics and by historical simulation, while at level 99% the estimation is made by historical simulation and by the historical method.

5. Conclusion
The variety of risk measurement approaches that have been developed in the financial market over the last decades raises a question about the validity of these measurements. One of the most popular measures in the literature is value at risk (VaR). Knowing the accuracy of the measurement is especially important for financial institutions, as they use VaR to estimate the amount of liquidity they need to reserve to cover potential losses. Any disability in the VaR model can mean that the institution does not hold sufficient reserves and could lead to significant losses, not only for the institution but potentially for its depositors and retail investors.

This research implements a VaR analysis for the BRICS countries (Brazil, Russia, India, China, and Africa from South) stock markets with market indices that represent the most relevant stocks in these countries. In addition, different performance measures for the assessment of the estimated VaR were discussed. The objective was to study the reliability of four methods (Historical Simulation, RiskMetrics, Historical Method, GARCH) in estimating market VaR. The use of a backtesting is a primary task, of which it consists in comparing the measure of the calculated VaR with the real losses (or gains) realized by the portfolio by the index. A Backtest is based on the level of confidence assumed in the calculation.

The results showed that in the five countries and at distinct levels of trust; the Historical Method and Historical Simulation were the most robust. The change of country and threshold having no effect on their reliability of VaR estimate. This means that there were two methods to estimate risk in emerging BRICS markets. While the GARCH model arrived last, it failed for all countries.

The results were obtained following the Kupiec POF Backtesting, but they can be confirmed by other tests, such as the Kupiec TUFF test (1995), the test of independence of Christoffersen
As a future line of research, it would be interesting to apply these methods to the ES (Expected Shortfall) which has become increasingly important in the field of financial market risk measurement. It is an alternative to value at risk which is more sensitive to the shape of the tail of the loss distribution. In addition, it would be useful to extend our analysis with additional VaR forecasting methods such as Monte Carlo simulation, Parametric Method, and EVT (Extreme Value Theory).

References


Salem, A. B., Safer, I., & Khefacha, I. (2021, December) Value at Risk Estimation For the BRICS Countries: A Comparative Study. In *8th TSFS International conference in Finance and Accounting*.


### Annex 1: market portfolios sorted by capitalization

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<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<tr>
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<td>RUSSIA 30/50</td>
<td>INDE 30/50</td>
<td>CHINE 20/1474</td>
<td>AFRIQUE DE SUD 20/40</td>
</tr>
<tr>
<td>2</td>
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<td>SUN PHARM.INDUSTRIES</td>
<td>BANK OF COMM.S'X</td>
<td>NASPERS</td>
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<tr>
<td>3</td>
<td>GAZPROM</td>
<td>TATA CONSULTANCY SYS</td>
<td>BANK OF CHINA 'A'</td>
<td>ASPIAN PRECIO</td>
</tr>
<tr>
<td>4</td>
<td>MMIC NORDILX NICKEL</td>
<td>TATA MOTORS</td>
<td>ALUMINIUM CORP.OF.CHINA 'A'</td>
<td>LOHMIN (JSP)</td>
</tr>
<tr>
<td>5</td>
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<td>AIR CHINA LIMITED 'A'</td>
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<td>SASOL</td>
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<tr>
<td>7</td>
<td>JSK FEES</td>
<td>MARUTI SUZUKI INDIA</td>
<td>CITIC SECURITIES 'A'</td>
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<td>NOVOL</td>
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Annex 2: Calculation of VaR by Historical Simulation
# Annex 3: Calculation of VaR by the Historical Method

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<th>Variance</th>
<th>Standard Deviation</th>
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<td>0.0012%</td>
<td>0.0012%</td>
<td>0.0012%</td>
<td>0.0012%</td>
</tr>
<tr>
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<td>0.0000%</td>
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<tr>
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# Russia:

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<td>26</td>
<td>01/26/11</td>
<td>100.23</td>
<td>3.83%</td>
<td>0.1332%</td>
<td>0.0123%</td>
<td>1.77%</td>
<td>-2.38%</td>
</tr>
<tr>
<td>27</td>
<td>01/27/11</td>
<td>100.23</td>
<td>3.83%</td>
<td>0.1332%</td>
<td>0.0123%</td>
<td>1.77%</td>
<td>-2.38%</td>
</tr>
<tr>
<td>28</td>
<td>01/28/11</td>
<td>100.23</td>
<td>3.83%</td>
<td>0.1332%</td>
<td>0.0123%</td>
<td>1.77%</td>
<td>-2.38%</td>
</tr>
</tbody>
</table>

# Annex 4: Calculation of VaR by RiskMetrics:

- Brazil:
- Russia:
Annex 4: following

- India

- China:

- South Africa:
Annex 5: Volatility in the BRICS group's market indices

**BVSP Closing Prices**

**MOEXI Closing Prices**

**SSE COMPOSITE Index Closing Prices**

**BSESN Closing Prices**

**FTSE/JSE Closing Price**
Annex 6: Daily returns of market indices

<table>
<thead>
<tr>
<th>Date</th>
<th>Return in percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Monthly Compound Returns</td>
</tr>
<tr>
<td></td>
<td>Brésil</td>
</tr>
<tr>
<td></td>
<td>Inde</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Index</th>
<th>FTSE</th>
<th>Brésil</th>
<th>Russie</th>
<th>Inde</th>
<th>Chine</th>
<th>Afrique de sud</th>
</tr>
</thead>
</table>
Annex 7: The best average model using ARIMA and the ARCH effect test

- **Brazil:**

  ```r
  > fit1 <- auto.arima(BV3PI.ret, trace=TRUE, test="kpss", ic="bic")
  
  Fitting models using approximations to speed things up...
  
  ARIMA(2,0,2) with non-zero mean : Inf
  ARIMA(0,0,0) with non-zero mean : 7098.224
  ARIMA(1,0,0) with non-zero mean : 7018.447
  ARIMA(0,0,1) with non-zero mean : 4536.125
  ARIMA(0,0,0) with zero mean : 7090.759
  ARIMA(1,0,1) with non-zero mean : Inf
  ARIMA(0,0,2) with non-zero mean : Inf
  ARIMA(1,0,2) with non-zero mean : Inf
  ARIMA(0,0,1) with zero mean : 4528.139
  ARIMA(1,0,1) with zero mean : Inf
  ARIMA(0,0,2) with zero mean : Inf
  ARIMA(1,0,2) with zero mean : Inf

  Now re-fitting the best model(s) without approximations...
  
  ARIMA(0,0,1) with zero mean : 7096.414

  Best model: ARIMA(0,0,1) with zero mean
  
  > Box.test(fit1$residuals^2, lag=12, type="Ljung-Box")
  
  Box-Ljung test
  
  data: fit1$residuals^2
  X-squared = 200.09, df = 12, p-value < 2.2e-16
  
- **Russia:**

  ```r
  > fit1 <- auto.arima(M0EXI.ret, trace=TRUE, test="kpss", ic="bic")
  
  Fitting models using approximations to speed things up...
  
  ARIMA(2,0,2) with non-zero mean : Inf
  ARIMA(0,0,0) with non-zero mean : 4479.892
  ARIMA(1,0,0) with non-zero mean : 4194.135
  ARIMA(0,0,1) with non-zero mean : Inf
  ARIMA(0,0,0) with zero mean : 4473.822
  ARIMA(2,0,0) with non-zero mean : 4202.272
  ARIMA(1,0,1) with non-zero mean : -24906.45
  ARIMA(2,0,1) with non-zero mean : Inf
  ARIMA(0,0,2) with non-zero mean : Inf
  ARIMA(0,0,1) with zero mean : Inf
  ARIMA(1,0,1) with zero mean : Inf

  Now re-fitting the best model(s) without approximations...
  
  ARIMA(1,0,1) with non-zero mean : 4493.034

  Best model: ARIMA(1,0,1) with non-zero mean
  
  > Box.test(fit1$residuals^2, lag=12, type="Ljung-Box")
  
  Box-Ljung test
  
  data: fit1$residuals^2
  X-squared = 47.401, df = 12, p-value = 3.972e-06
  ```
Annex 7: Following

- India:

```r
> fit1 <- auto.arima(BEESNI.ret, trace=TRUE, test="kps", ic="bic")

Fitting models using approximations to speed things up...

ARIMA(2,0,2) with non-zero mean : Inf
ARIMA(0,0,0) with non-zero mean : 6420.973
ARIMA(3,0,0) with non-zero mean : 5318.993
ARIMA(0,0,1) with non-zero mean : Inf
ARIMA(0,0,0) with zero mean : 5415.138
ARIMA(2,0,0) with non-zero mean : 5315.574
ARIMA(3,0,0) with non-zero mean : 5328.787
ARIMA(2,0,1) with non-zero mean : Inf
ARIMA(1,0,1) with non-zero mean : Inf
ARIMA(3,0,1) with non-zero mean : Inf
ARIMA(0,0,1) with zero mean : 5318.832
ARIMA(1,0,0) with zero mean : 5312.817
ARIMA(1,0,1) with zero mean : Inf
ARIMA(0,0,2) with zero mean : 6327.545
ARIMA(2,0,1) with zero mean : Inf

Now re-fitting the best model(s) without approximations...

ARIMA(0,0,0) with zero mean : 5421.754

Best model: ARIMA(0,0,0) with zero mean

> Box.test(fit1$residuals^2, lag=12, type="Ljung-Box")

Box-Ljung test

data: fit1$residuals^2
X-squared = 269.6, df = 12, p-value < 2.2e-16
```

- China:

```r
> fit1 <- auto.arima(SSE.ret, trace=TRUE, test="kps", ic="bic")

Fitting models using approximations to speed things up...

ARIMA(2,0,2) with non-zero mean : Inf
ARIMA(0,0,0) with non-zero mean : 6779.22
ARIMA(1,0,0) with non-zero mean : 6551.526
ARIMA(0,0,1) with non-zero mean : Inf
ARIMA(0,0,0) with zero mean : 6771.696
ARIMA(2,0,0) with non-zero mean : 6551.044
ARIMA(3,0,0) with non-zero mean : 6728.932
ARIMA(2,0,1) with non-zero mean : Inf
ARIMA(1,0,1) with non-zero mean : Inf
ARIMA(3,0,1) with non-zero mean : Inf
ARIMA(2,0,0) with zero mean : 6545.474
ARIMA(1,0,0) with zero mean : 6544.147
ARIMA(1,0,1) with zero mean : Inf
ARIMA(0,0,1) with zero mean : 2876.236
ARIMA(0,0,2) with zero mean : Inf
ARIMA(1,0,2) with zero mean : Inf

Now re-fitting the best model(s) without approximations...

ARIMA(0,0,1) with zero mean : 6779.133

Best model: ARIMA(0,0,1) with zero mean

> Box.test(fit1$residuals^2, lag=12, type="Ljung-Box")

Box-Ljung test

data: fit1$residuals^2
X-squared = 1104.7, df = 12, p-value < 2.2e-16
```
Annex 7: following

- South Africa:

```r
> fit1 <- auto.arima(FTSE.ret, trace=TRUE, test="kpss", ic="bic")

Fitting models using approximations to speed things up...

ARIMA[2,0,2] with non-zero mean : 5654.969
ARIMA[0,0,0] with non-zero mean : 5639.202
ARIMA[1,0,0] with non-zero mean : 5647.508
ARIMA[0,0,1] with non-zero mean : 5646.625
ARIMA[0,0,0] with zero mean : 5632.884
ARIMA[1,0,1] with non-zero mean : 5642.761

Now re-fitting the best model(s) without approximations...

ARIMA[0,0,0] with zero mean : 5632.884

Best model: ARIMA(0,0,0) with zero mean

> Box.test(fit1$residuals^2, lag=12, type="Ljung-Box")

Box-Ljung test

data: fit1$residuals^2
X-squared = 299.21, df = 12, p-value < 2.2e-16
```

Annex 8: Estimation of VaR by the GARCH model

- Brazil at 1%:

```r
> report(res_garch01_poll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)

VaR Backtest Report

--------------
Model: sGARCH-norm
Backtest Length: 1954
Data:

--------------------------------
alph: 1%
Expected Exceed: 18.5
Actual VaR Exceed: 32
Actual %: 1.74

Unconditional Coverage (Kupiec)
Null-Hypothesis: Correct Exceedances
LR.uc statistic: 8.11
LR.uc critical: 6.638
LR.uc p-value: 0.004
Reject Null: YES

Conditional Coverage (Christoffersen)
Null-Hypothesis: Correct Exceedances and Independence of Failures
LR.cc statistic: 10.497
LR.cc critical: 9.21
LR.cc p-value: 0.008
Reject Null: YES
```
Annex 8: following

- Brazil at 5%:

```
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report
=================================
Model:              sGARCH-norm
Backtest Length:    1854
Data:

alpha:      5%
Expected Exceed:  92.7
Actual VaR Exceed: 115
Actual %:   6.2%

Unconditional Coverage (Kupiec)
Null-Hypothesis: Correct Exceedances
LR.uc Statistic: 5.263
LR.uc Critical:  3.841
LR.uc p-value:   0.022
Reject Null: YES

Conditional Coverage (Christoffersen)
Null-Hypothesis: Correct Exceedances and Independence of Failures
LR.cc Statistic: 5.779
LR.cc Critical:  5.991
LR.cc p-value:   0.056
Reject Null: NO
```

- Russia at 1%:

```
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report
=================================
Model:              sGARCH-norm
Backtest Length:    1320
Data:

alpha:      1%
Expected Exceed:  13.1
Actual VaR Exceed: 20
Actual %:   1.5%

Unconditional Coverage (Kupiec)
Null-Hypothesis: Correct Exceedances
LR.uc Statistic: 6.635
LR.uc Critical:  6.635
LR.uc p-value:   0.075
Reject Null: NO

Conditional Coverage (Christoffersen)
Null-Hypothesis: Correct Exceedances and Independence of Failures
LR.cc Statistic: 4.194
LR.cc Critical:  9.21
LR.cc p-value:   0.123
Reject Null: NO
```
Annex 8: Following

- Russia at 5%:

```r
> report(res_garch11_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report
==================================
Model: sGARCH-norm
Backtest Length: 1310
Data:

----------------------------------------
alpha: 5%
Expected Exceed: 65.5
Actual VaR Exceed: 84
Actual %: 6.4%

Unconditional Coverage (Kupiec)
Null-Hypothesis: Correct Exceedances
LR.uc Statistic: 5.069
LR.uc Critical: 3.841
LR.uc p-value: 0.024
Reject Null: YES

Conditional Coverage (Christoffersen)
Null-Hypothesis: Correct Exceedances and
Independence of Failures
LR.cc Statistic: 6.948
LR.cc Critical: 5.991
LR.cc p-value: 0.042
Reject Null: YES
```

- India at 1%:

```r
> report(res_garch10_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report
==================================
Model: sGARCH-norm
Backtest Length: 1840
Data:

----------------------------------------
alpha: 1%
Expected Exceed: 18.4
Actual VaR Exceed: 72
Actual %: 3.94%

Unconditional Coverage (Kupiec)
Null-Hypothesis: Correct Exceedances
LR.uc Statistic: 90.854
LR.uc Critical: 6.635
LR.uc p-value: 0
Reject Null: YES

Conditional Coverage (Christoffersen)
Null-Hypothesis: Correct Exceedances and
Independence of Failures
LR.cc Statistic: 92.365
LR.cc Critical: 9.21
LR.cc p-value: 0
Reject Null: YES
```
Annex 8: Following

- India at 5%:

```r
> report(res_garch10_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report

Model: sGARCH-norm
Backtest Length: 1840
Data:

alpha: 5%
Expected Exceed: 92
Actual VaR Exceed: 149
Actual %: 8.1%

Unconditional Coverage (Kupiec)
Null-Hypothesis: Correct Exceedances
LR.uc Statistic: 31.562
LR.uc Critical: 3.841
LR.uc p-value: 0
Reject Null: YES

Conditional Coverage (Christoffersen)
Null-Hypothesis: Correct Exceedances and Independence of Failures
LR.cc Statistic: 31.563
LR.cc Critical: 5.991
LR.cc p-value: 0
Reject Null: YES
```

- China at 1%:

```r
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report

Model: sGARCH-norm
Backtest Length: 1824
Data:

alpha: 1%
Expected Exceed: 18.2
Actual VaR Exceed: 49
Actual %: 2.7%

Unconditional Coverage (Kupiec)
Null-Hypothesis: Correct Exceedances
LR.uc Statistic: 35.861
LR.uc Critical: 6.635
LR.uc p-value: 0
Reject Null: YES

Conditional Coverage (Christoffersen)
Null-Hypothesis: Correct Exceedances and Independence of Failures
LR.cc Statistic: 36.176
LR.cc Critical: 9.21
LR.cc p-value: 0
Reject Null: YES
```
Annex 8: Following

- China at 5%:

```r
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
```

**VaR Backtest Report**

<table>
<thead>
<tr>
<th>Model:</th>
<th>sGARCH-norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backtest Length:</td>
<td>1524</td>
</tr>
<tr>
<td>Data:</td>
<td></td>
</tr>
</tbody>
</table>

| alpha: | 5% |
| Expected Exceed: | 91.2 |
| Actual VaR Exceed: | 101 |
| Actual %: | 5.5% |

**Unconditional Coverage (Kupiec)**

<table>
<thead>
<tr>
<th>Null-Hypothesis:</th>
<th>Correct Exceedances</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR.uc Statistic:</td>
<td>1.073</td>
</tr>
<tr>
<td>LR.uc Critical:</td>
<td>3.541</td>
</tr>
<tr>
<td>LR.uc p-value:</td>
<td>0.3</td>
</tr>
<tr>
<td>Reject Null:</td>
<td>NO</td>
</tr>
</tbody>
</table>

**Conditional Coverage (Christoffersen)**

<table>
<thead>
<tr>
<th>Null-Hypothesis:</th>
<th>Correct Exceedances and Independence of Failures</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR.cc Statistic:</td>
<td>1.301</td>
</tr>
<tr>
<td>LR.cc Critical:</td>
<td>5.991</td>
</tr>
<tr>
<td>LR.cc p-value:</td>
<td>0.116</td>
</tr>
<tr>
<td>Reject Null:</td>
<td>NO</td>
</tr>
</tbody>
</table>

- South Africa at 1%:

```r
> report(res_garch11_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)
```

**VaR Backtest Report**

<table>
<thead>
<tr>
<th>Model:</th>
<th>sGARCH-norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backtest Length:</td>
<td>1965</td>
</tr>
<tr>
<td>Data:</td>
<td></td>
</tr>
</tbody>
</table>

| alpha: | 1% |
| Expected Exceed: | 19.7 |
| Actual VaR Exceed: | 50 |
| Actual %: | 2.5% |

**Unconditional Coverage (Kupiec)**

<table>
<thead>
<tr>
<th>Null-Hypothesis:</th>
<th>Correct Exceedances</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR.uc Statistic:</td>
<td>33.171</td>
</tr>
<tr>
<td>LR.uc Critical:</td>
<td>6.638</td>
</tr>
<tr>
<td>LR.uc p-value:</td>
<td>0</td>
</tr>
<tr>
<td>Reject Null:</td>
<td>YES</td>
</tr>
</tbody>
</table>

**Conditional Coverage (Christoffersen)**

<table>
<thead>
<tr>
<th>Null-Hypothesis:</th>
<th>Correct Exceedances and Independence of Failures</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR.cc Statistic:</td>
<td>33.237</td>
</tr>
<tr>
<td>LR.cc Critical:</td>
<td>9.21</td>
</tr>
<tr>
<td>LR.cc p-value:</td>
<td>0</td>
</tr>
<tr>
<td>Reject Null:</td>
<td>YES</td>
</tr>
</tbody>
</table>
Annex 8 : Following

- South Africa at 5% :

```r
> report(res_garch1_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
> VaR Backtest Report

Model:          sGARCH-norm
Backtest Length: 1965
Date:

========================================================================
alpha:          5%
Expected Exceed: 98.2
Actual VaR Exceed: 140
Actual %:       7.1%

Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:      16.596
LR.uc Critical:       3.841
LR.uc p-value:        0
Reject Null:           YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                        Independence of Failures
LR.cc Statistic:      21.460
LR.cc Critical:       5.991
LR.cc p-value:        0
Reject Null:           YES
```

Annex 9 : Chi-2 distribution

![Chi-2 distribution table]

Source: Passel, 2016