Inflation effects on capital accumulation in a model with residential and non-residential assets

Koka, Katerina

Brock University

January 2014

Inflation effects on capital accumulation in a model with residential and non-residential assets

Katerina Koka[1],

Department of Economics, Brock University

Abstract

We study aggregate effects on capital accumulation of changes in the permanent rate of inflation in a model that incorporates both residential and non-residential capital. The framework is a dynamic general equilibrium life-cycle economy populated by heterogeneous individuals with respect to age, income and homeownership status. Inflation raises the non-residential capital income tax burden, affects the user cost of housing capital and raises the opportunity cost of holding money. A numerical analysis is provided based on parameter values from the US economy. We find that housing capital and inflation exhibit a positive correlation while inflation reduces savings in business capital. It is shown that this outcome arises from the interaction of inflation with individual tenure decisions and a number of characteristics and tax provisions available in the housing market.

Keywords: Long run inflation, Housing capital, Tenure choice
JEL classification codes: E22; E31; H21

[☆]I would like to thank Brian Ferguson, Stephen Kosempel, Jim MacGee, Gregory Smith, Kathleen Rybczynski and participants in the Canadian Economic Association, Eastern Economic Association and Spring Meeting of Young Economists conferences for their valuable comments and suggestions. I have also greatly appreciated comments from the editor of this journal and two anonymous referees.

[1]Department of Economics, Brock University, 500 Glenridge Ave., St. Catharines, Ont., L2S 3A1, Canada. Email: kkoka@brocku.ca

Preprint resubmitted to The B.E. Journal of Macroeconomics *March 26, 2014*

1. Introduction

The objective of this paper is to explore the effects of inflation on capital accumulation in a model that incorporates residential and non-residential capital, tax differences across assets, housing characteristics and tenure choice. In recent literature it has been shown that separate treatment of housing capital can offer distinct channels through which inflation impacts economic activity. This fact is not surprising considering that the purchase of a home is one of the most important lifetime investments for average households. In the US, residential capital alone accounts for more than 50 percent of the total capital stock while the typical household allocates about one quarter of their income to housing expenditures.

In a recent study, Aruoba, Davis, and Wright (2012) document a positive relationship between the rate of inflation and the ratio of housing capital to GDP, persistent in the US economy from 1975 to 1999. The correlation between housing capital scaled by output and inflation is estimated to be about 0.62 while its semi-elasticity with respect to inflation is about 1.1 percent.[2] Similarly, between 1960 and 1990, homeownership rates in the US and the average annual inflation exhibit a positive correlation for most of the three decades. The correlation coefficient between inflation and homeownership rates for this period is 0.84. Even though the positive association between inflation and housing capital breaks down for a decade starting in 2000, Aruoba et al. (2012) argue that low inflation rates and high house prices in the US represent an asset bubble in the housing market and not a long term relationship. Particular practices in the banking sector, the presence of risky behaviour and lack of regulatory oversight were some of the factors that contributed to the housing bubble. This period experienced an increase in the number of home equity loans and financial innovation that led to higher levels of household borrowing. The collapse of house values with the financial crisis in 2007-2008 marks a return to the stationary path of the relationship between housing and inflation which in the long run exhibits a positive correlation.

In this paper we explore a model that provides an explanation for the empirical long run association of inflation and housing capital. Specifically, we show that inflation affects the stock of housing and business capital accumulation through its effects on household tenure decisions, by creating incentives in favour of homeownership. The mechanism is as follows: An increase in the rate of inflation is associated with a portfolio allocation effect aimed at adjusting previously optimal asset holdings. Individuals substitute away from money balances to business and housing capital, since inflation reduces the value of money holdings. They may also shift resources from non-residential to residential capital. Portfolio choices are influenced by two factors related to the form in which the two capital assets are taxed. Firstly, in most advanced

[2] Aruoba et al. (2012) also find a positive relationship between the ratio of the value of housing capital to the money stock and the rate of inflation. Due to lack of historical data on housing wealth for other advanced economies, the study reports country specific correlations of the ratio of house prices to GDP and the rate of inflation. These associations are positive for nine out of eleven documented countries, including Australia, Canada, Denmark, Finland, Italy, Japan, the Netherlands, Norway and Sweden.

economies housing is subject to a variety of tax subsidies, including non-taxation of housing rents available to homeowners and some housing capital gains. These provisions in the tax code create a gap between the taxation of residential and non-residential capital. Secondly, empirical evidence shows that non-residential capital taxes are not indexed to account for changes in inflation and since nominal and not real income is taxed, inflation increases the real income tax burden of business capital.[3] As a consequence, inflation raises the gap between the taxation of business and owner-occupied housing capital, which in turn induces households to switch from renting to buying housing services or, for individual homeowners, to increase the size of their housing investment.

Tenure decisions are particularly important in savings behaviour and business capital accumulation. The purchase of housing capital differs from the purchasing of other assets because housing is typically expensive and because it cannot be purchased in very small amounts. To obtain housing, an average household would necessitate a loan and an upfront downpayment requirement. Becoming a homeowner then requires sufficient savings which entails that tenure choices directly influence individual lifetime savings and consumption profiles. If this is the case, inflation would impact the formation of aggregate business capital and housing stock through its impact on household tenure decisions.

We develop a dynamic general equilibrium life-cycle economy with heterogeneous households and characteristics of the housing market in the US economy. Individual differences are captured with respect to age, level of income and homeownership status. The economy is populated by individuals who can, based on their earning ability, either own or rent housing services. Housing characteristics include borrowing constraints and tax subsidies in favour of homeownership. Inflation impacts the return on investment in business capital, the price of housing services and consequently household tenure decisions. The state of the economy, both on the aggregate and individual level, is compared in different inflationary regimes with high and low average inflation rates.

To be able to model tenure choice and differentiate between owning and renting housing services we incorporate several features present in the housing market. To begin with, housing capital is both a consumption and an investment good. For example we model the stock of owner-occupied dwellings to yield utility directly for the household.[4] In addition, investment in residential capital may require a minimum size to be purchased. Should households decide to buy housing capital, mortgage financing is available through financial institutions. When issuing mortgage loans, banks require a downpayment amount that is equal to a fraction of the value of the house. Thus to become a homeowner, households are expected to cover the

[3]Heer and Sussmuth (2007) use data from the US economy spanning the period 1965-1998 and find that the inflation rate positively impacts effective capital income tax rates, which suggests that nominal income tax rates are not adjusted for inflation.

[4]The homeowner can also exercise some control over its investment returns through maintenance of the housing capital, but in this paper we don't examine this possibility.

downpayment cost and be able to at least have the smallest allowable equity on the house.

In this framework, where residential capital is subjected to tax subsidies in favour of home-ownership, inflation affects tenure decisions and prompts a portfolio composition effect on households' accumulated assets. Since the model has no analytical solutions, a numerical approach is employed to estimate inflation effects on both residential and non-residential capital accumulation. We investigate distinct channels from existing literature on the effects of permanent changes in inflation and include a separate treatment of features in the housing market. The paper builds on a life-cycle model developed by Gervais (2002) to study the accumulation of residential and non-residential capital in the presence of distortionary tax provisions in the housing market. We focus our attention on how inflation interacts with the nominal taxation of capital gains, tenure decisions and characteristics of the housing market. Our model shows that on the aggregate level there is a positive correlation between the accumulation of residential capital and inflation. We also find that inflation reduces aggregate wealth in high inflationary regimes and that this effect is the least pronounced when housing capital receives preferential tax treatments. The welfare cost of inflation is unevenly distributed among income groups: we find that inflation negatively impacts the welfare of low income groups by more than high income groups.

This paper is closely related to literature that looks at effects of tax and monetary policies on capital accumulation and demand for housing. By reducing the cost of homeownership, inflation has disproportionate effects on the accumulation of residential and non-residential capital. In particular, the interaction of inflation and the tax code leads to a portfolio shift by providing greater incentives to invest in owner-occupied housing relative to other assets. Such an argument is present in both the static and dynamic analyses of Hendershott and Hu (1981), Hendershott and Hu (1983), Feldstein (1982), Titman (1982), Dougherty and Van Order (1982), Kau and Keenan (1983), Poterba (1984), and the empirical works of Summers (1981) and Ebrill and Possen (1982). Similarly, Nielsen and Sorensen (1994) look at the impact of inflation on capital income taxation and wealth distribution in a small open economy.[5] These studies omit the treatment of rental markets, individual income differences, constraints in the housing market as well as a quantitative analysis. We contribute to this literature by examining the effects of inflation in a framework which captures not only household heterogeneity in income, age and homeownership status but also lifetime tenure decisions.[6]

Piazzesi and Schneider (2009) use a temporary equilibrium model conceptually described in Grandmont (1977) to quantitatively assess portfolio shifts due to inflation of housing and

[5]The authors refrain from a quantitative analysis while showing that inflation induced effects in the model depend on their choice of structural parameter values.

[6]One exception in the literature, Follain and Ling (1988), simulate tenure decisions by assuming different scenarios of how inflation may affect interest rates and conclude that while inflation (under certain assumptions) increases the demand for housing it actually reduces homeownership rates. The authors use a partial equilibrium one period model to calculate from the data the user cost of renting versus the user cost of owning housing. This study however does not take into account dynamic choices in household tenure decisions.

financial capital and to explain the negative correlation between house and stock prices in the 1970s. One of the channels examined is the impact of inflation on after tax returns to equity and real estate capital. In their quantitative analysis, the authors show that changes in household portfolios due to this channel alone cannot generate the asset portfolio shift observed in 1970s.[7] Other important channels that are considered include demographic changes, unanticipated inflation and disagreement across population age groups on inflation expectations. We contribute to this study by estimating the effects of long run inflation in the presence of endogenous tenure choice. Tenure decisions are of key importance to include in an asset portfolio analysis: they affect the housing capital accumulation process, can impact within period portfolio allocations and eventually the lifetime distribution of consumption and savings. In addition, we also examine housing market characteristics which may impede the affordability of owner-occupied housing. Finally, since the model estimates a general equilibrium it can capture both inflation induced intra-generational and inter-generational income redistribution.

A few studies consider other important ways through which inflation is related to housing capital accumulation and house prices. Aruoba et al. (2012) develop a model in which housing is an input in household production and argue that because inflation can act as a tax on market activity but not on home production, it prompts households in high inflationary times to invest more in housing capital. The authors find that this channel is able to explain a fifth to a half of the long run relationship between housing capital and inflation in the data, depending on parameter values. Brunnermeier and Julliard (2008) postulate that inflation and house prices are interconnected through money illusion and the lack of agents taking into account real versus nominal housing costs when making renting or owning decisions. In this paper, changes in aggregate stocks of housing and business capital stem from the adjustments of tenure decisions in response to higher inflation. Due to characteristics in the housing and mortgage markets, we expect such changes to have a significant impact on household savings and consumption.

Recent studies have considered portfolio choice with housing within the framework of general equilibrium life-cycle models. For example, Gervais (2002) analyzes the impact of preferential taxation of housing capital on the accumulation of residential and business capital. Yao and Zhang (2005) quantify the effects of homeownership on the equity composition of individual investor portfolios. Consumption and welfare effects of varying house prices are analyzed by Li and Yao (2007). Yang (2009) looks at the role played by borrowing constraints and transaction costs in generating housing capital age profiles consistent with the data.[8] In a recent paper, Chen (2010) performs policy experiments to examine how housing capital and homeownership rates change when social security is eliminated.

[7] Average effective tax rates on equity returns are set at 20 percent. To generate the portfolio changes observed in the data a higher tax rate on equity is required, as high as 45 percent.

[8] Similarly, Chambers et al. (2009a) examine the impact of loan structures on individual mortgage choices and aggregate statistics, while Chambers et al. (2009b) look at the role of changing demographics and mortgage innovations in explaining the rise in homeownership rates between 1994 and 2005.

This paper is organized as follows: Section 2 introduces the dynamic general equilibrium model. In section 3, structural parameters are calibrated based on data from the US economy. Results from the numerical solution are presented in section 4. Section 5 concludes the discussion by also providing some ideas for future research.

2. The model

The economic environment that we explore is a dynamic general equilibrium economy populated by overlapping generations. Agents differ with respect to their age, average productivity type, asset holdings, working and homeownership status. Savings are invested in financial capital, housing capital and money holdings. The model economy has a financial intermediary which collects individual deposits and issues mortgage loans, facilitates investments in residential and business capital and provides rental accommodation services. Output is produced by firms using business capital and effective labour inputs. Finally, there is a government that conducts both fiscal and monetary policy. Taxes are levied on earnings from labour and financial assets while revenues from taxation, seigniorage and unintended bequests are distributed equally to all living members of the population.

2.1. Demographics

We assume that in each period t a new generation is born. A typical individual may live for a maximum of J model periods. The conditional probability of surviving from age j to age $j+1$ is $\psi_j \in (0,1)$, for $j = 1, .., J-1$. Therefore, the unconditional probability of becoming l-years of age is $\Pi_{j=1}^{l}\{\psi_j\}$. People are born and die with certainty, so $\psi_0 = 1$ and $\psi_J = 0$. The fraction of each cohort in the total population is constant and given by $\mu_{j+1} = (1+n)^{-1}\mu_j\psi_j$ where $\mu_1 = (1+\sum_{j=1}^{J-1}(1+n)^{-j}\Pi_{i=1}^{j}\psi_i)^{-1}$ and n is the growth rate of the population. All agents retire from working at the age of jr, which is mandatory. If individuals fail to survive until age J, their assets are collected by the government and redistributed via lump-sum transfers, tr_t.

2.2. Preferences

Period utility is derived from the consumption of a non-durable composite good (c), housing services (s) and real money balances (m). New-entrants in the economy maximize their discounted expected lifetime utility:

$$E_0\left\{\sum_{j=1}^{J}\beta^{j-1}\left(\prod_{i=1}^{j}\psi_i\right)U\left(c_{j,t}, m_{j,t}, s_{j,t}\right)\right\} \quad (1)$$

where β denotes the time discount factor.

The money in utility functional form is adopted for simplicity to generate a demand and distribution for the medium of exchange so it can be thought of as a shortcut to a more complicated transaction story.

2.3. Asset holdings, income and prices

Individuals can either own ($h_{j,t}$) or rent ($x_{j,t}$) housing services ($s_{j,t}$). We assume that renting and owning are perfect substitutes in consumption, $s_{j,t} = h_{j,t} + x_{j,t}$, and individual investments in residential capital are owner-occupied. Through a linear technology, one unit of housing capital can be transformed into one unit of housing services.

Homeownership involves two constraints. Firstly, it requires individuals to hold equity in the form of a downpayment, equal to a fraction γ of the value of the house. Secondly, purchasing housing involves a minimum quantity requirement. This is added to the model to take into account the fact that rental accommodation can be obtained in smaller quantities than typical residential properties for sale. The smallest amount of housing an individual may buy is denoted by $\underline{h}$. If the two constraints are met and an individual's own wealth is not sufficient to finance a house, individuals may borrow funds against the house to finance its payment. Borrowing in the form of a mortgage loan is the only type of borrowing that is allowed in the model. Since housing capital depreciates in every period, homeowners incur a maintenance cost δ_h per unit of housing capital.[9]

Individuals receive income from labour, financial assets, imputed rents and government transfers. Each individual is endowed with one unit of labour time per period which is supplied inelastically to work and transformed to ϵ_j efficiency units of labour. The economy is populated with individuals who have two types of earning ability: high and low productivity type agents. Each agent, denoted by $z \in (high, low)$, has a deterministic productivity profile. The measure of each agent type is $\lambda(z) \in (0, 1)$. One unit of effective labour is paid the average wage rate w_t. Labour income is taxed at the rate τ_w.

Savings take the form of non-residential assets net of borrowing ($a_{j,t}$), residential assets ($h_{j,t}$) and real money balances ($m_{j,t}$). Non-residential assets consist of deposits in financial institutions. Their return is the nominal interest rate i_t which is taxed at a constant tax rate τ_a. Housing receives income in the form of imputed rents since homeowners do not pay for housing services. The tax rate on imputed rents is denoted by τ_h.[10] The tax code allows for two streams of housing income to go untaxed: firstly, imputed rents which consist of homeowners enjoying free housing services, and secondly, capital gains acquired from selling housing capital.[11] In the model economy, capital gains on residential property equal the long run rate of inflation, and thus the value of residential capital is preserved in each period.

[9] We do not distinguish between depreciation rates of rental units and owner-occupied houses. Typically, however, rental accommodation involves higher maintenance costs than owner-occupied housing (see Chen (2010)).

[10] In the benchmark model, the tax on imputed rents is set equal to zero.

[11] Another provision includes full deductability of mortgage interest payments from taxable income. However, Gervais (2002) shows that mortgage interest rate deductability plays only a small role on the incentive to over-invest in housing capital, in particular, the removal of this provision is accompanied with changes in the composition of housing capital but insignificant changes of its quantity. For this reason, and to ease the computation of the model we have omitted this tax provision.

Let P_t denote the price of the composite good and P_t^h denote the price of housing services. Prices of output, residential and non-residential investment and maintenance goods are equal to the price of the composite consumption good, which is set to be the numeraire. This is the case since output can be converted at no cost to any of these components. Let $\frac{P_t}{P_{t-1}} = 1 + \pi_t$ denote the rate of inflation, $v_t = \frac{P_t^h}{P_t}$ the relative price of housing services and $m_{j,t} = \frac{M_{j,t}}{P_t}$ real money holdings, respectively. The nominal interest rate i_t is approximated by using the Fisher equation: $i_t = (1 + \pi_t)(1 + r_t) - 1$, where r_t is the before tax real interest rate determined endogenously in the model by individual optimal outcomes. We assume that the interest rate applied on financial assets is the same as the rate charged on borrowed funds.

Current period wealth is denoted by $\varpi_{j,t}$ and consists of non-residential assets net of loans, residential assets and money holdings. To simplify the notation, we have omitted indices that indicate the individual type (low and high average productivity profiles), which will be used when aggregating individual outcomes. The nominal budget constraint for an individual homeowner aged j in period t is given by:[12]

$$\begin{aligned} P_t c_{j,t} + P_t a_{j+1,t+1} + P_t h_{j+1,t+1} + M_{j+1,t+1} = (1-\tau_w) P_t \epsilon_j w_t + P_t tr_t + \\ + P_{t-1} \left(1 + (1-\tau_a) i_t\right) a_{j,t} - \tau_h P_t^h h_{j,t} + P_t (1-\delta_h) h_{j,t} + M_{j,t} \end{aligned} \quad (2)$$

And for the individual renter:

$$\begin{aligned} P_t c_{j,t} + P_t^h x_{j,t} + P_t a_{j+1,t+1} + P_t h_{j+1,t+1} + M_{j+1,t+1} = (1-\tau_w) P_t \epsilon_j w_t + P_t tr_t + \\ + P_{t-1} \left(1 + (1-\tau_a) i_t\right) a_{j,t} + M_{j,t} \end{aligned} \quad (3)$$

2.4. The individual problem

To solve the dynamic problem we divide it into an inter-temporal and intra-temporal optimization decision.[13] At each point in time the individual decides the quantity of savings to be transferred over to the next period, leaving out the decision of that period's allocation problem. In a framework without uncertainty, the composition of savings set aside in the current period will be the same, whether the decision is made in the current or the future period. This is the inter-temporal problem. The intra-temporal problem involves a portfolio allocation problem: given the amount of savings carried over from the previous period, current labour income and government transfers, the individual decides her optimal allocation of wealth into consumption goods, housing and money balances. She also decides on whether to rent or to own housing services, based on which option is attainable and yields the highest level of utility.

Tenure choice is determined by several factors. Since housing capital receives preferential tax treatment (for example when $\tau_h = 0$ and $\tau_a > 0$), its rate of return is higher than the

[12]For the homeowner's budget constraint, imputed rents $P_t^h h_{j,t}$ cancel out from both sides of equation 2.

[13]We follow a similar method to Krussell and Smith (1998), Heer (2004), Erosa and Ventura (2002) and Gervais (2002). A detailed description of the algorithm is found in the appendix.

return on business capital investments. Inflation increases the wedge between the two rates of return, which makes real estate capital an even more attractive investment. Consequently, in high inflationary regimes individual renters have an incentive to become homeowners, and homeowners have an incentive to increase their original investment in housing capital. However, a shift from renting to owning or investing in additional housing is only possible if an individual is able to accumulate sufficient savings and meet the downpayment requirement and to demand at least the smallest available house in the market.

The inter-temporal problem for an individual aged j at time t is given by:

$$V_t\left(\varpi_{j,t};j\right) = max_{\varpi_{j+1,t+1\in\Gamma}}\left\{G\left(\varpi_{j,t},\varpi_{j+1,t+1}\right)+\beta\psi_j V_{t+1}\left(\varpi_{j+1,t+1};j+1\right)\right\}$$

Here Γ is a finite set containing all possible values of ϖ_{j+1} in the next period.

Given a combination of wealth today and tomorrow $(\varpi_{j,t}, \varpi_{j+1,t+1}\in\Gamma)$, each individual solves the following intra-temporal problem:

$$G_t = max_{c_{j,t},s_{j,t},a_{j,t},h_{j,t},m_{j,t}} U\left(c_{j,t},m_{j,t},s_{j,t}\right)$$

subject to the following real budget constraints:

$$c_{j,t}+v_t s_{j,t}+\varpi_{j+1,t+1} = (1-\tau_w)\epsilon_j w_t + tr_t + \frac{1+(1-\tau_a)\,i_t}{1+\pi_t}a_{j,t}+\left(1+(1-\tau_h)v_t-\delta_h\right)h_{j,t}+m_{j,t} \quad (4)$$

$$\varpi_{j,t} = a_{j,t}+h_{j,t}+(1+\pi_{t-1})m_{j,t} \quad (5)$$

$$s_{j,t} = x_{j,t}+h_{j,t} \quad (6)$$

$$h_{j,t} \geq \underline{h} \quad (7)$$

$$a_{j,t} \geq -(1-\gamma)h_{j,t} \quad (8)$$

$$c_{j,t} > 0, m_{j,t} > 0, h_{j,t} > 0 \quad (9)$$

$$a_{1,t} = h_{1,t} = 0 \quad (10)$$

Within each period a tenure decision is made by comparing the value received when choosing to own (G_t^o) or to rent (G_t^r) housing capital:

$$G_t = max\left\{G_t^o, G_t^r\right\} \quad (11)$$

2.5. Firms

Firms combine a Cobb-Douglas technology, business capital K_t and effective labour N_t to produce output Y_t:

$$Y_t = T_t K_t^{\theta} N_t^{1-\theta} \quad (12)$$

where θ is capital's share of total output and T denotes the level of total factor productivity. Both factors of production are paid their marginal products:

$$w_t = (1-\theta)T_t K_t^{\theta} N_t^{-\theta} \quad (13)$$

$$r_t = \theta T_t K_t^{\theta-1} N_t^{1-\theta} - \delta_k \tag{14}$$

One unit of business capital depreciates at the rate δ_k. Output is consumed and invested in both residential and non-residential capital. The stocks of business capital and housing capital (H_t) evolve according to the following equations:

$$K_{t+1} = I_t^k + (1-\delta_k)K_t \tag{15}$$

$$H_{t+1} = I_t^h + (1-\delta_h)H_t \tag{16}$$

where I_t^h and I_t^k denote investment in residential and non-residential capital, respectively. Housing capital includes both owner-occupied housing H_t^o and rental housing H_t^r: $H_t = H_t^o + H_t^r$.

2.6. Financial Intermediary

We assume that the model economy has a financial firm which acts as an intermediary to facilitate the process of renting out residential capital to individual renters. The firm receives financial assets in the form of deposits net of any mortgage loans, A_t and uses them to purchase housing assets H_t^r and business capital K_t. Residential capital is then rented to households who we assume deliver rental payments at the end of each period. Therefore the budget constraint of the financial firm is given by:

$$(1-v_t)H_t^r + K_t \leq A_t \tag{17}$$

In each period the financial firm receives nominal interest returns on business capital $K_t(1+i_t)$ and rental housing net of maintenance costs, $H_t^r(1-\delta_h)$. The firm has to repay individual lenders their financial assets plus an interest, $A_t(1+i_t)$. The objective of the financial intermediary is to maximize its profit function:

$$\Pi_t = max_{A_t, H_t^r, K_t} \{H_t^r(1-\delta_h) + K_t(1+i_t) - A_t(1+i_t)\} \tag{18}$$

subject to the budget constraint given by equation 17.

Assuming competitive markets where the typical firm makes zero profits in equilibrium, and solving the problem of the financial agency, the price of housing services relative to the price of non-durable goods is given by:

$$v_t = \frac{i_t + \delta_h}{1 + i_t} \tag{19}$$

Another way of specifying the problem of the financial intermediary is to assume that real estate in the current period can only generate housing services in the next period, a specification similar to Gervais (2002). In the appendix we show that the results of the model are robust when the problem of the financial intermediary is formulated in this way.

2.7. Government

The government is responsible for carrying out both fiscal and monetary policy. As a monetary authority, the government prints money M_t which grows at the rate g_m:

$$\frac{M_t - M_{t-1}}{M_{t-1}} = g_m \tag{20}$$

The inflation rate is determined by the money growth rate and it also equals g_m. Government earnings include taxation of interest bearing assets and labour income, revenues from seigniorage, and unintended bequests. The government maintains a balanced budget by equally distributing revenues to agents via lump sum transfers:

$$\tau_w w_t \sum_z \sum_j \lambda(z)\mu_j \epsilon^z_{j,t} + \tau_h v_t \sum_z \sum_j \lambda(z)\mu_j h^z_{j,t} + \frac{\tau_a}{1+\pi_t} i_t \sum_z \sum_j \lambda(z)\mu_j a^z_{j,t} + beq_t + \frac{\pi_t}{1+\pi_t} m_{t-1} = tr_t \tag{21}$$

where the level of seigniorage and bequests are given by equations (22) and (23), respectively:

$$\frac{M_t - M_{t-1}}{P_t} = \frac{\pi_t}{1+\pi_t} m_{t-1} \tag{22}$$

$$beq_t = \sum_z \sum_j \lambda(z)\mu_j (1-\psi_j) \varpi^z_{j,t} \tag{23}$$

2.8. Definition of the competitive equilibrium

A stationary equilibrium for the benchmark economy, given a set of policy arrangements $\{g_m, \tau_h, \tau_a, \tau_w\}$, is a collection of value functions $V_t(\varpi_j, j)$, a set individual policy rules $c_j(\varpi_j, \epsilon_j)$, $x_j(\varpi_j, \epsilon_j)$, $m_j(\varpi_j, \epsilon_j)$, $a_j(\varpi_j, \epsilon_j)$, $h_j(\varpi_j, \epsilon_j)$ for each age $j = 1, ..., J$, a time invariant measure of agent types $\lambda(z)$ for each type $z \in (high, low)$, a production plan $\{Y, K, N\}$ for the representative firm, an allocation set $\{A, H^r, K\}$ for the financial intermediary, and a set of relative prices $\{v, r, w\}$ such that:

1. given relative prices and government policy parameters, individual decision rules solve the dynamic program.
2. relative prices solve the firm's problem.
3. relative prices solve the financial intermediary's problem.
4. the government budget is balanced.
5. individual and aggregate behaviour are consistent:
   $A = \sum_z \sum_j \lambda(z)\mu_j a^z_j \left(\varpi^z_j, \epsilon^z_j\right)$
   $N = \sum_z \sum_j \lambda(z)\mu_j \epsilon^z_j$
   $C = \sum_z \sum_j \lambda(z)\mu_j c^z_j \left(\varpi^z_j, \epsilon^z_j\right)$
   $H^o = \sum_z \sum_j \lambda(z)\mu_j h^z_j \left(\varpi^z_j, \epsilon^z_j\right)$
   $H^r = \sum_z \sum_j \lambda(z)\mu_j x^z_j \left(\varpi^z_j, \epsilon^z_j\right)$
   $\frac{M}{P} = \sum_z \sum_j \lambda(z)\mu_j m^z_j \left(\varpi^z_j, \epsilon^z_j\right)$
   $W = \sum_z \sum_j \lambda(z)\mu_j \varpi^z_j$
   where W denotes the aggregate level of wealth in the economy.

6. Markets clear:

   i) housing and rental markets clear:

   $H^r = H - H^o$

   ii) goods market clears:

   $C + I_h + I_k = Y$

   iii) the asset market clears:

   $K + H^r(1 - v) = A$

3. Calibration

The model is calibrated with US data to produce long run average statistics of this economy. Each model period amounts to one year in real time and the individual life-cycle may last for up to 60 periods ($J = 60$). Conditional survival probabilities, ψ_j, are obtained from the actuarial studies of the Social Security Administration (see Bell and Miller (2005)). The population growth rate n is set to 0.012 and equals the long run average value in the data. The mandatory retirement date is set at age 65 ($jr = 45$).

Average deterministic productivity series are obtained from Heathcote et al. (2008) who estimate the age contribution to labour productivity from the Panel Study of Income Dynamics (PSID) over the period 1967-1995. We have assumed that the population in the model includes two productivity types z, agents with low and high ability. High ability individuals are identified as those who have acquired a college or higher education degree. Workers with a high school degree or less are classified as low productivity workers. From the same study we obtain the reported average college wage premium and set the age dependent efficiency parameter for individuals in the high ability group 1.8 times higher than individuals in the low ability group. We also get the share of college graduates to the total number of working agents, $\lambda(z)$, which has a value of 0.3.

The functional form of the utility function is: $U = \frac{\left(c_j^{\alpha_1} m_j^{\alpha_2} h_j^{1-\alpha_1-\alpha_2}\right)^{1-\sigma}}{1-\sigma}$. The coefficient of relative risk aversion, σ, is set equal to 2, a value commonly used in the macroeconomics literature. Long run statistics from the US economy which are targeted in the calibration process are computed by Chen (2010) for the time period 1954 to 2000. We use a Cobb-Douglass production function of constant returns to scale: $Y = TK^\theta N^{1-\theta}$. To calculate the share of business capital income to output in the data, θ, Chen (2010) subtracts from total income service flows from housing capital. The value is estimated to be 0.274. We set the parameter measuring total factor productivity, T, equal to 1 and the productivity growth rate g equal to 0.015, the average output growth rate in the US data.

The discount factor β and two parameters in the utility function, α_1 and α_2, are set jointly to match as closely as possible the non-residential capital to output ratio $\frac{K}{Y}$ (a value of 1.729), the money balances to output ratio $\frac{M}{Y}$ (a value of 0.152) and the share of owner-occupied housing capital in total assets $\frac{H_o}{H_o+A}$ (a value of 0.322).[14] Their respective values are 0.991, 0.897 and 0.013. The depreciation rate for business capital δ_k takes the value 0.0951 so that the investment to business capital ratio equals the long run ratio $\frac{I_k}{K}$ of 0.1201 in the data.[15] Similarly, the depreciation rate for residential capital, δ_h, is set equal to 0.0205 to match the share of residential capital to total output in the data $\frac{H}{Y}$ which has a value of 1.073.[16] In addition, these parameters closely estimate the share of the value of housing services in total consumption expenditure.

The tax rate on business capital income τ_a and the tax rate on labour income τ_w are set equal to 0.29 and 0.20, respectively, and are estimates of average effective tax rates for the period 1980 to 2010 provided by McDaniel (2007).[17] For the benchmark model, the tax rate on housing services τ_h is set equal to zero. The minimum house size requirement to purchase housing capital is set equal to 0.7685 to match the homeownership rate in the US economy of 65.07 percent.[18] The average downpayment requirement for primary mortgage loans in the US is 20 percent, so we set $\gamma = 0.2$. Finally, the inflation rate, π, is set to 3 percent in the reference economy, the mean inflation rate over the period 1982 to 2012.[19]

4. Results

4.1. Policy experiments

In this section we initially describe both aggregate and individual outcomes that arise from the benchmark economy and then compare their response to changes in the long run inflation rate. We perform several policy experiments to test our hypothesis which relates increases in the rate of inflation to the accumulation of housing and business capital. The results are contrasted to outcomes from experiments with two alternative economies; in the first case we raise the tax rate on imputed rents obtained from owner-occupied housing capital and in the second case, we examine an economy where housing capital can only be rented.

4.2. Benchmark economy

Table 1 displays ratios of aggregate variables in the benchmark economy and US data. The model generates the average business capital to output ratio of 1.72, the average money to output ratio of 0.15, the share of owner-occupied housing capital to total assets of 0.32, and the

[14] The ratio of money balances to output is set equal to the long run share of M1 to GNP in the data calculated by Erosa and Ventura (2002).

[15] Given these aggregate ratios calculated by Chen (2010), the implied steady state real interest rate, $r = \frac{\theta Y}{K} - \delta_k$, is 6.34 percent.

[16] We make the assumption that housing capital used for rental services depreciates at the same rate as owner-occupied housing capital. In practice, investments in rental housing might involve higher depreciation rates than those of owner-occupied housing. However, these differences would only increase the incentive to invest in housing capital, and thus not affect the core argument of the paper.

[17] The author provides updated estimates of these tax rates upon request.

[18] Data obtained from US Census Bureau Statistics, 1960 to 2009 average.

[19] Historical inflation rates published monthly by the US Bureau of Labor Statistics.

homeownership rate of 65 percent. The value of the residential capital to output ratio is 1.05 in the model and 1.07 in data. While not set as targets in the calibration process, the share of the value of housing services in total consumption expenditure, $\frac{vH}{(C+vH)}$ is 0.1314 (0.1120 in the data), whereas the Gini coefficient for housing capital is 0.4316 (0.64 in the data).[20]

Figure 1 displays the age profile of wealth for the two individual productivity types in the reference economy. As it is common in life-cycle models, the net worth of the individual is low at the beginning stages of life, peaks prior to retirement and gradually declines until fully depleted. All agents rent housing services when young until enough savings are accumulated to afford purchasing a house. Figure 2 depicts the mean value of owner-occupied housing capital and shows that high income individuals are able to become homeowners at an earlier stage than their low income counterparts.

4.3. The effects of higher inflation: benchmark model

4.3.1. Relative and aggregate outcomes

Inflation effects on steady state relative outcomes are presented in table 2, panel A, for the model economy under three inflationary regimes where the average inflation rates are 1, 3, and 6 percent, respectively. An increase in inflation lowers the ratio of business capital to output and increases the ratio of housing capital to output. Homeownership rates increase as does the share of housing assets to total assets and average expenditure spent on housing services. Despite the increase in the ratio of housing assets to output, we observe that with higher inflation total wealth relative to output declines.

Panel A of table 3 provides a more detailed account of the response of aggregate variables and relative prices. To illustrate, we present outcomes obtained from the benchmark economy with an inflation rate of 3 percent to outcomes obtained by increasing the inflation rate to 6 percent. We observe that in the high inflation regime, the model economy is characterized by a higher level of housing capital, an increase in the stock of owner-occupied housing and a lower level of business capital. Since business capital is used in the production of goods and services, its decline, coupled with a constant supply of effective labour leads to a reduction in aggregate output. Given that labour is paid its marginal product, the decline in business capital also reduces the average wage rate. Inflation lowers the value of money, inducing households to reduce the stock of real money balances in their optimal portfolios. Similarly, aggregate consumption and rental housing services also decline. The nominal and real interest rates, as well as the relative price of housing are higher in the high inflation regime. Rental housing services decline due to an increase in the quantity of owner-occupied housing and the relative

[20] Aruoba et al. (2012) documents semi-elasticities of the value of housing capital scaled by GDP and M1 with respect to inflation from four different data sources, the most reliable of which comes from Davis and Heathcote (2005). The baseline model of Aruoba et al. (2012) generates a semi-elasticity of the housing capital to output ratio with respect to inflation of 0.2 percent while in our model we generate a semi-elasticity 0.53 percent. The value of this elasticity in the data is 1.1 percent.

price of renting housing services. Finally, the higher the average inflation rate is in the economy, the lower the aggregate level of wealth.

4.3.2. Individual outcomes

What explains these effects of inflation on the benchmark economy? To better understand aggregate outcomes, we closely examine individual responses in each of the two inflationary scenarios presented in panel A of table 4. A higher rate of inflation increases the opportunity cost of holding money and triggers a portfolio allocation effect. Individuals are induced to shift resources from money holdings to interest bearing assets, even though in equilibrium a certain amount of money balances will be optimal, given that money is valued in the utility function. Residential and non-residential investment decisions are based on the response of their respective rates of return to a higher rate of inflation. The real after tax rate of return to financial capital is given by: $R_a = \frac{1+(1-\tau_a)i_t}{1+\pi_t} = \frac{\tau_a}{1+\pi_t} + (1+r_t)(1-\tau_a)$. The real rate of return (net of implicit rental income) associated with investing in housing capital is: $R_h = 1 - \tau_h v_t - \delta_h$. Therefore if $\tau_h = 0$, $R_h = 1 - \delta_h$. In each period t housing also receives untaxed capital gains equal to π_t. Consequently, for a given level of the real interest rate, inflation reduces the real after tax rate of return to business capital: $\frac{\partial R_a}{\partial \pi_t} < 0$, but leaves unchanged the period rate of return to housing capital. Panel A of table 3 shows that, for business capital, the after tax real rate of return is lower in high inflation equilibria even when the real interest rate is allowed to respond to changes in inflation. Consequently, housing becomes more attractive providing an incentive for individual renters to modify tenure decisions and invest in owner-occupied housing, or individual homeowners to increase their investment in housing capital.

As mentioned above, on the aggregate level business capital declines and both housing capital and owner-occupied housing levels increase as we increase the rate of inflation. Residential capital growth comes from both bigger houses and new homeowners for both the high and low income individuals, even though due to restrictions in the housing market, homeownership rates increase more for high income households. Since groups who rent housing services in the reference economy are young individuals, new homeowners are then young households, for whom renting would have been initially optimal and who, due to inflation, alter previous tenure decisions in favour of buying housing capital. The reduction of business capital comes from lower savings in financial assets from both low and high income households. On one hand, savings decline due to the inflation induced higher taxation of financial assets. On the other hand, as inflation changes the relative rates of return to assets in favour of homeownership, the downpayment requirement to purchase housing capital lowers life-cycle savings and business capital accumulation for individual households. The remaining individual responses have the same sign as on the aggregate level: for both income groups inflation lowers wealth levels alongside rental housing services and real money balances.

4.4. Taxing imputed rents

If taxation of imputed rents from owner-occupied housing were allowed, inflation would also reduce the net returns to owner-occupied housing, thereby correcting in high inflation economies some of the portfolio allocation incentives to over-invest in housing capital. Specifically, holding the real interest rate constant, $\frac{\partial R_h}{\partial \pi_t} = \tau_h[(\delta_h - 1)(1+\pi)^{-2}(1+r)^{-1}] < 0$. To illustrate this point we raise the tax rate on imputed rents, τ_h, and set it equal to the tax on business capital income τ_a. So we run the same experiments with $\tau_h = 29\%$. The results are displayed in panels B of table 2 for relative ratios, table 3 for aggregate variables and table 4 for individual outcomes, respectively. Table 3 shows that the after tax rate of return to housing capital declines with inflation even when the real interest rate is allowed to adjust.

In panel B of table 2 we observe that an increase in inflation is associated with a lower business capital to output ratio and, contrary to our previous results, also with a lower housing capital to output ratio.[21] The share of housing capital to total assets and the share of housing expenditure to total expenditure, similar to the reference economy, increase with inflation. Homeownership rates initially increase when the inflation rate is raised from 1 to 3 percent but they remain relatively constant with further increases in inflation. Even though imputed rents are taxed at an equal rate to business capital income, initially there is still an incentive to invest in housing capital due to the fact that capital gains from housing capital are not taxed. However, incentives to adjust tenure decisions in favour of owner-occupied housing services, are greatly reduced in higher inflation regimes.

We display in panel B of table 3 the aggregate outcomes for the alternative economy in which imputed rents are taxed, when the inflation rate increases from 3 percent (the reference economy) to 6 percent. The results show that both business and housing capital fall. Noting that homeownership rates have slightly increased, we observe that young households are still induced to revise original tenure choices, however these rates increase by much less than in the benchmark economy and the size of investment in housing capital is also lower. Looking at mean individual outcomes in table 4, we notice that individuals of both low and high productivity types allocate fewer resources in housing capital.

4.5. Removing owner-occupied housing

To further test our results, we also perform an experiment where the possibility of owning housing capital is removed. This is done by increasing the minimum house size requirement so that no household, whether of a high or low income type, can invest in owner-occupied housing. The results for these experiments are presented in panels C of tables 2, 3 and 4. Similarly to previous cases, inflation reduces the after-tax rate of return to business capital and the degree of capital accumulation for production purposes in relation to total output. An increase in the relative price of housing is followed by a lower demand for renting services, thereby reducing

[21] As a result, the total wealth to output ratio also declines.

the allocation of housing capital. Household savings for each individual type are also lower in high inflationary regimes.

4.6. Wealth differences in alternative economies

Table 5 shows relative responses of the wealth to output ratio in the previous alternative experiments. The first and second rows depict changes resulting from increasing the inflation rate from 1 to 3 percent and from 1 to 6 percent, respectively. Recall that the reference economy holds the highest housing tax provisions and displays a positive relationship between the housing capital to output ratio and the average inflation rate. In the other two scenarios, the economy taxing imputed rents and the economy with only rental housing, inflation reduces the ratio of housing capital to output.

We observe that the decline of the aggregate wealth to output ratio is smallest in the reference economy. This is the case since in the benchmark economy agents are able to invest more in housing capital which in times of high inflation helps preserve the value of household resources.

4.7. Welfare

Welfare outcomes in the model are influenced by several factors. Both average consumption and money balances decline with higher inflation. In high inflation regimes, low income and younger households may not be able to achieve their desired portfolio composition of asset holdings due to restrictions in the housing market. Also, an increase in the nominal rate of return to financial assets increases the price of housing services: $\frac{\partial v_t}{\partial i_t} > 0$. As a result, the price of rental housing increases for renters who are predominately low income households.

At the same time, inflation generates an income redistribution effect which benefits low income households. Part of individual income consists of government transfers which are equally distributed among the population and financed by taxation and seigniorage. Since inflation raises the real income tax burden of financial assets and high income households have the highest holdings of financial capital, redistribution of income takes place from wealth-rich to wealth-poor households.

Welfare changes due to inflation are found by computing the compensating variation as the fixed percentage Δ of the consumption index defined as: $c^{\alpha_1} m^{\alpha_2} s^{1-\alpha_1-\alpha_2}$ in each period required to make individuals indifferent in the two inflationary regimes. Denoting π_0 and π_1 the economies with low and high inflation rates, respectively, the compensating variation for each individual of type z is computed in the following way:

$$\Delta = \left(\frac{V^z\{(c_j, m_j, s_j/\pi_0\}}{V^z\{(c_j, m_j, s_j/\pi_1\}} \right)^{\frac{1}{1-\sigma}} - 1 \tag{24}$$

We look at the percent steady state consumption index that is necessary to compensate individuals for an increase in the inflation rate from 1 to 3 percent. This is found to be 0.743 percent for high income individuals and 1.003 percent for low income individuals. We can see that the cost of inflation in the model is unevenly distributed and it negatively impacts low income households by more than high income households. Low income households pay higher housing rents and have a harder time achieving their desired asset portfolio composition in high inflationary regimes, even though they benefit from some income redistribution taking place from high to low income households.

5. Conclusion

In this study we present a model that explains the long run positive association between housing capital and inflation. The channels that we explore include the response to inflation of the real rates of return to residential and non-residential portfolio assets and tenure decisions on whether to own or to rent housing services. To illustrate the degree to which these identified channels play a role on the accumulation of housing and business capital, we also consider two alternative economies. In the first alternative scenario, the taxation of housing capital was revised, allowing for taxation of imputed rents on owner-occupied housing services. In the second scenario, we consider a one asset economy and remove the ability to accumulate wealth in the form of housing capital. Our analysis shows that the positive correlation between the accumulation of residential capital and inflation is only present in the benchmark economy, where inflation interacts with housing features and taxation, as well as the taxation of nominal income from business capital. The stock of business capital declines in high inflationary regimes in all of the experiments performed. We also show that the negative impact of inflation on aggregate wealth is mitigated when housing capital receives preferential tax treatment.

There are several questions not treated in the present analysis that can be addressed in future research. Following changes in inflation rates, a transitional analysis would be of particular interest to estimate outcomes in the process of individual portfolio adjustments. In addition, incorporating uncertainty about future incomes and investment returns may play an important role on household tenure decisions and the distributional effects of inflation.

References

S.B. Aruoba, M.A. Davis, and R. Wright. Homework in monetary economics: Inflation, home production, and the production of homes. Working Paper 18276, National Bureau of Economic Research, August 2012. URL `http://www.nber.org/papers/w18276`.

F.C. Bell and M.L. Miller. Life tables for the United States Social Security Area 1900-2100. *Office of the Chief Actuary, Social Security Administration, Actuarial study nr. 120*, 2005.

M. K. Brunnermeier and C. Julliard. Money illusion and housing frenzies. *Review of Financial Studies*, 21(1):135–180, 2008.

M. Chambers, C. Garriga, and D. Schlagenhauf. The loan structure and housing tenure decisions in an equilibrium model of mortgage choice. *Review of Economic Dynamics*, 12(3):444–468, July 2009a.

M. Chambers, C. Garriga, and D. E. Schlagenhauf. Accounting for changes in the homeownership rate. *International Economic Review*, 50(3):677–726, August 2009b.

K. Chen. A life-cycle analysis of social security with housing. *Review of Economic Dynamics*, 13:597–615, 2010.

M. A. Davis and J. Heathcote. Housing and the business cycle. *International Economic Review*, 46(3):751–784, 2005.

A. Dougherty and R. Van Order. Inflation, housing cost and the consumer price index. *The American Economic Review*, 72(1):154–164, 1982.

P.L. Ebrill and U.M. Possen. Inflation and taxation of equity in coorporations and owner-occupied housing. *Journal of Money, Credit and Banking*, 14(1):33–47, 1982.

A. Erosa and G. Ventura. On inflation as a regressive consumption tax. *Journal of Monetary Economics*, 49:761–795, 2002.

M. Feldstein. Inflation, tax rules and the accumulation of residential and nonresidential capital. *Scandinavian Journal of Economics*, 84(2):293–311, 1982.

J.R. Follain and D.C. Ling. Another look at tenure choice, inflation, and taxes. *AREUEA Journal*, 16(3):207–229, 1988.

M. Gervais. Housing taxation and capital accumulation. *Journal of Monetary Economics*, 49: 1461–1489, 2002.

J.M. Grandmont. Temporary general equilibrium. *Econometrica*, 45:535–572, 1977.

J. Heathcote, K. Storesletten, and G. L. Violante. The macroeconomic implications of rising wage inequality in the united states. Working Paper 14052, National Bureau of Economic Research, June 2008.

B. Heer. Nonsuperneutrality of money in the Sidrauski model with heterogenous agents. *Economics Bulletin*, 5(8):1–6, 2004.

B. Heer and B. Sussmuth. Effects of inflation on wealth distribution: Do stock market participation fees and capital income taxation matter? *Journal of Economic Dynamics and Control*, 31:277–303, 2007.

P.H. Hendershott and S.C. Hu. Inflation and extraordinary returns on owner-occupied housing: Some implications for capital allocation and productivity growth. *Journal of Macroeconomics*, 3:177–203, 1981.

P.H. Hendershott and S.C. Hu. The allocation of capital between residential and nonresidential uses: taxes, inflation, and capital market constraints. *Journal of Finance*, 38(3):795–812, 1983.

J.B. Kau and D. Keenan. Inflation, taxes and housing: a theoretical analysis. *Journal of Public economics*, 21:93–104, 1983.

P. Krussell and A.A. Smith. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106:867–896, 1998.

W. Li and R. Yao. The life-cycle effects of house price changes. *Journal of Money, Credit and Banking*, 39(6):1375–1409, 2007.

C. McDaniel. Average tax rates on consumption, investment, labor and capital in the oecd 1950-2003. *Working Paper, Arizona State University*, 2007.

S.B. Nielsen and P.B. Sorensen. Inflation, capital taxation and housing, the long run in a small open economy. *The Canadian Journal of Economics*, 27(1):198–217, 1994.

M. Piazzesi and M. Schneider. Inflation and the price of real assets. Staff Report 423, Federal Reserve Bank of Minneapolis, 2009. URL `http://ideas.repec.org/p/fip/fedmsr/423.html`.

J.M. Poterba. Tax-subsidies to owner-occupied housing: an asset-market approach. *The Quarterly Journal of Economics*, 99(4):729–752, 1984.

L.H. Summers. Inflation, the stock market and owner-occupied housing. *The American Economic Review*, 71(2):429–434, 1981.

Sh. Titman. The effects of anticipated inflation on housing market equilibrium. *Journal of Finance*, 37(3):827–842, 1982.

F. Yang. Consumption over the life cycle: How different is housing? *Review of Economic Dynamics*, 12(3):423–443, July 2009.

R. Yao and H. H. Zhang. Optimal consumption and portfolio choices with risky housing and borrowing constraints. *Review of Financial Studies*, 18(1):197–239, 2005.

Appendix

Alternative specification

An alternative specification for the problem of the financial firm is to assume that real estate in the current period can only generate housing services in the next period. This is the specification in Gervais (2002).

In this case the price of housing services relative to the price of non-durable goods is given by:

$$v_t = i_t + \delta_h \tag{25}$$

Table 6 shows outcomes obtained by applying this condition. The results are robust to this specification and all the main relationships discussed in the paper remain the same.

Solution algorithm

The equilibrium of the model economy is found in the following way:

Step 1: Guess the aggregate capital stock that is used in production. Given a constant efficient labour supply and a Cobb-Douglas production function, factor prices can be computed. Using the Fisher equation, the real rate of return to business capital and the long run inflation rate we can find the nominal interest rate. From the problem of the financial intermediary, the price of housing services is also determined.

Step 2: We iterate on the value function to solve the individual problem. The state variable in the model is the wealth of the individual household. We construct a finite state space: $\Gamma = [\varpi_{min}, \varpi_{max}]$ containing all points that an individual may choose in each period for her next period savings. The size of the grid is large enough to not be binding for choices of the household. We tabulate the value function over grid points starting from the last period of life and iterating backwards. For every j and a combination of $(\varpi_j, \varpi_{j+1}) \in \Gamma$, we solve the intra-temporal problem in order to find the level of consumption, housing services and the portfolio allocation of assets. When choosing housing, agents make a tenure choice and decide whether to own or to rent housing services.

Step 3: Choose ϖ_{j+1} given ϖ_j which maximizes:
$G(\varpi_j, \varpi_{j+1}) + \beta\psi_j V_{t+1}(\varpi_{j+1})$. We use two neighbouring points on the wealth grid to bracket the maximum of the value function. Then a Golden Section Search is applied to locate maximum of the Bellman equation. Linear interpolations are performed to obtain values of the value function off the grid points.

Step 4: From optimal choices of households, we compute the aggregate capital stock used in production. If the new level of capital matches our guess, then an equilibrium is found, otherwise update the guess until convergence.

The problem in step 2 can be summarized as follows: At the end of life, J, individuals know that they are not surviving in the next period and thus set $\varpi_{J+1} = 0$. Given this end period value and $\varpi_J \in \Gamma$, intra-temporal outcomes can be computed for each value of ϖ_J to obtain a vector of last period's value V_J. In period $J-1$, the individuals use combinations of $\varpi_j, \varpi_{j+1} \in \Gamma$ to find within period outcomes. Given $G(\varpi_{J-1}, \varpi_J)$ we choose the optimal amount of savings ϖ_J to be transferred over to the next period and compute V_{J-1}. We continue this way until the entire distribution is computed.

Figure 1: Mean wealth over the life-cycle by age and income type

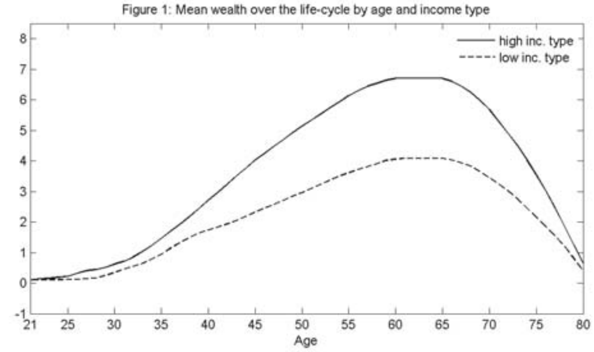

Figure 2: Mean owner-occupied housing by age and income type

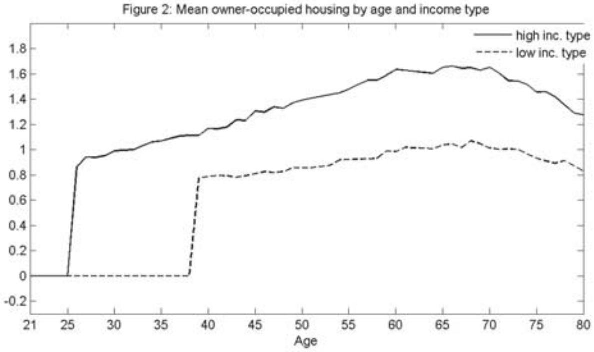

Table 1: Benchmark model and aggregate statistics of the US economy

	K/Y	M/Y	H/Y	$H^o/(H^o+A)$	$H.rate$	I_k/K
Model	1.7218	0.1503	1.0507	0.3170	65.17	0.1201
Data	1.7290	0.1520	1.0730	0.3220	65.07	0.1201

Table 2: Inflation effects on aggregate ratios in the benchmark model and alternative economies

Benchmark Economy

A	K/Y	H/Y	M/Y	$H.rate$	$H^o/(A+H^o)$	W/Y	$vH/(C+vH)$
Model infl. 1 %	1.7407	1.0396	0.1777	61.42	0.2936	2.9347	0.1095
Model infl. 3 %	1.7218	1.0507	0.1503	65.17	0.3170	2.9006	0.1314
Model infl. 6 %	1.6736	1.0960	0.1226	70.59	0.3518	2.8774	0.1665

Model with taxing imputed rents

B	K/Y	H/Y	M/Y	$H.rate$	$H^o/(A+H^o)$	W/Y	$vH/(C+vH)$
Model infl. 1 %	1.8519	0.9367	0.1965	-	-	2.8747	0.0922
Model infl. 3 %	1.8284	0.8668	0.1685	39.12	0.1714	2.8267	0.1041
Model infl. 6 %	1.8063	0.7970	0.1364	39.19	0.1761	2.6865	0.1190

Model with rental housing only

C	K/Y	H/Y	M/Y	$H.rate$	$H^o/(A+H^o)$	W/Y	$vH/(C+vH)$
Model infl. 1 %	1.8519	0.9367	0.1965	-	-	2.8747	0.0922
Model infl. 3 %	1.8321	0.8260	0.1670	-	-	2.7179	0.0993
Model infl. 6 %	1.7440	0.6183	0.1192	-	-	2.3780	0.0960

Table 3: Inflation effects on aggregate outcomes and average prices in the benchmark model and alternative economies

	A. Benchmark Economy		B. Taxing imputed rents		C. Rental housing only	
	Model 3% infl.	Model 6 % infl.	Model 3 % infl.	Model 6 % infl.	Model 3 % infl.	Model 6 % infl.
Aggregate outcomes						
Output Y	0.7844	0.7761	0.8052	0.7983	0.8040	0.7878
Wealth W	2.2752	2.2331	2.2759	2.1446	2.1853	1.8734
Business Capital K	1.3506	1.2989	1.4722	1.4419	1.4731	1.3739
Housing Capital H	0.8242	0.8506	0.6979	0.6362	0.6642	0.4871
Owner Occ. Housing H^o	0.6828	0.7502	0.3660	0.3573	-	-
Rental Housing H^r	0.1414	0.1004	0.3319	0.2790	0.6642	0.4871
Money Holdings M	0.1179	0.0951	0.1357	0.1089	0.1343	0.0939
Consumption C	0.5804	0.5771	0.5923	0.5920	0.5926	0.5969
Prices (Annual rates)						
Real interest rate r	0.0643	0.0689	0.0550	0.0569	0.0547	0.0623
Average wage w	0.5717	0.5657	0.5868	0.5818	0.5860	0.5742
Nominal interest rate i	0.0963	0.1331	0.0867	0.1203	0.0864	0.1260
Relative price of housing v	0.1065	0.1355	0.0986	0.1257	0.0984	0.1301
After tax rates of return						
Business Capital, R_a	1.0372	1.0325	1.0306	1.0240	1.0304	1.0278
Housing Capital, R_h	0.9795	0.9795	0.9509	0.9431	0.9795	0.9795
Homeownership rate	65.17	70.59	39.12	39.20	-	-

Table 4: Inflation effects on mean individual outcomes in the benchmark model and alternative economies

A. Benchmark Economy

	High Income		Low income	
	Model 3 % infl.	Model 6 % infl.	Model 3 % infl.	Model 6 % infl.
Owner-occ. housing	1.2346	1.2932	0.6417	0.6997
Rental housing	0.0423	0.0281	0.1186	0.0833
Money holdings	0.1693	0.1353	0.1079	0.0866
Business Capital	2.2135	2.1722	1.4329	1.3016
Wealth	3.6225	3.6088	2.1857	2.0931

B. Taxing imputed rents

	High Income		Low income	
	Model 3 % infl.	Model 6 % infl.	Model 3 % infl.	Model 6 % infl.
Owner-occ. housing	0.9189	0.9014	0.2554	0.2230
Rental housing	0.0859	0.0627	0.3805	0.3371
Money holdings	0.1876	0.1495	0.1263	0.0997
Business Capital	2.2934	2.1343	1.8574	1.7677
Wealth	3.4055	3.1942	2.2429	2.0965

C. Rental housing only

	High Income		Low income	
	Model 3 % infl.	Model 6 % infl.	Model 3 % infl.	Model 6 % infl.
Owner-occ. housing	-	-	-	-
Rental housing	0.9223	0.6934	0.6104	0.4396
Money holdings	0.1846	0.1325	0.1252	0.0848
Business Capital	3.0212	2.8147	2.0569	1.7551
Wealth	3.2113	2.9551	2.1859	1.8450

Table 5: Wealth to output ratio, relative changes from economy with an inflation rate of 1%

	Benchmark case	Taxing imputed rents	Rental housing only
Model 3% infl.	-1.1626	-1.6694	-5.4549
Model 6% infl.	-1.9530	-6.5472	-17.2771

Table 6: Benchmark economy-alternative specification for the financial firm

	K/Y	H/Y	M/Y	$H.rate$	$H^o/(A+H^o)$	W/Y	$vH/(C+vH)$
Model infl. 1 %	1.7540	0.9913	0.1749	60.72	0.2908	2.8927	0.1038
Model infl. 3 %	1.7257	1.1019	0.1575	71.40	0.3449	2.9723	0.1370
Model infl. 6 %	1.6538	1.1187	0.1247	76.25	0.3694	2.9269	0.1707