Cost coverage in subsistence production and in surplus production

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Abstract

Some examples are given showing how surplus production (and consequently its pursuit) can make reproduction more difficult, compared to the simpler case of subsistence production.

Keywords: reproduction; surplus

1 Introduction

Two definitions of production may be given. The first one refers to ”the ’making of things’ in the physical sense” i.e. ”a physical process by which certain physically measurable goods or services are combined in order to produce a physically measurable product or products” [1]. In [6, pp. 3–5], this definition – which disregards the extent to which it is possible to measure the value created by production – is taken as the necessary point of departure for a theory of production. According to the second definition, production is defined as ”the production of goods of which the sales receipts cover the costs” [14]. Hans Neisser notes that, by adopting this second definition 1 in his Cours complete d’économie politique pratique, Say reduced his Law to a tautology. If we see self-consumption as a sale to oneself, in subsistence production without credit the two definitions coincide: if a community – in which the different units produce exactly the quantities of products that are needed in order for the community to reproduce itself – does actually manage to reproduce itself in absence of credit, this means that the quantities of the different

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1”[...] one only truly produces when all the productive services being paid for, the product is worth its production costs” [20]
products are exchanged among the different units, according to a unique set of exchange-ratios which make production costs equal product worth, when expressed in the same unit. These exchange-ratios result implicitly from restoring the original distribution of products in order for the process to restart [22, p. 4]. At any different set of exchange-ratios, subsistence reproduction would imply the recourse to credit and for some units a product value not covering costs. In surplus production things may be different as, besides the quantities of product which re-enter the system in order to reproduce it, there are other quantities (physical surpluses) which are not needed for that purpose. If the physical production of some units increases due to a technical improvement, so that a physical surplus occurs, it is no guarantee that the latter can be exchanged for something else. The fact that self-replacing prices exist – as [26] shows to be the case also allowing for non uniform rates of profit – does not ensure per se that surplus goods are actually exchanged at those prices (in particular if prices are not determined simultaneously with the volume and composition of the product, see [7]). In order for the whole physical production to be exchanged in absence of credit, there must be some unit, which a) needs the product and b) has sufficient purchasing power – coming from the exchange of its unused production – to obtain the product.

If both conditions are fulfilled, the two definitions of production are consistent. With reference to the two conditions, we may have three different situations:

- Self-consumption, in which both conditions are fulfilled by definition.
- Production for subsistence, in which the first condition is always fulfilled.
- Production with a surplus, in which even the first condition may not always be satisfied.

In [27], it is shown that

...the introduction of credit and debt enlarges the domain of prices that allow the system to replicate the existing production structure, thus creating the necessary flexibility and structural viability within the system.

However, the emergence of credit and debt may indicate that the second condition is not satisfied, if the means of production are partially financed through debt: revenues do not cover costs and physical production does not amount to economic production as well. The exchange is not any more between products, but between something existing.

\[\text{Of course, greater efficiency could translate into less work rather than more production; on the reasons that may lead to the prevalence of the latter, Martin Orans observes that Smith’s often cited assumption of “universal insatiable needs”, while lacking evidence in non-market societies, is perhaps the result of “a confusion of the insatiable expansive tendencies of capitalist enterprise with the desire for consumption” [16] in market societies.}\]

\[\text{Obviously self-consumption and subsistence production do not coincide as the second may also include some exchange, through which what is necessary for subsistence is obtained [19, p. 83].}\]
and something which will only possibly exist in the future. Things can obviously be more complicated if (as in surplus production) even the first condition may not be satisfied: (physical) production that is not needed for the reproduction of the system (i.e. surplus) might be wished by no unit. In the next sections this will be shown by working on an example in [22, p. 4] (Section 2) and on some variants on it (Section 3).

2 Subsistence

We start from an example of production for subsistence in [22, p. 4]:

\[ \mathbf{A} = \begin{bmatrix} 240 & 12 & 18 \\ 90 & 6 & 12 \\ 120 & 3 & 30 \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 450 \\ 21 \\ 60 \end{bmatrix} \]

where \( a_{ij} \) is the part of output of unit \( j \) that is input of unit \( i \) and \( b_i \) is total output of unit \( i \); the goods of which the subsistence wage consists (there is no surplus wage as there is no surplus) appear among the means of production. In absence of credit, there is only one set of exchange ratios which allows the system to reproduce itself; this set of exchange ratios is implicitly defined by the price vector \( \mathbf{p} = [0.1, 1, 0.5] \) or by one of its scalar multiples. While we may assume that the first condition mentioned above

\[ ^{4}\text{The problem is similar to that raised in The Accumulation of Capital (1913) by Rosa Luxemburg, with reference to the transition from simple reproduction (all the surplus is consumed) to enlarged reproduction (part of the surplus is accumulated): "Once the commodity has been produced [manufactured], it must be realised, it must be converted into a form of pure value; that is, into money" [12, p. 36], but as "...in a capitalist system of production, it is not consideration of social needs which actuates the individual private producer who alone matters in this connection" [p. 37], the question arises "where is this continually increasing demand to come from?" [p. 150]. Since in [12] the problem is related to the enlarged production [25, p. 38], the answer has been given that higher demand to absorb surplus production would come from the increased means of production needed to expand production [5], [23, p. 204], [2, p. 203]. However this may not mean that the whole surplus would be absorbed [24, p. 23] and in any case in the transition from subsistence to surplus the use of those resources is increased that are "not themselves produced" [22, p.88] (e.g. in [8]): while the question raised in [12] and the proposed answer were focused on the last split (simple/enlarged reproduction) of the following taxonomy, the present work deals instead with the first dichotomy, the one between subsistence and surplus production.}

\[ ^{5}\text{In the original examples indices refer to industries (namely wheat, iron and pigs).}

\[ ^{6}\text{In [18, p. 37] the original example is further detailed (Table II.2) in order to single out total consumption; however the latter is referred to as including the surplus of the economic system, which is in contradiction with the production for subsistence described by the original example.}

\[ ^{7}\text{And the units to cover their costs.} \]
(there are some units needing the product) is met, the second condition (those units have sufficient purchasing power) might not, if for some reason the set of exchange ratios were not the one that guarantees reproduction (without credit); in this case the system could still reproduce itself, but – given the method of production – the resulting debt positions would only be settled (in a following period) with zero interest, by switching to a price vector whose changes with respect to \( \bar{p} \) are opposite to the ones generating those positions.  

3 Surplus

3.1 No surplus wage

We first consider the case in which all the surplus goes to profit: there is no surplus wage and, in line with the approach indicated as "more appropriate" in [22, p. 10], "the goods necessary for the subsistence of the workers continue to appear [...] among the means of production". We may increase in the original example the output of units 1 and 2, which now show a surplus, leaving all the other quantities unchanged:

\[
A = \begin{bmatrix} 240 & 12 & 18 \\ 90 & 6 & 12 \\ 120 & 3 & 30 \end{bmatrix}; b = \begin{bmatrix} 625 \\ 31 \\ 60 \end{bmatrix}
\]

As there is no guarantee that the surplus part of the outcome will actually be needed by some unit (differently from the part that replaces the means of production), it may be useful to ask to what extent cost coverage depends on the exchange of it.

Differently from the case of subsistence production, in which there is only one set of exchange ratios that ensures cost coverage, in surplus production there may be many. Unless the set of exchange ratios remains unchanged from that of subsistence production, there will be some units for which part of the cost of the means of production must be

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8 Differently from the more general case mentioned in [27], no abstention from consumption would be possible as there is no surplus.

9 The exercise is analogous to that in [22, p. 7], but for the fact that it is done on the three industries example.

10 In particular, this is the case if we allow for non-uniform rates of profit as in [26].

11 Under the assumption of this Section (no surplus wage), the original set of exchange ratios of subsistence production is consistent with a uniform rate of profit only if the rate of physical surplus \((\frac{b_i}{\sum_{k} a_{ki}} - 1)\) is also uniform: in \(\pi_i = \frac{p_{i} b_i}{\sum_{j} a_{ij} p_j} - 1\) (the rate of profit, which here coincides with the surplus rate) we have that \(\sum_j a_{ij} p_j = p_i \sum_k a_{ki}\) at the original set of exchange ratios of subsistence production.

The subsistence system mentioned in [21, p. 414] with reference to which "...prices prevailing in the subsistence system (III) must also prevail for the empirical system" is defined differently from the one dealt with in the present work. In [21] it is obtained by removing the surpluses from the standard system, in the present work it is obtained by removing the surpluses from the original (empirical in the terminology of [21]) system; as a consequence the respective sets of exchange ratios are also different.
covered by revenues coming from the exchange of the surplus. \(^{12}\) With reference to the previous example, Table 3.1 shows this part for the units with physical surplus. \(^{13}\)

### Table 3.1: Costs coverage in surplus production.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(p_i)</td>
<td>(r_i)</td>
<td>(\sum_j a_{ij} p_j)</td>
<td>(p_i \sum_k a_{ki})</td>
<td>(p_i (b_i - \sum_k a_{ki}))</td>
</tr>
<tr>
<td>1</td>
<td>0.10147</td>
<td>0.239</td>
<td>51.18</td>
<td>45.66</td>
<td>17.76</td>
</tr>
<tr>
<td>2</td>
<td>1.00000</td>
<td>0.239</td>
<td>25.02</td>
<td>21.00</td>
<td>10.00</td>
</tr>
<tr>
<td>3</td>
<td>0.82376</td>
<td>0.239</td>
<td>39.89</td>
<td>49.43</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Columns: (1) Unit; (2) Prices; (3) Rate of surplus; (4) Cost of the means of production; (5) Revenues from the exchange of the part of the product necessary for reproduction; (6) Revenues from the exchange of surplus product

If a part of the surplus production meets the additional "needs" of no units, \(^{14}\) a first obstacle to the reproduction of the system arises, which did not exist in subsistence. \(^{15}\) Consequences may be different depending on which part of the surplus meets additional requirements of no units. Table 3.2 shows for units 1 and 2 how physical surplus breaks down between the part that has to be exchanged for means of production and the remaining surplus:

### Table 3.2: Physical surplus decomposition.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2) (\sum_j a_{ij} p_j - \sum_k a_{ki})</th>
<th>(3) (b_i - \sum_j a_{ij} p_j / p_i)</th>
<th>(4) (b_i - \sum_k a_{ki})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54.4</td>
<td>120.6</td>
<td>175</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>6.0</td>
<td>10</td>
</tr>
</tbody>
</table>

Columns: (1) Unit; (2) Part of surplus that should be exchanged for means of production; (3) Part of surplus above that to be exchanged for means of production; (4) Total surplus

If further requirements of products 1 and 2 by some unit – above the amount needed for the reproduction of the system – reach the quantities of at least 54.4 and 4.0 respectively, but without covering the whole surplus production, the consequence would be a reduction

\(^{12}\) As in subsistence there is only one set of exchange ratios which allows the system to reproduce itself without credit, with any other series of prices there will be some unit for which the revenues from the part of the product included in the system’s means of production will not cover the costs of its own means of production.

\(^{13}\) The table shows this for the case in which the set of exchange ratios gives rise to a uniform rate of surplus, but an analogous situation arises whenever for some unit the rate of (value) surplus is different from the physical rate of surplus.

\(^{14}\) Naturally, reference is made here to possible additional requirements, higher than those relating to the reproduction of the system, the latter being already included in the means of production.

\(^{15}\) Planned obsolescence and disposals aim to expand the part of physical product to be classified as means of production, as already pointed out in [11, chap. 3].
of the rate of profit of units 1 and 2; if even those quantities were not reached, the two units would not cover their costs and the system could not reproduce itself unless the two units take the missing part of the means of production on credit.

On the other hand, even if all the surplus outcome meets the requirements of some units, the possibility for the exchanges to actually take place depends on purchasing power of these units given the set of exchange ratios and/or on their access to credit. The general relation between prices, profit rates, wage rates and credit changes that would allow the system to reproduce itself may be found in [27]. Given the exchange ratios of Table 3.1, a set of surplus requirements as in the following matrix

\[
\begin{bmatrix}
41.76 & 8.00 & 0.00 \\
49.11 & 1.00 & 0.00 \\
84.13 & 1.00 & 0.00 \\
\end{bmatrix}
\]

would ensure reproduction without the need for credit. If the the set of surplus requirements were to be as in the following matrix

16In this case the share of means of production in the face of unwanted surpluses, while fulfilling the first condition of Section 1 (there are some units needing the product), does not fulfill the second (those units have sufficient purchasing power).

17As regards credit, two clarifications are in order:

- in this work only the credit positions still open after the completion of the production process by all industries are considered; therefore the temporary debit/credit positions due to the different durations of the production processes are ignored (as in [22]: "commodities [...] are exchanged for one another at a market held after the harvest");
- debts and credits relating to financial intermediation are also disregarded: as shown in [17, par. 3] they can always be described as the result of (a sequence of) credits generated from commodity sales.

18In the more limited context of the present work, if \(y_{ij}\) is the part of surplus output of unit \(j\) that meets requirement of unit \(i\) (above its needs for means of production) two necessary conditions in order for all the units to cover their overall costs without resorting to credit (here we consider all the costs including – in addition to those of the means of production – also those for the purchase of surplus products), are

\[
\sum_{h \in H} y_{hi} \geq \sum_j a_{ij} p_j \frac{p_j}{p_h} - \sum_h a_{hi} \text{ where } H = \{ h : \sum_j y_{hj} p_j \leq b_h p_h - \sum_j a_{hj} p_j \}, \forall i
\]

(financing the purchase of means of production without credit)

and

\[
\sum_j y_{ij} p_j \leq b_i p_i - \sum_j a_{ij} p_j
\]

(financing the purchase of surplus production without credit)
\[
Y' = \begin{bmatrix}
41.76 & 0.00 & 0.00 \\
49.11 & 0.00 & 0.00 \\
84.13 & 10.00 & 0.00
\end{bmatrix}
\]

at the price set of Table 3.1, the reproduction of the system would depend on the possibility for unit 3 to obtain credit to finance its costs-revenues gap:

**Table 3.3: Cost-Revenue gap (debt/credit).**

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$\sum_j a_{ij}p_j$</td>
<td>$\sum_j y'_{ij}p_j$</td>
<td>$p_i \sum_k (a_{ki} + y'_{ki})$</td>
<td>$\Delta CD$</td>
</tr>
<tr>
<td>1</td>
<td>51.18</td>
<td>4.24</td>
<td>63.42</td>
<td>8.00</td>
</tr>
<tr>
<td>2</td>
<td>25.02</td>
<td>4.98</td>
<td>31.00</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>39.89</td>
<td>18.54</td>
<td>49.43</td>
<td>-9.00</td>
</tr>
</tbody>
</table>

Columns: (1) Unit; (2) Cost of the means of production; (3) Cost of surplus; (4) Total revenues; (5) Variations in credit and debt positions

Since the arising of debts amounts to an exchange between existing products and products that will only possibly exist in the future\(^{19}\), it may well be the case that the debts will not be repaid. If this happens, the final result would be as if the exchanges had occurred from the start at a different price set (namely $\bar{p}' = [0.09, 1, 1.04]$): same allocation of surplus production as in $Y'$, with no debt/credit positions. However, should this final result be achieved through the initial granting of the credit, followed by its non-repayment, the survival of the system could be compromised by the debtor’s bankruptcy.\(^{20}\)

### 3.2 Surplus wage

As the present work is not directly focused on distributive issues, in the previous section we have assumed that all the surplus goes to profit. In this section we try to show that even taking into account the presence of surplus wages, our basic conclusions do not change:\(^{21}\)

1. The emergence of a surplus can compromise the reproduction of the system in absence of credit, if the set of exchange ratios is different from the subsistence one\(^{22}\)

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19 According to the definition given in Section 1, for a product to exist, the physical process by which it is produced must also guarantee cost coverage.

20 Of course, things would be different if the deficit units were to be financed through some form of non-debt financing (such as those described in \([9]\) and \([4]\)).

21 However, in this section, no reference is made to “labour as a measurable quantity”, a concept which, according to \([10]\), Sraffa introduced when addressing the issue of worker participation in the surplus product (the implicit reduction of heterogeneous units of concrete labour to homogeneous units is considered “difficult if not impossible” in \([15, p. 78]\))

22 If $\alpha_i$ is the share of surplus going to wages in unit $i$, the original set of exchange ratios of subsistence
and the surplus production does not meet the requirements of some units;

2. even if the insolvency on the debts raised to buy parts of the surplus does not in itself affect the reproduction of the system, the legal consequences of bankruptcy do.

If a percentage \( \alpha \) of surplus goes to wages, as shown in Table 3.4, there is still the possibility that coverage of the cost of means of production depends on some unit (firm or family) requiring part of the surplus production: 23

| Table 3.4: Costs coverage in surplus production. |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| \( i \) | \( p_i \) | \( \pi_i \) | \( \alpha_i \) | \( \sum_j a_{ij}p_j \) | \( p_i \sum_k a_{ki} \) | \( p_i(b_i - \sum_k a_{ki}) \) |
| 1 | 0.09605 | 0.168 | 0.1 | 50.57 | 43.22 | 16.81 |
| 2 | 1.00000 | 0.168 | 0.3 | 24.99 | 21.00 | 10.00 |
| 3 | 0.86210 | 0.168 | 0.4 | 40.39 | 51.73 | 0.00 |

Columns: (1) Unit; (2) Prices; (3) Rate of profit; (4) percentage of surplus going to wages; (5) Cost of the means of production; (6) Revenues from the exchange of the part of the product necessary for reproduction; (7) Revenues from the exchange of surplus product

Again, credit may make up for the lack of purchasing power. 24 If surplus product requirements are now represented by matrix

would be consistent with a uniform rate of profit \( \bar{\pi} \) only if \( \alpha_i = 1 - \frac{\bar{\pi}}{R_i} \) (where \( R_i \) is the surplus rate) for all the units.

23 This happen whenever column (6) is lower than column (5), at the given price set (which here is consistent with the assumption of uniform rate of profit).

24 Conditions for non-debt reproduction analogous to those provided in footnote 18 would be

\[
\sum_{h \in H} y_{hi} + \theta(w - \sum_j y_{n+1,j}p_j)y_{n+1,i} \geq \sum_j a_{ij}p_j - \sum_{h} a_{hi}
\]  

where:

\( H = \{ h : \sum_j y_{h,j}p_j \leq (b_hp_h - \sum_j a_{h,j}p_j)(1 - \alpha_h) \} \)

\( y_{n+1,j} \) refers to surplus product requirements of workers, here considered as an additional single unit

\( w = \sum_i w_i = \sum_j (b_ip_i - \sum_j a_{ij})\alpha_i \)

\( \theta(\cdot) \) is the Heaviside step function

\( i, j = 1, \ldots, n \) with \( n \) number of (firm) units

(financing the purchase of means of production without credit)

and

\[
\sum_j y_{ij}p_j \leq (b_ip_i - \sum_j a_{ij}p_j)(1 - \alpha_i)
\]

\[
\sum_j y_{n+1,j}p_j \leq w
\]

(financing the purchase of surplus production without credit)
\[ Y'' = \begin{bmatrix} 10.00 & 1.55 & 0.00 \\ 18.34 & 7.44 & 0.00 \\ 70.82 & 0.00 & 0.00 \\ 75.84 & 1.00 & 0.00 \end{bmatrix} \]

(the last line of which shows surplus product requirements of workers, here considered as a single additional unit) starting from Table 3.4 the cost-revenue gap may calculated as in Table 3.5 (the last line of which compares surplus wage income with the cost of surplus products to workers):

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5) ( p_i \sum_k (a_{ki} + y_{ki}'') \cdot w )</th>
<th>(6) ( \Delta \text{CD} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.57</td>
<td>2.51</td>
<td>0.95</td>
<td>60.03</td>
<td>6.00</td>
</tr>
<tr>
<td>2</td>
<td>24.99</td>
<td>9.20</td>
<td>1.80</td>
<td>31.00</td>
<td>-5.00</td>
</tr>
<tr>
<td>3</td>
<td>40.39</td>
<td>6.80</td>
<td>4.53</td>
<td>51.72</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>8.28</td>
<td>0.00</td>
<td>7.28</td>
<td>-1.00</td>
</tr>
</tbody>
</table>

Columns: (1) Unit; (2) Cost of the means of production; (3) Cost of surplus; (4) Surplus wage (here as a costs for units 1,2,3); (5) Total revenues, it includes in the last row the Surplus wage for workers; (6) Variations in credit and debt positions

### 3.3 Reproduction flow-chart

The possibility of obtaining reproduction depends on the outcome of a series of alternatives which can be summarized as follows
While reproduction is ensured at the subsistence price set (alternative $\alpha$ in the flow chart), at any different set it depends on

- the possibility for all units facing a negative gap between proceeds of the sale of the part of own product which is included in the system’s means of production and the cost of its own means of production (including subsistence goods) to cover it with the proceeds from the exchange of surplus production (alternative $\beta$ in the flow chart),

or

- on the availability of credit (alternative $\gamma$ in the flow chart).

If we consider a sequence of production cycles, the possibility of having the necessary means of production at the beginning of each cycle also depends on the repayment of debts, as the insolvency of a unit can interrupt its activity; for this reason, while the unavailability of credit necessary to finance the means of production (in alternative $\gamma$) can hinder reproduction, the unavailability of credit to finance the purchase of surplus goods (alternative $\gamma'$ in the following flow chart) can play the opposite role, possibly avoiding insolvency in a subsequent productive cycle (in alternative $\delta$):  

$25$ The conditions for non-debt reproduction shown in alternative $\beta'$ are described in footnotes 18 and 24.  

$26$ More in general the double role of credit, which on the one hand has the function "to extend production and to facilitate exchange" and on the other hand "... destroys, during the crisis, the very productive forces it itself created", was highlighted by Rosa Luxemburg in [13, Chap. II].  

$27$ Subsequent production cycle is shown in green.
3.4 Conclusions

The decision on what and how much to (physically) produce is taken consistently with the economic balance of the unit, based on a hypothesis about the exchange possibilities of the (physical) product. However, while by definition the part of the product that is needed for the reproduction of the system (means of production including subsistence) could always meet the needs of some unit, this may not be the case for production above this part (surplus). Apart from the case that the set of exchange ratios is the single one which would assure reproduction also in absence of surplus, any other set would imply that there is some unit whose revenues from the sale of the part of its product that is needed for the reproduction of the system (means of production) do not cover the cost of its own means of production. So while in subsistence production there is only one path leading to the reproduction of the system, only provided that the exchange ratio is the one which “restore the original distribution of the products”, if there is a (physical) surplus, several paths may start, for which the possibility to achieve reproduction (in the current productive cycle and in the following) are conditional on assumptions related to the needs for surplus production, the availability of credit, the repayment of credit.

References


\(^{28}\)See [3, p. 74].

\(^{29}\)This may be the case also for those units whose products enter (directly or indirectly) into the production of all products (the examples in the previous sections do not deal with luxury products).

\(^{30}\)[22].


