Sovereign Bailouts and Moral Hazard with Strategic Default

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Abstract

Do bailouts create moral hazard, even when they come in the form of loans that do not involve any debt relief component? And what is the rationale for imposing ex-ante conditionality in terms of fiscal policy? I address these questions in a model of strategic severeign default, in which a debt crisis occurs after a bad fundamental shock. The market's willingness to lend is limited by the inability of the government to commit to future repayment; the government may decide to default although it would be willing to repay if it was able to borrow more and commit to repay. An International Financial Institution (IFI) is able to enforce repayment, and can therefore bail out the government by lending more than the markets are willing to do. I show that, if the IFI is ready to step in, markets lend more at lower spreads, and governments collect lower fiscal surplus and accumulate more debt. In a numerical example calibrated to Argentina, I show that, although the incidence of default is reduced in the presence of the IFI, bailouts are frequent and inevitable unless bailout access is subject to conditionality.

*Preliminary and incomplete

1 Introduction

One of the key questions after the sovereign defaults in emerging markets of the 1990's was to what extent the availability of IMF bailouts could result in moral hazard by investors and borrowing countries. Rogoff (2002) has casted doubt on the belief that moral hazard should be a big concern to taxpayers who finance IMF bailouts, noting that "IMF loans have had a stubborn habit of being repaid in full". Recently, the European debt crisis has renewed interest in the questions involving the role and the inintended consequences of bailouts.

A related question which stimulates vivid political debate is about the conditionality to be attached to bailouts. In December 2019, the Eurogroup agreed in principle on the elements related to a reform of the European Stability Mechanism (ESM). Part of the reform is an amendment of its Precautionary Conditioned Credit Line (PCCL), specifying that access to this credit line be possible only to countries respecting some ex-ante criteria in terms of debt and deficit levels. Countries not fulfilling these criteria would instead have access, in case of a crisis, to the Enhanced Credit Line (ECL), which might entail debt restructuring. Some European countries are averse to the ex-ante conditions attached to the PCCL: they see such conditions as an unreasonable request for "austerity", and perceive abiding to them as a loss of national sovereignty in fiscal policy matters.

In this paper I build a dynamic stochastic model of strategic sovereign default to address these two broad issues: the moral hazard created by the possibility of bailouts, even when the latter come in form of loans that do not involve any debt relief component, and the rationale for demanding ex-ante conditionality, in particular in terms of fiscal policy. In the model, the government's fiscal policy and repayment decision, labor supply and production in the private sector, the market's and the bailout institution's willingness to lend are all endogenous and interacting decisions.

The government enters a period with a certain debt level d, and, in the Eaton-Gersovitz (1981) tradition, decides whether or not to repay by comparing the value function in repayment and in default. If it repays, it decides how much to tax and how much to borrow. Markets are incomplete and only 1-period, non-contingent debt is available from international markets. The only available taxes are distorting taxes

on labor, implying that raising taxes has an increasing cost in terms of labor supply and output. Borrowing decisions are crucially constrained by the market's willingness to lend. The maximum amount that the markets are willing to lend is related to the maximum the government is expected to repay next period. The market's willingness to lend therefore interacts with the government's willingness to raise taxes and repay.

The central role of the market's willingness to lend is the novel aspect of my analysis, one that has not been emphasized in models of strategic default. The inability of the government to commit to future repayment implies that the market's willingness to lend is inferior to the present discounted value of the surpluses the government could collect. After a bad (fundamental) shock, the government might default although it would be willing to repay if it was able to borrow more and commit to future repayments.

Although international investors are individually risk-neutral, they suffer an aggregate negative externality in case of default. Curbing the negative externality is the rationale for setting up an International Financial Institution (IFI) of the sort of the IMF and the ESM. A bailout by the IFI has a cost, but I assume that this cost is lower than the negative externality associated with a default.

The defining characteristic of the IFI, as opposed to the markets, is its ability to enforce repayment. [1] Thanks to this, the IFI is willing to lend up to the maximum sustainable debt level, i.e. the maximum debt level consistent with the full intertemporal budget constraint of the government. A bailout occurs when the market cannot lend enough for the government to repay its past debt, but the IFI can. In this case I assume that the IFI imposes on the government a repayment schedule specified so that the value to the government of repaying the IFI is equal to the value function in default. This way, the government is willing to accept a bailout when (and only when) it would otherwise default on its debt. In the baseline model the IFI does not impose any conditionality on the access to a bailout, other than the sustainability of the debt level. This is meant to provide a benchmark against which the need for conditionality

[1] Historically, losses incurred by the IMF on its loans have been very small. As discussed for example by Aylward and Thorne (1998), the IMF employs a set of strategies to enforce repayment and to quickly resolve cases of overdue obligations. These strategies include conditionality on the use of resources, technical assistance in the design and implementation of adjustment programs, strong remedial measures in case of protracted arrear problems.

in terms of fiscal policy should be measured.

The core of my analysis consists in analyzing how the presence of the IFI affects the optimal decisions of the government and the lenders. If the maximum investors were willing to lend did not change, the presence of the IFI would reduce the risk of a default to almost zero. But in fact I show that the market's willingness to lend dramatically increases in the presence of the IFI. As a consequence, default risk persists, to the point that, if the government borrows as much as it can from the markets, the probability that it defaults next period (by exceeding even the threshold that makes it eligible for a bailout) is unchanged relative to the model without the IFI. In turn, the optimal response of the government changes: it borrows more, reduces its fiscal surplus for a given level of debt, and reduces the maximum surplus it is willing to raise before resorting to default (or asking for a bailout). Importantly, I show that, although the value function of accepting a bailout is equal to the value function in default, for the the government it is important to contain the risk of a default, but it is not important to avoid a bailout.

The intuition is the following: governments try to avoid default because it imposes a deadweight cost, on top of the fact that the investor participation constraint needs to be satisfied. In other words the possibility of a default next period imposes the double whammy of a credit spread in the states of repayment, and of the deadweight cost in case of default. If a bailout is available, instead, the government will repay either the IFI (in the bailout states) or the investors directly, and no credit spreads are charged in the limit in which the default probability is negligible.

In a numerical exercise calibrated to the Argentinian economy, in which I solve for the optimal fiscal and default policy of the government, I find that the fiscal surplus chosen by the government in the presence of the IFI, for any given level of debt, is 2 to more than 10 percentage points lower than in the absence of the IFI. The maximum primary surplus (MPS) is 6% in the absence of the IFI and less than 3% in the presence of the IFI. On the positive side, the IFI reduces the frequency of defaults: without the IFI, a country similar to Argentina defaults with probability 13% in a century; with the IFI, this probability is reduced to around 2%. However, the government needs a bailout with probability one, with an average time to bailout of about 14.5 years after starting with zero debt!

If bailouts were costless, this state of things would pose no problem. In this case the IFI would be an ideal solution to the inability of the government to commit to future repayments. If this is not the case, the cumulative cost of bailout might become prohibitive in the absence of conditionality.

In the last part of the paper, I explore the possibility that the IFI could impose some constraints on the government's fiscal policy as a precondition for the bailout guarantee, and I highlight that any conditionality entails a tradeoff between the welfare of the government and the welfare of the agents who bear the cost of the bailout and of the default. To find the conditions that the IFI should optimally impose, one would need to know the severity of these costs, and take a stand on the weight to assign to the welfare of the various agents, which is beyond the scope of this paper.

Literature review

This paper builds on the seminal paper on strategic sovereign default by Eaton and Gersovitz (1981), and on the subsequent quantitative contributions by Aguiar and Gopinath (2006), by Arellano (2008), and by Cuadra, Sanchez and Sapriza (2010) (the latter also endogenize the government's fiscal policy). Relative to this literature, the novel aspect of my paper is the central role of the market's willingness to lend, which endogenously interacts with the decisions of the other agents, notably with the government's fiscal decisions.

This paper is also related to the "excusable default" models by Collard, Habib and Rochet (2015) and Ghosh, Kim, Mendoza, Ostry and Qureshi (2013) , which provide an alternative to the Eaton-Gersovitz tradition. Like my paper, these two papers develop the concept of the market's willingness to lend, which depends on the MPS achievable by the government; however the MPS is exogenous in their models.

Models about sovereign bailouts include Zettelmeyer, Ostry and Jeanne (2008), Fink and Scholl (2016), Roch and Uhlig (2018), Corsetti, Guimaraes and Roubini (2006), Corsetti, Erce and Uhlig (2018). The bailout agency takes different roles in all these papers. In Zettelmeyer, Ostry and Jeanne (2008) the bailout agency can force the government to undertake ex-post fiscal reforms, and can therefore make solvent a previously insolvent government. In Roch and Uhlig (2018) and Corsetti, Erce and Uhlig (2018) the bailout agency can coordinate investors' expectations, and is therefore useful to avert self-fulfilling debt crises. In Corsetti, Guimaraes and Roubini (2006)

the agency can provide liquidity support to solvent countries and is therefore useful in case of liquidity runs. In Fink and Scholl (2008) the government, even before a debt crisis, can decide to switch from market investors to official lenders, if it commits to undertake some fiscal reforms. The novel role of the bailout agency in my paper lies in its ability to lend more than the markets, due to the fact that it can enforce repayment. In contrast, the markets' willingness to lend is limited by the possibility of future default. Also novel is the analysis about how optimal fiscal policy endogenously changes in the presence of the bailout agency, absent any conditionality.

2 Model

2.1 Households

A small open economy is populated by a representative household with preferences

$$\Sigma_{t=0}^{\infty}\beta^t\delta_t E_0[u(c_t, L_t)] \tag{1}$$

where c_t is consumption, L_t hours worked, β is the discount factor and δ_t is a variable that incorporates households' disutility in case of a government default. More details on the variable δ_t will be given in the next subsection. Production is

$$Y_t = A_t L_t \tag{2}$$

A_t is the stochastic productivity, following a process

$$\frac{A_t}{A_{t-1}} = g_t \tag{3}$$

The probability distribution of g_t depends on the history $g^{t-1} \equiv (g_0, g_1, ...g_{t-1})$ but not on the level of productivity.

Having no access to private saving instruments, the household decision is a purely static consumption/leisure choice: the budget constraint is simply

$$c_t = (1 - \tau_t)A_t L_t \tag{4}$$

where τ_t is the labor tax rate, imposed by the government and taken as given by the household. I use a Greenwood, Hercovitz and Huffman (1988) specification $u(c, L) =$

$u(c - g(L))$, and in particular

$$u(c_t, L_t; A_t) = \frac{\left(c_t - A_t \frac{L_t^{1+\psi}}{1+\psi}\right)^{1-\gamma}}{1-\gamma} \tag{5}$$

which results in the following policy functions

$$L(\tau) = (1-\tau)^{\frac{1}{\psi}} \tag{6}$$

$$c(\tau) = A_t(1-\tau)^{\frac{1+\psi}{\psi}} \tag{7}$$

We see that this specification of the utility function, in which labor disutility increases in the technology level A_t, results in a labor supply that reacts to changes in tax rates but not to changes in technology. Using the policy functions we can express the felicity function u as a function of τ only (given the exogenous A_t):

$$u(\tau; A_t) = \left(\frac{\psi A_t}{1+\psi}\right)^{1-\gamma} \frac{1}{1-\gamma}(1-\tau)^{\frac{(1+\psi)}{\psi}(1-\gamma)} \equiv \left(\frac{\psi A_t}{1+\psi}\right)^{1-\gamma} \tilde{u}(\tau) \tag{8}$$

where the rescaled felicity function $\tilde{u}$ is given by $\tilde{u} \equiv \frac{1}{1-\gamma}(1-\tau)^{\frac{(1+\psi)}{\psi}(1-\gamma)}$. Finally, using (3) and (8) the utility function (1) can be written as

$$\left(\frac{\psi A_0}{1+\psi}\right)^{1-\gamma} \Sigma_{t=0}^{\infty} \beta^t E_0 \left[\left(\Pi_{t'=1}^{t} g_{t'}\right)^{1-\gamma} \delta_t \tilde{u}(\tau_t)\right] \tag{9}$$

Since future growth rates g_t are independent of the initial technology level A_0, the latter is just a scaling factor, that does not affect decisions.

2.2 Government

The government sets the tax rate τ_t and consumes $G_t = \tau_g Y_t$, with τ_g constant. The government is benevolent and shares the same utility function (1) as the household. While the household makes the intratemporal decision between leisure and consumption, the government, which has access to international debt markets at the international rate r (constant), is responsible for the intertemporal decision.

More precisely the government starts period t with outstanding debt D_t. The first decision is whether or not to honor this debt. If it defaults, it exits the debt market and from then on the tax rate will be a constant $\tau = \tau_g$ (I assume the recovery rate to be zero for simplicity). In addition to exclusion from the international markets,

defaulting also entails an extra loss of utility: the variable δ_t in (1) is equal to 1 as long as the government has never defaulted, but if the government defaults at time t^{def}, δ_t is equal to a constant ξ for every period $t \geq t^{def}$.[2]Abstracting from the scaling factor $\left(\frac{\psi A_t}{1+\psi}\right)^{1-\gamma}$ (as I will always do in the following), the value function in default is thus

$$V^{def}(g^t) = \xi \Sigma_{t'=t}^{\infty} \beta^{t'-t} E_t \left[\left(\Pi_{s=t}^{t'} g_s \right)^{1-\gamma} \right] \tilde{u}(\tau_g) \tag{10}$$

If it does not default, the government needs to decide how much to borrow (B_t) and how much to tax, in order to finance its own expenditure and repay the outstanding debt, i.e. to satisfy the budget constraint

$$(\tau_t - \tau_g) A_t L_t + B_t = D_t \tag{11}$$

Defining

$$d_t \equiv \frac{D_t}{A_t} \tag{12}$$

$$b_t \equiv \frac{B_t}{A_t} \tag{13}$$

and using the policy function (6), (11) can be rewritten as

$$(\tau - \tau_g)(1-\tau)^{\frac{1}{\psi}} + b_t = d_t \tag{14}$$

d_t and b_t can be interpreted as debt and borrowing as a fraction of potential output (the maximum output level for a given technology level, which occurs when $\tau = 0$ and $L = 1$).

Notice that, given the household policy function for labor supply (6), the fiscal surplus (also scaled by A_t)

$$\mathcal{S}_t = (\tau_t - \tau_g) L_t = (\tau - \tau_g)(1-\tau)^{\frac{1}{\psi}} \tag{15}$$

follows a Laffer curve whose peak is reached at $\tau^{peak} = (\psi + \tau_g)/(1+\psi)$.[3] Surplus at the peak is

$$\mathcal{S}^{peak} = \psi \left(\frac{1-\tau_g}{1+\psi} \right)^{\frac{1+\psi}{\psi}} \tag{16}$$

[2]Assuming $\gamma > 1$, the value function is negative, and loss of utility occurs for $\xi > 1$.

[3]Total fiscal revenues $\tau A_t L_t = A_t \tau (1-\tau)^{\frac{1}{\psi}}$ also follow a Laffer curve, but with peak at a different tax rate $\tau = \frac{\psi}{1+\psi}$.

Maximal labor supply, L_{max}, occurs for zero taxes, and minimal labor supply, L_{min} occurs for $\tau = \tau^{peak}$ (higher tax rates will never be chosen by the government):

$$L_{max} = 1 \tag{17}$$

$$L_{min} = \left(\frac{1-\tau_g}{1+\psi}\right)^{\frac{1}{\psi}} \tag{18}$$

Since, as we will see below, the government has limited borrowing capacity, it has no choice but to default when the maximum surplus $\mathcal{S}^{peak}$, plus the maximum it can borrow from the market, is insufficient to repay.

Calling $\hat{b}_t$ the maximum level of borrowing achievable at time t, we can write the government's problem in recursive form as

$For\ d_t > \hat{b}_t + \mathcal{S}^{peak}$

$$V(d_t, g^t) = V^{def}(g^t)$$

$otherwise:$

$$V(d_t, g^t) = max(V^{def}(g^t), V^{no\ def}(d_t, g^t))$$

$$V^{no\ def}(d_t, g^t) = max_{\{b_t\},\{\tau_t\}} \left(u(\tau_t) + \beta E_t[g_{t+1}^{1-\gamma}\ V(d_{t+1}, g^{t+1})]\right)$$

$s.t.$

$$d_t - b_t = (\tau_t - \tau_g)(1-\tau_t)^{\frac{1}{\psi}} \tag{19}$$

$$d_{t+1}g_{t+1} = (1 + r + x_t)b_t \tag{20}$$

$$b_t \leq \hat{b}_t \tag{21}$$

Constraint (20) is the relationship between borrowing at t and debt at $t+1$. x is the credit spread set by investors. Constraint (21) reflects the fact that each period there is a (state-dependent) maximum that investors are willing to lend, as we will see in the investors' problem.

If, for every history of shocks g^t, imposing the maximum tax rate τ^{peak} and exhausting the borrowing capacity is preferrable to default, i.e.

$$\tilde{u}(\tau^{peak}) + \beta E_t[g_{t+1}^{1-\gamma}\ V(d_{t+1}, g^{t+1})|b_t = \hat{b}_t] > V^{def}(g^t) \tag{22}$$

then there is never a strategic default: the government only defaults when it is impossible to repay. In this limit model is effectively equivalent to a model of "excusable default", as e.g. the model of Collard Habib Rochet (2017).

2.3 Investors

Investors are risk neutral and perfectly competitive. Knowing that the government can default, and given b_t, they set the credit spread x_t so that

$$(1 + r + x_t)(1 - P^{def}) = (1 + r) \tag{23}$$

The borrowing capacity $\hat{b}(g^t)$ is the highest value of borrowing b for which (23) has a solution. It is then

$$\hat{b}(g^t) = \frac{1}{1+r} max_{d^e} E_t[(1 - P^{def}(d^e; g^t))d^e] \tag{24}$$

with $d^e \equiv (1 + r + x)b$, representing the face value of time-$t + 1$ debt as a fraction of time-t technology level.

2.4 Equilibrium

An equilibrium is given by:

- household policy functions $c(\tau)$, $L(\tau)$
- government policy functions: default decision $\mathcal{D}(d_t, g^t)$ ($\mathcal{D} = 1$ is default, $\mathcal{D} = 0$ is no default), tax and borrowing decisions in case of no default $\tau(d_t, g^t)$, $b_t(d_t, g^t)$
- borrowing capacity $\hat{b}(g^t)$, spread $x(d_t, g^t)$

such that: given τ, the labor/leisure policy functions $c(\tau)$ and $L(\tau)$ maximize intratemporal households' utility; given the exogenous shocks, given $\hat{b}$ and x chosen by investors, and given the household's choices, $\mathcal{D}$ and, in case of repayment, τ and b, maximize the government's value function; given the government's policy functions the credit spread x is such that that the investors' participation constraint is satisfied and $\hat{b}$ is the highest borrowing level for which the participation constraint can be satisfied.

3 Baseline case: i.i.d. growth rates

I now specialize the analysis of the previous section to an i.i.d. $g_t \sim \mathcal{N}(\mu, \sigma)$. This greatly simplifies the analysis of the government problem and the determination of the borrowing capacity. The history of shocks g^t is irrelevant to predict future shocks and

only affects the current technology level. Since the latter, as we saw in the previous section, does not affect the problem of the household, the government or the investors, g^t can be disregarded altogether and the only state variable is d_t. The value function in default is a constant, independent of the state, and can be easily computed as

$$V^{def} = \frac{\xi(1-\tau_g)^{(1+\psi)(1-\gamma)}}{(1-\gamma)\left(1-\beta exp\left(\mu(1-\gamma)+\frac{\sigma^2}{2}(1-\gamma)^2\right)\right)} \tag{25}$$

The borrowing capacity $\hat{b}$ is also a constant, whose determination will be addressed in Section 3.2.

As discussed in the previous section, default can happen for two reasons: either because the government cannot collect the resources to repay ("excusable default" case), or because the value function in case of repayment is lower than the value function in default ("strategic default" case). In the latter case default occurs when $V^{no\ def}(d_t) < V^{def}$. Since the value function depends only on the debt level and it is a decreasing function of d, default occurs for d_t bigger than a fixed threshold (see also Arellano (2008)). In sum the default threshold debt d^* is

$$d^* = min\{d\ s.t.\ V(d) = V^{def}, \mathcal{S}^{peak} + \hat{b}) \tag{26}$$

For debt levels lower than d^* the government decides to repay and makes the fiscal decision of how much to tax and how much to borrow.

3.1 Euler Equation and Policy Functions

Given the default threshold debt level d^*, whose value needs to be solved for, the Euler equation is

$$\begin{aligned} \frac{\tilde{u}'(\tau_t)}{f(\tau_t)} + \lambda_t &= \beta(1+r+P_{surv}(b_t)bx'(b_t))E_t\left[g_{t+1}^{-\gamma}\frac{\tilde{u}'(\tau_{t+1})}{f(\tau_{t+1})} \mid g_{t+1} \geq g^{th}\right] \\ &+ (g^{th})^{1-\gamma}(V^{def} - V(d^*))\mathrm{f}(g^{th})\ \frac{dg^{th}}{db} \end{aligned} \tag{27}$$

$$\text{with} \quad g^{th} = \frac{b_t(1+r+x(b_t))}{d^*} \tag{28}$$

$$\text{and} \quad f(\tau) = \frac{d\mathcal{S}}{d\tau} = (1-\tau)^{\frac{1}{\psi}} - \frac{1}{\psi}(\tau-\tau_g)(1-\tau)^{\frac{1}{\psi}-1} \tag{29}$$

b_t is related to τ_t by the budget constraint (19); $x'(b_t)$ is the derivative of the credit spread; λ_t is the Lagrange multiplier associated to the condition $b_t \leq \hat{b}$; g^{th} is the

threshold value of the growth factor below which default occurs next period; f(g) is the probability density of the growth factor; $P_{surv}(b_t)$ is the survival probability next period given the borrowing level b_t (i.e. the probability that g_{t+1} be above g^{th}).

The intuition conveyed by the Euler equation is the following: suppose the government at time t borrows b_t and imposes the tax rate τ_t, satisfying the budget constraint (19). What would be the marginal effect of increasing borrowing by an amount Δ? At time t, the tax rate could be decreased by an amount $\frac{\Delta}{f(\tau_t)}$, which would increase today's utility by an amount $\frac{u'(\tau_t)}{f(\tau_t)}\Delta$. Next period, two things would occur: first, default would occur in more states, as the probability of default would increase by f$(g^{th})\frac{dg^{th}}{db}\Delta$. In the new default states (those just below the default threshold d^*, where default would not have occurred without extra borrowing Δ) utility changes by an amount $(g^{th})^{1-\gamma}(V^{def} - V(d^*))$, which is zero in the "strategic default" case and negative in the "excusable default" case. Second, in the survival states, the amount the government needs to repay would increase from $(1 + r + x(b_t))b_t$ to $(1 + r + x(b_t + \Delta))(b_t + \Delta)$. To first order in Δ, and using (23), the increase can be written as $\frac{1+r+P_{surv}(b_t)x'(b_t)}{P_{surv}(b_t)}\Delta$.

The increase in default risk, reflected by a positive $x'(b)$, clearly makes the extra Δ amount borrowing less attractive: repayment in the (fewer) survival states increases by more than $\frac{(1+r)\Delta}{P_{surv}}$, while utility is at best unchanged in the (more numerous) default states.

A solution of the government's problem consists in finding the default threshold d^* and the policy functions $\tau(d)$ and $b(d)$ – tax rate and borrowing level for $d \leq d^*$ – satisfying the Euler equation (27). While a full solution needs to be computed numerically, the following properties of the policy functions and the default threshold can be proved analytically (all detailed proofs are in Appendix):

Proposition 1 *If, for a debt level d, $b(d) < \hat{b}$, then $\tau(d) < \tau^{peak}$.*

This proposition tells us that the government would never want to raise a surplus corresponding to the peak of the Laffer curve if it has not already exhausted its borrowing capacity. As the tax rate approaches the peak of the Laffer curve, raising taxes is more and more costly in terms of consumption, but the increase in the surplus gradually approaches zero. In fact $f(\tau)|_{\tau=\tau^{peak}} = 0$, making the LHS of the Euler equation infinitely negative at this point. The only case in which the government would want to

reach this tax rate is in the "excusable default" case, when default is so costly that anything that can be done to avoid it is worth it: borrow the maximum and collect the maximum possible surplus. In this case the Lagrange multiplier λ is infinite.

Proposition 2 *As long as $b(d) < \hat{b}$, both $\tau(d)$ and $b(d)$ are strictly increasing in d.*
This proposition implies that the maximum primary surplus (MPS) achieved by the government occurs at the default threshold: $MPS = \tau(d^*)$.

Proposition 3 $d^* \geq \hat{b} + (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}}$ *where τ^o is such that $\tilde{u}(\tau^o) = \xi \tilde{u}(\tau_g)$*
This proposition gives us a lower bound for the default threshold d^*. The lower bound comes as a consequence of the fact that the government does not default if it can repay by collecting a surplus such that its current-period utility is equal to utility in default, and borrowing the rest. By defaulting it would lose optionality without any utility gain.

Propositions 1-3 imply that the solution of the government problem falls in one of three cases. In a first case, $\tau(d)$ and $b(d)$ both increase for every debt level $d \leq d^*$, and we have $b(d^*) < \hat{b}$, $\tau(d^*) < \tau^{peak}$. In this case the government defaults before exhausting its fiscal and borrowing capacity. In a second case $\tau(d)$ and $b(d)$ both increase until $b(d)$ reaches the borrowing capacity $\hat{b}$ for a debt level $\tilde{d} < d^*$; for $\tilde{d} < d \leq d^*$ $\tau(d)$ continues to increase while $b(d)$ is constant at $\hat{b}$. At the default threshold we have $b(d^*) = \hat{b}$ and $\tau(d^*) < \tau^{peak}$. In a third case (the "excusable default" case) default occurs at the debt level $\hat{b} + \mathcal{S}^{peak}$, with $b(d^*) = \hat{b}$ and $\tau(d^*) = \tau^{peak}$, so that all the government's resources are exhausted.

3.2 The government's borrowing capacity

The government's borrowing capacity in general needs to be determined in conjunction with the default threshold d^*. Suppose we know the default threshold level d^*. The government enters period t with debt to repay d_t; if $d_t \leq d^*$ the government repays, by borrowing $b(d_t)$ and taxing at a rate $\tau(d_t)$. Call "end-of-period debt" $d_t^e \equiv b(d_t)(1 + r + x(b))$, where $x(b)$ is the credit spread. At the beginning of next period, after the tecnhological shock g_{t+1} is realized, debt will be $d_{t+1} = d_t^e / g_{t+1}$ and repayment will

occur if $d_{t+1} \leq d^*$, i.e. if $g_{t+1} > \frac{d^e}{d^*}$, hence

$$P^{surv}(b) = 1 - F\left(\frac{b(1+r+x(b))}{d^*}\right) \tag{30}$$

where $F(g)$ is the probability distribution of the growth factor. The investor participation constraint is

$$\begin{aligned} b(1+r) &= b(1+r+x(b))P^{surv}(b) \\ &= d^* \times \frac{d^e}{d^*}\left(1 - F\left(\frac{d^e}{d^*}\right)\right) \end{aligned} \tag{31}$$

where the last equality uses (30). The borrowing capacity is the maximum value of b for which the above equation has a solution

$$\hat{b} = \frac{d^*}{1+r} max\ g(1 - F(g)) \equiv \theta \frac{d^*}{1+r} \tag{32}$$

The borrowing capacity is therefore proportional to the default threshold debt level d^*, and the proportionality factor contains the constant $\theta \equiv max(g(1 - F(g)))$, which depends on the distribution of the growth factor.

- **Case 1 – "Strategic default"**

  In this case the default threshold d^* is determined by the condition $V(d^*) = V^{def}$. The borrowing capacity $\hat{b}$ and default threshold d^* need to be jointly solved for. Given $\hat{b}$, we can solve for the government's value function $V(d)$, and find d^* as the solution to $V(d^*) = V^{def}$. Given d^*, $\hat{b}$ solves (32).

- **Case 2 – "Excusable default"**

  In this case we know te default threshold to be $d^* = \mathcal{S}^{peak} + \hat{b}$. Using (32) we obtain the borrowing capacity

  $$\hat{b} = \frac{\theta}{1+r-\theta}\mathcal{S}^{peak} \tag{33}$$

  Given (16) we can fully express the borrowing capacity in terms of the model's exogenous parameters:

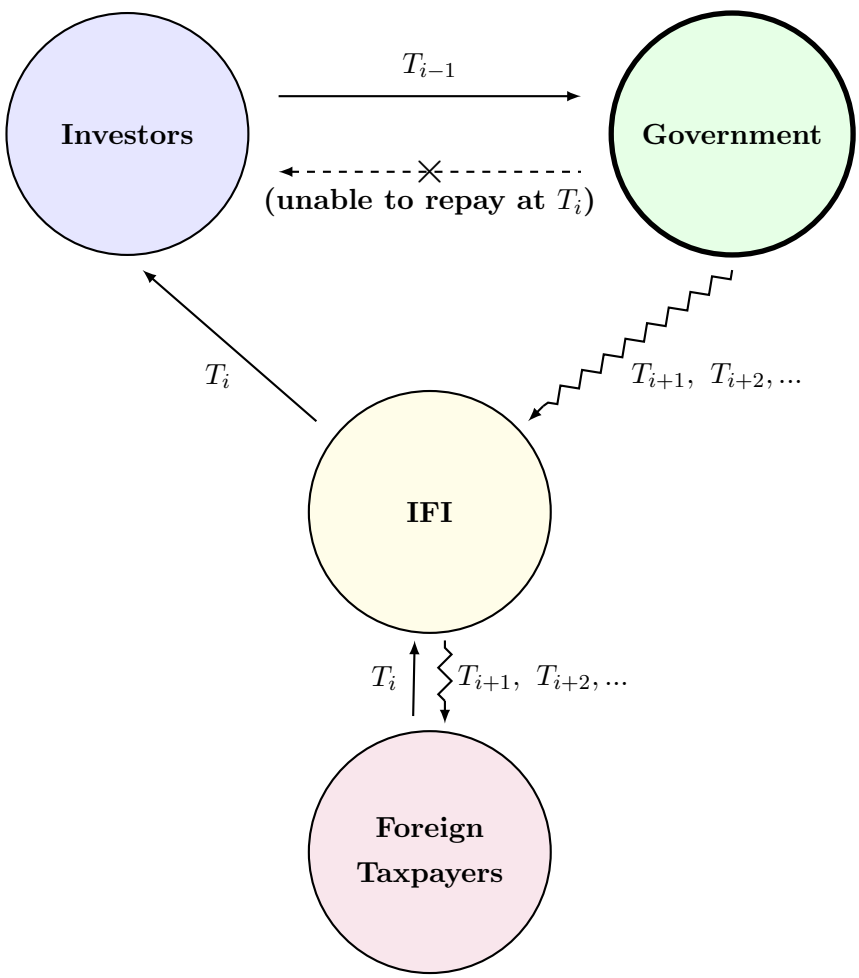
  $$\hat{b} = \frac{\theta}{1+r-\theta}\psi\left(\frac{1-\tau_g}{1+\psi}\right)^{\frac{1+\psi}{\psi}} \tag{34}$$

4 Bailouts

Why bailouts? I posit that investors, although risk-neutral, suffer a utility loss κ^{def} in case of default. This does not affect their willingness to lend, since default is not something they individually can affect. However, the disutility they suffer in case of default induces them to establish an International Financial Institution (IFI).

The IFI purports to avoid the deadweight cost associated with default by acting as an intermediary between a pool of (risk-neutral) foreign taxpayers and the government, as graphically represented in Figure 1.

Figure 1



Differently from uncoordinated investors, the IFI can enforce government repayment

and can propose an equity-like contract to the government: the latter will repay a fraction of its output in each period after the bailout, so that the expected value of all future repayments equals the bailout amount. [4] I assume that the IFI incurs a cost κ^{bail}, such that $0 < \kappa^{bail} < \kappa^{def}$, for each bailout, and that the government is free to accept or reject the bailout contract. The equilibrium between these two forces implies that the IFI offers the government a contract such that the government is indifferent between accepting it and rejecting it, when rejecting it would mean defaulting on the debt. Indeed, if the utility of the government after accepting a bailout, V^{bail}, was lower than V^{def}, the government would prefer default, and if $V^{bail} > V^{def}$ the government would accept a bailout also in situations where it would otherwise have repaid the debt, which would impose an unnecessary cost on the IFI.

Proposition 5 *Assuming $\xi > 1$ (i.e. assuming that default entails a loss of utility, on top of the exclusion from international markets), the maximum the IFI is willing to lend to the government is bigger than d^**

(Detailed proof is in Appendix). This is the crucial proposition which tells us that, while both individual investors and investors intermediated by the IFI are risk-neutral and want a fair return for their investment, "IFI investors" can lend more than individual ones (by Proposition 3, $d^* > \hat{b}$ if $\xi > 1$).

One way to see this is the following. Consider "scenario A": we are in the absence of the IFI and debt at the beginning of a period t_0 is equal to d^*. The government is willing to repay by imposing a tax rate $\tau(d^*)$ (and borrowing $b(d^*)$) in the same period, and a sequence of tax rates in the following periods, contingent on the realization of the productivity shock, until default. The investor participation constraint implies that the present discounted value of the sequence of surpluses (including the one at t_0) is equal to d^* After default, the tax rate would be τ_g but the felicity function would only be $\xi\tilde{u}(\tau_g)$.

The IFI could propose a contract to the government, so that, contingent on the

[4] Although repayment schedules to bailout agencies such as the IMF are usually defined in dollar terms, rather than in percentages of the debtor country's output, many countries experiencing difficulties, often political problems or civil unrests, have been able to run even protracted arrears without losing the IMF financial support. IMF loans thus contain in practice an element of state-contingency that is absent from ordinary market debt.

realization of the shock, the latter would collect the same surpluses to repay the IFI as in "scenario A" (including the one at the bailout time t_0). When the sequence of shocks is such that the government would be in default in "scenario A", the IFI would impose a payment equal to a fraction τ_o of income, with τ_o such that $\tilde{u}(\tau_o) = \xi\tilde{u}(\tau_g)$. If $\xi > 1$, the present discounted value of this sequence of surpluses is higher than the one in "scenario A", hence must be higher that d^*.

The above discussion shows that the IFI is able to lend the government more than the markets would, under a contract that respects the participation constraint of "foreign taxpayers" and that the government would take only if it is unable to repay by borrowing from the markets. The higher willingness to lend of the IFI is due to its enforcement ability, and to the fact that it can propose equity-like contracts to the government, whereas individual investors are restricted to 1-period, non-contingent bonds.

The core question I want to address is how agents change their behavior after the IFI is established. In particular:

1. Would investors be willing to lend more to the government and/or at lower spreads?
2. Would the government's fiscal decisions, in particular the MPS, change?
3. How often would the IFI need to intervene by bailing out the government?
4. Finally, how would the frequency of defaults change?

4.1 The market's willingness to lend

This subsection addresses the first of the above-listed questions: how the market's willingness to lend is affected by IFI's bailout guarantee.

The first observation is that market investors want to be repaid, regardless whether they are repaid by the government directly or through a bailout. The only case in which they are not repaid is if the country's debt at the beginning of next period exceeds b^{bail}. By the same reasoning used in Section 3.2, the market's willingness to lend b^{mkt} is

$$b^{mkt} = \frac{b^{bail}}{1+r} max\ g(1 - F(g)) \equiv \theta \frac{b^{bail}}{1+r} \tag{35}$$

Proposition 6 $\hat{b} < b^{mkt} < b^{bail}$. *For a given borrowing level $b \leq \hat{b}$, the spread $x(b)$ charged by the market is lower in the presence if the IFI.*

The first inequality, $\hat{b} < b^{mkt}$, is obvious if we compare (32) and (35), given that $d^* < b^{bail}$. The second inequality, $b^{mkt} < b^{bail}$ is proved in Appendix. Finally, for a given borrowing level, default next period occurs for fewer realizations of g in the presence of the IFI, therefore the spread charged by investors is lower.

In the next subsection, and especially in section 5 when I present the result of a numerical exercise, I will explore how the government changes its fiscal behavior in the presence of the IFI. However, the increase in the market's willingness to lend already allows us to draw one conclusion:

Proposition 7: *Suppose that an impatient (or constrained) government borrows the maximum it can from the market. Whether we are in the absence of the IFI (so that the impatient government borrows $\hat{b}$), or in the presence of the IFI (so that it borrows b^{mkt}), the probability of default next period does not change.*

While we still don't know how the government changes its fiscal behavior in the presence of the IFI, this proposition tells us that, given the increased willingness to lend of the market, the promise of an IFI intervention does not guarantee a decrease in the probability of default.

4.2 The Euler equation

The first step to address the question of how the government changes its behavior in the presence of the IFI is to look at the Euler equation. The Euler equation in the presence of the IFI is

$$\begin{aligned}
\frac{u'(\tau_t)}{f(\tau_t)} + \lambda_t &= \beta P_{surv}(b_t)(1 + r + x(b) + bx'(b_t))E_t\left[g_{t+1}^{-\gamma}\frac{u'(\tau_{t+1})}{f(\tau_{t+1})} \mid g_{t+1} \geq g^{th}\right] \\
&+ (V^{def} - V(d^*))f(g^{th})(g^{th})^{1-\gamma}\frac{dg^{th}}{db} \qquad (36) \\
\text{with} \quad g^{th} &= \frac{b_t(1 + r + x(b_t))}{d^*} \\
\text{and} \quad d^* &= min(d \ s.t. \ V(d) = V^{def}, b^{mkt} + \mathcal{S}^{peak})
\end{aligned}$$

Notice that d^* is now the threshold beyond which the government receives a bailout, rather than the default threshold. $f(\tau)$ is defined as in (29) and λ_t is the Lagrange multiplier associated with the borrowing constraint.

P_{surv} is here intended as the probability that the government repays its debt next period (with no need for a bailout), and $x(b_t)$ is the credit spread, reflecting the probability of default, i.e. the probability of debt exceeding the maximum lending capacity of the IFI b^{bail}. This is the most important difference between the Euler equation (27) without the IFI and (36). Without IFI, the Euler equation is very similar, but $P_{surv}(1+r+x(b))$ can be simplified to $(1+r)$ by the investor participation constraint. With the IFI, this expression is lower than $1+r$.

In the limit in which the probability of a default can be neglected, the government has an incentive to increase its borrowing as much as it can, since by borrowing one extra unit it will have to repay *less than* $1+r$ *units* next period directly to the investors. The investor participation constraint is satisfied because in the bailout states investors will be repaid by the IFI. Of course the government will then have to repay the IFI, but this payment does not affect utility in this states, which is always V_{def}. In sum, what may prevent the government from borrowing one extra unit is only the possibility of default, in particular the term proportional to $x'(b_t)$, which may become very high close to b^{mkt}, rather than the possibility of a bailout.

A solution of the government's problem, given b^{bail} and b^{mkt} related by (35), consists in finding the default threshold d^* and the policy functions $\tau(d)$ and $b(d)$ satisfying the Euler equation (36). Propositions 1-3 still hold for the solution with the IFI (provided that we reinterpret $\hat{b}$ as b^{mkt}). A full solution, and a comparison between the solution with and without the IFI, needs to be obtained numerically.

5 Quantitative Analysis

To fully address how the government chooses its fiscal behavior, in particular the MPS and the default decision, with and without the IFI; how effective is the IFI to reduce the frequency of defaults, and how often the IFI would have to actually intervene with a bailout (Questions 2-4 listed in Section 4), I will now turn to a numerical example.

5.1 Calibration

I calibrate model parameters to the Argentinian economy. An average growth rate of 2% and a standard deviation of 5% reflect the moments of Argentinian GDP (I use

Table 1: Baseline model parameters	
Parameter	Description
$\mu = 0.02$	average growth rate
$\sigma = 0.05$	volatility of growth rate
$\psi = 2$	inverse labor elasticity
$\gamma = 2$	relative risk aversion
$r = 0.04$	risk-free rate
$\xi = 1.03$	default cost
$\tau_g = 0.35$	government spending (fraction of GDP)

gross GDP data from 1962 to 2018). The default cost ξ is chosen so that, given the value of the other parameters, in particular the average growth rate and its standard deviation, MPS is 6% (primary surplus in Argentina was close to 6% in 2004). This value of ξ corresponds to a utility loss in consumption terms of about 3% (in addition to the utility loss of losing the ability to borrow). The government spending I choose, $\tau_g = 35\%$, reflects the average Argentian government spending over the last twenty years.

With these parameters, I find that the borrowing capacity in the absence of bailouts is 39.3%, which is lower, but not dramatically so, than Argentinian debt over GDP when Argentina defaulted in 2001 (it was 49%). The parameter ψ is chosen so that the peak of the revenue Laffer curve occurs at $\tau^{peak} = 0.67$.[5] Clearly, with these parameters we are in a regime of "strategic defaults": the maximum tax rate that the government is willing impose on households, τ_g+ MPS, is 0.41, well below the peak of the Laffer curve.

[5]This value of τ^{peak} is roughly in line with the labor tax rate at the peak of the Laffer curve for the US and the EU-14 countries, as estimated by Trabandt and Uhlig (2012).

5.2 Borrowing capacity and fiscal decisions

With the baseline parameters the government's borrowing capacity is $\hat{b} = 0.302$. This corresponds to 39.3% of GDP, when the government collects a surplus equal to the MPS (MPS is 6%, and when surplus is equal to the MPS, GDP reaches a minimum value $y_{min} = (1 - \tau_g - s^*)^{\frac{1}{\psi}}$, in this case 0.7681).

What happens in the presence of the IFI? For simplicity, for this analysis I assume that the IFI can only offer a simple contract to the government, in which the latter is required to repay the IFI a constant fraction τ of its income for N periods, starting in the period when the bailout occurs. Under these condition, the present value at bailout time of all the future government repayments (considering that the government still needs to finance its expenditure $\tau_g Y$) is

$$\tau(1 - (\tau + \tau_g))^{\frac{1}{\psi}} \left(1 + \Sigma_{i=1}^{N} \frac{(E[g])^i}{(1+r)^i}\right) \tag{37}$$

The maximum value of (37) that is consistent with the government's participation constaint $V^{def} = V^{bail}$ is obtained when $N = \infty$ and the total tax rate the government is required to collect, $\tau + \tau_g$, equals τ_o, the tax rate that makes the felicity function equal to the one in default (i.e. $u(\tau_o) = \xi u(\tau_g)$). The maximum the IFI can lend is therefore

$$b^{bail} = (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}} \left(1 + \Sigma_{i=1}^{\infty} \frac{(E[g])^i}{(1+r)^i}\right) = \frac{(\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}}}{1 - \frac{exp\left(\mu + \frac{\sigma^2}{2}\right)}{1+r}} \tag{38}$$

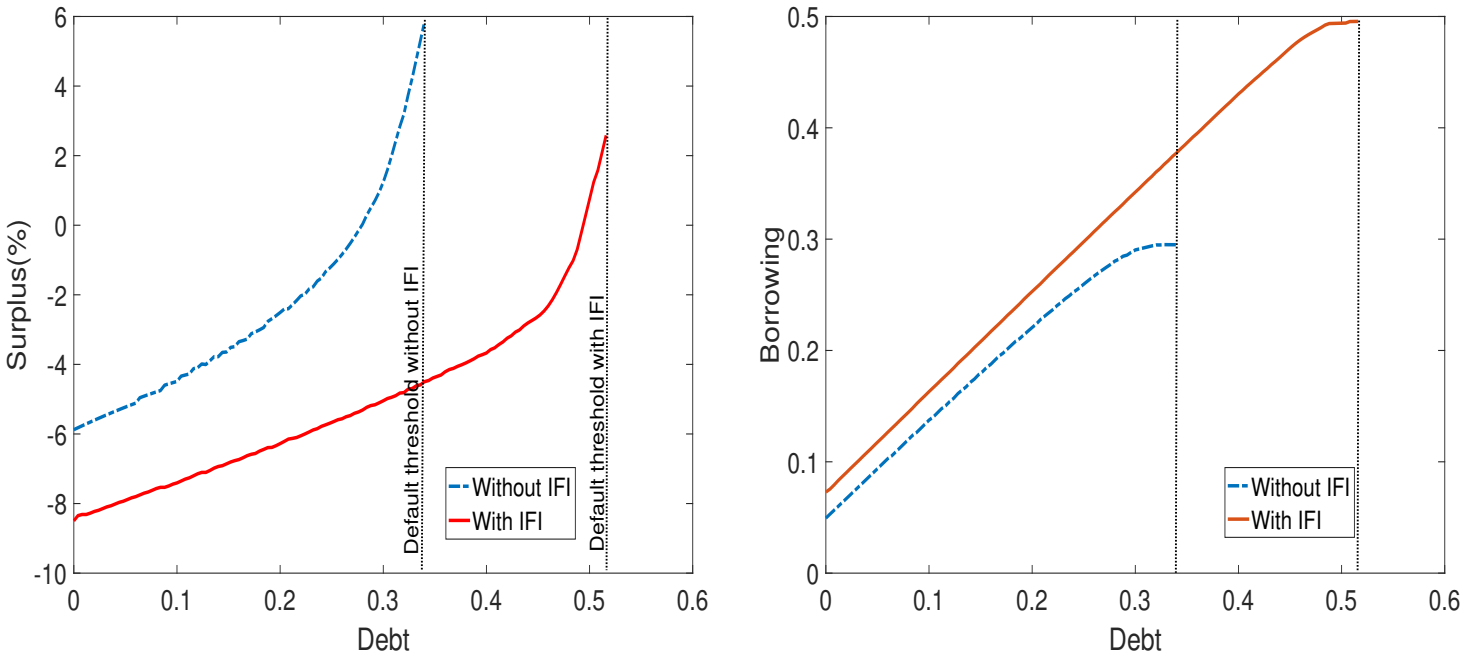
With the parameters I choose, $\tau_o = 0.3628$ and $b^{bail} = 0.574$. In the presence of the IFI willing to lend so much, from (35) I obtain that the market is willing to lend 0.498. The government, taking advantage of the higher borrowing capacity and less willing to raise taxes, reduces its MPS to only 2.8%, less than half of the MPS in the absence of the IFI. Table 2 summarizes the comparison between borrowing capacities and MPS with and without the IFI.

5.3 Fiscal policy functions

In this section I show in detail the fiscal policy of the government with and without the IFI, obtained by solving numerically the government's problem with a value function iteration method.

Table 2			
Without IFI		**With IFI**	
$\hat{b}$	0.302 (39.3 % y_{min})	b^{bail}	0.574 (72.8% y_{min})
		b^{mkt}	0.498 (63.1% y_{min})
d^*	0.341 (44.4% of y_{min})	d^*	0.518 (67.4% of y_{min})
$b(d^*)$	0.295 (38.4% of y_{min})	$b(d^*)$	0.496 (64.6% of y_{min})
MPS	6%	MPS	2.8%

Figure 3: Fiscal Policy

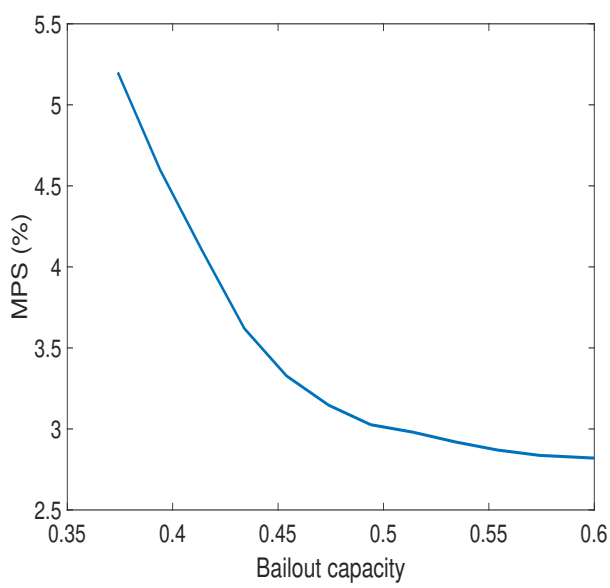


Without the IFI, the government has negative surplus for debt smaller than 0.28, and positive surplus between $d = 0.28$ and the default threshold $d^* = 0.341$. With the IFI negative surplus occurs for debt smaller than 0.494, so that positive surplus is achieved only in the narrow region between $d = 0.494$ and the default threshold $d^* = 0.518$. For a given level od debt, the difference in surplus achieved without and with the IFI is never smaller than 2%; this difference reaches a maximum at $d = 0.341$ (corresponding to the default threshold without the IFI), where it is 10.5%.

The right panel of Figure 3 shows the borrowing level. For a given debt level, borrowing and tax rates are linked by (11). So it is not surprising that, as we see a steepening of the surplus for debt levels close to the default threshold, we see a flattening of the borrowing level. As we see in Table 2, even at the default threshold the borrowing level is slightly below capacity, both without and with the IFI.

Finally, Figure 4 shows that the MPS is decreasing in b^{bail}. The intuition for this is the following: as proved by Proposition 2, the MPS is the surplus (as a fraction of GDP) achieved at the highest debt level, i.e. at the default threshold d^*. At this debt level, the value function of the government is equal to the value function in default by construction (except in the extreme case of "excusable default"). But as the bailout capacity, and hence the borrowing capacity from the market, increases, at the default threshold debt level the government borrows more, which means more debt for the future. How can utility be the same in spite of the different level of accumulated debt? The answer is that, when b^{bail} is higher, the burden of a higher future debt is compensated by the fact that the repayment of this debt can be better spread out in the future, so current consumption can be higher, i.e. current surplus can be lower.

Figure 4: Maximum Primary Surplus



5.4 Frequency of default and bailout

I first simulate the model without the IFI. I draw 10,000 paths for the productivity shock of 100 periods each, and use the above policy functions to observe the dynamic behavior of debt, and the frequency of default. I find the government enters a period with an average debt equal to 0.29 (37.8% of y_{min}), and defaults with probability 13% over 100 periods (1 period corresponds to 1 year).

The possibility of a bailout reduces the number of defaults: simulating the model with the IFI, I find that the default probability is 1.9% on a 100-period path. However, a bailout occurs on every path. If the government enters the first period with zero debt, it takes on average 14.5 years to need a bailout. The average debt with which the government enters a period is non-stationary and increasing in the time from inception.

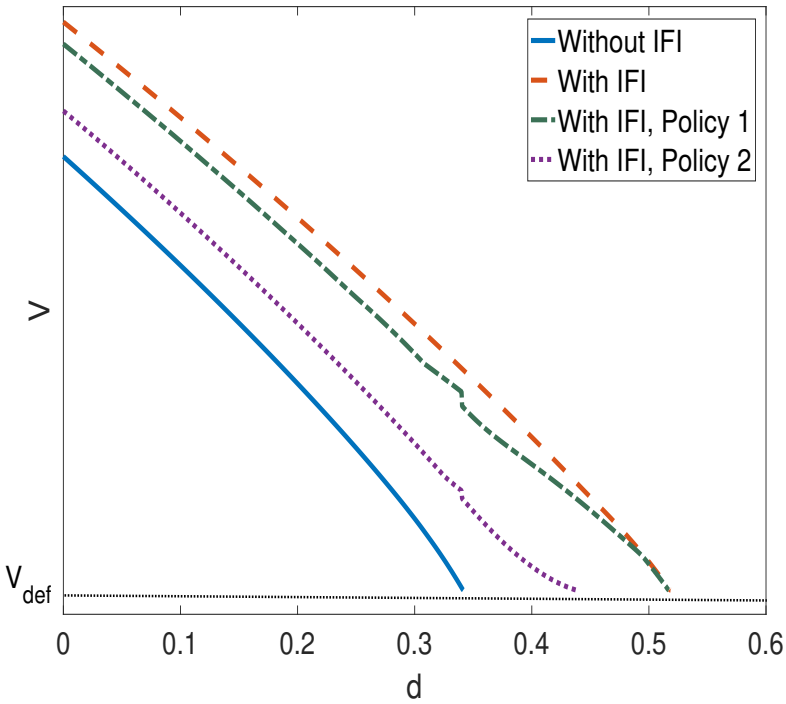
If $\kappa^{bail} = 0$, this state of things would cause no problem. The IFI could be thought as a device to enforce government's repayment and reduce the incidence of default, and thus the incidence of the associated externality. Otherwise, if κ^{bail} is significant, clearly the IFI needs to find a way to reduce the frequency of bailouts. One obvious way would be to make bailout conditional on some prior "good behavior" on the part of the government.

5.5 Conditionality

I now explore what happens if the IFI sets some constraints the government's fiscal policy, and denies access to a bailout to a government that violates the constraints. As an extreme case, the IFI could impose the government to keep the same tax schedule that government chooses without the IFI. In this case, once debt reaches $d^* = 0.341$, the default threshold without the IFI, the government would stop repaying and opt for a bailout. Thus, the bailout probability would be 13% within a century. The default probability would be zero. The value function of the government for every $d \leq d^*$ would be the same as without the IFI (solid line in Figure 5), but would result in a lower externality (κ^{bail} in place of κ^{def}).

Other, less extreme options would reduce less dramatically the bailout and default probability, but also would reduce less dramatically the government's welfare. I discuss

Figure 5: The effect of conditionality on the government's value function



here two examples of such policies.

As a first example, that I call "Policy 1", the IFI might impose the government a positive surplus (i.e. a tax rate above τ_g) any time the debt level rises above a threshold. For example I choose as a threshold level $d = 0.34$, approximately the debt level above which the government defaults in the absence of the IFI. A simulation shows that also in this case a bailout would occur on each of the 100-period paths, but, thanks to the fiscal constraint, the time to bailout would be 21.5 years, higher than the time to bailout when no conditions are imposed. The default probability in a century would be 1.7%, only slightly lower than in the case without conditionality. Figure 5 shows that the government would accept this condition in exchange for the bailout guarantee, as its value function (dash-dotted line) would be higher than the value function without the IFI for every debt value.

As a second example, the IFI could impose a more stringent condition, that I call "Policy 2": in this case above a debt threshold, again $d = 0.34$ in this example, the

government should collect a surplus at least equal to the interest on outstainding debt, i.e. the minimum tax rate above this debt level should be $\tau_g + rd$; in addition, for any debt level surplus could never be lower than a certain percentage level, e.g. 5%. A simulation shows that, if this policy is adopted, a bailout would occur on 81% of the 100-period paths, and the average time to bailout would be 48 years. The default probability would be 0. Also in this case the government would accept the conditions in exchange for the bailout guarantee, as its value function (dashed line in Figure 5), although lower than with "Policy 1", would still be higher than without the IFI for every debt value.

The optimal policy depends on the objective function of the IFI and the parameters of the model, in particular the value of the externality associated with a bailout and a default, κ_{bail} and κ. This is beyond the scope of this paper.

6 Conclusion

In this paper I model the fiscal and default decisions of a government whose borrowing ability is limited by its inability to commit to future repayment. An International Financial Institution is established to avoid the externality associated with default. The IFI can lend more than the market because it can enforce repayment and because loans have equity-like features.

In the presence of the IFI, the markets themselves are willing to lend more (about 80% more than in the absence of the IFI, in a numerical example calibrated to Argentina), and the government borrows more for the same level of debt: its fiscal surplus for a given level of debt is 2 to 10.5 percentage points lower and its maximum primary surplus is less than half than in the absence of the IFI.

In the presence of the IFI the frequency of defaults is reduced, from 13% in a century to 2% in a century, however a bailout occurs on average every 15 years. This would be a good outcome if a bailout was costless, but clearly provides a strong rational for imposing some fiscal conditionality if the cost of a bailout is significant.

Appendix

Proposition 1

If, for a debt level d, $b(d) < \hat{b}$, then $\tau(d) < \tau^{peak}$.

Since $f(\tau^{peak}) = 0$ and since the Lagrange multiplier $\lambda_t = 0$ for $b_t < \hat{b}$, if for a debt level d we had $b(d) < \hat{b}$ and $\tau(d) = \tau^{peak}$, the LHS of the Euler equation (27) would be infinite, which cannot be an equilibrium.

Proposition 2

Call $\tau(d)$ and $b(d)$ the policy functions adopted by the government for $d < \bar{d}$. As long as $\tau(d) < \tau^{peak}$ and $b(d) < \hat{b}$, both $\tau(d)$ and $b(d)$ are strictly increasing in d.

The government's optimization problem satisfies all the conditions specified in Stokey, Lucas and Prescott (1989) for the value function to be strictly concave in the argument d. The envelope condition is

$$V'(d) = \left.\frac{u'(\tau)}{f(\tau)}\right|_{\tau=\tau(d)} \tag{39}$$

Given that $u(\tau)$ is decreasing and strictly concave and that $f(\tau)$ is positive and strictly decreasing, the RHS of (39) is strictly decreasing in τ. Since V is strictly concave the LHS of (39) is strictly decreasing in d. For the equality to hold, $\tau(d)$ must be strictly increasing.

Now I'll show that $\tau(d)$ increasing implies $b(d)$ increasing if $b(d) < \hat{b}$. $\tau(d)$ strictly increasing implies that the LHS of the Euler equation (27) is strictly decreasing in the debt level d_t (strictly increasing in absolute value). Then also the RHS must be strictly increasing in absolute value. It is easy to see that all the terms in the RHS are strictly increasing (in absolute value) in b_t. For the RHS to be strictly increasing in absolute value in d_t, it must be that $b(d)$ is strictly increasing.

Proposition 3

$d^* \geq \hat{b} + (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}}$ *where τ^o is such that $\tilde{u}(\tau^o) = \xi\tilde{u}(\tau_g)$*

If $d = \hat{b} + \tau^o(1-\tau^o)^{\frac{1}{\psi}}$ the following strategy is available: repaying by taxing τ^o today and borrowing $\hat{b}$. This strategy gives utility
$V = u(\tau_o) + \beta\mathcal{C}(\hat{b}) \geq u(\tau_o) + \beta E[g^{1-\gamma}]V^{def} = V^{def}$, where $\mathcal{C}(b)$ is the continuation value after borrowing b.
The optimal strategy when $d = \hat{b} + (\tau^o - \tau_g)(1-\tau^o)^{\frac{1}{\psi}}$ must give at least this utility. Therefore, the threshold debt level d^*, which is such that $V(d^*) = V^{def}$ cannot be lower than $\hat{b} + (\tau^o - \tau_g)(1-\tau^o)^{\frac{1}{\psi}}$.

Proposition 4

Assuming $\xi > 0$, the maximum the IFI is willing to lend to the government is bigger than d^*

Consider what I called "scenario A" in Section 4.1: there is no IFI and outstanding debt at time t is equal to the default threshold level d^*. The government is willing to repay by imposing a tax rate $\tau(d^*)$ and borrowing $b(d^*)$, with

$$d^* = \mathcal{S}(d^*) + b(d^*) \tag{40}$$

where $\mathcal{S}(d) \equiv \tau(d)(1-\tau(d))^{\frac{1}{\psi}}$. We can rewrite (40) as

$$d^* = \mathcal{S}(d^*) + \frac{g_{t+1}d_{t+1}}{1+r+x(b(d^*))} = \mathcal{S}(d^*) + P^{surv}(g_{t+1})\frac{g_{t+1}d_{t+1}}{1+r} \tag{41}$$

In the last equality in (41) I highlighted that, given the borrowing level at time t, the survival probability at $t+1$ is a function of the shock g_{t+1}. Iterating forward I obtain

$$d^* = \mathcal{S}(d^*) + \frac{P^{surv}_{t+1}(g_{t+1})}{1+r}g_{t+1}\mathcal{S}(d_{t+1}) + \frac{P^{surv}_{t+2}(g_{t+1}, g_{t+2})}{(1+r)^2}g_{t+1}g_{t+2}\mathcal{S}(d_{t+2}) + ... \tag{42}$$

(42) shows that outstanding debt at time t is equal to the present discounted value of future surpluses. Notice also that, given the initial outstanding debt, future debt d_{t+1}, d_{t+2} and so on, and hence future surpluses and survival probabilities, are only functions of the realized shocks g_{t+1}, g_{t+2}....

Now, the IFI could write the following contract, that would clearly result in the same utility function for the government: conditional on the realized shocks, impose the same sequence of surpluses as in "Scenario A" when the shocks would imply survival in "scenario A", and impose surplus $\mathcal{S}_o = (\tau_o - \tau_g)(1-\tau_o)^{\frac{1}{\psi}}$, with τ_o defined as in

Proposition 3, when the sequence of shocks would imply default in "scenario A". The present value of such surpluses would be

$$\begin{aligned} & \mathcal{S}(d^*) + \frac{P_{t+1}^{surv}(g_{t+1})}{1+r} g_{t+1}\mathcal{S}(d_{t+1}) + \frac{1-P_{t+1}^{surv}(g_{t+1})}{1+r} g_{t+1}\mathcal{S}_o \\ + & \frac{P_{t+2}^{surv}(g_{t+1},g_{t+2})}{(1+r)^2} g_{t+1}g_{t+2}\mathcal{S}(d_{t+2}) + \frac{1-P_{t+2}^{surv}(g_{t+1},g_{t+2})}{(1+r)^2} g_{t+1}g_{t+2}\mathcal{S}_0 + ... \end{aligned} \tag{43}$$

which would clearly be higher than d^*.

Proposition 5

$\hat{b} < b^{mkt} < b^{bail}$. For a given borrowing level $b \leq \hat{b}$, the spread $x(b)$ charged by the market is lower in the presence if the IFI.

The first inequality $\hat{b} < b^{mkt}$ is obvious when comparing (32) with (35). To prove the second inequality, $b^{mkt} < b^{bail}$, I'll first show that $\theta < \bar{g}$, where $\bar{g} = exp(\mu + \sigma^2/2)$ is the average value of the growth rate. Remember that $\theta \equiv max_g g(1-F(g))$. But for every value of g

$$g(1-F(g)) = g\int_g^\infty f(g)dg < \int_g^\infty gf(g)dg < \int_{-\infty}^\infty gf(g)dg = \bar{g} \tag{44}$$

The inequality $b^{mkt} < b^{bail}$ follows from (35), together with the inequality $\theta < \bar{g}$ and the assumption $\bar{g} < 1+r$.

For a given borrowing level b, default next period in the absence of the IFI is $1 - F\left(\frac{d^*}{b(1+r+x(b))}\right)$ and with the IFI it is $1 - F\left(\frac{b^{bail}}{b(1+r+x^{IFI}(b))}\right)$. Since $b^{bail} > d^*$, and making the hypothesis that $x(b)$ (credit spread in the absence of the IFI) is bigger than $x^{IFI}(b)$ (credit spread in the presence of the IFI), the default probability given the borrowing level b is clearly higher without the IFI, which implies that the credit spread without the IFI is higher, which justifies the above hypothesis and proves the second statement of this proposition.

Proposition 6

$P^{def,IFI}(b^{mkt}) = P^{def}(\hat{b})$. Let us start with the case without the IFI. (32) can be written as

$$\hat{b} = \frac{d^*}{1+r} max_d \frac{d}{d^*} \int_{\frac{d}{d^*}}^\infty f(g)dg \tag{45}$$

Call $\bar{d}$ the value that maximizes the above expression. The probability of default after borrowing $\hat{b}$ is then $F\left(\frac{\bar{d}}{d^*}\right)$.

Let us consider now the case with the IFI. We can analogously write (35) as

$$b^{mkt} = \frac{b^{bail}}{1+r} max_d \frac{d}{b^{bail}} \int_{\frac{d}{b^{bail}}}^{\infty} f(g)dg \tag{46}$$

The value $\bar{d}'$ that maximizes the above expression is clearly such that $\frac{\bar{d}}{d^*} = \frac{\bar{d}'}{b^{bail}}$. Therefore also the default probability after borrowing b^{mkt}, which is $F\left(\frac{\bar{d}'}{b^{bail}}\right)$, is the same as in the case without the IFI after borrowing $\hat{b}$, which is $F\left(\frac{\bar{d}}{d^*}\right)$.

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